# Financial Pooling in a Supply Chain

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Trade credit is a common component in supply contracts. Large buyers with easier access to financial market often demand trade credit from small suppliers with higher financing cost. Moreover, many buyers also delay paying their suppliers beyond the agreed due day. Prior literature attributes this phenomenon to quality assurance or buyer's abuse of bargaining power. In this paper, we show that pooling could be another reason behind this practice. Using a game-theoretical model that explicitly captures the liquidity shocks faced by different supply chain partners, we analyze the total financing cost of the supply chain under endogenous supply contract and liquidity policies. We show that the embedded stretch option of trade credit allows supply chain partners to pool their liquidity buffers. Due to this pooling effect, even as the supplier's financing costs are strictly higher than the buyer's, the buyer may still demand for trade credit from the supplier for a purely financial reason. Under our framework, trade credit is more efficient than cash on delivery when the supplier's cost for collecting trade credit is low (e.g., when the retailer trusts the supplier) or when the supplier does not have access to a low-cost financing channel when facing liquidity shocks. While trade credit can only be more efficient than cash on delivery when the supplier's liquidity shock is sufficiently volatile, it may still benefit the supply chain when the buyer faces a deterministic liquidity shock. The benefit of pooling increases as the buyer has a more diversified supplier portfolio. As an innovative financing scheme, reverse factoring further enhances the efficiency of this pooling effect, and as a result reduces the overall supply chain financing cost.

*Key words*: trade credit, pooling, reverse factoring, supply chain finance, financial constraints, liquidity management, operations-finance interface

## 1. Introduction

Trade credit is commonly used in supply contracts. Many large retailers often receive trade credit from smaller suppliers, who normally face higher financing costs. For example, under *net 30*, a commonly-seen trade-credit term, the buyer is only obliged to pay the supplier 30 days after

the goods are delivered (Ng et al., 1999). Under similar terms, big box retailers such as Wal-Mart borrow billions of dollars from their suppliers. While such terms may have already been a financial burden for those small suppliers compared to other terms of trade such as cash-on-delivery (COD), buyers often stretch payment beyond the contractually agreed period. Indeed, as of 2015, in U.K. alone, small and medium-sized companies (SMEs) are owed more than £30 billions due to late payments (the payments that are not received by the contractually agreed due day), costing suppliers £8 billions in terms of higher financial cost (e.g., costs related to overdraft facilities) and administration time spent chasing late payments (Sukhraj, 2015). Such phenomenon is also documented by Giannetti et al. (2011) and Boissay and Gropp (2013). Practitioners and politicians condemn such late payment behaviors and attribute them to big buyers abusing their bargaining power. However, such abuse of bargaining power does not appear to be efficient: when a buyer faces a lower interest rate than her supplier, isn't it more beneficial for the buyer to use her bargaining power to demand for a lower wholesale price, instead of stretching trade credit terms?

In spite of their notorious payment behaviors, many large buyers, including retailers (e.g., Wal-Mart), manufacturers (e.g., Philips), and companies from emerging markets (e.g., JD.com, Tunca and Zhu 2014) started to offer a form of financing scheme, commonly known as reverse factoring or supply chain finance, to suppliers which allows them to use invoices approved by the buyer to obtain cash at a discount *before* the invoice is due. While the aim of such schemes is to lessen small suppliers' financial burden, its existence seems paradoxical: if the buyer intends to pay her suppliers earlier, why does not she simply shorten the trade credit period directly instead of offering the supplier an option to get paid faster?

Motivated by the above phenomenon, we examine the following three research questions in the paper. 1) How do different terms of trade such as cash-on-delivery (COD) and trade credit (TC) influence firms' working capital (cash holding) policies? 2) Can it be economically efficient for a supplier with higher financing costs to lend to a buyer with lower financing costs for a pure financial reason? If so, when and how? 3) How does reverse factoring further enhance the possible financial efficiency of trade credit?

To answer the above questions, we model a supply chain with one retailer and one supplier. Focusing on the financial pooling effect, we assume the retailer faces deterministic demand and hence procures an exogenous amount of goods from the supplier. To capture the stronger bargaining power of the retailer, we assume that the retailer acts as the Stackelberg leader and proposes the terms of trade and wholesale price to the supplier. On the financial side, we consider three types of terms of trade (i.e., financing schemes).

1. under Cash-on-Delivery (**COD**), the retailer pays the supplier at the time *when* the goods are delivered.
2. under Trade Credit (**TC**), the retailer pays the supplier *after* the goods are delivered. To capture the business practice that buyers often over-stretch payment (Brealey et al., 2008), we assume that the retailer always tries to postpone payments beyond the due day. To get paid on time, the supplier needs to incur a collection cost.
3. under Reverse Factoring (**RF**), while the retailer still receives trade credit, the supplier has the option to get paid before the trade credit due day.

In addition, we assume that both the retailer and supplier experience liquidity shocks after the goods are delivered. To maintain their liquidity, each firm has two financing channels: they can borrow through a regular channel before the liquidity shock is realized, and keep borrowed amount as a cash buffer, or borrow from an emergency channel after the liquidity shock is realized. Due to factors such as search cost and opportunity cost of holding cash for emergency lending, the interest rate of the regular channel is lower than that of the emergency channel. Further, to capture the stronger financial position of the retailer, we assume that the retailer faces lower interest rates.

Under this model, we first establish the two firms' working capital policy under COD. Due to the difference of financing cost between regular and emergency channels, both firms carry positive amounts of cash as buffers for their liquidity shocks. Under COD, as the payment transaction occurs before liquidity shocks are realized, the two firms' working capital policies can be decomposed, each holding a cash level analogous to a newsvendor solution, with the critical fractile determined by the financing costs from the two channels.

Under TC, the supplier holds more cash than under COD, in anticipation that he may need to incur additional costs to collect payment from the retailer, hence incurring higher financing cost. However, as the retailer can sometimes successfully over-stretch the payment (when the supplier does not collect), she can hold less working capital than under COD. In total, the supply chain's working capital can be lower under TC. This is because the cash raised by the supplier can be used to maintain not only the supplier's liquidity, but the retailer's. In other words, through the embedded over-stretching option of trade credit, the retailer can *pool* part of the supplier's working capital for her own use. This pooling benefit determines that even though the supplier faces higher interest rate, it can still be more efficient for the retailer to borrow from the supplier. In other words, even though working capital raised by the supplier has a higher marginal cost than by the retailer, it can also achieve a higher marginal benefit under trade credit. This finding provides a

novel theory of trade credit that explains why firms with higher financing costs extend trade credit for a pure financial reason.

Our theory suggests that the whole supply chain's financing cost is lower under TC than under COD when the supplier faces low collection fee or high interest rate from the emergency channel. The former point is consistent with empirical evidence that firms extend more trade credit to long-term supply chain partners – such long-term collaboration may significantly lower the information asymmetry between the two parties and enhance trust in the meantime, both resulting in lower collection cost. The latter point agrees with anecdotal observations that even the mostly financially disadvantaged firms, who may depend on overdraft facilities or personal credit cards to maintain liquidity, still offer trade credit to their buyers. We also find that while COD is in general more efficient when the supplier's liquidity shock is less volatile, TC can still dominate COD when the retailer's liquidity shock is completely deterministic. This suggests that buyers may actually finance long-term investments by systematically postponing trade credit payments. Further, we find that this pooling benefit is enhanced when the retailer faces multiple suppliers, as she can secure a more stable cash buffer by pooling unused cash from multiple suppliers.

For our third question, we find that RF is a valuable addition to the supply chain as it not only maintains the pooling benefit of trade credit, but also eliminates the collection fee associated with the retailer's over-stretching option. However, as RF transfers the option from the retailer to the supplier, in order to achieve the pooling value, it has to be associated with a longer trade credit period, which is also consistent with the anecdotal evidence that buyers often demand for longer trade credit period when offering RF to suppliers (Tanrisever et al., 2012).

The rest of the paper is organized as follows. Section 2 summarizes the related literature and positions our paper. The basic model is laid out in Section 3. Section 4 analyzes the optimal contract and cash holding policy under the cash-on-delivery case, which is used as a benchmark for the optimal decisions and performance of trade credit (Section 5). Section **??** extends the basic model to multiple suppliers or reverse factoring. The paper is concluded in Section 7. All proofs are in the Appendix, which also includes a list of notations used in the paper.

## 2. Related Literature

By explaining trade credit using a classic principle in Operations Management – pooling, our paper is closely related to three streams of literature: theories of trade credit (including reverse factoring), inventory pooling, and cash as inventory.

Within the trade credit literature, financing advantage is the one of the earliest theories economists offer to rationalize why suppliers extend credit to retailers. Schwartz (1974) first points

out that when a supplier faces a lower interest rate/opportunity cost than its buyer, trade credit allows the supplier to share its financing channel with its buyer. Emery (1984) shows that in the presence of financial friction, it is optimal for suppliers to offer trade credit to customers. Similar to the above two papers, we also show that trade credit may lower the overall financing cost of the supply chain. However, we differ from them mainly in two aspects. First, both Schwartz (1974) and Emery (1984) assume that the supplier has some form of financial advantage relative to the buyer, and thus rationalizing trade credit. In our model, however, the retailer actually have easier access to financing channels (in the form of lower financing costs). Second, while both earlier papers are built under a deterministic framework, stochastic liquidity shocks are essential in our model. Operations Management (OM) researchers have also contributed to the trade credit literature by examining the implication of trade credit on firms' operational decisions. Among them, Yang and Birge (2009) and Kouvelis and Zhao (2012) find that trade credit serves as a risk-sharing mechanism and allows the retailer to order more from the supplier, hence improving chain efficiency. Babich and Tang (2012) examine how buyers could use trade credit to deter suppliers' opportunism and ensure product quality. Chod (2015) shows that trade credit alleviates the buyer's risk-shifting behavior by tieing financing to a specific product assortment. Although our paper abstracts away from the supply chain partners' operational decisions, we argue one reason for the widespread usage of trade credit is a classic principle in OM – inventory pooling. More recently, researchers start to study reverse factoring in a supply chain setting. Tanrisever et al. (2012) quantify the value of reverse factoring in both make-to-order and make-to-stock supply chains through its impact on the supplier's inventory decision. Tunca and Zhu (2014) and Chen and Gupta (2014) also find that under different operational settings, reverse factoring can enhance the supply chain efficiency by inducing higher stocking levels. Our paper complements this literature by offering a unified explanation for both trade credit and reverse factoring.

Inventory pooling, which is central in our model, is first formalized by Eppen (1979). This principle is enriched by numerous related works. Among them, Tagaras and Cohen (1992), Corbett and Rajaram (2006) and Bimpikis and Markakis (2016) quantify the value of pooling under stochastic lead time and/or generalized demand distributions. Related to this literature, our theory of trade credit is hinged upon its embedded pooling value. We contribute to this literature in three aspects: first, we generalize the concept of pooling from inventory to a cash setting. Second, pooling is made possible in a decentralized fashion, i.e., the retailer has to provide incentive for the supplier to build a cash buffer for pooling purposes, and it is also costly for this pooling value to be realized as the supplier has to incur collection cost. This gives rise to a unique feature in our

model: pooling does not always lower cost, in contrast to the extant literature which finds pooling always valuable Gerchak and Mossman (1992). Finally, in trade credit, pooling is uni-directional. In other words, the natural direction of business transactions determines that only cash raised by the supplier can serve the pooling purpose through trade credit. Therefore, even though the buyer has access to financing at a lower cost, it may be more efficient for the supplier to raise cash and lend to the buyer.

The analog between cash and inventory has been drawn since at least Baumol (1952). Miller and Orr (1966) establish that in the presence of transaction costs, a firm should manage its cash holding according the classic $(s, S)$ inventory model. Recently, Luo and Shang (2013a) characterizes the optimal allocation of cash in a centralized multi-tier supply chain. Luo and Shang (2013b) design an innovative working capital policy by treating accounts payable and receivables as cash. Similar to the above papers, our model also draws an analogy between cash and inventory. However, unlike the the extant literature, which has only examined centralized decisions, our model studies the cash holding policy for multiple parties under a game-theoretical framework, highlighting the strategic interaction between the buyer and supplier(s).

## 3. The Model

Consider a supply chain that consists of one retailer ("she") and one supplier ("he").[1]

### 3.1. The supply chain

To focus on the financial pooling aspect, we assume that the retailer faces a deterministic demand, which we normalize to one. The retailer sources the product from the supplier, whose marginal cost of production is normalized to zero, and sells it to the market at price $p$. To capture the strong bargaining power of the retailer, we assume that the retailer acts as the Stackelberg leader and propose the wholesale price and the *terms of trade*, i.e., the financial arrangement between herself and the supplier. The supplier accepts the term if and only if the corresponding net profit, which includes both his operating margin and financing cost, which we will discuss in detail later, is no less than its outside option $\pi_s^o$.

### 3.2. Liquidity shocks and financing channels

After the order is delivered, both the supplier and the retailer experience liquidity shocks. Let the shock to the retailer be $\xi$ and to the supplier be $\xi_s$. The CDF of the joint distribution is $F(\xi, \xi_s)$. The marginal CDF is $F_r(\cdot)$ for the retailer and $F_s(\cdot)$ for the supplier. The liquidity shock can be seen as a stochastic cost.

[1] In Section 5.4, we extend the basic model to incorporate multiple suppliers.

To continue normal operation, firms need to maintain non-negative cash level *after* the liquidity shocks. To do so, firms have two financing channels: the retailer (supplier) can either build a cash buffer $k$ ($k_s$) before the liquidity shock by borrowing from a regular channel at rate $r$ ($r_s$), or borrow from an emergency channel at rate $\beta$ ($\beta_s$), or both. In addition, without loss of generality, we assume that holding cash generates zero return. For example, the total financing cost to the retailer with cash buffer $k$ and realized liquidity shock $\xi$ is $rk + \beta(\xi - k)^+$. To avoid trivial cases, we make the following assumptions.

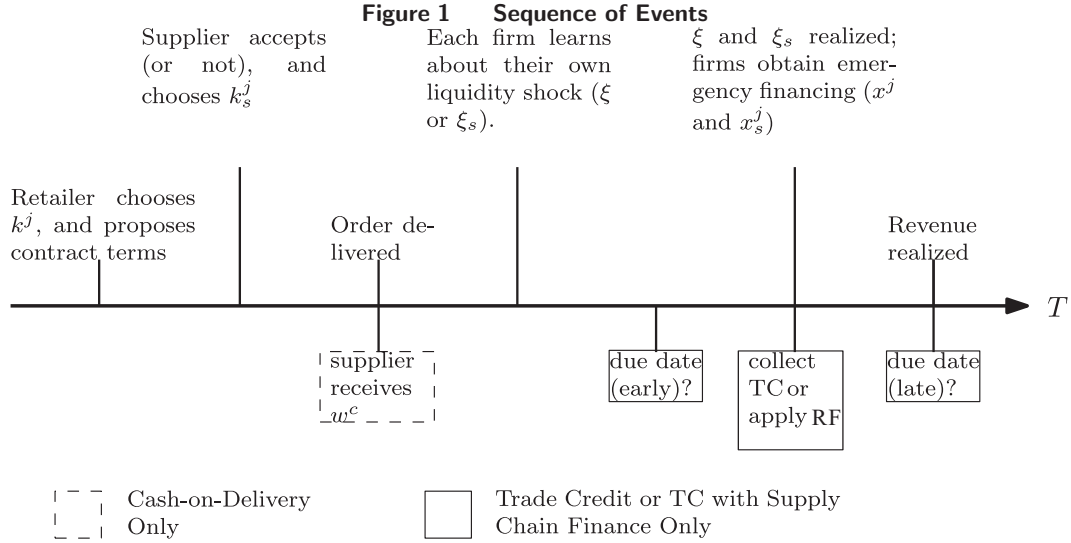
Assumption 1. *For the same channel, the retailer's rate is lower than the supplier's ($r < r_s$, $\beta < \beta_s$). For the same firm, the regular channel's rate is lower than the emergency channel ($r < \beta$, $r_s < \beta_s$).*

### 3.3. Terms of trade

In this paper, we study three modes of terms of trade commonly used in practice: cash on delivery (**COD**, all related quantities with superscript $c$), trade credit (**TC**, superscript $t$), and trade credit with reverse factoring (**RF**, superscript $f$).

Under cash on delivery, the retailer pays the wholesale price $w^c$ to the supplier *upon* the delivery of the goods. The sequence of events is illustrated in Figure 1.

**Figure 1 Sequence of Events**




Under trade credit, the retailer pays the wholesale price $w^t$ *after* the delivery of the goods. In this scenario, in addition to the wholesale price, the contract also specifies the due date of trade credit, i.e., when the retailer should pay back to the supplier. In addition, to capture the widespread practice of late payment, we assume that the retailer can over-stretch trade credit after it is due,

unless the supplier engages in some debt collection activity, which incurs a unit cost of $\alpha$. That is, the supplier can receive trade credit payment with amount $y$ with $y \leq w^t$ on the due date if she incurs cost $\alpha y$. The amount of trade credit that is not collected by the supplier will be paid at the end of the game. Finally, to focus on the pooling effect, we constrain our analysis to *net terms*, i.e., the retailer does not receive a discount even if she pays before the due date.

Under reverse factoring, the sequence of events is similar to TC. The only difference is that *before* the trade credit is due, the supplier can request to be paid earlier, by paying the retailer an interest rate $r^s$ for the requested amount. The rest is paid when the trade credit is due.

## 4. Cash on Delivery

Under COD, we characterize the cash holding decision for the supplier and the retailer by solving the model backward. On the supplier side, with initial cash level $k_s^c$, after the goods are delivered, his cash position will be $k_s^c + w^c$. The additional financing he needs to obtain is:

$$x_s^c = [\xi_s - (k_s^c + w^c)]^+,$$

where $z^+ = \max\{z, 0\}$. His net profit (operational profit minus financing cost) is:

$$\pi_s^c = w^c - r_s k_s^c - \beta_s \int_{-\infty}^{+\infty} x_s^c \; dF_s(\xi_s). \tag{1}$$

With the initial cash level $k^c$, the retailer's cash position after paying the wholesale price to the supplier is $k^c - w^c$. The additional financing she needs is:

$$x^c = [\xi - (k^c - w^c)]^+ .$$

The retailer's net profit is:

$$\pi^c = (p - w^c) - rk^c - \beta \int_{-\infty}^{+\infty} x^c \; dF_r(\xi). \tag{2}$$

Lemma 1. *Under COD with the wholesale price $w^c$, the supplier's optimal level of cash is*

$$k_s^c = \bar{F}_s^{-1}\left(\frac{r_s}{\beta_s}\right) - w^c;$$

*and the retailer's optimal level of cash is*

$$k^c = \bar{F}_r^{-1}\left(\frac{r}{\beta}\right) + w^c.$$

Two observations are notable. First, under COD, the optimal initial cash levels of the supplier and the retailer are independent of each other's cash level choice and liquidity shock. This is because the cash transaction between the supplier and the retailer is completed before the liquidity shock is realized. Therefore, the two firms' working capital policies can be completely decomposed. Second, the optimal cash levels share some similarities with the classic newsvendor critical fractile formula, consistent with our view that in managing liquidity shocks, firms use cash as inventory. Clearly, the retailer's (supplier's) optimal initial cash level increases in the emergency rate, and decreases in the regular rate.

Finally, we define the *total* financing cost (the sum of the two firms' financing cost) as follows:

$$FC^c = r_s k_s^c + r k^c + \beta_s \int_{-\infty}^{+\infty} x_s^c dF_s(\xi_s) + \beta \int_{-\infty}^{+\infty} x^c dF_r(\xi).$$

It is clear that the supply chain's profit equals to revenue $p$ minus $FC^c$. As $p$ does not vary between different terms of trade, the efficiency of different terms of trade is uniquely defined by the total financing cost. In the following sections, we investigate whether and under what circumstances trade credit without or with reverse factoring can lower this financing cost.

## 5. Financial Pooling with Trade Credit

In this section, we analyze the cost and benefit of using trade credit. Under trade credit, the retailer pays the supplier *after* the good is delivered.

### 5.1. The supplier's cash policy under TC

First, we can show that the retailer should never offer a trade credit contract that is due *after* the liquidity shock is realized.

Lemma 2. *The total financing cost under cash on delivery is less than that under trade credit with the late due date.*

The above result highlights that under the current model setting, the only possible value of trade credit is its pooling role, i.e., when the supplier extends trade credit to the retailer, he has the option to collect late payment in order to meet his own liquidity requirement. Therefore, in the rest of the section, we focus on trade credit with the *early* due date.

Under such a trade credit contract, the wholesale price is $w^t$, which will not be repaid by the retailer until the end of the game unless the supplier incurs a unit debt collection fee $\alpha$. Therefore, to ensure liquidity, the supplier has three sources of funds: ex-ante financing (internal cash) $k_s^t$, ex-post financing $x_s^t$, and trade credit collected $y_s^t$, which should satisfy $y_s^t \leq w^t$. Under a given $k_s^t$, the pecking order of the other two financing sources depends on the relative costs.

When $\alpha > \beta_s$, i.e., collecting trade credit is more expensive than ex-post external financing, the supplier always goes with external financing, i.e., $y_s^t = 0$, and $x_s^t = (\xi_s - k_s^t)^+$. One can show that this case is equivalent to trade credit due after liquidity shock, hence according to Lemma 2, the retailer again should not demand for trade credit. Therefore, throughout this section, we focus only on the following non-trivial scenario.

Assumption 2. *The trade credit collection fee $\alpha$ is no greater than the supplier's emergency rate $\beta_s$, i.e., $\alpha \leq \beta_s$.*

Under this assumption, the supplier's liquidity policy is as follows.

Lemma 3. *For given $k_s^t$ and $w^t$, the supplier's optimal choices of $x_s^t$ and $y_s^t$ are:*

$$x_s^t = [\xi_s - (k_s^t + w^t)]^+,$$
$$y_s^t = \min((\xi_s - k_s^t)^+, w^t).$$

The supplier's liquidity policy is intuitive: when his liquidity shock is smaller than his cash buffer, he neither collects trade credit nor borrows from the emergency channel. As the size of the liquidity shock increases, he collects part of trade credit to cover the liquidity shock, and subsequently, he collects the full amount of trade credit, and then fills the shortfall by borrowing from the emergency channel. This policy hints the potential pooling value of trade credit: the supplier only collects overdue trade credit when he needs to. When the liquidity shock is sufficiently small, or the supplier's cash level is sufficiently high, he does not need to collect trade credit, which in turn can be used by the retailer.

Following this liquidity policy, the supplier's net profit under $w^t$ and $k_s^t$ is:

$$\pi_s^t = w^t - r_s k_s^t - \alpha \int_{-\infty}^{+\infty} y_s^t dF_s(\xi_s) - \beta_s \int_{-\infty}^{+\infty} x_s^t dF_s(\xi_s). \tag{3}$$

Lemma 4. *Under trade credit wholesale price $w^t$, the supplier's optimal initial cash level $k_s^{*,t}$ satisfies:*

$$\alpha \bar{F}_s(k_s^{*,t}) + (\beta_s - \alpha)\bar{F}_s(k_s^{*,t} + w^t) = r_s. \tag{4}$$

Proposition 1 (Monotonicity of Supplier's Optimal Initial Cash Level). *The supplier's optimal initial cash level $k_s^{*,t}$ increases in $\alpha$ for $\alpha \in [0, \beta_s]$, and decreases in $w^t$, and increases in $\xi_s$ in the sense of the usual stochastic order.*

Proposition 1 confirms that the supplier adjusts his cash level in response to various parameters. As it is more costly to collect late payment than to raise funds from regular channel, the supplier holds more cash as the higher collection fee increases. This is because that as the supplier's collection fee increases, the "average" cost of being short of cash is higher, nudging the supplier to build up more cash upfront. Similarly, when facing a stochastically larger liquidity shock, the supplier would do the same.

### 5.2. The optimal wholesale price under TC

Anticipating the supplier's cash level $k_s^{*,t}(w^t)$ in the best response to $w^t$, the retailer would set $w^{*,t}$ accordingly so that the supplier's net profit under TC equals to his outside option $\pi_s^o$.

Proposition 2 (Monotonicity of TC's Wholesale Price). *The optimal trade credit wholesale price $w^{*,t}$, under which the supplier's net profit equals to $\pi_s^o$, satisfies:*

*(i)* $w^{*,t} = w^c$ *for* $\alpha = 0$.

*(ii)* $w^{*,t}$ *increases in* $\alpha \in [0, \beta_s]$.

*(iii)* $w^{*,t}$ *increases in* $\xi_s$ *in the sense of the usual, convex or increasing convex stochastic order.*

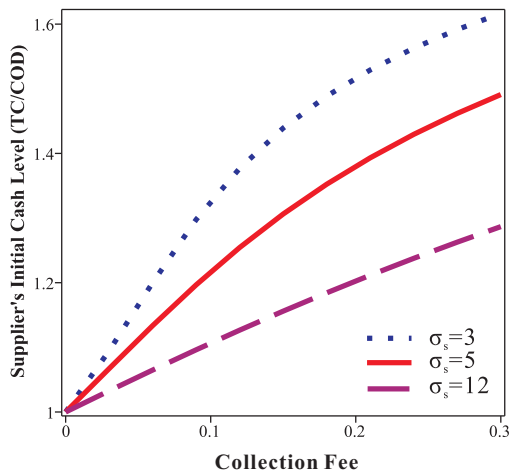
Several observations are notable. First, when the collection fee is zero, the wholesale price under TC is the same as that under COD. This is because that if the supplier can collect trade credit without incurring any cost, it is as if he has already received the payment. Therefore, his cash holding policy is exactly the same as under COD. Consequently, the retailer does not have to compensate him with a higher wholesale price. However, as one can imagine, the total financing cost under TC with $\alpha = 0$ should be different from that under COD. This is because as when the supplier does not need to fully collect the payment (when his liquidity shock is lower), the retailer can utilize part of the supplier's cash by delaying payment. By expecting this to occur, the retailer may lower her own cash level and/or the amount of emergency borrowing, and hence lowering the total financing cost.

Second, and related to the first point, the retailer has to compensate the supplier with a higher wholesale price as the supplier's collection fee increases. Similarly, when the supplier's liquidity shock increases stochastically, or becomes more volatile, the wholesale price also increases. This is because a stochastically larger liquidity shock means that the supplier is more likely to incur the collection cost and/or the ex-posting financing cost, and hence has to be compensated.

Third, the net effect of TC on supplier's optimal cash level $k_s^{*,t}$ is a result of two competing forces. By Proposition 1, an increasing collection fee or a (stochastically) growing liquidity shock would directly push the supplier to hold more cash. Nonetheless, the increasing collection fee or

growing liquidity shock would also push up the wholesale price paid to the supplier (by Proposition 2(ii) and (iii)). The higher wholesale price in turn would incentivize the supplier to lower his cash level (by Proposition 1). As a result, the net effect of TC on supplier's optimal cash level depends on the relative strengthes of these two competing forces and analytically it is indefinite.

**Figure 2 Supplier's Optimal Initial Cash Level under TC**




Notes. Parameters: the market price is $p = 10$, the wholesale price under COD is $w^c = 5$, the supplier's ex-ante financing cost is $r_s = 0.045$ and his ex-post financing cost is $\beta_s = 0.3$ , the retailer's ex-ante financing cost is $r = 0.02$ and her ex-post financing cost $\beta = 0.10$; both the supplier's and the retailer's liquidity shocks follow normal distributions, i.e., $\xi_s \sim N(10, \sigma_s^2)$ and $\xi \sim N(5, 10^2)$. We use $\sigma_s$ $(\sigma)$ to denote the standard deviations of the supplier's (retailer's) liquidity shock distribution.

The parameter values for the liquidity shock are chosen in such a way that on average, the supplier has a larger liquidity shock than the retailer, but the retailer's liquidity shock is more volatile than the supplier's. In addition, the average liquidity shocks are in the same magnitude as the market and wholesale prices. All the numerical examples shown in this paper follow the parameter values of the base case except for those parameters that are allowed to vary to illustrate comparative statics.

Figure 2 illustrates the ratio of the supplier's optimal initial cash level under TC to his optimal initial cash level under COD. Clearly, offering trade credit increases the supplier's cash burden relative to COD. Further, we note that the supplier is induced to carry more cash as it becomes more costly to collect trade credit. Finally, we observe that the negative impact of trade credit on the supplier's cash position actually decreases in the volatility of the liquidity shock he faces. This

is because when the supplier's liquidity shock is highly volatile, even under COD, he still need to maintain a large buffer of cash. As such, relatively, the additional cash he need to keep under TC becomes less significant. In other words, the option of collecting trade credit from the buyer becomes more valuable.

### 5.3. The pooling value of TC

Following the supplier's liquidity policy and the optimal wholesale price, we study the retailer's liquidity policy, and the potential value of TC.

LEMMA 5. *Given the realized $\xi$ and initial cash level $k^t$, after the supplier's collection of (part of) the over-due trade credit $y_s^t$, the amount of emergency fund the retailer obtains ($x^t$) follows:*

$$x^t(k^t;\xi,\xi_s) = [k^t;\xi + y_s^t(\xi_s, k_s^{*,t}, w^{*,t}) - k^t]^+.$$

Clearly, as $y_s^t$ depends on $k_s^{*,t}$ and $\xi_s$, the amount of emergency fund $x^t$ is also a function of the supplier's cash level $k_s^{*,t}$ and the realized liquidity shock $\xi_s$ as determined in Lemma 4.

Accordingly, the retailer's net profit is:

$$\pi^t(k^t) = p - w^{*,t} - rk^t - \beta \int_{-\infty}^{+\infty} x^t(k^t;\xi,\xi_s) dF(\xi,\xi_s).$$

PROPOSITION 3 (EFFICIENCY OF TC DUE TO LOW DEBT COLLECTION COST). *For a fixed $\beta > 0$, there exist two thresholds $0 \le \underline{\alpha} \le \overline{\alpha} < \beta_s$ such that,*

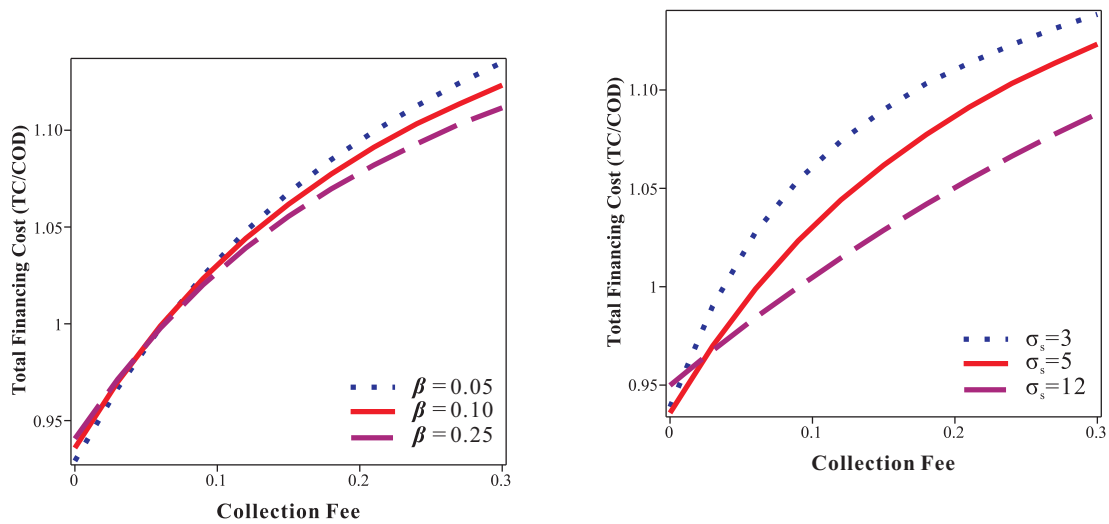
*(i) if $\alpha \le \underline{\alpha}$, TC is more efficient;*

*(ii) if $\alpha \ge \overline{\alpha}$, COD is more efficient.*

As shown, TC is more efficient than COD when collection fee is low. This is because when the supplier has low liquidity shock and does not need to costly force the retailer to pay the full amount, the retailer can use the stretch option as a buffer to deal with her own liquidity shock, which leads to a lower initial cash level. However, COD may be more efficient than any TC. This is because when debt collection fee is high, the supplier is forced to self-finance not only his own liquidity shock, but also the additional amount of $w^{*,t}$. As the supplier's financing costs (regular or emergency) are strictly higher than the retailer's corresponding rates, TC with high collection fee for the supplier would add cost to the total financing cost of the supply chain, as compared to COD.

Numerically, we observe that the total financing cost is strictly increasing in debt collection fee and exhibits a threshold effect: the total financing cost is lower (higher) under TC if the collection fee is lower (greater) than a threshold. Figures 3(a) and 3(b) are two representative examples of

these observations. The Y-axis of these figures is the ratio of the total financing cost under TC to that under COD, and we use it as the index of efficiency gain (loss) by TC, i.e., TC is more (less) efficient than COD if the ratio is smaller (greater) than 1. To illustrate comparative statics, Figure 3(a) allows the retailer's emergency financing cost to take three different values, i.e., $\beta$ equals to 0.05, 0.10 or 0.25, and Figure 3(b) allows the standard deviation of the supplier's liquidity shock distribution to take three different values, i.e., $\sigma_s$ equals to 3, 5 or 12.




(a) Retailer's Ex-post Financing Cost (b) Supplier's Liquidity Shock Volatility

**Figure 3 Total Financing Cost (TC/COD)**

Proposition 4 (Efficiency of COD Due to Low Financing Cost). *Fix any $\alpha > 0$, there exists a threshold $\underline{\beta} \geq 0$ such that if $\beta \leq \underline{\beta}$, COD is more efficient.*

The above result confirms that COD is more efficient if the retailer's emergency financing cost is sufficiently low. This is because the benefit of trade credit lies in saving the retailer's (part of) financing cost by allowing the retailer to use the supplier's cash to manage her liquidity shock. When the retailer's emergency financing cost is sufficiently low, the saving is little and insufficient to compensate the cost associated with the supplier's higher initial cash level and additional borrowed amount the supplier has to obtain. Furthermore, Figure 3(a) indicates that depending on the collection fee, $\underline{\beta}$ can be 0 or $\beta_s$, i.e., TC can be more efficient than COD regardless of retailer's emergency financing cost, or vice versa. Together with Proposition 3, this observation suggests that the main determining factor of efficiency by TC lies in the debt collection fee.

Proposition 5 (Efficiency of TC Due to Supplier's Liquidity Shock). *Within a class of random liquidity shocks that are comparable under the convex order, COD is more efficient*

*when the variance of supplier's liquidity shock is sufficiently small. In other words, TC can only be more efficient than COD when the variability of supplier's liquidity shock is sufficiently large.*

The intuition behind Proposition 5 highlights another aspect of the pooling role of trade credit: for trade credit to be cost efficient, there must be times where the supplier does not need the cash he prepared for his own liquidation. In the absence of variability in the supplier's liquidity shock, the supplier can always manage his liquidity precisely by preparing the exact amount of cash needed, and hence there will be no pooling between the supplier and the retailer, and thus TC holds no advantage over COD.

Proposition 5 asserts the pooling effect when the variability of the supplier's liquidity shock is present. An analogous question would then be: would the pooling effect diminish when the variability of the retailer's liquidity shock is absent? The following proposition shows that the answer is NO.

Proposition 6 (Efficiency of TC Under Deterministic Retailer's Liquidity Shock). *If $\frac{\beta}{r} < \frac{\beta_s}{r_s}$ and $\alpha$ is sufficiently low, TC is more efficient than COD even if the retailer's liquidity shock has no variability.*

By Proposition 5, as the variability of the supplier's liquidity shock vanishes, COD is more efficient than TC. In contrast, as Proposition 6 shows, when the retailer's liquidity shock is deterministic (or has low variability), the opposite can happen. That is, TC can be more efficient than COD, as long as that ex-post financing is relatively (compared to ex-ante financing) cheaper for the retailer than the supplier and the debt collection cost is low. In this sense, the retailer may in fact use trade credit to finance projects with deterministic investments.

Figure 4 shows illustrative numerical examples of Proposition 6, where the retailer's liquidity shock is deterministic at $\xi = 5$, while the retailer's ex-post financing cost $\beta$ varies from $(r =)0.02$ to $(\beta_s =)0.30$. As a result, the retailer's ratio of financing costs $\beta/r$ ranges from 1 to 15, while the supplier's ratio $\beta_i/r_i$ stays at $0.3/0.045 = 6.67$. Figure 4 suggests that when the retailer's liquidity shock is deterministic, as the retailer's ex-post financing cost increases, the supply chain's total financing cost under TC also increases. To allow the result that supply chain's total financing cost under TC is lower than that under COD to hold, the ratio $\beta/r$ must be be lower than the ratio $\beta_i/r_i$ (sufficient condition). Moreover, the smaller the debt collection fee $\alpha$, the more likely that result will hold. This observation, once again, stresses the pivotal role of debt collection fee in determining the supply chain's financial efficiency under TC.

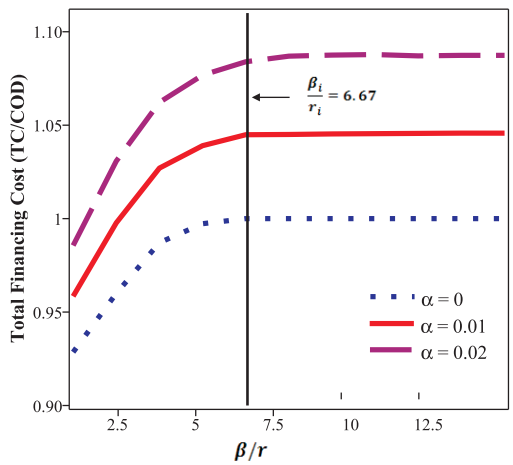



**Figure 4 Deterministic Retailer's Liquidity Shock**

We note that the condition $\frac{\beta}{r} < \frac{\beta_s}{r_s}$ is a reasonable scenario. For example, both the supplier and the retailer may have similar regular rates, for example, from bank loan. However, for emergency channels, as a larger company, the retailer may have access to line of credit, etc. The supplier, on the other hand, may need to liquidate its asset or borrow from credit card, which are more expensive. As a result, when the debt collection fee is not significant, it may be wise to free the retailer from committing a fixed amount of cash ex-ante, and only satisfies the supplier's cash need when it is needed ex-post. Proposition 6 shows that the ratio between the ex-ante and ex-post financing costs plays an important role in determining whether TC may be more efficient.

To complement the results in Propositions 5 and 6, Figure 3(b) and other numerical examples, that are not shown here for the sake of brevity, suggest that the efficiency gain by TC increases as either the supplier's or the retailer's liquidity shock volatility increases. This is sensible as a trade credit contract with stretched payment option allows the sharing of cash buffer between the supplier and the retailer and thus achieves a pooling effect in the supply chain's financing. The more volatile the liquidity shock from either party, the stronger the pooling effect.

Additionally, we also conduct numerical analysis on the comparative statics of supplier's and retailer's ex-ante financing costs on the supply chain's total financing cost under TC. The numerical analysis shows that the efficiency gain by TC decreases in the suppliers ex-ante financing cost, but increases in the retailer's ex-ante financing cost. This is because the supplier has to set a higher initial cash level under TC than under COD. It is more expensive for him to finance this higher initial cash level if her ex-ante financing cost is higher. On the contrary, because the retailer does not have to finance the wholesale price ex-ante under TC, the retailer would save more if his ex-ante financing cost is higher.

### 5.4. The benefit of horizontal pooling

The previous subsection has shown that due to the pooling effect of trade credit, even though the supplier's financing cost is higher than that of the retailer, and the supplier has to incur additional cost to collect trade credit payment from the retailer, it may still be more efficient for the supplier to lend to the retailer in the form of trade credit. In this subection, we extend the basic model to show that the trade credit can further lower total supply chain financing cost when the retailer faces multiple suppliers.

We assume instead of dealing with one supplier, now the retailer faces $n$ suppliers, each of which is identical to the supplier studied in the base model. We follow the same notation as before and use superscript $j$ to index a specific supplier $j$. All suppliers operate in their own markets and interact only with the retailer. Hence for a given wholesale price under the contract of COD and TC, they behave exactly the same as being characterized in the base model. The retailer would also pay exactly the same wholesale price, $w^{*,t}$, under TC for each supplier to keep them incentive compatible, regardless of the number of suppliers. The retailer's net profit under TC *per supplier* is:

$$\bar{\pi}_n^t(\bar{k}_n^t) = p - w^{*,t} - r\bar{k}_n^t - \beta \int \bar{x}_n^t(\bar{k}_n^t; \xi, \vec{\xi}_s) dF(\xi, \vec{\xi}_s),$$

where $\bar{k}_n^t = \frac{k^t}{n}$, $\vec{\xi}_s = (\xi_s^1, \xi_s^2, \ldots, \xi_s^n)$, and $\xi_s^j$, for all $j$, is independent and identically distributed copy of supplier's liquidity shock $\xi_s$ and

$$\bar{x}_n^t(\bar{k}_n^t; \xi, \vec{\xi}_s) = \left[ \frac{\xi + \sum_{j=1}^n y_s^t(\xi_s^j, k_s^{*,t}, w^{*,t})}{n} - \bar{k}_n^t \right]^+.$$

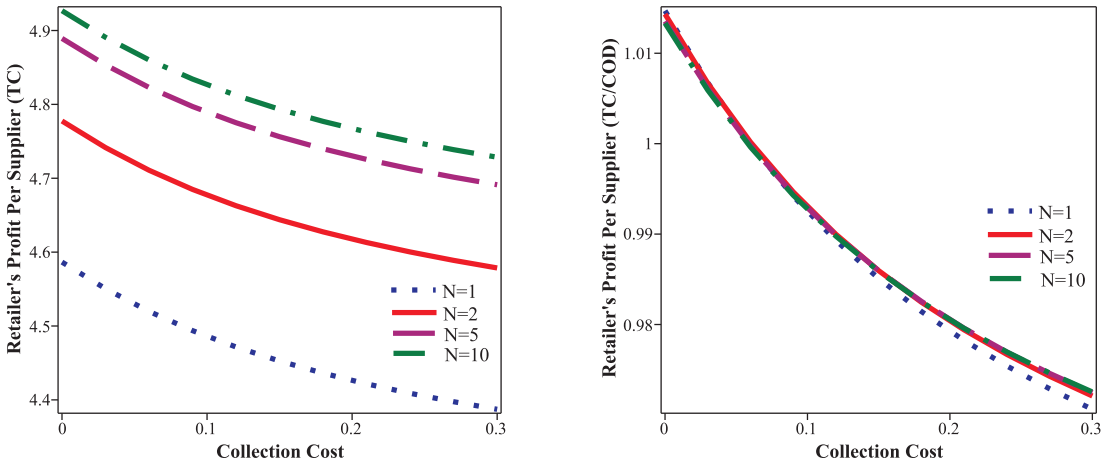
It is expected that as the retailer deals with more suppliers, there is less variability in the trade credit collected per supplier. In other words, the retailer faces a collectively more certain amount of trade credit collected (i.e., the coefficient of variation of the total amount of trade credit collected). This is due to the pooling of cash needs from different suppliers, which we call *horizontal pooling*.

Proposition 7 (Benefit of Horizontal Pooling). *Under TC, the retailer's optimal net profit per supplier $\bar{\pi}^t(\bar{k}_n^{*,t})$ increases in $n$, and the total financing cost of the supply chain per supplier decreases in $n$.*

When there are a large number of suppliers in the supply chain, there exists a strong horizontal pooling effect under TC so that the collective amount of debt requested by all suppliers has a very small variation relative to the debt collection amount. To meet this *stable* debt collection from suppliers, the retailer only needs to set aside an initial cash amount per supplier that is about the same level as the expected debt collection. The benefit of horizontal pooling under TC would

be greater if liquidity shocks among the suppliers are negatively correlated. The counterpart of Amazon in China, JD.com procures from a large number of small suppliers. Over a rolling horizon of account payables and receivables, the firm only needs to set aside an almost fixed amount of cash in the event of suppliers' debt collection. With a stable cash flow, the firm can invest in capital-intense, high-yield projects. In a highly competitive retail market, like in today's China, where e-retailers are burning cash to grab market shares, the stable returns from horizontal pooling under TC may be the only lucrative source of income which can only be enjoyed when the operational scale of the retailer is sufficiently large.

Figure 5 numerically illustrates the improvement of the retailer's profit as the number of suppliers increases. Figure 5(a) shows the retailer's absolute profit per supplier under TC with respect to collection cost and the number of suppliers $N$. As the number of suppliers increases, the horizontal pooling effect is strengthened and the retailer's profit per supplier strictly increases. On the other hand, note that as the number of suppliers increases, the retailer's profit per supplier under COD also increases. This is because the retailer's fixed total financing cost under COD is shared among more suppliers. Figure 5(b), therefore, shows the ratio of retailer's profit per supplier under TC versus under COD. Figure 5(b) indicates that the ratios are almost identical regardless of $N$, except when $\alpha \in (0.1,\ 0.3)$, there is a noticeable jump in the ratio as $N$ goes from 1 to 2. This may suggest that the benefit of horizontal pooling can be largely harvested with a small number of suppliers in the supply chain.




(a) Profit under TC

(b) Profit – **TC** vs COD

**Figure 5 Retailer's Profit Per Supplier**

## 6. The Value of Reverse Factoring

As an innovative financing scheme, reverse factoring (RF), also known as supply chain finance in practice, allows the supplier to receive payment, at a discount, before the due date of trade credit. Intuitively, as RF offers the supplier an additional option comparing with traditional trade credit contract, it is not surprising that relative to the trade credit only contract studied in the base model, RF should further lower the overall financing cost of the supply chain. However, it is not clear whether trade credit with RF is always more efficient than COD. Further, in practice, to implement RF, a large buyer often needs to rely on a bank or a third-party vendor. Therefore, the implementation is not cost-free. As such, in this section, we examine whether and under what circumstances RF adds value relative to TC and COD, which were examined previously.

Similar to Section 5, we first establish the necessary condition for the contract due date so that RF adds value to trade credit.

Lemma 6. *RF is only valuable when trade credit is due at the late due date.*

Lemma 6 suggests that in order for RF to add value, the trade credit duration has to be extended. This is commonly seen in practice. Therefore, in the rest of the section, we confine our analysis to a RF contract with the later due date. In addition, we also assume that the interest rate the supplier has to pay in order to receive earlier payment, $r_s^f$, is lower than the supplier's emergency rate $\beta_s$, i.e., $r_s^f \leq \beta_s$. Otherwise, the RF option becomes too expensive for the supplier to use.

We conduct our analysis backward in time. We start by characterizing the supplier's behavior under RF, which is analogous to his behavior under TC. The only difference is that when the supplier needs part or all of the payment payables, instead of insuring the debt collection rate $\alpha$, the supplier pays the early withdraw an interest rate of $r_s^f$. Under a given supplier's cash level $k_s^f$ and wholesale price $w^f$ of RF, the supplier's optimal choices of early withdrawal of $x_s^f$ from the retailer and the amount using ex-post external financing $y_s^f$ are given as:

$$x_s^f = \min((\xi_s - k_s^f)^+, w^f) \text{ and } y_s^f = \left[\xi_s - \left(k_s^f + w^f\right)\right]^+.$$

Following this liquidity policy, the supplier's net profit under $w^f$ and $k_s^f$ is:

$$\pi_s^f = w^f - r_s k_s^f - r_s^f \int_{-\infty}^{+\infty} y_s^f dF_s(\xi_s) - \beta_s \int_{-\infty}^{+\infty} x_s^f dF_s(\xi_s).$$

Analogous to Lemma 4, under reverse factoring price $w^f$, the supplier's initial cash level $k_s^{*,f}(r_s^f, \beta_s)$ satisfies:

$$r_s^f \bar{F}_s(k_s^f) + (\beta_s - r_s^f)\bar{F}_s(k_s^f + w^f) = r_s.$$

With an exogenous $r_s^f$, anticipating the supplier's cash level choice $k_s^{*,f}$, the retailer should set $w^{*,f}$ accordingly so that the supplier's net profit under RF equals to that under COD. Analogous to Propositions 1 and 2, we can derive analogous monotonicity properties of the supplier's optimal initial cash level $k_s^{*,f}(r_s^f)$ and the wholesale price $w^{*,f}(r_s^f)$ under RF to keep the supplier incentive compatible.

Lemma 7. *Fix $\beta_s$.*

*(i) $k_s^{*,f}(r_s^f)$ increases in $r_s^f \in [0, \beta_s]$.*

*(ii) $w^{*,f}(r_s^f)$ increases in $r_s^f \in [0, \beta_s]$, in particular, $w^c = w^{*,f}(r_s^f = 0) \leq w^{*,f}(r_s^f)$ for $r_s^f \in [0, \beta_s]$.*

Next, we shall evaluate the retailer's initial cash level and the corresponding net profit under RF. Following the supplier's liquidity policy, the retailer's liquidity policy can be characterized as the following, which is analogous to Lemma 5. Specifically, after the signal of $\xi$ and the supplier's RF amount $y_s^f$, the retailer obtains ex-post external financing $x^f$ as:

$$x^f(w^f; \xi, \xi_s) = [\xi + y_s^f(\xi_s, k_s^{*,f}, w^f) - k^f]^+.$$

Accordingly, the retailer's net profit under RF is:

$$\pi^f(k^f) = p - w^f - rk^f - \beta \int_{-\infty}^{+\infty} x^f(k^f; \xi, \xi_s) dF(\xi, \xi_s) + r_s^f \int_{-\infty}^{+\infty} y_s^f dF_s(\xi_s). \tag{5}$$

Unlike the retailer's net profit under TC, we have one additional term in the expression of the retailer's net profit under RF. This term is the profit gain in the event of the early withdraw by the supplier. A similar amount has been made to the debt collection agency under TC, which is a *deadweight loss* to the supply chain. The efficiency gain of RF is to formalize the practice of vertical pooling (i.e., the payment stretch of the retailer) and avoid the deadweight loss in the case that the supplier needs the money back before the due date.

With the above cash holding policies, we can compare the supply chain financing efficiency between RF and the other two terms of trade, namely, TC and COD.

Proposition 8 (Exogenous RF Rate). *Given a fixed $\beta > 0$, with an exogenously given $r_s^f$,*

*(i) If $r_s^f = \alpha$, RF is more efficient than TC.*

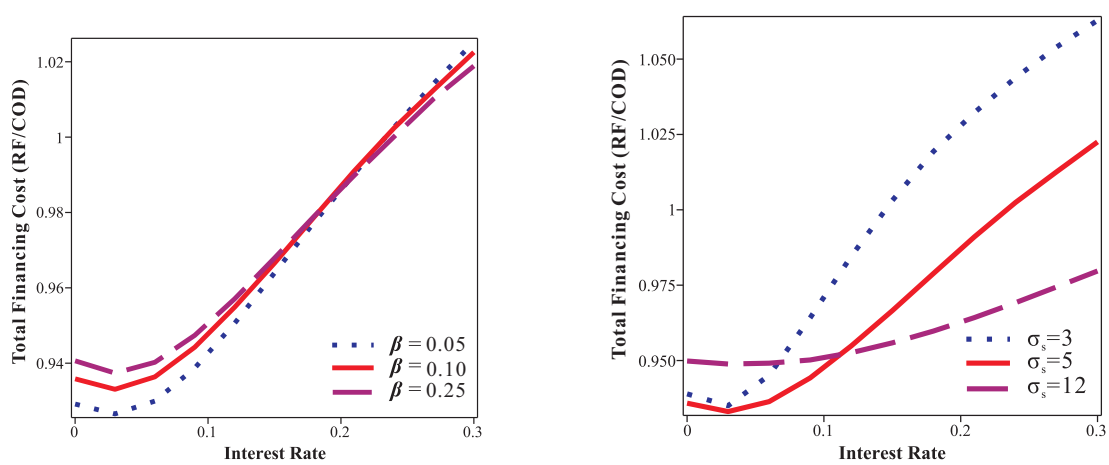
*(ii) There exist a largest threshold $\underline{r}_s^f \in [0, \beta_s)$ such that, if $r_s^f \leq \underline{r}_s^f$, RF is more efficient than COD. Moreover, $\underline{\alpha} \leq \underline{r}_s^f$.*

Clearly, when the retailer has the complete freedom to choose $r_s^f$, the efficiency under RF is always higher than that under COD. In this case, the efficiency gain is achieved by the retailer offering a low interest rate for the amount reverse factored. This is consistent with the anecdotal evidence that some buyers offer subsidized financing to suppliers via reverse factoring.

Proposition 9 (Endogenized RF Rate). *The optimal RF with an endogenized rate is more efficient than COD and the optimal rate $r_s^{*,f}$ is in the interior of the range* $(0, \beta_s)$.

Proposition 9 proves that it is optimal for the retailer to set a RF rate that is neither 0 nor $\beta_s$ and has to be a value in between. This is consistent with what we observe in practice.

Similar to the case of TC, we also conduct numerical analysis to study the efficiency gain by RF with respect to COD when the values of key parameters vary. Again, we use the ratio of total financing cost under RF to that under COD as the index of efficiency gain. Figures 6(a) and 6(b) are two representative examples of the numerical analysis. Most of the observations on efficiency gain under TC hold under RF. However, two differences are noted. First, although the threshold effect still exists, the total financing cost under RF is no longer a monotone function of interest rate. As suggested by Proposition 9, since the supplier's interest payment is part of the retailer's profit, it is not optimal for the retailer to set the interest rate to zero so that she loses the opportunity to make profit from supply chain finance. Second, RF is more efficient than COD for a wider range of interest rate than the range of collection fee under TC. As a result, the comparative statics that may be insignificant under TC can become material under RF. For example, Figure 3(a) shows that under TC, the total financing cost increases in retailer's ex-post financing cost $\beta$ only when the collection fee is in the range of $\alpha \in [0,\ 0.06]$, while this is true under RF when the interest rate is in the range of $r_f \in [0,\ 0.23]$ (Figure 6(a)). Similarly, Figure 3(b) shows that under TC, the supply chain's financing efficiency increases in the supplier's liquidity volatility when the collection fee is in the range of $\alpha \in (0.02, 0.30]$, while this is true under RF when the interest rate is in the range of $r_f \in (0.12, 0.30]$ (Figure 6(b))




(a) Retailer's Ex-post Financing Cost

(b) Supplier's Liquidity Shock Volatility

**Figure 6 Total Financing Cost (RF/COD)**

## 7. Conclusion

Trade credit is a commonly used terms of trade. In this paper, we offer a novel theory of trade credit using a classic principle in operations management – pooling. Motivated by the common phenomenon that buyers often stretch trade credit payment beyond the contractually agreed due date, our theory argues that this embedded stretching option of trade credit allows the supplier and buyer to pool their liquidity buffers, and hence lowers the overall financing cost of the supply chain. Because of this pooling benefit, it may be efficient for the buyer to borrow from a supplier with higher financing cost. In addition to offering a new explanation for trade credit, our results also offer an economic rationale of reverse factoring, a novel financing scheme aimed to alleviate suppliers' financial burden due to late payment of trade credit. We find that by granting the supplier an option to receive payment before trade credit due date, reverse factoring further enhances the benefit of pooling.

Our paper gives rise to a number of related questions for future research. First, our findings lead to testable hypotheses for future empirical work. For example, we find that the efficiency of trade credit is closely linked to the collection fee the supplier needs to incur. As such, we hypothesize that when the retailer trusts the supplier (which may result from repeated interactions) or when the legal enforcement of payment is strong, the supplier offers more trade credit to the retailer. Further, our result also predicts that after the introduction of reverse factoring, one should expect that a longer duration of trade credit and a lower wholesale price. As the first attempt to treat cash as inventory under a game-theoretical framework, our paper can also be extended analytically. For instance, one can generalize the model from a two-tier supply chain/network to a multi-tier one, or consider repeated interaction between the supplier and the retailer. While it is conjectured that the main insights of the paper should remain robust, a detailed examination may lead to additional findings.

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## Appendix A: Notation

**Table 1 Notational Glossary**

| | |
|---|---|
| **Parameters** | |
| $c, t, f$ | Subscripts for Cash on delivery (COD), trade credit (TC) and reverse factoring (RF) |
| $i$ | Supplier $i$, $i = 1, \ldots, N$, in the case of horizontal pooling |
| $p$ | Retail price |
| $w^o$ | Supplier's outside option, with $w^o \in [0, p)$ |
| $\alpha$ | unit collection fee for supplier under trade credit |
| $r/r_s$ | Ex-ante financing cost for the retailer/supplier, with $r < r_s$ |
| $\beta/\beta_s$ | Ex-post financing cost for the retailer/supplier, with $r < \beta$, $r_s < \beta_s$ and $\beta < \beta_s$ |
| $\xi/\xi_s$ | Random liquidity shock for the retailer/supplier |
| $F(\xi, \xi_s)$ | Cumulative distribution function of $\xi$ and $\xi_s$ |
| $F_r(\cdot)/F_s(\cdot)$ | Marginal CDF of liquidity shock for the retailer/supplier |
| **Decision variables** | |
| $w^j$ | Wholesale price to supplier, $j = c, t, f$ |
| $k/k_s$ | Cash level for the retailer/supplier |
| $x/x_s$ | The amount of external financing used by retailer/supplier |
| $y_s$ | The amount of trade credits collected by supplier |
| $r^s$ | rate under reverse factoring charged by the retailer to supplier |

Table 1 summarizes a list of notations used in the paper. The subscripts $j$ represent the terms of trade: $c$ stands for cash on delivery (COD), $t$ for trade credit (TC), and $s$ for trade credit with reverse factoring (RF).

## Appendix B: Proofs

### Proof of Lemma 1

The supplier's (retailer's) optimal cash level follows directly by taking derivative of $\pi_s^c$ in (1) with respect to $k_s^c$ ($\pi^c$ in (2) with respect to $k^c$).

### Proof of Lemma 2

Under TC with the late due date, the optimal level of cash is $\bar{F}_s^{-1}\left(\frac{r_s}{\beta_s}\right)$ for the supplier and $\bar{F}_r^{-1}\left(\frac{r}{\beta}\right)$ for the retailer. This is because the supplier cannot rely on the contract payment to cope with his liquidity shock. The retailer makes her payment to the supplier after liquidity shocks materialize and the revenue is realized. We then have

$$\underbrace{\beta_s \bar{F}_s^{-1}\left(\frac{r_s}{\beta_s}\right) + \beta \bar{F}_r^{-1}\left(\frac{r}{\beta}\right)}_{\mathbf{A}} = \underbrace{\beta_s\left[\bar{F}_s^{-1}\left(\frac{r_s}{\beta_s}\right) - w^c\right] + \beta\left[\bar{F}_r^{-1}\left(\frac{r}{\beta}\right) + w^c\right]}_{\mathbf{B}} + \underbrace{(\beta_s - \beta)w^c}_{>0}$$

where **A** is the ex-ante financing cost under TC with the late due date, which is greater than **B**, the ex-ante financing cost under COD.

The ex-post financing cost is the same under COD and under TC with the late due date for both the supplier and the retailer. As a result, the supply chain's total financing cost is greater under TC with the late due date than under COD. Note that the retailer needs to compensate for the increased ex-ante financing cost born by the supplier, to make TC with the late due date incentive compatible for the supplier. As a result, the retailer would prefer COD over TC with the late due date.

### Proof of Lemma 4

It follows directly from setting $\frac{\partial \pi_s^t}{\partial k_s^t} = 0$ where $\pi_s^t$ follows from (3).

### Proof of Lemma 6

As the supplier never needs to raise capital under TC with the early due date, he will never exercise the option of reverse factoring. Therefore, the supply chain's total financing cost cannot be lowered when trade credit is due at the early date.

### Proof of Lemma 7

The proof is analogous to the proof of Proposition 1 and the proof of Proposition 2(ii).

**Proof of Proposition 1**

By (3), we have

$$\frac{\partial^2 \pi_s^t}{\partial k_s^t \partial \alpha} = F_s(k_s^t + w^t) - F_s(k_s^t) \geq 0.$$

According to Theorem 2.8.1 of Topkis (1998), $k_s^{*,t}(\alpha; w^t)$ is increasing in $\alpha$. Analogously, $k_s^{*,t}(\alpha; w^t)$ is decreasing in $w^t$, because

$$\frac{\partial^2 \pi_s^t}{\partial k_s^t \partial w^t} = -(\beta_s - \alpha) f_s(k_s^t + w^t) \leq 0.$$

Note that the left-hand-side of (4) is increasing in $\xi_s$ in the sense of the usual stochastic order. Because $\bar{F}(x)$ is decreasing in $x$, $k_s^{*,t}(\xi_s; w^t)$, as the fixed point of (4), is increasing in $\xi_s$ in the sense of the usual stochastic order.

**Proof of Proposition 2**

(i) By (4), when $\alpha = 0$, we have $k_s^{*,t}(\alpha = 0; w^t) + w^t = \bar{F}_s^{-1}\left(\frac{r_s}{\beta_s}\right) = k_s^c + w^c$, where the last identity is due to Lemma 1. Since the retailer has an incentive to set the wholesale price $w^t$ as low as possible, while satisfying the incentive compatibility for the supplier benchmarked against COD, we must have $w^{*,t}(\alpha = 0) = w^c$.

(ii) Since $\frac{\partial \pi_s^t}{\partial \alpha} = -\int_{-\infty}^{+\infty} y_s^t dF_s(\xi_s) \leq 0$, then for $0 \leq \alpha_1 < \alpha_2 \leq \beta_s$, with the same $k_s^t$, we must have

$$\pi_s^t(\alpha_1, k_s^t; w^t) \geq \pi_s^t(\alpha_2, k_s^t; w^t). \tag{6}$$

As a result,

$$\pi_s^t(\alpha_1, k_s^{*,t}(\alpha_1); w^t) \geq \pi_s^t(\alpha_1, k_s^{*,t}(\alpha_2); w^t) \geq \pi_s^t(\alpha_2, k_s^{*,t}(\alpha_2); w^t), \tag{7}$$

where the first inequality is due to that $k_s^{*,t}(\alpha_1)$ is the optimal cash level corresponding to the collection fee $\alpha_1$, and the second inequality is due to (6). Since $\alpha \leq \beta_s$, we must have

$$\frac{\partial \pi_s^t}{\partial w^t} = 1 + (\beta_s - \alpha)[1 - F_s(k_s^t + w^t)] > 0. \tag{8}$$

Because the retailer sets $w^{*,t}$ such that the supplier's net profit under TC is equal to that under COD (i.e., $\alpha = 0$) in anticipation of the supplier's cash level $k_s^{*,t}$, then for any $\alpha \in [0, \beta_s]$, we have

$$\pi_s^t(\alpha, k_s^{*,t}(\alpha); w^{*,t}(\alpha)) = \pi_s^c = \pi_s^t(\alpha = 0, k_s^{*,t}(\alpha = 0); w^{*,t}(\alpha = 0)). \tag{9}$$

In view of (7) and (8), for $0 \leq \alpha_1 < \alpha_2 \leq \beta_s$, we must have $w^c = w^{*,t}(\alpha = 0) \leq w^{*,t}(\alpha_1) \leq w^{*,t}(\alpha_2)$. Otherwise, if $w^{*,t}(\alpha_1) > w^{*,t}(\alpha_2)$, we must have $\pi_s^t(\alpha_1, k_s^{*,t}(\alpha_1); w^{*,t}(\alpha_1)) \geq \pi_s^t(\alpha_2, k_s^{*,t}(\alpha_2); w^{*,t}(\alpha_1)) >$

$\pi_s^t(\alpha_2, k_s^{*,t}(\alpha_2); w^{*,t}(\alpha_2))$, where the first inequality is due to (7) and the second strict inequality is due to (8). This result contradicts (9).

(iii) Define the ex-post debt collection and financing cost function $z_s^t(\xi_s)$ as follows:

$$z_s^t(\xi_s) = \begin{cases} 0 & \text{if } \xi_s \in [0, k_s^t); \\ \alpha(\xi_s - k_s^t) & \text{if } \xi_s \in (k_s^t, k_s^t + w^t); \\ \alpha w^t + \beta_s[\xi_s - (k_s^t + w^t)] & \text{if } \xi_s \in [k_s^t + w^t, \infty); \end{cases}$$

Note that $z_s^t(\xi_s)$ is an increasingly convex function in $\xi_s$, for any $\alpha \in [0, \beta_s]$. Then $\int_0^\infty z_s^t(\xi_s)\, dF_s(\xi_s)$ increases in the liquidity shock $\xi_s$ in the sense of the usual, convex, or increasing convex stochastic order. As a result, $\pi_s^t(k_s^t; \xi_s; w^t) = w^t - r_s k_s^t - \int_{-\infty}^{+\infty} z_s^t(\xi_s)\, dF_s(\xi_s)$ is decreasing in $\xi_s$ in those stochastic orders. For a fixed $w^t$, $\max_{k_s^t} \pi_s^t(k_s^t; \xi_s)$ is decreasing in $\xi_s$ in those stochastic orders: for $\xi_s \preceq \xi_s'$ in one of those stochastic orders, $\max_{k_s^t} \pi_s^t(k_s^t; \xi_s) = \pi_s^t(k_s^{*,t}(\xi_s); \xi_s) \geq \pi_s^t(k_s^{*,t}(\xi_s'); \xi_s) \geq \pi_s^t(k_s^{*,t}(\xi_s'); \xi_s') = \max_{k_s^t} \pi_s^t(k_s^t; \xi_s')$. As the retailer adjusts the wholesale price such that the profit of the supplier stays the same under different liquidity shocks, $w^{*,t}(\xi_s)$ is increasing in $\xi_s$ in the sense of the usual, convex, or increasing convex stochastic order.

**Proof of Proposition 3**

According to Proposition 2(i), when $\alpha = 0$, we have $w^{*,t}(\alpha = 0) = w^c$ and $k_s^{*,t}(\alpha = 0) = k_s^c$. Moreover, the supplier earns the same profit under TC as under COD. Note that $w^c = w^{*,t}(\alpha = 0) \geq y_s^t(\xi_s, k_s^{*,t}(\alpha = 0), w^{*,t}(\alpha = 0))$ for any $\xi_s$. We have

$$\pi^t(k^c; \alpha = 0) - \pi^c(k^c) = \beta \int_{-\infty}^{+\infty} \left\{ [\xi + w^c - k^c]^+ - [\xi + y_s^t(\xi_s, k_s^{*,t}(\alpha = 0), w^{*,t}(\alpha = 0)) - k^c]^+ \right\} dF(\xi, \xi_s) \geq 0,$$

and hence $\max_{k^t} \pi^t(k^t; \alpha = 0) \geq \pi^t(k^c; \alpha = 0) \geq \pi^c(k^c; \alpha = 0)$. Therefore, when $\alpha = 0$, the retailer's profit under TC is no less than her profit under COD.

By Lemma 4, when $\alpha = \beta_s$, we have $k_s^{*,t}(\alpha = \beta_s) = k_s^c + w^c$ and thus $w^{*,t} \geq w^c + r_s w^c$. This is because compared to COD, the retailer should at least raise the wholesale price under TC by $r_s w^c$ to compensate the supplier for his loss incurred by increasing his cash level from $k_s^c$ to $k_s^c + w^c$.

The following inequality must hold:

$$\begin{aligned}
&\pi^t(k^{*,t}; \alpha = \beta_s) - \pi^c(k^c) \\
&= -(w^{*,t} - w^c) - \left( r k^{*,t} + \beta \int_{-\infty}^{+\infty} [\xi - (k^{*,t} - y_s^t)]^+ \, dF(\xi, \xi_s) \right) + \left( r k^c + \beta \int_{-\infty}^{+\infty} [\xi - (k^c - w^c)]^+ \, dF_r(\xi) \right) \\
&\leq -(w^{*,t} - w^c) - \left( r k^{*,t} + \beta \int_{-\infty}^{+\infty} [\xi - k^{*,t}]^+ \, dF_r(\xi) \right) + \left( r k^c + \beta \int_{-\infty}^{+\infty} \left[ \xi - \bar{F}_r^{-1}\left(\frac{r}{\beta}\right) \right]^+ dF_r(\xi) \right) \\
&\leq -(w^{*,t} - w^c) - \left( r \bar{F}_r^{-1}\left(\frac{r}{\beta}\right) + \beta \int_{-\infty}^{+\infty} \left[ \xi - \bar{F}_r^{-1}\left(\frac{r}{\beta}\right) \right]^+ dF_r(\xi) \right) + \left( r k^c + \beta \int_{-\infty}^{+\infty} \left[ \xi - \bar{F}_r^{-1}\left(\frac{r}{\beta}\right) \right]^+ dF_r(\xi) \right)
\end{aligned}$$

$= -(w^{*,t} - w^c) + rw^c < 0,$

where the second inequality follows because $\bar{F}_r^{-1}\left(\frac{r}{\beta}\right)$ is the cash level that achieves the lowest financing cost when dealing with a single random liquidity shock $\xi$; the second equality follows because $k^c = \bar{F}_r^{-1}\left(\frac{r}{\beta}\right) + w^c$ according to Lemma 1; and the last inequality follows because $w^{*,t} \geq w^c + r_s w^c > w^c + rw^c$. Therefore, when $\alpha = \beta_s$, the retailer earns strictly more under COD than under TC.

Consider the retailer's optimal profit under TC with respect to $\alpha$, i.e., $\pi^t(\alpha) = p - w^{*,t}(\alpha) - rk^{*,t}(\alpha) - \beta \int_{-\infty}^{+\infty} [\xi - (k^{*,t}(\alpha) - y_s^t(\xi_s, k_s^{*,t}(\alpha), w^{*,t}(\alpha)))]^+ \, dF(\xi, \xi_s)$. We have the desired result by the continuity of the function $\pi^t(\alpha)$ in $\alpha$.

**Proof of Proposition 4**

For any $k$, we have $\pi^t(k;\beta) - \pi^c(k;\beta) = -w^{*,t} + w^c - \beta[\int_{-\infty}^{+\infty} [\xi + y_s^t - k]^+ \, dF(\xi, \xi_s) - \int_{-\infty}^{+\infty} [\xi + w^c - k]^+ \, dF_r(\xi)]$. By Proposition 2, we have $w^c \leq w^{*,t}$. Moreover, we have the following bounds, independent of $k$:

$$-(E[\xi] + w^c) = 0 - \int_{-\infty}^{+\infty} (\xi + w^c) \, dF_r(\xi) \leq \int_{-\infty}^{+\infty} [\xi + y_s^t - k]^+ \, dF(\xi, \xi_s) - \int_{-\infty}^{+\infty} [\xi + w^c - k]^+ \, dF_r(\xi). \quad (10)$$

When $\beta = 0$, $\pi^t(k;\beta) - \pi^c(k;\beta) = -w^{*,t} + w^c \leq 0$. Because of the uniform bound (10), there exists a threshold $\underline{\beta} \geq 0$, such that if $\beta \leq \underline{\beta}$, we have $\pi^t(k;\beta) \leq \pi^c(k;\beta)$ for any $k$. Therefore, for any $\beta \leq \underline{\beta}$, we have $\pi^t(k^{*,t};\beta) = \max_k \pi^t(k;\beta) \leq \max_k \pi^c(k;\beta) = \pi^c(k^c;\beta)$.

**Proof of Proposition 5**

Let $\overline{\xi}_s$ and $\sigma_s$ denote the mean and variance of the supplier's liquidity shock. Consider the case where $\sigma_s = 0$, then the supplier faces a determinsitic liquidity shock $\overline{\xi}_s$. The strategies of the supplier and the retailer under COD and TC are as follows.

Under COD, the supplier has to keep an initial cash level $\left(\overline{\xi}_s - w^c\right)$ and thus incurs a cost of $r_s\left(\overline{\xi}_s - w^c\right)$.

Under TC, consider two scenarios as follows.

First, if $r_s \leq \alpha$, then the supplier would not collect debt from the retailer, i.e., $y_s^t = 0$, and would self-finance his deterministic shock. Since $r_s \geq r$, under TC, it is not efficient for the supplier to self-finance his own liquidity shock while the retailer compensates for that. Hence, COD is more efficient than TC for this case.

Second, if $r_s > \alpha$, then the supplier collects $y_s^t = \min\left\{\overline{\xi}_s, w^t\right\}$. In this case, we need to further consider two subclasses.

(1) If $\overline{\xi}_s > \frac{1+r_s}{1+r_s-\alpha}w^c$, then it is easy to show that $w^{*,t} = \frac{1+r_s}{1+r_s-\alpha}w^c$ and $y_s^t = w^{*,t}$. The retailer's optimal initial cash level is $k^{*,t} = k^c - w^c + w^{*,t}$. We have

$$\begin{aligned}
&\pi^t(k^{*,t}) - \pi^c(k^c) \\
=& - w^{*,t} + w^c - \beta\left\{\int_{-\infty}^{+\infty}[\xi - (k^{*,t} - y_s^t)]^+ dF(\xi,\xi_s) - \int_{-\infty}^{+\infty}[\xi - (k^c - w^c)]^+ dF_r(\xi)\right\} - rk^{*,t} + rk^c \\
=&(1+r)(w^c - w^{*,t}) < 0.
\end{aligned}$$

(2) If $\overline{\xi}_s \leq \frac{1+r_s}{1+r_s-\alpha}w^c$, then it is easy to show that $w^{*,t} = (1+r_s)w^c - (r_s-\alpha)\overline{\xi}_s$ and $y_s^t = \overline{\xi}_s$. The retailer's optimal initial cash level is $k^{*,t} = k^c - w^c + \overline{\xi}_s$. We have

$$\begin{aligned}
&\pi^t(k^{*,t}) - \pi^c(k^c) \\
=& - w^{*,t} + w^c - \beta\left\{\int_{-\infty}^{+\infty}[\xi - (k^{*,t} - y_s^t)]^+ dF(\xi,\xi_s) - \int_{-\infty}^{+\infty}[\xi - (k^c - w^c)]^+ dF_r(\xi)\right\} - rk^{*,t} + rk^c \\
=&(r-r_s)w^c + (r_s-\alpha-r)\overline{\xi}_s
\end{aligned}$$

If $(r_s-\alpha-r) \leq 0$, then we must have $(r-r_s)w^c + (r_s-\alpha-r)\overline{\xi}_s < 0$; otherwise, we have $(r-r_s)w^c + (r_s-\alpha-r)\overline{\xi}_s \leq (r-r_s)w^c + (r_s-\alpha-r)\frac{1+r_s}{1+r_s-\alpha}w^c = -\frac{(1+r_s)\alpha}{1+r_s-\alpha}w^c < 0$.

Now consider a class of continuous random liquidity shocks that share the same mean and are parameterized by the variance $\sigma_s$. Within this class, $G(\sigma_s) = \pi^t(k^{*,t}|\sigma_s) - \pi^c(k^c|\sigma_s)$ is a function of $\sigma_s$ and $G(0) < 0$. Since the random shocks are continuously parameterized by $\sigma_s$, $G(\sigma_s)$ is continuous in $\sigma_s$. Therefore, $G(\sigma_s) < 0$ must hold in a neighbourhood of $\sigma_s = 0$. That is, $G(\sigma_s) \geq 0$ is possible only when $\sigma_s$ is sufficiently large.

**Proof of Proposition 6**

First, we show that when the retailer's liquidity shock has no variability and $\alpha = 0$, TC is more efficient than COD, given that the condition $\frac{\beta}{r} < \frac{\beta_s}{r_s}$ is satisfied.

Consider COD: when the retailer's liquidity shock has no variability, the retailer faces a fixed liquidity shock $\overline{\xi}$ and pays $w^c$ to the supplier, so the retailer's initial cash level is $\overline{\xi} + w^c$. The supplier's initial cash level is $k_s^c$.

Consider TC: when $\alpha = 0$, we have $w^{*,t} = w^c$ and $k_s^{*,t} = k_s^c$. When the retailer's liquidity shock has no variability, the retailer faces a fixed liquidity shock $\overline{\xi}$ and a random debt collection as

$$y_s^t = \begin{cases} 0, & \xi_s \in [0, k_s^c]; \\ \xi_s - k_s^c, & \xi_s \in (k_s^c, k_s^c + w^c); \\ w^c, & \xi_s \in [k_s^c + w^c, \infty), \end{cases}$$

with complementary probability distribution

$$\mathbb{P}\left(y_s^t \geq q\right) = \begin{cases} 1, & q = 0; \\ \bar{F}_s(k_s^c + q), & q \in (0, w^c]; \\ 0, & q \in (w^c, \infty). \end{cases}$$

Since $w^{*,t} = w^c$, as long as the retailer's optimal initial cash level under TC, $k^{*,t}$, is lower than $\overline{\xi} + w^c$, the retailer's financing cost under TC is lower than that under COD.

The optimal initial cash levels $k_s^c$ and $k^{*,t}$ satisfy the following two identities:

$$\frac{r_s}{\beta_s} = \mathbb{P}\left(\xi_s \geq k_s^c + w^c\right) = \mathbb{P}\left(y_s^t \geq w^c\right),$$
$$\frac{r}{\beta} = \mathbb{P}\left(y_s^t \geq k^{*,t} - \overline{\xi}\right).$$

If $\frac{\beta}{r} < \frac{\beta_s}{r_s}$, then $\mathbb{P}\left(y_s^t \geq k^{*,t} - \overline{\xi}\right) = \frac{r}{\beta} > \frac{r_s}{\beta_s} = \mathbb{P}\left(y_s^t \geq w^c\right)$. Hence, $k^{*,t} - \overline{\xi} < w^c$ and then $k^{*,t} < \overline{\xi} + w^c$.

Consider the retailer's optimal profit under TC with respect to $\alpha$, i.e., $\pi^t(\alpha)$. We have the desired result by the continuity of the function $\pi^t(\alpha)$ in $\alpha$.

**Proof of Proposition 7**

By Shaked and Shanthikumar (2007, Theorem 3.A.30), we have

$$\frac{\sum_{j=1}^{n+1} y_s^t(\xi_s^j, k_s^{*,t}, w^{*,t})}{n+1} \preceq_{\text{cx}} \frac{\sum_{j=1}^{n} y_s^t(\xi_s^j, k_s^{*,t}, w^{*,t})}{n},$$

where $\preceq_{\text{cx}}$ is the convex order. Since the convex order is closed under convolutions (Shaked and Shanthikumar 2007, Theorem 3.A.12(d)), we must have

$$\xi + \frac{\sum_{j=1}^{n+1} y_s^t(\xi_s^j, k_s^{*,t}, w^{*,t})}{n+1} \preceq_{\text{cx}} \xi + \frac{\sum_{j=1}^{n} y_s^t(\xi_s^j, k_s^{*,t}, w^{*,t})}{n}.$$

Since $[x-k]^+$ is a convex function for any $k$, by the definition of the convex order, we have $E[\bar{x}_{n+1}^t(k)] \leq E[\bar{x}_n^t(k)]$ and hence $\bar{\pi}_{n+1}^t(k) \geq \bar{\pi}_n^t(k)$ for any $k$. Thus it is easy to see that $\max_k \bar{\pi}_{n+1}^t(k) \geq \max_{\tilde{k}} \bar{\pi}_n^t(\tilde{k})$. In other words, the retailer's optimal net profit per supplier increases in $n$. Since each supplier's financing cost is unchanged regardless of the number of suppliers, the total financing cost per supplier decreases in $n$.

**Proof of Proposition 8**

(i) Comparing the formulations of the supplier's and the retailer's optimization problems under TC and under RF, it is easy to see that they are exactly the same, except that there is an additional positive term $r_s^f \int_{-\infty}^{+\infty} y_s^f dF_s(\xi_s)$ in retailer's profit function (5). Consequently, if $r_s^f = \alpha$ and $w^f = w^t$, then we must have $k_s^{*,f} = k_s^{*,t}$. That is, the supplier's optimal profits under TC and RF are the same. Since $w^{*,t}$ and $w^{*,f}$ are set in such a way that $\pi_s^{*,t} = \pi_s^c$ and $\pi_s^{*,f} = \pi_s^c$, we must have $w^{*,f} = w^{*,t}$.

Given $r_s^f = \alpha$ and thus $w^{*,f} = w^{*,t}$, we argue that the retailer's optimal profit under RF must be no less than that under TC, i.e., $\pi^{*,f} \geq \pi^{*,t}$. This is because the retailer's profit under RF with $k^f = k^{*,t}$ is already no less than $\pi^{*,t}$. Therefore, we can conclude that the supply chain is at least as efficient under RF as under TC.

(ii) The proof is exactly the same as that of Proposition 3. This is because when $r_s^f = 0$, the supplier behaves exactly the same as under COD, while the retailer earns no less under RF than under COD. Therefore, when $r_s^f = 0$, RF is at least as efficient as COD. Moreover, by part (i), we must have $\underline{\alpha} \leq \underline{r}_s^f$. Otherwise, suppose $\underline{\alpha} > \underline{r}_s^f$. For any $\alpha^o \in [\underline{r}_s^f, \underline{\alpha})$, by Proposition 3, TC with $\alpha = \alpha^o$ is more efficient than COD. By part (i), RF with with $r_s^f = \alpha^o$ is more efficient than TC with $\alpha = \alpha^o$ and hence more efficient than COD. This contradicts to the definition of $\underline{r}_s^f$.

**Proof of Proposition 9**

By Proposition 8(ii), the retailer does not want to set $r_s^f = \beta_s$, at which point COD is strictly more efficient. It remains to show that the retailer can earn strictly more with $r_s^f > 0$ than with $r_s^f = 0$.

Recall that $x^f(w^f;\xi,\xi_s) = [\xi + y_s^f(\xi_s, k_s^{*,f}, w^f) - k^f]^+$ and $y_s^f = \min((\xi_s - k_s^f)^+, w^f)$. For any given $w^f$, by Lemma 7, $k_s^{*,f}$ increases in $r_s^f$ and hence $x^f(w^f;\xi,\xi_s)$ decreases in $r_s^f$ for any realization of $\xi$ and $\xi_s$. For any $k^f$ and $w^f = w^c = w^{*,f}(r_s^f = 0)$, we have

$$\begin{aligned}
\left.\frac{\partial \pi^f}{\partial r_s^f}\right|_{r_s^f=0} &= -\beta \int_{-\infty}^{+\infty} \frac{\partial x^f(\xi,\xi_s)}{\partial r_s^f} dF(\xi,\xi_s) + \int_{-\infty}^{+\infty} y_s^f dF_s(\xi_s) + r_s^f \int_{-\infty}^{+\infty} \left.\frac{\partial y_s^f}{\partial r_s^f} dF_s(\xi_s)\right|_{r_s^f=0} \\
&= -\beta \int_{-\infty}^{+\infty} \frac{\partial x^f(\xi,\xi_s)}{\partial r_s^f} dF(\xi,\xi_s) + \int_{-\infty}^{+\infty} \left. y_s^f dF_s(\xi_s)\right|_{r_s^f=0} \\
&\geq \int_{-\infty}^{+\infty} \left. y_s^f dF_s(\xi_s)\right|_{r_s^f=0} > 0,
\end{aligned}$$

where the second equality is due to $r_s^f = 0$, the first inequality holds because $x^f(w^f;\xi,\xi_s)$ decreases in $r_s^f$ for any $\xi$ and $\xi_s$ and for any $w^f$ (in particular for $w^f = w^c$), and the last strict inequality holds because when $r_s^f = 0$, we have $k_s^{*,f} = k_s^c$ and $w^{*,f} = w^c$ and since $\mathrm{P}(\xi_s \geq k_s^c + w^c) = r_s/\beta_s > 0$, we have that $\int_{-\infty}^{+\infty} y_s^f dF_s(\xi_s) \geq w^c\ \mathrm{P}(\xi_s \geq k_s^c + w^c) > 0$ holds for $r_s^f = 0$ and $w^f = w^c$. In conclusion, there is an incentive for the retailer to raise the optimal RF rate above 0.

**QQ: There seems to be a problem here. Note that** $\pi_s^f = p - w^f - r_s k_s^f + r_s^f \int_{-\infty}^{+\infty} y_s^f dF_s(\xi_s) - \beta_s \int_{-\infty}^{+\infty} x_s^f dF_s(\xi_s)$**., so**

$$\left.\frac{\partial \pi^f}{\partial r_s^f}\right|_{r_s^f=0} = -\left.\frac{\partial w^f}{\partial r_s^f}\right|_{r_s^f=0} - r\left.\frac{\partial k^f}{\partial r_s^f}\right|_{r_s^f=0} - \beta \int_{-\infty}^{+\infty} \frac{\partial x^f(\xi,\xi_s)}{\partial r_s^f} dF(\xi,\xi_s) + \int_{-\infty}^{+\infty} y_s^f dF_s(\xi_s) + r_s^f \int_{-\infty}^{+\infty} \left.\frac{\partial y_s^f}{\partial r_s^f} dF_s(\xi_s)\right|_{r_s^f=0}.$$

**The sign of** $\left(-\left.\frac{\partial w^f}{\partial r_s^f}\right|_{r_s^f=0} - r\left.\frac{\partial k^f}{\partial r_s^f}\right|_{r_s^f=0}\right)$ **is not definite here.**