MEANS OF PAYMENT AND TIMING OF MERGERS AND ACQUISITIONS IN A DYNAMIC ECONOMY*

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Abstract

This paper studies the interaction between takeover activity, means of payment (cash versus stock), and premiums in a dynamic model. The timing of takeovers and the equilibrium bids in contests with multiple bidders are driven by three factors: synergies of the bidder with the target, cash constraints of the bidder, and cash constraints of its competitors. The model produces many testable hypotheses, both novel and consistent with existing studies. First, winning stock bids leave fewer gains to the bidder, so higher-synergy targets are approached when they are small and acquired in cash. The resulting sample average takeover premiums in stock deals can be lower than in cash deals. Second, cash constraints need not have a monotonic effect on incentives of bidders to approach the target. Finally, high deal activity can be caused by shocks not only to valuations but also to cash constraints.

Keywords: Auctions, financial constraints, cash constraints, mergers and acquisitions, real options, security design.
The decision to merge is clearly one of the most important choices that the firm’s management and board of directors face, with the potential to gain or lose millions and billions in profit. It is therefore important to understand how these multifaceted decisions are made, what factors affect them. Among the most important choices in mergers are timing and the medium of exchange: a bidder must decide when to approach the target with an offer and which form of payment to use. These choices are affected by various firm-specific and economy-wide changes. In particular, it is known that mergers often occur in waves that are correlated with periods of economic expansion and easy access to external financial markets, or financing constraints. In this paper, we provide a theoretical analysis of mergers based on two simple ideas: (i) a bidder can choose when to approach the target with an offer; (ii) her ability to pay cash is limited by a financing constraint. We show that a simple real options model of mergers has the potential to match much of the empirical evidence on the relation of merger activity to economic shocks, financing constraints, medium of exchange of the offers, and the distribution of gains among the contest participants. In addition, we provide a few novel predictions that relate to this interdependence.

More specifically, we consider a dynamic model in which there are three agents: a target and two potential bidders. The target is a growth firm: its assets and cash flows grow over time with some uncertainty. Both bidders are mature companies: their assets and cash flows do not grow without acquiring the target. The bidders have synergies with the target: an acquisition improves productivity of the target in a combined company by a bidder-specific multiple. At any time each bidder can approach the target with an offer. Once a bidder makes a bid, the auction between the first bidder and the competitor is initiated, and the bidder who submits the highest bid wins the contest. A bidder’s decision when to approach the target reflects the following trade-off. On one hand, approaching the target early leads to an earlier increase in its productivity; on the other hand, a deal involves a cost: the post-merger market share of the losing bidder diminishes. If the synergy parameter of a bidder is small, it is optimal to wait until the target grows in size so that the increase in its productivity outweighs the cost of acquisition.

The second building block of the model is information asymmetry between the target and the

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1 In 2007 alone, the value in dollars of world-wide deal volume exceeded $4.8 trillion.
3 An alternative interpretation of the framework is that the target’s assets and cash flows change relative to those of the bidders.
bidders. Similarly to the literature that studies takeovers as auctions but unlike the prior literature that considers takeovers in the real options framework, we assume that potential synergies from acquiring the target is the private information of the bidder. As shown in the literature on securities auctions, this feature makes bids in stock and in cash not equivalent, in contrast to the case when bidders do not have any private information. Specifically, because the value of a bid in stock (but not in cash) depends on the bidder’s private information, in a stock auction, it is more costly for a bidder to separate itself from a marginally lower type. Even if both bidders offer the same proportion of the combined company, the bidder with the higher valuation will end up paying more. Because of this effect, which is a version of the linkage effect (Milgrom and Weber, 1982), each bidder wants to bid in cash whenever possible. The ability to do this is, however, limited by imperfect access of the bidder to the financial market: she cannot pay in cash above a certain limit.

We initially solve for the equilibrium initiation strategies and terms of takeovers when means of payments are exogenous: both bidders submit bids in cash; in stock; or one bidder submits bids in cash and the other in stock. These cases correspond to special cases of the general model, in which financing constraints of each bidder are either infinite or zero. Then, we provide solution of the general model with arbitrary cash constraints. The model delivers a number of implications relating timing of mergers, synergies, and means of payments.

We show that high-synergy targets are typically acquired young and with cash, while low-synergy targets are typically acquired old (if at all, despite positive synergies) and with stock. Intuitively, if the target is high-synergy, it does not pay off to wait and the target is acquired when small. As a result, for an acquirer, the required payment is likely to be below the financing constraint, leading to deals done in cash. Because of high synergies, such deals are also likely to result in high takeover premiums (relative to the current value of the target under its current management). Thus, the model predicts that in a sample of deals, cash deals can be associated with higher takeover premiums, despite stock deals being perceived as more expensive by bidders. This finding is broadly consistent with empirical evidence (e.g., Betton, Eckbo, and Thorburn, 2007). While this evidence may seem inconsistent with predictions of security-bid auctions literature, it becomes consistent once dynamic self-selection of targets into cash and stock deals is taken into account.

The model delivers interesting comparative statics as to what deals are likely to be done in cash versus in stock and when. For example, all else equal, the option to delay approaching the target is higher if the value of the target’s assets is more volatile. Thus, such targets are acquired later, when the financial constraint of the acquirer is less likely to be satisfied, and hence, are more likely to occur
in stock. All else equal, stock deals for these targets are also, on average, better than stock deals for lower-risk targets: they have higher average synergies and higher average takeover premiums.

Surprisingly, the financing constraint of a bidder need not have a monotonic effect on her initiation strategy. There are two effects in play. On one hand, if the financing constraint of a bidder is only slightly relaxed, she is able to finance best deals with cash if they are sufficiently small, which leads to her higher surplus and consequently to an earlier optimal initiation. On the other hand, if the target already does most deals in cash, the bidder is less under pressure to “fit into” cash: further relaxing the constraint gives her the ability to postpone the deal without incurring the cost of a stock deal.

If the two bidders have asymmetric financing constraints, their initiation strategies are asymmetric as well. Thus, conditional on the contest being initiated, participating bidders with different constraints can have asymmetric distributions of synergies even if those are symmetric ex ante. In equilibrium, the initiating bidder does not always win. Consistent with empirical evidence (Betton, Eckbo, and Thorburn (2007)), for some parameters of the model contests initiated in cash are less often rejected in favor of a competing bid due to the link between synergies and the resulting means of payment: contests are usually completed in cash by the strongest acquirers against the weakest competitors.

Taken together, our results suggest that the decision to initiate a takeover contest is driven by three fundamental factors: the technological shocks that affect the state process of the synergies from the acquisition, the cash constraints of the bidder, and the bidder’s perception of the cash constraints of other potential bidders. While the importance of technology shocks has often been acknowledged in the prior literature (e.g., Mitchell and Mulherin, 1996, Jovanovic and Rousseau, 2002, Lambrecht, 2004), the effect of cash constraints is more subtle. Indeed, a naive argument suggests that in a dynamic setting, the presence of cash constraints should either accelerate or have no effect on the decision to acquire. The logic is as follows. First, if a bidder is cash constrained, her only chance to acquire the target is when it is relatively small, which could be a suboptimal decision for an unconstrained bidder. Second, if a constrained bidder can use stock, it can simply offer the target the proportion of the stock of the combined firm that is equal in value to its potential cash bid. We show that the naive argument is not valid when the bidders have privately known synergies. In this case, cash constraints can significantly delay mergers, consistent with the findings of Harford (2005).

Our paper is related to two strands of research. First, it is related to the literature that studies the role of means of payment in mergers and acquisitions and, more generally, in auctions, in which bidders can make bids in securities.4 Many of our empirical implications on stock versus cash payments in

4Some real estate sales and long-term lease offerings attract competitive bids and result in a buyer paying an equivalent
mergers are also delivered in Fishman (1989) via a different mechanism. Fishman (1989) provides a static model of means of payments that features a two-sided information asymmetry between bidders and the target.\(^5\) The advantage of a stock bid is that it reduces the adverse selection problem, inducing a more efficient accept/reject decision of the target. A cash bid is, however, used when a bidder has a high enough valuation to preempt competition by signaling a high valuation. In contrast to Fishman (1989), our paper shows that a one-sided information asymmetry where only bidders have private information is sufficient to capture empirical evidence on means of payments, once dynamic aspects are taken into account. It also explains why stock bids are often perceived as more expensive by bidders but look smaller in the data. Clearly, both stories manifest in the data. The way to test the relative importance of the two explanations for the observed means of payments would be to account for the timing of acquisitions, such as size of the target and its age, and financing constraints of bidders. More generally, our paper is related to the literature on security-bid auctions (Hansen, 1985; Rhodes-Kropf and Viswanathan, 2000, and DeMarzo, Kremer, and Skrzypacz, 2005).

Second, our paper is related to the literature that studies mergers as real options. Lambrecht (2004) provides a setting in which mergers are driven by economies of scale and shows that the merger takes place once the price of the industry output raises to a sufficiently high threshold, thereby providing a rationale for the procyclicality of mergers. Morelec and Zhdanov (2005) build on the framework of Lambrecht to study mergers in a setting with incomplete information between the market and the merging firms. Hackbarth and Morelec (2008) apply a real options framework to analyze the dynamics of stock returns and risk in mergers and acquisitions. Versions of a real options framework have also been applied to study the effects of bidders’ financial structure (Morelec and Zhdanov (2008)) and product market competition (Hackbarth and Miao (2011)). Our paper differs from the extant literature in one important aspect: while this literature assumes that all bidders and the target have perfect information about the gains from acquisition, we follow the traditional literature on auctions in assuming that bidders have private information about their synergies with the target. This assumption has a crucial effect on the role of means of payment in takeovers. If all bidders and the target have perfect information about the gains from the synergies, takeover battles in stock and in cash are equivalent, as discussed in Morelec and Zhdanov (2008): stock bids are not subject to the linkage effect, and thus there is no difference in the values of stock bids and cash bids. Because we of cash and securities (e.g., a fixed payment stream plus a claim on earnings). The choice of timing is also important in these deals.

\(^5\)In addition to Fishman (1989), models of means of payments that are based on two-sided information asymmetry are provided by Hansen (1987), Eckbo, Giammarino, and Heinkel (1990), and Berkovitch and Narayanan (1990).
allow for bidders’ private information about their synergies, battles in cash and in stock are no longer equivalent, which allows us to discriminate between takeovers in cash and in stock.

The remainder of the paper is organized in the following way. Section I outlines the setup of the model. Section II solves the model with exogenous means of payments, or equivalently, special cases of the general model in which cash constraints are either perfect or non-existent. Section III solves the general model. Section IV analyzes the properties of the equilibrium and the predictions of the model, and discusses testable hypotheses. Section V concludes. All proofs appear in Appendix A. Appendix B contains the details of the numerical solution of the general model.

I Model Setup

We consider a setting in which the risk-neutral target attracts two potential risk-neutral acquirers, or bidders. The roles of the target and the bidders are exogenous. The value of the target as a separate entity at time \( t \) is given by \( X_t \), where \( X_t \) evolves as a geometric Brownian motion:

\[
dX_t = \mu X_t dt + \sigma X_t dB_t, \quad X_0 = x.
\]

Here, \( \mu \) and \( \sigma > 0 \) are constant growth rate and volatility, and \( dB_t \) is the increment of a standard Brownian motion. The discount rate is equal to \( r > \mu \). Process \( (X_t)_{t \geq 0} \) is a reduced-form specification of the present value of the target’s assets.\(^6\) We interpret it as the current size of the target. It accounts for all exogenous shocks to their value, such as changes in the price of the final product and inputs, as well as for the endogenous response of the target firm to them. The value of each bidder as a separate entity is constant at \( \Pi_b \). If bidder \( i \) acquires the target at time \( t \), the value if the combined firm will be

\[
\Pi_b + v_i X_t,
\]

where \( v_i \in [\underline{v}, \bar{v}] \), \( \bar{v} > v > 1 \) is the multiple that characterizes an improvement in operations of the target due to a change in ownership from a stand-alone firm to bidder \( i \).\(^7\) We refer to \( v_i \) as the synergy parameter. Importantly, each bidder’s synergy with the target is the bidder’s private information that is known to her before the start of the acquisition process.\(^8\) The target and the bidders share

\(^6\)For example, one gets this value if the target firm produces cash flow \( (r - \mu) X_t \) per unit of time.

\(^7\)Allowing \( v \) below 1 does not add to the model intuition in any way. However, in the model, we show that even some bidders with \( v > 1 \) will choose to never initiate the acquisition process.

\(^8\)Introducing the additional private information that the bidder can learn at the beginning of the contest does not affect the results of the model qualitatively. It is only the ex-ante private information that defines bidders’ strategies to
common knowledge about the distribution of synergies unknown to them: they are distributed with p.d.f. \( f(v) > 0 \) on \([v, \bar{v}]\). We assume that the distribution of valuations satisfies the restriction that the expected payoff of a winning bidder with valuation \( v \) monotonically increases in \( v \) in all specifications.\(^9\)

To represent the potential change in the industry structure after the acquisition, we assume that the losing bidder is also affected by the acquisition: its value changes from \( \Pi_b \) to \( \Pi_o < \Pi_b \). The loss in the losing bidder’s value will be a source of delay in the acquisition contests in the model. Of course, other potential sources of delay such as direct costs of initiating the takeover contest are possible too. We denote the value loss of the losing bidder as \( \Delta \equiv P_o - P_b \).

The main goal of the paper is to study how initiation of takeovers interacts with the parameters of economic environment and the outcomes of the takeover contests. In practice, takeover contests are typically initiated when a potential bidder approaches the target’s board with a formal or informal offer. To reflect this practice, we assume that each bidder has a real option to approach the target with a take-it-or-leave-it offer at any time. If a bidder approaches the target at time \( t \), the takeover contest is initiated and both bidders compete for the target in an open ascending-bid (English) auction.

The term English auction refers to a wide variety of open ascending-bid auction formats, which can differ in their precise rules. The most convenient formalization of English auctions is the “button” auction formalized by Milgrom and Weber (1982). Initially, all bidders are active. An auctioneer sets the price at zero and gradually raises it. All bidders observe the current price and the number of still active bidders. A bidder confirms her participation continuously until the raising price forces her to withdraw from the auction. As soon as only one bidder remains, she is declared the winner and pays the current price.

Each bidder can submit bids in the form of cash or stock of the combined company, or any mixture of the two.\(^{10}\) Thus, in contrast to the static auction literature and natural to the dynamic setting of our problem, each bidder needs to decide not only how much to bid, but also when to make an offer for the target. The assumption that bidders but not the target determine the timing of a takeover can be justified by the absence of commitment to sell on the part of the target. If the target announces that it wants to sell itself before any bidder is ready to make a move, the bidders can always delay the acquisition by not participating in the target-initiated process. On the other hand, when the bidder with positive value is ready to make an offer then, consistent with the “Revlon Duty,” the target’s

\(^9\)For example, in the model of Section II.B this restriction is equivalent to a restriction that \( v - \mathbb{E}[w|w \leq v] \) is a strictly increasing function of \( v \). An example of distribution that satisfies these restrictions is uniform distribution.

\(^{10}\)We generalize the “button” auction to the cases of stock and mixed bids in Sections II.B and II.C.
board would be responsible to consider all offers and accept the highest bid offered provided that it exceeds the value of the target under the current management.

Because all private information about future synergies is on the side of the bidders, they have incentives to pay in cash whenever possible. In the prior literature on mergers and acquisitions, stock bids are rationalized using either adverse selection about the acquirer’s assets in place or private information of the acquirer about her own firm. In this paper, we abstract from these issues and instead focus on another reason why bidders make bids in stock of the combined company - cash constraints of the bidder. Specifically, other things equal, a bidder wants to make a bid in cash because it is cheaper to separate from lower types in cash than in stock. However, the ability to submit bids in cash is limited, because the bidder’s internal cash is finite and borrowing from the outside investors may be expensive for various reasons. For simplicity, we suppose that bidder \( i \) has only \( C_i \) units of cash, and the cash constraints are infinitely rigid after that: in other words, bidder \( i \) can bid up to \( C_i \) units of cash, but cannot bid above that at any cost. First, we consider a model with exogenous means of payments. Then, we endogenize them by introducing cash constraints into the model.

II Initiation with Exogenous Means of Payments

First, we solve the model with exogenous means of payments. In particular we consider three cases. In the first case, both bidders always compete in cash bids. In the second case, both bidders always compete in stock bids. Finally, in the third case, one bidder competes in cash bids and the other bidder competes in stock bids. While the means of payments are exogenous in this part of the paper, this model will provide much of the intuition behind the model with endogenous means of payments.

II.A Two Cash Bidders

Consider the case in which both bidders make offers in cash. Suppose that a takeover contest is initiated at time \( t \) and both bidders compete for the target in an English auction. A weakly dominant

\[ \text{This result was established first by Hansen (1985). See Rhodes-Kropf and Viswanathan (2000) and DeMarzo, Kremer, and Skrzypacz (2005) for generalizations of it.} \]

\[ \text{According to the first reason, bidders use stock bids whenever it is overpriced by the market. See Myers and Majluf (1984) for the general argument and} \]

\[ \text{Rhodes-Kropf and Viswanathan (2004) for a specific model. According to the second reason, stock bids induce targets to make more efficient acceptance decisions. See} \]

\[ \text{Hansen (1987) and Fishman (1989).} \]

\[ \text{Our qualitative results hold as long as cash limits of the bidders grow at a slower rate than the target.} \]
strategy for bidder \(i\) is to bid up to \(b_i\), where

\[ \Pi_b + v_i X_t - b_i = \Pi_o. \] (3)

Intuitively, each bidder is willing to bid up to a point at which her payoff from acquiring the target at this bid \((\Pi_b + v_i X_t - b_i)\) is exactly equal to her payoff from losing the contest to the rival \((\Pi_o)\). Therefore, the bidder with the highest valuation acquires the target and pays \(\min(v_1, v_2) X_t + \Delta\).

Conjecture that the bidder with valuation \(v\) finds it optimal to initiate the contest at threshold \(\bar{X}_c(v)\) provided that the other bidder has not initiated the contest yet, where \(\bar{X}_c(v)\) is decreasing in \(v\). If the bidder with valuation \(v\) wins the contest for target \(X_t\) against the bidder with valuation \(w\), the change in her value relative to the stand-alone level is

\[ (v - w) X_t - \Delta. \] (4)

If, on the other hand, the bidder loses, the corresponding difference is \(-\Delta\). If the bidder with valuation \(v\) decides to initiate the contest at threshold \(\bar{X}_c(z), z \leq v\), her expected value at time 0 is

\[
\left(\frac{X_0}{\bar{X}_c(z)}\right)^\beta \int_z^v ((v - w) \bar{X}_c(z) - \Delta) f(w) \, dw \\
+ \int_z^v \left(\frac{X_0}{\bar{X}_c(w)}\right)^\beta ((v - w) \bar{X}_c(w) - \Delta) f(w) \, dw \\
- \int_v^\beta \left(\frac{X_0}{\bar{X}_c(w)}\right)^\beta \Delta f(w) \, dw,
\] (5)

where \(\beta > 1\) is the positive root of the fundamental quadratic equation \(\frac{1}{2}\sigma^2 \beta (\beta - 1) + \mu \beta - r = 0\). The second multiplicand in each term in (5) is the change in the value of the bidder from the takeover contest relative to the stand-alone level, and the first multiplicand is the price of the contingent claim that pays $1 at the time when the takeover contest is initiated. The first term of (5) corresponds to the case when the bidder with valuation \(v\), pretending to be the bidder with valuation \(z \leq v\), initiates the contest and wins it. The second term corresponds to the case when the competitor with valuation \(w \in (z, v)\) initiates the contest but the bidder with valuation \(v\) still wins. The third term corresponds to the case when the competitor with valuation \(w > z\) initiates the contest and wins it. Maximizing
with respect to $z$ and applying the equilibrium condition that $z = v$, we get

$$
\bar{X}_c(v) = \frac{\beta}{\beta - 1} \frac{\Delta}{v - \mathbb{E}[w|w \leq v]}.
$$

This equation is intuitive. Because of the option to delay approaching the target, a bidder approaches the target only at a point when her expected surplus from initiating the contest exceed the costs by a high enough margin. The increase in the target’s efficiency that is captured by the acquirer in expectation is $(v - \mathbb{E}[w|w \leq v])X_t$, and the cost of approaching the target is $\Delta$. The term $\beta/(\beta - 1) > 1$ captures the degree with which the option to delay approaching the target is important.

By assumption, the bidder’s surplus from winning the contest less what she pays in expectation, $v - \mathbb{E}[w|w \leq v]$, is increasing in $v$. Hence, $\bar{X}_c(v)$ is indeed decreasing in the synergy parameter $v$. This monotonicity has two implications. First, it implies that targets with higher potential synergies are acquired earlier. Second, it implies that the bidder that approaches the target is the bidder with the higher valuation. In this model, it follows that in equilibrium, the bidder that approaches the target always wins the auction.\(^{15}\) In a more general setting, in which bidders can update their valuations after the contest initiation (e.g., during due diligence), this result would not hold, but the bidder that initiates the contest would always win with a higher probability that her competitor, provided that the degree of initial information is the same for both bidders.

Another interesting property of (6) is that bidders with valuations $v > \bar{v}$ find it optimal to initiate the takeover contest at some finite $\bar{X}_c(v)$. This is because, as (4) shows, there always exists high enough $X_t$ such that the winning bidder receives a positive surplus for any $w < v$. As long as $v > \bar{v}$, the competitors have lower valuations than the initiating bidder with probability one which results in a positive expected surplus for high enough $X_t$ and, a result, in a finite initiation threshold.

The equilibrium is summarized in the following proposition:

**Proposition 1.** The symmetric equilibrium in the joint entry-bidding game when both bidders always make bids in cash is as follows. A bidder with the valuation parameter $v$ approaches the target at threshold $\bar{X}_c(v)$, given by (6), provided that no bidder has approached the target before. In the auction, bidder $i$ progressively increases her bid until it reaches $b_i = v_iX_t + \Delta$ or until the other bidder drops out. The bidder with the higher synergy parameter initiates the contest and acquires the target.

\(^{14}\)One gets an equivalent condition by writing down the value function in the range $\bar{X}_c(z)$, $z \geq v$, and maximizing it with respect to $z$. See the derivation of Eq. (22), for example.

\(^{15}\)This result will not hold in a model, in which the bidders are asymmetric with respect to their means of payments or cash constraints.
In the special case of the uniform distribution of \( v \) over \([v, \bar{v}]\), \( \mathbb{E}[w|w < v] = (\bar{v} + v)/2 \). Therefore,

\[
\bar{X}_c(v) = \frac{\beta}{\beta - 1} \frac{2\Delta}{v - \bar{v}}.
\]

(7)

It is easy to see that \( \bar{X}_c(v) \) is indeed a decreasing function of \( v \).

II.B Two Stock Bidders

Now, consider the case in which both bidders make offers in stock. It is instructive to expand the definition of a “button” auction to the case of stock bids. An auctioneer sets the initial fraction of the combined company to zero and gradually raises it. Each bidder confirms her participation continuously until the fraction of the combined company to be paid is too high and forces her to withdraw from the auction. As soon as only one bidder remains, she is declared the winner and pays the current fraction of the combined company. It immediately follows that a weakly dominant strategy for bidder \( i \) competing for target \( X_t \) is to bid up to \( \alpha_i \), where

\[
(1 - \alpha_i) (\Pi_b + v_i X_t) = \Pi_o
\]

(8)

Intuitively, as in the case of cash offers, each bidder is willing to bid up to a level at which her remaining payoff \( ((1 - \alpha_i) (\Pi_b + v_i X_t)) \) is exactly equal to her payoff from losing the contest to the rival (\( \Pi_o \)). It is easy to show that the value of this offer to the target is equal to (3). Suppose that a bidder with the valuation parameter \( v \) wins against the bidder with the valuation parameter \( w \leq v \). Then, she pays a fraction

\[
1 - \frac{\Pi_o}{\Pi_b + w X_t}
\]

(9)

to the target. Note that the winner’s payment is a proportion of the combined entity under the new management and therefore depends on the winner’s synergies, while the share she pays is determined by the synergies of her competitor. Because the value of the stock payment increases in the bidder’s valuation, the value of the winner’s payment in this case is higher than in the case of an all-cash contest against the same competitor. More precisely, the change in the winner’s value relative to the stand-alone level is

\[
\frac{\Pi_o (v - w) X_t}{\Pi_b + w X_t} - \Delta.
\]

(10)
Note that $\Pi_0 / (\Pi_b + wX_t) < 1$. It immediately follows that a bidder always gets less surplus from competing in stock than in cash.

Conjecture that bidder with valuation $v$ finds it optimal to initiate the contest at threshold $X_s(v)$ provided that the other bidder has not initiated the contest yet, where $X_s(v)$ is decreasing in $v$. If the bidder with the synergy parameter $v$ decides to initiate the contest at threshold $X_s(z)$, $z \leq v$, her expected value at time 0 is

$$
\left( \frac{X_0}{X_s(z)} \right)^\beta \int_z^v \left( \frac{\Pi_o (v - w) X_s(z) - \Delta}{\Pi_b + wX_s(z)} \right) f(w) \, dw
+ \int_z^v \left( \frac{X_0}{X_s(w)} \right)^\beta \left( \frac{\Pi_o (v - w) X_s(w) - \Delta}{\Pi_b + wX_s(w)} \right) f(w) \, dw
- \int_v^\infty \left( \frac{X_0}{X_s(w)} \right)^\beta \Delta f(w) \, dw.
$$

The first term of (11) corresponds to the case when the bidder with valuation $v$, pretending to be the bidder with valuation $z \leq v$, initiates the contest and wins it. The second term corresponds to the case when the competitor with valuation $w \in (z, v)$ initiates the contest but the bidder with valuation $v$ still wins. The third term corresponds to the case when the competitor with valuation $w > z$ initiates the contest and wins it. Maximizing (11) with respect to $z$ and applying the equilibrium condition that $z = v$, we obtain

$$
E \left[ \frac{\Pi_o \left( \Pi_b + wX_s(v) \right)}{(\Pi_b + wX_s(v))^2} (v - w) \mid v \leq v \right] X_s(v) = \frac{\beta}{\beta - 1} \Delta.
$$

As with (6), the left-hand side is a strictly increasing function of $X$, which verifies the conjecture that the optimal approaching policy of each bidder is given by the upper trigger $X_s(v)$. In particular, monotonicity implies that if the trigger exists, it is unique. However, (12) does not have a solution for some $v \in [\underline{v}, \bar{v}]$. By monotonicity, the highest value of the left-hand side of (12) is

$$
\lim_{X \to \infty} E \left[ \frac{\Pi_o \left( \Pi_b + \frac{\beta}{\beta - 1} wX(v) \right)}{(\Pi_b + wX(v))^2} (v - w) \mid v \leq v \right] = \frac{\beta}{\beta - 1} \Pi_o E \left[ \frac{v - w}{w} \mid v \leq v \right].
$$

This value decreases in $v$ and reaches zero when $v = \underline{v}$. Thus, once $v$ decreases to a sufficiently low

\[\text{value decreases in } v \text{ and reaches zero when } v = \underline{v}.\]

\[\text{Thus, once } v \text{ decreases to a sufficiently low}\]
level $v^*$, given by
\begin{equation}
E \left[ \frac{v^* - w}{w} \big| w \leq v^* \right] = \frac{\Delta}{\Pi_o},
\end{equation}

no bidder finds it optimal to approach the target, even though it is socially optimal to do so when $X_t$ is high enough. The intuition can be seen from (10). As $X_t \to \infty$, the bidder with valuation $v$ only gets a limited revenue, $\Pi_o \frac{v - w}{w} - \Delta$, in a contest against the bidder with valuation $w$. The reason for the limited bidder revenue in the case of stock bids is that in the model, the relative size of the bidders and the target changes in time. As $X_t$ increases, for the same $v$, the bidder has to give away a larger portion of the combined company to the target. As a result, the expected revenue of the bidder with valuation $v$ is also limited from above as $X_t \to \infty$. For sufficiently low $v$, the bidder prefers to remain standalone; the threshold $v^*$ is given above.

The equilibrium is summarized in the following proposition:

**Proposition 2.** The symmetric equilibrium in the joint entry-bidding game when both bidders always make bids in stock is as follows. If the synergy parameter of a bidder is $v > v^*$, where $v^*$ is defined by (14), then she approaches the target at threshold $\bar{X}_s(v)$, given by (12), provided that no bidder has approached the target before. If $v \leq v^*$, then a bidder never approaches the target first. In the auction, bidder $i$ progressively increases her bid until it reaches $\alpha_i(v_i)$, given by (8) or until the other bidder drops out. If $\max(v_1, v_2) > v^*$, the bidder with the higher synergy parameter initiates the contest and acquires the target. If $\max(v_1, v_2) \leq v^*$, the takeover never occurs.

While there is no analytical solution for $\bar{X}_s(v)$, it is easy to study its properties. In particular, it is interesting to see how (12) relates to (6). For this purpose, it is convenient to decompose (12) into two parts:
\begin{equation}
E \left[ \frac{\Pi_o (v - w) \bar{X}}{\Pi_b + w \bar{X}} \big| w \leq v \right] + \frac{1}{\beta - 1} E \left[ \frac{\Pi_o (v - w) w \bar{X}^2}{(\Pi_b + w \bar{X})^2} \big| w \leq v \right] = \frac{\beta}{\beta - 1} \Delta.
\end{equation}

The left-hand side of (15) consists of two components. The first component is the surplus that the bidder gets in expectation. It is always below the left-hand side of (6), because separation is costlier is stock than in cash. If this were the only term on the left-hand side of (15), then each bidder would always find it optimal to approach the target later if she bids in stock. However, (15) contains an additional positive second term. It corresponds to the effect that the delay causes the surplus of the
bidder to increase at a slower pace when the bidder make bids in stock. Alternatively, one can think of this term as a part of the delay cost on the right-hand side of (15): when $X_t$ is higher, further delay is less costly to the bidder as further increase in $X_t$ has a negative effect of a smaller magnitude on the bidder revenue. The magnitude of this effect depends on the value of delay parameter $\beta / (\beta - 1)$. The following proposition shows that if $\beta / (\beta - 1)$ is not too high, then this additional effect is dominated by the first effect, so the bidder always approaches the target earlier if she bids in cash:

**Proposition 3.** Suppose that the measure of the option value of delay, $\beta / (\beta - 1)$, is not too high:

\[
\frac{\beta}{\beta - 1} < 2 \frac{\Pi_b}{\Pi_o}, \tag{16}
\]

Then, $\bar{X}_s(v) > \bar{X}_c(v)$ for any $v$.

For standard parameters, the multiplier of the delay option, $\beta / (\beta - 1)$, does not exceed 2. As a consequence, condition (16) holds, so other things equal the bidder approaches the target later if she bids in stock. However, if the multiplier of the delay option is very high, then the stock bidder can approach the target earlier than the cash bidder, despite getting a lower fraction of the total surplus in expectation. As an example, consider an extreme case in which the multiplier of the delay option is infinite, $\beta / (\beta - 1) \rightarrow \infty$ (or equivalently, $\beta \rightarrow 1$). Then, the optimal threshold of the cash bidder is $\bar{X}_c(v) \rightarrow \infty$ for all $v$. By contrast, (15) implies that the optimal threshold of the stock bidder $\bar{X}_s(v)$ solves

\[
\mathbb{E} \left[ \Pi_o \frac{(v - w) w \bar{X}_s(v)^2}{(\Pi_b + w \bar{X}_s(v))^2} | w \leq v \right] = \Delta. \tag{17}
\]

In particular, it is finite for all $v > v^*$. Thus, if $\beta / (\beta - 1)$ is very high, then stock bidders with high enough $v$ approach the target earlier than cash bidders. However, it is never the case that stock bidders approach the target earlier than cash bidders uniformly for all $v$: if $v$ is low enough, $\bar{X}_c(v)$ is always below $\bar{X}_s(v)$ even if condition (16) does not hold.

For simplicity, in the rest of the paper we assume that condition (16) holds. We refer to this case as the “normal” case. In Section 4 we discuss what implications may change if the parameters of the model are such that the approaching threshold of a stock bidder is lower than in a cash bidder for some $v$. 

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II.C  Cash vs. Stock Bidder

Finally, consider the remaining case with exogenous means of payments. Suppose that one bidder makes bids in cash and the other bidder makes bids in stock. While in this case means of payments are exogenous, the bidders choose their bids from different sets. Therefore it is important to formalize the “button” auction for bids in combinations of stock and cash, allowing for asymmetric cash constraints, of which the above is a special case. The challenge with such auction is that bids can be multi-dimensional, so it is not obvious what raising a single price means for different auction participants.

We propose the following modification of the “button” auction for bids in mixes of stock and cash:

1. The auctioneer sets the starting price to zero and gradually rises it. A price $p$ corresponds to either a payment of $p$ dollars in cash or a payment of any $b \in [0, p]$ dollars in cash and a fraction $\alpha (b, p)$ in the stock of the company, to be determined later.

2. A bidder confirms her participation continuously until the combination $(b, \alpha)$ forces her to withdraw from the auction.

3. As soon as only one bidder remains, she is declared the winner and pays any element of her choice from the set $(b, \alpha)$ corresponding to the current price $p$, i.e., any $b \in [0, p]$ in cash and a fraction $\alpha (b, p)$ in the stock of the combined company.

4. $\alpha (b, p)$ is such that a bidder who withdraws at price $p$ is indifferent between all elements of the set $(b, \alpha)$. It is shown below that

$$\alpha (b, p) = \frac{p - b}{\Pi_o + p}.$$  (18)

To obtain (18), note that the bidder with valuation $v$ withdraws at price $p$ if and only if

$$(1 - \alpha) (\Pi_b + vX_t) = b + \Pi_0 \quad \Rightarrow \quad vX_t = \frac{b + \Pi_o}{1 - \alpha} - \Pi_b.$$  (19)

The indifference condition among the elements of set $(b, \alpha)$ is

$$b + \alpha (\Pi_b + vX_t) = p.$$  (20)

Plugging in $vX_t$ from (19) and solving for $\alpha$ yields (18). In the current case, there are no restrictions on bidding in cash for the cash bidder. However, an extra restriction is imposed on the stock bidder: she cannot pay any positive amount in cash so $b = 0$, $\alpha (b, p) = \frac{p}{\Pi_o + p}$ for such bidder.
Without loss of generality, we call the cash bidder “bidder 1” and the stock bidder “bidder 2.” Suppose that an auction takes place at time \( t \). If the synergy parameter of bidder 1 is equal to \( v_1 \), bidder 1 is willing to offer up to \( b(v_1) = p(v_1) = v_1X_t + \Delta \). If the synergy parameter of bidder 2 is equal to \( v_2 \), bidder 2 is willing to offer up to \( \alpha(v_2) \), given by (8). The cash value of this bid is equal to \( p(v_2) = v_2X_t + \Delta \). If \( v_1 > v_2 \), then bidder 1 is the winning bidder, and she pays according to the maximum value that bidder 2 is willing to offer, \( v_1X_t + \Delta \). If \( v_2 > v_1 \), then bidder 2 is the winning bidder, and she has to pay a fraction of the combined company corresponding to the maximum cash bid that bidder 1 is willing to offer, \( \alpha(v_1) \equiv \alpha(b(v_1), p(v_1)). \) In other words, the winning bidder of each type (cash or stock) always makes the same payment as if she competes against the bidder of her own type.

Conjecture that bidder \( i \) with valuation \( v \) approaches the target at threshold \( \bar{X}_i(v) \), where \( \bar{X}_1(v) < \bar{X}_2(v) \) for each \( v \), and which is a decreasing function of \( v \) for both \( i = 1, 2 \). First, consider bidder 1 with valuation \( v \), approaching the target at threshold \( \bar{X} \) in the neighborhood of \( \bar{X}_1(v) \). Because \( \bar{X}_1(v) < \bar{X}_2(v) \), \( \bar{X}_1^{-1}(\bar{X}) < \bar{X}_2^{-1}(\bar{X}) \). If bidder 1 with valuation \( v \) decides to approach the target at threshold \( \bar{X} \) in the neighborhood of \( \bar{X}_1(v) \), imitating type \( z \) in the neighborhood of \( v \), her expected payoff at time 0 is equal to

\[
\left( \frac{X_0}{\bar{X}} \right)^\beta \int_v^\bar{X} ((v - w)\bar{X} - \Delta) f(w) \, dw \\
- \left( \frac{X_0}{\bar{X}} \right)^\beta \int_v^{\min(\bar{X}_2^{-1}(\bar{X}), v)} \Delta f(w) \, dw \\
- \int_{\min(\bar{X}_2^{-1}(\bar{X}), v)}^{\bar{v}} \left( \frac{X_0}{\bar{X}_2(w)} \right)^\beta \Delta f(w) \, dw.
\]  

(21)

Intuitively, if valuation of bidder 2 is below valuation of bidder 1, then bidder 1 initiates the contest and acquires the target. This case corresponds to the first term in (21). If valuation of bidder 2 is above valuation of bidder 1 but below \( \bar{X}_2^{-1}(\bar{X}) \), then the contest is initiated by bidder 1 but won by bidder 2. This case corresponds to the second term in (21). Finally, if valuation of bidder 2 is above \( \bar{X}_2^{-1}(\bar{X}) \), then bidder 2 initiates and wins the contest. This case corresponds to the last term in (21). Maximizing (21) with respect to \( \bar{X} \) and applying the equilibrium condition that the maximum is reached at \( \bar{X}_1(v) \), we obtain

\[
\bar{X}_1(v) = \frac{\beta}{\beta - 1} v - E[w|w \leq v] \Psi(v),
\]

(22)
where $\Psi(v) = \frac{f(\bar{X}_2^{-1}(\bar{X}_1(v)))}{f(v)}$. Note that (22) is the same as (6) with an additional multiplier $\Psi(v)$. Because $\bar{X}_1(v) < \bar{X}_2(v)$, this multiple is greater or equal to 1. Consequently, bidder 1 delays her decision to approach the target compared to the case in which she faces another cash bidder: $\bar{X}_1(v) \leq \bar{X}_2(v)$. Intuitively, because other things equal bidder 2 approaches the target later than bidder 1, at the time when bidder 1 approaches the target, for any $v$, she faces a stronger competitor than if she faced a cash bidder. Because of this, bidder 1 faces a lower probability of winning the auction, which decreases her expected surplus. Consequently, she delays her decision to approach the target more.

Second, consider bidder 2 with valuation $v$, approaching the target at threshold $\bar{X}$ in the neighborhood of $\bar{X}_2(v)$. Her expected payoff at time 0 is equal to

$$
\left(\frac{X_0}{\bar{X}}\right)^\beta \int_{\bar{X}}^{X_1^{-1}(X)} \left(\Pi_o \Pi_b + v\bar{X} - \Pi_b\right) f(w) \, dw \\
+ \int_{X_1^{-1}(\bar{X})}^{v} \left(\frac{X_0}{X_1(w)}\right)^\beta \left(\Pi_o \Pi_b + v\bar{X}_1(w) - \Pi_b\right) f(w) \, dw - \int_{v}^{\bar{X}_1} \left(\frac{X_0}{X_1(w)}\right)^\beta \Delta f(w) \, dw.
$$

Intuitively, if valuation of bidder 1 is below $\bar{X}_1^{-1}(\bar{X})$, bidder 2 initiates the contest and acquires the target. This case corresponds to the first term in (23). If valuation of bidder 1 is between $\bar{X}_1^{-1}(\bar{X})$ and $v$, then the contest is initiated by bidder 1 but won by bidder 2, as she is the stronger bidder. This case corresponds to the second term in (23). Finally, if valuation of bidder 1 is above $v$, then the contest is initiated and won by bidder 2. This case corresponds to the last term in (23). Maximizing (23) with respect to $\bar{X}$ and applying the equilibrium condition that the maximum is reached at $\bar{X}_2(v)$, we get

$$
\mathbb{E} \left[ \left(\frac{\Pi_o \left(\Pi_b + \frac{\beta}{\beta-1} w\bar{X}_2(v)\right)}{\Pi_b + w\bar{X}_2(v)^2}\right) (v - w) \, \right]_{w \leq \Omega(v)} = \frac{\beta}{\beta-1} \Delta,
$$

where $\Omega(v) = \bar{X}_1^{-1}(\bar{X}_2(v))$. Note that (24) is the same as (12) with the difference that the conditional expectation on the left-hand side is computed over $w \leq \Omega(v)$, not over $w \leq v$. Because $\bar{X}_1(v) < \bar{X}_2(v)$, $\Omega(v) < v$ and $w$ takes lower values compared to the case in which bidder 2 faces another stock bidder: $\bar{X}_2(v) \geq \bar{X}_s(v)$. Consequently, bidder 2 accelerates her decision to approach the target. Intuitively, because other things equal bidder 1 approaches the target earlier than bidder 2, at the time when
bidder 2 approaches the target, for any \( v \), she faces a weaker competitor than if she faced another stock bidder. Because of this, bidder 2 gets a higher expected surplus from the auction, which leads to a lower optimal delay of the decision to approach the target.

The equilibrium is summarized in the following proposition:

**Proposition 4.** The separating equilibrium in the joint entry-bidding game between the stock and the cash bidder takes the following form. The initiation strategy of bidder 1 (the cash bidder) with the synergy parameter \( v_1 \) is to approach the target at threshold \( \bar{X}_1(v_1) \), given by (22), provided that no bidder has approached the target before. The initiation strategy of bidder 2 (the stock bidder) with the synergy parameter \( v_2 > v^*_2 \) is to approach the target at threshold \( \bar{X}_2(v_2) \), given by (24), provided that no bidder has approached the target before. If \( v_2 \leq v^*_2 \), then bidder 2 never approaches the target first. The boundary type \( v^* \) is given by

\[
v^*_2 = \frac{\Pi_b}{\Pi_o} v > v.
\]  

In the auction, bidder 1 progressively increases her cash bid until it reaches \( b(v_1) = v_1 X_t + \Delta \) or until bidder 2 drops out; bidder 2 progressively increases her stock bid until it reaches fraction \( \alpha(v_2) \), given by (8) or until the other bidder drops out.

As in the case of two stock bidders, expecting low revenue from acquiring the target in stock, the stock bidder does not initiate the takeover contest for valuations equal to or below \( v^*_2 \). There is no analytical solution for the jointly determined \( \bar{X}_1(v) \) and \( \bar{X}_2(v) \) but two closed form equations can be obtained for \( \bar{X}_1^{-1}(X) \) and \( \bar{X}_2^{-1}(X) \) which make the numerical analysis of the strategies easy. Appendix B provides more detail.

**II.D Comparative Statics**

We investigate the effects of bidders’ valuations as well as target and bidder characteristics on initiation strategies. First, we formalize the conclusions about the relationship between the strategies in the three cases of exogenous means of payment:

**Proposition 5.** If the separating equilibrium \( \bar{X}_1(v) < \bar{X}_2(v) \) exists, the initiation strategies in the
three cases are ordered: for any $v$,

$$
\bar{X}_c(v) < \bar{X}_1(v) < \bar{X}_2(v) < \bar{X}_s(v).
$$

Figure 1: Initiation strategies of cash and stock bidders facing different types of competitors. The figure shows the optimal initiation strategies of bidders as a function of their valuations, $v$. The thin solid (thin dashed) line is the strategy of a cash (stock) bidder facing another cash (stock) bidder; the thick solid (thick dashed) line is the strategy of a cash (stock) bidder facing a stock (cash) bidder.

The proof of the proposition is straightforward and follows the discussion of Sections II.A – II.C. For the numerical example, we choose the baseline model parametrization: $r = 0.05, \mu = 0.01, \sigma = 0.25, \underline{\bar{v}} = 1.1, \bar{v} = 1.5, v \sim \text{Uniform}[\underline{\bar{v}}, \bar{\bar{v}}], \Pi_b = 100, \Pi_o = 95$. Specifically, the baseline case considers acquisition of a target whose assets grow at rate $\mu$ typically used in dynamic models of the firm, and has the average COMPUSTAT asset volatility $\sigma$. The losing bidder’s profits are 5% below the pre-acquisition levels. The average synergies are equal to 30% of the target’s core business. The interest rate is set at constant 5%.

Figure 1 shows the four thresholds for our baseline parametrization as a function of bidders’ valuations, $v$. A higher probability of losing the takeover contest makes a cash bidder that competes against a stock bidder more cautious compared to the case when she competes against another cash bidder. The opposite is also true: a lower probability of losing the takeover contest makes a stock bidder more aggressive when she competes against a cash bidder. Another interesting result is that competing against a cash bidder not only directly accelerates the initiation by a stock bidder but also makes stock bidders with lower valuations willing to initiate in the first place: $v^*_2 < v^*$. As a result,
both effects combine and initiation of takeover contests is sped up and happens more frequently when at least one of the bidders is able to bid cash.

Figure 2: Initiation strategies of cash and stock bidders as a function of model parameters. The figure shows the comparative statics of the four initiation strategies for the baseline model parametrization as a function of (i) the growth rate of a target’s assets, $\mu$, (ii) the volatility of a target’s assets, $\sigma$, (iii) the interest rate, $r$, (iv) the value loss of the losing bidder, $\Delta$, (v) the starting value of bidders, $P_b$ (keeping $P_o/P_b$ fixed), and (vi) the standard deviation of bidder synergies, $std(v)$. The comparative statics are calculated for the bidder with the average valuation, $v = 1.3$. The thin solid (thin dashed) line is the strategy of a cash (stock) bidder facing another cash (stock) bidder; the thick solid (thick dashed) line is the strategy of a cash (stock) bidder facing a stock (cash) bidder.

Next, we discuss the effect of the change in other model parameters on initiation strategies:

**Proposition 6.** Initiation strategies are affected by the change in the model parameters: for any $v$, $\tilde{X}_i(v)$, $i \in \{c, s, 1, 2\}$:

1. increases in $\mu$;
2. increases in $\sigma$;
3. decreases in $r$;
4. increases in $\Delta$ – the value loss of the losing bidder;
5. increases in $\Pi_b$ – the original bidder value, keeping $\Delta$ fixed;
6. decreases in std(v) – the standard deviation of the volatility of bidder synergies.

The results of Proposition 6 are intuitive. (i) When \( \mu \) is higher, the bidders are ready to wait longer before initiating the contest for a target with stochastic value, because the extra delay is compensated by the higher probability that the target will eventually be acquired. (ii) For the same reason, when the discount rate \( r \) is lower (so that late acquisition is not discounted heavily), the takeover contest is initiated later. (iii) Higher \( \sigma \) implies higher probability of very negative shocks to the target value, which in turn makes initiation of a takeover contest at the same threshold costlier; as a result, a bidder initiates later. (iv) When the value loss of the losing bidder, \( \Delta \), is high, the winning bidder has to pay more to separate himself from the losing bidder: the value of the winning bidder’s outside option (losing the contest) is a negative function of \( \Delta \). As a result, the bidders’ expected revenues decrease in \( \Delta \) and they initiate a contest later. (v) The initiation strategies of two cash bidders competing against each other are constant in \( \Pi_b \) keeping \( \Delta \) fixed. However, when \( \Pi_b \) is larger, the equilibrium stock bid is for a smaller portion of the combined company, which leads to earlier initiation, no matter the type of the competing bidder. Surprisingly, when the cash bidder competes against the stock bidder, higher \( \Pi_b \) makes the average valuation of the competing stock bidder lower as for any \( v \), its initiation strategy is sped up, thus increasing the expected revenue of the cash bidder. As a result, the cash bidder also initiates a contest earlier. This interesting and novel result suggests that the linkage effect reflects not only in the stock bidder’s revenues, but also in their initiation strategies. Moreover, the linkage effect spreads to initiation strategies of less cash constrained competitors, also making them initiate later. (vi) As the volatility of bidder synergies grows, the standard result in auction theory is that a bidder with synergy \( v \) becomes better separated from bidders with lower synergies, and therefore on average pays less in a successful contest. As a result, the bidder initiates the contest earlier.

Figure 2 shows the comparative statics of the four optimal acquisition thresholds for the baseline model parametrization as a function of the six model parameters considered in Proposition 6. The thresholds are calculated for the bidder with the average valuation, \( v = 1.3 \). The acquisition threshold seems to be particularly sensitive to the value loss of the losing bidder and the volatility of bidder synergies. In fact, when the loss of profit is large enough or the volatility of synergies is low enough, the stock bidder with the average valuation never initiates a contest: its valuation is below the threshold \( v^* (\nu_2^*) \), as given in Proposition 2 (3).

In results not reported in the paper, we extend the model for the case of more than two bidders, under the assumption that all bidders are either cash or stock bidders. We show that a higher number of bidders delays initiation: if a bidder faces more potential competitors, it expects to keep a smaller
share of total revenues and thus initiates the contest at a higher threshold.

III Initiation with Endogenous Means of Payments

In the previous section we solved for the optimal initiation thresholds assuming that the means of payments of each bidder is exogenous. In this section, we endogenize them by introducing cash constraints of bidders to the economy: specifically, bidder $i$ can only bid up to $C_i \geq 0$ in cash.

Following our definition of the “button” auction for bids in combinations of stock and cash, if bidder $i$ with valuation $v$ is still in the contest when the value of its bid in cash is $p$, she should be willing to offer $b \in [0, C]$ in cash and proportion $\alpha(b, p) = \frac{b-p}{\Pi + p}$ of the combined company to the target. Suppose that an auction takes place at time $t$. If the valuation bidder $i$, $v_i$, is such that

$$
    b(v_i) = v_i X_t + \Delta < C_i,
$$

then bidder $i$ is willing to offer up to $b(v_i)$ in cash. If $v_i$ is such that the opposite of (26) holds, then bidder $i$ is willing to offer up to $C_i$ in cash and, slightly abusing notation, $\alpha(v_i, C_i)$ in stock, where $\alpha(v, C)$ is given implicitly by

$$
    -C + (1 - \alpha(v, C)) (\Pi_b + vX_t) = \Pi_o
$$

$$
    \Rightarrow \quad \alpha(v, C) = 1 - \frac{\Pi_o + C}{\Pi_b + vX_t}.
$$

Suppose that bidders $i$ and $j$ have synergy parameters $v$ and $w$ such that $v > w$. Then, bidder $i$ wins the takeover contest and acquires the company. Because separating is costlier in stock than in cash, bidders submit bids in cash whenever possible and only switch to positive combinations in stock after the cash constraints bind. If $w$ is below a certain level given by

$$
    wX_t + \Delta = C_i,
$$

then bidder $i$ has enough cash to win the takeover contest by submitting a cash bid that is marginally higher than the maximum surplus that bidder $j$ is willing to offer to the target, given by $wX_t + \Delta$. Note that this surplus does not depend on the cash constraints of bidder $j$. In this case, the change
in the value of bidder \( i \) relative to the stand-alone level is

\[
(v - w) X_t - \Delta. \tag{30}
\]

In contrast, if \( w \) is above this level, then bidder \( i \) does not have enough cash to win the takeover contest by submitting a cash bid. As a consequence, bidder \( i \) wins the contest by paying a cash amount \( C_i \) and the stock amount that just exceeds \( \alpha (w, C_i) \). In this case, the change in the value of bidder 1 relative to the stand-alone level is

\[
(1 - \alpha (w, C_i)) (\Pi_b + vX_t) - C_1 - \Pi_b
= \frac{\Pi_o + C_i}{\Pi_b + wX_t} (v - w) X_t - \Delta.
\tag{31}
\]

Combining (30) with (31), the change in the value of bidder 1 relative to the stand-alone level is

\[
\min \left\{ \frac{\Pi_o + C_i}{\Pi_b + wX_t}, 1 \right\} (v - w) X_t - \Delta. \tag{32}
\]

Without loss of generality, suppose that bidder 1 is less cash constrained than bidder 2: \( C_1 \geq C_2 \). Conjecture that bidder \( i \) with valuation \( v \) approaches the target at threshold \( \bar{X}_i (v) \), where \( \bar{X}_1 (v) \leq \bar{X}_2 (v) \), and which is a decreasing function of \( v \) for both \( i = 1, 2 \). First, consider bidder 1 with valuation \( v \), who decides to approach the target at threshold \( \bar{X} \) in the neighborhood of \( \bar{X}_1 (v) \). Because \( \bar{X}_1 (v) \leq \bar{X}_2 (v) \), \( \bar{X} \leq \bar{X}_2^{-1} (\bar{X}) \). Bidder 1’s expected payoff at time 0 is then equal to

\[
\left( \frac{X_0}{\bar{X}} \right)^\beta \int_2^v \left( \min \left\{ \frac{\Pi_o + C_i}{\Pi_b + wX_t}, 1 \right\} (v - w) \bar{X} - \Delta \right) f (w) dw
- \left( \frac{X_0}{\bar{X}} \right)^\beta \int_v^\min (\bar{X}_2^{-1} (\bar{X}), v) \Delta f (w) dw
- \int_{\min (\bar{X}_2^{-1} (\bar{X}), v)}^v \left( \frac{X_0}{\bar{X}_2 (w)} \right)^\beta \Delta f (w) dw. \tag{33}
\]

Intuitively, if valuation of bidder 2 is below valuation of bidder 1, then bidder 1 initiates the contest and acquires the target. This case corresponds to the first term in (33). If valuation of bidder 2 is above valuation of bidder 1 but below \( \bar{X}_2^{-1} (\bar{X}) \), then the contest is initiated by bidder 1 but won by bidder 2. This case corresponds to the second term in (33). Finally, if valuation of bidder 2 is above \( \bar{X}_2^{-1} (\bar{X}) \), then bidder 2 initiates and wins the contest. This case corresponds to the last term in
Maximizing (33) with respect to $\bar{X}$ and applying the equilibrium condition that the maximum is reached at $\bar{X}_1(v)$, we obtain

$$
\mathbb{E} \left[ \min \left\{ \frac{\Pi_o + C_1}{\Pi_b + w\bar{X}_1(v)}, 1 \right\} (v - w) \right| \begin{array}{c} w \leq v \end{array} \bar{X}_1(v) 
+ \frac{1}{\beta - 1} \int_{\min \left( \frac{c_1 - \Delta}{\bar{X}_1(v)}, v \right)}^{v} \left( \Pi_o + C_1 \right) \frac{(v - w) w \bar{X}_1(v)^2 f(w)}{\left( \Pi_b + w\bar{X}_1(v) \right)^2} F(v) \, dw
$$

$$
= \frac{\beta}{\beta - 1} \Delta \Psi(v),
$$

where $\Psi(v) = \frac{F(X^{-1}_2(v))}{F(v)} > 1$. Note that this solution embeds solutions for two special cases, $C_1 = 0$ when the bidder always bids in stock and $C_1 = \infty$ when the bidder always bids in cash. In the former case, (34) becomes equivalent to (12). In the latter case, (34) becomes equivalent to (22).

Second, consider bidder 2 with valuation $v$, approaching the target at threshold $\bar{X}$ in the neighborhood of $\bar{X}_2(v)$. Her expected payoff at time 0 is equal to

$$
\left( \frac{X_0}{X} \right)^\beta \int_{\bar{X}}^{X^{-1}(X)} \left( \min \left\{ \frac{\Pi_o + C_2}{\Pi_b + w\bar{X}}, 1 \right\} (v - w) \bar{X} - \Delta \right) f(w) \, dw
+ \int_{X^{-1}(X)}^{v} \left( \frac{X_0}{X_1(w)} \right)^\beta \left( \min \left\{ \frac{\Pi_o + C_2}{\Pi_b + w\bar{X}}, 1 \right\} (v - w) \bar{X} - \Delta \right) f(w) \, dw
- \int_{v}^{X^{-1}(X)} \left( \frac{X_0}{X_1(w)} \right)^\beta \Delta f(w) \, dw.
$$

Intuitively, if valuation of bidder 1 is below $X^{-1}_1(\bar{X})$, bidder 2 initiates the contest and acquires the target. This case corresponds to the first term in (35). If valuation of bidder 1 is between $X^{-1}_1(\bar{X})$ and $v$, then the contest is initiated by bidder 1 but won by bidder 2, as she is the stronger bidder. This case corresponds to the second term in (35). Finally, if valuation of bidder 1 is above $v$, then the contest is initiated and won by bidder 2. This case corresponds to the last term in (35). Maximizing (35) with respect to $\bar{X}$ and applying the equilibrium condition that the maximum is reached at $\bar{X}_2(v)$, we obtain

$$
\mathbb{E} \left[ \min \left\{ \frac{\Pi_o + C_2}{\Pi_b + w\bar{X}_2(v)}, 1 \right\} (v - w) \right| \begin{array}{c} w \leq \Omega(v) \end{array} \bar{X}_2(v) 
+ \frac{1}{\beta - 1} \int_{\min \left( \frac{c_2 - \Delta}{\bar{X}_2(v)}, \Omega(v) \right)}^{\Omega(v)} \left( \Pi_o + C_2 \right) \frac{(v - w) w \bar{X}_2(v)^2 f(w)}{\left( \Pi_b + w\bar{X}_2(v) \right)^2} F(\Omega(v)) \, dw
$$

$$
= \frac{\beta}{\beta - 1} \Delta,
$$

(36)
where $\Omega(v) = \bar{X}^{-1}_1 (\bar{X}_2 (v)) < v$. The equilibrium is summarized in the following proposition:

**Proposition 7.** The separating equilibrium in the joint entry-bidding game between the two bidders with cash positions $C_1$ and $C_2$ takes the following form. The initiation strategy of bidder $i$ with the synergy parameter $v_i > v_i^*$ is to approach the target at threshold $\bar{X}_i (v_i)$, given by (34) if $C_i \geq C_{-i}$ or (36) if $C_i \leq C_{-i}$, provided that no bidder has approached the target before. If $v_i \leq v_i^*$, where $v_i^*$ is defined in Appendix A, then bidder $i$ never approaches the target first. In the auction, bidder $i$ progressively increases her bid in cash until it reaches $v_i X_t + \Pi_b - \Pi_o$ or $C_i$ or until the other bidder drops out. In the first case, bidder $i$ drops out. In the second case, bidder $i$ offers combination of cash $C_i$ and stock, progressively increasing the stock component until it reaches $\alpha (v_i, C_i)$ or until the other bidder drops out. If the other bidder drops out, bidder $i$ wins the auction and pays according to her latest bid.

Figure 3: Initiation strategies of cash and stock bidders facing different types of competitors. Panel A shows the optimal initiation strategies of bidders as a function of their valuations, $v$. The thin solid (thin dashed) line is the strategy of a cash (stock) bidder facing another cash (stock) bidder; the thick solid (thick dashed) line is the strategy of a bidder with internal cash $C_1 = 125$ ($C_1 = 0$) facing a bidder with internal cash $C_2 = 0$ ($C_2 = 125$). Panels B–D show the cash and non-cash part of bidder revenue (first lines of (34) and (36)), and relative value component for non-cash deals (second lines of (34) and (36)) for bidders with internal cash $C_1 = 125$ and $C_2 = 0$.

As long as $C_1 < \infty$ and $C_2 < \infty$, both bidders do not initiate the takeover contest for valuations
equal to or below, correspondingly, \( v_1^* \) and \( v_2^* \). Appendix B provides more detail on the numerical solution for \( X_1(v) \) and \( X_2(v) \).

Figure 4: Initiation strategies of cash and stock bidders as a function of model parameters. The figure shows the comparative statics of the four initiation strategies for the baseline model parametrization as a function of (i) the growth rate of a target’s assets, \( \mu \), (ii) the volatility of a target’s assets, \( \sigma \), (iii) the interest rate, \( r \), (iv) the value loss of the losing bidder, \( \Delta \), (v) the starting value of bidders, \( P_b \) (keeping \( P_o/P_b \) fixed), and (vi) the standard deviation of bidder synergies, \( std(v) \). The comparative statics are calculated for the bidder with the average valuation, \( v = 1.3 \). The thin solid (thin dashed) line is the strategy of a cash (stock) bidder facing another cash (stock) bidder; the thick solid (thick dashed) line is the strategy of a bidder with internal cash \( C_1 = 125 \) (\( C_1 = 0 \)) facing a bidder with internal cash \( C_2 = 0 \) (\( C_2 = 125 \)).

Figure 3, Panel A shows the four thresholds (cash vs. cash bidders, stock vs. stock bidders, and bidders with internal cash \( C_1 = 125 \) and \( C_2 = 0 \) competing against each other) for our baseline parametrization as a function of bidders’ valuations, \( v \). An interesting new effect compared to the case of exogenous means of payment is that for intermediate valuations, the bidders can choose to accelerate initiation of a contest even compared to the case of two cash bidders. This happens because they try to “fit into” their cash constraints. Consider Figure 3, Panels B and C which show bidder revenue from cash only and non-cash deals. As the valuation of bidder 1 decreases, she initiates contests for a larger target and eventually finds herself unable to complete all deals in cash (the dashed vertical line on the right-hand side). At this stage, bidder 1 trades off costs of inefficiently early initiation compared to the unconstrained case against its benefits (a smaller proportion of deals is non-cash, resulting in a higher bidder revenue). If the latter dominates, bidder 1 can initiate a contest for a smaller target.
compared to the case when she is unconstrained ($C_1 = 0$) or even to the case when both bidders are unconstrained. As the valuation of bidder 1 decreases even further (beyond the dashed vertical line on the left-hand side), all contests it wins require payment of at least $C_1$ to the target, so bidder 1’s initiation threshold increases faster, similarly to an all-stock bidder.

If bidder 2 follows the initiation strategy assuming that $C_1 \rightarrow \infty$, she becomes on average a stronger bidder with higher expected revenues because bidder 1 with $C_1 < \infty$ accelerates initiation. It is therefore optimal for bidder 2 to also accelerate initiation for intermediate valuations.

Figure 4 shows the comparative statics of the four optimal initiation strategies (cash vs. cash bidders, stock vs. stock bidders, and bidders with internal cash $C_1 = 125$ and $C_2 = 0$ competing against each other) for the baseline model parametrization as a function of the six model parameters. The thresholds are calculated for the bidder with the average valuation, $v = 1.3$. Incentives to “fit into” cash constraints are particularly strong when $\mu, \sigma$ or $P_0$ are large, or when $r$ is small: in all these cases, the combined company has a higher expected value. When means of payment are endogenous, the bidders are unwilling to share this upside with the target at the cost of earlier initialization.

Figure 5 shows the comparative statics of the optimal initiation strategies for the baseline model parametrization and bidders with cash constraints $C_1$ and $C_2 = 0$ as a function of $C_1$. The thresholds are calculated for the bidder with the average valuation, $v = 1.3$. For intermediate ranges of $C_1$, bidder 1 has incentives to “fit into” cash constraints and bidder 2 follows her by decreasing her own initiation threshold. For low and high values of $C_1$, all deals either require all available cash to be done or are always done in cash only, resulting in initiation thresholds of both bidders between the thresholds of two cash and two stock bidders competing against each other.

IV Analysis

The results obtained in the previous two sections yield a rich set of predictions. Below, we list and discuss each of them.

IV.A Initiation of Takeover Contests, Means of Payment, and Premiums

A1. Companies acquired in stock are larger and older than companies acquired in cash.

Bidders with low synergies have higher benefits to wait and end up paying in stock, as compared to bidders with intermediate to high synergies who wait less, possibly trying to “fit into” their cash constraints, and pay in cash. This result is particularly important for empirical research
as it establishes a reverse link between means of payment in takeover contests and size/age of targets. Not only are large companies acquired in non-cash deals because the bidders do not have sufficient cash; such companies may be allowed to grow large exactly because potential bidders are cash constrained and are unwilling to initiate the contest inefficiently early.

Figure 6 shows, for the baseline parametrization and $C_1 = 125$, $C_2 = 0$, probabilities that cash and non-cash deals are completed in year 1, 2–5, 6–10, 11–25, and 26–100 as well as average acquisition size in deals completed by the end of year 1, 5, 10, 25, and 100. The starting value of the target is such that it is on the verge of being acquired by the highest bidder with the

$\footnote{Formally, for each given realization of the two bidder valuations, $v_1$ and $v_2$, the conditional probability that a contest is initiated over a finite time horizon $T$ is}$

$\Pr[\text{acquisition}|v_1, v_2, X_t, T] = \min \left\{ 1, N \left( \frac{-\log \min \{X_1(v_1), X_2(v_2)\}}{\sigma \sqrt{T}} + (\mu - \sigma^2/2)T \right) \right\}$

$+ \max \left\{ \frac{2(\mu - \sigma^2/2) \log \min \{X_1(v_1), X_2(v_2)\}}{\sigma^2} \right\} N \left( \frac{-\log \min \{X_1(v_1), X_2(v_2)\}}{\sigma \sqrt{T}} - (\mu - \sigma^2/2)T \right)$.

Then, the conditional probability that a contest is initiated over a finite time horizon $T$ for any $v_1$ and $v_2$ is

$\Pr[\text{acquisition}|X_t, T] = \mathbb{E}_{v_1, v_2} \left[ \mathbb{I}[v_1 > v_1^*, v_2 > v_2^*] \Pr[\text{acquisition}|v_1, v_2, X_t, T] \right]$, where $\mathbb{I}[:]$ is the indicator function equal to one if the condition in brackets is satisfied and zero otherwise.
lowest cash constraints: \( X_0 = \bar{X}_1(\bar{v}) \). Cash deals mostly happen within the first five years of the target’s life while stock deals reach their peak in years 2–5 and continue to be dominant types of acquisition in years 6–10. Cash deals are on average smaller and the gap in average size of cash and stock deals increases with the sample horizon as more and more stock deals are made for large targets by bidders with the lowest synergies.

While we do not directly model shocks to cash constraints, the above results make it evident that takeover activity can be spurred by two types of shocks: technology shocks that affect the potential synergies from a takeover and shocks to bidders’ cash constraints. The effect of technology shocks is clear: a contest is triggered once technology shocks shoot the cash flow variable \( X_t \) up to the upper acquisition threshold. The effect of shocks to cash constraints is more subtle. According to a naive argument, cash constraints should have no effect, because even if a bidder is cash constrained, it can always pay the target the proportion of the stock of the combined firm. In the setting with bidders’ private information about synergies, this naive argument is not valid, because a cash constrained bidder initiates a contest at a higher threshold than an unconstrained bidder. As a result, the change in economic environment that relaxes the bidders’ cash constraints decreases the threshold on the level of cash flows at which each bidder initiates an acquisition and thereby sparks merger activity.

A2. *Companies acquired in cash generate higher total premiums (target+bidder) than companies acquired in stock.*

Because bidders with intermediate to high synergies acquire targets in cash, the total premium in cash deals (as a percentage of the target’s value) is higher.

A3. *For some parametrizations of the model, bidders pay higher takeover premiums to acquire companies in cash.*

Despite the fact that bidders give away a smaller portion of synergies in cash acquisitions, they are the bidders with higher synergies. They give away a smaller portion of a larger pie. As a result, there exist parametrizations for which the effect of a pie increase dominates the effect of a smaller pie share and cash bidders on average pay higher takeover premiums (as a percentage of the target’s value).\(^\text{18}\)

Figure 7 shows the average takeover premiums in cash and non-cash deals, both conditional on

\(^{18}\)Note that this result in our paper is obtained despite the fact that we do not assume asymmetric tax treatment of cash and stock offers.
observing the highest bidder valuation and sample-wide unconditional where the sample consists of takeover contests that differ only in valuations of participating bidders. As expected, the conditional takeover premiums are higher in non-cash deals for any value of highest valuation. However, in the case when both bidders have non-zero internal cash (Panels B and D), best deals are done exclusively in stock while worst deals are done exclusively in combinations on cash and stock which leads to an inverse relationship between the sample-wide unconditional average takeover premiums. Note that this result is obtained without assuming either adverse selection about the bidders’ assets or private information of the acquirer about her own firm as in the previous literature. It is the endogenously determined positive correlation between cash deals and high synergy deals that is responsible for the result.

An empirical implication of A3 is that, if a good proxy of bidder’s synergies can be found, then conditional on this proxy, takeover premiums in cash deals should be lower than those in non-cash deals.

A4. Stock bidders receive lower acquirer premiums than cash bidders.

Not only do stock bidders give away a larger portion of synergies in acquisitions, but also they
have lower synergies, so the two effects complement each other.

A5. *The target premium and the target revenue is higher when the bidder has an access to outside funding.*

When the bidder with low synergies can use its equity to finance the deal, it will initiate the acquisition later, pay a larger proportion of its synergies to the target, which also means paying more in absolute terms.

![Figure 7: Conditional and unconditional takeover premiums in cash and non-cash deals.](image)

The figure corresponds to prediction A3. Panels A and C show, for the two cases: (i) $C_1 = 125, C_2 = 0$, (ii) $C_1 = 125, C_2 = 125$, the probability that a takeover contest is completed in cash as a function of the highest bidder valuation. Panels B and D show, for the same two cases, the average takeover premiums in cash and non-cash deals, both conditional on observing the highest bidder valuation (thick solid and dashed lines) and sample-wide unconditional (extra thick solid and dashed lines).

These properties of our model can already reconcile a wealth of empirical findings with the theory. The first property is consistent with the result of Betton, Eckbo, and Thorburn (2008) that takeovers paid in cash are for smaller firms than those partially or fully paid in stock. The third property is consistent with a number of studies (e.g., Franks, Harris, and Mayer (1988), Eckbo and Langohr (1989)) that historically, offer premiums were greater in all-cash offers, even controlling for the differential tax impact. The fourth property is consistent with the findings by Eckbo, Giammarino, and Heinkel (1990) and Berkovitch and Narayanan (1990) who show in cross-section that the larger the cash portion of the deal, the higher are acquirer abnormal announcement returns. The second property is the combination
of the two described above. Finally, the fifth property is consistent with the result of Betton, Eckbo, and Thorburn (2008) that both the initial and final offer premiums are higher when the bidder is a public company and thus likely has an access to equity markets.

IV.B Properties of Initiating and Winning Bidders

B1. *If the impact of an acquisition on the losing bidder is negative, bidders with sufficiently low synergies and cash constraints never initiate a contest.*

As a result, there is a non-zero probability that neither cash constrained bidder initiates a contest and a valuable target continues as standalone. Figure 8, region (1) shows that for the baseline model parametrization and cash constraints $C_1 = 125, C_2 = 0$, the probability that a valuable target is never acquired is approximately 4%. In the case of two stock bidders competing against each other, this probability is almost 9%.

B2. *In initiated contests, the distribution of participating bidders’ valuations is determined endogenously and can be asymmetric.*

This result holds true even if the unconditional distribution of synergies is the same for the bidders. Figure 8, left-most dashed line shows valuations of bidders 1 and 2, $v$ and $w$, at which they initiate contest at the same threshold, $\bar{X}_1(v) = \bar{X}_2(w)$. In contests initiated by a bidder with any valuation, the highest possible valuation of the more constrained bidder is higher than that of the less constrained bidder; the less constrained bidder also faces a stronger competitor on average. Interestingly, in the sample of takeovers that differ only in valuations of participating bidders, this result is reversed: because more constrained bidders are less likely to initiate takeover contests in the first place, their average valuation across all initiated contests is lower than that of less constrained bidders. Figure 9 shows how average valuations of the bidders with cash constraints $C_1 = 125, C_2 = 0$ change with respect to the parameters that have the strongest effect on the probability that a contest is never initiated: the value of the losing bidder, $P_0$ and the cash constraint of one of the bidders, specifically, $C_1$. Lower $P_0$ and $C_1$ correspond to larger gap between $v^*_1$ and $v^*_2$ and result in a larger difference between average valuations in the sample of similar takeovers.

B3. *Some initial bids of a less constrained bidder will be rejected in favor of a more constrained bidder. Under some parametrizations of the model, initial bids in cash have a smaller probability to be rejected compared to initial bids which include stock.*
The two predictions might seem contradictory at first. However, a less constrained bidder and a bidder who completes the deal in cash are not equivalent. The latter bidder is more likely to have both high cash balances and high valuation so that she approaches the target while the deal can still be sealed in cash. For the baseline parametrization and $C_1 = 125, C_2 = 0$, Figure 8, regions (2) and (4) show contests initiated by the less constrained bidder 1 in which the initial bidder bids in combinations of cash and stock. Region (4) shows contests in which such bidder loses to bidder 2 who bids in stock. Region (6) shows contests initiated by bidder 2 who wins in stock. The conditional probability of the initiating non-cash bidder losing the contest is the area of region (4) divided by the combined areas of regions (2), (4), and (6) and is equal to approximately 10%. In contrast, regions (3) and (5) show contests initiated by the less constrained bidder 1 in which the initial bidder bids in cash. Region (5) shows contests in which such bidder loses to bidder 2 who bids in stock. The conditional probability of the initiating cash bidder losing the contest is the area of region (5) divided by the combined area of regions (3) and (5) and is equal to approximately 2.6%. Hence, for a given parametrization, cash bids by the initiating bidder indeed have a smaller probability to be rejected compared to non-cash bids. It is easy to construct an example in which the opposite is true: take $C_1 \rightarrow \infty, C_2 = 0$. In this case, there is zero correlation between cash bids and cash bidder valuations and only cash bids by the initiating bidder can be rejected.

IV.C Target’s Preference for Cash versus Stock Bids

A famous result in the static security design literature (see, e.g., Hansen (1985) and DeMarzo, Kremer, and Skrzypacz (2005)) is that contests in stock dominate contests in cash in terms of seller revenue. As a result, if in a static setting the target can commit to accept only stock bids, she will do so. However, practical cases of such commitment in takeover contests are rare. An interesting question is to study whether the target would have incentives to commit to accept only stock bids in a dynamic setting, when bidders can time an acquisition. In this paper, we do not aim to provide a rigorous answer to this question. One of the complications that can arise is that the target, upon learning about bidder valuations from their initiation (and non-initiation) decisions, can change the preferred security design of the takeover contest dynamically. Instead, to provide a flavor of the more general case, we consider a simpler setting in which the seller has to commit to the security design at time zero. We also focus

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\(^{19}\)The results remain the same for any $X_0$ below the lowest initiation threshold of the bidders: target revenue takes the form $\alpha + \gamma_j X_0^3$ where $j \in \{c, s\}$ corresponds to the case of cash and stock bids. This also means that as long as
Figure 8: Initiation, acquisition and means of payment in takeover contests with cash constrained bidders. For the baseline parametrization and cash constraints of bidders 1 and 2 equal to $C_1 = 125$, $C_2 = 0$, the figure shows regions of valuations for which bidders initiate and win takeover contests, as well as the resulting type of the deal (cash, cash and stock, stock). The dash-dotted line separates the cases in which bidder 1 makes cash and non-cash final bids.

Figure 9: Average valuations of cash constrained bidders in initiated contests. The figure shows average valuations cash constrained bidders for the baseline parametrization as a function of (i) the value of the losing bidder, $P_0$, assuming cash constraints $C_1 = 125$, $C_2 = 0$, and (ii) cash constraint of bidder 1, $C_1$, assuming $C_2 = 0$ and $P_0 = 85$. The solid (dashed) line is the average valuation of bidder 1 (2).
on the case in which both bidders are exogenously unconstrained: $C_1 \rightarrow \infty, C_2 \rightarrow \infty$. In this setting, Proposition 8 characterizes the target’s preference for contests in stock versus cash as a function of the mean $\mu$ and the volatility $\sigma$ of target’s assets, as well as the interest rate $r$:

**Proposition 8.** The target prefers to conduct contests in stock if

1. $\mu$ is high;
2. $\sigma$ is high;
3. $r$ is low.

\[ \text{Figure 10: The ratio of the target revenue (present value) from contests in cash and in stock.} \]

For the baseline parametrization, the figure shows the ratio of present values of target revenues in cash and stock deals as a function of (i) the growth rate of the target’s assets, $\mu$, (ii) the volatility of the target’s assets, $\sigma$, (iii) the interest rate $r$.

Intuitively, if a target has a higher growth rate or higher volatility of assets (or interest rate is lower), the difference between initiation thresholds of cash and stock bidders is passed quicker (or affects the present value of target revenues less). As a result, the effect of extra delay is less important for the present value of high-growth targets, which leads to their preference for battles in stock. For the baseline parameters, Figure 10 shows the ratio of present values of target revenues in cash and stock contests as a function of $\mu$, $\sigma$, and $r$. For realistic parameters, the target prefers not to commit to restricting bids to stock. When $\mu$ and $\sigma$ are well above realistic parameters (or $r$ is very low), contests in stock start to dominate contests in cash in terms of target revenue.\(^{20}\)

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\(^{20}\)If the target can choose whether to restrict the type of bids at any point of time, learning about bidder valuations from $X_t$ strengthens her incentives to commit to restricting bids. This is because, as the support of possible bidder valuations shrinks, stock bids extract an increasingly higher proportion of revenues from the bidders.
This result suggests that in a dynamic setting, most targets (including targets with “standard” characteristics which are similar to an average COMPUSTAT firm) may have aligned incentives with the bidders: both the bidders and the target may prefer cash deals. This is in line with the observation that there are very few (if any) practical cases in which the target attempts to restrict the type of bids. However, a small fraction of firms with either high growth or high volatility of assets can have misaligned incentives with the bidders. If there is any evidence regarding the target’s ability to restrict the type of bids in takeover contests, it is likely to be found among high-μ, high-σ targets.\footnote{In contests for growth targets and targets from hi-tech industries, it is common that target managers are major stockholders in their company and have much control over its decision making, including the potential to negotiate terms of a potential takeover. In many cases, they eventually become large stockholders of the combined company which is consistent with our result.}

\section*{V Concluding Remarks}

The co-dependence of the timing of takeover contests, means of payment used by the winner to compensate target shareholders, and division of acquisition gains between the winner and the target has long been recognized in the literature as a potential cause of bias in empirical inference (Eckbo, Maksimovic, and Williams (1990), Betton, Eckbo, and Thorburn (2008)). This paper provides an initial step to solve this problem by studying the effect of privately observable bidder synergies with the target, as well as bidders’ cash constraints on their choice of timing, optimal bid, and means of payment in a takeover.

The results of our general model are consistent with a wide variety of cross-sectional and time-series empirical findings. In addition, we offer a number of novel testable predictions that relate takeover timing and outcomes to the three key characteristics of competing bidders: the technological shocks that affect the synergies from the acquisition, the cash constraints of the bidder, and the bidder’s perception of the cash constraints of other potential bidders. Importantly, the testable predictions recognize the effect of private information on decision making and hence can help deal with problems of self-selection in cross-sectional analysis.

One direction of future research could be studying the effect of less obvious contingent forms of payment on timing and outcomes of takeovers. In particular, these means of payment include stock swaps, clawbacks, earnouts, and collars that share option-like features. It is unclear whether such forms of payment generally arise due to bidders’ cash constraints, the ability of the target to force bidders to submit contingent bids, or due to separation of ownership and control in the bidding firm.
Because our framework is not restricted to bids in cash and stock, it seems like a natural starting point to analyze these more complex deal structures.

**Appendix A  Proofs**

**Proof of Proposition 1.** Maximizing (5) with respect to \( z \) yields

\[
-\beta \frac{X'(z)}{X(z)} \int_U^v (v - w) \bar{X}_c(z - \Delta) f(w) \, dw \thinspace + \thinspace \frac{1}{X(z)} \int_U^v (v - w) \bar{X}_c(z) f(w) \, dw \\
+ \frac{1}{X(z)} \left[ (v - w) \bar{X}_c(z - \Delta) f(z) - \frac{1}{X(z)} \left( (v - w) \bar{X}_c(z - \Delta) f(z) \right) \right] = 0
\]

(A1)

\[
\Leftrightarrow \bar{X}_c(z) (\beta - 1) \int_U^v (v - w) f(w) \, dw = \beta \Delta \int_U^v f(w) \, dw
\]

(A2)

\[
\Leftrightarrow \bar{X}_c(z) = \frac{\beta}{\beta - 1} \frac{\Delta}{v - \mathbb{E}[w|w \leq z]}
\]

(A3)

In the rational expectations equilibrium, it must be the case that \( z = v \). Hence,

\[
\bar{X}_c(v) = \frac{\beta}{\beta - 1} \frac{\Delta}{v - \mathbb{E}[w|w \leq v]}
\]

(A4)

By assumption, \( v - \mathbb{E}[w|w \leq v] \) is increasing in \( v \). Therefore, \( \bar{X}_c(v) \) is indeed decreasing in \( v \).

**Proof of Proposition 2.** Maximizing (11) with respect to \( z \) and applying the equilibrium condition that \( z = v \) yields

\[
0 = -\beta \int_U^v \left( \Pi_b + \frac{v \bar{X}_s}{\Pi_b + w \bar{X}_s} - \Pi_b \right) f(w) \, dw \\
+ \bar{X}_s \int_U^v \Pi_b \left[ \frac{\Pi_b + v \bar{X}_s}{\Pi_b + w \bar{X}_s} \right]' f(w) \, dw.
\]

(A5)

The derivative is equal to

\[
\begin{bmatrix} \Pi_b + v \bar{X}_s \\ \Pi_b + w \bar{X}_s \end{bmatrix}' = \frac{v (\Pi_b + w \bar{X}_s) - w (\Pi_b + v \bar{X}_s)}{(\Pi_b + w \bar{X}_s)^2}
\]

(A6)
Plugging it into (A5) and dividing by $F(v)$ yields

$$0 = -\beta \Pi_b \mathbb{E} \left[ \frac{\Pi_b + v \bar{X}_s}{\Pi_b + w \bar{X}_s} \right] + \beta \Pi_b + \Pi_o \Pi_b \mathbb{E} \left[ \frac{(v - w) \bar{X}_s}{(\Pi_b + w \bar{X}_s)^2} \right].$$

(A7)

Rewriting, we get (12).

**Proof of Proposition 3.** We need to compare

$$\mathbb{E}[v - w | w \leq v] \text{ and } \mathbb{E} \left[ \frac{\Pi_o \left( \Pi_b + \frac{\beta}{\beta - 1} w \bar{X} \right)}{(\Pi_b + w \bar{X})^2} (v - w) | w \leq v \right].$$

(A8)

Consider the following difference:

$$1 - \frac{\Pi_o \left( \Pi_b + \frac{\beta}{\beta - 1} w \bar{X} \right)}{(\Pi_b + w \bar{X})^2} = \frac{\Pi_b^2 + 2 \Pi_b w \bar{X} + w^2 \bar{X}^2 - \Pi_o \Pi_b - \frac{\beta}{\beta - 1} \Pi_o w \bar{X}}{(\Pi_b + w \bar{X})^2}$$

(A9)

The first term in the numerator is positive because $\Pi_b > \Pi_o$. The second term in the numerator is positive because of (16). Therefore, (A9) is positive for all $w$ and $\bar{X}$. Consequently,

$$\mathbb{E}[v - w | w \leq v] > \mathbb{E} \left[ \frac{\Pi_o \left( \Pi_b + \frac{\beta}{\beta - 1} w \bar{X} \right)}{(\Pi_b + w \bar{X})^2} (v - w) | w \leq v \right].$$

(A10)

Because of this and monotonicity of the left-hand side of (12) with respect to $\bar{X}$, the unique solution of (12), $v > v^*$ is higher than the unique solution of (6).

**Proof of Proposition 4.** First, we maximize (21) with respect to threshold $\bar{X}$. Analogously to the proof of proposition 1, we get (22). Second, we maximize (23) with respect to threshold $\bar{X}$:

$$0 = -\frac{\beta}{X^{\beta + 1}} \int_0^{X^{-1}(\bar{X})} \left( \Pi_o \left( \frac{v - w) \bar{X}}{\Pi_b + w \bar{X}} - \Delta \right) f(w) \right) \left( \frac{(v - w) \bar{X}}{\Pi_b + w \bar{X}} \right) f(w) \, dw \right)$$

(A11)
Equivalently,

\[ 0 = -\beta \int_{\bar{X}}^{\bar{X}^{-1}(\bar{X})} \frac{(v - w) \bar{X}}{\Pi_o + wX} f(w) \, dw + \beta \Delta \bar{F} \left( \bar{X}_1^{-1}(\bar{X}) \right) \]

\[ + \bar{X} \int_{\bar{X}}^{\bar{X}^{-1}(\bar{X})} \frac{(v - w) \Pi_b}{(\Pi_b + wX)^2} f(w) \, dw. \]  

(A12)

Dividing by \( \bar{F} \left( \bar{X}_1^{-1}(\bar{X}) \right) \):

\[ 0 = -\beta \Pi_o \mathbb{E} \left[ \frac{(v - w) \bar{X}}{\Pi_b + wX} \middle| w \leq \bar{X}_1^{-1}(\bar{X}) \right] + \beta \Delta \]

\[ + \Pi_o \mathbb{E} \left[ \frac{(v - w) \bar{X} \Pi_b}{(\Pi_b + wX)^2} \middle| w \leq \bar{X}_1^{-1}(\bar{X}) \right]. \]  

(A13)

Equivalently,

\[ \mathbb{E} \left[ \frac{\beta v - w}{\Pi_b + wX} - \Pi_b \frac{v - w}{(\Pi_b + wX)^2} \middle| w \leq \bar{X}_1^{-1}(\bar{X}) \right] \bar{X} = \beta \frac{\Delta}{\Pi_o}. \]  

(A14)

Rewriting yields (24). Finally, we need to determine the synergy parameter \( v^* \) such that bidder 2 never approaches the target if \( v \leq v^* \). Consider \( \bar{X} \to \infty \). Because \( \bar{X}_1(v) \) is finite as \( v > 1 \), \( \bar{X}_1^{-1}(\bar{X}) = \bar{X} \).

Therefore, the left-hand side of (A14) is

\[ \mathbb{E} \left[ \frac{\beta v - w}{w} \middle| w \leq v \right] = \beta \frac{v - v}{v}. \]  

(A15)

Point \( v^* \) is such that

\[ \beta \frac{v^* - v}{v} = \beta \frac{\Delta}{\Pi_o}, \]  

(A16)

which yields

\[ v^* = \frac{\Pi_b}{\Pi_o} v. \]  

(A17)

**Proof of Proposition 7.** Maximizing (33) with respect to the threshold:

\[ 0 = -\frac{\beta}{\bar{X}^{\beta+1}} \int_{\bar{X}}^{v} \left( \min \left\{ \frac{\Pi_o + C_1}{\Pi_b + wX}, 1 \right\} (v - w) \bar{X} - \Delta \right) f(w) \, dw \]

\[ - \frac{\beta}{\bar{X}^{\beta+1}} \int_{\min(\bar{X}_2^{-1}(\bar{X}), v)}^{v} \Delta f(w) \, dw \]  

\[ + \frac{1}{\bar{X}^{\beta+1}} \int_{\bar{X}}^{v} \left( \min \left\{ \frac{\Pi_o + C_1}{\Pi_b + wX}, 1 \right\} (v - w) \bar{X} \right) f(w) \, dw. \]  

(A18)
Equivalently,

\[ 0 = -\beta \int_{v}^{\bar{v}} \min \left\{ \frac{\Pi_o + C_1}{\Pi_b + w\bar{X}}, 1 \right\} (v - w) \bar{X} f(w) \, dw + \beta \Delta F \left( \min \left( \bar{X}^{-1}(\bar{X}), \bar{v} \right) \right) \]
\[ + \bar{X} \int_{v}^{\bar{v}} \left[ \min \left\{ \frac{\Pi_o + C_1}{\Pi_b + w\bar{X}}, 1 \right\} (v - w) \bar{X} \right] f(w) \, dw. \]  

(A20)

Dividing by \( F(v) \):

\[ 0 = -\beta \mathbb{E} \left[ \min \left\{ \frac{\Pi_o + C_1}{\Pi_b + w\bar{X}}, 1 \right\} (v - w) \bar{X} \big| w \leq v \right] + \beta \Delta \frac{F(\min (\bar{X}^{-1}(\bar{X}), \bar{v}))}{F(v)} \]
\[ + \mathbb{E} \left[ \left[ \min \left\{ \frac{\Pi_o + C_1}{\Pi_b + w\bar{X}}, 1 \right\} (v - w) \bar{X} \right] \bar{X} \big| w \leq v \right]. \]  

(A21)

Equivalently,

\[ \mathbb{E} \left[ \beta \min \left\{ \frac{\Pi_o + C_1}{\Pi_b + w\bar{X}}, 1 \right\} (v - w) \big| w \leq v \right] - \left[ \min \left\{ \frac{\Pi_o + C_1}{\Pi_b + w\bar{X}}, 1 \right\} (v - w) \bar{X} \right] \bar{X} \]
\[ = \beta \Delta \frac{F(\min (\bar{X}^{-1}(\bar{X}), \bar{v}))}{F(v)}. \]  

(A22)

Let us decompose this expression into two intervals:

- if \( w < \frac{C_1 - \Delta}{\bar{X}} \), then the expression under the expectation operator is

\[ \beta (v - w) - \left[ (v - w) \bar{X} \right] = (\beta - 1) (v - w); \]  

(A23)

- if \( w > \frac{C_1 - \Delta}{\bar{X}} \), then the expression under the expectation operator is

\[ \left( \Pi_o + C_1 \right) \left( \frac{\beta (v - w)}{\Pi_b + w\bar{X}} - \left[ \frac{(v - w) \bar{X}}{\Pi_b + w\bar{X}} \right] \right) \]
\[ = \left( \Pi_o + C_1 \right) \left( \frac{\beta v - w}{\Pi_b + w\bar{X}} - \Pi_b \frac{v - w}{(\Pi_b + w\bar{X})^2} \right) \]
\[ = \left( \Pi_o + C_1 \right) \frac{(v - w)}{(\Pi_b + w\bar{X})^2} \left( \frac{\beta (\Pi_b + w\bar{X}) - \Pi_b}{\Pi_b + w\bar{X}} \right) \]
\[ = (\beta - 1) \left( \Pi_o + C_1 \right) \frac{(v - w)}{\Pi_b + w\bar{X}} \frac{\beta}{\beta - 1} \frac{w\bar{X}}{\Pi_b + w\bar{X}} \]
\[ = (\beta - 1) \left( \Pi_o + C_1 \right) \frac{(v - w)}{\Pi_b + w\bar{X}} + (\beta - 1) \left( \Pi_o + C_1 \right) \frac{(v - w)}{\Pi_b + w\bar{X}} \frac{1}{\Pi_b + w\bar{X}}. \]  

(A24)
Hence, we can rewrite (A22) as

\[
E\left[\min \left\{ \frac{\Pi_o + C_1}{\Pi_b + w\bar{X}}, 1 \right\} (v - w) | v \leq \bar{v} \right] \bar{X} \\
+ \frac{1}{\beta - 1} \int_0^v \left( \Pi_o + C_1 \right) \frac{(v - w) w\bar{X}^2 f(w)}{(\Pi_b + w\bar{X})^2 F(v)} dw \\
= \frac{\beta}{\beta - 1} \Delta \frac{F\left( \min \left( \bar{X}_2^{-1}(\bar{X}), \bar{v} \right) \right)}{F(v)}.
\] (A25)

Next, we differentiate (35) with respect to \( \bar{X} \):

\[
0 = -\frac{\beta}{\bar{X}^{\alpha+1}} \int_{\frac{\bar{X}}{\beta}}^{X_1^{-1}(\bar{X})} \left( \min \left\{ \frac{\Pi_o + C_2}{\Pi_b + w\bar{X}}, 1 \right\} (v - w) \bar{X} - \Delta \right) f(w) dw \\
+ \frac{1}{\bar{X}^\alpha} \int_{\frac{\bar{X}}{\beta}}^{X_1^{-1}(\bar{X})} \left( \min \left\{ \frac{\Pi_o + C_2}{\Pi_b + w\bar{X}}, 1 \right\} (v - w) \bar{X} \right)' f(w) dw.
\] (A26)

Equivalently,

\[
0 = -\beta \int_{\frac{\bar{X}}{\beta}}^{X_1^{-1}(\bar{X})} \min \left\{ \frac{\Pi_o + C_2}{\Pi_b + w\bar{X}}, 1 \right\} (v - w) \bar{X} f(w) dw \\
+ \beta \Delta F\left( \max \left( X_1^{-1}(\bar{X}), \bar{X} \right) \right) \\
+ \bar{X} \int_{\frac{\bar{X}}{\beta}}^{X_1^{-1}(\bar{X})} \left( \min \left\{ \frac{\Pi_o + C_2}{\Pi_b + w\bar{X}}, 1 \right\} (v - w) \bar{X} \right)' f(w) dw.
\] (A27)

Dividing by \( F\left( X_1^{-1}(\bar{X}) \right) \):

\[
0 = -\beta E\left[ \min \left\{ \frac{\Pi_o + C_2}{\Pi_b + w\bar{X}}, 1 \right\} (v - w) | v \leq X_1^{-1}(\bar{X}) \right] \bar{X} + \beta \Delta \\
+ E\left( \min \left\{ \frac{\Pi_o + C_2}{\Pi_b + w\bar{X}}, 1 \right\} (v - w) \bar{X} \right)' | v \leq X_1^{-1}(\bar{X}) \right] \bar{X}.
\]

Equivalently,

\[
E\left[ \beta \min \left\{ \frac{\Pi_o + C_2}{\Pi_b + w\bar{X}}, 1 \right\} (v - w) - \min \left\{ \frac{\Pi_o + C_2}{\Pi_b + w\bar{X}}, 1 \right\} (v - w) \bar{X} \right]' | v \leq X_1^{-1}(\bar{X}) \right] \bar{X} \\
= \beta \Delta.
\] (A28)
Using the decomposition of the derivative from the above, this equation is equivalent to

\[
E \left[ \min \left\{ \frac{\Pi_o + C_2}{\Pi_b + wX}, 1 \right\} | (v - w) | w \leq \bar{X}_1^{-1}(X) \right] X 
+ \frac{1}{\beta - 1} \int_{\min\left(\frac{C_2}{X}, 1\right)}^{\bar{X}_1^{-1}(X)} \left( \frac{\Pi_o + C_2}{\Pi_b + wX} \right) \frac{(v - w)wX^2}{F(X)} \frac{f(w)}{f(X)} \, dw 
= \frac{\beta}{\beta - 1} \Delta.
\] (A29)

Similar to Section II.B, equations (A25), (A29) do not have solutions for low enough \( v \). Let \( v^*_i \) be such that \( \lim_{v \to v^*_i} \bar{X}_i(v) = \infty \). Then, writing equations (A25), (A29) at these limit points yields

\[
\int_{v}^{v^*_i} \frac{v^*_i - w}{w} dF(w) = \Delta \frac{F(v^*_i)}{\Pi_o + C_1},
\] (A30)

\[
\int_{v}^{v^*_i} \frac{v^*_2 - w}{w} dF(w) = \Delta \frac{F(v^*_1)}{\Pi_o + C_2}.
\] (A31)

In the case of symmetric cash constraints, \( C_1 = C_2 = C \), \( v^*_1 = v^*_2 = v^* \) is given by

\[
E \left[ \frac{v^* - w}{w} | w \leq v^* \right] = \frac{\Delta}{\Pi_o + C}.
\] (A32)

It is easy to see that in the special cases of \( C = 0 \) and \( C \to \infty \), we get \( v^* \) and \( \bar{v} \), correspondingly.

**Appendix B  Asymmetric Initiation: Numerical Procedure**

Consider the case of cash versus stock bidder, \( C_1 \to \infty, C_2 = 0 \). The cases of endogenous means of payment are numerically solved in the same fashion. We use substitution of variables to express the first order conditions for the two asymmetrically constrained bidders in terms of \( \bar{X}_1^{-1}(X), \bar{X}_2^{-1}(X) \) only. This approach is similar in spirit to the approach employed to solve for joint bidding strategies in first price auctions with heterogeneous bidders (e.g., Brendstrup and Paarsch (2007)). Specifically, let

\[
a \equiv \bar{X}_1(v) \Rightarrow v = \bar{X}_1^{-1}(a), \bar{X}_2^{-1}(\bar{X}_1(v)) = \bar{X}_2^{-1}(a);
\]

\[
b \equiv \bar{X}_2(v) \Rightarrow v = \bar{X}_2^{-1}(b), \bar{X}_1^{-1}(\bar{X}_2(v)) = \bar{X}_1^{-1}(b).
\] (B1)
Then, the system of equations (22), (24) becomes
\[ a = \frac{\beta}{\beta - 1} \frac{\Delta}{X_1^{-1}(a) - \int_{\Xi} X_1^{-1}(a) w \frac{f(w)}{F(X_1^{-1}(a))} dw F(X_1^{-1}(a))}, \]  
(B2)

\[ b \int_{\Xi} X_1^{-1}(b) \Pi_o \left( \frac{\Pi_b + \frac{\beta}{\beta - 1} wb}{(\Pi_b + wb)^2} \right) \frac{f(w)}{F(X_1^{-1}(b))} dw = \frac{\beta}{\beta - 1} \Delta. \]  
(B3)

We have two equations and four different combinations of functions and arguments as unknowns. We consider the interior case \( \bar{X}_1^{-1}(j) \in [v, \bar{v}] \) for \( i \in \{1, 2\}, j \in \{a, b\} \). Assume that both boundaries are equal, \( \bar{X}_1(v) = a = b = \bar{X}_2(w) \), for some \( v = \bar{X}_1^{-1}(a), w = \bar{X}_2^{-1}(a) \). This allows to simplify the system to two non-linear equations and two functions of one argument as unknowns which can be easily solved with a mathematical package.

Note that the above algorithm does not provide corner solution for \( v > \bar{v} = \bar{X}_1^{-1}(X_2(\bar{v})) \). Observe, however, that (B2) in this case can be rewritten as
\[ a = \frac{\beta}{\beta - 1} \frac{\Delta}{X_1^{-1}(a) - \int_{\Xi} X_1^{-1}(a) w \frac{f(w)}{F(X_1^{-1}(a))} dw F(X_1^{-1}(a))}, \]  
(B4)

and does not depend on \( \bar{X}_2^{-1}(a) \). As a result, a single non-linear equation with a single unknown is easily solved numerically. Combinations \( (\bar{X}_1^{-1}(a), a) \) and \( (\bar{X}_2^{-1}(a), a) \) constitute pairs of valuations and equilibrium initiation strategies for the two bidders.

As an example, when bidder valuations are uniformly distributed on \([v, \bar{v}]\), in the interior case
\[ a = \frac{\beta}{\beta - 1} \frac{\Delta}{(X_1^{-1}(a) - v) / 2 X_1^{-1}(a) - \bar{v}}, \]  
(B5)

\[ a \int_{\Xi} X_1^{-1}(a) \Pi_o \left( \frac{\Pi_b + \frac{\beta}{\beta - 1} wa}{(\Pi_b + wa)^2} \right) \frac{\bar{X}_2^{-1}(a) - w}{\bar{X}_1^{-1}(a) - w} dw = \frac{\beta}{\beta - 1} \Delta. \]  
(B6)

The integral in (B6) has a closed form representation.

References


