Data-Driven Investment Management

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Based on joint research with

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EDHEC Business School

Keynote talk
Eleventh International Conference on Computational Management Science
Lisbon, May 2014
“The motivation behind my dissertation was to apply mathematics to the stock market” Harry Markowitz

“This is not a dissertation in economics, and we cannot give you a PhD in economics for this” Milton Friedman
Efficient frontier: Investor concerned only about mean and variance of returns chooses portfolio on **efficient frontier**.
Mean Variance portfolio optimization

\[
\begin{align*}
\min_{w} & \quad w^\top \Sigma w - \frac{w^\top \mu}{\gamma} \\
\text{s.t.} & \quad w \in C
\end{align*}
\]

- **w** portfolio weight vector
- **\Sigma** covariance matrix of asset returns
- **\mu** mean asset returns
- **\gamma** risk aversion parameter
Implementation: estimation and rebalancing

Portfolio for January 2013

Jan 03 - Dec 12

Estimation window

Next month

Time
Portfolio for January 2013

Estimation window

Jan 03  Dec 12

Next month

Portfolio for February 2013

Estimation window

Feb 03  Jan 13

Next month

Will the portfolios for January 2013 and February 2013 be similar?
Problem: unstable portfolios

- The portfolios for consecutive months usually differ greatly:
  - **Unstable portfolios**: Extreme weights that fluctuate a lot as we rebalance the portfolio
  - **Why?**
Problem: unstable portfolios

- The portfolios for consecutive months usually differ greatly:
  - **Unstable portfolios**: Extreme weights that fluctuate a lot as we rebalance the portfolio
  - Why?
  - Estimation error!!!
The Markowitz Optimization Enigma: Is ‘Optimized’ Optimal?

See also [Chopra and Ziemba, 1993] and [Broadie, 1993].
The 1/N paper
The 1/N paper
DeMiguel, Garlappi, Uppal (RFS, 2009)

Objective: Quantify impact of estimation error by comparing mean-variance and equal-weighted portfolio (1/N).

Why use the 1/N portfolio as a benchmark?

- **Estimation-error free,**
- **Simple** but not simplistic:
  - Does have **some diversification**, though not “optimally” diversified;
  - With rebalancing $\Rightarrow$ contrarian;
  - Without rebalancing $\Rightarrow$ momentum.
- **Ancient wisdom**
  - Rabbi Issac bar Aha (Talmud, 4th Century): Equal allocation A third in land, a third in merchandise, a third in cash.
- **Investors use it**, even nowadays.
Thomson Reuters equal weighted commodity index
What we do

- **Empirically:** Compare fourteen portfolio rules to $1/N$ across seven datasets
  - Sharpe ratio
    \[
    SR = \frac{mean}{std.\, dev}.
    \]

- **Analytically:** Derive critical estimation window length for mean-variance strategy to outperform $1/N$.

- **Simulations:** Extend analytical results to models designed to handle estimation error.
What we find

▶ **Empirically:** None of the fourteen portfolio models consistently dominates $1/N$ across seven separate datasets (SR and turnover).

▶ **Analytically:** Based on U.S. stock market data, critical estimation window for sample-based mean variance (MV) to outperform $1/N$ is
  - Approximately 3,000 months for 25-asset portfolio
  - Approximately 6,000 months for 50-asset portfolio

▶ **Simulations:** Even models designed to handle estimation error need unreasonably large estimation windows to outperform $1/N$. 
Portfolios tested

- Bayesian portfolios
Historical return data

Sample mean
Bayes rule

Mean prior distribution

Mean posterior distribution

Historical return data

Sample mean
Portfolios tested

- **Bayesian portfolios**
  - Diffuse prior [Barry, 1974], [Klein and Bawa, 1976], [Brown, 1979],
  - Informative empirical prior [Jorion, 1986]
  - Prior belief on asset pricing model [Pástor, 2000] and [Pástor and Stambaugh, 2000]
Portfolios tested

- **Bayesian portfolios**

- **Portfolios with moment restrictions**
  - Minimum-variance portfolio often outperforms mean-variance portfolios; [Jagannathan and Ma, 2003],
  - Value-weighted market portfolio optimal in a CAPM world,
    \[
    \Sigma = \nu \mu \mu^\top + \sigma^2 I_N.
    \]
Portfolios tested

- Bayesian portfolios
- Portfolios with moment restrictions
- Portfolios subject to shortsale constraints

[Jagannathan and Ma, 2003] show they perform well in practice.
Portfolios tested

- Bayesian portfolios
- Portfolios with moment restrictions
- Portfolios subject to shortsale constraints
- Optimal combinations of portfolios
  
  [Kan and Zhou, 2007]

\[ w_{KZ} = a \cdot w_{\text{mean-variance}} + b \cdot w_{\text{minimum-variance}}, \]

where \( a \) and \( b \) minimize portfolio loss.
Empirical results: Out-of-sample Sharpe ratios - I

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1/N</td>
<td>0.19</td>
<td>0.14</td>
<td>0.13</td>
<td>0.22</td>
<td>0.16</td>
<td>0.18</td>
</tr>
<tr>
<td>mean var (in sample)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean variance</td>
<td>0.38</td>
<td>0.21</td>
<td>0.21</td>
<td>0.29</td>
<td>0.51</td>
<td>0.54</td>
</tr>
</tbody>
</table>

**Classical approach that ignores estimation error**

| mean variance          | 0.08          | -0.04          | -0.07          | 0.22       | -0.07      | 0.00       |

**Bayesian approach to estimation error**

| Bayes Stein            | 0.08          | -0.03          | -0.05          | 0.25       | -0.06      | 0.00       |
| data and model         | 0.14          | -0.05          | 0.10           | 0.02       | 0.07       | 0.24       |

**Moment restrictions**

| minimum variance       | 0.08          | 0.16           | 0.15           | 0.25       | **0.28**   | -0.02      |
| market portfolio       | 0.14          | 0.11           | 0.12           | 0.11       | 0.11       | 0.11       |
| missing factor         | 0.19          | 0.13           | 0.12           | 0.06       | 0.15       | 0.15       |
Why does minimum-variance portfolio outperform mean-variance portfolio?

- **In sample**, min-var portfolio has smallest expected return.
Why does minimum-variance portfolio outperform mean-variance portfolio?

- **In sample**, min-var portfolio smallest expected return.

  - Estimation error in mean larger than in variance; [Merton, 1980].
  - Jagannathan and Ma (2003): “estimation error in the sample mean is so large that nothing much is lost in ignoring the mean altogether”.

![Expected Return vs Variance Chart](chart.png)

- **in-sample frontier**
- **out-of-sample frontier**
Mean variance portfolio returns are extreme
### Empirical results: Out-of-sample Sharpe ratios - II

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<tbody>
<tr>
<td>$1/N$</td>
<td>0.19</td>
<td>0.14</td>
<td>0.13</td>
<td>0.22</td>
<td>0.16</td>
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<td>0.29</td>
<td>0.51</td>
<td>0.54</td>
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**Shortsale constraints**

<table>
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<tr>
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<th>mean variance</th>
<th>Bayes Stein</th>
<th>minimum variance</th>
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<tr>
<td>mean variance</td>
<td>0.09</td>
<td>0.07</td>
<td>0.08</td>
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<tr>
<td>Bayes Stein</td>
<td>0.11</td>
<td>0.08</td>
<td>0.08</td>
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<tr>
<td>minimum variance</td>
<td>0.08</td>
<td>0.14</td>
<td>0.15</td>
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**Optimal combinations of portfolios**

<table>
<thead>
<tr>
<th></th>
<th>mean &amp; min-var</th>
<th>1/$N$ &amp; min-var</th>
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<tbody>
<tr>
<td>mean &amp; min-var</td>
<td>0.07</td>
<td>-0.03</td>
</tr>
<tr>
<td>1/$N$ &amp; min-var</td>
<td>0.12</td>
<td>0.16</td>
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<tbody>
<tr>
<td>mean &amp; min-var</td>
<td>0.06</td>
<td>0.25</td>
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<td>1/$N$ &amp; min-var</td>
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<td>0.25</td>
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<tr>
<td>mean &amp; min-var</td>
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<td>1/$N$ &amp; min-var</td>
<td>0.26</td>
<td>-0.02</td>
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Why do shortselling constraints help?

- **Intuitively**, they prevent extreme (large) positive and negative weights in the portfolio.

- **Theoretically**, imposing shortselling constraints is like reducing the covariances assets that we would short; Jagannathan and Ma (2003).

\[ \Sigma = \hat{\Sigma} - \lambda e^\top - e\lambda^\top \]
### Results: Turnover relative to $1/N$

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<tbody>
<tr>
<td>$1/N$</td>
<td>3.05%</td>
<td>2.16%</td>
<td>2.93%</td>
<td>2.37%</td>
<td>1.62%</td>
<td>1.98%</td>
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**Mean variance and Bayesian portfolios**

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<thead>
<tr>
<th></th>
<th>mean variance</th>
<th>Bayes Stein</th>
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<td>mean variance</td>
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<td>22.41</td>
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<tr>
<td>Bayes Stein</td>
<td>607479</td>
<td>10092</td>
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**Moment restrictions**

<table>
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<tr>
<td>mean variance</td>
<td>6.54</td>
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<tr>
<td>Bayes Stein</td>
<td>21.65</td>
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**Shortsale constraints**

<table>
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<tr>
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<tr>
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<tr>
<td>Bayes Stein</td>
<td>7.17</td>
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<tr>
<td>minimum variance</td>
<td>2.47</td>
</tr>
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</table>
Empirical results: Summary

- **In-sample**, mean-var strategy has highest Sharpe ratio.

- **Out-of-sample**
  - $1/N$ does quite well in terms of Sharpe ratio and turnover.
  
  - Sample-based mean-var strategy has worst Sharpe ratio.
  
  - Bayesian policies typically do not out-perform $1/N$.
  
  - Constrained policies do better than unconstrained, but constraints alone do not help (need extra moment restrictions).

- Min-var-constrained does well, but:
  
  - Sharpe Ratios and CEQ statistically indistinguishable from $1/N$.
  
  - Better than $1/N$ in only one dataset (20-size/bm portfolios).

  - Turnover is 2-3 times higher than $1/N$. 
When simplicity is a real asset

By Tim Harford

Perhaps the whole ‘don’t put all your eggs in one basket’ school of portfolio allocation is financial wisdom enough

When James Tobin won the Nobel memorial prize in 1981, a journalist asked him to summarise his research in simple language. The great macroeconomist attempted to respond to this challenge, and one wire service dutifully reported that Professor Tobin had won the prize “for his work on the principle of not putting all your eggs in one basket”.

A newspaper cartoon then appeared announcing the award of a Nobel prize for “an apple a day keeps the doctor away”.
Beating 1/N
Beating $1/N$

\[
\begin{align*}
\min_{w} & \quad w^\top \hat{\Sigma} w - w^\top \hat{\mu} / \gamma \\
\text{s.t.} & \quad w \in C
\end{align*}
\]

**Improve portfolio performance using**

I. better covariance matrix,

II. better mean,

III. better constraints.
I. Better covariance matrix

To estimate covariance matrix better:

1. Use higher-frequency data
   ▶ more accurate estimates of covariance matrix; [Merton, 1980].
I. Better covariance matrix

To estimate covariance matrix better:

1. Use higher-frequency data

2. Use factor models

\[ r = a + Bf + \epsilon. \]

- more parsimonious estimates; [Chan et al., 1999].
I. Better covariance matrix

To estimate covariance matrix better:

1. Use higher-frequency data
2. Use factor models
3. Use shrinkage estimators
   ▶ “Honey, I have shrunk the sample covariance matrix”
   [Ledoit and Wolf, 2004b, Ledoit and Wolf, 2004a]

\[
\Sigma_{LW} = \alpha \hat{\Sigma} + (1 - \alpha)I.
\]
Shrinkage estimators

Distribution of deterministic covariance matrix estimator

Distribution of sample covariance matrix estimator

Distribution of shrunk covariance matrix estimator

Bias

Variance $\Sigma$

Variance $\Sigma$

$\Sigma_{LW}$

Variance $\Sigma$
I. Better covariance matrix

To obtain better estimates of covariance matrix:

1. Use higher-frequency data

2. Use factor models

3. Use shrinkage estimators

4. Use robust optimization
   ▶ [Goldfarb and Iyengar, 2003], [Tütüncü and Koenig, 2003], and others

\[
\begin{align*}
\min_w \quad & \max_{\Sigma \in \mathcal{U}} \quad w^T \Sigma w - w^T \hat{\mu} / \gamma \\
\text{s.t.} \quad & w \in C
\end{align*}
\]

▶ Worst-case CVaR
   [Zhu and Fukushima, 2009], [Gotoh et al., 2013].
I. Better covariance matrix

To obtain better estimates of covariance matrix:

1. Use higher-frequency data
2. Use factor models
3. Use shrinkage estimators
4. Use robust optimization
5. Use robust estimation
Portfolio Selection with Robust Estimation

[DeMiguel and Nogales, 2009]

- Square function amplifies the impact of outliers
Absolute value function reduces impact of outliers: [Konno and Yamazaki, 1991]
M-estimators use smooth error function
Portfolio Selection with Robust Estimation

- S-estimators: the impact of outliers is bounded
II. Better mean

To obtain better estimates of mean:

1. Ignore means
II. Better mean

To obtain better estimates of mean:

1. Ignore means

2. Use Bayesian estimates
II. Better mean

To obtain better estimates of mean:

1. Ignore means

2. Use Bayesian estimates

3. Use robust optimization

\[
\min_w \ w^T \Sigma w - \min_{\mu \in \mathcal{U}} \ w^T \mu / \gamma \\
\text{s.t.} \quad w \in C
\]
II. Better mean

To obtain better estimates of mean:

1. Ignore means
2. Use Bayesian estimates
3. Use robust optimization
4. Use option-implied information
Improving Portfolio Selection Using Option-Implied Volatility and Skewness

DeMiguel, Plyakha, Uppal, and Vilkov, JFQA (2013)

<table>
<thead>
<tr>
<th>Call Quote</th>
<th>XYZ</th>
<th>Put Quote</th>
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<tbody>
<tr>
<td>* 6.50-7.00</td>
<td>* APR25</td>
<td>* 0.15-0.25*</td>
</tr>
<tr>
<td>* 1.55-1.90</td>
<td>* APR30</td>
<td>* 0.15-0.25*</td>
</tr>
<tr>
<td>* 0.15-0.25</td>
<td>* APR35</td>
<td>* 3.10-3.50*</td>
</tr>
<tr>
<td>* 6.50-7.00</td>
<td>* MAY25</td>
<td>* 0.05-0.15*</td>
</tr>
<tr>
<td>* 4.10-4.50</td>
<td>* MAY27 $\frac{1}{2}$</td>
<td>* 0.15-0.25*</td>
</tr>
<tr>
<td>* 1.75-2.00</td>
<td>* MAY30</td>
<td>* 0.20-0.45*</td>
</tr>
<tr>
<td>* 0.45-0.70</td>
<td>* MAY32 $\frac{1}{2}$</td>
<td>* 1.15-1.40*</td>
</tr>
<tr>
<td>* 0.15-0.25</td>
<td>* MAY35</td>
<td>* 3.10-3.50*</td>
</tr>
<tr>
<td>* 6.90-7.40</td>
<td>* AUG25</td>
<td>* 0.15-0.25*</td>
</tr>
<tr>
<td>* 2.75-3.10</td>
<td>* AUG30</td>
<td>* 0.90-1.00*</td>
</tr>
<tr>
<td>* 0.50-0.75</td>
<td>* AUG35</td>
<td>* 3.40-3.80*</td>
</tr>
</tbody>
</table>

- **Implied volatilities** improve volatility by 10-20%.
- **Implied correlations** do not improve performance.
- **Implied skewness and volatility risk premium** proxy mean returns.
  - Improve Sharpe ratio even with moderate transactions costs for weekly and monthly rebalancing.
II. Better mean

To obtain better estimates of mean:

1. Ignore means
2. Use Bayesian estimates
3. Use robust optimization
4. Use option-implied information
5. Use stock return serial dependence
Stock Return Serial Dependence and Out Of Sample Performance

DeMiguel, Nogales, Uppal, RFS (2013)

- Stock return serial dependence
  - Contrarian: “buy losers and sell winners”.
  - Momentum: “buy winners and sell losers”.

- Can this be exploited systematically with many assets?
  - Yes, for transaction costs below 10 basis points.

tomorrow

(noun)

The best time to do everything you had planned for today.
II. Better mean

To obtain better estimates of mean:

1. Ignore historical means
2. Use Bayesian estimates
3. Use robust optimization
4. Use option-implied information
5. Use stock return serial dependence
6. Exploit anomalies
Exploit anomalies

- **size**: small firms outperform large firms,
- **momentum**: past winners outperform past losers,
III. Better constraints

To improve performance impose:

1. Shortsale constraints
III. Better constraints

To improve performance impose:

1. **Shortsale constraints**

2. **Parametric portfolios**
   [Brandt et al., 2009] impose constraint
   \[
   \begin{pmatrix}
   w_1 \\
   w_2 \\
   \vdots \\
   w_N
   \end{pmatrix}
   =
   \begin{pmatrix}
   w_1^M \\
   w_2^M \\
   \vdots \\
   w_N^M
   \end{pmatrix}
   +
   \begin{pmatrix}
   me_1 & btm_1 & mom_1 \\
   me_2 & btm_2 & mom_2 \\
   \vdots & \vdots & \vdots \\
   me_N & btm_N & mom_N
   \end{pmatrix}
   \begin{pmatrix}
   \theta_{me} \\
   \theta_{btm} \\
   \theta_{mom}
   \end{pmatrix},
   \]

   me = market equity,
   btm = book-to-market ratio,
   mom = momentum or average return over past 12 months.
III. Better constraints

To improve performance impose:

1. Shortsaling constraints

2. Parametric portfolios

3. Performance-based regularization
   ▶ Constrain variance of estimators of portfolio mean and CVaR.
   ▶ “Performance-based regularization in mean-CVaR portfolio optimization”, [Karouï et al., 2011].
III. Better constraints

To improve performance impose:

1. Shortsale constraints
2. Parametric portfolios
3. Performance-based regularization
4. Norm constraints
A Generalized Approach to Portfolio Optimization: Improving Performance by Constraining Portfolio Norms

DGNU (MS, 2009)

\[
\begin{align*}
\min_{w} & \quad w^\top \hat{\Sigma} w \\
\text{s.t.} & \quad \|w\| \leq \delta
\end{align*}
\]

- **Diversification**: 2-norm,
- **Leverage constraint**: 1-norm,
- **Shortsale constraint**: tight 1-norm constraint,
- **Sparsity constraint**: quasi-norm ($l_p$ with $p < 1$); [Chen et al., 2013, Fengmin et al., 2014b],
- **Cardinality constraint**: 0-norm; [Brito and Vicente, 2012, Ruiz-Torrubiano and Suárez, 2009, Fengmin et al., 2014a].
Help Wanted
Help wanted

▶ Optimization can make a difference in portfolio selection.

▶ Research opportunities
Help wanted

- **Optimization** can make a difference in portfolio selection.

- **Research opportunities**
  - Integer variables
    - **VaR:**
      - [Gaivoronski and Pflug, 2005], [Benati and Rizzi, 2007], Vera and Zuluaga (2013),
    - fixed costs and cardinality constraints.
Help wanted

- **Optimization** can make a difference in portfolio selection.

- **Research opportunities**
  - Integer variables
  - Multistage optimization
    - return predictability,
    - transaction costs,
Help wanted

- **Optimization** can make a difference in portfolio selection.

- **Research opportunities**
  - Integer variables
  - Multistage optimization
  - Calibration, calibration, calibration:
Same dog, different collar?
Devil Is in One Detail: Calibration

Constraints
- Gotoh & Takeda
  CMS 2011

Robust optimization
- Goldfarb
  Iyengar
  MOR 2003
- Caramanis
  Mannor
  Xu (2011)

Bayesian portfolios
- Jaganathan
  Ma
  JoF, 2003
- DGNU
  MS 2009

Shrinkage estimators
- Jorion
  JFQA 1986
Help wanted

- **Optimization** can make a difference in portfolio selection.
- **Research opportunities**
  - Integer variables
  - Multistage optimization
  - Calibration, calibration, calibration
    - "Size Matters: Optimal Calibration of Shrinkage Estimators for Portfolio Selection", [Martin-Utrera, D., Nogales, 2013],
    - choose criterion carefully, get nonparametric.

Statistics/big data/real-time estimation and optimization:
- stock return prediction: [Welch and Goyal, 2008] show that 15 predictors from the literature fail out of sample, and [Rapach et al., 2010] show that combining forecasts helps.
- hedge fund replication: [Roncalli and Weisang, 2011] show that $l_1$ regularization helps when replicating hedge fund returns using 12 factors.
  - high-frequency trading.
Help wanted

- **Optimization** can make a difference in portfolio selection.

- **Research opportunities**
  - Integer variables
  - Multistage optimization
  - Calibration, calibration, calibration

  - Statistics/big data/real-time estimation and optimization:
    - **stock return prediction**: [Welch and Goyal, 2008] show that 15 predictors from the literature fail out of sample, and [Rapach et al., 2010] show that combining forecasts helps.
    - **hedge fund replication**: [Roncalli and Weisang, 2011] show that $l_1$ regularization helps when replicating hedge fund returns using 12 factors.
    - High-frequency trading.
High frequency trading
High frequency trading

**SPEED TRADING**

HIGH-FREQUENCY TRADING AS SHARE OF ALL STOCK TRADING

<table>
<thead>
<tr>
<th>Country</th>
<th>Share</th>
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<tr>
<td>US</td>
<td>65%</td>
</tr>
<tr>
<td>Europe</td>
<td>45%</td>
</tr>
<tr>
<td>Japan</td>
<td>40%</td>
</tr>
<tr>
<td>Australia</td>
<td>30%</td>
</tr>
<tr>
<td>Canada</td>
<td>24%</td>
</tr>
<tr>
<td>Brazil</td>
<td>14%</td>
</tr>
<tr>
<td>Asia without Japan</td>
<td>12%</td>
</tr>
</tbody>
</table>

SOURCE: NYT
On May 6, 2010, Wall Street plunged suddenly and losses gained speed as high speed trading attempted to prevent losses. But almost as quickly the market recovered much of the decline.
Thank you

Victor DeMiguel
http://www.london.edu/avmiguel/


Sparse portfolio selection via quasi-norm regularization.

The effect of errors in means, variances, and covariances on optimal portfolio choice.

A Generalized Approach to Portfolio Optimization: Improving Performance By Constraining Portfolio Norms.

Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy?

Parameter uncertainty in multiperiod portfolio optimization with transaction costs.
*Forthcoming in Journal of Financial and Quantitative Analysis*.

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