Price Discovery in the European Bond Market

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Abstract

What is a benchmark bond? We provide a formal theoretical treatment of this concept and derive its implications. We describe a rich but little used econometric technique for identifying the benchmark that is congruent with our theoretical framework. We apply this to the natural experiment that occurred when benchmark status was contested in the European bond market following the introduction of the euro. We show that France unambiguously provides the benchmark at most maturities while benchmark status is contested between France and Germany at the longest maturity.

Keywords: Price discovery, benchmark, euro government bonds, cointegration

JEL Classification: F36, G12, H63

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1. Introduction

The introduction of the euro on 1 January 1999 eliminated exchange risk between the currencies of participating member states and thereby created the conditions for a substantially more integrated public debt market in the euro area. The euro-area member states agreed that from the outset, all new issuance should be in euro and outstanding stocks of debt should be re-denominated into euro. As a result, the euro-area debt market is comparable to the US treasuries market both in terms of size and issuance volume (see Galati and Tsatsaronis 2001; Blanco 2001). Unlike in the United States, however, public debt management in the euro area is decentralised under the responsibility of 12 separate national agencies.

This decentralised management of the euro-area public debt market is one reason for cross-country yield spreads. McCauley (1999) draws some comparisons between the US municipal bond market and the euro government bond markets. But the evidence of differentiation across countries has not been thoroughly explored (see Codogno et al. 2003; Portes 2003). It is clear from Blanco (2001), however, that in the initial years of the euro, yields were normally lowest for German bonds; that there was an inner periphery of countries centred on France for which yields were consistently higher; and that the outer periphery centred on Italy displayed the highest yields.

Our main contribution comes in examining benchmark status rather than resolving these yield spreads. In this decentralised euro government bond market, there is no official designation of benchmark securities, nor any established market convention. Indeed, benchmark status is more or less explicitly contested among countries.
One might ask why this should be so, aside from national pride. What are the benefits of achieving benchmark status? This leads us to consider the appropriate definition of ‘benchmark’. If the ‘benchmark’ were simply the security with lowest yield, the question would answer itself: clearly governments wish to borrow at the lowest possible yields; and there is an obvious welfare consequence, if foreigners hold any significant share of domestic government securities.

If indeed lowest yield were all that mattered for benchmark status, then the German market would provide the benchmark at all maturities. Analysts who take this view accept that the appropriate underlying criterion for benchmark status is that this is the security against which others are priced, and they simply assume that the security with lowest yield takes that role (e.g., Favero et al., 2000, pages 25-26). A plausible alternative, however, is to interpret benchmark to mean the most liquid security (see Blanco 2002), which is therefore most capable of providing a reference point for the market. But the Italian market, not the German, is easily the largest and arguably the most liquid.

Liquidity is to some extent quantifiable, but liquidity alone is unlikely to be a reliable identifier of benchmark status. For example, the Italian long yield is probably too variable to be a good reference point, or a suitable hedge, for other parts of the market. We believe that the characteristic of being a reference point for the market is closely related to Yuan’s (2005) definition of a benchmark. This definition highlights the benchmark security’s sensitivity to systematic risk as opposed to country-specific risk. Modifying it appropriately, we can arrive at a criterion that will enable us to distinguish the benchmark empirically. It is the systematic risk that guides the price discovery process. See Hasbrouck (1995) for a treatment in the context of equity markets.
While Yuan’s model employs an exogenously determined benchmark, we attribute similar characteristics to an endogenously determined benchmark. We modify Yuan’s model accordingly to fit the Euro-area bond market in this and other respects. It is important to note, however, that endogenous emergence of the benchmark is not of central importance to our identification methodology.

Our model and empirical approach associate benchmark status with the price discovery process. Once in existence, the benchmark security provides an information externality to the market as a whole because it best represents common movements of the entire market. The benchmark bond is the instrument to which the prices of other bonds react. The identification of benchmark status must therefore emerge from empirical analysis of price formation; it cannot just be asserted or simply read off the data. In essence a benchmark security concentrates the aggregation of information and reduces the cost of information acquisition in all markets where a security is traded against the benchmark.

The existing empirical approaches to identifying the price discovery process are limited. Scalia and Vacca (1999), for example, use Granger-causality tests to determine whether price discovery occurs in the cash or futures market in Italian bonds. In the context of identifying benchmark status, however, we believe that Granger-Causality testing exhibits significant weaknesses, particularly for high-frequency transaction data with variable liquidity. First, it can be inconclusive because series can Granger-cause each other. Second, Granger-causality is about dynamics: it has nothing to say about long-run relationships between series.1

Our alternative empirical method exploits the fact that yields are non-stationary for every country and at every maturity. If there were a unique benchmark at every maturity, then we would expect that the yields of other bonds would be
cointegrated with that benchmark. Indeed, there should be multiple cointegrating vectors centering on the benchmark bond. Our empirical approach relies on a result, based on Davidson (1998), that the structural nature of the cointegrating relationship between a benchmark bond and other bonds can be identified even in the context of quite a general theoretical framework.

An additional motivation for our study arises from the implementation of monetary policy in the euro area. One legacy of the introduction of the euro has been the growing recognition of the need to broaden the scope of open market operations. The European Central Bank currently concentrates on the swaps and repo market to implement its monetary policy. In its ‘General documentation on Eurosystem monetary policy instruments and procedures (2002)’, however, the ECB refers to the possible need for structural operations that may be required to influence the market’s liquidity position over long horizons. Our analysis has a bearing on the choice of policy instrument.

In the next section, we provide an explicit theoretical framework within which a benchmark security is defined. Section 3 presents the novel empirical methodology. The results from applying this to the euro-area bond market are presented in section 4. Section 5 contains concluding remarks and directions for future research.
2. Benchmark securities: a framework

Yuan (2005) formalises the concept of a benchmark security. Adopting her definition to our context, consider a country-specific security as having a yield with the following factor structure:

$$r_i = \tilde{r} + \beta_i \gamma + \tilde{\epsilon}_i \quad i = 1, \ldots, n$$

(1)

where $r_i$ is the nominal return on the ith country’s security, $\tilde{r}$ is the risk-free rate, $\gamma$ is euro-zone wide risk and $\beta_i$ is country i’s sensitivity to that risk. $\tilde{\epsilon}_i$ is the country-specific shock.

Conventionally, factor pricing models place very little emphasis on issues of stationarity. This is surprising since bond yields are typically non-stationary. In this respect, we must go beyond Yuan’s model. We specifically identify the source of the non-stationarity as the systematic risk $\gamma$ which is a general I(1) process. The equations can be motivated, for example, as inverse money demand functions with nonstationary velocity. In a multi-currency setting, such as the European Monetary System, this would have been implausible: there would have been as many non-stationarity factors as currencies. The proposed one-factor structure is designed to capture the essential character of the new monetary union. Consequently, all of the yields are themselves non-stationary.

The risk-free rate need not be constant, so long as it is stationary. This assumption would not be tenable during inflationary periods. It is reasonable, however, when inflation is credibly low, as in the euro-zone, so long as the rate of return on capital is stationary. Any stationary time variation in the risk-free rate is systematic and is included in the stationary component of systematic risk $\gamma$. 


The country specific shocks $\tilde{\epsilon}_i \forall i = 1, \ldots, n$ are stationary ARMA processes:

$$\tilde{\epsilon}_i = B_i(L)\tilde{\eta}_i$$ \hfill (2)

The parameters of the ARMA process $B_i(L)$ are country-specific, and the $\tilde{\eta}_i$ are independently distributed with mean zero and constant variance $\sigma_i^2$. Any country-specific dynamics in the risk-free rate are included here. Specifically, we are modelling country default and credit risk as stationary. If this were not so, the eurozone would not be a credible monetary union. This is what distinguishes the eurozone from a mere system of national currency boards. We also assume that $E(\tilde{\eta}_i\tilde{\mu}_i) = 0 \ \forall i$. This implies that no country in the union is large enough for its risk factors to become systematic.

At this point, we need the following Lemma:

Lemma 1. All pairs of country yields $\{r_i, \ i = 1, \ldots, n\}$ are cointegrated. Proof: For any $r_i$ and $r_j$ equation (1) implies that

$$\frac{r_i}{\beta_i} - \frac{r_j}{\beta_j} = r_i^f \left(\frac{1}{\beta_i} - \frac{1}{\beta_j}\right) + \left(\frac{\tilde{\epsilon}_i - \tilde{\epsilon}_j}{\beta_i - \beta_j}\right)$$

The right hand side is stationary by assumption. The cointegrating vector is

$$\begin{pmatrix} \frac{1}{\beta_i} - \frac{1}{\beta_j} \end{pmatrix}$$

The variance of the cointegrating residual is:

$$\text{Var}\left(\frac{r_i}{\beta_i} - \frac{r_j}{\beta_j}\right) = \frac{\{B_i(L)\}^2}{\beta_i^2} \sigma_i^2 + \frac{\{B_j(L)\}^2}{\beta_j^2} \sigma_j^2$$ \hfill (3)

We are now able to define a benchmark security:

Definition 1 (Yuan): A benchmark security has the following two properties:
(i) it has no sensitivity to country-specific risk,
(ii) it has unit sensitivity to systematic risk.

In our case, systematic risk is the euro-zone risk $\tilde{\gamma}$. The benchmark security can be constructed as follows. Form the following basket of country-specific securities:

$$r_b = \sum_{i=1}^{n} w_i \bar{r}_i + \sum_{i=1}^{n} w_i \bar{\beta}_i \tilde{\gamma} + \sum_{i=1}^{n} w_i \bar{\epsilon}_i$$

with $\lim_{n \to \infty} \sum_{i=1}^{n} w_i \bar{\epsilon}_i = 0$ and $\lim_{n \to \infty} \sum_{i=1}^{n} w_i \bar{\beta}_i = 1$

where $w_i$ for $i = 1, \ldots, n$ are the weights on each country’s security with $0 < w_i < 1$ and $\sum_{i=1}^{n} w_i = 1$. In effect, the benchmark security’s yield, $r_b$, is:

$$r_b = \bar{r}_b + \tilde{\gamma}$$

Note that this definition gives no explicit role to the level of the yield. While benchmark status may give rise to lower yields, here it is assumed that benchmark attributes stem purely from characteristics related to the security’s information content.

Note also that the benchmark here is a basket of bonds. The concept of a benchmark security as a basket of bonds is not entirely new. Galati and Tsatsaronis (2001) raise the idea in the context of euro-area government bonds, only to dismiss it immediately: ‘Market participants, however, are not yet ready to accept a benchmark yield curve made up of more than one issuer, being wary of the problems posed by small but persistent technical differences between the issues that complicate hedging and arbitrage across the maturity spectrum (p. 10).’ In applying the definition, we in fact consider individually the bonds of three countries in the euro area.

Lemma 2. All country yields \{r_i, i = 1, ..., n\} are pairwise cointegrated with the benchmark yield $r_b$. 


Proof:

From equations (1) and (5),

\[ \frac{r_i}{\beta_i} - r_p = \bar{r}^j \left( \frac{1}{\beta_i} - 1 \right) + \frac{\varepsilon_i}{\beta_i} \]

The right hand side is stationary by assumption. The cointegrating vector is

\[ \left( \frac{1}{\beta_i} - 1 \right) \]

The variance of the cointegrating residual is:

\[ \text{Var} \left\{ \bar{r}^j \left( \frac{1}{\beta_i} - 1 \right) + \frac{\varepsilon_i}{\beta_i} \right\} = \left\{ \frac{B_i(L)}{\beta_i} \right\}^2 \sigma_i^2 \]  

(6)

We can now state the main result:

Theorem 1:

The variance of the residual error in the cointegrating vector between country i’s yield and any other country-specific security j=1,...,n is always greater than the variance of the residual error in the cointegrating vector between country j’s yield and the benchmark yield.

Proof: Compare equations (3) and (6). ■

The analysis of this section starts from an implicit assumption that the benchmark bond is issued exogenously. This is not essential, however, and indeed in the euro-area bond market, this cannot occur. We argue, that a particular country’s bond emerges endogenously as the benchmark, at each maturity, with the characteristics specified in Definition 1. Regardless of whether the benchmark is endogenously determined, our analysis regarding its characteristics is applicable.

The contest for benchmark status may itself be worth modelling, but here we restrict attention to the more modest task of identifying the benchmarks at each
maturity. Our empirical approach can identify the benchmark independently of any contest for benchmark status.

3. Econometric Methodology

The factor definition of a benchmark in Section 2, along with Lemmas 1 and 2 and Theorem 1, suggests that the benchmark should be identified from an analysis of the cointegration properties of the yield series. Section 2 is framed in terms of a basket of bonds as benchmark, but here we take the special case in which a single country’s bond could possess benchmark characteristics. If a particular country provides the benchmark at a given maturity, then there should be two cointegrating vectors in the three-variable system of country yields. For example, if Germany were the benchmark, then the cointegrating vectors could be

\[
\text{Italian yield} = \gamma \text{German yield} + \text{nuisance parameters}
\]

\[
\text{French yield} = \delta \text{German yield} + \text{nuisance parameters}
\]

where \( \gamma \) and \( \delta \) are parameters.

The difficulty with this analysis lies in the identification problem. Even if we are satisfied that such cointegration vectors exist, we still cannot draw any immediate conclusion about the structure of the relationships between yields such as the identity of the benchmark. The reason is that any linear combination of multiple cointegrating vectors is itself a cointegrating vector. Consider the following example:

\[
\text{Italian yield} = \frac{\gamma}{\delta} \text{French yield} + \text{nuisance parameters}
\]
This provides us with a perfectly valid cointegrating vector, but it is simply a derivative of the two relations we posit as the structural cointegrating relations. On the face of it, any one of the three yields can provide the benchmark and we have made gone nowhere.

A recent development in non-stationary econometrics due to Davidson (1998) and developed by Barassi, Caporale and Hall (2000 a,b) \[\text{BCH}\] enables us to progress further. This approach tests for irreducibility of cointegrating relations and ranks them according to the criterion of minimum variance. The interesting feature of this method is that it allows us to learn about the structural relationship that links cointegrated series from the data alone, without imposing any arbitrary identifying conditions. In this case, the ‘structural’ relationship that we are exploring is the identity of the benchmark in a set of bond yields.

There is a risk of confusion in the use of the word ‘structure’, which has many different uses in the literature. In Davidson’s approach, it refers to parameters or relations that have a direct economic interpretation and may therefore satisfy restrictions based on economic theory. It need not mean a relationship that is regime-invariant. The possibility that “incredible assumptions” (Sims, 1980) need not always be the price of obtaining structural estimates turns out to be a distinctive feature of models with stochastic trends.

Let us therefore define an irreducible cointegrating vector.

**Definition 2 (Davidson):** A set of I(1) variables is called irreducibly cointegrated (IC) if they are cointegrated, but dropping any of the variables leaves a set that is not cointegrated.

IC vectors can be divided into two classes: *structural* and *solved*. A structural IC vector is one that has a direct economic interpretation.
Theorem 2 (Davidson). If an IC relation contains a variable which appears in no other IC relation, it is structural.

The less interesting solved cointegrating vectors are defined as follows:

Definition 3 (Davidson). A solved vector is a linear combination of structural vectors from which one or more common variables are eliminated by choice of offsetting weights such that the included variables are not a superset of any of the component relations.

A solved vector is an IC vector which is a linear combination of structural IC vectors. Once an IC relation is found, we wish to distinguish between structural and solved forms. Of course, the theoretical model might answer this question for us, but this would then simply be using the theory to identify the model, so in the absence of overidentifying restrictions we could learn nothing about the validity of the theory itself. The key issue is whether we can identify the structure from the data directly.

BCH introduce an extension of Davidson's framework that can be illustrated concretely with our problem as follows. In our system made up of three I(1) variables, the French, German and Italian bond yields, consider the case where the pairs (German yields, French yields) and (German yields, Italian yields) are both cointegrated. It follows necessarily that the pair (French yields, Italian yields) is also cointegrated. The cointegrating rank of these three variables is 2, and one of these three IC relations necessarily is solved from the other two. The problem is that we cannot know which, without a prior theory. Here is where the BCH extension of Davidson's methodology gives an (almost) unambiguous answer. In order to detect which of the cointegrating relations is the solved one and which of the vectors are irreducible and structural, we calculate the descriptive statistics of each cointegrating relation and rank these vectors on the basis of the magnitude of the variance of their residual errors. The structural vectors emerge as those corresponding to the lowest
variance. The reason for this is suggested by standard statistical theory and can be illustrated as follows: Let \( x, y \) and \( z \) be our cointegrated series and let

\[
\begin{align*}
    y - \alpha x &= e_1 \\
    y - \gamma z &= e_2 \\
    x - \delta z &= e_3
\end{align*}
\]

be the three irreducible cointegrating relations. Now assume that the structural relationships are the first two in equation (7), with \( e_1 \) and \( e_2 \) being the structural error terms from the first two which are therefore assumed\(^7\) to be distributed independently \( N\left(0, \sigma_i^2, i = 1, 2\right) \). The third equation is just solved from the first two\(^8\). This implies that \( e_3 \) is a function of \( e_1 \) and \( e_2 \), and therefore we expect it to be distributed

\[
N\left(0, \frac{\sigma_1^2 + \sigma_2^2}{\alpha^2}\right): \text{if } \alpha \leq 1 \text{ then } \sigma_3^2 > \text{Max}\{\sigma_1^2, \sigma_2^2\}.
\]

Therefore cointegrating relations whose residuals display lower variance should be the structural ones, the remaining others being just solved cointegrating relations. This result simply mirrors the statement of Theorem 1 in Section 2.

The requirement that \( \alpha \leq 1 \) appears arbitrary and dependant on normalisation. Even if the outcome of a specific analysis is dependant on the choice of normalisation, the theory of the previous section still provides a good guide. Suppose that the Italian bond is proposed as a benchmark. Lemma 2 requires that all equations, involving this proposed benchmark, should be normalised on the Italian yield. For the cross-equation, involving the German and French bonds, Lemma 1 requires that we calculate:

\[
\text{Var}\left(\frac{r_G}{\beta_G} - \frac{r_F}{\beta_F}\right) = \frac{1}{\beta_G^2} \text{Var}\left(r_G - \frac{\beta_G}{\beta_F} r_F\right) = \frac{1}{\beta_F^2} \text{Var}\left(r_F - \frac{\beta_F}{\beta_G} r_G\right)
\]

(8)
In practice, the last two expressions in equation (8) can be interpreted as derived from estimation using the two possible normalisations, *i.e.*, regressing $r_F$ on $r_G$ and $r_G$ on $r_F$ respectively. The resulting residual variances are deflated by $\frac{1}{\beta_G^2}$ and $\frac{1}{\beta_F^2}$ respectively. These can be interpreted as being derived from regressing the German return on the proposed Italian benchmark $\hat{\beta}_G$ and similarly for $\hat{\beta}_F$. The final step is to compare the three variances. Using the BCH maximum variance criterion, this either confirms or refutes the hypothesis that (in this case) the benchmark is the Italian bond.

There are two obvious points here. First,

$$\frac{1}{\hat{\beta}_G^2} \text{Var} \left( r_G - \frac{\hat{\beta}_G}{\hat{\beta}_F} r_F \right) = \frac{1}{\hat{\beta}_F^2} \text{Var} \left( r_F - \frac{\hat{\beta}_F}{\hat{\beta}_G} r_G \right)$$

is only true asymptotically. Consequently, the ranking could be ambiguous. In this case, we could not conclude that the benchmark is Italian. For the same reason, two different bonds could be identified as ‘the’ benchmark, and in this case we could not determine the identity of the benchmark. This would occur if benchmark status were contested. We show below that this applies in the case of one particular maturity below.
4. Application and Results

4.1 Data

We have a unique data set from Euro-MTS for 1 April 2003 to 31 March 2005. We use daily data for this period. Since the creation of the euro in 1999, Euro-MTS has emerged as the principal electronic trading platform for bonds denominated in euros. Already by the end of 2000, it was handling over 40% of total transactions volume (Galati and Tsatsaronis, 2001). Government bonds traded on Euro-MTS must have an issue size of at least €5 billion. For a discussion of MTS, see Scalia and Vacca (1999).

We selected data from France, Germany and Italy. Together the three countries account for over 70% of the euro-zone market (Blanco, 2001). We found that the coverage of the data for the other euro-zone countries was too sparse to get a consistently clear picture of even daily activity. In the analysis below we examine bonds from within four maturity buckets. Within each maturity, we select the bond that is most recently issued for each country. This rule is applied on a 3-month basis, so that the bond used in the analysis can change a number of times over the two years of our study. At the long and very long maturities, for each country, the same bond was available and liquid throughout the two-year sample. For the short and medium maturities, there were frequent changes of bonds, both because bonds changed maturity bracket or went off the run. In our empirical work we included shift dummies to account for bond changes of this kind. The daily observation for each country and maturity was chosen as the final mid-quote at the 16.30 close. We used
the mid-quote to avoid spurious bid-ask bounce. It is important to emphasise that the quotes are not indicative but executable.

4.2 Results

For each maturity, each bond yield is subjected to a Dickey-Fuller test for stationarity. The results are reported in Table 1. The outcome of the tests is simple to summarise. In every case, the yield is non-stationary.

In this light, our empirical strategy is as follows. First, we use the Johansen procedure to identify the number of cointegrating vectors at each maturity in our three-variable system. Then, we use Phillips-Hansen fully modified estimation to estimate the irreducible cointegrating vectors as recommended by Davidson. Finally we rank the irreducible cointegrating vectors using the variance-ranking criterion of BCH. From this we identify the structural vectors and therefore the benchmark. The latter must be the common yield in the two structural irreducible cointegrating vectors. The results are shown for each maturity in Tables 2 to 5.

(i) Johansen Procedure:

In Tables 2, 3, 4 and 5, it is clear that that there are two cointegrating vectors among the three yields at the short, medium, long and very long maturities. This can be read from comparing the \( \lambda \)-max and trace test statistics to their critical values which we reproduce for convenience. Consequently, all yields are pairwise cointegrated.
(ii) Irreducible cointegration vectors and BCH minimum variance ranking:

Regressions for each pair of yields were carried out using Phillips Hansen Fully Modified Estimation. Two sets of estimates were obtained for each pair, one for each possible normalisation. This amounts to six regressions for each maturity, 24 in all. Full details of the regressions are available on request. For each maturity, Tables 2 to 5 report the residual standard error of each regression. The results are summarised here by each maturity.

**Short:** The standard deviation of the residuals of the six cointegrating vectors varies from 9.10 to 30.97 basis points. The highest two arise from the regression of the Italian on the German yield and vice versa. From this we conclude that that the French-German and Italian-French relationships are structural and that the French yield provides the benchmark at the short end.

**Medium:** The standard deviation of the residuals of the six cointegrating vectors varies from 8.78 to 12.02 basis points. As with the short maturity, the highest two arise from the regression of the Italian on the German yield and vice versa. From this we conclude that that the French-German and Italian-French relationships are structural and that the French yield provides the benchmark at the medium maturity.

**Long:** The standard deviation of the residuals of the six cointegrating vectors varies from 7.57 to 13.80 basis points. Once again, the highest two arise from the regression of the Italian on the German yield and vice versa. From this we conclude that that the
French-German and Italian-French relationships are structural and that the French yield provides the benchmark at the long end.

Very Long: It is clear that French-German pair provides us with the lowest residual standard deviation irrespective of normalisation. However, the range of the residual variances for the Italian-French and the German-Italian pairings overlap. So far, all we can definitely conclude is that Italy does not provide the benchmark. This is the situation anticipated in the discussion at the conclusion of section 3.

Consider the possibility that Germany provides the benchmark. Then from Lemma 1, we use the standard deviations from the regressions with Germany as the right hand side variable (French-German, 7.71 and Italian-German, 13.16). The discussion following equation (8) shows that one cannot choose between the two normalisations from the Italy-France pair. Regardless of normalisation, however, France is chosen as the benchmark, which generates a contradiction.

Now consider that France provides the benchmark. This time, Lemma 1 tells us to start with the German-French (7.62) and Italian-French (12.26) residual standard deviations. The regressions involving the third pair of countries, Italy and Germany, give a range of values for the residual standard deviation between 12.18 and 13.16 basis points. This means that we do not have an unambiguous ranking and that though the assumption that France is the benchmark is not contradicted, we cannot reach a definite ranking of pairs. From this we conclude that benchmark status is contested at the very long maturity between France and Germany.
5. Conclusion

We focus on the meaning of ‘benchmark’ bond in the context of the market for euro-area government securities, extending the theoretical definition of a benchmark. We show that the Modified Davidson Method is an econometric technique that enables us to identify the benchmarks in the European bond market.

The simple idea that the security with the lowest yield provides the benchmark has no role to play in the analysis. Instead it is the information content of the security that is key. Our analysis undermines the conventional view of Germany as the benchmark issuer. What is striking is that France now dominates at all but the longest maturity10.

Both the theoretical framework and the econometric methodology presented here are completely general and are not specific to the particular application that is offered as a detailed illustration. Whenever a market displays a concentration of liquidity, a benchmark asset can potentially be identified. This is true in any national bond market, where our analysis can be used to pinpoint the benchmark. It can be used to analyse municipal and state bonds along the lines of the market in the US. Corporate bond markets also display a concentration of price discovery, and our approach can be applied there as well. Going beyond bonds, the analysis can be used to uncover benchmarks in any market where information acquisition is concentrated.
References


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Table 1: Stationarity test results.

<table>
<thead>
<tr>
<th>SERIES</th>
<th>t-value</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHORT YIELD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Italian</td>
<td>-0.787</td>
<td>Non-stationary</td>
</tr>
<tr>
<td>French</td>
<td>-1.148</td>
<td>Non-stationary</td>
</tr>
<tr>
<td>German</td>
<td>-0.776</td>
<td>Non-stationary</td>
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<tr>
<td>MEDIUM YIELD</td>
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<td></td>
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<td>Italian</td>
<td>-0.861</td>
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<tr>
<td>French</td>
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<tr>
<td>German</td>
<td>-0.880</td>
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<tr>
<td>LONG YIELD</td>
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</tr>
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<td>-1.019</td>
<td>Non-stationary</td>
</tr>
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</table>

The Dickey Fuller Test was used in each case with neither trend nor lags of differences. In the cases of the short and medium yields, dummies were included to account for changes in bonds as they matured.
Table 2: Cointegration: Short Maturity

**Johansen Test of Cointegrating Rank.**

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>λ-max</th>
<th>Trace</th>
<th>H0: rank</th>
<th>λ-max Crit. 90%</th>
<th>Trace Crit. 90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7220</td>
<td>638.81</td>
<td>700.2</td>
<td>0</td>
<td>11.23</td>
<td>21.58</td>
</tr>
<tr>
<td>0.1154</td>
<td>61.20</td>
<td>61.35</td>
<td>1</td>
<td>7.37</td>
<td>10.35</td>
</tr>
<tr>
<td>0.0003</td>
<td>0.16</td>
<td>0.16</td>
<td>2</td>
<td>2.98</td>
<td>2.98</td>
</tr>
</tbody>
</table>

**Conclusion**

Both the λ-max and Trace statistics imply that there are two cointegrating vectors.

**Philips-Hansen pair-wise cointegrating regressions:**

**Standard Deviation in basis points of residual for all regression pairings.**
Dependent variable listed first – all regressions include intercept, trend and bond-change dummies.

<table>
<thead>
<tr>
<th>French-German</th>
<th>German-French</th>
<th>Italian-French</th>
<th>French-Italian</th>
<th>Italian-German</th>
<th>German-Italian</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.08</td>
<td>21.98</td>
<td>9.10</td>
<td>9.29</td>
<td>30.97</td>
<td>27.21</td>
</tr>
</tbody>
</table>

**Conclusion:**

French bond provides the benchmark.
Table 3: Cointegration: Medium Maturity

<table>
<thead>
<tr>
<th>Johansen Test of Cointegrating Rank.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endogenous Variables: Italian, French and German Yields.</td>
</tr>
<tr>
<td>Exogenous variables in cointegration space: Drift and dummies for changes in bonds</td>
</tr>
<tr>
<td>Unrestricted constant outside cointegration space.</td>
</tr>
<tr>
<td>Lag length: 2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>λ-max</th>
<th>Trace</th>
<th>H0: rank</th>
<th>λ-max Crit. 90%</th>
<th>Trace Crit. 90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8566</td>
<td>969.08</td>
<td>1319.5</td>
<td>0</td>
<td>16.13</td>
<td>39.08</td>
</tr>
<tr>
<td>0.4954</td>
<td>341.33</td>
<td>350.38</td>
<td>1</td>
<td>12.39</td>
<td>22.95</td>
</tr>
<tr>
<td>0.0180</td>
<td>9.05</td>
<td>9.05</td>
<td>2</td>
<td>10.56</td>
<td>10.56</td>
</tr>
</tbody>
</table>

**Conclusion**

Both the λ-max and Trace statistics imply that there are two cointegrating vectors.

Philips-Hansen pair-wise cointegrating regressions:

**Standard Deviation in basis points of residual for all regression pairings.**
Dependent variable listed first – all regressions include intercept, trend and bond-change dummies.

<table>
<thead>
<tr>
<th>French-German</th>
<th>German-French</th>
<th>Italian-French</th>
<th>French-Italian</th>
<th>Italian-German</th>
<th>German-Italian</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.81</td>
<td>8.78</td>
<td>10.16</td>
<td>10.09</td>
<td>12.02</td>
<td>11.81</td>
</tr>
</tbody>
</table>

**Conclusion:**

French bond provides the benchmark.
Table 4: Cointegration: Long Maturity

### Johansen Test of Cointegrating Rank.

Endogenous Variables: Italian, French and German Yields.
Exogenous variables in cointegration space: Drift.
Unrestricted constant outside cointegration space.
Lag length: 2 Effective sample size: 498

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>λ-max</th>
<th>Trace</th>
<th>H0: rank</th>
<th>λ-max Crit. 90%</th>
<th>Trace Crit. 90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0602</td>
<td>30.98</td>
<td>60.44</td>
<td>0</td>
<td>16.13</td>
<td>39.08</td>
</tr>
<tr>
<td>0.0511</td>
<td>26.19</td>
<td>29.46</td>
<td>1</td>
<td>12.39</td>
<td>22.95</td>
</tr>
<tr>
<td>0.0065</td>
<td>3.27</td>
<td>3.27</td>
<td>2</td>
<td>10.56</td>
<td>10.56</td>
</tr>
</tbody>
</table>

**Conclusion**
Both the λ-max and Trace statistics imply that there are two cointegrating vectors.

**Philips-Hansen pair-wise cointegrating regressions:**

**Standard Deviation in basis points of residual for all regression pairings.**
Dependent variable listed first – all regressions include intercept and trend. There are no bond-changes.

<table>
<thead>
<tr>
<th>French-German</th>
<th>German-French</th>
<th>Italian-French</th>
<th>French-Italian</th>
<th>Italian-German</th>
<th>German-Italian</th>
</tr>
</thead>
</table>

**Conclusion:**
French bond provides the benchmark.
Table 5: Cointegration: Very-Long Maturity

Johansen Test of Cointegrating Rank.

Endogenous Variables: Italian, French and German Yields.
Exogenous variables in cointegration space: Drift.
Unrestricted constant outside cointegration space.
Lag length: 2  Effective sample size: 498

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>λ-max</th>
<th>Trace</th>
<th>H0: rank</th>
<th>λ-max Crit. 90%</th>
<th>Trace Crit. 90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1581</td>
<td>85.72</td>
<td>125.0</td>
<td>0</td>
<td>16.13</td>
<td>39.08</td>
</tr>
<tr>
<td>0.0702</td>
<td>36.23</td>
<td>39.27</td>
<td>1</td>
<td>12.39</td>
<td>22.95</td>
</tr>
<tr>
<td>0.0061</td>
<td>3.04</td>
<td>3.04</td>
<td>2</td>
<td>10.56</td>
<td>10.56</td>
</tr>
</tbody>
</table>

**Conclusion**

Both the λ-max and Trace statistics imply that there are two cointegrating vectors.

**Philips-Hansen pair-wise cointegrating regressions:**

**Standard Deviation in basis points of residual for all regression pairings.**
Dependent variable listed first – all regressions include intercept and trend. There are no bond-changes.

<table>
<thead>
<tr>
<th></th>
<th>French-German</th>
<th>German-French</th>
<th>Italian-French</th>
<th>French-Italian</th>
<th>Italian-German</th>
<th>German-Italian</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conclusion:</td>
<td>7.71</td>
<td>7.62</td>
<td>12.26</td>
<td>11.47</td>
<td>13.16</td>
<td>12.18</td>
</tr>
</tbody>
</table>
We did in fact carry out Granger-causality tests. The results are generally inconclusive.

In what follows, all variables are implicitly indexed by time. To avoid cluttering the notation, we suppress the time subscripts.


For example: \( r_i = -\beta \log v + \text{constant} + \text{noise} \) where \( v \) is the velocity of money. The latter is typically non-stationary. \( \beta \) is the inverse of the interest semi-elasticity of the demand for money and is country-specific.

This also implies that our model is an unconditional factor model. A conditional factor model can be expressed as an unconditional factor model with additional factors equal to the existing factors scaled by the conditioning variables. We could introduce a multi-factor model, but it is not indicated by our empirical results. In Section 4, we compare three countries. If there were two nonstationary factors there would be at most one cointegrating vector between the three. If pairwise cointegration is rejected, this amounts to a refutation of the one-factor model.

For example, for Cobb-Douglas production technology, all that is required is that the capital-output ratio be trend-stationary.

If residuals from the structural vectors are not orthogonal, then it is not clear what ‘structural’ means in this context. It is essential one way or the other to make some assumption about the covariance between the structural relations. Any assumption other than a zero value, however, makes the application of the irreducible cointegrating vector approach inconclusive.

Note that \( e_3 = \frac{e_2}{\alpha} - \frac{e_1}{\alpha} \) and \( \delta = \frac{\gamma}{\alpha} \)

Short-dated bonds have maturities between 1.25 and 3.5 years. Medium, long and very long bonds have maturity spans of 3.5-6.5 years, 6.6-13.5 years and >13.5 years respectively. There is also a fifth category for bills: securities with maturity less than 1.25 years. However, until recently, only Italy was significantly trading such instruments on Euro-MTS.

For early support of this view, at least at the shorter maturities, see Jessen and Matzen (1999). See also Favero, Pagano and von Thadden (2005).