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Consumption choice and asset pricing with a non-price-taking agent★

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Summary. This paper develops a pure-exchange model to study the consumption-portfolio problem of an agent who acts as a non-price-taker, and to analyze the implications of his behavior on equilibrium security prices. The non-price-taker is modeled as a price leader in all markets; his price impact is then recast as a dependence of the Arrow-Debreu prices on his consumption, allowing a tractable formulation. Besides the aggregate consumption, the endowment of the non-price-taker appears as an additional factor in driving equilibrium allocations and prices. Comparisons of equilibria between a price-taking and a non-price-taking economy are carried out.

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1 Introduction

Central to the equilibrium-based asset pricing models is the competitive agents paradigm: each agent is atomistic relative to the market, and takes prices to be unaffected by his actions. However, an observation of today's security markets (and especially government bond markets) reveals the ever-increasing importance of large pension funds and financial institutions in the market-place."Large" investors are particularly prevalent in smaller security markets outside the U.S.A. A large investor may have a significant effect on prices, and hence may prefer to choose a strategy taking the price impact of his own behavior into account. It is well-known that large trades do have a

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permanent price impact. This is attributable partially to the information a large trade reveals about future cash flows (Kraus and Stoll (1972), Holthausen, Leftwich, and Mayers (1987, 1990), Seppi (1992)), but conceivably also to the effect of such a large position on security supply and demand. Internationally, there is widespread anecdotal evidence that large trades can affect prices independent of any information they may contain.

The objective of this paper is to study the optimization problem and effect on market equilibrium of a non-price-taking investor, who takes account of the price impact of the position he takes on (independent of any information revealed by his trading). Part of this objective is for the formulation to be fully consistent with no arbitrage, market clearing, and rational expectations. Our approach is to adapt a standard asset pricing environment to include a non-price-taking agent, while retaining the usual assumptions of complete and frictionless markets, and symmetric information. Accordingly, we develop an equilibrium model of an Arrow-Debreu pure-exchange economy consisting of one or more price-taking agents and one non-price-taking agent. The presence of the non-price-taker is specified exogenously and not generated endogenously, for example, by his superior information over the other agents (e.g., Kyle (1985)) or by the size of his security holdings.[1] We focus on a single non-price-taker to abstract away from strategic issues between multiple non-price-takers. Our emphasis is on characterization of equilibrium, while assuming the existence of optimal policies and an equilibrium.

Our notion of equilibrium is in the spirit of the price leadership model in the oligopoly literature (e.g., Varian (1992, Chapter 16)). The non-price-taker chooses his consumption-portfolio strategy, aware that prices must adjust so that the remaining (price-taking) agents' demands clear all security and consumption good markets. We recast the non-price-taker's impact on security market prices in terms of an equivalent impact of his consumption choice on the Arrow-Debreu prices, making the analysis highly tractable. If there is only one price-taker or the price-takers' preferences are such that their representative agent's utility is independent of individual wealth allocations (i.e., allow Gorman (1953) aggregation), we show that the non-price-taker's consumption in any state affects the state price (the price of one unit of consumption) only in that state; otherwise the state prices across all states are affected. The main results of the paper are presented for the single price-taker case, but are directly applicable to multiple price-takers whose preferences allow Gorman aggregation.

Solving for the equilibrium consumption allocations reveals that the non-price-taking agent deviates from his price-taking behavior by tending to keep

[1] In a frictionless economy such as ours, the size of agents' security holdings is not restricted by their wealth, so strictly it should not be argued that an investor with large net wealth can affect prices more than a small investor; however, in practice given the presence of short sales constraints and transaction costs in the marketplace, it is plausible that some investors are able to make larger trades and hence affect prices more than others.

his consumption closer to his endowment payoff. The extent of this deviation depends on how much of a net trader the non-price-taker is, and on the risk aversion of the other agents in the economy. In the polar case of risk-neutral price-takers, the non-price-taker cannot influence prices at all and hence does not deviate. In addition to the aggregate consumption, the non-price-taker's endowment appears as an extra factor in explaining the equilibrium asset and Arrow-Debreu prices. This leads an asset's risk premium to depend on two factors: the covariance of its return with changes in the non-price-taking agent's endowment stream as well as with changes in the aggregate consumption.

To derive further implications of non-price-taking behavior, we specialize to the case of preferences exhibiting constant absolute risk aversion (CARA). The non-price-taker is found to react less to changes (on moving across states) in the aggregate consumption than if he were a price-taker, but also to react positively to changes in his own endowment, to which he would not react as a price-taker. Consequently, as compared with an economy of all price-takers, the Arrow-Debreu prices react more to changes in the aggregate consumption, and also react to changes in the non-price-taker's endowment. The non-price-taker pushes up the price of consumption in states where he has a high endowment, and tends to decrease the risk premium on assets having a payoff highly correlated with his own endowment.

The closest work related to this paper is by Lindenberg (1979) and Grinblatt and Ross (1985). In contrast to our Arrow-Debreu formulation (allowing us to work with general preferences and asset payoffs), these papers work in a two-period mean-variance framework. One (or some) of the agents in these models recognizes that the market clearing security prices depend on his portfolio strategy and formulates his optimization problem accordingly. For general mean-variance investors, Lindenberg finds the optimal portfolio of the non-price-taking investors to contain differing percentages of the supply of each security, hence two-fund separation fails. A two-factor CAPM results, in which an asset's risk premium is driven by the covariance of its return with the market return and with the return on the aggregate portfolio of the price-affecting investors. For tractability, Grinblatt and Ross specialize to the case of a risk-neutral non-price-taker, and restrict his optimal demand function yet further to be a function of initial price (and signals and extraneous risk). For this special case, with symmetric information, they assert no effect on prices is attributable to the non-price-taker *per se*. Under asymmetric information, significant price effects are demonstrated but not fully characterized.

Other related work includes Cvitanić (1995), Cuoco and Cvitanić (1996) and El Karoui, Peng and Quenez (1996), who exogenously specify a price dependence on a "large" investor's strategy and study such an investor's continuous-time dynamic optimization problem at a partial equilibrium level. While (in our dynamic extension) we allow the non-price-taker's consumption-portfolio choice to affect the whole process of the price system, these papers specify his portfolio holding to only affect the concurrent security price drift (and also not the price volatility). While we endogenously derive the price-dependence from general equilibrium restrictions, these papers do not impose market clearing (and hence equilibrium) and so may allow a general form of the price-dependence. Working in discrete-time, Jarrow (1992) also exogenously specifies a dependence of asset prices on the non-price-taker's trading strategy, and focuses on market manipulation strategies which generate arbitrage opportunities for the non-price-taker.

Section 2 outlines the pure-exchange two-period two-agent framework of our model. In Section 3 we present the Arrow-Debreu formulation of the non-price-taking equilibrium, and in Section 4 characterize agents' equilibrium consumption allocations and asset risk premia. In Section 5 we present the example with CARA utility, in Section 6 the extension to multiple price-takers, and in Section 7 the dynamic extension in continuous-time. Section 8 concludes, and the Appendix provides all proofs.

2 The economy

We consider a pure-exchange economy with a single consumption good (the numeraire) and two time periods, 0 and 1. Uncertainty is resolved at time 1, represented by $L+1$ states of nature indexed by $\omega = \{1, \ldots, L+1\}$ having probability $\mathscr{P}(\omega)$.[2] Trade takes place at time 0, and returns and investors' utility from consumption are calculated at time 1. All time-1 symbols lacking the ω argument are understood to be random variables. All equalities involving random variables are assumed to hold $\mathscr{P}$-a.s.

Securities. We assume there are $L+1$ securities: a riskless bond in zero net supply, with initial price 1 and time-1 certain payoff $1+r$, and L risky stocks each in constant net supply of 1 and providing a time-1 random payoff of $\delta_i, i = 1, \ldots, L$. The aggregate payoff is defined by $\delta \equiv \sum_{i=1}^{L} \delta_i$. The $(L+1) \times (L+1)$ bond-risky stock payoff matrix is assumed invertible. The interest rate, r, and the time-0 price, S_i, of each stock are to be determined endogenously in equilibrium.

Agents' Preferences and Endowments. We assume there are two agents in the economy. Agent N is a non-price-taker in the sense that he takes into account the fact that his consumption-portfolio choice affects the whole price system $(r, S_i;\ i = 1, \ldots, L)$. The other agent P is a price-taker. Each agent, n, is endowed at time zero with e_{ni} shares of each risky security i, which he trades for a portfolio holding of α_{ni} shares of each risky security i and α_{n0} shares of the bond, held until time 1, and then converted into consumption. Preferences of each agent n are represented by a von Neumann-Morgenstern utility

[2] It is straightforward to extend the current analysis to an infinite dimensional state space, as long as enough zero net supply securities for market completeness are assumed to exist. In Section 7 we provide an extension to an infinite dimensional state space with continuous trading in a multi-period setting.

function over time-1 consumption, $E[u_n(c_n)]$. The functions $u_n(\cdot)$ are assumed three times continuously differentiable, strictly increasing and strictly concave.

In this article, we analyze quantities of interest by appealing to general equilibrium restrictions. Our equilibrium notion is based on a standard price leadership model and formally defined as follows.

Definition. *Equilibrium* in an economy with one non-price-taker and one price-taker is defined as a price system $(r, S_i;\ i = 1, \ldots, L)$ and consumption-portfolio policies $(c_n^*, \alpha_n^*), n = N, P$, such that (i) the price-taker chooses his optimal (utility-maximizing) consumption-portfolio strategy at the given prices, (ii) the non-price-taker chooses his optimal consumption-portfolio policy taking account of the fact that the price system responds to clear the markets, and (iii) the price system is such that the good and security markets do clear, i.e.,

$$\sum_{n=N,P} c_n^* = \delta; \quad \sum_{n=N,P} \alpha_{ni}^* = 1, \quad i = 1, \ldots, L; \quad \sum_{n=N,P} \alpha_{n0}^* = 0\ . \tag{1}$$

The non-price-taker acts as a price leader in all markets, observing the other agent's demands as a function of prices, and then choosing prices and his own consumption-portfolio policy so as to maximize his objective function subject to the condition that all markets must clear.

Throughout the paper, a symbol with a caret ($\hat{}$) denotes an optimal quantity. A symbol with an asterisk (*) denotes equilibrium in a non-price-taking economy; a symbol with an overbar ($\bar{}$), equilibrium in a price-taking economy (where all agents are price-takers).

3 Agents' optimization

This section presents agents' optimization problems according to our equilibrium notion. The assumed market completeness allows the construction of a unique system of Arrow-Debreu securities consistent with no arbitrage. Accordingly, we may define the state price density ξ, where $\xi(\omega)$ represents the Arrow-Debreu price per unit of probability $\mathscr{P}(\omega)$, of a unit of consumption in state ω. The time-0 value of stock i is, then, given by

$$S_i = E[\xi \delta_i], \quad i = 1, \ldots, L\ . \tag{2}$$

Hence each agent's initial wealth is $\sum_{i=1}^{L} e_{ni} S_i = E[\xi \epsilon_n]$, where $\epsilon_n \equiv \sum_{i=1}^{L} e_{ni} \delta_i$ is the payoff from agent n's initial stock endowment.

The price-taker maximizes his expected utility subject to his budget constraint, taking the price system as given. In this Arrow-Debreu economy, his optimization problem is

$$\max_{c_P} E[u_P(c_P)] \quad \text{subject to} \quad E[\xi c_P] \le E[\xi \epsilon_P]\ .$$

Using the Lagrangian method, the first order condition for optimality of the price-taker is

$$u'_P(\hat{c}_P) = y_P \xi \ , \tag{3}$$

where y_P satisfies

$$E[\xi I_P(y_P \xi)] = E[\xi \epsilon_P] \ , \tag{4}$$

where $I_P(\cdot)$ is the inverse of agent P's marginal utility. Equation (3) states that for the price-taker, the marginal benefit from an extra unit of consumption in state ω is proportional to the cost $\xi(\omega)$ of that extra unit of consumption.

We now formulate the non-price-taker's optimization problem. According to the definition of equilibrium, the non-price-taker simultaneously solves for his optimal strategy and the equilibrium prices. He acts as a price leader in all (good and security) markets. However, since all agents' budget constraints hold with equality, in a market with no redundant securities, it suffices to simply clear the consumption good market to ensure clearing in all markets. Hence, we need only explicitly enforce price leadership in the consumption good market. Recalling the price-taker's demand (3), clearing in the consumption good (1) implies

$$c_N = \delta - I_P(y_P \xi) \ . \tag{5}$$

This expression can be interpreted as the "residual supply curve", analogous to the notion of residual demand in the price leadership model of oligopoly theory (e.g., Varian (1992, Chapter 16)). Hence we have the following relationship between ξ and the non-price-taker's consumption:

$$\xi = u'_P(\delta - c_N)/y_P \ . \tag{6}$$

So, as a price leader, the non-price-taker's influence on prices manifests itself, via (6), as his consumption affecting the Arrow-Debreu prices. Recasting the non-price-taker's effect in this consumption-based way allows the analysis of general preferences and general asset payoffs, thus extending the current literature. Since the non-price-taker finances his consumption through his portfolio strategy, and since asset prices are determined from Arrow-Debreu prices via (2), this non-price-taking price dependence is exactly equivalent to a dependence of the asset prices on the trading strategy.

The non-price-taker, then, solves the following optimization problem:

$$\begin{aligned} &\max_{c_N, \xi} E[u_N(c_N)] \\ &\text{subject to} \quad E[\xi c_N] \leq E[\xi \epsilon_N] \\ &\text{and} \qquad \xi = u'_P(\delta - c_N)/y_P, \quad \text{where } y_P \text{ satisfies (4).} \end{aligned} \tag{7}$$

We note that, in contrast to the price-taker the non-price-taker's budget constraint is nonlinear in his consumption choice.

Remark 1. The non-price-taker's constraint set is not necessarily convex, so showing optimality to problem (7) and hence the existence of equilibrium is not straightforward. A set of sufficient conditions for convexity of the budget set (and sufficiency of the first order conditions and uniqueness of the solution later stated to this problem) is: (i) $u_P''' > 0$ (implying decreasing absolute risk aversion of the price-taker), (ii) $-cu_P'''/u_P'' < 2$ (relative prudence of the price-taker less than 2), and (iii) $\epsilon_N < \delta$. An example of a utility function satisfying (i) and (ii) is the HARA utility, $u_P(c) = \frac{1-\gamma}{\gamma}\left(\frac{\beta c}{1-\gamma} + \eta\right)^{\gamma}$, with $\gamma \in (0,1)$, including constant relative risk aversion preferences with relative risk aversion less than 1. (The full HARA utility family has $\beta > 0,\ \gamma \neq 1,\ \eta \geq 0$, with $\eta = 1$ when $\gamma = -\infty$, and is defined over the domain where $\left(\frac{\beta c}{1-\gamma} + \eta\right) > 0$, and includes power utility ($\beta = 1,\ \eta = 0,\ \gamma < 1$), logarithmic utility ($\beta = 1,\ \eta = 0,\ \gamma = 0$), negative exponential utility ($\eta = 1,\ \gamma = -\infty$) and quadratic utility ($\gamma = 2$).)

Remark 2. We could equivalently work from the price leadership in the securities markets by calculating the price-taker's asset demands as a function of the entire price system, writing the residual supplies as $\alpha_{Ni} = 1 - \hat{\alpha}_{Pi}(r, \{S_j\}),\ i = 1, \ldots, L;\ \ \alpha_{N0} = -\hat{\alpha}_{P0}(r, \{S_j\})$, and then attempting to deduce the effect of α_N on the price system. However, a derivation of the price-taker's asset demands in our general set-up does not yield explicit expressions, and inversion does not in general yield a simple non-price-taking effect such as increased asset holdings leading to a higher asset price. On the other hand, if instead of following a price leadership model, we were to specify a particular price-dependence in an asset market [as, for example, in Jarrow (1992)], for an equilibrium analysis it would be necessary to ensure consistency of this specification with market clearing and optimality. This is not a straightforward exercise and in general may not be possible.

4 Characterization of equilibrium

It can be shown that in equilibrium the vector (y_N, y_P) is only determined up to a multiplicative constant, so we may let $y_P = 1$, thereby expressing both agents' weights relative to the price-taker's. Hence, from (6) we obtain the mapping

$$\xi(\omega) = u_P'(\delta(\omega) - c_N(\omega)) \ , \tag{8}$$

revealing that the non-price-taker's state-ω consumption only affects the state price in that state, with $\xi(\omega)$ increasing in $c_N(\omega)$.

4.1 Equilibrium consumption allocations

Assuming equilibrium exists,[3] Proposition 1 characterizes the non-price-taker's solution to his optimization problem, and hence the equilibrium consumption allocations and state prices.

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[3] Existence of equilibrium requires demonstrating: (i) existence of a solution (c_N^*, y_N) to (9)–(10), (ii) optimality of the non-price-taker's policy (c_N^*, α_N^*), (iii) existence of y_P solving (4), and (iv) optimality of the price-taker's policy. (i) and (ii) are nontrivial for general utility, but have been shown for HARA preferences with $\gamma \in (0,1)$ of the price-taker (details available from the author upon request). (iii) and (iv) are standard.

Proposition 1. *If an equilibrium exists, then the non-price-taker's equilibrium consumption and weight, c_N^* and y_N, satisfy*

$$u_N'(c_N^*) = y_N\left[u_P'(\delta - c_N^*) - u_P''(\delta - c_N^*)(c_N^* - \epsilon_N)\right] \ , \tag{9}$$

and

$$E\left[u_P'(\delta - c_N^*)c_N^*\right] = E\left[u_P'(\delta - c_N^*)\epsilon_N\right] \ . \tag{10}$$

Subsequently, the equilibrium state price density is determined from (8) and the price-taker's equilibrium consumption and weight from (3) and (4).

The effect of agent N being a non-price-taker is an extra term in his first order condition (9). His marginal benefit from an extra unit of consumption in state ω is proportional to the cost $\xi(\omega) = u_P'(\delta(\omega) - c_N(\omega))$ of that extra unit, plus an additional "cost" term, $-u_P''(\delta - c_N^*)(c_N^* - \epsilon_N)$, due to the direct effect of this extra unit of consumption on the price of consumption. Since $u_P''(\cdot) < 0$, when $(c_N^* - \epsilon_N)$ is positive the additional term in (9) is positive, hence increasing $u_N'(c_N^*)$ or decreasing c_N^*; and vice versa when $(c_N^* - \epsilon_N)$ is negative. So the presence of the additional term tends to induce the non-price-taker to deviate towards his "own" endowment payoff, ϵ_N.[4] The intuition is that when the non-price-taker is, say, a net "buyer" of consumption $(c_N^* > \epsilon_N)$, it is in his interest to reduce the price of consumption in that state, and he recognizes that he can do this by decreasing his own consumption (and vice versa).

Rewriting equation (9) as

$$u_N'(c_N^*) = y_N u_P'(\delta - c_N^*)\left[1 - \frac{u_P''(c_P^*)}{u_P'(c_P^*)}(c_N^* - \epsilon_N)\right] \ , \tag{11}$$

we see that the extent of agent N's deviation towards ϵ_N depends both on how much of a net consumer he is $(c_N^* - \epsilon_N)$ and on the absolute risk aversion of the price-taker $(-u_P''/u_P')$. The more risk averse the price-taker, the less his consumption reacts to changes in the state price or, conversely, the more the state price reacts to his (and hence also the non-price-taker's) consumption and so the more incentive the non-price-taker has to deviate. In the limit of a risk neutral price-taker, the non-price-taker cannot affect the state price at all and so does not deviate from his price-taking behavior.

The following proposition establishes formally when there is or is not an effect of making agent N a non-price-taker.

Proposition 2.

(a) Suppose agents' endowments are such that agent N does not trade in equilibrium in the economy where both are price-takers. Then in equilib-

[4] Since y_N and $u_P'(\delta - c_N^*)$ differ across economies we cannot say that c_N^* is always closer to ϵ_N than is N's consumption in the price-taking economy, but it is reasonable to assume that this will be the case when agent N is enough of a net buyer or seller of consumption good.

rium in the economy where agent N is a non-price-taker, he does not deviate from his price-taking consumption-portfolio strategy.

(*b*) *If the initial endowments are such that agent N does trade in equilibrium in the economy where both are price-takers, in the non-price-taking equilibrium the non-price-taker N does deviate from his price-taking strategy.*

If the price-taking optimal behavior of agent N is to not trade and to consume his "own" dividend, it turns out that he always makes himself worse off by deviating from the price-taking case. Suppose in some state he increases his consumption and so becomes a net buyer of consumption good. By doing so, he simultaneously raises the price of consumption in that state, which would affect him adversely. If he becomes a net seller of consumption, he at the same time reduces its price; deviation in either direction has an adverse effect.

We note that the non-price-taker's influence on the equilibrium does not depend on how wealthy he is, but on how much of a trader he is. Even if agent N has no initial wealth ($e_N = 0$) he may still change the equilibrium by being a non-price-taker. For constant relative risk aversion agents, if agent N has no initial wealth, it happens that he never trades and so there is no deviation from the price-taking economy. However, for CARA agents, it is not $e_{Ni} = 0$ but $e_{Ni} = \delta_i/2, \ i = 1, \ldots, L$ for which there is (no trading and hence) no deviation from the price-taking equilibrium. As long as agent N trades in the benchmark economy, he does deviate from his price-taking strategy, which causes a distinct deviation in the state prices. This contrasts with the Grinblatt and Ross (1985) result of no influence of a non-price-taker under symmetric information. Their non-price-taker is restricted to be risk neutral and also to only have risky asset demands linear in the initial price.

An appropriate question is whether the non-price-taker gains any advantage through taking into account his effect on prices. Proposition 3 states that indeed the non-price-taker is at least as well off as if he were a price-taker. This is because when solving his optimization problem the non-price-taking agent always has the option of choosing his price-taking equilibrium consumption.

Proposition 3. *In equilibrium in the non-price-taking economy, agent N's expected lifetime utility from consumption is greater than or equal to that in equilibrium in the price-taking economy, i.e.,* $E[u_N(c_N^*)] \geq E[u_N(\bar{c}_N)]$.

We have seen that, unlike the price-taker, the non-price-taker's marginal utility of consumption is not proportional to the state price. Hence his marginal rate of substitution between consumption across states does not equal the state price ratio, whereas the price-taker's does. This discrepancy between agents' marginal rates of substitution, implies that the non-price-taking equilibrium consumption allocations are not pareto optimal, as stated in Proposition 4.

Proposition 4. *If the non-price-taking equilibrium differs from the price-taking equilibrium, then the non-price-taking equilibrium allocations are not pareto optimal.*

4.2 Equilibrium asset risk premia

It is well-known that in the economy with agent N also a price-taker, the equilibrium consumption allocations, $\bar{c}_N(\delta;\bar{y}_N)$ and $\bar{c}_P(\delta;\bar{y}_N)$, are only a function of the aggregate consumption δ. Hence, all agents' consumption are perfectly correlated. Furthermore, the equilibrium state price density $\bar{\xi}(\delta;\bar{y}_N)$ is also driven only by the aggregate consumption. Consequently, the risk premium of an asset, $\bar{\mu}_i - \bar{r}$, where $\bar{\mu}_i \equiv E[\delta_i]/\bar{S}_i - 1$, is positively related to the covariance of its return with the aggregate consumption:

$$\bar{\mu}_i - \bar{r} = -\mathrm{cov}\left(\frac{\delta_i}{\bar{S}_i}, U'(\delta;\bar{y}_N)\right) \approx \bar{\lambda}_\delta \,\mathrm{cov}\left(\frac{\delta_i}{\bar{P}_i}, \delta\right), \quad i = 1, \ldots, L\,,$$

where $\approx$ (here and elsewhere) denotes employment in the covariance or variance operator of a first order approximation,[5] and where $\bar{r} = 1/E[U'(\delta;\bar{y}_N)] - 1$ and $\bar{\lambda}_\delta \equiv -U''(\mu_\delta;\bar{y}_N) > 0$. Here $U(\cdot;\bar{y}_N)$ is the standard utility function of the representative agent of both agents (analogous to the one defined in Section 6), and $\mu_\delta \equiv E[\delta]$.

In our non-price-taking economy, the solutions (if they exist) for the equilibrium consumption allocations from Proposition 1, $c_N^*(\delta, \epsilon_N; y_N)$ and $c_P^*(\delta, \epsilon_N; y_N)$, are driven by two factors, the aggregate consumption and the payoff from N's initial endowment, ϵ_N. A representative agent over the two agents can no longer be defined in the standard manner. Consequently, the agents' consumption are no longer perfectly correlated with each other, nor with the aggregate consumption. The equilibrium state price density $\xi(\delta, \epsilon_N; y_N)$ and asset risk premia are also driven by the two factors, as summarized in Proposition 5.

Proposition 5. *Assume an equilibrium exists. The equilibrium asset risk premia are given by:*

$$\mu_i^* - r^* = \mathrm{cov}\left(\frac{\delta_i}{S_i^*}, u_P'(\delta - c_N^*)\right) \approx \lambda_\delta^* \,\mathrm{cov}\left(\frac{\delta_i}{S_i^*}, \delta\right) - \lambda_{\epsilon_N}^* \,\mathrm{cov}\left(\frac{\delta_i}{S_i^*}, \epsilon_N\right)\,, \quad i = 1, \ldots, L, \tag{12}$$

where

[5] The nonlinear function $F(\delta, \epsilon_N)$ in the operator is linearized around the means of δ and ϵ_N, μ_δ and μ_{ϵ_N}, using a first order Taylor's expansion $F(\delta, \epsilon_N) \approx F(\mu_\delta, \mu_{\epsilon_N}) + F_1(\mu_\delta, \mu_{\epsilon_N})(\delta - \mu_\delta) + F_2(\mu_\delta, \mu_{\epsilon_N})(\epsilon_N - \mu_{\epsilon_N})$. This local approximation is valid for small deviations of δ and ϵ_N from their means.

$$\lambda_\delta^* \equiv -u_P''(\mu_\delta - c_N^E)\left\{\frac{u_N''(c_N^E) + y_N u_P''(\mu_\delta - c_N^E)}{u_N''(c_N^E) - y_N u_P'''(\mu_\delta - c_N^E)(c_N^E - \mu_{\epsilon_N}) + 2y_N u_P''(\mu_\delta - c_N^E)}\right\}$$

$$\lambda_{\epsilon_N}^* \equiv \frac{y_N u_P''(\mu_\delta - c_N^E)}{u_N''(c_N^E) + y_N u_P''(\mu_\delta - c_N^E)}\lambda_\delta^* ,$$

$r^* = 1/E[u_P'(\delta - c_N^*)] - 1$, $\mu_\delta \equiv E[\delta], \mu_{\epsilon_N} \equiv E[\epsilon_N]$, *and* c_N^E *is the solution to* $u_N'(c_N^E) = y_N[u_P'(\mu_\delta - c_N^E) - u_P''(\mu_\delta - c_N^E)(c_N^E - \mu_{\epsilon_N})]$, *i.e., the equilibrium consumption if the aggregate consumption and agent N's endowment were to take on their expected values.*

An asset's risk premium is now driven by the covariance of its return with both the aggregate consumption and the payoff from the non-price-taker's endowment. This extends the Lindenberg (1979) result beyond a mean-variance framework. The λ^*'s are nonzero and have the same sign, so the dependencies on the two covariances are of opposite sign. It can be show that λ_δ^* and $\lambda_{\epsilon_N}^*$ are positive for the price-taker having HARA and the non-price-taker having any utility function defined over some domain (c_∞, ∞) $(c_\infty \geq -\infty)$, satisfying $\lim_{c\to\infty} u_N'(c) = 0$ and $\lim_{c\to c_\infty} u_N' = \infty$.[6] In this case, an asset whose return is positively correlated with the aggregate consumption, may yet demand a negative risk premium if its return is highly correlated with the non-price-taker's endowment. Indeed, in the CARA example of Section 5 we show that this nonstandard result is possible. The non-price-taker wishes to push up the price of his own endowment portfolio (or of portfolios highly correlated with his endowment) so as to increase his initial wealth; hence the risk premia of such portfolios are reduced. It is an open question whether for other utility functions the λ's can go negative, depending on u_P''' and $(c_N^* - \epsilon_N)$. A traditional one-factor asset pricing formula arises when δ and ϵ_N are perfectly correlated, collapsing the two terms together.

5 CARA utility example

In order to derive further implications of non-price-taking behavior, we now specialize our general set-up to the case where both the price-taker and the non-price-taker exhibit CARA preferences, $u(c) = -\exp\{-ac\}/a$, $a > 0$, defined over the domain $(-\infty, \infty)$. The results for the benchmark economy are straightforward to derive and are often quoted without proof.

5.1 Equilibrium consumption allocations

In the benchmark price-taking economy, the equilibrium consumption allocations are

[6] For $\lambda_\delta^*, \lambda_{\epsilon_N}^* > 0$, we also need to assume that agents' endowment payoffs, ϵ_N and ϵ_P lie within the domains of their respective utility functions, i.e., $\epsilon_N > c_\infty$ and $\left(\frac{\beta\epsilon_P}{1-\gamma} + \eta\right) > 0$.

$$\bar{c}_N = \frac{1}{2}\delta - \frac{1}{2a}\ln(\bar{y}_N), \quad \bar{c}_P = \frac{1}{2}\delta + \frac{1}{2a}\ln(\bar{y}_N) , \tag{13}$$

where $\bar{y}_N$ is given by

$$\frac{1}{2a}\ln(\bar{y}_N) = \frac{E\left[(\delta - \epsilon_N)\exp\{-\frac{1}{2}a\delta\}\right]}{E\left[\exp\{-\frac{1}{2}a\delta\}\right]} . \tag{14}$$

Each agent shares the risk equally, consuming half the aggregate consumption plus a constant depending on his relative initial wealth.

In the non-price-taking economy the equilibrium consumptions are given by

$$c_N^* = \frac{1}{2}\delta - \frac{1}{2a}\ln[1 + a(c_N^* - \epsilon_N)] - \frac{1}{2a}\ln(y_N) , \tag{15}$$

$$c_P^* = \frac{1}{2}\delta - \frac{1}{2a}\ln[1 + a(c_N^* - \epsilon_N)] + \frac{1}{2a}\ln(y_N) , \tag{16}$$

where y_N is such that c_N^* satisfies

$$E\left[c_N^* \exp\{-a(\delta - c_N^*)\}\right] = E\left[\epsilon_N \exp\{-a(\delta - c_N^*)\}\right].$$

Agents consume half the aggregate consumption plus additional random terms. For fixed y_N, [7]

$$\partial c_N^*/\partial\delta = f/(2f+1) < 1/2 = \partial\bar{c}_N/\partial\delta ,$$
$$\partial c_N^*/\partial\epsilon_N = 1/(2f+1) > 0 = \partial\bar{c}_N/\partial\epsilon_N ,$$

where $f \equiv \exp\{-2a(c_N^* - \delta/2)\}/y_N > 0$. In contrast to the benchmark economy, agent N chooses his consumption to react positively (in moving across states) to changes in his endowment; he wants to push up the price of consumption in states where his endowment's payoff is high to increase his endowment's value. The non-price-taker reacts less to changes in aggregate consumption, while the price-taker reacts more. Proposition 6 compares the levels of agents' consumption across and within economies.

Proposition 6.

(a) For given ϵ_N, $\quad \text{sign}\{c_N^* - \bar{c}_N\} = \text{sign}\{\delta_{crit}(\epsilon_N) - \delta\}$,
where $\delta_{\text{crit}}(\epsilon_N) \equiv 2(\bar{y}_N/y_N - 1)/a + \ln(\bar{y}_N)/a + 2\epsilon_N$;
for given δ, $\quad \text{sign}\{c_N^* - \bar{c}_N\} = \text{sign}\{\epsilon_N - \epsilon_{N\text{crit}}(\delta)\}$,
where $\epsilon_{N\text{crit}}(\delta) \equiv \delta/2 - (\bar{y}_N/y_N - 1)/a - \ln(\bar{y}_N)/(2a)$.

(b) In the case of $e_{Ni} = e_N$, $i = 1, \ldots, L$ *(perfect correlation of* ϵ_N *and* δ*), when* $e_N > 1/2$: $\bar{c}_N > \bar{c}_P$ *a.s., but we may have* $c_N^* < c_P^*$. *When* $e_N < 1/2$: $\bar{c}_N < \bar{c}_P$ *a.s., but we may have* $c_N^* > c_P^*$.

Part (a) reveals that for a given endowment, as the aggregate consumption decreases below a critical level the non-price-taker consumes more than

---

[7] We abuse notation by using c_N^* to denote the mapping $c_N^*(\delta, \epsilon_N; y_N)$ implied in (15), not the random variable.

if he were a price-taker, and as it increases above the critical level he consumes less; as discussed above, he is reacting less to changes in aggregate consumption. On the other hand, he reacts more to changes in his own endowment. Part (b) compares agents N and P within economies, providing an example in which even if the non-price-taker initially owns more than half of the market, there may be states in which he chooses to consume less than the less-endowed price-taker.

We next compare the variability in agents' consumption. In the benchmark equilibrium the volatilities of both agents' consumption are equal, $\mathrm{var}(\bar{c}_N) = \mathrm{var}(\bar{c}_p) = \mathrm{var}(\delta)/4$. Consequently, $\mathrm{var}(\bar{c}_N - \epsilon_N) = \mathrm{var}(\delta/2 - \epsilon_N)$.

Proposition 7. *In the non-price-taking equilibrium, the non-price-taking agent's consumption variability is approximated by*

$$\mathrm{var}(c_N^*) \approx \left(\frac{f^E}{2f^E+1}\right)^2 \mathrm{var}(\delta) + \left(\frac{1}{2f^E+1}\right)^2 \mathrm{var}(\epsilon_N) + \frac{2f^E}{(2f^E+1)^2}\,\mathrm{cov}(\delta, \epsilon_N)\ , \tag{17}$$

where $f^E \equiv \exp\{-2a(c_N^E - \mu_\delta/2)\}/y_N > 0$ *and* c_N^E *is as given in Proposition 5. Consequently,*

$$\mathrm{var}(c_N^* - \epsilon_N) \approx \left(\frac{2f^E}{2f^E+1}\right)^2 \mathrm{var}\left(\frac{\delta}{2} - \epsilon_N\right)\ .$$

The non-price-taker's consumption volatility tends to be higher in the non-price-taking economy than in the benchmark economy if his own endowment volatility is high compared with the aggregate consumption volatility and/or if his endowment is highly positively correlated with the aggregate consumption. The second result in Proposition 7 reveals that (under this linear approximation) the volatility of the difference between the non-price-taker's consumption and his own endowment payoff is reduced when he is a non-price-taker. This formalizes the intuition discussed in Section 4.1 that he deviates towards his own endowment payoff.

5.2 Equilibrium state prices and asset risk premia

In the benchmark economy, the equilibrium state price density is $\bar{\xi} = \exp\{-a\delta/2\}\bar{y}_N^{-1/2}$. In the non-price-taking economy, ξ^* satisfies equation (8). Hence, for fixed y_N we have

$$\partial \ln(\xi^*)/\partial\delta = -a(f+1)/(2f+1) > -a/2 = \partial \ln(\bar{\xi})/\partial\delta\ ,$$
$$\partial \ln(\xi^*)/\partial\epsilon_N = a/(2f+1) > 0 = \partial \ln(\xi)/\partial\epsilon_N\ .$$

The state price density reacts less (negatively) to changes in the aggregate consumption than in the benchmark economy, while now reacting positively to the non-price-taker's endowment. The non-price-taker desires to push the price of consumption up in states where his own endowment is high so as to increase the value of his endowment.

The variability in the logarithm of the state price density in the price-taking economy is given by $\mathrm{var}(\ln(\bar{\xi})) = a^2\sigma_\delta^2/4$. Proposition 8 presents the corresponding expression for the non-price-taking economy, and the related expression for asset risk premia.

Proposition 8. *In the non-price-taking economy, the variability in the (log) state price density is approximated by*

$$\mathrm{var}(\ln(\xi^*)) \approx \left(\frac{f^E+1}{2f^E+1}\right)^2 a^2 \mathrm{var}(\delta) + \left(\frac{1}{2f^E+1}\right)^2 a^2\ \mathrm{var}(\epsilon_N)$$
$$- 2\frac{(f^E+1)}{(2f^E+1)^2} a^2 \,\mathrm{cov}(\delta, \epsilon_N)\ ,$$

and the asset risk premia are approximately expressed as

$$\mu_i^* - r^* \approx - u_P''(\mu_\delta - c_N^E)\left(\frac{f^E+1}{2f^E+1}\right)\mathrm{cov}\left(\frac{\delta_i}{S_i^*}, \delta\right)$$
$$+ u_P''(\mu_\delta - c_N^E)\frac{1}{2f^E+1}\mathrm{cov}\left(\frac{\delta_i}{S_i^*}, \epsilon_N\right)\ ,$$

where f^E and c_N^E *are as defined in Proposition 7.*

Under this approximation, the state price density is more volatile than in the benchmark economy, if the non-price-taker's endowment covaries negatively with aggregate consumption or if his endowment is volatile enough. The state price density is less volatile for highly positively covarying non-price-taker's endowment and aggregate consumption.

As in a price-taking economy, an asset's risk premium is positively related to the covariance of its return with aggregate consumption, but now is also negatively related to the covariance of its return with the non-price-taker's endowment. It is possible, then, for an asset whose payoff covaries positively with the aggregate consumption to demand a negative risk premium (if $\mathrm{cov}(\delta_i/S_i^*, \epsilon_N) > (f^E+1)\ \mathrm{cov}(\delta_i/S_i^*, \delta)$). When an asset's payoff covaries highly with the non-price-taker's endowment, he reduces its risk premium; hence the non-price-taker lowers the risk premia (raises the initial price) of assets he is endowed with.

6 Extension to multiple price-takers

We now assume there are N agents in the economy; agent N is still a non-price-taker while $n = 1, \ldots, N-1$ are price-takers. The definition of equilibrium is appropriately modified for N agents. Each price-taker's optimization is identical to that in Section 3. For analytical convenience, we introduce a representative price-taking agent (e.g., Magill and Quinzii (1996, Chapter 3)), whose utility function is defined by

$$U_P(c;\Lambda) \equiv \max_{c_1,\ldots,c_{N-1}} \sum_{n=1}^{N-1} \lambda_n u_n(c_n)\ ; \quad \Lambda \equiv (\lambda_1, \ldots, \lambda_{N-1}) \in \mathscr{R}_{++}^{N-1}\ ,$$

subject to $\sum_{n=1}^{N-1} c_n = c$. The inverse of $U_P'(c;\Lambda)$ is given by $J_P(h;\Lambda) \equiv \sum_{n=1}^{N-1} I_n(h/\lambda_n)$. Identifying $\Lambda = (1/y_1 \ldots, 1/y_{N-1})$, the aggregate optimal price-taker consumption is given from (3) by

$$\sum_{n=1}^{N-1} \hat{c}_n = J_P(\xi; 1/y_1, \ldots, 1/y_{N-1}) \ . \tag{18}$$

Clearing in the consumption good yields the "residual supply curve" analogous to (5), $c_N = \delta - J_P(\xi; 1/y_1, \ldots, 1/y_{N-1})$, and hence

$$\xi = U_P'(\delta - c_N; 1/y_1, \ldots, 1/y_{N-1}) \ , \tag{19}$$

where $(1/y_1, \ldots, 1/y_{N-1})$ satisfy the analogues of (4). The non-price-taker's problem (7) is readily modified for this dependence of state prices on his consumption choice. The remainder of our analysis depends upon the nature of the representative price-taker's utility function, divided into two cases.

6.1 Representative price-taker independent of individual weights

Here, we consider the case where the representative price-taker's utility function can be written as

$$U_P(c;\Lambda) = h(\Lambda)U_P(c) \ .$$

This essentially restricts the representative price-taker's utility function to be independent of the individual weights $\lambda_n, n = 1, \ldots, N-1$. The set of individual price-taker's utility functions satisfying this property consists of those belonging to the HARA class, $u_n(c_n) = \frac{1-\gamma}{\gamma}\left(\frac{\beta_n c_n}{1-\gamma} + \eta_n\right)^{\gamma}$, with identical marginal risk tolerance $1/(1-\gamma)$ across all price takers [e.g., Magill and Quinzii (1996, Chapter 3)]. This includes the familiar cases of all price-takers having the same power or logarithmic utility, or negative exponential or quadratic utility with possibly differing absolute risk aversion. Under the assumption of von Neumann-Morgenstern preferences, this set of preferences provides well-known conditions (Milne (1979)) for the classical notion of Gorman (1953) aggregation, which more generally requires agents to exhibit quasi-homothetic preferences with a specific relationship between all agents.

We may let $h(\Lambda) = 1$ without loss of generality, so that $\xi(\omega) = U_P'(\delta(\omega) - c_N(\omega))$; the non-price-taker's state-ω consumption again only affects the state price in that state. In this case all the previous propositions and intuition of the two-agent case go through, with the single price-taker's utility u_P simply replaced by the representative price-taker's utility U_P.

6.2 Representative price-taker dependent on individual weights

If the representative price-taker's utility function is not independent of the individual price-takers' weights, according to (19) agent N's consumption in state ω, depends not only on $\xi(\omega)$ but also on the vector of weights Λ. Since the weights are driven by the whole random variable ξ, $c_N(\omega)$ now affects the random variable ξ in all states; there is no longer a one-to-one mapping between $c_N(\omega)$ and $\xi(\omega)$. Now the non-price-taker has to worry about the externalities he imposes on the other agents by his choice of consumption (and hence state prices), since he influences the distribution of wealth across agents. The analysis of his optimization problem becomes much more complicated, as seen in his first order condition, presented in Proposition 9.

Proposition 9. *If an equilibrium exists, then the non-price-taker's consumption and all agents' weights,* c_N^* and $(y_1, \dots, y_N)$, *satisfy*

$$
\begin{aligned}
u_N'(c_N^*) = & y_N[U_P'(\delta - c_N^*;\Lambda) - U_P''(\delta - c_N^*;\Lambda)(c_N^* - \epsilon_N)] \\
& - \sum_{n=1}^{N-1} K_n \frac{U_P''(\delta - c_N^*;\Lambda)}{u_n''(I_n(y_n U_P'(\delta - c_N^*;\Lambda)))} \\
& * \left[y_n U_P'(\delta - c_N^*;\Lambda) + u_n''(I_n(y_n U_P'(\delta - c_N^*;\Lambda))) \right. \\
& \left. * (I_n(y_n U_P'(\delta - c_N^*;\Lambda)) - \epsilon_n)\right] ,
\end{aligned} \tag{20}
$$

$$
E[U_P'(\delta - c_N^*;\Lambda)c_N^*] = E[U_P'(\delta - c_N^*;\Lambda)\epsilon_N] , \tag{21}
$$

and

$$
\begin{aligned}
& E[U_P'(\delta - c_N^*;\Lambda) I_n(y_n U_P'(\delta - c_N^*;\Lambda))] \\
& = E[U_P'(\delta - c_N^*;\Lambda)\epsilon_n], \quad n = 1, \dots, N-1 ,
\end{aligned} \tag{22}
$$

where K_n is as defined in the Appendix. Subsequently, the equilibrium state price density is determined from (19), and the price-takers' equilibrium consumption from (3). If $U_P(c;\Lambda) = h(\Lambda)U_P(c)$, then $K_n = 0$ for all $n = 1, \dots, N-1$.

Equation (20) is similar to (9) but with $N-1$ extra terms. Again, the marginal benefit to the non-price-taker from an extra unit of consumption in state ω must equal the total "costliness" to him of that extra unit. The first and second terms on the right-hand-side of (20) are again the cost of that extra unit of consumption, and the costliness to him due to the direct effect of $c_N(\omega)$ on $\xi(\omega)$. In this case, however, an extra unit of consumption is costly to agent N in a third way. The non-price-taker also realizes that an extra unit of $c_N(\omega)$ now affects the effective wealth distribution of the other agents which in turn affects the whole random state price density ξ, and hence the non-price-taker's satisfaction in all other states. We argue in the proof of Proposition 9 that the extra terms in (20) are indeed the indirect incremental change in agent N's expected utility $E[u_N(c_N)]$ via the effect of an extra unit of $c_N(\omega)$ on each of the other agents' budget constraints. The last statement of Proposition 9 shows that (20) does collapse to (9) when the price-taker representative agent utility function is independent of individual weights.

Propositions 3 (the non-price-taker better off) and 4 (pareto inefficiency) carry through to the more general multiple price-taker case of this Section 6.2. Proposition 2 (conditions for the non-price-taker to not deviate), however, fails because of the complexity of the state price dependence on agent N's allocation. Referring back to the intuition for Proposition 2, here if agent N were consuming exactly his "own" dividend, by deviating to become a net buyer of consumption good in some states, he may not necessarily increase the state price in that state and he may also affect the state price in all other states. Hence unlike in the single price-taker case, this deviation may not be detrimental to him. The asset risk premia expression of Proposition 5 fails, since, from equation (20), the consumption allocations, state prices and asset prices depend on N driving factors $(\delta, \epsilon_2, \ldots, \epsilon_N)$. In this case a simple representation for the asset risk premia may not be derived as it was in Proposition 5 or its continuous-time limit in Section 7.

7 Dynamic extension in continuous-time

We now consider a dynamic extension of the two-agent economy, a continuous-time variation on the Lucas (1978) pure-exchange economy.[8] The static analysis carries through readily to the dynamic case; recasting the non-price-taker's price impact as an impact of his consumption on state prices combines naturally with martingale techniques, allowing conversion of the dynamic non-price-taking problem to an effectively static one. The non-price-taking strategy we solve for is a "self-commitment" strategy in which the non-price-taker chooses a plan initially and then does not deviate from that plan. A major advantage of the continuous-time formulation is that the approximate expressions in the static formulation for asset risk premia, consumption and state price density variabilities, collapse to exact formulae in continuous-time with continuous trading.

The economy has a finite horizon $[0, T]$. Uncertainty is represented by a probability space $(\Omega, \mathscr{F}, \mathscr{P})$ on which is defined an L-dimensional Brownian motion, $W = (W_1, \ldots, W_L)^{\top}$. All stochastic processes are assumed adapted to the filtration generated by W. The risky stocks pay out a dividend stream at rate δ_i, with aggregate dividend $\delta(t) \equiv \sum_{i=1}^{L} \delta_i(t)$ following an Itô process

$$d\delta(t) = \mu_\delta(t)dt + \sum_{j=1}^{L} \sigma_{\delta j}(t)dW_j(t) \ .$$

The bond and (ex-dividend) stock price dynamics are posited to follow

$$dS_0(t) = S_0(t)r(t)dt \ ,$$

$$dS_i(t) + \delta_i(t)dt = S_i(t)\left[\mu_i(t)dt + \sum_{j=1}^{L} \sigma_{ij}(t)dW_j(t)\right] \ , \quad i = 1, \ldots, L,$$

with $S_i(T) = 0$, and the volatility matrix $\sigma \equiv \{\sigma_{ij}\}$ assumed invertible. The endogenous price system is represented by the stochastic processes

[8] For more details on this extension, see Basak (1996). The formulation follows the continuous-time pure-exchange general equilibrium models recently developed by Duffie and Huang (1985), Duffie (1986), Huang (1987), Duffie and Zame (1989), and Karatzas, Lehoczky and Shreve (1990).

(r, μ, σ), $\mu \equiv (\mu_1, \ldots, \mu_L)^\top$. The posited dynamic market completeness allows the construction of a unique state price density process ξ with dynamics

$$d\xi(t) = -\xi(t)[r(t)dt + \theta(t)^\top dW(t)] \ , \tag{23}$$

where θ is the market price of risk process, $\theta(t) \equiv \sigma(t)^{-1}[\mu(t) - r(t)\mathbf{1}]$, and $\mathbf{1}$ is an L-dimensional vector with every component equal to 1. Agents now derive time-additive state-independent utility $u_n(c_n(t))$ from intertemporal consumption in $[0, T]$, and finance their consumption through an $(L+1)$-dimensional portfolio process $\alpha_n \equiv (\alpha_{n0}, \ldots, \alpha_{nL})^\top$, denoting units of shares held.

Using the martingale representation approach, the price-taker's dynamic optimization problem is converted into a static variational problem (Cox and Huang (1989), Karatzas, Lehoczky and Shreve (1987)). Consequently, the price-taker's first order condition (3) here holds at all states and times. Equilibrium is defined as in the two-period model except with (r, μ, σ) as the price system. Analogous to the static case, in dynamically complete markets, to ensure clearing in all markets at agents' optima, it suffices to simply clear the consumption good market (e.g., Karatzas, Lehoczky and Shreve (1990), Basak (1995)). Hence the price leadership of agent N is fully captured by equation (6), now holding at all states and times. In the continuous-time model, recasting the non-price-taker's effect in this way combines naturally with the martingale method of solution of agents' optimization problems.

The non-price-taker maximizes expected lifetime utility over consumption-portfolio policies and price systems (r, μ, σ) subject to his dynamic budget constraint. Similarly to the case of price-takers, we may convert the non-price-taker's dynamic optimization problem into a static one. In the well-known case of price-taking agents, their dynamic budget constraint is equivalent to the static budget constraint for all price systems. Hence this result can certainly be applied to the non-price-taker under the particular price system restricted to obey equation (6). So, the non-price-taker's optimization problem is rewritten as

$$\max_{c_N, \xi} E\left[\int_0^T u_N(c_N(t))dt\right]$$

$$\text{subject to } E\left[\int_0^T \xi(t)c_N(t)dt\right] \le E\left[\int_0^T \xi(t)\epsilon_N(t)dt\right]$$

$$\text{and } \xi(t) = u_P'(\delta(t) - c_N(t))/y_P$$

where y_P satisfies $E[\int_0^T \xi(t)I_P(y_P\xi(t))dt] = E[\int_0^T \xi(t)\epsilon_P(t)dt]$. Here the agents' endowment streams ϵ_n are defined by $\epsilon_n(t) \equiv \sum_{i=1}^L e_{ni}\delta_i(t)$, $t \in [0, T], n = N, P$. From this static variational problem, Propositions 1 to 4 follow through for the dynamic case, equations (9)–(10) holding for all states and times. An advantage of the continuous-time formulation is the convenient representation (23) for the state price density process in terms of the interest rate and market price of risk processes. This leads to a two-factor modification of the consumption-based CAPM (Breeden (1979)) yielding an exact formula for the asset risk premia:

$$\mu_i^*(t) - r^*(t) = \frac{\lambda_\delta^*(t)}{u_P'(\delta(t) - c_N^*(t))} \operatorname{cov}\left(\frac{dS_i^*(t)}{S_i^*(t)}, d\delta(t)\right) - \frac{\lambda_{\epsilon_N}^*(t)}{u_P'(\delta(t) - c_N^*(t))} \operatorname{cov}\left(\frac{dS_i^*(t)}{S_i^*(t)}, d\epsilon_N(t)\right),$$

where λ_δ^* and $\lambda_{\epsilon_N}^*$ are as in Proposition 5 but evaluated at $\delta(t), \epsilon_N(t)$ rather than their expected values. Moreover, the interest rate can be shown to be driven by the dynamics of the two factors, aggregate consumption and the non-price-taker's endowment.

The CARA utility example also follows through analogously to the two-period case, with the consumption and state price volatility expressions in Propositions 7 and 8 holding exactly. In the continuous-time model, further representations and results can be derived on agents' portfolio strategies and asset price volatilities and risk premia, employing tools from Malliavin calculus, in particular the Clark-Ocone formula (Basak (1996)).

Remark 3. In this dynamic framework, the non-price-taker's optimal consumption-portfolio strategy is time-inconsistent (e.g., Sargent (1987, p.11)), in the sense that the non-price-taker has an incentive to deviate at a later date from the strategy chosen at time 0. The intuition is similar to the familiar time-inconsistency arising in the analysis of pricing by a durable-good monopolist (e.g., Tirole (1988, Chapter 1)) and is related to the Coase Conjecture (1972). The financial assets in our model are similar to durable goods in that their value is durable over many periods. The reason for the time-inconsistency is that at time 0, the non-price-taker takes account of the effect of his future (say time s) actions on the current asset prices (through the multi-period version of equation (2)). However, when agent N gets to time s he no longer cares about his effect on the past prices and so changes his optimal strategy. An alternative to the self-commitment strategy would be to solve for the non-price-taker's subgame perfect strategy, obtained by backward induction, which is a time-consistent strategy. This can be thought of as the "short-sighted" strategy, since the non-price-taker reoptimizes every period. Our preliminary analysis suggests that this problem is, however, intractable in our general framework.[9] Also, since the non-price-taker is restricted to only follow strategies which are optimal in all subgames, he must be worse off in the short-sighted strategy than the self-commitment strategy. In fact, of all strategies, the "self-commitment" strategy is the best the non-price-taker can do to maximize his lifetime utility. Accordingly, the non-price-taker may want to create some mechanism to force himself to commit (e.g., hire an agent at time 0 to implement the optimal strategy and disallow subsequent changes). We finally note that, due to the time-inconsistency of

[9] Kihlstrom (1996) constructs a three-period securities market example containing a risk-neutral non-price-taker in a risky asset market who cannot make future commitments. He illustrates that Coase's argument may extend to a dynamic securities market, and investigates conditions under which monopoly power cannot be fully exercised.

the non-price-taker's strategy, this problem provides us with an example where dynamic programming cannot be used to solve for the commitment solution. In this case, to our knowledge, the martingale method is the only way to solve this problem.

8 Conclusion

In this paper we develop a pure-exchange, general equilibrium model to include an agent who acts as a price leader in the security and good markets. We analyze the equilibrium consumption-portfolio choice of this non-price-taking agent for general state-independent utility functions and in more detail for the special case of CARA utility. We investigate the effect of the presence of the non-price-taker on asset and state prices.

A methodological contribution of this paper is to demonstrate that the non-price-taker's price-impact manifests itself through an impact of his consumption choice on Arrow-Debreu prices, making the analysis highly tractable. A main conclusion of this work is that, in addition to the aggregate consumption, the non-price-taking-taker's endowment is an extra factor driving the equilibrium allocations and prices. This leads to modified formulae for the asset risk premia. Further comparisons of the price-taking and non-price-taking equilibria are carried out.

Further work related to this paper may include the following. An extension to multiple non-price-taking agents would be of interest. One could initially formulate this extension as a one-shot Cournot game played in the Arrow-Debreu securities market. The main results of the single non-price-taker economy would extend qualitatively, with multiple additional factors now driving the economy, the endowments of each non-price-taking agent. An application of this paper would be to study non-price-taking behavior in the currency markets, a natural non-price-taking environment. Here, one could have a central bank representing a country, and model each central bank as a non-price-taker in its own currency.

9 Appendix

Proof of Proposition 1: Applying the Lagrangian method to (7) implies (9) and (10). *Q.E.D.*

Proof of Proposition 2:

(a) By assumption, $\bar{c}_N = \epsilon_N$ is a solution to equilibrium in the price-taking economy. Hence there exists a constant $\bar{y}_N$ such that $u_N'(\epsilon_N) = \bar{y}_N \xi = \bar{y}_N u_P'(\delta - \epsilon_N)$. Adding a term equaling zero to the right hand side of this equation, we obtain:

$$u_N'(\epsilon_N) = \bar{y}_N[u_P'(\delta - \epsilon_N) - u_P''(\delta - \epsilon_N)(\epsilon_N - \epsilon_N)] \ ,$$

the sufficient condition for equilibrium in the non-price-taking economy, equation (9), for $c_N^* = \epsilon_N$ and $y_N = \bar{y}_N$. Clearly agent N's budget constraint holds with equality for $c_N^* = \epsilon_N$. So $c_N^* = \epsilon_N$ is also an equilibrium in the non-price-taking economy.

(b) By assumption, $\bar{c}_N$ is a solution to equilibrium in the price-taking economy. Hence there exists a $\bar{y}_N$ such that $u_N'(\bar{c}_N) = \bar{y}_N u_P'(\delta - \bar{c}_N)$. Assume $\bar{c}_N$ is also a solution to equilibrium in the non-price taking economy. Then there exists a constant y_N such that

$$u_N'(\bar{c}_N) = y_N[u_P'(\delta - \bar{c}_N) - u_P''(\delta - \bar{c}_N)(\bar{c}_N - \epsilon_N)] \ ,$$

and $\bar{c}_N$ satisfies N's budget constraint with equality. The above expressions imply

$$\frac{\bar{y}_N - y_N}{\bar{y}_N} = \frac{u_P''(\delta - \bar{c}_N)(\bar{c}_N - \epsilon_N)}{u_P'(\delta - \bar{c}_N)} \ .$$

Since by assumption $\bar{c}_N - \epsilon_N \neq 0$ with positive probability, for the right hand side to be a constant, we must have either (i) $\bar{c}_N - \epsilon_N > 0$, or (ii) $\bar{c}_N - \epsilon_N < 0$, a.s., either of which contradicts N's budget constraint holding with equality. *Q.E.D.*

Proof of Proposition 3: Define the set of price-taking equilibrium agent-N consumption:

$$\mathscr{A}_N \equiv \{c_N; \quad \text{there exists } \bar{y}_N, \bar{y}_P, \bar{\xi} \quad \text{such that} \quad \sum_{n=N,P} I_n(\bar{y}_N \bar{\xi}) = \delta,$$
$$E[(I_n(\bar{y}_n \bar{\xi}) - \epsilon_n)\bar{\xi}] = 0, \quad n = N, P, \quad \text{and} \quad c_N = I_N(\bar{y}_N \bar{\xi}).\}$$

In a non-price-taking economy, agent N solves

$$\max_{c_N \in \mathscr{B}_N} E[u_N(c_N)],$$

where

$$\mathscr{B}_N \equiv \{c_N; \quad \text{there exists } \tilde{y}_N, \tilde{y}_P, \tilde{\xi} \quad \text{such that } c_N = \delta - I_P(\tilde{y}_P \tilde{\xi}),$$
$$E[(I_P(\tilde{y}_P \tilde{\xi}) - \epsilon_P)\tilde{\xi}] = 0, \quad \text{and } E[(c_N - \epsilon_N)\tilde{\xi}] = 0.\}$$

We show that $\mathscr{A}_N \subset \mathscr{B}_N$ so agent N could have chosen any price-taking equilibrium consumption. Take any $\bar{c}_N \in \mathscr{A}_N$. There exists $\bar{y}_N, \bar{y}_P, \bar{\xi}$ such that $\bar{c}_N = \delta - I_P(\bar{y}_P \bar{\xi})$ and $E[(\bar{c}_N - \epsilon_N)\bar{\xi}] = 0$. So $\bar{c}_N \in \mathscr{B}_N$. *Q.E.D.*

Proof of Proposition 4: We first argue that if the non-price-taking equilibrium differs from the price-taking equilibrium, then there exist subsets of state space $A, B \subset \{1, \ldots, L+1\}$ such that $c_N^*(\omega) > \epsilon_N(\omega), \omega \in A$ and $c_N^*(\omega) < \epsilon_N(\omega), \omega \in B$. Assume not, and that $c_N^* = \epsilon_N$ a.s. Then, from (9), $u_N'(\epsilon_N) = y_N u_P'(\delta - \epsilon_N)$, and c_N^* satisfies N's budget constraint, so c_N^* is also the solution to equilibrium in the price-taking economy. Hence, by the contrapositive there must exist states such that $c_N^* \neq \epsilon_N$. By the continuity of $c_N^*(\omega)$ and $\epsilon_N(\omega)$ and by (10), there must exist states in which $c_N^* > \epsilon_N$ and in which $c_N^* < \epsilon_N$.

Define a random variable ϕ by

$$\phi \equiv \frac{1}{2}\left(\frac{u_N'(c_N^*)}{y_N} + \xi^*\right).$$

Then by concavity of $u_N(\cdot)$ and $u_P(\cdot)$, and continuity of $u_N'(\cdot)$ and $u_P'(\cdot)$, and by (8) and (9) there exists $\Upsilon > 0$ such that, for all $v \in (0, \Upsilon)$,

$$\frac{u_N'(c_N^*(\omega))}{y_N} > \frac{u_N'(c_N^*(\omega) + v)}{y_n} > \phi(\omega) > u_P'(\delta(\omega) - c_N^*(\omega) - v)$$
$$> u_P'(\delta(\omega) - c_N^*(\omega)), \quad \omega \in A \qquad (24)$$

$$\frac{u_N'(c_N^*(\omega))}{y_N} < \frac{u_N'(c_N^*(\omega) - v)}{y_n} < \phi(\omega) < u_P'(\delta(\omega) - c_N^*(\omega) + v)$$
$$< u_P'(\delta(\omega) - c_N^*(\omega)), \quad \omega \in B. \qquad (25)$$

Next define a random variable ψ where $\psi(\omega) > 0$ for $\omega \in A$; $\psi(\omega) < 0$ for $\omega \in B$; $\psi(\omega) = 0$ otherwise; and ψ satisfies $E[\phi\psi] = 0$, implying

$$E_B[\phi\psi] = E_A[\phi\psi], \qquad (26)$$

where E_A and E_B denote expectations over the subsets A and B, respectively. Then we choose $\epsilon > 0$ such that $\epsilon|\psi| < \Upsilon$, a.s.

We perturb c_N^* to $\tilde{c}_N \equiv c_N^* + \epsilon\psi$ and c_P^* to $\delta - \tilde{c}_N$, which is feasible. Agent N and P's expected utility become

$$E[u_N(\tilde{c}_N)] = E[u_N(c_N^*)] + E_A\left[\int_{c_N^*}^{c_N^*+\epsilon\psi} u_N'(c)dc\right] + E_B\left[-\int_{c_N^*+\epsilon\psi}^{c_N^*} u_N'(c)dc\right]$$
$$> E[u_N(c_N^*)] + E_A[\epsilon\phi\psi] - E_B[\epsilon\phi\psi] = E[u_N(c_N^*)] \ ,$$
$$E[u_P(\delta - \tilde{c}_N)] = E[u_P(\delta - c_N^*)]$$
$$- E_A\left[\int_{c_N^*}^{c_N^*+\epsilon\psi} u_P'(\delta - c)dc\right] + E_B\left[\int_{c_N^*+\epsilon\psi}^{c_N^*} u_P'(\delta - c)dc\right]$$
$$> E[u_P(\delta - c_N^*)] - E_A[\epsilon\phi\psi] + E_B[\epsilon\phi\psi] = E[u_p(\delta - c_N^*)] \ ,$$

using (24)–(26). Hence the non-price-taking equilibrium is not pareto optimal. *Q.E.D.*

Proof of Proposition 5: Manipulating (2) yields

$$\frac{E[\delta_i]}{S_i^*} - (1 + r^*) = -\operatorname{cov}\left(\frac{\delta_i}{S_i^*}, \xi^*\right) \ .$$

From (8) and (9) we have $\xi^* = g(\delta, \epsilon_N)$. A first order Taylor's expansion about $\mu_\delta, \mu_{\epsilon_N}$, yields

$$\xi^* \approx g(\mu_\delta, \mu_{\epsilon_N}) + g_1(\mu_\delta, \mu_{\epsilon_N})(\delta - \mu_\delta) + g_2(\mu_\delta, \mu_{\epsilon_N})(\epsilon_N - \mu_{\epsilon_N}) \ .$$

Implicit differentiation of (9) with respect to δ and ϵ_N combined with differentiation of (8) with respect to δ and c_N^* yields $g_1(\mu_\delta, \mu_{\epsilon_N}) = -\lambda_\delta^*$;

$g_2(\mu_\delta, \mu_{\epsilon_N}) = -\lambda^*_{\epsilon_N}$, where λ^*_δ and $\lambda^*_{\epsilon_N}$ are as quoted. Hence we deduce (12). *Q.E.D.*

Proof of Proposition 6: From (13) $\bar{c}_N - \delta/2$ is a constant. Differentiating (15) implicitly (state by state) with respect to $\delta(\omega)$ and then $\epsilon_N(\omega)$ yields the partial derivatives quoted in the text. We conclude that $(c^*_N(\omega) - \delta(\omega)/2)$ is strictly monotonically increasing in $\delta(\omega)$ for given $\epsilon_N(\omega)$, and strictly monotonically increasing in $\epsilon_N(\omega)$ for given $\delta(\omega)$.

For given ϵ_N, let us now show there exists a $\delta_{crit}(\epsilon_N)$ such that $c^*_N - \delta/2 = \bar{c}_N - \delta/2$. At $\delta_{crit}(\epsilon_N)$ we have $c^*_N = \bar{c}_N$, so we substitute (13) into (15) and rearrange to yield the expression quoted. For given δ we similarly derive $\epsilon_{Ncrit}(\delta)$ for which $c^*_N - \delta/2 = \bar{c}_N - \delta/2$. Then, since $(\bar{c}_N - \delta/2)$ is constant and $(c^*_N - \delta/2)$ is monotonically increasing in δ, for $\delta > \delta_{crit}(\epsilon_N), c^*_N - \delta/2 > \bar{c}_N - \delta/2$ and for $\delta < \delta_{crit}(\epsilon_N), c^*_N - \delta/2 < \bar{c}_N - \delta/2$. The analogous results for $\epsilon_{Ncrit}(\delta)$ complete part (a).

For part (b), substituting for $\epsilon_N = e_N\delta$ in (15) and taking the total derivative with respect to δ reveals that $(c^*_N - \delta/2)$ is monotonically increasing $(e_N > 1/2)$, decreasing $(e_N < 1/2)$ or constant $(e_N = 1/2)$. Define $\bar{\delta} = \frac{y_{N-1}}{ay_N(e_N - 1/2)}$ where, from (15), $c^*_N - \delta/2 = 0$. Hence for $\delta < \bar{\delta}, c^*_N < \delta/2$ for $e_N > 1/2, c^*_N > \delta/2$ for $e_N < 1/2$, and $c^*_N = \delta/2$ for $e_N = 1/2$. Since $c^*_P = \delta - c^*_N$, we deduce part (b). *Q.E.D.*

Proof of Proposition 7: From equation (15) we have $c^*_N = c^*_N(\delta, \epsilon_N)$. Expanding as a Taylor's series about $\mu_\delta, \mu_{\epsilon_N}$ and making use of the partial derivatives quoted, we obtain to first order

$$c^*_N(\delta, \epsilon_N) \approx c^*_N(\mu_\delta, \mu_{\epsilon_N}) + \frac{f^E}{2f^E + 1}(\delta - \mu_\delta) + \frac{1}{2f^E + 1}(\epsilon_N - \mu_{\epsilon_N}) \ ,$$

implying (17). Moreover, $\mathrm{var}(c^*_N - \epsilon_N) \approx \mathrm{var}\{f^E\delta/(2f^E + 1) - 2f^E\epsilon_N/(2f^E + 1)\}$, yielding the second result. *Q.E.D.*

Proof of Proposition 8: From (8) and (15) we may express $\xi^* = \xi^*(\delta, \epsilon_N)$. Expanding $\ln(\xi^*)$ as a first order Taylor's series about $\mu_\delta, \mu_{\epsilon_N}$ yields $\mathrm{var}(\ln(\xi^*))$. Substitution for CARA utility into (12) and use of (15) yields the approximate expression for an asset's risk premium. *Q.E.D.*

Proof of Proposition 9: The optimality of $c^*_n, n = 1, \ldots, N-1$, in (3), the expression for ξ in (6), and the N agents' budget constraints holding with equality imply (21)–(22). Define the random variable $g \equiv U'_P(\delta - c_N; \Lambda)(c_N - \epsilon_N)$ and the mappings G, G^{-1} and $H_n, n = 1, \ldots, N-1$, by:

$$G(c, y_1, \ldots, y_{N-1}; \omega) = U'_P(\delta(\omega) - c; 1/y_1, \ldots, 1/y_{N-1})(c - \epsilon_N(\omega)) \ ,$$

$$g = U'_P(\delta(\omega) - G^{-1}(g, y_1, \ldots, y_{N-1}; \omega); 1/y_1, \ldots, 1/y_{N-1}) \\ * (G^{-1}(g, y_1, \ldots, y_{N-1}; \omega) - \epsilon_N(\omega)) \ ,$$

$$H_n(g, y_1, \dots, y_{N-1}; \omega) = U_P'(\delta(\omega) - G^{-1}(g, y_1, \dots, y_{N-1}; \omega); 1/y_1, \dots, 1/y_{N-1}) * \left(I_n(y_n U_P'(\delta(\omega) - G^{-1}(g, y_1, \dots, Y_{N-1}; \omega); 1/y_1, \dots, 1/y_{N-1})\right) - \epsilon_n(\omega)\right) .$$

Using the notation G_k, G_k^{-1} and H_{nk} to denote the derivatives of these mappings with respect to their kth argument, agent N's optimization problem can be written as

$$\max_g E\left[u_N\left(G^{-1}(g, y_1, \dots, y_{N-1})\right)\right]$$

subject to $E[g] = 0$ and $E[H_n(g, y_1, \dots, y_{N-1})] = 0, \quad n = 1, \dots, N-1.$

Suppose c_N^* is an equilibrium agent-N consumption process with weights $y_1^*, \dots, y_{N-1}^*$ and $g^* = G(c_N^*, y_1^*, \dots, y_{N-1}^*)$. We use the notation $G(*), G^{-1}(*)$, etc., to denote the mappings evaluated at this equilibrium. Then perturb g^* to $g^\epsilon = g^* + \epsilon\eta$. Since g^* is optimal, we have

$$\lim_{\epsilon \to 0} \frac{d}{d\epsilon} E\left[u_N\left(G^{-1}(g^\epsilon, y_1(\epsilon), \dots, y_{N-1}(\epsilon); \omega)\right)\right] = E\left[u_N'(c_N^*) G_1^{-1}(*)\eta\right] + \sum_{j=1}^{N-1} y_j'(0) E\left[u_N'(c_N^*) G_{j+1}^{-1}(*)\right] = 0 , \tag{27}$$

for all random variables η satisfying $E[\eta] = 0$, where the functions $y_j(\epsilon)$ are determined from

$$E[H_n(g^\epsilon, y_1(\epsilon), \dots, y_{N-1}(\epsilon); \omega)] = 0, \quad n = 1, \dots, N-1 . \tag{28}$$

If we define a matrix $\mathscr{H}$ by

$$\mathscr{H}_{nj} \equiv E\left[H_{jn}(g^*, y_1^*, \dots, y_{N-1}^*; \omega)\right], \quad n, j = 1, \dots, N-1 ,$$

take derivatives of (28), evaluate at $\epsilon = 0$ and rearrange we solve for the $y_j'(0)$ as

$$y_j'(0) = -\sum_{n=1}^{N-1} \left(\mathscr{H}^{-1}\right)_{jn} E\left[H_{n1}(g^*, y_1^*, \dots, y_{N-1}^*; \omega)\eta\right], \quad j = 1, \dots, N-1 .$$

Substituting into (27) we obtain the condition

$$E\left[\left\{u_N'(c_N^*) G_1^{-1}(*) - \sum_{j=1}^{N-1} E\left[u_N'(c_N^*) G_{j+1}^{-1}(*)\right] \sum_{n=1}^{N-1} \left(\mathscr{H}^{-1}\right)_{jn} H_{n1}(*)\right\}\eta\right] = 0 ,$$

for all η satisfying $E[\eta] = 0$. Hence we must have

$$u_N'(c_N^*) G_1^{-1}(*) - \sum_{n=1}^{N-1} \sum_{j=1}^{N-1} E\left[u_N'(c_N^*) G_{j+1}^{-1}(*)\right] \left(\mathscr{H}^{-1}\right)_{jn} H_{n1}(*) = y_N .$$

Evaluating the G_1^{-1} and H_{n1} terms and rearranging we arrive at (20) with

$$K_n \equiv \sum_{j=1}^{N-1} E\left[u_N'(c_N^*) G_{j+1}^{-1}(g^*, y_1^*, \dots, y_{N-1}^*; \tilde{\omega})\right] \left(\mathscr{H}^{-1}\right)_{jn} .$$

The extra terms compared with (9) are $-K_n H_{n1}/G_1^{-1}$. Each expectation in K_n is the marginal utility to N due to a change in agent j's weight y_j. The $\mathscr{H}_{nj}$ represent the sensitivity of agent n's budget constraint to agent j's weight y_j. Hence K_n is the sensitivity of agent N's expected utility to agent n's budget constraint. Then H_{n1} is the sensitivity of agent n's budget constraint to agent N's cost of "net" consumption, and G_1^{-1} is the sensitivity of agent N's state ω-consumption to his cost of net consumption. Hence H_{n1}/G_1^{-1} is the sensitivity of agent n's budget constraint to agent N's state-ω consumption. Therefore each extra term $-K_n H_{n1}/G_1^{-1}$ represents the indirect marginal disutility to N of an extra unit of $c_N(\omega)$ via agent n's budget constraint.

Finally, we show that (20) collapses to (9) when $U_P(c;\Lambda) = h(\Lambda)U_P(c)$. For $j = 1, \ldots, N-1$,

$$E\left[u'_N(c^*_N)G^{-1}_{j+1}(*)\right] = E\left[\frac{u'_N(c^*_N)U'_{yj}(\delta - c_N;\Lambda)(c_N - \epsilon_N)}{U'_P(\delta - c_N;\Lambda) - U''_P(\delta - c_N;\Lambda)(c_N - \epsilon_N)}\right]$$
$$= y_n \frac{\partial h(\Lambda)}{\partial y_j} E\left[U'_P(\delta - c_N;\Lambda)(c_N - \epsilon_N)\right] = 0 \ ,$$

where we have used $U'_{yj}(c;\Lambda) = (\partial h(\Lambda)/\partial y_j)U'_P(c)$, have supposed that (9) does hold, and finally used agent N's budget constraint. Hence $K_n = 0$. *Q.E.D.*

References

Basak, S.: A general equilibrium model of portfolio insurance. R. Financ. Studies **8**, 1059–1090 (1995)

Basak, S.: Dynamic consumption-portfolio choice and asset pricing with non-price-taking agents. University of Pennsylvania, Finance Department, mimeo (1996)

Breeden, D. T.: An intertemporal asset pricing model with stochastic consumption and investment opportunities. J. Financ. Econ. **7**, 265–296 (1979)

Coase, R. H.: Durability and monopoly. J. Law Econ. **15**, 143–149 (1972)

Cox, J. C., Huang, C. F.: Optimal consumption and portfolio policies when asset prices follow a diffusion process. J. Econ. Theory **49**, 33–83 (1989)

Cuoco, D., Cvitanić, J.: Optimal consumption choices for a 'large' investor. Columbia University, Department of Statistics, mimeo (1996)

Cvitanić, J.: Nonlinear financial markets: hedging and portfolio optimization. Columbia University, Department of Statistics, mimeo (1995)

Duffie, D.: Stochastic equilibria: existence, spanning number, and the 'no expected financial gain from trade' hypothesis. Econometrica **54**, 1161–1383 (1986)

Duffie, D., Huang, C. F.: Implementing Arrow-Debreu equilibria by continuous trading of a few long-lived securities. Econometrica **53**, 1337–1356 (1985)

Duffie, D., Zame, W.: The consumption-based capital asset pricing model. Econometrica **57**, 1279–1297 (1989)

El Karoui, N., Peng, S., Quenez, M. C.: Backwards stochastic differential equations in finance and optimization. Math. Finance, forthcoming (1996)

Gorman, W.: Community preference fields. Econometrica **21**, 63–80 (1953)

Grinblatt, M. S., Ross, S. A.: Market power in a securities market with endogenous information. Quarterly J. Econ. **100**, 143–167 (1985)

Holthausen, R., Leftwich, R., Mayers, D.: Large block transactions, the speed of response, and temporary and permanent stock-price effects. J. Financ, Econ. **26**, 71–95 (1987)

Holthausen, R., Leftwich, R., Mayers, D.: The effect of large block transactions on security prices. J. Financ. Econ. **19**, 237–267 (1990)

Huang, C. F.: An intertemporal general equilibrium asset pricing model: the case of diffusion information. Econometrica **55**, 117–142 (1987)

Jarrow, R. A.: Market manipulation, bubbles, corners, and short squeezes. J. Financ. Quan. Analysis **27**, 311–336 (1992)

Karatzas, I., Lehoczky, J. P. Shreve, S. E.: Optimal portfolio and consumption decisions for a 'small investor' on a finite horizon. SIAM J. Control Optim. **25**, 1557–1586 (1987)

Karatzas, I., Lehoczky, J. P., Shreve, S. E.: Existence and uniqueness of multi-agent equilibrium in a stochastic, dynamic consumption/investment model. Math. Oper. Res. **15**, 80–128 (1990)

Kihlstrom, R.: Monopoly power in dynamic securities markets. University of Pennsylvania, Finance Department, mimeo (1996)

Kraus, A., Stoll, H.: Price impacts of block trading on the New York Stock Exchange. J. Finance **27**, 569–588 (1972)

Kyle, A. S.: Continuous auctions and insider trading. Econometrica **53**, 1315–1335 (1985)

Lindenberg, E. B.: Capital market equilibrium with price affecting institutional investors. In: Elton, Gruber (eds.) Portfolio theory 25 years later, pp. 109–124. Amsterdam: North Holland 1979

Lucas, R.: Asset prices in an exchange economy. Econometrica **46**, 1429–1445 (1978)

Magill, M., Quinzii, M.: Theory of incomplete markets. Cambridge: MIT Press 1996

Milne, F.: Consumer preferences, linear demand functions and aggregation in competitive asset markets. R. Econ. Studies **46**, 407–417 (1979)

Sargent, T.: Dynamic macroeconomic theory. Cambridge, Massachusetts: Harvard University Press 1987

Seppi, D. J.: Block trading and information revelation around quarterly earnings announcements. R. Financ Studies **5**, 281–305 (1992)

Tirole, J.: The theory of industrial organization. Cambridge, Massachusetts: MIT Press 1988

Varian, H. R.: Microeconomic analysis. New York: Norton 1992