

5 Addendum: Extension with Continuum of Correlation

In this section, we extend the benchmark model by allowing banks to allocate any proportion of their loans to the common industry. This extension enables us to confirm that our herding results are not just an artifact of the assumption that each bank invests in only one industry. We keep the same return structure as in the n -bank case, but with only two banks now, that is $n = 2$. More specifically, we have the common industry and the two bank-specific industries with the same return structure as before.

Bank i chooses $x_i \in [0, 1]$, the amount of loans it lends to the common industry and the rest of the loans, that is $[1 - x_i]$, go to the bank-specific industry i . We consider the symmetric equilibrium and hence denote x_i 's simply as x .

For bank i , the four possible outcomes for the return from its investments are $(R_c, R_i) \in \{(R, R), (R, 0), (0, R), (0, 0)\}$ and the return for its loan portfolio is given as:

$$\text{Return} = \begin{cases} R & \text{if } (R, R) \\ xR & \text{if } (R, 0) \\ (1-x)R & \text{if } (0, R) \\ 0 & \text{if } (0, 0) \end{cases} . \quad (32)$$

Let x^* be the critical proportion of loans to the common industry at and above which the bank is able to pay the promised return $r_0(x)$ when the return from the common industry is high, that is, $x^*R = r_0(x^*)$. By symmetry, when $x \in [0, 1 - x^*]$, the bank can pay the promised return when the specific industry performs well. We assume a pro-rata sharing rule: when the bank cannot pay the promised return, depositors share whatever is available, in particular, returns from the first investment and proceeds from asset sales.

One important difference from the benchmark model is that whether x^* is greater than or less than $1/2$ affects the probability of individual survival of a bank. In particular, this may be different from α_t for an intermediate range of x values. As we show in the Appendix, there exists a threshold R^* , given as

$$R^* = \frac{2[1 + (1 - \alpha_0)^2 (\alpha_1 \Delta + 1)]}{\alpha_0(2 - \alpha_0) + 2\alpha_1(1 - \alpha_0)^2}, \quad (33)$$

such that, if $R \geq R^*$, then $x^* \leq 1/2$, and otherwise $x^* > 1/2$. We analyze the case where $x^* \leq 1/2$ in this section and in the Appendix.¹²

When $x^* \leq 1/2$, the region where it is sufficient for a bank's survival that only the idiosyncratic industry to perform well ($x \in [0, 1 - x^*]$) and the region where it is sufficiently

¹²For $x^* > 1/2$, we obtain similar results that are available upon request.

that only the common industry to perform well ($x \in [x^*, 1]$) overlap at $x \in [x^*, 1 - x^*]$. In this intermediate range, if only one of the industries the bank invests in performs well, the bank can pay depositors the promised return. This gives us the following probabilities $P(k)$ for there being k bank failures:

- $x \in [0, x^*)$: Bank can pay the promised return if and only if the return from the idiosyncratic industry is high. We have a Binomial distribution:

$$\Pr(k) = C(2, k) \alpha_0^{2-k} (1 - \alpha_0)^k \text{ for } k = 0, 1, 2. \quad (34)$$

Note that, in this region the probability that the bank pays back its depositors and continues operating one more period is equal to α_0 .

- $x \in [x^*, 1 - x^*]$: Bank can pay the promised return to depositors when either of the industries performs well. We have:

$$\Pr(k) = \begin{cases} \alpha_0 + (1 - \alpha_0)\alpha_0^2 & \text{for } k = 0 \\ 2\alpha_0(1 - \alpha_0)^2 & \text{for } k = 1 \\ (1 - \alpha_0)^3 & \text{for } k = 2 \end{cases}. \quad (35)$$

In this region, the individual probability of survival is equal to $[\alpha_0(2 - \alpha_0)]$.

- $x \in (1 - x^*, 1]$: Bank can pay the promised return to depositors if and only if the common industry performs well. We have:

$$\Pr(k) = \begin{cases} \alpha_0 & \text{for } k = 0 \\ 0 & \text{for } k = 1 \\ 1 - \alpha_0 & \text{for } k = 2 \end{cases}, \quad (36)$$

and the individual probability of survival is α_0 .¹³

As we have shown above, the states that define the success and failure of banks depend on the realization of returns in common and idiosyncratic industries, as well as banks' choice of x .

¹³When $x^* > 1/2$, we have $x^* > (1 - x^*)$. For the region where $x \in [0, 1 - x^*]$, bank can pay the promised return if and only if the idiosyncratic return is high and for the region where $x \in [x^*, 1]$, bank can pay the promised return if and only if the common industry performs well. The only difference is the intermediate region $x \in (1 - x^*, x^*)$, where the bank can pay the promised return if and only if *both* of these industries perform well. Thus, for this region we have:

$$\Pr(k) = \begin{cases} \alpha_0^3 & \text{for } k = 0 \\ 2\alpha_0^2(1 - \alpha_0) & \text{for } k = 1 \\ (1 - \alpha_0) + \alpha_0(1 - \alpha_0)^2 & \text{for } k = 2 \end{cases}. \quad (37)$$

Thus, there is a mapping from $(\tilde{R}_c, \tilde{R}_1, \tilde{R}_2, x)$ to the set of possible states $\{SS, SF, FS, FF\}$. For a full description of this mapping, see Tables 4 and 5 in the Appendix.

An important difference from the benchmark case is the potential change in regulator's actions in the individual survival states. In the benchmark model, in states SS , SF and FS , there is no need for a regulatory intervention. However, in this case, the surviving bank may not have enough funds to purchase the failed bank's assets in the individual survival states.¹⁴ If both industries a bank invests in have the high return, then as in the benchmark case, the bank's profits are sufficient for the asset purchase. However, it may be the case that only one of the industries has the high return and the bank has invested enough amount in the high-return industry to pay back its depositors but not large enough for the first-period profits to be sufficient for the asset purchase. In these cases, the failed bank's assets may have to be sold to outsiders and we may observe social welfare losses even in individual failure states. Thus, the regulator may have incentives for intervention even in states SF and FS .

In deciding whether to liquidate or bail out in states SF , FS and FF , the regulator first determines the funds needed for the bailout and ranks banks according to these funds. Differently from the benchmark case, if a bank fails, it may have some funds available to pay to the depositors. Since bailouts are costly, the funds the failed bank(s) have affect the regulator's bailout decision. Formally, let F_1 and F_2 be the funds needed to bail out the bank(s), respectively. Thus,

$$F_i = r_0(x) - c_i \text{ for } i = 1, 2, \quad (38)$$

where c_i represents the funds the failed bank has from the first investment. Note that c_i depends on $(\tilde{R}_c, \tilde{R}_1, \tilde{R}_2, x)$. Without loss of generality we assume that $F_1 \leq F_2$. The regulator first makes the bailout decision for the bank that needs a smaller amount of funds, that is, F_1 . Furthermore, to simplify our analysis, we assume that the regulator takes the same equity share of β in all bailed-out banks. This gives us a result analogous to Lemma 2:

Lemma 7 *In any subgame perfect equilibrium, in states SF and FS , if the surviving bank has enough funds to purchase the failed bank's assets, it does so and the regulator does not intervene. If the surviving bank does not have enough funds to purchase the failed bank's assets and if $\alpha_1 \Delta > f(F_1)$, then the regulator bails out the failed bank. Otherwise, the regulator liquidates the failed bank. In State FF , the regulator does the following:*

- (i) *If $\alpha_1 \Delta \leq f(F_1)$, then both banks are sold to outsiders.*
- (ii) *If $f(F_1) < \alpha_1 \Delta \leq f(F_2 + F_1) - f(F_1)$, then the regulator bails out the bank that requires smaller funds for bailout. If both banks need the same funds, $F_1 = F_2$, then the bank to*

¹⁴For simplicity, we rule out partial asset sale in this case.

be bailed out is chosen randomly with equal probability. The other bank's assets are sold to outsiders.

(iii) If $\alpha_1\Delta > f(F_2 + F_1) - f(F_1)$, then the regulator bails out both banks.

Next, we investigate banks' ex-ante choice of x , the amount to invest in the common industry, and the unique subgame perfect equilibrium of the game. Rather than describing the equilibrium fully, we provide cases where the banks' choice of x may be different from the socially optimal level of x . A full characterization of the equilibrium is provided in the Appendix. We have the following Proposition:

Proposition 8 Let $\hat{\beta}_1^* = 1 - \frac{\Delta}{(\alpha_1 R - 1)}$ and $\beta_1^* = 1 - \frac{\alpha_1 \Delta}{(\alpha_1 R - 1)}$, where $\hat{\beta}_1^* < \beta_1^*$. In any subgame perfect equilibrium, we have:

- (i) If $(1 - \bar{\theta}) < \hat{\beta}_1^*$, then banks choose an $x \in (1 - x^*, 1]$ that is arbitrarily close to $1 - x^*$.
- (ii) If $\hat{\beta}_1^* \leq (1 - \bar{\theta}) < \beta_1^*$ and if for some $x \in (1 - x^*, 1]$, we have that $f(r_0(x)) < \alpha_1 \Delta$, then banks choose an $x \in (1 - x^*, 1]$ that is arbitrarily close to $1 - x^*$.

Proof: See Appendix.

Next we make welfare comparisons to investigate whether the banks' choice of x generates the socially optimal outcome. First we state the following Proposition:

Proposition 9 For all parameter values, the expected output generated by the banking industry net of any bailout or liquidation costs is strictly greater for $x = x^*$ than for any $x \in (1 - x^*, 1]$. Thus, an $x \in (1 - x^*, 1]$ cannot be the socially optimal outcome.

Proof: See Appendix.

Propositions 8 and 9 show that for a robust set of parameter values, banks' investment choices will be different from the socially optimal ones. As in the benchmark case, to take advantage of the too-many-to-fail guarantees, banks choose to be over-exposed to the common risk factor: they choose to invest $(1 - x^*)$ in the common asset though the socially optimal investment in the common asset is $x^* < (1 - x^*)$. This results in inefficient levels of expected output from the banking sector as liquidation and bailout costs are incurred too often compared to the first-best outcomes.

Proofs for Section 5:

Derivation of x^* and R^* :

We know that x^* satisfies $x^*R = r_0(x^*)$ so we first derive the expression for $r_0(x)$. When $x^* \leq 1/2$, we have:

- In the region where $x \in [0, x^*)$, depositors get the promised return of $r_0(x)$ if and only if the idiosyncratic industry performs well. Otherwise, they share whatever is available, including the proceeds from asset sale. From the individual rationality of depositors, we have:

$$\alpha_0 r_0 + \alpha_0(1 - \alpha_0) xR + (1 - \alpha_0)[\alpha_1(R - \Delta) - 1] = 1.$$

- When $x \in [x^*, 1 - x^*]$, one industry performing well is sufficient for depositors to get their promised return. That gives us:

$$[1 - (1 - \alpha_0)^2] r_0 + (1 - \alpha_0)^2 [\alpha_1(R - \Delta) - 1] = 1.$$

- When $x \in (1 - x^*, 1]$, depositors get the promised return if and only if the common industry performs well. This gives us:

$$\alpha_0 r_0 + \alpha_0(1 - \alpha_0)(1 - x)R + (1 - \alpha_0) [\alpha_1(R - \Delta) - 1] = 1.$$

Combining these results, we get:

$$r_0(x) = \begin{cases} \frac{1}{\alpha_0} - (1 - \alpha_0)xR - \frac{1 - \alpha_0}{\alpha_0} [\alpha_1(R - \Delta) - 1] & \text{for } x \in [0, x^*) \\ \frac{1}{\alpha_0(2 - \alpha_0)} - \frac{(1 - \alpha_0)^2}{\alpha_0(2 - \alpha_0)} [\alpha_1(R - \Delta) - 1] & \text{for } x \in [x^*, 1 - x^*] \\ \frac{1}{\alpha_0} - (1 - \alpha_0)(1 - x)R - \frac{(1 - \alpha_0)}{\alpha_0} [\alpha_1(R - \Delta) - 1] & \text{for } x \in (1 - x^*, 1] \end{cases} .$$

At $x = x^*$, the return from the common industry is just enough to pay back depositors, that is, $x^*R = r_0(x^*)$. Plugging this into the above equations, we get:

$$x^* = \frac{1}{\alpha_0(2 - \alpha_0)R} - \frac{(1 - \alpha_0)^2}{\alpha_0(2 - \alpha_0)R} [\alpha_1(R - \Delta) - 1].$$

Thus, $x^* \leq 1/2$, iff $\alpha_0(2 - \alpha_0)R \geq 2(1 - (1 - \alpha_0)^2 [\alpha_1(R - \Delta) - 1])$. It can be written as:

$$[\alpha_0(2 - \alpha_0) + 2\alpha_1(1 - \alpha_0)^2] R \geq 2[1 + (1 - \alpha_0)^2 (\alpha_1\Delta + 1)], \text{ which gives us:}$$

$$R \geq R^* = \frac{2[1 + (1 - \alpha_0)^2 (\alpha_1\Delta + 1)]}{\alpha_0(2 - \alpha_0) + 2\alpha_1(1 - \alpha_0)^2}.$$

Derivation of Expected Profits:

Next, we derive the expected profit of banks for different values of x . We have the three regions as before:

- When $x \in [0, x^*)$, we have

$$\begin{aligned} E(\pi_1(x)) &= \alpha_0^2 [R - r_0(x)] + \alpha_0(1 - \alpha_0)[(1 - x)R - r_0(x)] \\ &= \alpha_0 R [1 - (1 - \alpha_0)x] - \alpha_0 r_0(x). \end{aligned}$$

- When $x \in [x^*, 1 - x^*]$, we have

$$\begin{aligned} E(\pi_1(x)) &= \alpha_0^2 [R - r_0(x)] + \alpha_0(1 - \alpha_0)[(1 - x)R - r_0(x)] + \alpha_0(1 - \alpha_0)[xR - r_0(x)] \\ &= \alpha_0^2 [R - r_0(x)] + \alpha_0(1 - \alpha_0)[R - 2r_0(x)] \\ &= \alpha_0 R - \alpha_0(2 - \alpha_0)r_0(x). \end{aligned}$$

- When $x \in (1 - x^*, 1]$, we have

$$\begin{aligned} E(\pi_1(x)) &= \alpha_0^2 [R - r_0(x)] + \alpha_0(1 - \alpha_0)[xR - r_0(x)] \\ &= \alpha_0 R [\alpha_0 + (1 - \alpha_0)x] - \alpha_0 r_0(x). \end{aligned}$$

Using the values for $r_0(x)$, we get:

$$E(\pi_1(x)) = \begin{cases} \alpha_0 R - 1 + (1 - \alpha_0) [\alpha_1(R - \Delta) - 1] & \text{if } x \in [0, x^*) \\ \alpha_0 R - 1 + (1 - \alpha_0)^2 [\alpha_1(R - \Delta) - 1] & \text{if } x \in [x^*, 1 - x^*] \\ \alpha_0 R - 1 + (1 - \alpha_0) [\alpha_1(R - \Delta) - 1] & \text{if } x \in (1 - x^*, 1] \end{cases}$$

Note that the expected profit from the first period investment is lower when $x \in [x^*, 1 - x^*]$ and the exact difference is $\alpha_0(1 - \alpha_0) [\alpha_1(R - \Delta) - 1]$. As we show below, this difference can also be written as $\alpha_0(1 - \alpha_0) [E(\pi_2^{ss}) - \alpha_1 \Delta]$.

Next, we calculate the expected profit from the second period investment. Note that, at $t = 2$, there is no further investment opportunity and no asset sale. If the bank performs well, depositors get their promised return of r_1 , if not, they share whatever the bank has received from its investments. This gives us:

- When $x \in [0, x^*)$, from the individual rationality of depositors, we have: $\alpha_1 r_1 + \alpha_1(1 - \alpha_1) xR = 1$. We also have:

$$\begin{aligned} E(\pi_2^{ss}(x)) &= \alpha_1^2 [R - r_1(x)] + \alpha_1(1 - \alpha_1)[(1 - x)R - r_1(x)] \\ &= \alpha_1 R [1 - (1 - \alpha_1)x] - \alpha_1 r_1(x). \end{aligned}$$

- When $x \in [x^*, 1 - x^*]$, we have: $[1 - (1 - \alpha_1)^2] r_1 = 1$. We also have:

$$E(\pi_2^{ss}(x)) = \alpha_1 R - \alpha_1(2 - \alpha_1)r_1(x).$$

- When $x \in (1 - x^*, 1]$, we have: $\alpha r_1 + \alpha_1(1 - \alpha_1)(1 - x)R = 1$. We also have:

$$E(\pi_2^{ss}(x)) = \alpha_1 R[\alpha_1 + (1 - \alpha_1)x] - \alpha_1 r_1(x).$$

Combining these results, we get:

$$r_1(x) = \begin{cases} \frac{1}{\alpha_1} - (1 - \alpha_1)xR & \text{for } x \in [0, x^*) \\ \frac{1}{\alpha_1(2 - \alpha_1)} & \text{for } x \in [x^*, 1 - x^*] \\ \frac{1}{\alpha_1} - (1 - \alpha_1)(1 - x)R & \text{for } x \in (1 - x^*, 1] \end{cases} .$$

and using these values, we get $E(\pi_2^{ss}(x)) = \alpha_1 R - 1$ for all $x \in [0, 1]$.

BANKS' CHOICE:

Since f is continuous and $f(0) = 0$, there exists an $\varepsilon > 0$, where $f(2\varepsilon) - f(\varepsilon) < \alpha_1 \Delta = 0$. Therefore, when the funds needed for a bail out is less than or equal to ε , the regulator will bail out both banks.

Banks try to maximize their total profit from the first and second period investments. Therefore banks' problem is:

$$\underset{x \in [0, 1]}{Max} \quad E(\pi_1(x)) + E(\pi_2(x)) = E(\pi_1(x)) + \sum_i \Pr(i) E(\pi_2^i(\rho))$$

where i represents the eight possible outcomes of returns $(\tilde{R}_c, \tilde{R}_1, \tilde{R}_2)$.

When all three industries perform poorly, which occurs with a probability of $(1 - \alpha_0)^3$, both banks fail irrespective of the choice of x and have zero funds. In that case, the regulator's problem is exactly the same as in the benchmark case. Since for all values of x this case has the same probability of $(1 - \alpha_0)^3$ and the expected second period profit is independent of x , in the rest of the analysis, we drop this case from our calculations.

We also showed that $E(\pi_1(x))$ depends on x . Since we make comparisons among different values of x , we just add (subtract) the differences in $E(\pi_1(x))$ to (from) the overall payoff function. More specifically, we just subtract $\alpha_0(1 - \alpha_0)[\alpha_1(R - \Delta) - 1]$, that is $\alpha_0(1 - \alpha_0) [E(\pi_2^{ss}) - \alpha_1 \Delta]$ for $x \in [x^*, 1 - x^*]$.

Note that, when the return is $(R, R, 0)$ or $(R, 0, R)$, the surviving bank(s) has profits of $R - r_0(x)$ from the first investment. As in the benchmark model, these funds are enough to

purchase the failed banks assets, so that, if it is the case, $E(\pi_2^{sf}(R, R, 0)) = E(\pi_2^{ss}) + \alpha_1\Delta$, and $E(\pi_2^{fs}(R, R, 0)) = 0$.

Next, we investigate the three regions separately.

REGION I: $x \in [0, x^*)$:

Note that both $r_0(x)$ and $r_1(x)$ are decreasing in x in this region.

When the return is (R, R, R) or $(0, R, R)$, both banks survive.

When the return is $(R, 0, 0)$ or $(0, 0, 0)$, both banks fail. We discussed the case when the return is $(0, 0, 0)$, before. When the outcome is $(R, 0, 0)$, both banks have some funds, namely, xR . Note that as x increases, the funds that the failed banks have increase while what it owes to its depositors, namely $r_0(x)$, decreases. Thus, the funds needed for a bail out decreases as x increases, and there exists a critical value of x , denoted by \bar{x} , such that for all $x \in [\bar{x}, x^*)$, both banks will be bailed out when the return is $(R, 0, 0)$.

When the return is $(R, R, 0)$, $(R, 0, R)$, $(0, R, 0)$ or $(0, 0, R)$, one bank survives while the other fails. When the return is $(R, R, 0)$ or $(R, 0, R)$, the surviving bank has enough funds to purchase the failed bank's assets. However, when the return is $(0, R, 0)$ or $(0, 0, R)$, the surviving bank may not have enough funds for the asset purchase. For the surviving bank to have the necessary funds, we need $(1 - x)R - r_0(x) \geq \alpha_1(R - \Delta) - 1$, that is, $x \leq 1 - \frac{r_0(x) + \alpha_1(R - \Delta) - 1}{R}$. The critical value of x , denoted by \tilde{x} , below which the bank can purchase the failed bank's assets satisfies: $\tilde{x} = 1 - \frac{r_0(\tilde{x}) + \alpha_1(R - \Delta) - 1}{R}$.

In this region, we have¹⁵:

$$E(\pi_2(x)) = \alpha_0^2 E(\pi_2^{ss}) + \alpha_0^2(1 - \alpha_0)[E(\pi_2^{ss}) + \alpha_1\Delta] + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{sf}(0, R, 0)) + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{fs}(0, 0, R)) + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{ff}(R, 0, 0)).$$

The relationship between \tilde{x} , \bar{x} and x^* affects banks' choices through its impact on bail outs and the surviving bank's capability in purchasing the failed bank's assets. Note that, we always have $\bar{x} < x^*$. Thus, different values of \tilde{x} may have different implications for banks' choices. In particular, we have three different cases: Case 1: $x^* \leq \tilde{x} < 1 - x^*$, Case 2: $\bar{x} \leq \tilde{x} < x^*$ and Case 3: $\tilde{x} < \bar{x}$. Next we investigate these three cases in this region.

Case 1: $x^* \leq \tilde{x} < 1 - x^*$

In this case, when the return is $(0, R, 0)$ or $(0, 0, R)$, the surviving bank always has the necessary funds for the asset purchase, so that, $E(\pi_2^{sf}(0, R, 0)) = E(\pi_2^{ss}) + \alpha_1\Delta$, and $E(\pi_2^{fs}(0, R, 0)) = 0$. Thus, there is a potential bail out only when both banks fail. And banks

¹⁵Note that this is not the actual payoff but the components of the payoff that will appear when we compare different x values.

choose \bar{x} to guarantee a bail out when the return is $(R, 0, 0)$. We have:

$$\begin{aligned} E(\pi_2(\bar{x})) &= \alpha_0^2 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)[E(\pi_2^{ss}) + \alpha_1 \Delta] + \alpha_0(1 - \alpha_0)^2 \bar{\theta} E(\pi_2^{ss}) \\ &= \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)[\alpha_1 \Delta] + \alpha_0(1 - \alpha_0)^2 \bar{\theta} E(\pi_2^{ss}). \end{aligned}$$

Case 2: $\bar{x} < \tilde{x} < x^*$

Note that, for any $x \in [0, \tilde{x}]$, the surviving bank can purchase the failed bank's assets, however, a bail out is more likely for higher values of x . Thus, in this case, as well as Case 3, \tilde{x} weakly dominates any $x \in [0, \tilde{x})$ and banks never choose an $x < \tilde{x}$.

By choosing an $x \in [\bar{x}, \tilde{x}]$, the bank guarantees a bail out when the return is $(R, 0, 0)$ and also has enough funds for the asset purchase. Thus, for any $x \in (\bar{x}, \tilde{x}]$, we have $E(\pi_2^{sf}(R, R, 0)) = E(\pi_2^{sf}(0, R, 0)) = E(\pi_2^{ss}) + \alpha_1 \Delta$, and

$$E(\pi_2(x)) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)[\alpha_1 \Delta] + \alpha_0(1 - \alpha_0)^2 \bar{\theta} E(\pi_2^{ss}).$$

If the bank chooses $x \in (\tilde{x}, x^*)$, it guarantees a bail out when the return is $(R, 0, 0)$ but does not have enough funds for the asset purchase. However, depending on the cost, the regulator may bail out the failed bank even when the other bank survives since now purchase by the surviving bank is not an option. Thus, for $x \in (\tilde{x}, x^*)$, we have:

$$E(\pi_2(x)) = \alpha_0 E(\pi_2^{ss}) + \alpha_0^2(1 - \alpha_0)[\alpha_1 \Delta] + \alpha_0(1 - \alpha_0)^2 [E(\pi_2^{fs}(0, 0, R))] + \alpha_0(1 - \alpha_0)^2 \bar{\theta} E(\pi_2^{ss}).$$

Case 3: $\tilde{x} \leq \bar{x}$

In this case, banks choose either \tilde{x} or \bar{x} . From Case 1, we have

$$E(\pi_2(\bar{x})) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)[\alpha_1 \Delta] + \alpha_0(1 - \alpha_0)^2 \bar{\theta} E(\pi_2^{ss}).$$

When banks choose $x = \tilde{x}$, the surviving bank always has enough funds to acquire the failed bank's assets but a bail out may not be guaranteed when the return is $(R, 0, 0)$. Thus, we have:

$$\begin{aligned} E(\pi_2(\tilde{x})) &= \alpha_0^2 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)[E(\pi_2^{ss}) + \alpha_1 \Delta] + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{ff}(R, 0, 0)) \\ &= \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)[\alpha_1 \Delta] + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{ff}(R, 0, 0)). \end{aligned}$$

REGION II: $x \in [x^*, 1 - x^*]$:

Note that, in this region, both $r_0(x)$ and $r_1(x)$ are independent of x .

When the return is (R, R, R) , $(R, R, 0)$, $(R, 0, R)$, $(R, 0, 0)$ or $(0, R, R)$, both banks succeed

Both banks fail only when the return is $(0, 0, 0)$, a case we discussed before.

When the return is $(0, R, 0)$ or $(0, R, 0)$, only one bank survives. The surviving bank has funds of $[(1-x)R - r_0(x)]$, which decreases in x . Note that, the surviving bank will have the highest funds for the asset purchase when $x = x^*$.

In this region, we can write banks' profits as:

$$E(\pi_2(x)) = \alpha_0 E(\pi_2^{ss}) + \alpha_0^2(1 - \alpha_0)E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{sf}(0, R, 0)) + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{fs}(0, 0, R)) - \underbrace{\alpha_0(1 - \alpha_0) [E(\pi_2^{ss}) - \alpha_1 \Delta]}_{\text{Difference from } E(\pi_1)}$$

Note that, for this region, Case 2 and Case 3 have the same implications. Therefore, effectively, we have two cases to examine:

Case 1: $x^* \leq \tilde{x} < 1 - x^*$

Note that, for all $x \in [x^*, \tilde{x}]$, the surviving bank always has enough funds to purchase the failed bank's assets, which gives us $E(\pi_2^{sf}(0, R, 0)) = E(\pi_2^{ss}) + \alpha_1 \Delta$ and $E(\pi_2^{fs}(0, 0, R)) = 0$. Note that, for all $x \in [x^*, \tilde{x}]$, the expected profit is the same, so, for simplicity, let's assume that the banks choose $x = x^*$, which gives us:

$$\begin{aligned} E(\pi_2(x^*)) &= \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)^2[\alpha_1 \Delta] - \alpha_0(1 - \alpha_0) [E(\pi_2^{ss}) - \alpha_1 \Delta] \\ &= \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)[\alpha_1 \Delta] + \alpha_0(1 - \alpha_0)^2[\alpha_1 \Delta]. \end{aligned}$$

And if the banks choose $x > \tilde{x}$, the surviving bank will not have enough funds for the asset purchase when the return is $(0, R, 0)$ but may benefit from a potential bail out when the return is $(0, 0, R)$. Thus, for $x \in (\tilde{x}, 1 - x^*]$ we have

$$\begin{aligned} E(\pi_2(x)) &= \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{fs}(0, 0, R)) - \alpha_0(1 - \alpha_0) [E(\pi_2^{ss}) - \alpha_1 \Delta] \\ &= \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{fs}(0, 0, R)) + \alpha_0(1 - \alpha_0) [\alpha_1 \Delta] \end{aligned}$$

Case 2 and Case 3:

In these cases, when the return is $(0, R, 0)$ and $(0, 0, R)$, the surviving bank cannot purchase the failed bank's assets. And in this region, banks fail only when both industries they invested in performs poorly, in which case, they will have zero funds. Thus, for all $x \in [x^*, 1 - x^*]$, the expected profit is the same. And, for simplicity, let's assume that the banks choose $x = x^*$, which gives us:

$$E(\pi_2(x^*)) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{fs}(0, 0, R)) + \alpha_0(1 - \alpha_0) [\alpha_1 \Delta].$$

REGION III: $x \in (1 - x^*, 1]$:

In this region, both $r_0(x)$ and $r_1(x)$ are increasing in x .

Both banks succeed when the common industry performs well and both banks fail when the common industry performs poorly. The probability of individual survival is zero and there is no expected profit from an asset purchase. Thus, \tilde{x} does not have an impact on banks' profits and the banks' expected profit is the same for all three cases.

Both banks succeed when the common industry performs well, that is when the return is (R, R, R) , $(R, R, 0)$, $(R, 0, R)$ or $(R, 0, 0)$.

Both banks fail when the return is $(0, R, R)$, $(0, R, 0)$, $(0, 0, R)$ or $(0, 0, 0)$. When the outcome is $(0, R, R)$, $(0, R, 0)$ or $(0, 0, R)$, the bank(s) with the high return from its specific industry has funds of $(1 - x)R$. Note that as x decreases, these funds increase so that the cost of a bail out goes down. And there exists a critical value of x , denoted by \hat{x} , such that for all $x \in (1 - x^*, \hat{x}]$, the bank(s) with the high return from the idiosyncratic industry will be bailed out.

As a result, banks will choose any $x \in (1 - x^*, \hat{x}]$, for simplicity say \hat{x} .

We can write banks' expected profit as:

$$E(\pi_2(\hat{x})) = \alpha_0 E(\pi_2^{ss}) + \alpha_0^2(1 - \alpha_0) E(\pi_2^{ff}(0, R, R)) + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{ff}(0, R, 0)) + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{ff}(0, 0, R)).$$

Note that, when the return is $(0, R, R)$ or $(0, R, 0)$ (or $(0, 0, R)$ for the second bank), the first bank will be bailed out, that is, $E(\pi_2^{ff}(0, R, R)) = E(\pi_2^{ff}(0, R, 0)) = \bar{\theta}E(\pi_2^{ss})$. This gives us:

$$E(\pi_2(\hat{x})) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0) \bar{\theta}E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{ff}(0, 0, R)).$$

Whether the bank with zero funds will be bailed out or not, depends on the cost of bail out.

Let \bar{F}_1 be the maximum amount of funds needed for a bail out, above which the regulator will liquidate both banks, that is, $f(\bar{F}_1) = \alpha_1\Delta$. And let $\bar{F}_2(F_1)$ be the maximum amount of funds needed, above which the regulator will not bail out the second bank when the funds needed to bail out the first bank was F_1 , that is $f(\bar{F}_2(F_1) + F_1) - f(F_1) = \alpha_1\Delta$. Since f is convex, \bar{F}_2 is a function of F_1 , and $\bar{F}_2(F_1) \leq \bar{F}_1$, for all values of F_1 and $\bar{F}_2(F_1) < \bar{F}_1$, when $F_1 > 0$.

Note that, when the return is $(0, 0, R)$, both banks fail and Bank B will be bailed out since it had a high return from its idiosyncratic industry. The funds needed to bail out Bank B is given as $F_1 = F(\hat{x}R) = r_0(\hat{x}) - \hat{x}R$, which can be arbitrarily small since F is continuous and decreasing in x , and $\lim_{x \rightarrow (1-x^*)} F(xR) = 0$. Thus, if $F(0) < \bar{F}_1$, then $F(0) < \bar{F}_2(F(\hat{x}R))$.

Now, for this region, we have two different cases:

(3.1) If $F(0) > \bar{F}_2(F(\hat{x}R))$, then the failed bank with zero funds will not be bailed out and $E(\pi_2^{ff}(0, 0, R, \hat{x})) = 0$. In that case, we have:

$$E(\pi_2(\hat{x})) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0) \bar{\theta} E(\pi_2^{ss}).$$

(3.2) Otherwise, we have:

$$E(\pi_2(\hat{x})) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)(2 - \alpha_0) \bar{\theta} E(\pi_2^{ss}).$$

Banks' Choice and the Socially Optimal Outcome

Now we can find banks' choice in each case and compare it with the socially optimal outcome. We investigate each case separately.

Case 1: $\tilde{x} \geq x^*$

First, we investigate banks' choice and then compare it with the socially optimal level of x . When $\tilde{x} \geq x^*$, we have:

$$E(\pi_2(x)) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0) [\alpha_1 \Delta] + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{fs}(0, 0, R)) \text{ for } x \in (\tilde{x}, 1 - x^*), \text{ and}$$

$$E(\pi_2(\bar{x})) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0) [\alpha_1 \Delta] + \alpha_0(1 - \alpha_0)^2 \bar{\theta} E(\pi_2^{ss}).$$

Note that, if a bank is bailed out, the maximum expected second period profit is $\bar{\theta} E(\pi_2^{ss})$, so that, $\bar{\theta} E(\pi_2^{ss}) \geq E(\pi_2^{fs}(0, 0, R))$. Thus, for $x \in (\tilde{x}, 1 - x^*)$, $E(\pi_2(\bar{x})) \geq E(\pi_2(x))$, and banks choose \bar{x} over any $x \in (\tilde{x}, 1 - x^*)$. So, the choice is between \hat{x} , x^* and \bar{x} . We have:

$$E(\pi_2(x^*)) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0) [\alpha_1 \Delta] + \alpha_0(1 - \alpha_0)^2 [\alpha_1 \Delta], \text{ and}$$

$$E(\pi_2(\hat{x})) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0) \bar{\theta} E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{ff}(0, 0, R)).$$

We have the following cases:

- If $\bar{\theta} E(\pi_2^{ss}) < \alpha_1 \Delta$, banks choose $x = x^*$. Note that $\bar{\theta} E(\pi_2^{ss}) < \alpha_1 \Delta$ if and only if $1 - \bar{\theta} \geq 1 - \frac{\alpha_1 \Delta}{\alpha_1 R - 1} = \beta_1^*$.
- Otherwise, that is, when $\bar{\theta} E(\pi_2^{ss}) \geq \alpha_1 \Delta$ (which can also be written as $1 - \bar{\theta} < \beta_1^*$), banks prefer \bar{x} over x^* . Thus, the choice is between \bar{x} over \hat{x} and it depends on the bail out decision of the regulator:

– If $F(0) < \bar{F}_2(F(\hat{x}R))$, we have:

$$E(\pi_2(\hat{x})) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0) \bar{\theta} E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)^2 \bar{\theta} E(\pi_2^{ss}),$$

and banks choose $x = \hat{x}$.

– If $F(0) > \bar{F}_2(F(\hat{x}R))$, we have:

$$E(\pi_2(\hat{x})) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0) \bar{\theta} E(\pi_2^{ss}).$$

Note that $E(\pi_2(\hat{x})) > E(\pi_2(\bar{x}))$ if and only if $\alpha_0 \bar{\theta} E(\pi_2^{ss}) > \alpha_1 \Delta$, that is,

$$1 - \bar{\theta} < 1 - \frac{\Delta}{\alpha_1 R - 1} = \hat{\beta}_1^*.$$

Thus, if $1 - \bar{\theta} < \hat{\beta}_1^*$, banks choose $x = \hat{x}$, otherwise, they choose $x = \bar{x}$.

Next, we compare banks' choice with the socially optimal level of x . Note that, in this case, the only return where there is a social welfare loss is when the return is $(0, 0, 0)$, a case common for all x values. However, we know that for other values of x , we may have social welfare losses, either through liquidation or bail out, for other returns. Thus, in this case, the socially optimal level of x is x^* . However, as we showed, banks may choose other values of x , in particular $\hat{x} > x^*$, and get over-exposed to the common risk factor.

Case 2: $\bar{x} \leq \tilde{x} < x^*$

We have:

$$E(\pi_2(x^*)) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{fs}(0, 0, R)) + \alpha_0(1 - \alpha_0) [\alpha_1 \Delta], \text{ and}$$

$$E(\pi_2(\tilde{x})) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)^2 \bar{\theta} E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0) [\alpha_1 \Delta].$$

Note that, if a bank is bailed out, the maximum expected second period profit is $\bar{\theta} E(\pi_2^{ss})$. Thus, banks always choose \tilde{x} over x^* .

So the choice is between \hat{x} , \tilde{x} and $x \in (\tilde{x}, x^*)$, where, we have:

$$E(\pi_2(\hat{x})) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0) \bar{\theta} E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{ff}(0, 0, R)), \text{ and, for } x \in (\tilde{x}, x^*),$$

$$E(\pi_2(x)) = \alpha_0 E(\pi_2^{ss}) + \alpha_0^2(1 - \alpha_0) [\alpha_1 \Delta] + \alpha_0(1 - \alpha_0)^2 [E(\pi_2^{fs}(0, 0, R))] + \alpha_0(1 - \alpha_0)^2 \bar{\theta} E(\pi_2^{ss})$$

We have the following cases:

- If $\bar{\theta} E(\pi_2^{ss}) < \alpha_1 \Delta$, banks choose $x = \tilde{x}$. Note that $\bar{\theta} E(\pi_2^{ss}) < \alpha_1 \Delta$ if and only if $1 - \bar{\theta} \geq \hat{\beta}_1^*$.
- Otherwise, that is, when $\bar{\theta} E(\pi_2^{ss}) \geq \alpha_1 \Delta$ (which can also be written as $1 - \bar{\theta} < \hat{\beta}_1^*$), banks' choice depends on the bail out decision of the regulator:

– If $F(0) < \bar{F}_2(F(\hat{x}R))$, we have:

$$E(\pi_2(\hat{x})) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0) \bar{\theta} E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)^2 \bar{\theta} E(\pi_2^{ss}),$$

and banks choose $x = \hat{x}$.

– If $F(0) > \bar{F}_2(F(\hat{x}R))$, we have:

$$E(\pi_2(\hat{x})) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0) \bar{\theta} E(\pi_2^{ss}).$$

Note that, as we argued when we discussed banks' expected profit in Region III, when banks choose $x = \hat{x}$ and the return is $(0, 0, R)$, both banks fail and Bank B will be bailed out since it had a high return from its idiosyncratic industry. The funds needed to bail out Bank B is $F_1 = F(\hat{x}R) = r_0(\hat{x}) - \hat{x}R$, which can be arbitrarily small since F is continuous and decreasing in x , and $\lim_{x \rightarrow (1-x^*)} F(xR) = 0$.

Thus, $F(0) > \bar{F}_2(F(\hat{x}R))$ implies that $F(0) \geq \bar{F}_1$, in which case, the regulator does not bail out the first bank when it has zero funds. Therefore, for $x \in (\tilde{x}, x^*)$, we have $[E(\pi_2^{fs}(0, 0, R))]$. So, since in this case $\bar{\theta} E(\pi_2^{ss}) \geq \alpha_1 \Delta$, banks always choose \hat{x} over $x \in (\tilde{x}, x^*)$. Thus, the choice is between \hat{x} and \tilde{x} . Note that $E(\pi_2(\hat{x})) > E(\pi_2(\tilde{x}))$ if and only if $\alpha_0 \bar{\theta} E(\pi_2^{ss}) > \alpha_1 \Delta$, that is, $1 - \bar{\theta} < \hat{\beta}_1^*$. Thus, if $1 - \bar{\theta} < \hat{\beta}_1^*$, banks choose $x = \hat{x}$, otherwise, they choose $x = \tilde{x}$.

Depending on the cost of bail out and liquidation, the socially optimal level of x is either x^* or \tilde{x} , but never \hat{x} . Thus, as in Case 1, banks' choice may not result with the socially optimal outcome, in particular, banks may get over-exposed to the common risk factor.

Case 3: $\tilde{x} < \bar{x}$

We have:

$$E(\pi_2(x^*)) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{fs}(0, 0, R)) + \alpha_0(1 - \alpha_0) [\alpha_1 \Delta], \text{ and}$$

$$E(\pi_2(\tilde{x})) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0) [\alpha_1 \Delta] + \alpha_0(1 - \alpha_0)^2 \bar{\theta} E(\pi_2^{ss}).$$

Note that, $\bar{\theta} E(\pi_2^{ss}) \geq E(\pi_2^{fs}(0, 0, R))$. Thus, we have $E(\pi_2(\tilde{x})) \geq E(\pi_2(x^*))$ and banks choose \tilde{x} over x^* . So, the choice is between \hat{x} , \bar{x} and \tilde{x} . We have:

$$E(\pi_2(\bar{x})) = \alpha_0 E(\pi_2^{ss}) + \alpha_0^2(1 - \alpha_0) [\alpha_1 \Delta] + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{fs}(0, 0, R)) + \alpha_0(1 - \alpha_0)^2 \bar{\theta} E(\pi_2^{ss}), \text{ and}$$

$$E(\pi_2(\hat{x})) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0) \bar{\theta} E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)^2 E(\pi_2^{ff}(0, 0, R)).$$

- If $\bar{\theta} E(\pi_2^{ss}) < \alpha_1 \Delta$, banks choose $x = \tilde{x}$. Note that $\bar{\theta} E(\pi_2^{ss}) < \alpha_1 \Delta$ if and only if $1 - \bar{\theta} \geq \beta_1^*$.
- Otherwise, that is, when $\bar{\theta} E(\pi_2^{ss}) \geq \alpha_1 \Delta$ (which can also be written as $1 - \bar{\theta} < \beta_1^*$), banks' choice depends on the bail out decision of the regulator:

- If $F(0) = r_0(\hat{x}) < \bar{F}_2(F(\hat{x}R))$, the regulator bails out both banks when banks choose $x = \hat{x}$ and the return is $(0, 0, R)$. Thus, we have:

$$E(\pi_2(\hat{x})) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0) \bar{\theta} E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0)^2 \bar{\theta} E(\pi_2^{ss}),$$

and banks choose $x = \hat{x}$.

- If $F(0) = r_0(\hat{x}) > \bar{F}_2(F(\hat{x}R))$, we have:

$$E(\pi_2(\hat{x})) = \alpha_0 E(\pi_2^{ss}) + \alpha_0(1 - \alpha_0) \bar{\theta} E(\pi_2^{ss}).$$

But as we discussed in Case 2, we know that when banks choose $x = \bar{x}$, so that, $E(\pi_2^{fs}(0, 0, R)) = 0$. Thus, the choice is between \hat{x} and \tilde{x} . Note that $E(\pi_2(\hat{x})) > E(\pi_2(\tilde{x}))$ if and only if $\alpha_0 \bar{\theta} E(\pi_2^{ss}) > \alpha_1 \Delta$, that is, $1 - \bar{\theta} < \hat{\beta}_1^*$. Thus, if $1 - \bar{\theta} < \hat{\beta}_1^*$, banks choose $x = \hat{x}$, otherwise, they choose $x = \tilde{x}$.

Depending on the cost of bail out and liquidation, the socially optimal level of x is either x^* or \tilde{x} , but never \hat{x} . Thus, as in Case 1, banks' choice may not result with the socially optimal outcome, in particular, banks may get over-exposed to the common risk factor.

Finally, we can summarize our results as follows. For all parameter values, the expected output generated by the banking industry net of any bailout or liquidation costs is strictly greater for $x = x^*$ than for any $x \in (1 - x^*, 1]$. Thus, an $x \in (1 - x^*, 1]$ can never be the socially optimal outcome.

However, we showed that when $1 - \bar{\theta} < \hat{\beta}_1^*$, banks choose an $x \in (1 - x^*, 1]$ that is arbitrarily close to $1 - x^*$. Thus, banks are over-exposed to the common risk factor and the expected output generated by the banking sector net of any liquidation and/or bailout costs is strictly lower than its socially optimal level.

And, when $1 - \bar{\theta} < \beta_1^*$, if $f(F_1(r_0(x))) < \alpha_1 \Delta$, banks choose an $x \in (1 - x^*, 1]$ that is arbitrarily close to $1 - x^*$, with the same implication as above. \diamond

Table 4: Banks' returns and states for all possible outcomes when $x^* \leq 1/2$

(R_c, R_1, R_2)	Probability	$[0, x^*]$	$[x^*, 1 - x^*]$	$(1 - x^*, 1]$	Bank 1	Bank 2
R, R, R	α^3	SS	SS	SS	R	R
$R, R, 0$	$\alpha^2(1 - \alpha)$	SF	SS	SS	R	xR
$R, 0, R$	$\alpha^2(1 - \alpha)$	FS	SS	SS	xR	R
$R, 0, 0$	$\alpha(1 - \alpha)^2$	FF	SS	SS	xR	xR
$0, R, R$	$\alpha^2(1 - \alpha)$	SS	SS	FF	$(1 - x)R$	$(1 - x)R$
$0, R, 0$	$\alpha(1 - \alpha)^2$	SF	SF	FF	$(1 - x)R$	0
$0, 0, R$	$\alpha(1 - \alpha)^2$	FS	FS	FF	0	$(1 - x)R$
$0, 0, 0$	$(1 - \alpha)^3$	FF	FF	FF	0	0

Table 5: Banks' returns and states for all possible outcomes when $x^* \geq 1/2$

(R_c, R_1, R_2)	Probability	$[0, 1 - x^*]$	$(1 - x^*, x^*)$	$[x^*, 1]$	Bank 1	Bank 2
R, R, R	α^3	SS	SS	SS	R	R
$R, R, 0$	$\alpha^2(1 - \alpha)$	SF	SF	SS	R	xR
$R, 0, R$	$\alpha^2(1 - \alpha)$	FS	FS	SS	xR	R
$R, 0, 0$	$\alpha(1 - \alpha)^2$	FF	FF	SS	xR	xR
$0, R, R$	$\alpha^2(1 - \alpha)$	SS	FF	FF	$(1 - x)R$	$(1 - x)R$
$0, R, 0$	$\alpha(1 - \alpha)^2$	SF	FF	FF	$(1 - x)R$	0
$0, 0, R$	$\alpha(1 - \alpha)^2$	FS	FF	FF	0	$(1 - x)R$
$0, 0, 0$	$(1 - \alpha)^3$	FF	FF	FF	0	0