Fiscal Policy and Asset Prices with Incomplete Markets

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Abstract

We study the simultaneous impact of fiscal policy decisions on macroeconomic activity, wealth distribution and asset prices. We consider a general equilibrium, overlapping generations model with incomplete markets and heterogeneous agents, where government debt and capital are imperfect substitutes. Increases in public debt lead to significant increases in the riskless rate and to a reduction in the equity premium, while higher capital income tax rates lead to a higher equity premium. The crowding out effects (on capital and output) are much higher than in models where government debt and capital are perfect substitutes, which thus ignore households’ portfolio reallocation decisions.

JEL Classification: E21, E62, G12.

1 Introduction

The continuing 2011 eurozone sovereign debt crisis and the recent large increase in government debt in the developed world has sparked renewed interest on how government debt can affect macroeconomic quantities and asset prices. Between 2007 and 2011, total U.S. government debt rose from 63.8% to 97.8% of GDP, while debt held domestically rose from 35.9% to 67.1%. Similar, and sometimes larger, increases have been witnessed in many European countries over this period, accompanied by significant movements in tax rates. How should policy makers and investors expect asset prices and the macroeconomy to be affected by these large movements in government debt and taxes? We address this question by studying fiscal policy decisions in a general equilibrium, overlapping generations (OLG) model with incomplete markets and heterogeneous agents, where government debt and capital are imperfect substitutes. Our model presents a unified framework for studying the quantitative impact of changes in tax rates and government debt on macroeconomic activity and asset prices. As a result, our assessment explicitly takes into account the important links between these different elements, and how they might interact in reaction to policy decisions.

In our analysis, markets are incomplete due to both aggregate uncertainty and idiosyncratic productivity shocks. Following recent macro evidence, aggregate uncertainty includes shocks both to the level and volatility of aggregate productivity, and investment-specific shocks. The idiosyncratic shocks are not perfectly diversifiable due to the presence of borrowing constraints. Moreover, the OLG structure with a hump shape in labor income and borrowing constraint prevents households from borrowing to invest in the stock market and can generate a low risk free rate. In addition, we capture another important empirical fact: a significant fraction of households do not participate in the stock market, either directly or through pension funds. Furthermore, non-participation is much more pervasive among poor households, and the model is calibrated to match this fact.

The government must finance a given level of government expenditures (and interest on previously-issued debt) from taxes or by issuing new debt. We consider taxes on labor, consumption (sales), and capital income, and all of these (taxes, expenditures and public debt) are calibrated to match the data. In the model tax rates are explicitly modeled as a stochastic process so that, in addition to comparing steady-states with different fiscal policy parameters, we can compute the responses to transitory fiscal policy shocks under rational expectations. Finally, it is important to mention that, even though we study the impact of fiscal policy decisions, we are not solving for optimal fiscal policy rules.

We calibrate the model to match both unconditional asset pricing and macro moments. In addition we show that the model captures reasonably well the cross-sectional wealth distribution and generates significant time variation in conditional asset pricing moments. We then consider different fiscal policy experiments. First we focus on steady-state changes in both tax rates and

\[ \text{For example, in the 2007 SCF the overall participation rate is 51.2% and rises to 72.5% for households with wealth above the median, and falls to 30% for those with wealth below the median.} \]
government debt. Since, for tractability reasons, we do not include a household labor supply decision, we only study changes in capital income taxes. We find that a 5% increase in the capital income tax rate leads to 2.5% decrease in the aggregate capital stock. As household wealth (and therefore total saving) falls, the return on capital must increase to clear financial markets. The same effect also takes place in the bond market, but we also have an increase in the relative supply of bonds, since bond supply relative to GDP is now higher, as GDP is now lower. Overall these two effects are very close to canceling each other out and the riskfree rate remains almost unchanged. As a result, the equity premium increases by 26 basis points, corresponding to a semi-elasticity of 5.2 basis points for each 1% change in the capital income tax rate.

An increase in the ratio of government debt to GDP of approximately 25% causes a permanent reduction in the capital stock of 2.1%. To induce households to hold the extra government debt, the interest rate on government bonds increases by 28 basis points, and the cost of capital increases (by 14 basis points). The return on capital increases by less than the riskless rate, as the supply of capital is now lower while the supply of bonds is now higher, hence the equity premium falls. It is hard to find identical empirical counterparts for these numbers. However, Engen and Hubbard (2004), Laubach (2008), Greenwood and Vayanos (2010a, 2010b) and Krishnamurthy and Vissing-Jorgensen (2012) document empirical changes in bond prices which are close to the ones in our model. Krishnamurthy and Vissing-Jorgensen (2012) focus on how debt to GDP changes affect credit spreads (corporate to treasuries, for example), while Greenwood and Vayanos (2010b) focus on the relative supply of government bonds by maturity and how this variable affects bond returns. For tractability reasons we do not include defaultable corporate bonds or bonds of different maturities and therefore we cannot exactly replicate the regressions in Krishnamurthy and Vissing-Jorgensen (2012) or Greenwood and Vayanos (2010b), respectively. However, an advantage of our approach is that we can isolate how truly exogenous changes in government debt affect bond yields through the posited structural model. In that respect, our results are more directly comparable to Greenwood and Vayanos (2010a) who use an event-study analysis through debt buyback policies to illustrate how changes in the supply for long term bonds affect bond yields. Their results are consistent with what we find through an incomplete markets structural model. Sialm (2009) also documents a negative relationship between tax rate shocks and equity valuations. We therefore view our analysis as complementary to this empirical literature.

We next consider an increase of 75% in the level of debt, which brings the debt-to-gdp ratio to 64%, approximately the 2010 number for the US economy. The crowding out effect on capital is now 5.55%, while the cost of government debt goes up by almost 50% (to 2.57%), and the equity premium falls by 38 basis points. Naturally the increase in the cost of debt would be even larger if we had default risk in the model. Nevertheless, even in our economy, the semi-elasticity of interest expenses to a 1% change in government debt is approximately 1.1 basis points.

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2We write “approximately 25%” since the increase in debt to GDP ratio is endogenous since GDP is an endogenous variable in the model.
We show that modeling capital and government debt as separate assets with different, and more realistic, risk-return profiles is crucial for being able to study the impact of fiscal policy decisions on asset prices and macro-economic variables. Our results identify the portfolio re-allocation behavior of households (asset substitution channel) as an important channel for determining the impact of fiscal policy decisions on capital accumulation, aggregate economic activity and asset prices. To demonstrate this we repeat our policy experiments in an otherwise identical economy with very low aggregate uncertainty and consequently with a negligible equity risk premium. We find that the crowding out effect on capital is under-estimated by almost 1/3 in both experiments (increases in the capital tax rate and in government debt) and consequently the measured drop in consumption is 40% smaller than in the baseline economy. This highlights the importance of the portfolio re-allocation behavior of households for determining the total crowding out effect of fiscal policy decisions.

In the final part of the paper we consider the impact of temporary fiscal policy changes. An increase in the capital income tax rate leads to crowding out effects that have asset pricing implications. As the capital stock falls, the return on capital increases and wages decrease. Lower aggregate saving generates an increase in the risk-free rate to clear the bond market and a reduction in stockholders’ consumption due to crowding out. As we consider longer-lasting shocks, or an increase their expected duration, all these effects become larger.

Our model is also part of the literature studying fiscal policy decisions in a production economy setting. Baxter and King (1993) and Ludvigson (1996) consider infinite-horizon representative-agent models with and without aggregate uncertainty. Aiyagari (1995), Aiyagari and McGrattan (1998), and Conesa, Kitao, and Krueger (2009) study economies with heterogeneous agents, idiosyncratic shocks and borrowing constraints, but without aggregate uncertainty. Domeij and Heathcote (2004) study the transition between steady states following a capital gains tax reform. However, none of these models capture the asset substitution channel discussed in our paper since, in these economies, government bonds earn the same rate of return as the capital stock. Chari, Christiano and Kehoe (1994) characterize optimal fiscal policy in a model with heterogeneous agents and aggregate uncertainty. However, in their set-up, idiosyncratic risk is perfectly diversifiable, allowing them to determine the optimal allocations by solving the corresponding Ramsey problem. Most of these papers, however, incorporate a labor-leisure decision which is absent in our analysis, but on the other hand, they do not consider limited stock market participation. Heathcote (2005) also considers an incomplete markets production economy with heterogeneous agents, aggregate uncertainty, and no labor supply decision. As in our model, incomplete markets arise because of idiosyncratic productivity shocks and liquidity constraints. However, in that model, aggregate uncertainty is exclusively driven by tax rate shocks and therefore capital and government bonds are perfect substitutes. Sialm (2006) studies the impact of tax rate shocks on asset prices in an exchange economy setting. Consistent with our results, he also finds that stock market valuations in high-tax persistent regimes tend to be lower than valuations in low-tax regimes. In addition, our work complements the models studying the impact on asset prices of uncertainty about (future) fiscal policy decisions.
(Croce, Nguyen and Schmid (2012) and Croce et al. (2012)), and of policy uncertainty (Pastor and Veronesi (2012a)), and of time-varying political uncertainty (Pastor and Veronesi (2012b)).

The paper is structured as follows. Section 2 outlines the model and section 3 discusses the baseline calibration. In section 4 we present the results for different specifications of the model. In section 5 we study the impact of permanent fiscal policy changes (in government debt and capital taxation) on asset prices. Finally in section 6 we consider the impulse responses to tax shocks to analyze the effects of capital tax reform. Section 7 concludes. Technical details of the computational procedure are provided in Appendix A.

2 The Model

Our baseline quantitative model features overlapping generations with limited stock market participation and heterogeneous preferences. We use that model because it can account well for life-cycle consumption, saving and portfolio choices, asset prices, macroeconomic variables and cross-sectional distributions of wealth and consumption in the data (see Gomes and Michaelides (2008)).

Households receive wage income, subject to uninsurable idiosyncratic shocks, against which they cannot borrow. They can invest in two alternative assets, the risky capital stock (equity) and a (one-period) riskless government bond. Firms are perfectly competitive and combine capital and labor using a constant returns to scale technology to produce a non-durable consumption good. The government taxes wages, capital income and consumption to finance both government expenditures and the interest payments on public debt.

2.1 Production technology

Technology is characterized by a standard Cobb-Douglas production function

\[ Y_t = Z_t K_t^\alpha L_t^{1-\alpha}, \]

where \( K \) is the total capital stock in the economy, \( L \) is the total labor supply, and \( Z \) is a stochastic productivity which follows the process

\[ Z_t = G_t U_t \]
\[ G_t = (1 + g)^t \]

Secular growth in the economy is determined by the constant \( g > 0 \), while the productivity shocks \( U_t \) are persistent and stochastic and, consistent with recent macro-evidence (e.g. Bianchi (2012) and Fernández-Villaverde and Rubio-Ramírez (2011)), exhibit heteroskedasticity:

\[ U_t \sim N(\mu_v, \sigma^2_{u,v}), \quad v = L, H \]

\(^3\)The same qualitative predictions are also obtained in an infinite-horizon model without preference heterogeneity and without limited participation. However such a model cannot match all important moments as well as the more general version considered here.
where \( v = H \) and \( v = L \) denotes the high volatility/low mean and low volatility/high mean regimes respectively, i.e.

\[
\sigma^2_{u,H} > \sigma^2_{u,L} \\
\mu_H < \mu_L
\]

Firms make decisions after observing aggregate shocks. Therefore, they solve a sequence of static maximization problems with no uncertainty, and factor prices (wages, \( W_t \), and return on capital, \( R^K_t \)) are given by their first-order conditions

\[
W_t = (1 - \alpha)Z_t(K_t/L_t)^\alpha \\
R^K_t = \alpha Z_t(L_t/K_t)^{1-\alpha} - \delta_t
\]

where \( \delta_t \) is the depreciation rate.

Standard frictionless production economies cannot generate sufficient return volatility, since agents can adjust their investment plans to smooth consumption over time (see Jermann (1998) or Boldrin, Christiano and Fisher (2001)). This usually motivates adjustment costs for capital, which create fluctuations in the price of capital and increase return volatility. Since we have incomplete markets, different stockholders have different stochastic discount factors. They will therefore disagree on the solution to the optimal intertemporal decision problem of the firm (see Grossman and Hart (1979)). This is not a concern here because there is no intertemporal dimension to the firm’s problem, but introducing adjustment costs would change that.\(^4\) Recent papers with production economies and incomplete markets have captured the effect of adjustment costs by assuming a stochastic depreciation rate for capital (e.g. Gomes and Michaelides (2008), Storesletten et al. (2007), Krueger and Kubler (2006)). Here we follow the same route and assume that the depreciation rate is given by

\[
\delta_t = \delta_v + s_v \times \eta_t
\]

where \( \eta_t \) is an i.i.d. standard normal, while \( s_v \) and \( \delta_v \) depend on the aggregate volatility state. Just like for the aggregate productivity shocks we have \( s_H > s_L \), and also \( \delta_H > \delta_L \). Therefore, \( \delta_t \) is a general measure of economic depreciation, combining physical depreciation, adjustment costs, capital utilization and investment-specific productivity shocks.\(^5\) In the baseline case we assume that \( \eta_t \) is uncorrelated with the productivity shock \( U_t \).

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\(^4\) Guvenen (2005) introduces adjustment costs in a model with restricted stock market participation, but in his model there is perfect risk sharing among stockholders. Therefore, there is a unique stochastic discount factor for pricing capital. Favilukis, Ludvigson and Van Nieuwerburgh (2012) assume that the average pricing kernel across all agents provides a sensible solution and utilize it to compute the present-value of firm profits.

2.2 Government sector

The government’s budget constraint is

\[ B_{t+1} = (1 + R^B_t)B_t + G^c_t - T_t \]  \hspace{1cm} (8)

where \( G^c \) is government consumption, \( B \) is public debt, \( R^B \) is the interest rate on government bonds, and \( T \) denotes tax revenues. Tax proceeds arise from proportional taxation on capital (tax rate \( \tau_K \)), proportional taxation on labor (tax rate \( \tau_L \)) and a proportional consumption tax (tax rate \( \tau_C \)). We also assume that capital income tax rates are time varying as this will allow us to perform comparative statics along transition dynamics and to study the effects of temporary fiscal policy shocks on asset prices. More precisely \( \tau_K \) follows a Markov process with two values \( \tau^H_K \) and \( \tau^L_K \), where \( \tau^H_K > \tau^L_K \). The same modeling approach is taken in Sialm (2006).

In these types of models government debt can become non-stationary since \( B_{t+1} \) depends on \( B_t \) through a multiplication by a time-varying coefficient that is on average greater than one, since the riskless rate has a positive mean. As a result, if taxes and government consumption are stationary, then government debt becomes non-stationary. Moreover, it is not obvious what normalization may be used to make \( B_t \) stationary. One solution is offered by Heathcote (2005) who makes taxes (and household decisions) depend on government debt: high government debt relative to its long run average implies higher taxation. This requires the addition of one extra state variable in the model, and more importantly it imposes a restriction on the path of tax rates in response to other shocks in the economy.\(^6\) To avoid these complications, and to gain a better understanding of the model’s predictions, we instead assume that the government debt is constant over time with government consumption adjusting endogenously to satisfy (8) period-by-period.

2.3 Households and Financial Markets

Households have a finite horizon and Epstein-Zin preferences (Epstein-Zin (1989)). Let \( C_t \) denote consumption in period \( t \), then preferences are defined by

\[ V_t = \left\{ (1 - \beta)C_t^{1 - 1/\psi} + \beta \left( E_t(V_{t+1}^{1 - \rho}) \right)^{1 - 1/\psi} \right\}^{1 - \psi} \]  \hspace{1cm} (9)

where \( \rho \) is the coefficient of relative risk aversion, \( \psi \) is the elasticity of intertemporal substitution and \( \beta \) is the discount factor.

The household’s life cycle is divided in two periods: working life and retirement. During working life all households supply labor inelastically.

\(^6\)While still feasible in the setting without aggregate productivity or depreciation shocks, the computational burden of the additional state variable required by this method is a serious obstacle when we consider a model with aggregate shocks.
2.3.1 Labor income process and retirement transfers

Let \( i \) index households and \( a \) index age/cohorts. All households supply labor inelastically, and are subject to idiosyncratic productivity shocks. The stochastic process for individual labor income \((H^i_{at})\) is given by:

\[
H^i_{at} = W_t L^i_a,
\]

where \( L^i_a \) is the household’s labor endowment (labor supply scaled by productivity), and \( W_t \) is the aggregate wage per unit of productivity. The household’s labor productivity is age-dependent and specified to match the standard stochastic earnings profile in life-cycle models. More precisely, labor income productivity combines both permanent \((P^i_a)\) and transitory \((\varepsilon^i)\) shocks with a deterministic age-specific profile:

\[
L^i_a = P^i_a \varepsilon^i
\]

\[
P^i_a = \exp(f(a))P^i_{a-1} \xi^i
\]

where \( f(a) \) is a deterministic function of age, capturing the typical hump-shape profile in life-cycle earnings. We assume that \( \ln \varepsilon^i \), and \( \ln \xi^i \) are each independent and identically distributed with mean \((-0.5 \star \sigma^2_{\varepsilon}, -0.5 \star \sigma^2_{\xi})\), and variances \( \sigma^2_{\varepsilon} \) and \( \sigma^2_{\xi} \), respectively. As indicated by the subscript \( v \), the volatility of permanent income shocks is heteroskedastic with \( \sigma^2_{\xi,H} > \sigma^2_{\xi,L} \) (as in Storesletten, Telmer and Yaron (2007)). As discussed below, once calibrated to the data this feature does not play a significant role in the model.

Retirement is exogenous and deterministic. All households retire at age 65 \((a^R = 46)\) and retirement earnings are given by: \( \lambda P^i_{aR} W_t \), where \( \lambda \) is the (exogenous) replacement ratio. The retirement income is funded by a proportional social security tax \( \tau_s \). Including a social security system is important to provide the model with a realistic labor income process. If we ignore social security transfers we significantly increase households’ income risk and wealth accumulation.

2.3.2 Wealth accumulation

Total liquid wealth (cash-on-hand) can be consumed or invested in the two assets: a one-period riskless asset (government bond), and a risky investment opportunity (capital stock). The riskless asset return is \( R^B_t = \frac{1}{P^B_t} - 1 \), where \( P^B_t \) denotes the government bond price. The return on the risky asset is denoted by \( R^K_t \). At each age \((a)\), households enter the period with wealth invested in the bond market, \( B^i_{at} \), and (potentially) in stocks, \( K^i_{at} \), and receive \( L^i_a W_t \) as labor income. Cash-on-hand at time \( t \) is given by:

\[
X^i_{at} = K^i_{at} (1 + (1 - \tau_{K,t}) R^K_t) + B^i_{at} (1 + (1 - \tau_{K,t}) R^K_t) + L^i_a (1 - \tau_s - \tau_L) W_t
\]

before retirement \((a < a^R)\), and by:

\[
X^i_{at} = K^i_{at} (1 + (1 - \tau_{K,t}) R^K_t) + B^i_{at} (1 + (1 - \tau_{K,t}) R^K_t) + \lambda P^i_{aR} (1 - \tau_s - \tau_L) W_t
\]
during retirement \((a \geq a^R)\).

Households cannot borrow against their future labor income \((B^i_{at} \geq 0)\), and cannot short the risky asset \((K^i_{at} \geq 0)\). We normalize all variables both by aggregate productivity growth and by the permanent component of labor income during working life so that

\[k^i_{a,t+1} = \frac{K^{i}_{a,t+1}}{P^G_{a,t}}, \quad b^i_{at+1} = \frac{B^i_{at+1}}{P^G_{a,t}}, \quad c^i_{at} = \frac{C^i_{a,t}}{P^G_{a,t}}, \quad x^i_{at} = \frac{X^i_{a,t}}{P^G_{a,t}}.\]

The individual budget constraint can then be written as

\[(1 + \tau_C)c^i_{at} + k^i_{a,t+1} + b^i_{at+1} = x^i_{at} = (1 + R_{K,t}(1 - \tau_K,t))k^i_{at} + (1 + R_{B,t}(1 - \tau_K,t))b^i_{at} + \xi^i_w(1 - \tau_s - \tau_L)\]

where \(\omega_a = \exp(f(a))\xi^i\). After retirement, the equation looks the same except for the retirement benefit:

\[x^i_{at} = \frac{(1 + R^K_{i}(1 - \tau_{K,t}))k^i_{at}}{\omega_l\omega_a} + \frac{(1 + R^B_{i}(1 - \tau_{K,t}))b^i_{at}}{\omega_l\omega_a} + \frac{\xi^i_w(1 - \tau_s - \tau_L)}{\omega_t}\]

where \(\omega_a = 1\).

Labor taxes are non-distortionary in our model because there is no household labor-leisure decision. As a result we will preferentially refer to them as lump-sum taxes, which is what they effectively are. Naturally, it would also be interesting to include distortionary labor income taxes in the model, however this would require the inclusion of a labor supply decision, a substantial additional complexity in the presence of aggregate uncertainty.

### 2.4 Equilibrium

The equilibrium is characterized by a set endogenously determined prices (bond prices, wages, and equity returns), a set of cohort specific value functions, policy functions, \(\{V_a, b_a, k_a\}_{a=1}^A\), and rational expectations about the evolution of the endogenously determined variables, such that:

1. Firms maximize profits by equating marginal products of capital and labor to their respective marginal costs: equations (5) and (6).
2. Individuals choose their consumption and asset allocation by maximizing (9).
3. Markets clear and aggregate quantities result from individual decisions. Specifically:

\[k_t = \int_a \int_i P^i_{a-1}k^i_{at}dadi, \quad b_t = \int_a \int_i P^i_{a-1}b^i_{at}dadi.\]  \(17\)

The aggregation equation for labor supply is redundant since there is no labor-leisure choice (aggregate labor supply is normalized to one).

4. Once (3) is satisfied, Walras’ law implies that total expenditure (government consumption, investment, and household consumption) must equal total output:

\[c^G_t + k_{t+1} - \frac{(1 - \delta_t)k^*_t}{\omega_t} + \int_i \int_a P^i_{a-1}c^i_{at}dadi = U_t k^\alpha_t L^1 - \alpha(1 + g)\frac{1}{\omega_t}.\]  \(18\)
5. The social security system is balanced at all times:

$$\int I \int a \in I_W \tau_s L^i a w_i dadi = \int I \int a \in I_R [\lambda \exp(f(a^R))w_i P^i_{aR}] dadi,$$

where the left-hand side is integrated over all workers \((a \in I_W)\), while the right-hand side is integrated over retirees \((a \in I_R)\). This equation determines \(\tau_s\) for a given value of \(\lambda\).

6. The government budget [equation (8)] is balanced every period to sustain a given ratio of government debt to GDP. Specifically

$$b_{t+1} = \left(1 + R^B_t\right)b_t + c^G_t - \frac{k_t R^K_t \tau_K}{\omega_t} - \frac{b_t R^K_t \tau_{K,t}}{\omega_t} - \frac{w_t (1 - \tau_s) \tau_L}{\omega_t},$$

7. Market prices expectations are verified in equilibrium.

Analytical solutions to this problem do not exist and we therefore use a numerical solution method (details are given in Appendix A).

### 2.4.1 The dynamic programming problem

In the presence of aggregate uncertainty the model is similar to Krusell and Smith (1997), with the addition of stochastic depreciation and tax rates. Households are price takers and maximize utility given their expectations about future asset returns and aggregate wages. Under rational expectations, the latter are given by equations (5) and (6): returns and wages are determined by future capital and labor, and by the realizations of aggregate shocks. Labor supply is exogenous, as are the distributions of the aggregate shocks. The capital stock, however, is endogenous. Forming rational expectations of future returns and wages is, therefore, essentially equivalent to forecasting the future mean capital stock. As shown by Krusell and Smith (1998), for this “simple” class of incomplete-markets economies, it is possible to accurately forecast the one-period ahead capital stock using its current value \((k_t)\) and the state-contingent realizations of the four aggregate shocks (aggregate regime, \(v_t\), productivity shock, \(U_t\), stochastic depreciation, \(\eta_t\), and tax rate, \(\tau_{K,t}\)):

$$k_{t+1} = \Gamma_K(k_t, U_t, \eta_t, v_t, \tau_{K,t})$$

Since government bonds are only riskless over one period, households must forecast future bond prices \((P^B_t)\). The forecasting rule for \(P^B_t\) is

$$P^B_{t+1} = \Gamma_P(P^B_t, k_t, U_t, \eta_t, v_t, \tau_{K,t})$$

This process introduces six additional state variables in the individual’s maximization problem \((P^B_t, k_t, U_t, v_t, \tau_{K,t}\) and \(\eta_t\)).

The individual optimization problem now becomes:

$$V_a(x^i, k_t, U_t, \eta_t, v_t, \tau_{K,t}, P^B_t) = \max_{\{k_{a+1,t+1}; t_{a+1,t+1}\} a=1} \left\{ (1 - \beta)(c^i)_{a+1}^{1-1/\psi} + \beta (E_t [(\omega_{a+1,t+1})^{1-\rho} P_{a+1} V_{a+1}^{1-\rho} (x^i_{a+1,t+1}; k_{t+1}, U_{t+1}, \eta_{t+1}, v_{t+1}, \tau_{K,t+1}, P^B_{t+1})])^{1-1/\psi}) \right\}^{1-1/\psi}$$
subject to the constraints,

\[ k_{a,t+1}^i \geq 0, \quad b_{a,t+1}^i \geq 0, \quad (1 + \tau_C)c_{a,t}^i + b_{a,t+1}^i + k_{a,t+1}^i = x_{a,t}^i \]

and with the laws of motion,

\[
\begin{align*}
   x_{a,t}^i &= \begin{cases} 
   \frac{(1+R^K(1-\tau_{K,t}))k_{a,t}}{\omega_{a,t}} + \frac{(1+R^K(1-\tau_{K,t}))b_{a,t}}{\omega_{a,t}} + \frac{\epsilon_{a,t}^i(1-\tau_{s}-\tau_L)}{\omega_{a,t}} & a < 65 \\
   \frac{(1+R^K(1-\tau_{K,t}))k_{a,t}}{\omega_{a,t}} + \frac{(1+R^K(1-\tau_{K,t}))b_{a,t}}{\omega_{a,t}} + \frac{\epsilon_{a,t}^i(1-\tau_{s}-\tau_L)}{\omega_{a,t}} & a \geq 65
   \end{cases}
\end{align*}
\]

\[
\begin{align*}
   P_{t+1}^K &= R(k_{t+1}, U_{t+1}, v_t, \delta_{t+1}, \tau_{K,t}) \\
   w_{t+1} &= W(k_{t+1}, U_{t+1}, v_t, \tau_{K,t}) \\
   k_{t+1} &= \Gamma_K(k_t, U_t, \eta_t, v_t, \tau_{K,t}) \\
   P_{t+1}^B &= \Gamma_P(k_t, U_t, \eta_t, v_t, \tau_{K,t}, P_t^B)
\end{align*}
\]

3 Calibration

Decisions are made at an annual frequency. The household earnings processes and social security are calibrated from evidence based on micro-economic data (PSID), while the other parameters are used to match several empirical moments. The government sector variables are calibrated to match the ratios of government bonds, government expenditures and tax revenues to GDP. The technological and preference parameters are chosen to try to replicate, as close as possible, moments such as the consumption and investment shares of GDP, consumption volatility, wealth distribution, limited participation, and the mean and volatility of returns.

3.1 Labor income and social security

Agents begin working life at age 20, retire at age 65, and can live up to 90 years. The parameters for the household earnings processes are taken from the previous studies using the PSID. The variances of the idiosyncratic shocks are taken from Carroll (1992): 10 percent per year for \(\sigma_\epsilon\) and 8 percent per year for \(\sigma_\xi\). We set the ratio of volatilities of the permanent shocks across the two aggregate states \((\sigma^2_\xi,H/\sigma^2_\xi,L)\) to 4.5 from Storesletten, Telmer and Yaron (2007). The parameter values for the deterministic labor income profile, reflecting the hump shape of earnings over the life-cycle, are the ones estimated in Cocco, Gomes and Maenhout (2005).

For tractability we assume that the social security budget is balanced in all periods. Given a value for the replacement ratio of working life earnings (\(\lambda\)), the social security tax rate (\(\tau_s\)) is determined endogenously. This tax rate ensures that social security taxes are equal to total retirement benefits, taking into account the demographic weights. Consistent with the empirical evidence with regards to median replacement rates from the U.S. social security system, we use a 40% replacement rate (as in Cagetti and De Nardi (2006)), which implies an endogenous social security tax (\(\tau_s\)) of approximately 17.5% to maintain social security balance period by period.
3.2 Technology

Capital’s share of output ($\alpha$) is set to 34%, and the average annual depreciation rate ($\delta$) is 8% to match the investment to output ratio, with $\delta_H = 10\%$ and $\delta_L = 6\%$. To match stock market return volatility we set the unconditional standard deviation of the stochastic depreciation shock at 12%. The ratio of volatilities across the two states ($s_H/s_L$) is calibrated from Bianchi (2012), who estimates an heteroskedasticity of 1.8 in shocks to the marginal efficiency of investment and 1.5 in shocks to the relative price of investment. Based on these we calibrate $s_H/s_L$ to 2.

The aggregate productivity state follows a two-state Markov Chain with a transition probability across states set to 0.4 to match the duration of business cycles. The ratio of volatilities across these two states ($\sigma^2_{u,H}/\sigma^2_{u,L}$) is set to 2, also based on estimation results in Bianchi (2012). In addition we normalize $\mu_H - \mu_L$ such that the bad $U_t$ realization in the good state ($\mu_H - 0.5\sigma^2_{u,H}$) coincides with the good $U_t$ realization in the bad state ($\mu_L + 0.5\sigma^2_{u,L}$). This leaves us with only three aggregate states and one variable to calibrate, the unconditional standard deviation of the shocks which is set to 2.5% to match the standard deviation of aggregate output growth (annual U.S. GDP volatility since 1930).

3.3 Government sector

The aggregate supply of bonds is set to 36% of GDP, which is the average value of U.S. Treasury securities held by the U.S. public, as reported by the Congressional Budget Office (from 1960 to 2010). The ratio of total outstanding debt to GDP is higher, but the difference is due to the significant amount of US government bonds that is being held abroad. Including these in the model would lead to an extremely incorrect calibration of either total wealth or the capital stock in our economy. Of course excluding them also has a cost, since we are ignoring the interest payments on these bonds in the government’s budget constraint. However, we can simply interpret these as an additional exogenous source of government expenditures. Using the average historical values for both the cost of debt and total debt outstanding, this corresponds to an additional 0.6% of GDP, which has a fairly negligible impact on our baseline calibration.

We also want to match the share of government expenditures in GDP, which is an endogenous quantity in the model. This is achieved through an appropriate calibration of the tax rates. Even ignoring this extra constraint, the calibration of each tax rate already requires a compromise between matching two different features of the data: the tax rate itself or the corresponding share of tax revenues in GDP. We compute the tax shares using data from the Bureau of Economic Analysis from 1929 until 2010.\footnote{The BEA data does not provide a disaggregation of total personal income taxes, and therefore we combine it with data from the IRS to compute estimates of the relative percentages of labor income and capital income taxation in this category.} For capital income taxes we set the unconditional mean to 40%, following Trabandt and Uhlig (2011), Carey and Rabesona (2002) and Mendoza, Razin and Tesar (1994) and, as shown in table 1, the implied share of capital income revenues over GDP in the model is 5.41%,
which is extremely close to the value in the data. For the calibration of the two states ($\tau^H_K$ and $\tau^L_K$) we set the standard deviation of the tax shock ($\sigma_\tau$) to 2.5% and assume them to be i.i.d. in the baseline case.

With respect to the tax rate on labor income, the calibration decision is clear: since we do not have a labor supply decision in the model, these are effectively lump-sum taxes, and therefore we want to match the revenue share, as opposed to the tax rate. It turns out that this is actually an advantage of our model. As shown in table 1, a flat tax rate of 10% generates tax revenues which are in line with the empirical numbers.\(^8\) However, in reality the marginal tax rate on labor income is much higher than 10%. This shows that, with a linear tax schedule, researchers face an important trade-off: either match the marginal tax rate and dramatically over-estimate the importance of labor tax revenues, or match the revenues themselves and significantly under-estimate the distortion at the margin.\(^9\) In models with exogenous labor supply, such as ours, this is not an issue. As previously discussed, the choice is very clear: match the revenue share. However, in models with an endogenous labor supply this represents a serious concern, unless we also carefully incorporate different sources of non-linearity in the tax system (as in Castaneda et al. (2003)), which represents a significant additional computational challenge.

Finally, it is important to point out that this tension depends on one assumption in the model: Cobb-Douglas production technology. With such a technology

$$\frac{WL}{Y} = 1 - \alpha \Rightarrow \tau_L \frac{WL}{Y} = \tau_L (1 - \alpha)$$

and, in a model without retirement, the left-hand-side denotes the share of labor income revenues in GDP.\(^{10}\) This will hold regardless of most other features of the model (namely whether we have endogenous labor supply or not). Therefore, any tax rate higher than 10% will over-estimate total labor income revenues.

This still leaves us with one parameter left to calibrate: the tax rate on consumption. As previously discussed we want the model to match the share of government expenditures in GDP, so this is not a free parameter either. We set $\tau_C = 13\%$ to match $G/Y$ given the other tax rates and the calibration of $B/Y$. It turns out that this number delivers total tax revenues which, as a share of GDP, are fairly close to their empirical counterpart as shown in 1.

\(^8\)As we can see from the table, the ratio of labor tax revenues to GDP has increased over time. We want our calibration to reflect both the long time-series as much as possible, but we also want the fiscal policy conditions in our baseline economy to be fairly close to the current values, so that our results are directly applicable to the current US economy. Therefore, we try to match a target in between the 2007 (pre-crisis) value (9.38%) and the 1929-2010 average (6.80%).

\(^9\)This simply reflects the multiple sources of deductions and exemptions that are not being modeled with a linear tax schedule.

\(^{10}\)In a model with retirement, such as ours, the comparison is even worse.
3.4 Preference heterogeneity and limited participation

We consider two groups \((A\) and \(B\)) of households in the model: stock market participants and non-participants. In the recent data, the two groups are almost identical in size (51.2% and 48.8% respectively, using the data from the 2007 SCF).\(^{11}\) However, they have very different wealth accumulation profiles: the participation rate is 72.5% among households with wealth above the median, and only 30% for those with wealth below the median. In the model we treat limited participation as exogenous for tractability reasons (as in Basak and Cuoco (1998)), but make sure that the wealth accumulation differences are consistent with the data.\(^{12}\) We use ex-ante preference heterogeneity in the discount factor and the elasticity of intertemporal substitution to endogenously generate different wealth accumulation profiles, and we assume stockholders make up 50% of the population, consistent with the empirical magnitudes in the U.S. economy.

We rely primarily on discount factor and EIS heterogeneity to generate different wealth profiles. Type-\(A\) (non-stockholders) have a very low discount factor \((\beta = 0.7)\) and never accumulate much wealth over the life cycle, while type-\(B\) (stockholders) have a higher discount factor \((\beta = 0.99)\) chosen to match the historical risk free rate.\(^{13}\) There is strong evidence that stockholders have a higher EIS than non-stockholders (see, for example, Vissing-Jorgensen (2002)). Therefore, we assume that non-stockholders have a lower EIS in the model as well. We pick \(\psi^A = 0.45\) to match the wealth accumulation of this group, in combination with the discount factor. The value of the EIS stockholders is chosen to match, as close as possible, two different moments: the volatility of consumption growth for this group, and the volatility of the riskless rate. This gives us \(\psi^B = 0.7\) and, as we will see later, a good calibration of both of these moments. Finally, both types have the same risk aversion coefficient \((\rho = 5)\). The risk aversion coefficient is picked to generate the highest possible equity premium in the range of plausible coefficients and we view \(\rho = 5\) as a sensible upper bound.

\(^{11}\)These numbers take into account households that participate in the stock market indirectly through pension funds. These proportions are also similar to their 2001 counterparts.

\(^{12}\)Given the low wealth accumulation of non-stockholders, a small one-time entry cost would suffice to endogeneize the non-participation decision. For example, Alan (2006) estimates a structural participation model and finds that a one-time entry cost equal to approximately 2-3% of average annual income explains limited stock market participation. Gomes and Michaelides (2008) show that a one-time cost of 5% of average annual income or lower would deter participation for the poorer households. We leave such an entry cost out of the model to reduce the computational burden.

\(^{13}\)We emphasize that the quantitative results are almost identical regardless of the method we use to generate “poor” non-stockholders. What really matters is that we replicate poor households within the model. The same quantitative results would be obtained under alternative specifications, as long as these two groups are calibrated to match the same heterogeneity in wealth accumulation. For example, Gomes and Michaelides (2008) consider heterogeneity in risk aversion and EIS, with \(\beta = 0.99\) for both groups, among other combinations.
4 Asset Prices and Macro Moments

In this section we first compare the moments generated from the baseline model with the data. Next we discuss important comparative statics where we shut down some channels in our artificial economy to better understand the different mechanisms in the model.

4.1 Baseline Model

4.1.1 Unconditional asset prices and macro moments

Table 2 reports the main macroeconomic quantities. The shares of consumption, investment and government expenditures and debt relative to GDP match their empirical counterparts quite accurately (panel A). The annual empirical moments are taken from the National Income and Product Accounts (NIPA) published by the Bureau of Economic Analysis and span 1929 to 2010. Following Castaneda et al. (2003) we classify 75% of durable consumption expenditures as investment and 25% as consumption. Panel B shows that the model matches extremely well the volatilities of aggregate consumption growth. Panel B also shows that the consumption growth of stockholders is more volatile than the consumption growth of non-stockholders, consistent with the empirical evidence in Malloy, Moskowitz and Vissing-Jorgensen (2009).

Table 3 reports the main asset pricing moments implied by the model, along with their empirical U.S. counterparts. The consumption and output series are taken from the NIPA tables. The returns series are taken from CRSP. The equity return is the real return on the CRSP value-weighted index (including dividends), and the rate of return on government bonds is the real return on 1-year government bonds. The observed volatility of the riskless rate reflects both shocks to realized inflation and shocks to the unobserved ex-ante real rate, so we need to remove the former to have a model counterpart. It is beyond the scope of our paper to estimate these latent variables, instead we simply divide the observed volatility by two. Since firms in the model are not levered, our return on capital corresponds to the return of unlevered equity in the data. Therefore, we have to adjust the moments of our return series by the average leverage ratio of US corporations (25%) to make them comparable with the CRSP data (a similar approach is taken by Kaltenbrunner and Lochstoer (2010) and Favilukis, Ludvigson and Van Niewerburgh (2012)). Since we are implicitly assuming risk-free corporate debt, expected levered returns are computed using the simple Modigliani-Miller formula:

\[ r_{equity} = r_{levered} = r_{unlevered} + \frac{D}{E} \left( r_{unlevered} - r_f \right) \]  

(25)

Along the same lines, the Sharpe ratio on levered equity must be identical to the Sharpe ratio on

\[ We consider 1-year bonds because we have a yearly model where government bonds are risk free over 1 period. In the data, the average maturity for government debt has changed over time, but it is close to 5 years. If we use the price series for 5-year government bonds, which also include an additional risk premium, we would obtain a similar average return: 2.03% versus 1.58%.}
unlevered equity, allowing us to compute the standard deviation on the levered claim from:

\[ \sigma_{K_{\text{Levered}}} = \sigma_{K_{\text{unlevered}}} \frac{r_{K_{\text{Levered}}} - r_f}{r_{K_{\text{unlevered}}} - r_f} \]  

(26)

The risk free rate is matched closely (1.79% versus 1.58%) while the equity premium is still lower than in the data (4.45% versus 6.74%) as this model is not able to fully address the equity premium puzzle. The standard deviation of equity returns is also lower than in the data (16.57% versus 19.81%), so we could increase it to obtain a higher risk premium but in a production economy this also implies higher consumption volatility. Finally, the volatility of the riskless rate is similar to the one in the data: 1.27% versus 2.67%.

4.1.2 Conditional Asset Prices and Wealth Distribution

Next we turn our attention to the conditional asset pricing moments implied by the model. Given the distribution of shocks in our economy, we have 8 equally likely aggregate states. In Table 4 we report the maximum, the minimum and the spread across different moment returns within the model. Specifically, we focus on the conditional expected return on capital, the risk-free rate, the conditional risk premium, their conditional volatilities, and the conditional Sharpe ratio on capital. To generate these statistics, we compute average returns and their volatilities for simulations following a given aggregate state.

We find significant time-variation in conditional moments in our economy. In the best state of the world (low aggregate volatilities and positive aggregate shocks) next-period’s expected return on capital is 4.42%, while in the worst state of the world (high aggregate volatilities and negative aggregate shocks) it reaches 7.42%. It is important to point out again, that we only have 8 equally likely aggregate states so each of these events has a 12.5% unconditional probability, these are not extremely unlikely states of the world. The risk-free rate is also significantly different across these two states (2.39% versus 1.26%) but much less than the equity return, leading to a strongly counter-cyclical equity premium: 5.69% in the worst state versus 3.16% in the best state.\footnote{For completeness, the full distribution of the conditional equity premium across all 8 states is: 3.16%, 3.55%, 3.86%, 4.41%, 4.56%, 5.17%, 5.38% and 5.69%.

Consistent with our results for time-variation in conditional moments we find that the wealth distribution is linked with the conditional risk premium. As previously shown, fluctuations in the wealth share of non-participants plays no role in determining asset prices since they are poor, but stockholder wealth plays an important role. In the presence of borrowing constraints we obtain the standard result that the optimal share invested in equities decreases with financial wealth, since labor income is a close substitute for riskless asset holdings. As a result, even though we have homogeneous preferences among stockholders, there is significant heterogeneity in their optimal demand for equities. Following negative aggregate shocks, even though the relative wealth of stockholders will naturally fall, within this class the relative wealth of the richer stockholders will actually increase since they invest a lower fraction of their wealth in equities. Market clearing now implies
that these households must increase their share of wealth invested in the risky asset, which in turn requires an increase in its expected return. This is effectively the same mechanism as in models with preference heterogeneity, but here the differences in optimal portfolio shares are driven by market incompleteness. To illustrate this link we consider the stockholders’ wealth distribution for each of the eight aggregate states, and compute the correlation between expected returns and the share of wealth held by different percentiles of the stockholders’ wealth distribution. As expected, the highest correlation is obtained for the top percentile (0.324), while the corresponding correlation for the top 20th percentile is only 0.1.

4.1.3 Consumption and wealth inequality

Table 5 reports the shares of wealth held by different percentiles of the wealth distribution in the model and in the 2007 SCF data.\(^{10}\) We also report wealth distributions conditional on stockholding status since, as previously argued, matching the relative wealth of stockholders and non-stockholders is important for consistency. In the data, stockholders are defined as households owning stocks directly or through mutual funds either in taxable accounts or in pension plans. Overall, the model captures relatively well the wealth distribution. In particular, it replicates the fact that wealth below the median is negligible, while households in the top quintile hold 68% of total assets in our economy versus 83% in the data. The model also matches well the wealth distribution of non-stockholders. For stockholders, the wealth distribution is not as skewed as in the data, since our economy does not capture the rich entrepreneurs that dominate the top end of the distribution. Therefore, to match the capital stock, the model overshoots wealth accumulation in the intermediate (50-80) percentiles. Finally, the model’s results can also be recast in terms of aggregate gini coefficients. Aggregate wealth inequality in the data is 0.8, while consumption is much more evenly distributed, with a gini coefficient of 0.25.\(^{17}\) These numbers compare very well with those in the model, which are 0.7 and 0.29, respectively.

4.1.4 Life cycle profiles

The combination of idiosyncratic shocks, preference heterogeneity and differences in stock market participation status induces significant cross-sectional heterogeneity in wealth accumulation and consumption over the life cycle. Figure 1 (left panel) plots the gini coefficient for consumption, conditional on age. Consistent with the empirical evidence in Deaton and Paxson (1994), and more recently in Krueger and Perri (2006), consumption inequality tends to increase with age, as

\(^{10}\)In the SCF, wealth is defined as liquid assets net of all non-real estate loans plus real estate equity. Liquid wealth is made up of all types of transaction accounts, certificates of deposit, total directly-held mutual funds, stocks, bonds, total quasi-liquid financial assets, savings bonds, the cash value of whole life insurance, other managed assets (trusts, annuities and managed investment accounts) and other financial assets. Home equity is defined as the value of the home less the amount still owed on the first and 2nd/3rd mortgages and the amount owed on home equity lines of credit. Debts include all uncollateralized loans (credit cards, consumer installment loans) and loans against pensions.

\(^{17}\)The wealth gini coefficient is computed from the 2007 Survey of Consumer Finances, while the consumption gini coefficient is taken from Krueger and Perri (2006).
households are hit by different labor income shocks and also start saving for retirement. Total consumption inequality is much more pronounced during retirement because a significant fraction of the population (mostly non-stockholders) saves very little wealth during working years, due to their high discount rate, and thus have to rapidly scale down consumption towards their pension income.

Figure 1 (right panel) plots the same graph for wealth inequality over the life cycle. Overall, there is substantial wealth inequality in the economy reflecting the differential savings behavior across the two different groups. Initially wealth inequality is reduced a bit as stockholders start saving aggressively. Wealth inequality then rises from age 25 onwards as stockholders accumulate substantial amounts of wealth. Close to age 65 there is a significant decrease in inequality as non-stockholders finally decide to save something for retirement. Since they do not actually save much, this wealth is quickly consumed and thus the aggregate gini coefficient rapidly increases again.

4.2 Comparative Statics

In Table 6 we present results for alternative simplified versions of the baseline model to better understand the contribution of different model ingredients for our results.

4.2.1 No limited participation

In our first comparative statics we first allow the (previous) non-stockholders to invest in equities. The results are shown in column (2) of Table 6 (Model A), while column (1) reports the results for the baseline model. Consistent with previous literature (Gomes and Michaelides (2008)), the aggregate moments and asset prices are not significantly affected: nonstockholders have much less wealth than stockholders, hence allowing them to invest in capital has almost no impact on aggregate quantities and prices. The only slight change is a marginal increase in the mean risk free rate, from 1.79% to 1.82%. This slightly higher risk free rate is necessary to clear the bond market as former non-stockholders reallocate most of their (relatively small) wealth from bonds to equities. The rich buy the extra bonds, thereby reducing very slightly the aggregate capital stock by 0.19% and increasing the mean return on the stock market by one basis point. The mean equity premium is almost unchanged falling by one basis point only.

4.2.2 Homogeneous preferences

We next assume that the two groups of agents (participants and non-participants) have the same preferences. To facilitate comparison with the baseline economy we consider the preference parameters of the stock market participants for both. The results are reported in column (3) of Table 6 (Model B). Given their "new" preferences stock market non-participants accumulate much more wealth, and as a result the equilibrium risk free-rate is significantly lower (1.11%), and consequently the equilibrium risk premium is higher (5.12%). Additionally, overall consumption volatility
is slightly lower for two reasons: first non-stockholders are now smoothing their consumption much more, and second they are now responsible for a larger fraction of aggregate consumption. Allowing this type of limited participation can therefore generate a higher equity premium with a lower total consumption volatility. This experiment again illustrates the importance of calibrating the relative wealth of participants and non-participants to match the data. Failing to impose this discipline in the model makes it easier to match asset prices but through a counterfactual channel.

With regards to the conditional moments, the maximum spread in the riskless rate is now lower: 0.82% versus 1.13% in the baseline economy. Intuitively, since non-stockholders are now wealthier, a significantly larger fraction of the demand for bonds is less exposed to aggregate shocks. The maximum spread in the conditional return on capital across states is very similar: 2.92% versus 3.00% in the baseline economy.

4.2.3 Lower stockholder EIS

In the previous experiment we changed the preference parameters of the non-participants; we now instead lower the EIS of stockholders to 0.6 (from 0.7). The results are reported in column (4) of Table 6 (Model C). The lower savings incentives are reflected in a lower capital stock (−5.21%) and lower investment-output ratio (−3.46%). In equilibrium the riskless rate must increase to clear the bond market (0.42%) and the equity return also rises as a result of the lower capital stock (0.50%), with the net effect leading to a slightly higher equity premium (0.08%). Finally, consumption volatility becomes lower than in the baseline economy as stockholders’ wealth is less volatile: the fraction financed by stocks/capital has decreased relative to the fraction financed by the more stable human capital. This offers an interesting alternative calibration where the economy would have a higher equity premium and lower consumption volatility. The downside, and the reason why we choose a baseline calibration with EIS (for stockholders) of 0.7, is because we also want to match the risk-free rate.

Interestingly, the conditional moments are almost unaffected by the EIS. The maximum spread in conditional equity premia across states is now 2.56% versus 2.53% in the baseline economy, while the maximum spread in the riskless rate across states is 1.16% versus 1.13% in the baseline economy.

4.2.4 Homoskedastic Economy

In this final subsection we discuss the role of the heteroskedastic shocks. As we will see, they have a negligible impact on unconditional moments but they are important for generating additional variation in conditional asset prices. In column (5) in Table 6 (Model D) we report results for a version of the model where we shut down all sources of heteroskedasticity:

$$\sigma^2_{\xi,H} = \sigma^2_{\xi,L}, \sigma^2_{\xi,H} = \sigma^2_{\xi,L}, \sigma^2_{u,H} = \sigma^2_{u,L}, s_L = s_H, \mu_L = \mu_H, \delta_L = \delta_H$$  \hspace{1cm} (27)

and all of these parameters are set equal to their unconditional means in the baseline model. Without the additional uncertainty consumption volatility decreases, agents save less but invest a
higher fraction of their portfolio in stocks. Consequently the equilibrium capital stock is higher and market clearing implies a lower return on capital and a higher risk free rate. Quantitatively though the effects are small as the equity premium is only 0.15% lower than in the baseline economy.

Naturally we find the biggest differences in terms of conditional moments, particularly for equity returns (reported in table 7). It can be seen from panel A in table 7 that the spread in conditional expected stock returns across states is now only 1.70% versus 3.00% in the baseline model, and the spread in the conditional equity premium is only 0.95% versus 2.53% in the baseline economy. Despite the differences it is important to point out that we still have economically meaningful time variation in conditional moments, even in the homoskedastic economy. This is driven by the combination of borrowing constraints and market incompleteness in the model. In a production economy model following a negative aggregate shock the expected return on capital increases. In equilibrium this induces agents to invest more, such that in a frictionless economy we would have constant expected returns. However, due to the combination of borrowing constraints and uninsurable labor income risk, agents increase their savings less than in the frictionless benchmark. As a result, the equilibrium capital stock remains below the frictionless steady-state and the expected return on capital remains above its unconditional average.

Finally, in our last experiments we keep the idiosyncratic heteroskedasticity, so restriction (27) still holds except that $\sigma^2_{\xi,H} > \sigma^2_{\xi,L}$, as in the baseline model. As expected, the results (column (6) in Table 6) are in between the ones in an economy without any heteroskedasticity (column (5)), and the ones for the baseline economy (column(1)), and in particular asset prices are almost identical to the ones in the previous economy. The same conclusion holds when considering conditional moments shown in panel B of Table 7. We again find that the results are almost identical to the ones obtained in the previous economy. Realistically calibrated heteroskedasticity in idiosyncratic risk has a negligible impact on asset prices; heteroskedasticity in macroeconomic shocks generates quantitatively much stronger effects.

5 Permanent fiscal policy shocks

We first consider permanent changes in the fiscal policy variables: tax rates and government debt, while in the subsequent section we consider temporary fiscal policy shocks. To demonstrate the importance of explicitly taking into account risk premia in the model, and thus capturing the asset substitution channel, we compare our results with those obtained in an otherwise identical economy where we significantly decrease the volatility of stochastic depreciation (to 5%).\textsuperscript{18} In such an economy government debt and riskless capital become close substitutes and the corresponding equity premium converges to zero, approaching the classical macro fiscal policy framework.

\textsuperscript{18}To make the results comparable we re-calibrate the discount factor so that the two economies have the same steady-state average capital stock.
5.1 Capital income taxes

We first study the impact of changes in the capital income tax rate. More precisely, in Table 8 we consider an increase of both 2.5 and 5 percentage points (hereafter $pp$) in $\tau_K$ (from 40% in the baseline economy to 42.5% and 45%, respectively).

A higher capital income tax rate crowds out investment. In the new equilibrium the capital stock falls by 1.08% and 2.53%, respectively. As a result, consumption also falls and more than output, as households now save a higher fraction of their income, thus the consumption share of GDP decreases by 0.34% and 0.63% respectively. As consumers/investors reduce investment, the return on capital must increase to clear financial markets. The same pressure also exists in the bond market, as total households savings are lower, but here we also have an increase of the bond supply relative to GDP, since the latter is now lower. We find that the two effects are very close to canceling each other out with the riskfree rate remaining almost unchanged. Combining those two responses, the equity premium therefore increases: by 0.13% and 0.26% for the two experiments, respectively. This corresponds to a semi-elasticity of 5.2 basis points for each 1% change in the capital income tax rate.

In column three we report the results of an otherwise identical increase of 5 percentage points in the capital income tax rate, but in the economy with low aggregate volatility, where the two assets are very close substitutes and the risk premium is negligible. As we can see, the crowding out effect on capital is under-estimated by almost $\frac{1}{3}$, and consequently the (unreported) measured drop in consumption is 40% lower than in the baseline economy. This highlights the importance of the portfolio re-allocation behavior of households (asset substitution channel) for determining the total crowding out effect of fiscal policy decisions. Finally, in column four we repeat this experiment in the context of Model D (the homoskedastic economy). The results are similar to the ones in the baseline economy, with a slightly higher crowding out (the capital stock falls by 3.1% as opposed by 2.53%). Consequently, the increase in the return on capital is also higher (0.34% vs 0.26%).

5.2 Public debt

Now we consider changes in the level of public debt. We first consider an increase in the steady-state level of government debt by 25% which, from an historical perspective, is a relatively modest increase. The results (column 1 of Table 9) show that the riskless rate increases by 28 basis points to 2.08%, to induce households to hold these additional bonds. As the riskless rate increases and households shift their portfolios towards government debt, the capital stock falls and the cost of capital increases. We find that the capital stock falls by 2.1% so even more than in the previous experiment when we increased the capital income tax rate by 2.5%, while the return on capital goes up by 0.14% thus leading to a lower equity premium ($-0.14\%$): the supply of capital is now lower while the supply of bonds is now higher, therefore the riskless rate increases more than the return on capital.
In the second column we show the results of an increase of 75% in the level of debt. This brings the debt-to-gdp ratio to 64%, approximately the 2010 number for the US economy. The crowding out effect on capital increases to 5.55%, while the cost of government debt goes up by almost 50% (to 2.57%), leading to a 0.38% reduction in the equity premium. Naturally the increase in the cost of debt would be even larger if we had default risk in the model. Nevertheless, the semi-elasticity of interest expenses to changes in government debt is approximately 1.1 basis points.

In the third column we present the results of an increase of 25% in the steady-state level of debt, but in the economy with low aggregate volatility, where the two assets are very close substitutes and the risk premium is negligible. The conclusions are strikingly close to the ones obtained in the previous section: the measured crowding out effect is being under-estimated by approximately one third when we limit the impact on asset prices and their corresponding feedback effect. Lastly, in column four we repeat this experiment for the homoskedastic economy (Model D). The results are similar to the debt increase in the baseline economy. Compared to the first column of the table, there is marginally less crowding out (the capital stock falls by 1.86% as opposed by 2.07%). As a result, the increase in the return on capital is slightly higher (0.14% vs 0.13%).

6 Temporary fiscal policy shocks

In this section we study the impact of temporary shocks to the capital income tax rate. As before, these can also be viewed as capturing government expenditure shocks that are financed by variations in the capital income tax rate. We will compute responses to a change from a low to a high tax shock state lasting for \( T^* \) years and under different scenarios for \( \pi^\tau \), the expected duration of the shock.

Since we are solving for an economy with aggregate uncertainty, our steady-state is stochastic, and therefore we cannot compute responses either from a given steady-state level, or by setting all variables equal to their unconditional means (since this is a highly non-linear model). Therefore, we must compute average responses across a large range of alternative starting points (and wealth distributions), all taken from the actual stochastic equilibrium that we have solved for. More precisely, we consider 1000 different points in our simulation as alternative initial conditions. We therefore have \( N = 1000 \) initial conditions characterized by the values of the aggregate variables/states and the individual wealth of all agents in the economy:

\[
\{\{x_{i,a,t}^i\}_{a=1}^{80}\}_{i=1}^{500}, P_t^B, k_t, U_t, \eta_t, v_t, \tau_{K,t}\}_{t=1}^N
\]

For each of these we then simulate further under two hypothetical paths for a further 36 periods. One with \( \tau_{K,t+h} = \tau_{K}^H \) for the first \( T^* \) periods and one with \( \tau_{K,t+h} = \tau_{K}^L \) for those same initial periods. Subtracting one response from the other we can then compare the behavior of the economy under a high tax regime with its behavior under a low tax regime. Although the actual tax shock
only last for $T^*$ periods we trace the impulse responses further ahead, as the economy starts to converge back to the steady-state (a total of 36 years).

To simplify the notation let us define

$$\{\tau_K\}^H_t = \{\tau_{K,t} = \tau_{K,H} \}^{t+T^*}_t,$$
$$\{\tau_K\}^L_t = \{\tau_{K,t} = \tau_{K,L} \}^{t+T^*}_t,$$

Then for each of these two subsequent paths, and starting from each of the 1000 initial conditions, we consider all possible realizations of the other exogenous aggregate variables (productivity and depreciation), and all those possibilities are again averaged using their unconditional probabilities. The differential behavior of a variable $X_t$ under these two alternative series gives us the average response to the fiscal policy shock:

$$\frac{1}{N} \sum_{i=1}^{N} E_t \{ X_t | \{ \tau_K \}^H \} - \frac{1}{N} \sum_{i=1}^{N} E_t \{ X_t | \{ \tau_K \}^L \}$$

where the expectations are computed over the future realizations of the different shocks in the model. Further details can be found in the Appendix A.  

### 6.1 I.i.d. expected tax shocks

For a direct comparison with the baseline case we first consider i.i.d expected tax shocks ($\pi^T = 0.5$). In such a world, we study the impact of a tax shock lasting for four years ($T^* = 4$), as this is the typical duration of a presidential cycle. The results are shown in figure 2, where we plot the responses of aggregate consumption (total and for each subgroup), output, capital, wages, the riskless rate and the return on capital. The return responses are measured in basis points, while all other variables are reported in percentage changes “from the steady-state”, i.e. subtracting the responses to the low tax shock from the responses to the high tax shock.

The higher capital income tax rate leads to an immediate reduction in the stockholder’s consumption since their disposable income is now lower. The risk-free rate also responds within the same period since aggregate savings are now reduced. The lower savings lead to a decrease in next-period’s capital stock which in turn induces an additional reduction in the stockholder’s consumption. Moreover, as the capital stock falls, the return on capital increases and wages decrease. Lower wages now lead to a reduction in non-stockholders’ consumption as well, which combined with the additional fall in stockholders’ consumption (described above), implies a second year percentage drop in total aggregate consumption that is almost double its first year value. Once the tax rate reverts to its average value, we naturally observe a steady convergence of all variables to their stochastic steady states.
The magnitudes are perfectly consistent with those obtained in the previous section. There we found that a 5% permanent increase in the capital income tax rate induces a 26 basis points rise in the return on capital and a 2.53% drop in the capital stock. Here we find when that same 5% increase in the tax rate only lasts 4 periods, the return on capital is 6 basis points higher, while the drop in the capital stock is 0.77% (both measured at the peak).

6.2 Persistent Expected Tax Shocks

Next we consider the other extreme of highly persistent tax shocks, $\pi^\tau = 0.95$ (thus expected duration of 20 years), and an actual shock that will last exactly that expected duration: $T^* = 20$. Results with this higher persistence are very interesting because they come close to replicating a transition dynamics across two stochastic steady-states: naturally we converge to that limit as we let $\pi^\tau \to 1$ and $T^* \to \infty$. Since we are moving away from our baseline calibration in this experiment we have also decreased the capital income tax rate to 35% so that we study the response of changing taxes from 30% to 40%. These figures (in reverse) are within a range of various recent proposals to reduce corporate tax rates in the US.\textsuperscript{19}

In figure 3 we plot the responses of aggregate variables. Now we naturally observe strong hump-shaped responses for all variables, as the impact of the tax rate change continues to accumulate over many years. At the peak, the capital stock now decreases by almost 2.5%, and the return on capital increases by 35 basis points. These values are almost identical, or even slightly larger than the ones obtained when we considered permanent shocks to the tax rates. The explanation for this is that under the current calibration agents actually expect the tax shocks to last for (an average of) 20 years, a very long period.

7 Conclusion

We analyze the implications of fiscal policy changes in a heterogeneous agent OLG model with incomplete markets, and where the stock market and government debt are not perfect substitutes. The model is calibrated to fit the main macroeconomic and asset pricing moments, and to generate wealth and consumption heterogeneity consistent with the data. We quantify the impact of changes in tax rates and government debt on the macro-economy and on rates of returns. We find a 28 basis points increase in the riskless rate resulting from a 25% increase in government debt, which represents a 16% increase in the government’s borrowing costs, a profound increase considering that we do not have sovereign default in the model. An increase in public debt to 64% of GDP, close to the current US number, leads to a 77 basis points increase in the riskless rate, accompanied by

\textsuperscript{19}The current federal corporate tax rate is 35%, and the combined average tax rate including state taxes is close to 40%. Recent proposals to reduce the federal corporate income tax below 30% have come from the Simpson-Bowles commission, the President, several Republican presidential candidates, and academic economists (e.g. Martin Feldstein’s WSJ editorial on February 15, 2011). Although the exact numbers vary, most proposals would bring the combined federal and state tax burden closer to 30%.
a 38 basis points reduction in the equity premium. Furthermore, we identify household portfolio rebalancing decisions as a quantitatively important channel (the asset substitution channel) for determining the macro-economic impact of fiscal policy measures. The crowding-out effects of taxes and government debt is 50% higher than the ones obtained in an otherwise identical economy where capital and government debt are perfect substitutes. Our results are qualitatively (and sometimes quantitatively) consistent with the recent empirical evidence in Laubach (2008), Greenwood and Vayanos (2010a, 2010b) and Krishnamurthy and Vissing-Jorgensen (2012) on the effects of government debt on bond yields. Unfortunately, the complexity of the model does not allow us to include defaultable corporate debt to study the effects of debt to GDP changes on credit spreads, as is done in some of this recent empirical literature.
Appendix A  Solving the OLG model

A.1 Solution method outline


The numerical sequence works as follows:

i. Specify a set of forecasting equations ($\Gamma_K$ and $\Gamma_P$).

ii. Solve the household’s decision problem, taking prices as given, and using the forecasting equations to form expectations (details in A.2).

iii. Given the policy functions, simulate the model (10100 periods) while computing the market clearing variables at each period (details in A.3).

iv. Use the last 10000 periods to update the coefficients in the forecasting equations (details in A.4).

v. Repeat steps 1, 2, 3, 4, with the new forecasting equations until convergence. We have two convergence criteria:
   - Stable coefficients in the forecasting equations.
   - Forecasting equations with regression $R^2$ at 99.9%.

A.2 Solving the household’s decision problem

A.2.1 Discretization of the state space

Age ($a$) is a discrete state variable taking 71 possible values. We discretize the cash-on-hand dimension ($x^t_i$) using 51 points, with denser grids closer to zero to take into account the higher curvature of the value function in this region. The discrete aggregate state variables (the depreciation shock ($\eta^t_i$) and the aggregate productivity shock ($U^t_i$)) each take only two possible values. With respect to the other two continuous aggregate state variables, we use an adaptable grid that takes into account the availability of high or low capital in the economy and allows higher accuracy with a fewer number of grid points. The grid is based on the idea that the expected conditional equity premium has to be positive and therefore the price of the bond is an increasing function of the available capital stock. This adaptive grid (as opposed to a fixed, rectangular grid) allows greater accuracy since it neglects points in the state space that, according to the economics of the problem, will never be visited conditional on being at a particular level of a capital stock at a given point in time. This is a guess and verify method and the simulated bond prices are confirmed ex post (after convergence) to lie within the specified range. Typically, the R-squared statistic from the
bond regression is below 99.9% when the price of the bond hits the edges of this grid during the simulation. We use 15 points to discretize $k_t$, and 15 points to discretize $P_t^B$.

The grid range for the continuous state variables is verified ex-post by comparing with the values obtained in the simulations, and with the results obtained when this range is increased. A smaller number of grid points for $k_t$ and for $P_t^B$ would not affect the policy functions directly. It would, however, affect the R-squared of the forecasting equations and the convergence of their respective coefficients.

A.2.2 Maximization

We solve the maximization problem for each agent type using backward induction. For every age $a$ prior to $A$, and for each point in the state space, we optimize using grid search. We need to compute the value associated with each set of controls (consumption, and share of wealth invested in stocks). From the Bellman equation,

$$V_a(x_{at}; k_t, U_t, \eta_t, v_t, \tau_{K,t}, P_t^B) = \max_{\{k_{a+1,t+1}, k_{a+1,t+1}\}} \left\{ (1 - \beta)(c^{a+1}_{at})^{1-1/\psi} + \beta \left( E_1 \left[ (\omega_{a+1} \omega_{t+1})^{1-\rho} p_{a+1} V^{1-\rho}_{a+1} \left( x_{a+1,t+1}; k_{t+1}, U_{t+1}, \eta_{t+1}, v_{t+1}, \tau_{K,t+1}, P_{t+1}^B \right) \right] \right)^{1-1/\psi} \right\}^{1-1/\psi}$$

these values are given as a weighted sum of current utility $((c^{a+1}_{at})^{1-1/\psi})$ and the expected continuation value $(E_a V_{a+1}(\cdot))$, which we can compute once we have obtained $V_{a+1}$. In the last period the policy functions are trivial and the value function corresponds to the indirect utility function. This gives us the terminal condition for our backward induction procedure. Once we have computed the value of all the alternatives we pick the maximum, thus obtaining the policy rules for the current period. Substituting these decision rules in the Bellman equation we obtain this period’s value function $(V_a(\cdot))$, which is then used to solve the previous period’s maximization problem. This process is iterated until $a = 1$.

We use the forecasting equations ($\Gamma_K$ and $\Gamma_P$) to form expectations of the aggregate variables, and we perform all numerical integrations using Gaussian quadrature to approximate the distributions of the innovations to the labor income process ($\varepsilon^i$ and $\xi^i$) and the aggregate shocks ($\eta_t$, $v_t$, $U_t$, and $\tau_{K,t}$). For points which do not lie on state space grid, we evaluate the value function using a cubic spline interpolation along the cash-on-hand dimension, and a bi-linear interpolation along the other two continuous state variables ($k_t$ and $P_t^B$). Bi-linear interpolation works well along these two dimensions because households are price takers, and therefore these state variables are not affected by the control variables.
A.3 Simulating the model and clearing markets

A.3.1 Simulation

We use the policy functions for the two agent types (A and B) to simulate the behavior of 500 agents of each type in each of the 71 cohorts over 10500 periods. The realizations of the aggregate random variables (aggregate regime \( v_t \), stochastic depreciation \( \eta_t \), aggregate productivity \( U_t \) and capital income tax rates \( \tau_{K,t} \)) are drawn from their original discrete distributions, while the idiosyncratic productivity shocks \( (\varepsilon^i_t \text{ and } \xi^i_t) \) are drawn from the corresponding log-normal distributions. All other random variables are endogenous to the model. The realizations of the exogenous random variables are held constant within the outer loop, i.e. across iterations, so as not to affect the convergence criteria.

A.3.2 Market clearing

For every time period we simulate the households’ behavior for every possible bond price (i.e. every point in the grid for \( P_t^B \)). We then aggregate the individual bond demands and use a linear interpolation to determine the market clearing bond price. All household equilibrium allocations (consumption and asset holdings) are then obtained from a linear interpolation with the same coefficients, while the aggregate variables (capital and output) are computed by aggregating these market clearing allocations. This then determines the state variables for simulating the next period’s decisions.

A.4 Updating the forecasting equations

Using the simulated time-series (after discarding the first 500 observations) we estimate the following OLS regressions, for every combination of aggregate regime \( (v_t) \), productivity shock \( (U_{t+1}) \) depreciation shock \( (\eta_{t+1}) \) and capital income tax rate \( (\tau_{K,t}) \) realizations,

\[
\ln(k_{t+1}) = q_{10} + q_{11} \ln(k_t) \]  \hspace{1cm} (A.2)

and

\[
\ln(P_{t+1}^B) = q_{20} + q_{21} \ln(k_t) + q_{22} \ln(P_t^B) \]  \hspace{1cm} (A.3)

This gives us 16 equations and 16 sets of coefficients that forecast state-contingent capital \( (k_{t+1}) \) and bond prices \( (P_{t+1}^B) \). We iterate the code until we have converged on the coefficients and on the R-squared of these regressions. For the first set of equations (A.2) we obtain R-squared values around 99.99%. For the second set of equations (A.3), the R-squared values are in the 90%−95% range when we only use \( \ln(k_t) \) as a regressor, increase to about 99.9% when we add \( \ln(P_t^B) \).

A.5 Computing non-linear impulse responses

1) We will average over 1000 possible different initial distributions of wealth (bond holdings for two agents across their lifetimes and stockholdings of stockholders across their lifetimes). We have
these simulated wealth distributions as part of solving the model. Associated with each of these wealth distributions there are market clearing bond prices and aggregate capital we can use in the first simulation.

2) We then want to compute the conditional expectation of how endogenous variables move after a shock hits minus the conditional expectations of no shocks hitting for the next, say, 36 periods starting from each of these initial distributions. In our main experiment we compare the difference between a high tax rate on capital versus a low tax rate on capital. We therefore want to compute

\[ E_{t-1}(X_{t+h}|\tau_K = \text{high}) - E_{t-1}(X_{t+h}|\tau_K = \text{low}) \]

for \( h = 1, \ldots, 36 \), and where the tax rate is high for a certain number of periods (4 or 20 in our actual application).

3) We start from the low tax shock case. We compute all possible realizations of the exogenous states sequentially, market clear along each realization and then take the average. We then take the average of all endogenously evolving variables for these 1000 different cases. We save the evolution of the endogenous variables as \( X_{0} \).

4) We then compute the high tax shock case in a similar way and save the evolution of the endogenous variables as \( X_{1} \).

5) Our percent deviation from steady state is then the average difference over the 1000 simulations between (3) and (4).
References


Table 1: This table reports tax revenue (in percent of output) by source from the baseline model and aggregate U.S. data from the BEA National accounts. We report both the time-series average and the latest pre-crisis values.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>5.41</td>
<td>5.19</td>
<td>5.72</td>
</tr>
<tr>
<td>Labor</td>
<td>8.00</td>
<td>6.80</td>
<td>9.38</td>
</tr>
<tr>
<td>Consumption</td>
<td>7.64</td>
<td>8.43</td>
<td>8.01</td>
</tr>
</tbody>
</table>

Table 2: Panel A reports values from the baseline model and aggregate U.S. data from the BEA National Accounts (1929-2009). Following Castaneda et al. (2003) we classify 75 percent of durable consumption expenditures as investment and the remaining 25 percent as consumption. Government debt is the U.S. federal debt held by the public between 1960 and 2010. Output excludes net exports. Panel B reports the standard deviation of consumption and output (NIPA tables from the BEA, 1929-2010). Panel B also reports the standard deviation of stockholders’ and non-stockholders’ consumption growth rates from the baseline model and from the data. We use the values of consumption growth volatilities reported by Malloy, Moskowitz and Vissing-Jørgensen (2009) of 1.4% and 3.6% for non-stockholders and stockholders (from CEX data) and scale them by the ratio of the aggregate consumption volatilities from the CEX sample (1.7%) and the longer aggregate sample from 1929 to 2010 (3.3%).

Panel A: Share of Output (percent)

<table>
<thead>
<tr>
<th>Source</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>58.5</td>
<td>59.5</td>
</tr>
<tr>
<td>Investment</td>
<td>21.3</td>
<td>20.3</td>
</tr>
<tr>
<td>Government</td>
<td>20.2</td>
<td>20.4</td>
</tr>
<tr>
<td>Government Debt</td>
<td>35.9</td>
<td>35.8</td>
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</tbody>
</table>

Panel B: Standard deviation of growth rates (percent)

<table>
<thead>
<tr>
<th>Source</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate Output</td>
<td>3.80</td>
<td>4.28</td>
</tr>
<tr>
<td>Aggregate Consumption</td>
<td>3.09</td>
<td>3.28</td>
</tr>
<tr>
<td>Stockholders Consumption</td>
<td>4.37</td>
<td>5.80</td>
</tr>
<tr>
<td>Non-Stockholders Consumption</td>
<td>3.18</td>
<td>3.96</td>
</tr>
</tbody>
</table>
Table 3: Unconditional asset pricing moments from the data (CRSP) and baseline model.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Moment</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_f$</td>
<td>Mean</td>
<td>1.79</td>
<td>1.58</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>1.27</td>
<td>2.67</td>
</tr>
<tr>
<td>$r_m$</td>
<td>Mean</td>
<td>6.25</td>
<td>8.31</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>16.57</td>
<td>19.81</td>
</tr>
<tr>
<td>$r_m - r_f$</td>
<td>Mean</td>
<td>4.45</td>
<td>6.74</td>
</tr>
</tbody>
</table>

Table 4: Conditional asset pricing moments from the model. Reported are averages of simulations conditional on a given aggregate state. Returns and standard deviations are in percent.

<table>
<thead>
<tr>
<th></th>
<th>$E_t(R^K_{t+1})$</th>
<th>$R^f_{t+1}$</th>
<th>$E_t(R^K_{t+1} - R^f_{t+1})$</th>
<th>$\sigma_t(R^K_{t+1})$</th>
<th>$\frac{E_t(R^K_{t+1} - R^f_{t+1})}{\sigma_t(R^K_{t+1})}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>7.42</td>
<td>2.39</td>
<td>5.69</td>
<td>18.08</td>
<td>0.379</td>
</tr>
<tr>
<td>Min</td>
<td>4.42</td>
<td>1.26</td>
<td>3.16</td>
<td>15.04</td>
<td>0.185</td>
</tr>
<tr>
<td>Spread</td>
<td>3.00</td>
<td>1.13</td>
<td>2.53</td>
<td>3.04</td>
<td>0.194</td>
</tr>
</tbody>
</table>

Table 5: Wealth Distribution. The table reports the percentage of each group’s total wealth held within a given percentile range for this group. Data source: 2007 Survey of Consumer Finances. Wealth is the net worth of households as defined in the text and stockholders are defined as households who own stocks directly or through mutual funds either in taxable accounts or in tax-deferred pension plans. Figures from the model are averages over the last 200 simulations.

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Non-stockholders</th>
<th>Stockholders</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>0-20</td>
<td>0.01</td>
<td>0.45</td>
<td>2.63</td>
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<tr>
<td>20-50</td>
<td>0.49</td>
<td>5.35</td>
<td>18.25</td>
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<td>50-80</td>
<td>7.51</td>
<td>15.82</td>
<td>36.31</td>
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<tr>
<td>80-100</td>
<td>91.49</td>
<td>78.38</td>
<td>42.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>68.40</td>
</tr>
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</table>
Table 6: Comparative statics for different versions of the model. Model A denotes an economy without limited participation. Model B denotes an economy where all agents have the same preferences as the stockholders in the baseline economy. Model C denotes an economy where the stockholders have lower EIS (0.6). Model D denotes an economy without any heteroskedasticity. Model E denotes an economy without heteroskedasticity in aggregate variables. The Table shows the long-run averages of the different variables, and the differences (in square brackets) relative to the baseline model. The differences are in percentages relative to the baseline case (except return and growth rate volatility changes which are in percentage points). \(\sigma(\Delta LnC)\) denotes the volatility of log growth of aggregate consumption.

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Model A</th>
<th>Model B</th>
<th>Model C</th>
<th>Model D</th>
<th>Model E</th>
</tr>
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<tbody>
<tr>
<td>(\sigma(\Delta LnC))%</td>
<td>3.09</td>
<td>3.11</td>
<td>3.04</td>
<td>2.95</td>
<td>2.81</td>
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<tr>
<td></td>
<td>[0.02]</td>
<td>[-0.05]</td>
<td>[-0.14]</td>
<td>[-0.28]</td>
<td>[-0.23]</td>
<td></td>
</tr>
<tr>
<td>(K)</td>
<td>4.55</td>
<td>4.54</td>
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<td>4.31</td>
<td>4.63</td>
<td>4.64</td>
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<tr>
<td></td>
<td>[-0.19]</td>
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<td>[-5.21]</td>
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<td>[1.99]</td>
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<tr>
<td>(K/Y)</td>
<td>2.65</td>
<td>2.65</td>
<td>2.68</td>
<td>2.55</td>
<td>2.67</td>
<td>2.68</td>
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<td>(C/Y)</td>
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<td>[0.88]</td>
<td>[0.25]</td>
<td>[0.20]</td>
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<tr>
<td>(I/Y)</td>
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<td>0.212</td>
<td>0.215</td>
<td>0.206</td>
<td>0.211</td>
<td>0.211</td>
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<tr>
<td></td>
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<td>[-3.46]</td>
<td>[-1.10]</td>
<td>[-0.97]</td>
<td></td>
</tr>
<tr>
<td>(r_m) (%)</td>
<td>6.25</td>
<td>6.26</td>
<td>6.22</td>
<td>6.74</td>
<td>6.19</td>
<td>6.18</td>
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<tr>
<td></td>
<td>[0.01]</td>
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<td>[-0.06]</td>
<td>[-0.07]</td>
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<tr>
<td>(r_f) (%)</td>
<td>1.79</td>
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<td>2.21</td>
<td>1.88</td>
<td>1.87</td>
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<td>[0.42]</td>
<td>[0.09]</td>
<td>[0.07]</td>
<td></td>
</tr>
<tr>
<td>(r_m - r_f) (%)</td>
<td>4.45</td>
<td>4.44</td>
<td>5.12</td>
<td>4.53</td>
<td>4.30</td>
<td>4.31</td>
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<td>[0.66]</td>
<td>[0.08]</td>
<td>[-0.15]</td>
<td>[-0.14]</td>
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</table>
Table 7: Conditional asset pricing moments from the model. Reported are averages of simulations conditional on a given aggregate state. Returns and standard deviations are in percent.

<table>
<thead>
<tr>
<th></th>
<th>$E_t(R^K_{t+1})$</th>
<th>$R^f_{t+1}$</th>
<th>$E_t(R^K_{t+1} - R^f_{t+1})$</th>
<th>$\sigma_t(R^K_{t+1})$</th>
<th>$\frac{E_t(R^K_{t+1} - R^f_{t+1})}{\sigma_t(R^K_{t+1})}$</th>
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<tbody>
<tr>
<td><strong>Panel A: No Heteroskedasticity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td>6.93</td>
<td>2.28</td>
<td>4.72</td>
<td>16.37</td>
<td>0.293</td>
</tr>
<tr>
<td>Min</td>
<td>5.23</td>
<td>1.41</td>
<td>3.77</td>
<td>16.09</td>
<td>0.234</td>
</tr>
<tr>
<td>Spread</td>
<td>1.70</td>
<td>0.87</td>
<td>0.95</td>
<td>0.28</td>
<td>0.059</td>
</tr>
<tr>
<td><strong>Panel B: No Agg. Heteroskedasticity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max</td>
<td>6.93</td>
<td>2.26</td>
<td>4.72</td>
<td>16.37</td>
<td>0.293</td>
</tr>
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<td>1.40</td>
<td>3.77</td>
<td>16.09</td>
<td>0.234</td>
</tr>
<tr>
<td>Spread</td>
<td>1.71</td>
<td>0.86</td>
<td>0.95</td>
<td>0.28</td>
<td>0.059</td>
</tr>
</tbody>
</table>

Table 8: Comparative statics for changes in the capital income tax rate. Results are shown for the calibrations of shocks as in the baseline model and for the economies with low aggregate risk (s=5%) or with no heteroskedasticity in any of the variables. The table shows differences relative to the baseline case with tax rate $\tau_K = 40\%$, reported in percentages, except return and growth rate volatility changes which are in percentage points. $\sigma(\Delta \log(C))$ denotes the volatility of log growth aggregate consumption. (n/a): results not reported because they are not meaningful as consumption volatility and the risk premium are very close to zero in the economy with low aggregate risk.

<table>
<thead>
<tr>
<th>Shocks structure</th>
<th>Baseline $\tau_K = 42.5%$</th>
<th>Baseline $\tau_K = 45.0%$</th>
<th>Low Risk $\tau_K = 45.0%$</th>
<th>No heterosk. $\tau_K = 45.0%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \sigma(\Delta \log(C))$ (%pts)</td>
<td>-0.18</td>
<td>-0.27</td>
<td>(n/a)</td>
<td>-0.27</td>
</tr>
<tr>
<td>$\Delta K$</td>
<td>-1.08</td>
<td>-2.53</td>
<td>-1.73</td>
<td>-3.14</td>
</tr>
<tr>
<td>$\Delta K/Y$</td>
<td>-0.69</td>
<td>-1.64</td>
<td>-1.14</td>
<td>-2.04</td>
</tr>
<tr>
<td>$\Delta C/Y$</td>
<td>-0.34</td>
<td>-0.63</td>
<td>-0.30</td>
<td>-0.57</td>
</tr>
<tr>
<td>$\Delta I/Y$</td>
<td>-0.78</td>
<td>-1.82</td>
<td>-1.16</td>
<td>-2.21</td>
</tr>
<tr>
<td>$\Delta r_m$ (%)</td>
<td>0.11</td>
<td>0.26</td>
<td>(n/a)</td>
<td>0.34</td>
</tr>
<tr>
<td>$\Delta r_f$ (%)</td>
<td>-0.02</td>
<td>0.00</td>
<td>1.80</td>
<td>0.02</td>
</tr>
<tr>
<td>$\Delta r_m - r_f$ (%)</td>
<td>0.13</td>
<td>0.26</td>
<td>(n/a)</td>
<td>0.31</td>
</tr>
</tbody>
</table>
Table 9: Comparative statics for changes in the supply of government bonds. Results are shown both for the baseline calibration and for the economies with low aggregate risk ($s = 5\%$) or with no heteroskedasticity in any of the variables. The table shows differences relative to the baseline tax rate calibration ($B/Y = 35.9\%$) reported in percentages, except for return and growth rate volatility changes which are in percentage points. $\sigma(\Delta \ln C)$ denotes the volatility of log growth aggregate consumption. (n/a): not reported since they are not particularly meaningful as consumption volatility and the risk premium are very close to zero in the economy with low aggregate risk.

<table>
<thead>
<tr>
<th>Shocks structure</th>
<th>Baseline $B25%$ higher</th>
<th>Baseline $B75%$ higher</th>
<th>Low Risk Calibration $B25%$ higher</th>
<th>No heterosk. $B25%$ higher</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gov. debt</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New $B/Y$</td>
<td>0.452</td>
<td>0.640</td>
<td>0.450</td>
<td>0.449</td>
</tr>
<tr>
<td>$\Delta \sigma (\Delta \log(C))$ (%)</td>
<td>-0.03</td>
<td>-0.12</td>
<td>(n/a)</td>
<td>-0.03</td>
</tr>
<tr>
<td>$\Delta K$</td>
<td>-2.07</td>
<td>-5.55</td>
<td>-1.40</td>
<td>-1.86</td>
</tr>
<tr>
<td>$\Delta K/Y$</td>
<td>-1.36</td>
<td>-3.68</td>
<td>-0.92</td>
<td>-1.23</td>
</tr>
<tr>
<td>$\Delta C/Y$</td>
<td>0.63</td>
<td>1.96</td>
<td>0.50</td>
<td>0.58</td>
</tr>
<tr>
<td>$\Delta I/Y$</td>
<td>-1.38</td>
<td>-3.72</td>
<td>-0.92</td>
<td>-1.23</td>
</tr>
<tr>
<td>$\Delta r_m$ (%)</td>
<td>0.14</td>
<td>0.40</td>
<td>(n/a)</td>
<td>0.13</td>
</tr>
<tr>
<td>$\Delta r_f$ (%)</td>
<td>0.28</td>
<td>0.77</td>
<td>0.12</td>
<td>0.23</td>
</tr>
<tr>
<td>$\Delta r_m - r_f$ (%)</td>
<td>-0.14</td>
<td>-0.38</td>
<td>(n/a)</td>
<td>-0.10</td>
</tr>
</tbody>
</table>
Figure 1: Consumption and wealth inequality over the life-cycle. The figure shows the cross-sectional gini coefficients for consumption (left panel) and wealth (right panel).

Figure 2: Impulse responses to i.i.d. tax shocks.
Figure 3: Impulse responses to persistent tax shocks.