Sourcing through Intermediaries: 
The Role of Competition

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We study the joint impact of horizontal and vertical competition in retailer-driven global supply chains with intermediaries. We show that, as a consequence of the retailers leading, intermediaries prefer products for which the supplier base (existing production capacity) is neither too narrow nor too broad. We also find that the “right” balance of horizontal and vertical competition can entirely offset the double marginalization effect caused by the existence of an additional intermediary tier, and thus lead to supply chain efficiency. On accounting for intermediaries’ private information about the supply side, we find that it has the indirect effect of attenuating competition between retailers, who may therefore be better off (under some scenarios) relative to the case with complete information. Finally, we show how the classical transaction cost rationale for the existence of intermediaries can be incorporated to our competition framework, and find that retailers are more likely to use intermediaries when manufacturing costs increase, and that the threat that retailers may procure directly from the suppliers pushes intermediaries to expand their supply base.

Key words: Supply chain, vertical and horizontal competition, Stackelberg leader, intermediation.

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1. Introduction

For several decades, intense competition has driven retailers in developed countries to source products from low-cost international suppliers. For commodity-type products, which tend to have long life cycles, the retailer’s in-house procurement department often establishes a long-term relationship with one or more suitable suppliers that can fulfill demand. For specialized products such as fashion apparel, fashion shoes, toys, and housewares, however, retailers typically rely on intermediaries. These industries are characterized by high frequency of new product introduction coupled with short product life cycles, and thus require the use of a large and complex network of low-cost international suppliers. Under these circumstances, retailers generally find it economical to
outsourcing the maintenance of this network to intermediaries with deep knowledge of the product market and international supplier base. For instance, sourcing through trading companies is typical in fashion apparel (Masson et al. 2007, Ha-Brookshire and Dyer 2008, and Purvis et al. 2013). Specifically, [Masson et al. 2007, p. 247] study several UK clothing retailers and find that: “With the increasing number of new products introduced more frequently as well as the smaller volumes per product, the pool of skills required for clothing manufacturing is becoming more complex, requiring a larger global network of suppliers every season. For most retailers, developing a global sourcing network was not effective. We found that the common norm was simply for the retailers to make use of third party indirect sourcing import/export agencies or what many choose to call intermediaries.”

When an intermediary receives an order from a retailer, it identifies from its supplier base the firms with appropriate expertise and spare capacity to fulfill the order, and charges a margin to the retailer for its mediation. In addition, intermediaries often offer a variety of network coordination services such as procuring raw material; monitoring of compliance with ethical, safety, and quality standards; and arranging logistics and shipping.

This form of intermediation has recently received media attention due to the increasing globalization of supply chains and the resulting prominence of mega intermediaries such as Hong-Kong-based Li & Fung Limited (The Telegraph 2012). However, most intermediaries or trading companies are small, yet they play a critical role in facilitating procurement in international supply chains; see Rauch [2001]. For instance, Hsing [1999] explains that trading companies were the predominant conduit for fashion shoes produced during Taiwan’s manufacturing boom between the mid-1970s and the mid-1980s, and that “most Taiwan trading companies were small, with an average of seven employees”.

Vertical and horizontal competition are inherent aspects of global supply chains with intermediaries. By definition, when a retailer outsources procurement to an intermediary, the relationship between them is one of vertical competition. Similarly, intermediaries and suppliers engage in vertical competition—indeed Hsing [1999] argues that intermediaries like to keep their distance with suppliers so that retailers feel comfortable delegating quality control to them. Horizontal competition is also rife. Small trading companies are subject to fierce horizontal competition; e.g., [Hsing 1999, p. 112] explains that “a manufacturer usually had more than one partner trading company”, and [Ha-Brookshire and Dyer 2008, p. 11] document that industry executives describe the environment of US apparel import intermediaries as one of “deadly competition”. Likewise, retailers, which often compete for consumer demand, compete also to source from the same set of intermediaries; e.g., [Masson et al. 2007, p. 247] mention that intermediaries “work for multiple customers”.

Competition is clearly a prominent aspect of global supply chains with intermediaries, yet most of the existent literature has by-and-large ignored this aspect and focused instead on identifying
rationales for the existence of intermediaries. In this paper, we aim to address this gap in the literature by studying the joint impact of horizontal and vertical competition on the performance of a given supply chain with intermediaries. To do so, we model a three-tier supply chain, where the middle tier consists of a set of intermediaries who compete in quantities to mediate between the other two tiers, which consist of quantity-competing retailers and capacity-constrained suppliers.

A critical difference between our model and most existing models of multi-tier supply-chain competition is that our model portrays the retailers as Stackelberg leaders. Portraying retailers as followers is reasonable for many real-world supply chains (e.g., Dell and Coca-Cola may well lead the supply chains for distribution of their products), but may not be realistic in the context of intermediation firms that mediate between retailers in industries such as fashion apparel or shoes and low-cost international suppliers. For instance, fashion retailers continuously monitor market trends and generate orders as a response to these rapidly changing trends. Depending on the specific order, the intermediary thereafter selects the suppliers with the technical capability and spare capacity to fulfill demand. In other words, intermediaries orchestrate retailer-driven global supply chains as a response to a specific order from a retailer facing incidental demand; see Masson et al. [2007] and Knowledge@Wharton [2007]. Chronologically, the retailer order precedes the orchestration of the supply chain, and thus it makes sense to portray the retailers as leaders.

We provide a complete analytical characterization of the symmetric supply chain equilibrium, and use the closed-form expressions to answer four research questions. First, how does the competitive environment affect the profits of retailers and intermediaries? In particular, can the retailers leverage their leadership position to increase their market power and seize a large share of the overall supply chain profits? And, under which competitive circumstances can intermediaries retain a substantial share of the overall profits? Second, how does the presence of an intermediary tier affect the efficiency of the decentralized supply chain? The literature shows that intermediaries help retailers to overcome informational and transactional barriers, and thus they generally help to improve the overall performance of global supply chains. Nevertheless, the question remains whether the vertical competition established between retailers and intermediaries in global supply chains results in the double marginalization effect first identified by Spengler [1950], and thus brings an element of inefficiency. Third, can intermediaries exploit their private information about the supply side to alter their relative bargaining position with respect to the retailers? As mentioned above, intermediaries use their knowledge of the international supplier base to help retailers overcome their informational barriers, but as noted in Babich and Yang [2014], when suppliers have private information and procurement service providers (PSPs) are better informed than buyers, “it is not obvious that PSPs would share benefits of better information with the buyer.” We consider the case where intermediaries help retailers to identify suitable suppliers, but they withhold information about the suppliers’ cost function, in an attempt to improve their bargaining position. Fourth, how does competition affect the well-documented transactional benefits of intermediation? To answer this question, we consider a variant of our model where retailers have the option to
deal directly with the suppliers (without an intermediary) provided they are willing to pay a fixed transaction cost per supplier, and study how competition affects the retailers decision to either source directly or through intermediaries.

With respect to the impact of competition on retailer and intermediary profits, we find that the intermediary profits are unimodal with respect to the number of suppliers in its base. This is in direct contrast with the insight from existing models of supply chain competition, in which suppliers lead. Based on that literature, one might have expected that the larger the supplier base, the larger the market power of the intermediaries and thus the larger their profits. This intuition does bear out when the size of the supplier base is “small”. However, in a world where retailers lead, when the supplier base is “large enough”, we show that the weakness of the suppliers becomes the weakness of the intermediaries, and the retailers exploit their leadership position to increase their market power and retain greater supply chain profits. A crucial implication of this result is that intermediaries in retailer-driven global supply chains prefer products for which the supplier base (existing production capacity) is neither too narrow nor too broad, because (ceteris paribus) products for which there is an intermediate production capacity available generate larger intermediary profits. The result also offers some insight into how the financial performance of trading companies, and consequently of economies reliant on this sector, depends on the available production capacity. This capacity is a function of various economic and environmental factors. For instance, Barrie [2013] reports a shortfall in available capacity for 2013 in the fashion apparel sector, whereas Zhao [2013] points to endemic overcapacity in the Chinese fashion industry during 1980s and 1990s. Our analysis shows that intermediary profits will be squeezed in either of these two eventualities; that is, in case of shortfall or excess in available capacity.

With respect to supply chain efficiency, we observe that the presence of an additional tier of intermediaries does not necessarily introduce an element of inefficiency to the decentralized supply chain; that is, the aggregate supply chain profits in the decentralized three-tier chain is not necessarily smaller than that in the centralized (integrated) supply chain. The classic result on double marginalization would have suggested otherwise (Spengler 1950). However, the differentiating feature of our analysis is that, along with vertical competition, we simultaneously account for horizontal competition. It is well known that the relative bargaining strength of players in a vertical relationship significantly determines the extent of double marginalization. Further, increasing the number of within-tier competitors reduces a specific player’s bargaining power in the vertical interaction. Such adjustments to competitive intensity may be carried out at each tier. We find that there always exists an appropriate balance between horizontal and vertical competition that completely offsets the effect of double marginalization, and leads to supply chain efficiency. An implication of this result is that regulators may try and improve the efficiency of global supply chains by taking measures to encourage a healthy level of competition at each of the three tiers. Our analysis demonstrates, however, that in order for regulatory intervention to be successful, it must be carefully tailored to the structure of the supply chain in question.
We mentioned earlier that the retailers exploit their leadership position to increase their market power with respect to the intermediaries. This leads one to wonder whether *intermediaries can exploit their private information about the supplier’s cost function to extract greater rents*. To answer this question, we consider the case when the intermediaries know whether the supply sensitivity to price is high or low, but the retailers believe the sensitivity follows a certain prior distribution—in our model, higher sensitivity of supply corresponds to a steeper marginal cost function for the suppliers. We then characterize the impact of asymmetric information on both the expected and the realized equilibrium profits. The comparison of the *expected equilibrium profits* shows that on average the intermediaries are indeed able to exploit their private information at the expense of the retailers.

The comparison of the *realized equilibrium profits*, however, shows that the *realized* profits of both intermediaries and retailers could be higher or lower depending on the realized supply sensitivity. In particular, *when the realized sensitivity is low*, the intermediary profits are lower than with complete information because the retailers’ prior belief is that the supply sensitivity is higher than it actually is and, as a result, they select a low quantity which results in low intermediary profits. An implication of this result is that, when the realized sensitivity is low, intermediaries would benefit from disclosing their private information to the retailers, if they can do so credibly.

The impact of asymmetric information on the realized retailer profits for the low-sensitivity scenario depends on the retailers’ prior probability of low sensitivity. When the prior probability of low sensitivity is very small, realized retailer profits are smaller, but when this probability is moderate, they are *larger*. The reason for this is that the presence of asymmetric information has a dual effect on the realized retailer profits: a negative *incomplete information* effect, and a positive *competition mitigation* effect. The negative effect is that the retailers believe the supply is more sensitive than it really is, and thus they select a lower than optimal (for retailers) quantity. The positive effect, however, is that the missing information attenuates the intensity of competition among retailers—because the retailers believe supply sensitivity is higher than it actually is. As a result, when the prior probability of low supply sensitivity is small, the *incomplete information* effect dominates, and otherwise the *competition mitigation* effect dominates.

Finally, although our main focus is the impact of competition in global supply chain with intermediaries, we also show how the classical transaction cost rationale for the existence of intermediaries identified in the Economics literature can be incorporated into our competition framework. Specifically, we consider a variant of our model where retailers have the option to deal directly with the suppliers (without the intervention of an intermediary) provided they are willing to pay a fixed transaction cost per supplier. We use this enhanced framework to understand how the retailer

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1 Expected profits correspond to the ex-ante perspective (when neither the intermediaries nor the retailers know the supply sensitivity). Realized profits refer to the interim perspective (when the intermediaries know the supply sensitivity, but the retailers do not) and the ex-post perspective (when the intermediaries and the retailers know the supply sensitivity).
decision to either source directly or via intermediaries depends on the competitive environment. Specifically, we identify two insights. First, rising manufacturing costs in low-cost international locations, e.g., China, may hurt the operating margin of intermediation firms, but that should not discourage retailers from using intermediaries. Second, the threat of retailers sourcing directly from suppliers may induce intermediaries to widen their supplier base, relative to what would be optimal for them otherwise.

We make two main contributions. First, we propose a model of competition in global supply chains with intermediaries (with and without complete information) that incorporates both horizontal and vertical competition and portrays the retailers as leaders. Second, we use this framework to shed new light on aspects of supply chain sourcing such as intermediary profitability, supply chain efficiency, and the impact of asymmetric information. In the process, we synthesize and extend two parallel streams of literature: one on intermediation and the other on supply chain competition.

The remainder of this manuscript is organized as follows. Section 2 discusses how our work relates to the existing literature. Section 3 describes our model of competition in sourcing supply chains. Section 4 characterizes the equilibrium, and discusses its properties in terms of intermediary profits and supply chain efficiency. Section 5 studies the effect of asymmetric information, Section 6 studies the model with transaction costs and a direct procurement option, and Section 7 concludes. Appendix A contains tables and Appendix B contains figures. A supplemental file includes several appendices. Appendix D contains proofs; Appendix E shows that our results are robust to the case where intermediaries have access to both shared and exclusive suppliers; Appendix F shows that our results are robust to the use of a nonlinear marginal cost function; Appendix G shows that our results are robust to the presence of stochasticity in the demand function; and Appendix H compares the equilibrium for our model with retailers as leaders, with those for other models in the literature.

2. Relation to the literature

We now discuss how our work is related to the literature on intermediation and the literature on supply chain competition.

2.1. Literature on intermediation

Our work is related to the Economics literature on intermediation, which according to Wu [2004] “studies the economic agents who coordinate and arbitrage transactions in between a group of suppliers and customers.” As mentioned before, the main distinguishing feature of our work is that while the Economics literature has focused on justifying the existence of middlemen through their ability to reduce transaction costs (Rubinstein and Wolinsky [1987] and Biglaiser [1993]), we focus on understanding the joint impact of horizontal and vertical competition on sourcing in a three-tier
supply chain. Our modeling framework differs substantially from the modeling frameworks used in this literature. The intermediation literature generally assumes that each buyer and seller is interested in a single unit for which they have idiosyncratic valuations, and price is determined through bilateral bargaining. In contrast, in order to capture the key features of global sourcing arrangements, we assume that the interaction between various players is governed by a market mechanism which is retailer-driven. Accordingly, we model a consumer demand function, as well as quantity competition a la Cournot-Stackelberg. Thus, we are able to track not only the intermediary margin but also the overall efficiency of the supply chain. In summary, our model is closer to the multi-tier competition models developed in recent Operations Management literature, and our focus is on the operational aspects of supply chain sourcing.

Our work is also related to Belavina and Girotra [2012], who study intermediation in a supply chain with two suppliers, one intermediary, and two buyers, with players in the same tier not competing directly. They provide a new rationale for the existence of intermediaries. Specifically, they show that in a multi-period setting the intermediary is more effective in inducing efficient decisions from the suppliers (e.g., quality related), because the intermediary has access to the pooled demand of both buyers, and therefore superior ability to commit to future business with each supplier. Babich and Yang [2014] consider a supply chain with one retailer, one intermediary, and two suppliers. They consider the case where the suppliers possess private information about their reliability and costs, and they justify the existence of the intermediary because of the informational benefits it offers to the retailer. Again, the main difference between both of these papers and our work is that they focus on explaining the existence of an intermediary tier, while we focus on the impact of competition.

2.2. Literature on supply chain competition

In contrast to our manuscript, most existing models of multi-tier supply-chain competition assume the retailers are followers. A prominent example is Corbett and Karmarkar [2001], herein C&K, who consider entry in a multi-tier supply chain with vertical competition across tiers and horizontal quantity-competition within each tier. C&K assume the retailers face a deterministic linear demand function, and they are followers with respect to the suppliers who face constant marginal costs. Several papers use variants of the multi-tier supply chain proposed by C&K with quantity competition at every tier and where the retailers are followers: Carr and Karmarkar [2005] consider the case where there is assembly, Adida and DeMiguel [2011] consider a two-tier supply chain with multiple differentiated products and risk-averse retailers facing uncertain demand, Federgruen and Hu [2013] consider a multiple-tier supply network with differentiated products, and Cho [2013] uses the framework by C&K to study the effect of horizontal mergers on consumer prices. Even for two-tier supply chains, several papers consider models in which a single supplier leads several competing retailers: Bernstein and Federgruen [2005] consider one manufacturer and multiple retailers who compete by choosing their retail prices (they assume that the demand faced by each retailer
is stochastic with a distribution that depends on the retail prices of all retailers), Netessine and Zhang [2005] consider a supply chain with one manufacturer and quantity-competing retailers who face an exogenously determined retail price and a stochastic demand whose distribution depends on the order quantities of all retailers, Cachon and Lariviere [2005] consider one supplier who leads competing retailers (their results hold both for the case where the retailers are competitive newsvendors and when the retailers compete a la Cournot).

Very few papers in the existing literature model the retailers as leaders. For example, Choi [1991] considers a model in which one retailer leads two suppliers. This model assumes that the suppliers possess complete information about the demand function facing the retailer, and that they exploit this information strategically when making their production decisions. This assumption imposes a level of sophistication on the suppliers' strategic capabilities, and endows them with a degree of information, that does not seem appropriate for the context of low-cost international suppliers interacting with procurement firms. Moreover, we show in Appendix H that the equilibrium in Choi's model with the retailer as leader is equivalent to that of the model by C&K, where the retailers are followers, in the sense that the equilibrium quantity, retail price, and supply chain aggregate profits are identical for both models. Overall, we think that assuming suppliers can strategically exploit their complete knowledge of the retailers demand function, or for that matter even strategically compete with numerous other similar suppliers, is not realistic in our setting. This is crucial, because as we demonstrate in this paper, incorporating a more apt model for suppliers, along with retailers leading the interaction, results in substantially different insights than those suggested by the existing literature on supply chain competition.

Majumder and Srinivasan [2008], herein M&S, consider a model where any of the firms in a network supply chain could be the leader, and study the effect of leadership on supply chain efficiency as well as the effect of competition between network supply chains. Their model is closely related to C&K’s, but the two models differ in three important aspects. First, while C&K consider a serial multi-tier supply chain, M&S consider a network supply chain. Second, while C&K consider both vertical competition across tiers and horizontal competition within tiers, M&S consider only vertical competition within networks, and they consider horizontal competition only between networks. Third, while C&K assume constant marginal cost of manufacturing, M&S assume increasing marginal cost of manufacturing, and they argue that, with wholesale price contracts, this is the only assumption that results in equilibrium when suppliers follow.²

²Perakis and Roels [2007] study efficiency in supply chains with price-only contracts, and consider a comprehensive range of models, including both a push and a pull supply chain where the retailer leads. For the push chain (where the retailer keeps the inventory) they assume that the retailer decides both the wholesale price and the quantity, which results in the retailer keeping all the profits. In contrast, our model allows the intermediary to keep a positive margin, as do most procurement firms. Their pull supply chain does not capture the business model of intermediaries since it requires inventory to be held by the intermediaries, something that is not observed in practice (Fung et al. 2008). Another key difference between our model and the push and pull models by Perakis and Roels [2007] is that
3. The competition model

Modeling simultaneous horizontal and vertical competition in a multi-tier supply chain is a challenging problem. Fortunately, the seminal paper by C&K and the more recent paper by M&S provide a parsimonious framework to model supply chain competition. We build on these well-established models and justify any departure warranted by our specific context of intermediation in retailer-driven global supply chains.

We consider a model of competition in a supply chain with three tiers: (i) retailers, (ii) intermediaries, and (iii) suppliers. The first tier consists of $R$ retailers who face the consumer demand captured by a linear demand function. The retailers compete a la Cournot with each other, and act as Stackelberg leaders with respect to the intermediaries. Specifically, each retailer chooses its order quantity in order to maximize its profits, assuming the other retailers keep their order quantities fixed, and anticipating the reaction of the intermediaries as well as the intermediary market-clearing price. The second tier consists of $I$ intermediaries who compete a la Cournot with each other, and act as Stackelberg leaders with respect to the suppliers. Each intermediary chooses its order quantity in order to maximize its profit, assuming the other intermediaries keep their order quantities fixed, and anticipating the reaction of the suppliers as well as the supplier-market-clearing price. The third tier consists of $S$ capacity-constrained suppliers who choose their production quantities in order to maximize their profits. The sequence of events defining the game is illustrated in Figure 1.

We focus on a static (one-shot) game because we study situations where retailers in settings such as fashion apparel and shoes contact intermediaries to satisfy incidental demand for new products. As mentioned in the introduction, in these settings retailers constantly monitor the rapidly changing market trends, and place specific orders through intermediaries as a response to these trends. Moreover, in response to a product request from the retailer, intermediaries typically orchestrate a one-time order-specific supply chain. For instance, Purvis et al. [2013] explain that “Kopczak and Johnson [2003] state that in sectors in which product and process technology evolve rapidly and product lives are short, with each new generation of products the components and process technologies that are specified may change dramatically. Likewise, Christopher et al. [2004] state that retailers have to act these days as network orchestrators, working with a team of actors closely for a while but that will, however, be disbanded and a new one assembled for the next play.” This context is adequately captured with a static game.

Also, our model is based on wholesale price contracts. Our motivation to do this is first that there is empirical support for their widespread usage and popularity (Lafontaine and Slade 2012), and second that there is ample precedence in the supply chain literature (see, for instance, Lariviere they model horizontal competition in only a single tier, while we consider simultaneous horizontal competition in multiple tiers of the supply chain.
and Porteus [2001a] and Perakis and Roels [2007]). An added advantage of considering a static model with wholesale price contracts is that these are standard assumptions in the literature on supply chain competition (C&K, Choi 1991, M&S, Perakis and Roels 2007), and therefore a direct comparison with that literature is possible.

In the remainder of this section, we study the equilibrium via backward induction, starting with the suppliers in Section 3.1, the intermediaries in Section 3.2, and the retailers in Section 3.3.

3.1. The suppliers

Given a supplier price $p_s$, the $j$th supplier chooses its production quantity $q_{s,j}$ to maximize its profit:

$$\max_{q_{s,j}} \pi_{s,j} = p_s q_{s,j} - c(q_{s,j}),$$

where $p_s q_{s,j}$ is the supplier’s revenue from sales, and $c(q_{s,j})$ is the production cost. Like M&S, we assume the production cost is convex quadratic: $c(q_{s,j}) = s_1 q_{s,j} + (s_2/2)q_{s,j}^2$, or equivalently that the $j$th supplier has linearly increasing marginal cost:

$$c'(q_{s,j}) = s_1 + s_2 q_{s,j},$$

where $s_1 > 0$ is the intercept, and $s_2 \geq 0$ is the sensitivity.

The $j$th supplier optimally chooses to produce the quantity such that its marginal cost equals the supply price; that is, the quantity $q_{s,j}$ such that

$$p_s = s_1 + s_2 q_{s,j}.$$  (2)

This implies that the supplier profit is

$$\pi_{s,j} = s_2 q_{s,j}^2.$$  (3)

Because the suppliers are symmetric, it follows from (2) that the aggregate supply function; that is, the total quantity produced by the suppliers for a given supplier price $p_s$ is

$$Q_s(p_s) = S(p_s - s_1)/s_2.$$  (4)

We have also considered a model where the retailers offer a two-part tariff to the intermediaries, but we find that as the retailers set the contract terms to maximize their own profits while guaranteeing a reservation profit to intermediaries, the intermediaries earn exactly their reservation profit, leaving all surplus to the retailers. Thus, this is not adequate to model intermediation because, as explained in Rauch [2001] (p. 1196), intermediaries do keep a margin, but they have little leverage to raise their payoff through side payments or other means.

4 An alternative model would be to have the intermediary offer the suppliers a price equal to the suppliers’ average cost (i.e., $p_s = s_1 + s_2 q_{s,j}/2$), but this would result in zero supplier profits, which does not seem realistic. Besides, this alternate formulation would not affect the qualitative nature of our insights.
Remark 3.1. Note that the equilibrium among suppliers is completely characterized by the aggregate supply function given in (4). There are two implications from this. First, our analysis applies also to the case when suppliers are asymmetric, provided that their aggregate supply function is approximately linearly increasing, or (equivalently) that their aggregate marginal cost function is approximately linearly increasing.\footnote{To see this, note that confronted with a heterogeneous (in cost) supply base, the intermediaries would first order from the cheapest supplier (up to its maximum capacity) and then would engage with progressively more expensive suppliers. Such a supplier selection procedure would result in an increasing aggregate marginal cost function (common to all intermediaries) that could be approximated with a linearly increasing marginal cost function: $c'(Q) = s_1 + \hat{s}_2 \cdot Q$ with $\hat{s}_2 > 0$. It is easy to see that the equilibrium and the insights from our model would not change much if we used this aggregate supply function provided that $\hat{s}_2 = s_2/S$.} Second, because the aggregate supply function depends on the number of suppliers and their sensitivity only through the ratio $S/s$, the impact on the equilibrium of an increase in the number of suppliers $S$ is equivalent to the impact of a certain decrease in the supply sensitivity $s$. Essentially, in our model both $S$ and $s$ affect the total production capacity in the supply chain.

A few additional comments are in order. First, the suppliers in our model are price takers with respect to the supplier price $p_s$, and thus they do not compete strategically with each other. Nevertheless, suppliers do compete implicitly in our model because both the market clearing supplier price and their production quantity ultimately depend on the number of suppliers in the market. We believe this is an accurate representation of the decision process followed by the type of low-cost international suppliers that intermediary firms deal with. The alternative would be to model suppliers as being cognizant of their strategic interaction with numerous other suppliers and possessing knowledge of the retailers' demand function. Neither of these assumptions seem very palatable in our context. Second, although we choose a linear marginal cost function for tractability and clarity of exposition, in Appendix D we study the robustness of our results to the use of a nonlinear marginal opportunity cost function, and we show that the insight that the intermediary profits are unimodal with respect to the number of suppliers holds also for a convex monomial marginal cost function. Third, although we do not explicitly include the supplier capacity constraints in our model, they are implicitly considered because in equilibrium a supplier would never produce a quantity larger than $(d_1 - s_1)/s$, where $d_1 (> s_1)$ is the risk-adjusted intercept of the demand function.

Finally, like M&S we consider linearly increasing marginal costs of supply. In addition to M&S, other authors who have assumed increasing marginal costs include Anand and Mendelson [1997], Correa et al. [2013], and Ha et al. [2011]. M&S motivate their assumption of increasing marginal cost arguing that this assumption is required to achieve an equilibrium when the suppliers are followers in the supply chain. Specifically, [Majumder and Srinivasan 2008, p. 1190] claim: “Since we have models in which the manufacturer can be at the receiving end of a wholesale price contract, if she had a constant marginal cost, she would choose to either not produce (if the wholesale price is
lower than his marginal cost), produce an arbitrarily large quantity (if the wholesale price is higher) or produce an indeterminate quantity (if they are equal). In addition, we believe this is the most realistic assumption in the context of intermediary firms that use the existing production capacity of their network of suppliers to satisfy incidental demand from retailers. Because the suppliers use existing capacity, they do not make any additional capacity investments and thus they do not incur any additional fixed costs. One of the key rationales for modeling decreasing marginal cost of supply (economies of scale) is that any fixed cost of capacity investment can be defrayed over multiple units. In the absence of incremental fixed costs, it is sufficient for the purposes of decision making to capture the variable costs. In this setting, although marginal variable costs could be constant for small quantities, they will inevitably increase as the order quantities approach the capacity constraint of the suppliers. In addition, linearly increasing marginal costs can also be motivated by relaxing the assumption that suppliers are symmetric, and adopting an asymmetric aggregate view instead. As discussed in Remark 3.1 and Footnote 5, this would result in an increasing aggregate marginal cost function (common to all intermediaries) that could be approximated with a linear function.

3.2. The intermediaries

Given an intermediary price \( p_i \), the \( l \)th intermediary chooses its order quantity \( q_{i,l} \) to maximize its profit, assuming the rest of the intermediaries keep their order quantities fixed, and anticipating the reaction of the suppliers as well as the supplier price resulting from the supplier-market-clearing condition. The \( l \)th intermediary decision may be written as:

\[
\max_{q_{i,l}, p_s} (p_i - p_s) q_{i,l} \\
\text{s.t. } q_{i,l} + Q_{i,-l} = Q_s(p_s),
\]

where \( Q_{i,-l} \) is the total quantity ordered by the rest of the intermediaries, \( Q_s(p_s) \) is the total quantity produced by suppliers when the supplier price is \( p_s \), and Constraint (6) is the supplier-market-clearing condition.

Using Equation (4) to eliminate the supplier price from the intermediary decision problem, we obtain the following equivalent decision problem:

\[
\max_{q_{i,l}} \left[ p_i - \left( s_1 + s_2 \frac{q_{i,l} + Q_{i,-l}}{S} \right) \right] q_{i,l}.
\]

6 Citing a popular Economics textbook [Varian 1992, Section 5.2]: “When we are near to capacity, we need to use more than a proportional amount of the variable inputs to increase output. Thus, the average variable cost function should eventually increase as output increases.”

7 Note that we assume in our base case model that all suppliers are shared by all intermediaries. In Appendix E, however, we show that our qualitative results are robust to the general case where some of the suppliers may be shared by some of the intermediaries and others exclusive to a single intermediary.
Finally, we show in Appendix A that for a given intermediary price $p_i$, the total quantity produced by intermediaries at equilibrium is

$$Q_i(p_i) = \frac{SI(p_i - s_1)}{s_2(I + 1)}. \quad (8)$$

### 3.3. The retailers

To simplify the exposition, we model demand with a deterministic demand function, although we show in Appendix G that our results generally hold also for the case with a stochastic demand function. Concretely, we model demand with the following **linear inverse demand function**:\(^8\)

$$p_r = d_1 - d_2 Q, \quad (9)$$

where $d_1$ is the demand intercept and $d_2$ is the demand sensitivity.

The $k$th retailer profit is

$$\pi_{r,k} = (p_r - p_i)q_{r,k} = (d_1 - d_2(q_{r,k} + Q_{r,-k}) - p_i)q_{r,k},$$

where $q_{r,k}$ is the order quantity selected by the $k$th retailer and $Q_{r,-k}$ is the total quantity ordered by all other retailers.

Then the $k$th retailer chooses its order quantity $q_{r,k}$ to maximize its profit, assuming the rest of the retailers keep their order quantities fixed, and anticipating the intermediary reaction and the intermediary-market-clearing price $p_i$:

$$\max_{q_{r,k}, p_i} [d_1 - d_2(q_{r,k} + Q_{r,-k}) - p_i]q_{r,k} \quad (10)$$

s.t. $q_{r,k} + Q_{r,-k} = Q_i(p_i), \quad (11)$

where $Q_{r,-k}$ is the total quantity ordered by the rest of retailers, $Q_i(p_i)$ is the intermediary equilibrium quantity for a price $p_i$, and Constraint (11) is the intermediary-market-clearing condition.

Finally, using (8), one can eliminate the intermediary price from the problem and obtain the following equivalent retailer decision problem:

$$\max_{q_{r,k}} \left[ d_1 - d_2(q_{r,k} + Q_{r,-k}) - \left( s_1 + s_2 \frac{I + 1}{SI}(q_{r,k} + Q_{r,-k}) \right) \right] q_{r,k}. \quad (10)$$

---

\(^8\)Linear demand models have been widely used both in the Economics literature (see, for instance, Singh and Vives [1984] and Häckner [2003]) as well as in the Operations Management literature (Farahat and Perakis [2011], and Farahat and Perakis [2009]). C&K assume a deterministic linear inverse demand function. Other authors have also used it in the context of supply chain competition. Cachon and Lariviere [2005], for instance, mention that the results for their model with competing retailers can be applied for the particular case of “Cournot competition with deterministic linear demand”, and they give as an example a deterministic linear inverse demand function for a single homogeneous product with retailer differentiation.
4. The equilibrium

We now give closed-form expressions for the equilibrium quantities, and analyze their properties.

4.1. Closed-form expressions

Theorem 4.1 shows that the equilibrium quantities are as given in Table 1, Theorem 4.2 shows that the monotonicity properties of the equilibrium quantities are as given in Table 2, and Theorem 4.4 gives the closed-form expression for the supply chain efficiency and characterizes its monotonicity properties.

**Theorem 4.1.** Let \( d_1 \geq s_1 \), then there exists a unique equilibrium for the symmetric supply chain, the equilibrium is symmetric, and the equilibrium quantities are given by Table 1.

**Theorem 4.2.** Let \( d_1 \geq s_1 \), the monotonicity properties of the equilibrium quantities are as stated in Table 2. Moreover, the intermediary profit \( \pi_i \) achieves a maximum with respect to the number of suppliers for \( S = s_2(I + 1)/(d_2I) \).

Following the supply chain literature (Lariviere and Porteus [2001b], Netessine and Zhang [2005], Farahat and Perakis [2009], Adida and DeMiguel [2011]), we define the supply chain efficiency as the ratio between the decentralized supply chain profits to the centralized profits, which correspond to the case when a system planner chooses all quantities to maximize the total supply chain profits. The following proposition gives the optimal quantity and aggregate profit in the centralized supply chain.

**Proposition 4.3.** The optimal production quantity and aggregate profit in the centralized supply chain are
\[
Q = \frac{(d_1 - s_1)}{2d_2 + s_2/S} \quad \text{and} \quad \pi_c = \frac{(d_1 - s_1)^2}{2(2d_2 + s_2/S)}.
\]

The following proposition gives closed-form expressions of the supply chain efficiency, and characterizes its monotonicity properties.

**Theorem 4.4.** The supply chain efficiency is
\[
\text{Efficiency} = \frac{RI[s_2R(I + 2) + 2(d_2SI + s_2(I + 1))] 2d_2S + s_2}{(d_2SI + s_2(I + 1))^2} \frac{2d_2S + s_2}{(R + 1)^2}.
\]

Moreover the monotonicity properties of the efficiency with respect to the number of retailers, intermediaries, and suppliers are as follows:

1. The efficiency is unimodal with respect to the number of suppliers \( S \) and reaches a maximum equal to one for \( S = s_2(R + I + 1)/(d_2I(R - 1)) \) provided that \( R > 1 \). If \( R = 1 \), then the efficiency is monotonically increasing in the number of suppliers, and tends to one as \( S \to \infty \).

2. The efficiency is unimodal with respect to the number of intermediaries \( I \) and reaches a maximum equal to one for \( I = s_2(R + 1)/(d_2S(R - 1) - s_2) \) provided that \( d_2S(R - 1) - s_2 > 0 \). Otherwise, the efficiency is monotonically increasing in the number of intermediaries, and tends to \( R(2d_2S + s_2)(2d_2S + s_2(R + 2))/((R + 1)^2(d_2S + s_2)^2) \) as \( I \to \infty \).
3. The efficiency is unimodal with respect to the number of retailers $R$ and reaches a maximum\(^9\) equal to one for $R = (d_2SI + s_2(I + 1))/(d_2SI - s_2)$ provided that $d_2SI - s_2 > 0$. Otherwise, the efficiency is monotonically increasing in the number of retailers, and tends to $s_2I(I + 2)(2d_2S + s_2)/(d_2SI + s_2(I + 1))^2$ as $R \to \infty$.

4.2. Discussion

Table 2 summarizes the monotonicity properties of the equilibrium quantities. The table gives several intuitive results: (i) the total quantity produced in the supply chain is increasing in the number of retailers, intermediaries, and suppliers, and decreasing in the demand and supply sensitivities, (ii) the prices at each tier are decreasing in the number of players in following tiers, increasing in the number of players in leading tiers, increasing in the supply function sensitivity, and decreasing in the demand function sensitivity, and (iv) the retailer profit is increasing in the number of intermediaries and suppliers, but decreasing in the number of retailers, and decreasing in both the supply and demand sensitivities.

4.2.1. Intermediary profits. Compared to the above findings, the results about the intermediary profits are arguably more interesting. Although expectedly the intermediary margin decreases in the number of intermediaries and increases in the number of retailers, surprisingly the intermediary margin decreases in the number of suppliers. Moreover, since the overall order quantity is monotonically increasing in the number of suppliers, therefore the intermediary profits are unimodal with respect to the number of suppliers, reaching their maximum for a finite number of suppliers. Based on the results in C&K and Choi [1991], one might have expected that the larger the supplier base, the larger the market power of the intermediaries and thus the larger their margin and profits. However, in a world where retailers lead, when the supplier base is “large enough”, we show that the weakness of the suppliers becomes the weakness of the intermediaries, and the retailers exploit their leadership position to increase their market power and retain greater supply chain profits. Specifically, in the presence of an infinite number of suppliers, the retailers know that the intermediaries can get an unlimited quantity of the product at a price of $s_1$ per unit. As a result, the retailers can exploit their leader advantage to drive the intermediary price to the suppliers’ marginal cost $s_1$. In this limiting case, the retailers have all the market power and keep all profits.

A crucial implication of this result is that intermediaries in global supply chains prefer products for which the supplier base (existing production capacity) is neither too narrow nor too broad, because (ceteris paribus) products for which there is an intermediate production capacity available generate larger intermediary profits. The result also offers some insight into how the financial

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\(^9\) Note that the values of $R, S$ and $I$ respectively that maximize the efficiency may not be integers, in which case the maximum would be reached at the integer value just below or above it. To keep the exposition simple, we approximate the true integer maximum with the possibly non integer expressions above.
performance of trading companies, and consequently of economies reliant on this sector, depends on the available production capacity. This capacity is a function of various economic and environmental factors. For instance, Barrie [2013] reports a shortfall in available capacity for 2013 in the fashion apparel sector, whereas Zhao [2013] points to endemic overcapacity in the Chinese fashion industry during 1980s and 1990s. Our analysis shows that intermediary profits will be squeezed in either of these two eventualities; that is, in case of shortfall or excess in available capacity.

4.2.2. Efficiency. Theorem 4.4 gives the closed-form expression for the supply chain efficiency and characterizes its monotonicity properties. Our main observation is that a supply chain efficiency equal to one (that is, an efficient supply chain) can always be achieved in the presence of intermediaries provided there is the right balance of competition at the three tiers. To see this, note from Point 1 in Theorem 4.4 that provided the number of retailers is greater than one, we can always adjust the number of suppliers so that efficiency one is achieved. Also, note that provided that there is a sufficiently large number of suppliers and retailers, the condition $d_2S(R-1) - s_2 > 0$ will be satisfied, and thus from Point 2 we have that there exists a number of intermediaries for which the supply chain efficiency is equal to one. Finally, from Point 3 we see that provided there is a sufficiently large number of suppliers and intermediaries we can always satisfy the condition $d_2SI - s_2 > 0$ and thus there will exist a number of retailers for which the efficiency of the supply chain is equal to one.

The main takeaway from this analysis is that the presence of an additional tier of intermediaries in the supply chain does not necessarily introduce an element of inefficiency to the supply chain; that is, the aggregate supply chain profit in the decentralized chain with intermediaries is not necessarily smaller than that in the centralized (integrated) supply chain. The classic result on double marginalization would have suggested otherwise (Spengler 1950). However, accounting for competition in our model is the differentiating feature. It is well known that the relative bargaining strength of players in a vertical relationship significantly determines the extent of double marginalization. Further, increasing the number of within-tier competitors reduces a specific player’s bargaining power in the vertical interaction. Such adjustments to competitive intensity may be carried out at each tier. We find that there always exists an appropriate balance between horizontal and vertical competition that completely offsets the effect of double marginalization, and leads to supply chain efficiency. An implication of this result is that regulators may try and improve the efficiency of global supply chains by taking measures to encourage a healthy level of competition at each of the three tiers.

We find, however, that if regulatory intervention is to succeed at improving the overall supply chain efficiency, it must be carefully tailored to the structure of the supply chain in question. To see this, note that Proposition 4.2 shows that intermediaries would choose a number of suppliers $S = s_2(I + 1)/(d_2I)$ in order to maximize their profit. Part 1 of Theorem 4.4, on the other hand, shows that overall efficiency is maximized for a different number of suppliers $S = s_2(R + I + 1)/(d_2I(R -$
1). For \( R = 2 \), the number of suppliers that maximizes the overall supply-chain efficiency is larger than the number that maximizes intermediary profits. Also, the number of suppliers that maximizes efficiency is monotonically decreasing in \( R \). This suggests that with high concentration in the retail tier (small \( R \)), a decentralized supply chain is likely to yield a smaller-than-efficient size of the supply base, and thus subsidizing the intermediaries to enlarge their supply base is likely to improve the overall efficiency. However, for supply chains with low retailer concentration (large \( R \)), such subsidies are unlikely to help.

Interestingly, Rauch [2001] cites the trade-creating impact of immigrants, expatriates, and foreign direct investment, as evidence to suggest that intermediaries may not be adequately connecting buyers and sellers internationally, and thereby makes a case for regulatory intervention to encourage a larger supply base. However, Rauch [2001] also points to contradictory evidence: while the governments of Japan, Korea and Turkey improved trade by encouraging intermediaries to maintain larger supply bases through subsidies, similar attempts were unsuccessful in Taiwan and the U.S. Our analysis above offers a possible explanation for this in terms of retailer tier concentration. Specifically, we find that regulatory intervention will be effective only if it is tailored to the specific characteristics of the supply chains.

5. The case with asymmetric information

An important insight from our analysis is that in a supply chain where retailers lead, they take advantage of their leading position to increase their market power with respect to the intermediaries. This leads one to wonder whether intermediaries can exploit their private information to extract greater rents. For instance, although intermediaries help the retailers overcome some of their information barriers (e.g., by identifying appropriate suppliers), they may also find it advantageous to withhold certain information from the retailers such as the supply sensitivity. As noted in the literature on asymmetric information, truthful information sharing must be incentivized: an agent with private information may choose not to disclose it if beneficial. To answer this question, in this section we study how the presence of asymmetric information about the supply sensitivity alters the balance of market power between the retailers and the intermediaries.

5.1. The model and equilibrium

To model asymmetric information, we assume that intermediaries know whether the supply sensitivity is high \( s_2 = s^H_2 \) or low \( s_2 = s^L_2 \), but the retailers share the prior belief that there is a probability \( \nu \) that the sensitivity is low \( (s_2 = s^L_2) \) and \( 1 - \nu \) that it is high \( (s_2 = s^H_2) \).

The \( k \)th retailer chooses its order quantity to maximize its expected profit assuming the rest of the retailers keep their order quantities fixed, and anticipating the reaction of the intermediaries
as well as the intermediary-market-clearing price for each scenario (with low or high sensitivity). The \( k \)th retailer’s expected profit is:

\[
E[\pi_{r,k}] = \nu (d_1 - d_2(q_{r,k} + Q_{r,-k}) - p_i^L) q_{r,k} + (1 - \nu) (d_1 - d_2(q_{r,k} + Q_{r,-k}) - p_i^H) q_{r,k},
\]

(13)

where the first (second) term on the right-hand side of (13) corresponds to the profit for the case with low (high) sensitivity, and the intermediary market clearing prices for the scenarios with low and high sensitivity, \( p_i^L \) and \( p_i^H \), are given by Theorem 4.1 as in the case with complete information; that is, \( p_i^L = s_1 + (s_{L2}/S)((I + 1)/I)(q_{r,k} + Q_{r,-k}) \) and \( p_i^H = s_1 + (s_{H2}/S)((I + 1)/I)(q_{r,k} + Q_{r,-k}) \).

The following proposition gives closed-form expressions for the expected equilibrium quantities, where the expectation is taken over the prior distribution, as well as for the realized equilibrium quantities corresponding to each of the two scenarios with low and high sensitivity.

**Proposition 5.1.** Let \( d_1 \geq s_1 \), then there exists a unique equilibrium for the supply chain with asymmetric information and the equilibrium is symmetric. Moreover:

1. The aggregate quantity ordered by the retailers is

\[
Q = \frac{RSI}{(d_2SI + s_2(I + 1))(R + 1)},
\]

(14)

where \( s_2 \) is the expected supply sensitivity; that is, \( s_2 = \nu s_{L2} + (1 - \nu)s_{H2} \).

2. The realized equilibrium quantities for the scenario with low (high) sensitivity are given by the last column of Table 1 after replacing \( Q \) with the aggregate quantity given by (14), and the supply sensitivity \( s_2 \) with the sensitivity corresponding to the scenario with low \( s_{L2} \) (high \( s_{H2} \)) sensitivity.

3. The expected equilibrium quantities are given by the same expressions as for the case with perfect information (that is, by the third column of Table 1) after replacing the supply sensitivity \( s_2 \) by the expected supply sensitivity \( s_2 \).

**5.2. The impact of asymmetric information**

We characterize the impact of asymmetric information on both the expected and the realized equilibrium quantities.\(^{11}\) The impact of asymmetric information on the expected equilibrium quantities is important from the perspective of the retailer. Specifically, the impact of asymmetric information on the expected retailer profits is the value of perfect information. If the value of information is large, then the retailers have an incentive to gather further information about the supply sensitivity.

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\(^{10}\)This is the equivalent of a pooling contract in the standard treatment of games with asymmetric information. Screening of the different types of intermediary is not possible in the context of wholesale price contracts because only one contracting variable (price) is available. Note that we restrict our analysis to wholesale price contracts for the reasons argued in Section 3.

\(^{11}\)For expected equilibrium quantities, we compare the expected equilibrium quantities for the case with asymmetric information (given by Part 3 of Proposition 5.1), with the expected equilibrium quantities for the case with perfect information (computed by taking expectations over the prior distribution of the perfect information equilibrium quantities corresponding to the scenarios with low and high sensitivity).
The impact of asymmetric information on the realized equilibrium quantities is more interesting from the perspective of the intermediaries, because they have perfect information and thus their actions are always contingent on the realized sensitivity.

The following proposition characterizes how the presence of asymmetric information affects the equilibrium. To simplify the exposition we focus on the case when the high sensitivity \( s_{2H} \) is sufficiently large.\(^{12}\)

**Proposition 5.2.** Let \( d_1 \geq s_1 \) and let \( s_{2H} \) be sufficiently large, then:

1. The comparison between the expected equilibrium quantities in the supply chains with asymmetric and complete information is as indicated in the third column of Table 3.

2. The comparison between the realized equilibrium quantities in the supply chains with asymmetric and complete information is as indicated in the fourth, fifth, and sixth columns of Table 3 for the cases with high realized sensitivity, low realized sensitivity and prior probability of low sensitivity \( \nu > \nu_0 \), and low realized sensitivity and \( \nu < \nu_0 \), respectively.

The comparison of the *expected equilibrium quantities* results in the intuitive insight that ex-ante (in expectation) the intermediaries are indeed able to exploit their private information about the supply sensitivity to improve their bargaining position with respect to the retailers. Note, in particular, that the expected intermediary profits are larger and the expected retailer profits lower in the presence of asymmetric information.

The comparison of the *realized equilibrium quantities*, however, shows that the realized profits of both intermediaries and retailers could be higher or lower depending on the degree of information asymmetry, the intensity of retailer competition, and the realized sensitivity. *When the realized sensitivity is high*, intermediaries facing high-sensitivity suppliers can exploit their superior knowledge of the supply sensitivity to increase their profits at the expense of the retailers. Specifically, we find that in this case the retailers order more than they would with complete information, and this results in a lower retail price, lower retailer margin and profits, and higher intermediary margin and profits.

The result for the case when *the realized sensitivity is low* is more interesting. In this case the retailers order less than with perfect information. This lower quantity results in lower intermediary-market-clearing price and lower intermediary profits. This implies that intermediaries facing low-sensitivity supply may benefit from disclosing their private information to the retailers—if they can do so in a credible manner. In other words, it is not always optimal for intermediaries to keep their private information private. Even more surprising is that, when the realized sensitivity is low, the retailer profits could be *larger or smaller* depending on their prior beliefs. One would expect that in the presence of asymmetric information the retailers profits should be smaller than with

\(^{12}\)See the proof in Appendix D for the exact threshold value. We have also analyzed the case where this value is below the threshold, and the insights from the analysis are similar.
perfect information, but Table 3 shows that, when the realized sensitivity is low, the retailer profit is lower if and only its prior probability of low sensitivity ($\nu$) is lower than a certain threshold ($\nu_0$):

$$\nu < \nu_0 \equiv \frac{(I + 1)(s^H_2 - Rs^L_2) - d_2SI(R - 1)}{(I + 1)(s^H_2 - s^L_2)}.$$

For the case with a single retailer we have that $\nu_0 = 1$, and thus monopolist retailer profits are always lower in the case with asymmetric information. Indeed, a monopolist retailer with perfect information on supply sensitivity must be able to extract larger profits from the supply chain than in the absence of perfect information, regardless of whether the realized supply sensitivity is low or high.

A more interesting result occurs when there are several competing retailers. For this case, when the retailers’ prior probability of low sensitivity is small ($\nu < \nu_0$), the retailer profits are smaller than in the case with complete information. When the retailers’ prior probability of low sensitivity is moderate ($\nu > \nu_0$), however, their profits are larger in the presence of asymmetric information. The explanation for this is that in the case with multiple competing retailers, the presence of asymmetric information has a negative and a positive effect on retailer profits. The negative effect is that the retailers lack information about the supply sensitivity that would help to identify the optimal quantity to select. The positive effect, however, is that this missing information attenuates the intensity of competition among retailers because the retailers’ prior belief is that the supply sensitivity is on average higher than it actually is and thus they choose a lower quantity, which results in lower intermediary-market-clearing price. For the case where the prior probability of low sensitivity is low, the negative effect (incomplete information) dominates; for the case where the prior probability of low sensitivity is moderate, the positive effect of asymmetric information (competition mitigation) dominates. This result is illustrated in Figure 2.

Finally, the following proposition shows that, although the insight that intermediary profits are unimodal with respect to the number of suppliers is robust to the presence of asymmetric information, the number of suppliers that maximizes intermediary profits in the case with asymmetric information is larger (smaller) than in the case with complete information depending on whether the realized sensitivity is low (high).

**Proposition 5.3.** Let $d_1 \geq s_1$, then the intermediary profits in the presence of asymmetric information are unimodal with respect to the number of suppliers, and the number of suppliers that maximizes intermediary profits in the case with asymmetric information is larger (smaller) than in the case with complete information depending on whether the realized sensitivity is low (high).

The main insight from Proposition 5.3 is that the presence of asymmetric information can result in alterations to the intermediaries’ preferred product portfolio. When the realized sensitivity is low (high), the preferred product portfolio is biased towards products with more (less) available production capacity.
6. Why supply chain intermediation?

Although our main focus is the impact of competition in a supply chain with intermediaries, we now also show how the traditional rationales identified in the Economics literature for the existence of intermediaries can be incorporated into our competition model. To do so, we consider the case where retailers can choose to either use the services of intermediaries, or deal directly with the suppliers. We assume that if a retailer chooses to deal directly with the suppliers, the retailer incurs a fixed cost per supplier $F$, which represents the cost associated with establishing a working relation with each supplier. In addition, if a retailer chooses to deal directly with the suppliers, the retailer incurs an additional variable cost per unit $v_h$. This additional variable cost may capture the fact that it may be more expensive for the retailer to validate the quality of each unit purchased (compared to the experienced intermediary). If, on the other hand, the retailer decides to use the services of the intermediary, it simply pays a price per unit to the intermediary, who keeps a certain margin, and the intermediary places its orders from the suppliers without incurring any fixed costs because it has an established relationship with its supplier base.

6.1. The case without intermediaries

We model the case when the retailers deal directly with the suppliers as a two-stage game. In the first stage, the retailers choose the number of suppliers to deal with $S_R$, incurring a fixed cost $S_R F$. In the second stage, the retailers choose their order quantities.

The following proposition characterizes the second-stage equilibrium.

**Proposition 6.1.** In the second stage (that is, for a given number of suppliers $S_R$), the equilibrium with no intermediaries coincides with the equilibrium with an infinite number of intermediaries and where the intercept of the inverse demand function equals $d_1 - v_h$.

In the first stage, the retailer chooses a number of suppliers $S_R$ to maximize her profits net of fixed costs. Using Proposition 6.1 and Table 1 we can write the first-stage retailer decision as:

$$\max_{S_R \geq 0} \bar{\pi}_r = \frac{S_R}{d_2 S_R + s_2} \left(\frac{(d_1 - v_h - s_1)^2}{(R + 1)^2} - S_R F\right).$$

(15)

The following proposition provides the optimal number of suppliers selected by retailers in the first stage and the associated retailer profit net of fixed costs.

---

Note that although $S_R$ should be integer, for tractability we approximate the integer problem with its continuous relaxation and do not impose integrality constraints on $S_R$. Due to the concavity of the formulation, as shown in Proposition 6.2, the optimal integer value is the integer below or above the solution of the continuous relaxation.
Proposition 6.2. The retailer profit in the first stage $\bar{\pi}_r$ is a strictly concave function of $S_R$. Moreover, the optimal number of suppliers selected by the retailers $S_R$ and the corresponding retailer profit net of transaction costs $\bar{\pi}_r$ are:

$$S_R = \frac{1}{d_2} \left[ \frac{(d_1 - v_h - s_1)\sqrt{s_2}}{(R + 1)\sqrt{F}} - s_2 \right]^+, \quad \text{and} \quad \bar{\pi}_r = \frac{s_2}{d_2} \left( \frac{d_1 - v_h - s_1}{(R + 1)\sqrt{s_2} - \sqrt{F}} \right)^2,$$

where $[.]^+$ is the positive part.

6.2. To intermediate or not to intermediate

The retailers benefit from utilizing the network of intermediaries when their profit in the case with intermediaries exceed their profit without, i.e. when

$$SI(d_1 - s_1)^2 \leq \frac{s_2}{d_2} \left( \frac{d_1 - v_h - s_1}{(R + 1)\sqrt{s_2} - \sqrt{F}} \right)^2.$$

Proposition 6.3. Retailers use the intermediaries when one of the following holds:

1. the fixed cost $F$ is sufficiently large,
2. the retailer additional variable cost $v_h$ is sufficiently large,
3. the number of suppliers is sufficiently large,
4. the number of intermediaries is sufficiently large,
5. the number of retailers is sufficiently large, or
6. the first unit margin $d_1 - s_1$ is sufficiently small.

The results in Proposition 6.3 are consistent with intuition. When the retailers fixed costs or additional variable costs of dealing directly with the suppliers are high, they prefer working with the intermediaries to avoid these costs. When the number of suppliers is high, the aggregate supply function faced by the intermediaries $Q_s(p_s) = S(p_s - s_1)/s_2$ is relatively steep because $S$ is large, and thus the intermediaries will benefit from low supply costs. Should the retailers choose to bypass the intermediaries, they would probably choose not to work with all suppliers, because there is a fixed cost per supplier, and thus the supply function faced by the retailers would not be as steep as that faced by the intermediaries, and thus they would face higher supply costs. Therefore, the retailers will choose to use the services of the intermediaries when the number of suppliers in the market is high.\(^\text{14}\)

When the number of intermediaries is high, there is intense competition among them, and thus the intermediary margin will be relatively low, which implies that the retailers will be better off paying the small intermediary margin rather than the fixed costs. When the number of retailers is large, each retailer will supply a small quantity to the consumers and they will retain a relatively

---

\(^{14}\)Note that as explained in Section 3.1 the effect of an increase in $s_2$ on the willingness of a retailer to use the services of intermediaries is equivalent to that of a decrease in $S$, because the aggregate supply function depends only on $s_2/S$. 
small margin. Under these circumstances, it will most likely not be profitable for the retailers to pay any fixed costs. Finally, when the first unit margin \( d_1 - s_1 \) is small, the retailer margin will be small (as it must be smaller than the first unit margin), and therefore it may not be profitable to pay any fixed costs.

### 6.3. The squeezed middleman

Proposition 6.3 throws some light over the decline in profitability of supply chain intermediaries in recent years. Some speculate that this may be the result of retailers increasingly sourcing directly from suppliers; Chu [2012] claims: “children apparel companies such as Carter’s Inc. and Gymboree Corp. have said they plan to obtain more of their products directly in coming years”. Others, such as Hughes and Kirk [2013], point at the low margins of supply chain intermediaries due to increasing labor costs in countries such as China.

Our analysis shows that while increasing manufacturing costs in China may indeed be hurting the operating profits of intermediaries, that is not likely to be the reason behind the increasing number of retailers that choose to source directly from the suppliers. Specifically, Part 6 in Proposition 6.3 shows that increasing manufacturing costs, which result in lower margins, can only encourage retailers to use the services of intermediaries. Therefore the motivation for retailers increasingly sourcing directly from low-cost suppliers across Asia is likely to be driven by a reduction in the fixed costs associated with retailers working with low-cost suppliers, or as Hughes and Kirk [2013] put it: “China is no longer the mystery it was to western buyers a decade ago”.

### 6.4. Intermediary supplier base and product portfolio

We now argue that the threat that the retailers may work directly with the suppliers, may encourage intermediaries to expand the supplier base for the products they carry, or to carry products with a broader supplier base. To see this, note that in the absence of this threat, the intermediary has the incentive to work with the number of suppliers \( S^* \) that maximizes its profits, which by Theorem 4.2 is \( S^* = s_2(I + 1)/(d_2 I) \). However, from Part 3 of Proposition 6.3 we know that there is \( S_{\text{min}} \) such that if the number of suppliers is smaller than \( S_{\text{min}} \), then the retailers would choose to work directly with the suppliers, and this would result in zero profit for the intermediaries. Faced with this threat, the intermediary may optimally choose to work with a number of suppliers \( S > S^* \). This intuition is formalized in the following Proposition.

**Proposition 6.4.** Suppose retailers have the option to deal directly with retailers, then the number of suppliers that maximizes the intermediary profit is

\[
\bar{S} = \begin{cases} 
S_{\text{min}} > S^* & \text{if } S_{\text{min}} > 0 \text{ and } (\sqrt{2} - 1)(d_1 - s_1) > \sqrt{2}(v_h + (R + 1)\sqrt{F}s_2), \\
S^* & \text{otherwise},
\end{cases}
\]

where

\[
S_{\text{min}} = \frac{s_2^2(I + 1)(R + 1)^2((d_1 - v_h - s_1) - \sqrt{F})^2}{d_2I[(d_1 - s_1)^2 - s_2(R + 1)^2((d_1 - v_h - s_1) - \sqrt{F})^2]^2}.
\]
Proposition 6.4 implies that, when the condition in (18) holds, the threat that the retailers may go deal directly with suppliers gives the intermediaries an incentive to either expand their supplier base for products they are carrying, or carry products with a broader supplier base, in order to ensure the retailers make use of their services.

Figure 3 shows the intermediary profit as a function of the number of suppliers for two cases with different fixed cost $F$. The left panel has a higher fixed cost, and in this case the intermediary is better off working with $S^*$ suppliers. The right panel has a lower fixed cost, and this pushes the intermediary to prefer working with a higher number of suppliers $S_{min}$.

7. Conclusion

We study the impact of horizontal and vertical competition on sourcing arrangements that operate through intermediaries. Our focus on the role of competition distinguishes our work from the bulk of the prior literature on intermediation, which provides rationales for the existence of intermediaries. Also, our considering a model with retailers as Stackelberg leaders distinguishes our work from the prior literature on supply chain competition. Our main result suggests that intermediaries prefer products for which the supply base is neither too broad nor too narrow. This result not only helps clarify the characteristics of products which are likely to be favored by intermediaries, but also helps explain how financial performance of intermediaries, and hence, of economies reliant on this sector, may vary over time, depending upon the availability of supply.

The intermediary tier in global supply chains has seen a number of acquisitions of late, accompanied with speculation about further acquisitions (Reuters 2012). A natural question then is: What is the impact of these acquisitions on not only the acquiring firm, but also on the efficiency of the entire supply chain? Our competition model provides a framework to think about these issues. Specifically, we find that right balance of horizontal and vertical competition can entirely offset the adverse effect of double marginalization.

Our analysis provides additional perspective on the puzzle identified by Rauch [2001] that while the governments of Japan, Korea, and Turkey succeeded in improving trade by subsidizing the intermediaries to widen their supplier base, similar attempts failed in Taiwan and the U.S. Concretely, our analysis show thats in order for regulatory intervention in a specific tier in the supply chain to be successful, it must be carefully tailored to the structure of the entire supply chain in question.

Finally, we study the impact of asymmetric information in the context of supply chain intermediation, and find that when the realized supply sensitivity is low, competing retailers may be better off at the expense of the intermediaries compared to the case with perfect information. In fact, intermediaries facing low-sensitivity demand may prefer to reveal their private information to the retailers if they can do so credibly.

Appendix A: Tables
Table 1  Equilibrium quantities with complete and asymmetric information
This table gives the equilibrium quantities for the cases with complete and asymmetric information. The first column in the table lists the different equilibrium quantities reported: the aggregate supply, the supply price, the intermediary price, the retail price, the intermediary margin, the retailer margin, the aggregate supplier profit, the aggregate intermediary profit, the aggregate retailer profit, and the aggregate supply chain profit. The second column reports the different symbols used to represent the equilibrium quantities. The third and fourth columns give the expression for the equilibrium quantity for the case with complete information—the third column in terms of aggregate quantity and the fourth column after replacing $s_2$ with $s_2^d$ ($s_2^{h+1}$). The fifth column gives the expression of the realized equilibrium quantity with $s_2$ replacing $s_2$ with $s_2^d$ ($s_2^{h+1}$). These results are proven in Theorem 4.1 and Proposition 5.1.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>In terms of $Q$</th>
<th>Closed-form</th>
<th>Asymmetric info (realized)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate quantity</td>
<td>$Q$</td>
<td>$Q$</td>
<td>$\frac{RSI}{(d_2SI + s_2(I+W))} d_1$</td>
<td>$Q$</td>
</tr>
<tr>
<td>Supply price</td>
<td>$p_s$</td>
<td>$s_1 + \frac{s_2}{S} Q$</td>
<td>$s_1 + \frac{s_2}{d_2SI + s_2(I+W)} d_1$</td>
<td>$s_1 + \frac{s_2}{S} Q$</td>
</tr>
<tr>
<td>Intermediary price</td>
<td>$p_i$</td>
<td>$s_1 + \frac{s_2}{S} \frac{I+1}{I} Q$</td>
<td>$s_1 + \frac{s_2}{d_2SI + s_2(I+W)} d_1$</td>
<td>$s_1 + \frac{s_2}{S} \frac{I+1}{I} Q$</td>
</tr>
<tr>
<td>Retail price</td>
<td>$p_r$</td>
<td>$d_1 - d_2 Q$</td>
<td>$d_1 - d_2$</td>
<td>$d_1 - d_2 Q$</td>
</tr>
<tr>
<td>Intermediary margin</td>
<td>$m_i$</td>
<td>$\frac{s_2 Q}{SI}$</td>
<td>$s_2 \frac{R}{d_2SI + s_2(I+W)} d_1$</td>
<td>$s_2 Q$</td>
</tr>
<tr>
<td>Retailer margin</td>
<td>$m_r$</td>
<td>$\frac{d_1 - s_1}{R+I}$</td>
<td>$\frac{d_1 - s_1}{R+I}$</td>
<td>$d_1 - s_1 - \left( d_2 + \frac{s_2}{S} \frac{I+1}{I} \right) Q$</td>
</tr>
<tr>
<td>Agg. supplier profit</td>
<td>$\pi_s$</td>
<td>$\frac{2sQ^2}{2S}$</td>
<td>$\frac{2sQR^2}{2(d_2SI + s_2(I+W))} \frac{(d_1 - s_1)^2}{(R+I)^2}$</td>
<td>$\frac{2sQ^2}{2S}$</td>
</tr>
<tr>
<td>Agg. intermediary profit</td>
<td>$\pi_i$</td>
<td>$\frac{2sQ^2}{SI}$</td>
<td>$\frac{2sQR^2}{(d_2SI + s_2(I+W))} \frac{(d_1 - s_1)^2}{(R+I)^2}$</td>
<td>$\frac{2sQ^2}{SI}$</td>
</tr>
<tr>
<td>Agg. retailer profit</td>
<td>$\pi_r$</td>
<td>$\frac{d_1 - s_1}{R+I} Q$</td>
<td>$\frac{RSI}{(d_2SI + s_2(I+W))} \frac{(d_1 - s_1)^2}{(R+I)^2}$</td>
<td>$(d_1 - s_1 - (d_2 + \frac{s_2}{S} \frac{I+1}{I}) Q) Q$</td>
</tr>
<tr>
<td>Agg. chain profit</td>
<td>$\pi_a$</td>
<td>$S\pi_s + I\pi_i + R\pi_R$</td>
<td>$\frac{RSI[2R(1+2R) + 2(d_2SI + s_2(I+W))]}{2(d_2SI + s_2(I+W))} \frac{(d_1 - s_1)^2}{(R+I)^2}$</td>
<td>$S\pi_s + I\pi_i + R\pi_R$</td>
</tr>
</tbody>
</table>
Table 2  Monotonicity properties
This table gives the monotonicity properties of the equilibrium quantities. The first column in the table lists the different equilibrium quantities for which we report the monotonicity properties: the aggregate supply, the supply price, the intermediary price, the retail price, the intermediary margin, the retailer margin, the supplier profit, the intermediary profit, and the retailer profit. The second column reports the different symbols used to represent the equilibrium quantities. The next five columns give the monotonicity relation of each equilibrium quantity to the number of suppliers (S), the number of intermediaries (I), the number of retailers (R), the supply sensitivity (s_2), and the demand sensitivity (d_2), respectively, where the symbol “+” (“−”) indicates that the equilibrium quantity increases (decreases) with respect to the parameter, and “∩” indicates that the equilibrium quantity is unimodal with respect to the parameter. The results in this table are proven in Theorem 4.2.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>S</th>
<th>I</th>
<th>R</th>
<th>s_2</th>
<th>d_1</th>
<th>d_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate quantity</td>
<td>Q</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>Supply price</td>
<td>p_s</td>
<td>−</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>Intermediary price</td>
<td>p_i</td>
<td>−</td>
<td>−</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>Retail price</td>
<td>p_r</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>+</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>Intermediary margin</td>
<td>m_i</td>
<td>−</td>
<td>−</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>Retailer margin</td>
<td>m_r</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>Supplier profit</td>
<td>π_s</td>
<td>−</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>Intermediary profit</td>
<td>π_i</td>
<td>−</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>Retailer profit</td>
<td>π_r</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>+</td>
<td>−</td>
</tr>
</tbody>
</table>

Table 3  Effect of asymmetric information on equilibrium quantities
This table shows the effect of asymmetric information about supply sensitivity on the equilibrium quantities. The first column lists the different equilibrium quantities for which we report the relation: the aggregate supply, the supply price, the intermediary price, the retail price, the intermediary margin, the retailer margin, the supplier profit, the intermediary profit, and the retailer profit. The second column reports the different symbols used to represent the equilibrium quantities. The third column shows the effect on the expected equilibrium quantities. The last three columns report the effect of asymmetric information on the realized equilibrium quantities. The fourth column shows the effect for the case where the realized sensitivity is high (s_2 = s_2^H). The fourth column for the case where the realized sensitivity is low and the prior probability of low sensitivity is low (s_2 = s_2^L and ν < ν_0). The fifth column for the case where the realized sensitivity is low and the prior probability of low sensitivity is high (s_2 = s_2^L and ν > ν_0). The symbol “+” (“−”) indicates that the equilibrium quantity is larger (smaller) in the supply chain with asymmetric information, and “0” means it does not change. The results in this table are proven in Proposition 5.2.

<table>
<thead>
<tr>
<th>Expected</th>
<th>Realized</th>
</tr>
</thead>
<tbody>
<tr>
<td>High sensitivity</td>
<td>Low sensitivity</td>
</tr>
<tr>
<td>ν &gt; ν_0</td>
<td>ν &lt; ν_0</td>
</tr>
<tr>
<td>Aggregate quantity</td>
<td>Q</td>
</tr>
<tr>
<td>Supply price</td>
<td>p_s</td>
</tr>
<tr>
<td>Intermediary price</td>
<td>p_i</td>
</tr>
<tr>
<td>Retail price</td>
<td>p_r</td>
</tr>
<tr>
<td>Intermediary margin</td>
<td>m_i</td>
</tr>
<tr>
<td>Retailer margin</td>
<td>m_r</td>
</tr>
<tr>
<td>Supplier profit</td>
<td>π_s</td>
</tr>
<tr>
<td>Intermediary profit</td>
<td>π_i</td>
</tr>
<tr>
<td>Retailer profit</td>
<td>π_r</td>
</tr>
</tbody>
</table>
Appendix B: Figures

Figure 1  Timeline of events

![Timeline of events diagram]

Figure 2  Equilibrium prices and profits with asymmetric information

This figure depicts the equilibrium prices and profits in the presence of asymmetric information when the true supply sensitivity is low. We assume \( d_2 = 1, s_1 = 3, d_1 = 5, I = 2, S = 3, R = 2, s_H^L = 6, s_L^L = 1 \). The horizontal axis gives the retailers’ prior belief probability \( \nu \) that the supply sensitivity is low (\( \nu = 1 \) corresponds to the case with complete information). The left vertical axis gives the realized retail and intermediary prices, and the right vertical axis gives the realized aggregate retailer and intermediary profits. A vertical line indicates the value of the probability such that the retailer profits with asymmetric information equal the retailer profits with complete information.
Figure 3  Aggregate intermediary profit depending on number of suppliers

This figure depicts the aggregate intermediary profit for a number of suppliers ranging between 1 and 12, and for two different values of the fixed search cost $F$ when retailers have the option to deal directly with suppliers. We assume $d_2 = 0.25, s_2 = 1, s_1 = 3, d_1 = 5, I = 2, R = 2, v_h = 0.1$ and $F = 0.05$ for the case depicted in the left panel, while $F = 0.02$ for the case in the right panel. The horizontal axis gives the number of suppliers and the vertical axis gives the aggregate intermediary profit. $S^*$ indicates the number of suppliers that maximizes the intermediary profit when retailers do not have the option to deal directly with suppliers. $S_{\min}$ indicates the minimum number of suppliers required for retailers to choose using the intermediaries.
References


The Telegraph. 2012. Li & Fung—the Made in China giant you have never heard of. The Telegraph.


Supplemental file: Proofs, robustness checks and additional analysis

Appendix D: Proofs for the results in the paper

Proof of Theorem 4.1. We prove the result in three steps. First, we characterize the best response of the intermediaries to the retailers. Second, we characterize the retailer equilibrium as leaders with respect to the intermediaries. Third, simple substitution into the best response functions leads to the closed form expressions.

Step 1. The intermediary best response. We first show that the intermediary equilibrium best response exists, is unique, and symmetric. It is easy to see from equation (7) that the intermediary decision problem is a strictly concave problem that can be equivalently rewritten as a linear complementarity problem (LCP); see Cottle et al. [2009] for an introduction to complementarity problems. Hence, the intermediary order vector \( q_i = (q_{i,1}, \ldots, q_{i,I}) \) is an intermediary equilibrium if and only if it solves the following LCP, which is obtained by concatenating the LCPs characterizing the best response of the \( I \) intermediaries:

\[
0 \leq (-p_i + s_1)e + (s_2/S)M_i q_i \perp q_i \geq 0,
\]

where \( e \) is the \( I \)-dimensional vector of ones, and \( M_i \in \mathbb{R}^{I \times I} \) is a positive definite matrix whose diagonal elements are all equal to one and its off-diagonal elements are all equal to two. Thus this LCP has a unique solution which is the unique intermediary equilibrium best response. Because the intermediary equilibrium is unique and the game is symmetric with respect to all intermediaries, the intermediary equilibrium must be symmetric. Indeed, if the equilibrium was not symmetric, because the game is symmetric with respect to all intermediaries, it would be possible to permute the strategies among intermediaries and obtain a different equilibrium, hereby contradicting the uniqueness of the equilibrium.

We now characterize the intermediary equilibrium best response. To avoid the trivial case where the quantity produced equals zero, we assume the equilibrium production quantity is nonzero. In this case, for the symmetric equilibrium, the first-order optimality conditions for the intermediary are:

\[
 p_i = s_1 + s_2 I + 1 ST Q.
\]

Note that the intermediary margin is therefore \( m_i = p_i - p_s = s_2 Q/(SI) \), and the intermediary profit is

\[
\pi_i = m_i Q = \frac{s_2 Q^2}{SI}.
\]

Step 2. The retailer equilibrium. We first show that the retailer equilibrium exists, is unique, and symmetric. The \( k \)th retailer decision is

\[
\max_{q_{r,k}} \left[ d_1 - d_2(q_{r,k} + Q_{r,-k}) - \left( s_1 + s_2 I + 1 ST (q_{r,k} + Q_{r,-k}) \right) \right] q_{r,k}.
\]

It is easy to see from (22) that the retailer problem is a strictly concave problem that can be equivalently rewritten as an LCP. Hence, the retailer order vector \( q_r = (q_{r,1}, \ldots, q_{r,R}) \) is a retailer equilibrium if and only if it solves the following LCP, which is obtained by concatenating the LCPs characterizing the optimal strategy of the \( R \) retailers:

\[
0 \leq (-d_1 + s_1)e + (d_2 + s_2(I + 1)/(SI)) M_r q_r \perp q_r \geq 0,
\]

where \( e \) is the \( R \)-dimensional vector of ones, and \( M_r \in \mathbb{R}^{R \times R} \) is a positive definite matrix.
whose diagonal elements are all equal to two and whose off-diagonal elements are all equal to one. Thus this LCP has a unique solution which is the unique retailer equilibrium. Using an argument similar to the intermediary equilibrium, the retailer equilibrium must be symmetric.

We now characterize the retailer equilibrium. To avoid the trivial case where the quantity produced equals zero, we focus on the more interesting case with non zero quantities. For the symmetric equilibrium, the first-order optimality conditions for the retailer can be written as $d_1 - s_1 - (d_2 + s_2(I + 1)/(SI))(R + 1)q_{r,k} = 0$, and therefore assuming $d_1 \geq s_1$, we have that the optimal retailer order quantity is

$$q_{r,k} = \frac{d_1 - s_1}{(R + 1)(d_2 + s_2(I + 1)/SI)} = \frac{SI}{(d_2SI + s_2(I + 1))(R + 1)}.$$  \hspace{1cm} (23)

the intermediary price is

$$p_i = s_1 + s_2 \frac{R(I + 1)}{d_2SI + s_2(I + 1)(R + 1)}$$

and the retailer profit is

$$\pi_r = \left[ d_1 - s_1 - (d_2 + s_2(I + 1)/SI) \right] \frac{R}{R + 1} \frac{d_1 - s_1}{(d_2 + s_2(I + 1)/SI)}.$$  \hspace{1cm} (25)

Step 3. Derivation of the final results. It follows from (23) that

$$Q = Rq_{r,k} = \frac{RSI}{(d_2SI + s_2(I + 1))(R + 1)}.$$  \hspace{1cm} (24)

Since $q_{s,j} = Q/S$, the expression for the supply price follows from (1). The expression for the intermediary price follows from (20) and (25). The retailer price is obtained by substituting (25) into (9). Expressions for $m_i = p_i - p_s$ and $m_r = p_r - p_i$ are obtained by direct substitution. Using $q_{s,j} = Q/S$, (3) and (25), we obtain the supplier profit. Substituting (25) into (21) leads to the expression for the intermediary profit. The expression for $\pi_r$ was found in (24). Finally, the total aggregate profit follows from straightforward algebra.

Proof of Theorem 4.2. The results follows from straightforward calculus from the expressions in Table 1.

Proof of Proposition 4.3. The objective of a central planner is to maximize the sum of the profits of all supply chain members:

$$\max \pi_c = S(p_s - s_1 - \frac{s_2}{2}q_s)q_s + I(p_i - p_s)q_I + R(p_r - p_i)q_R$$

where $q_s, q_I$ and $q_R$ are respectively the quantities selected by each supplier, intermediary and retailer. Since $Q = S q_s = I q_I = R q_R$, the central planner’s problem is equivalent to

$$\max \pi_c = (p_r - s_1 - \frac{s_2 Q}{2})Q = (d_1 - d_2 Q - s_1 - \frac{s_2 Q}{2})$$

The first-order optimality conditions are $d_1 - s_1 - 2(d_2 + s_2/(2S))Q = 0$, which result in $Q = (d_1 - s_1)/(2d_2 + s_2/S)$. Therefore the total supply chain profit in the centralized chain is

$$\pi_c = \left[ d_1 - s_1 - \left(\frac{s_2}{2S}\right)\frac{d_1 - s_1}{2d_2 + \frac{s_2}{S}}\right] \frac{d_1 - s_1}{2d_2 + \frac{s_2}{S}} = \frac{(d_1 - s_1)^2}{2(2d_2 + \frac{s_2}{S})}.$$  \hspace{1cm} (26)
Proof of Theorem 4.4. Equation (12) is obtained by direct substitution of the aggregate profit given in Table 1 and (26). The monotonicity properties follow by applying straightforward algebra to the partial derivatives of the efficiency with respect to $R$, $S$, and $I$.

Proof of Proposition 5.1.

Part 1. The $k$th retailer’s expected profit is:

$$E[\pi_{r,k}] = \nu \left( d_1 - d_2 (q_{r,k} + Q_{r,-k}) - p_i^L \right) q_{r,k} + (1 - \nu) \left( d_1 - d_2 (q_{r,k} + Q_{r,-k}) - p_i^H \right) q_{r,k}$$

where $p_i^L = s_1 + (s_2^L / S) ((I + 1) / I) (q_{r,k} + Q_{r,-k})$ and $p_i^H = s_1 + (s_2^H / S) ((I + 1) / I) (q_{r,k} + Q_{r,-k})$. Denoting $\bar{s}_2 = \nu s_2^L + (1 - \nu) s_2^H$, we can rewrite the $k$th retailer’s expected profit as:

$$E[\pi_{r,k}] = \left( d_1 - d_2 (q_{r,k} + Q_{r,-k}) \right) q_{r,k} - \left( s_1 + \bar{s}_2 \frac{I + 1}{SI} (q_{r,k} + Q_{r,-k}) \right) q_{r,k}$$

which is identical to the $k$th retailer’s objective (22) when information is symmetric and $\bar{s}_2 = s_2$.

As a result, it follows from Step 2 of the proof of Theorem 4.1 that the retailer equilibrium exists, is unique, and symmetric with

$$q_{r,k} = \frac{d_1 - s_1}{(R + 1) \left( d_2 + \bar{s}_2 \frac{I + 1}{SI} \right)} = \frac{SI}{(d_2SI + \bar{s}_2(I + 1))} \frac{d_1 - s_1}{(R + 1)}$$

It follows that the aggregate quantity is

$$Q = Rq_{r,k} = \frac{RSI}{(d_2SI + \bar{s}_2(I + 1))} \frac{d_1 - s_1}{(R + 1)}.$$  \hspace{1cm} (27)

Part 2. The expressions for the realized equilibrium quantities for each of the scenarios ($p_i$, $p_s$, $p_r$, $m_r$, $m_i$, $\pi_s$, $\pi_i$ and $\pi_r$) follow similarly to the symmetric information case by using the aggregate quantity $Q$ and realized price sensitivity ($s_2^L$ or $s_2^H$).

Part 3. Note that, given the aggregate quantity in (27), the expressions for the realized equilibrium quantities given by the fourth column in Table 1 are linear in $s_2$. Then, straightforward algebra shows that the expressions for the expected equilibrium quantities are given by the third column in Table 1 after replacing $s_2$ with $\bar{s}_2$.

Proof of Proposition 5.2.

Part 1. The result follows from Jensen’s inequality. To see this, note first that the closed-form expressions for the expected equilibrium quantities in the case with asymmetric information and the realized equilibrium quantities in the case with perfect information coincide, and are given by the fourth column in Table 1. In the case with complete information, we replace $s_2$ by its realized value ($s_2^L$ or $s_2^H$) to obtain the realized equilibrium quantity. In the case with asymmetric information, we replace $s_2$ with $\bar{s}_2$ to obtain the expected quantity.

Let the expression for a particular equilibrium quantity as a function of $s_2$ by $f(s_2)$. Then, the expected equilibrium quantity with complete information is simply $\nu f(s_2^L) + (1 - \nu) f(s_2^H)$. Also, the expected equilibrium quantity with asymmetric information is $f(\bar{s}_2) = f(\nu s_2^L + (1 - \nu) s_2^H)$. Then
the result follows from Jensen’s inequality because it is easy to show that the aggregate quantity and the ex-ante expected retailer profits are strictly convex in the supply sensitivity $s_2$, but the ex-ante expected supply, intermediary, and retail prices, the intermediary margin, and the aggregate supplier and intermediary profits are strictly concave in the supply sensitivity $s_2$. Finally, from Table 1 it is clear that the retailer margin does not depend on the supply sensitivity $s_2$, and thus it is not affected by asymmetric information.

**Part 2.** In this proof, for each quantity in the table, we determine the sign of the difference between the symmetric and asymmetric cases. Let $Q^{L,a}$ be the aggregate quantity produced with asymmetric information and $Q^{L,s}$ the aggregate quantity produced with symmetric information when $s_2 = s_2^L$. We have

$$Q^{L,a} - Q^{L,s} = \frac{RSI}{(d_2SI + s_2^L(I + 1))(R + 1)} - \frac{RSI}{(d_2SI + s_2^L(I + 1))(R + 1)},$$

and because $s_2^L < s_2^H$, it follows that $s_2 > s_2^L$ and thus $Q^{L,a} - Q^{L,s} < 0$. Clearly, a similar reasoning leads to $Q^{H,a} - Q^{H,s} > 0$.

Let $p^{L,a}_s$ be the supply price with asymmetric information and $p^{L,s}_s$ the supply price with symmetric information when $s_2 = s_2^L$. Using the last column of Table 1, we have

$$p^{L,a}_s - p^{L,s}_s = s_1 + s_2^L Q^{L,a} - (s_1 + s_2^L Q^{L,s}) = s_2^L(Q^{L,a} - Q^{L,s}) < 0,$$

$$p^{H,a}_s - p^{H,s}_s = s_1 + s_2^H Q^{H,a} - (s_1 + s_2^H Q^{H,s}) = s_2^H(Q^{H,a} - Q^{H,s}) > 0.$$

The comparisons for $p$, $p_1$, $m$, $\pi$, and $\pi_1$ follow similarly from the aggregate quantity comparison and Table 1.

Let $m^{L,a}_r$ be the retailer margin with asymmetric information and $m^{L,s}_r$ the retailer margin with symmetric information when $s_2 = s_2^L$. We have

$$m^{L,a}_r - m^{L,s}_r = d_1 - s_1 - \left(\frac{d_2 + s_2^L I + 1}{SI}\right) \frac{d_1 - s_1}{R + 1} \frac{RSI}{d_2SI + s_2^L(I + 1)} - \frac{d_1 - s_1}{R + 1},$$

and thus $m^{L,a}_r > m^{L,s}_r$. Similarly, we obtain $m^{H,a}_r < m^{H,s}_r$.

Let $\pi^{H,a}_r$ be the retailer profit with asymmetric information and $\pi^{H,s}_r$ the retailer profit with symmetric information when $s_2 = s_2^H$. We have

$$\pi^{H,s}_r - \pi^{H,a}_r = \frac{(d_1 - s_1)^2}{(R + 1)^2} \frac{SI}{d_2SI + s_2^H(I + 1)} - \frac{d_1 - s_1}{R + 1} \frac{SI}{d_2SI + s_2^H(I + 1)} \left(\frac{d_1 - s_1}{(d_2 + s_2^H I + 1)SI} \frac{d_1 - s_1}{R + 1} \frac{RSI}{d_2SI + s_2^H(I + 1)}\right),$$

after simplification. Note that since $s_2 < s_2^H$ and $R \geq 1$, then $s_2 < Rs_2^H$ and it follows that $\pi^{H,s}_r > \pi^{H,a}_r$.

Let $\pi^{L,a}_r$ be the retailer profit with asymmetric information and $\pi^{L,s}_r$ the retailer profit with symmetric information when $s_2 = s_2^L$. We obtain similarly

$$\pi^{L,s}_r - \pi^{L,a}_r = \frac{(d_1 - s_1)^2}{(R + 1)^2} \frac{SI(I + 1)(s_2^L - s_2^L)}{d_2SI + s_2^L(I + 1)(d_2SI + s_2^L(I + 1))} ((I + 1)(s_2 - Rs_2^L) - d_2SI(R - 1)).$$
Thus we want to determine the sign of
\[
\Delta \pi_r^L = (I + 1)(s_2 - Rs_2^L) - d_2SI(R - 1)
= (I + 1)(\nu s_2^L + (1 - \nu)s_2^H - Rs_2^L) - d_2SI(R - 1)
= -\nu(I + 1)(s_2^H - s_2^L) + (I + 1)(s_2^H - Rs_2^L) - d_2SI(R - 1).
\]
Because \(s_2 > s_2^L\), the sign of \(s_2 - Rs_2^L\) (and thus of \(\Delta \pi_r^L\)) varies depending on the input parameters. We have
\[
\Delta \pi_r^L \leq 0 \iff \nu \geq \frac{(I + 1)(s_2^H - Rs_2^L) - d_2SI(R - 1)}{(I + 1)(s_2^H - s_2^L)} \equiv \nu_0.
\]
It is easy to observe that \(\nu_0 \leq 1\). Moreover, \(\nu_0 \geq 0\) when
\[
s_2^H \geq Rs_2^L + \frac{d_2SI(R - 1)}{I + 1} \equiv s_2.
\]
As a result, \(\Delta \pi_r^L \leq 0\) (and \(\pi_r^{L,s} \leq \pi_r^{L,a}\)) for all values of \(\nu \in [0, 1]\) when \(s_2^H \leq s_2\); but when \(s_2^H > s_2\), \(\Delta \pi_r^L \leq 0\) (and \(\pi_r^{L,s} \leq \pi_r^{L,a}\)) for \(\nu > \nu_0\) and \(\Delta \pi_r^L \geq 0\) (and \(\pi_r^{L,s} \geq \pi_r^{L,a}\)) for \(\nu \leq \nu_0\).

**Proof of Proposition 5.3.** If \(s_2 = s_2^L\), the actual intermediary profit can be written as
\[
\pi_i = \frac{R^2S(d_1 - s_1)^2s_2^L}{(R + 1)^2(d_2SI + s_2(I + 1))^2}.
\]
It is easy to find that the derivative of the intermediary profit is, after simplifications,
\[
\frac{\partial \pi_i}{\partial S} = \frac{R^2(d_1 - s_1)^2s_2^L}{(R + 1)^2} \cdot \frac{s_2(I + 1) - d_2SI}{(d_2SI + s_2(I + 1))^3}.
\]
It is clear that the intermediary profit is unimodal and reaches its maximum at
\[
S = \frac{s_2(I + 1)}{d_2I}.
\]
The result follows from the fact that for the case with symmetric information, Theorem 4.2 shows that the intermediary profits are maximized for \(S = s_2(I + 1)/(d_2I)\) and \(s_2 > s_2^L\),.

The case when \(s_2 = s_2^H\) follows similarly.

**Proof of Proposition 6.1.** We assume that there are \(S_R\) symmetric suppliers and \(R\) symmetric retailers, but no intermediaries. The demand function remains \(p_r = d_1 - d_2Q\). Following the reasoning detailed in previous sections, the kth retailer’s decision problem can be formulated as
\[
\max_{q_r,k} [d_1 - d_2(q_r,k + Q_r,-k) - (s_1 + (s_2/S_R)(q_r,k + Q_r,-k)) - v_h]q_r,k,
\]
which is identical to (22) with \((I + 1)/I = 1\), i.e. \(I = \infty\), and with an intercept of the consumer inverse demand function equal to \(d_1 - v_h\).

**Proof of Proposition 6.2.** We have
\[
\frac{\partial \pi_r}{\partial S_R} = \frac{(d_1 - v_h - s_1)^2}{(R + 1)^2} \cdot \frac{s_2}{(d_2S_R + s_2)^2} - F \quad \text{and} \quad \frac{\partial^2 \pi_r}{\partial S_R^2} = -\frac{(d_1 - v_h - s_1)^2}{(R + 1)^2} \cdot \frac{2s_2d_2}{(d_2S_R + s_2)^3} < 0,
\]
and thus the concavity result follows. Then, using first-order optimality conditions in the presence of a non negativity constraint, we obtain that the optimal value of \(S_R\) is solution to \(s_2(d_1 - v_h - s_1)^2/(d_2S_R + s_2)^2(R + 1)^2) = F\) if this solution is non negative, and otherwise \(S_R = 0\).
Proof of Proposition 6.3. The result follows by straightforward algebraic manipulation of (17).

Proof of Proposition 6.4. It is straightforward that $\bar{S} = \max\{S^*, S_{\min}\}$. Algebraic manipulation of (19) lead to finding that $S_{\min} = \gamma S^*$ where

$$
\gamma = \frac{s_2 (R + 1)^2 (d_1 - s_1)}{(d_1 - s_1)^2 - s_2 (R + 1)^2 (d_1 - s_1)}.
$$

It is easy to find that $\gamma > 1$ (i.e., $S_{\min} > S^*$) iff

$$
\left[\frac{d_1 - v_h - s_1}{(R + 1)^2} - \sqrt{F}\right]^+ > 0
$$

and

$$
2(d_1 - s_1 - v_h - (R + 1)\sqrt{Fs_2})^2 > (d_1 - s_1)^2,
$$

which is equivalent to condition 18.

Appendix E: The general case with shared and exclusive suppliers

In our competition model, we assume that all $S$ suppliers work with all $I$ intermediaries. We now show that the results from our analysis are robust to the general case where the $j$th supplier works with a subset of $I_j$ intermediaries. In particular, we show that the equilibrium for this general case is equivalent to the equilibrium for a supply chain where all $S$ suppliers work with a different number of intermediaries $\bar{I}$. Therefore, the qualitative insights obtained from our base case model hold also for the general case where suppliers work with a subset of intermediaries.

Proposition E.1. The equilibrium for the general case where the $j$th supplier works with a subset of $I_j$ intermediaries is equivalent to the equilibrium for a supply chain where all $S$ suppliers work with $\bar{I} \in [1, I]$ intermediaries, where $\bar{I} \in [0.5, 1]$.

Proof. Note that given an intermediary price, it is easy to show that the intermediary decisions for different suppliers are decoupled. Moreover, it is easy to show that the intermediary equilibrium for the $j$th supplier satisfies

$$
p_i = s_1 + s_2 \frac{I_j + 1}{I_j} q_{s,j},
$$

where $q_{s,j}$ is the optimal order quantity from the $j$th supplier aggregated over all intermediaries, and therefore

$$
q_{s,j} = \frac{p_i - s_1}{s_2} \frac{I_j}{I_j + 1}.
$$

Hence the total order quantity aggregated over all suppliers is

$$
Q = \sum_{j=1}^{S} q_{s,j} = \frac{p_i - s_1}{s_2} \sum_{j=1}^{S} \frac{I_j}{I_j + 1}.
$$

It is easy to show that there exists $\bar{I} \in [1, I]$ such that $\frac{\bar{I} + 1}{\bar{I}} = \frac{1}{S} \sum_{j=1}^{S} \frac{I_j}{I_j + 1} \in [0.5, 1]$. Then we have

$$
Q = \frac{p_i - s_1}{s_2} S \frac{\bar{I}}{\bar{I} + 1}.
$$

Therefore, the retailer equilibrium and the intermediary price in the general case where each supplier works with $I_j$ intermediaries are the same as for the case where all suppliers work with $\bar{I}$ intermediaries.
Appendix F: The case with convex increasing marginal opportunity cost

We now show that the insight that the intermediary profit $\pi_i$ is unimodal in the number of suppliers $S$ holds also for the following more general marginal opportunity cost function:

$$p_s(q) = s_1 + s_2 q^\theta; \text{for } \theta \geq 1.$$ 

Note that this is a convex increasing monomial function.

The supplier’s profit is:

$$\pi_{s,j} = p_s(q_{s,j})q_{s,j} - \int_0^{q_{s,j}} (s_1 + s_2 q^\theta) dq = s_2 q_{s,j}^{\theta+1}/\theta + 1.$$ 

The intermediary’s decision is:

$$\max_{q_{i,l}} \left[ p_i - \left[ s_1 + s_2 \left( \frac{q_{i,l} + Q_{i,-l}}{S} \right)^\theta \right] \right] q_{i,l}.$$ 

The first-order conditions imply:

$$p_i - \left[ s_1 + s_2 \left( \frac{q_{i,l} + Q_{i,-l}}{S} \right)^\theta \right] - s_2 \theta \left( \frac{q_{i,l} + Q_{i,-l}}{S} \right)^{\theta-1} q_{i,l} = 0;$$ 

which implies:

$$p_i = s_1 + s_2 \left( \frac{Q}{S} \right)^\theta + s_2 \theta \left( \frac{Q}{S} \right)^\theta;$$ 

and the margin, $m_i$, of the intermediary is:

$$m_i = p_i - p_s = \frac{s_2 \theta Q}{I} \left( \frac{Q}{S} \right)^\theta;$$ 

and the profit, $\pi_i$, of each intermediary is:

$$\pi_i = m_i \frac{Q}{I} = \frac{s_2 \theta Q^2}{I^2} \left( \frac{Q}{S} \right)^{\theta+1}.$$ 

The retailer’s decision problem can be stated as:

$$\max_{q_{r,k}} (p_r - p_i) q_{r,k} = \left[ d_1 - d_2 (q_{r,k} + Q_{r,-k}) - \left[ s_1 + s_2 \left( \frac{Q}{S} \right)^\theta + s_2 \theta \left( \frac{Q}{S} \right)^\theta \right] \right] q_{r,k}.$$ 

The first order conditions then imply:

$$\Phi(Q, S) = d_2 \left( 1 + \frac{1}{R} \right) Q + s_2 \left( 1 + \frac{\theta}{I} \right) \left( 1 + \frac{\theta}{R} \right) \left( \frac{Q}{S} \right)^\theta - (d_1 - s_1) = 0.$$ 

Now we are ready to determine whether $\pi_i$ is unimodal in $S$. We first calculate $d\pi_i/dS$:

$$\frac{d\pi_i}{dS} = \left( \frac{\partial \pi_i}{\partial Q} \right) \frac{dQ}{dS} + \frac{\partial \pi_i}{\partial S} = \left( \frac{Q}{S} \right)^\theta \frac{s_2 \theta Q}{I^2} \left[ (\theta + 1) \frac{dQ}{dS} - \frac{Q}{S} \right].$$
where \(\frac{dQ}{dS}\) can be determined using the following relationship:

\[
\frac{d\Phi}{dS} = \frac{\partial \Phi}{\partial Q} \frac{dQ}{dS} + \frac{\partial \Phi}{\partial S} = 0;
\]

which implies:

\[
\frac{dQ}{dS} = \frac{s_2 \theta \left(1 + \frac{\theta}{I}\right) \left(1 + \frac{\theta}{R}\right) \left(Q^\theta \right)}{d_2 \left(1 + \frac{1}{R}\right) + s_2 \theta \left(1 + \frac{1}{I}\right) \left(1 + \frac{\theta}{R}\right) \left(Q^\theta \right)} = \frac{Q}{S} \left(1 + \left(\frac{d_2 (R+1)}{s_2 \theta R (1+\frac{1}{I})(1+\frac{1}{R})}\right) \left(\frac{s}{Q}\right)^{\theta-1}\right) < \frac{Q}{S}. \tag{28}
\]

Substituting the above relationship in the expression for \(\frac{d\pi_i}{dS}\) and evaluating the terms we conclude that \(\frac{d\pi_i}{dS} < 0\) if and only if \(\frac{dQ}{dS} < \frac{\theta Q}{(\theta + 1)S}\) or equivalently, if and only if:

\[
\frac{1}{1 + \left(\frac{d_2 (R+1)}{s_2 \theta R (1+\frac{1}{I})(1+\frac{1}{R})}\right) \left(\frac{s}{Q}\right)^{\theta-1} S} < \frac{\theta}{\theta + 1}.
\]

Also note that:

\[
\frac{d\left(\frac{Q}{S}\right)}{dS} = \frac{1}{S} \left(\frac{dQ}{dS} - \frac{Q}{S}\right) < 0 \implies \frac{d\left(\frac{s}{Q}\right)}{dS} > 0.
\]

Hence, it is easy to verify that:

\[
\frac{d}{dS} \left[ \frac{1}{1 + \left(\frac{d_2 (R+1)}{s_2 \theta R (1+\frac{1}{I})(1+\frac{1}{R})}\right) \left(\frac{s}{Q}\right)^{\theta-1} S} \right] < 0.
\]

This implies that, as \(S\) is increased, then \(\frac{d\pi_i}{dS}\) can change sign from positive to negative at most once. In other words \(\pi_i\) is unimodal in \(S\).

We also show that for this marginal opportunity cost function the retailer margin does depend on the number of suppliers and of intermediaries, as opposed to the case of a linear function. Indeed,

\[
m_r = d_1 - d_2 Q - s_1 - s_2 \left(1 + \frac{\theta}{I}\right) \left(\frac{Q}{S}\right)^\theta
\]

and

\[
\frac{dm_r}{dS} = \frac{\partial m_r}{\partial Q} \frac{dQ}{dS} + \frac{\partial m_r}{\partial S},
\]

where

\[
\frac{\partial m_r}{\partial S} = \frac{s_2}{S} \left(1 + \frac{\theta}{I}\right) \left(\frac{Q}{S}\right)^\theta,
\]

and

\[
\frac{\partial m_r}{\partial Q} = -d_2 - \frac{s_2}{S} \left(1 + \frac{\theta}{I}\right) \left(\frac{Q}{S}\right)^{\theta-1}.
\]

Since both of the two expressions above are independent of \(R\) but \(\frac{dQ}{dS}\) does depend on \(R\) when \(\theta > 1\), as is apparent from (28), it is clear that \(\frac{dm_r}{dS} \neq 0\) in general.

Similarly,

\[
\frac{dm_r}{dI} = \frac{\partial m_r}{\partial Q} \frac{dQ}{dI} + \frac{\partial m_r}{\partial I},
\]

\[
\frac{dm_r}{dI} = \frac{\partial m_r}{\partial Q} \frac{dQ}{dI} + \frac{\partial m_r}{\partial I},
\]
where
\[ \frac{\partial m_r}{\partial I} = \frac{s_2 \theta}{T^2} \left( \frac{Q}{S} \right)^\theta. \]

\( dQ/dI \) can be deduced from the first order conditions
\[ \Psi(Q, I) = d_2 \left( 1 + \frac{1}{R} \right) Q + s_2 \left( 1 + \frac{\theta}{T} \right) \left( 1 + \frac{\theta}{R} \right) \left( \frac{Q}{S} \right)^\theta - (d_1 - s_1) = 0 \]

and the relationship:
\[ \frac{d\Psi}{dI} = \frac{\partial \Psi}{\partial Q} \frac{dQ}{dI} + \frac{\partial \Phi}{\partial I} = 0; \]

which implies:
\[ \frac{dQ}{dI} = \frac{\frac{s_2 \theta}{T^2} \left( 1 + \frac{\theta}{T} \right) \left( \frac{Q}{S} \right)^\theta}{d_2 \left( 1 + \frac{1}{R} \right) + s_2 \theta \left( 1 + \frac{\theta}{T} \right) \left( 1 + \frac{\theta}{R} \right) \left( \frac{Q^{\theta-1}}{S^{\theta-1}} \right) + \frac{Q}{P^2} + \frac{d_2 (R+1)}{s_2 \theta (R+\theta)} \left( \frac{S^\theta}{Q^{\theta-1}} \right)} \].

Since both \( \partial m_r/\partial I \) and \( \partial m_r/\partial Q \) are independent of \( R \) but \( dQ/dI \) does depend on \( R \) when \( \theta > 1 \), as is apparent from (29), it is clear that \( dm_r/dI \neq 0 \) in general.

**Appendix G: The case with stochastic demand**

We now show that our results are generally robust to the presence of stochasticity in the demand function. We first consider the case with stochastic additive demand, and then with multiplicative stochastic demand.

**G.1. The case with stochastic additive demand**

Consider the following stochastic additive linear inverse demand function:
\[ p_r = d_1 - d_2 Q + \epsilon, \]

where \( d_1 \) is the demand intercept and the random variable \( \epsilon \), which has zero mean and standard deviation \( \sigma \), represents a random additive perturbation to the demand function. In addition to the conditions we impose on \( d_1 \) to ensure that the expected retail price is non negative at equilibrium, we assume the amplitude of perturbation \( \epsilon \) is assumed to be small enough so that at equilibrium, the realized retail price remains non negative.

The \( k \)th retailer profit is
\[ \pi_{r,k} = (p_r - p_i) q_{r,k} = (d_1 - d_2 (q_{r,k} + Q_{r,-k}) + \epsilon - p_i) q_{r,k}. \]

Note that the only uncertainty in the retailers profit function is the random variable \( \epsilon \), which determines the realized demand function. The retailer order quantities and market-clearing intermediary price \( p_i \) are deterministic because retailers make their orders before demand is realized.

As in Adida and DeMiguel [2011], we assume that retailers have the following mean-standard-deviation utility function:
\[ E[\pi_{r,k}] - \gamma \text{ St.Dev.}[\pi_{r,k}] = (d_1 - \gamma \sigma - d_2 (q_{r,k} + Q_{r,-k}) - p_i) q_{r,k}. \]
This utility function allows for risk-averse retailers facing a stochastic demand function \((\gamma > 0\) and \(\sigma > 0)\), but it also covers the case when retailers are risk-neutral \((\gamma = 0)\), or when demand is deterministic \((\sigma = 0)\).

Then the \(k\)th retailer chooses its order quantity \(q_{r,k}\) to maximize its mean-standard deviation utility, assuming the rest of the retailers keep their order quantities fixed, and anticipating the intermediary reaction and the intermediary-market-clearing price \(p_i\):

\[
\max_{q_{r,k}, p_i} \left[ \tilde{d}_1 - d_2(q_{r,k} + Q_{r,-k}) - p_i \right] q_{r,k}
\]

\[
\text{s.t. } q_{r,k} + Q_{r,-k} = Q_i(p_i),
\]

where \(Q_{r,-k}\) is the total quantity ordered by the rest of retailers, \(Q_i(p_i)\) is the intermediary equilibrium quantity for a price \(p_i\). Constraint (32) is the intermediary-market-clearing condition, and we define \(\tilde{d}_1 \equiv d_1 - \gamma \sigma\) to be the risk-adjusted demand intercept.

From the retailer’s objective function (31), it is apparent that the impact of risk on the equilibrium is equivalent to a reduction of the demand intercept to \(\tilde{d}_1 \equiv d_1 - \gamma \sigma\). Therefore, the analysis in the main body of our manuscript applies to the case with stochastic additive demand after replacing the intercept with the risk-adjusted demand intercept.

**G.2. The case with stochastic multiplicative demand**

Consider the following stochastic multiplicative linear inverse demand function:

\[ p_r = (d_1 - d_2 Q) \epsilon \]

with \(E[\epsilon] = 1\) and \(Var[\epsilon] = \sigma^2\).

Assuming retailers are risk averse with a mean-standard-deviation utility function, the \(k\)th retailer’s objective is to maximize

\[
[\tilde{d}_1 - d_2(q_{r,k} + Q_{r,-k}) - p_i(q_{r,k} + Q_{r,-k})] q_{r,k} - \gamma \sigma (d_1 - d_2(q_{r,k} + Q_{r,-k})) q_{r,k}
\]

\[
= \left[ \tilde{d}_1 - \tilde{d}_2(q_{r,k} + Q_{r,-k}) - p_i(q_{r,k} + Q_{r,-k}) \right] q_{r,k},
\]

where \(\tilde{d}_1 = d_1(1 - \gamma \sigma)\) and \(\tilde{d}_2 = d_2(1 - \gamma \sigma)\).

From the retailer’s objective function, it is apparent that the impact of risk on the equilibrium is equivalent to replacing \(d_1\) and \(d_2\) with \(d_1(1 - \gamma \sigma)\) and \(d_2(1 - \gamma \sigma)\), respectively, provided that \(\gamma \sigma \leq 1\) and \(d_1 - s_1 \geq 0\). Therefore, the analysis in the main body of our manuscript applies to the case with stochastic multiplicative demand after replacing the intercept and slope with their risk-adjusted counterparts.

**Appendix H: Comparison to other models in the literature**

We now give a detailed comparison of the equilibria for our proposed model and for the models proposed by C&K and Choi [1991]. To do so, we first briefly state three-tier variants of the models of C&K and Choi that are similar to our model, and then we compare the equilibria of the three models.

As discussed in Section 2.2, our model also shares some common elements with that proposed by M&S. Their model, however, does not consider horizontal competition within tiers, and instead focuses on competition between supply networks. Moreover, M&S focus on how the equilibrium depends on the position of the leader within the supply network, whereas we fix the leader position
to be at the retailer, which is the situation faced by the supply chain intermediary firms that we are interested in. For these reasons, the equilibrium for M&$S$’s model is not comparable to that for our model, and thus we focus in this section on the comparison with the equilibria for the models by C&K and Choi, who consider serial multi-tier supply chain models with vertical and horizontal competition similar to our model.


We first consider a three-tier version of C&K’s model. The first tier consists of $S$ suppliers who lead the second tier consisting of $I$ intermediaries who lead the third tier consisting of $R$ retailers. There is quantity competition at all three tiers. We focus on the case where only the first tier of suppliers face production costs, which is the closest to our proposed model of intermediation.

The $k$th retailer chooses its order quantity $q_{r,k}$ to maximize its profit given an inverse demand function $p_r = d_1 - d_2 Q$ and for a given price requested by the intermediary $p_i$:

$$\max_{q_{r,k}} [d_1 - d_2(q_{r,k} + Q_{r,-k}) - p_i] q_{r,k}.$$  

The $l$th intermediary chooses its order quantity $q_{i,l}$ to maximize its profit for a given price requested by the suppliers $p_s$, and anticipating the price that the retailers are willing to pay for a total quantity $q_{i,l} + Q_{i,-l}$:

$$\max_{q_{i,l}} [p_i(q_{i,l} + Q_{i,-l}) - p_s] q_{i,l}.$$  

Finally, the $j$th supplier chooses its production quantity $q_{s,j}$ to maximize its profit anticipating the price that the intermediaries are willing to pay for a total quantity $q_{s,j} + Q_{s,-j}$ and given its unit variable cost is $s_1$:

$$\max_{q_{s,j}} [p_s(q_{s,j} + Q_{s,-j}) - s_1] q_{s,j}.$$  

H.2. A Choi-type model.

We now consider a three-tier version of Choi’s model with the retailers as leaders. Choi assumes that suppliers know the demand function and exploit this knowledge strategically when making production decisions. Moreover, Choi assumes that the suppliers are margin takers with respect to the intermediaries. Thus the $j$th supplier’s decision problem is

$$\max_{q_{s,j}} (d_1 - d_2(q_{s,j} + Q_{s,-j}) - m_r - m_i - s_1) q_{s,j},$$

where $s_1$ is the unit production cost, and $m_r$ and $m_i$ are the retailer and intermediary margins, respectively. Note that because $p_i = m_r + m_i + s_1$ it is apparent that the supplier’s decision problem for the Choi-type model is exactly equivalent to the retailer’s decision problem for the C&K-type model.

Following the spirit of Choi’s model with retailers as leaders, we assume that the intermediary also knows the demand function and exploits this knowledge strategically when making order quantity decisions. On the other hand, the intermediary is a follower with respect to the retailers and thus is a margin taker with respect to the retailers, but the intermediary is a leader with
respect to the suppliers and thus anticipates the price requested by the suppliers to deliver a given quantity. Therefore the \( l \)th intermediary decision can be written as

\[
\max_{q_{i,l}} (d_1 - d_2 (q_{i,l} + Q_{i,-l}) - m_r - p_s(q_i,l + Q_{i,-l}))q_{i,l}.
\]

Finally, the \( k \)th retailer in Choi’s model chooses its order quantity to maximize the profit given the demand function and anticipating the price required by the intermediaries to supply a total quantity \( q_{k,r} + Q_{r,-k} \):

\[
\max_{q_{k,r}} (d_1 - d_2 (q_{k,r} + Q_{r,-k}) - p_i(q_{k,r} + Q_{r,-k}))q_{k,r}.
\]

**H.3. Comparing the C&K and Choi models.**

C&K give closed-form expressions for the equilibrium quantities for their model. Choi also gives closed-form expressions for the equilibrium quantities for his two-tier model with retailers as leaders, and it is straightforward to extend these closed-form expressions for the three-tier variant of his model that we consider. The resulting closed-form expressions are collected in the second and third columns of Table 4.

The striking realization when comparing the second and third columns in Table 4 is that the equilibrium total order quantities in the C&K and Choi models coincide. Moreover, the aggregate intermediary profits also coincide. Furthermore, a careful look at the expressions for the aggregate profits of the retailers and suppliers reveals that the aggregate retailer profit in C&K’s model coincides with the aggregate supplier profit in Choi’s model if one replaces the number of retailers by the number of suppliers. Likewise, the aggregate supplier profit in C&K’s model coincides with the aggregate retailer profit in Choi’s model if one replaces the number of suppliers by the number of retailers. In other words, the equilibria of the C&K and Choi model are equivalent. We believe the reason for this is the assumption in Choi’s model that the suppliers have perfect information about the retailer demand function and exploit this strategically when making decisions. This is not a realistic assumption for the intermediation context that we study.

**H.4. Comparing our model to the C&K and Choi models.**

The most important difference between the equilibrium to our model and those to the C&K and Choi models is in the intermediary margins and profits. For all three models, the suppliers market power decreases in the number of suppliers, and their profits become zero in the limit when there is an infinite number of suppliers. The intermediaries’ margin and profit, however, behave quite differently for the three models. In C&K’s model, the intermediaries’ margin increases with the number of suppliers; in Choi’s model, it remains constant; and in our model, it decreases. As a result, for the C&K and Choi models, the intermediary profits increase in the number of suppliers, because order quantities also increase. In our model, on the other hand, the increase in the order quantities is not sufficient to offset the decrease in margin and, as a result, the intermediary profits are unimodal in the number of suppliers. The reason for this is that as the number of suppliers grows large, the retailers know that intermediaries and suppliers will agree to produce at any price above \( s_1 \), and therefore the retailers can take advantage of their leading position to extract higher rents, leaving the intermediaries with zero margin and profit for the limiting case where the number of suppliers is infinite.
Comparing the aggregate retailer profits in all three models, we observe that the equilibrium retailer margins for our proposed model and the model by Choi are equal and they are both larger than the equilibrium retailer margin for the model by C&K. Moreover, because the equilibrium total quantity in the models by C&K and Choi are identical, this implies that the aggregate retailer profit in the model by Choi is larger than that in the model by C&K. This is not surprising as it is well known that a leading position often results in larger profits for a given player; see Vives [1999].

The question of whether the aggregate retailer profits is larger in our model than in those by C&K and Choi is a bit harder to answer. Note that there is an additional parameter in our model: the marginal cost sensitivity $s_2$. This parameter enters the closed-form expression for the equilibrium quantity in our model and thus it is difficult to compare to the other two models. However, assuming the marginal cost sensitivity $s_2$ equals the demand sensitivity $d_2$, it is easy to show that the equilibrium quantity in our model is larger than in the C&K and Choi models. This implies that the equilibrium aggregate retailer profit utilities in our model are larger not only than those in the C&K model, but also than those in Choi’s model (for the case $s_2 = d_2$). Two comments are in order. First, since in our proposed model the retailers act as leaders, they are able to capture greater profit utilities than in the model by C&K. Second, while Choi also captures the retailers as leaders, he assumes that intermediaries and suppliers know the retailer demand function and exploit this knowledge strategically. This assumption leaves the retailers in a weaker position compared to our model.

We conclude that there are significant differences between the models by C&K and Choi [1991] and our proposed model, particularly the leadership positions and information available within the game, which result in different insights. Our model fits best situations when retailers act as leaders, while C&K’s model is more appropriate when suppliers can be considered leaders. Choi’s model makes sense when the retail demand function can realistically be known by all players.

\[15\] Note that the retailer margin and profits in Choi’s model coincide with the supplier margin and profits in C&K’s model after replacing the number of retailers with the number of suppliers, so as we argue above both models are essentially equivalent.
Table 4  Comparison of the equilibrium for the models by C&K, Choi [1991], and the proposed model.

This table gives the equilibrium quantities for variants of the models proposed by C&K and Choi [1991], as well as for our proposed model. The first column in the table lists the different equilibrium quantities reported: the aggregate supply quantity, the supply price, the intermediary price, the retail price, the supplier margin, the intermediary margin, the retailer margin, the aggregate supplier profit, the aggregate intermediary profit, the aggregate retailer profit, the aggregate supply chain profit, and the efficiency. The second, third, and fourth columns give the expression of the equilibrium quantity for the variants of the models proposed by C&K and Choi [1991], and for our proposed model, respectively. Note that for our proposed model the supplier marginal opportunity cost is not constant, and thus we report the average supplier margin.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Corbett and Karmarkar</th>
<th>Choi</th>
<th>Retailers lead</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate quantity</td>
<td>( \frac{RSI}{d_2(S+1)(I+1)} \frac{d_1 - s_1}{R+1} )</td>
<td>( \frac{RSI}{d_2(S+1)(I+1)} \frac{d_1 - s_1}{R+1} )</td>
<td>( \frac{RSI}{(d_2SI + s_2(I+1))} \frac{d_1 - s_1}{R+1} )</td>
</tr>
<tr>
<td>Supply price</td>
<td>( d_1 - \frac{d_2(R+1)(I+1)}{R} Q )</td>
<td>( s_1 + \frac{d_2}{S} Q )</td>
<td>( s_1 + \frac{s_2}{S} Q )</td>
</tr>
<tr>
<td>Intermediary price</td>
<td>( d_1 - \frac{d_2(R+1)}{R} Q )</td>
<td>( s_1 + \frac{d_2 S + I + 1}{I} Q )</td>
<td>( s_1 + \frac{s_2 I + 1}{I} Q )</td>
</tr>
<tr>
<td>Retail price</td>
<td>( d_1 - d_2 Q )</td>
<td>( d_1 - d_2 Q )</td>
<td>( d_1 - d_2 Q )</td>
</tr>
<tr>
<td>Supplier margin</td>
<td>( \frac{d_1 - s_1}{S+1} \frac{I}{R+1} )</td>
<td>( \frac{d_2 Q}{S} \frac{(S+1)}{(I+1)} )</td>
<td>( \frac{s_2 Q}{2S} )</td>
</tr>
<tr>
<td>Intermediary margin</td>
<td>( \frac{d_2 Q(R+1)}{R} )</td>
<td>( \frac{d_2 Q(S+1)}{S} )</td>
<td>( \frac{s_2 Q}{2S} )</td>
</tr>
<tr>
<td>Retailer margin</td>
<td>( \frac{d_1 - s_1}{R+1} \frac{I}{(I+1)(S+1)} )</td>
<td>( \frac{d_1 - s_1}{R+1} )</td>
<td>( \frac{d_1 - s_1}{R+1} )</td>
</tr>
<tr>
<td>Agg. supplier profit</td>
<td>( \frac{d_1 - s_1}{S+1} \frac{Q}{S} )</td>
<td>( \frac{d_2 Q^2}{S} \frac{(S+1)}{(I+1)} )</td>
<td>( \frac{s_2 Q^2}{2S} )</td>
</tr>
<tr>
<td>Agg. interm. profit</td>
<td>( \frac{d_2 Q^2(R+1)}{R} )</td>
<td>( \frac{d_2 Q^2(S+1)}{S} )</td>
<td>( \frac{s_2 Q^2}{2S} )</td>
</tr>
<tr>
<td>Agg. retailer profit</td>
<td>( \frac{d_1 - s_1}{R+1} \frac{Q}{R} )</td>
<td>( \frac{d_1 - s_1}{R+1} \frac{Q}{R} )</td>
<td>( \frac{d_1 - s_1}{R+1} \frac{Q}{R} )</td>
</tr>
<tr>
<td>Agg. chain profit</td>
<td>( S \pi_s + I \pi_i + R \pi_R )</td>
<td>( S \pi_s + I \pi_i + R \pi_R )</td>
<td>( S \pi_s + I \pi_i + R \pi_R )</td>
</tr>
<tr>
<td>Efficiency</td>
<td>( \frac{4STR(RI + RS + SI + R + S + I + 1)}{(S+1)^2(I+1)^2(R+1)^2} )</td>
<td>( \frac{4STR(RI + RS + SI + R + S + I + 1)}{(S+1)^2(I+1)^2(R+1)^2} )</td>
<td>( \frac{RI [d_2 B (I+2) + 2(d_2 SI + s_2(I+1))] 2d_3 S + s_2}{(d_2 SI + s_2(I+1))^2 (R+1)^2} )</td>
</tr>
</tbody>
</table>