Understanding Retail Assortments In Competitive Markets

October 13, 2010
Abstract

Assortment planning, where a retailer decides which products to place on their store shelves, is one of the most fundamental decisions in retailing. With the ever increasing number of products available to retailers, category managers face a complex decision to select the “right” assortment for their categories. These assortment decisions are impacted by local consumer preferences, competition and costs. Under these conditions, a retailer’s optimization problem is mathematically daunting and empirically challenging.

In this paper we address the analytical and empirical challenges of modeling retailers’ assortment decisions. The analytical model shows that the optimal retail assortment can be described by ranking products based on a combination of demand and cost parameters. The advantage of this representation is that it can be used to study retail categories with a large number of available products. In the empirical study, we use these analytical results to model supermarket assortment decisions across 10 categories and 34 local markets.

Our empirical estimates can be used to advise retailers, manufacturers and policy makers. For retailers, we estimate that an optimal 25% reduction in category size results in only a 3% reduction in category profits. For manufacturers, we find that wholesale price discounts can increase distribution, however these are most effective with retailers who have large assortments. For policy makers we show (a) the introduction of manufacturer tying contracts can have a large negative impact on retailers’ profits and small manufacturers’ distribution and (b) that assortment and pricing decisions are important when considering mergers between retailers.
1 Introduction

Assortment planning, where retailers decide which products to place on their store shelves, is one of the most fundamental decisions in retailing. Consumers view the assortment as an important category management service output that drives their decisions on where to shop (Kok and Fisher [2007]). Consistent with this view, in a recent meta analysis, Pan and Zinkhan [2006] reviewed 14 papers all concluding that assortments are an important driver for consumers’ purchase decisions. A recent survey by Nielsen found that store assortment is the second most important factor driving consumers’ decisions (Nielsen, Dec 17, 2007). Moreover, category managers believe that their assortments are important competitive tools that allow them to differentiate. Understanding the mechanics behind assortment decisions is important for retailers, manufacturers and policy makers alike.

Over the last few decades there has been an sharp increase in the number of products sold in retail stores, from 14k in 1980 to 49k in 1999 (Food Marketing Institute [2004]). With this increase in available products, retail category managers face a complex decision when selecting the “right” assortment for their categories. These assortment decisions are impacted by local consumer preferences, competition and costs. Given the large number of available options, the optimization problem is mathematically daunting and empirically challenging. Researchers often consider simplifications to the problem to make it more tractable, for example, considering only a small number of products in a subcategory (Draganska et al. [2009]) or relying on heuristics that the retailer can use (Kok and Fisher [2007]).

In this paper we develop a different approach to address the analytical and empirical challenges of modeling retailers’ assortment decisions. In the analytical model, we consider the two decisions made by a retail store category manager: category assortment and retail prices. We show that the optimal retail assortment can be described by ranking the products based on a combination of demand and cost parameters. The advantage of this representation is that it can be used to study a large number of options in the retailer’s consideration set. This provides a realistic view of managerial decision making, and there is evidence in the literature that retailers do indeed use a form of rank ordering to make their assortment decisions (Esbjerg et al. [2004]).

When considering the empirical analysis, we take the view that data are generated from a market equilibrium (Bresnahan [1987]) where demand is based on consumer purchase decisions and supply is based on retailers’ pricing and assortment decisions. Therefore in each market and category we simultaneously model the demand and supply processes. A key advantage of the derived analytical result is that it allows us to consider the retailer’s assortment decision as a sorting problem. This allows us to include information about products not in the assortment when modeling demand and supply parameters. For example, the fact that a store in an affluent neighborhood does not store value products informs us that consumer preferences for value products are low in this area.

Our empirical study models supermarket assortment decisions across 10 categories and 34 local markets across the United States. Studying multiple categories allows us to make generalizations about retailers’ assortment decisions. In our model we introduce a fixed cost of introducing a
product on the retail shelf. This fixed cost represents a minimum threshold profit that an introduced product must add to the category profits. Our estimates suggest that the differences in fixed costs across categories are driven by the size of products in the category and whether or not they require refrigeration. In our framework we empirically estimate the three main drivers of local retail store assortments: demand, costs, and competition. This allows us to consider the impact of changes in any of these characteristics on retailer assortment decisions.

With our analytical framework and econometric estimates we can quantify the effect of changes in assortment sizes on demand or profits (Broniarczyk et al. [1998], Boatwright and Nunes [2001], Kk and Xu [2010]). When considering changes to the size of the assortment, we describe which UPCs a retailer should store and the prices the retailer should charge. In our data, we estimate that a 25% reduction in total assortment size results in about a 3% reduction in total profits. This suggests that retailers might strategically reduce their assortments with a small impact on category profits if the additional shelf space created can be used to carry additional categories. This methodology can be a powerful tool that retailers can use to make assortment decisions across multiple categories.

Our model considers competition between retailers in their assortment and pricing decisions. We model this as a full information pure strategy Nash Equilibrium where each retailer plays a best response to competitors’ actions. We estimate that, in our data, competition between retailers can lead to an increase in the total number of UPCs available to consumers as well as the predicted decrease in prices. Moreover, the magnitude of this effect depends on the space allocated to the category in retail stores. This can be an important result for policy makers, as mergers can have two negative influences on consumer welfare: higher prices and smaller selection.

We model the effect of changes in manufacturers’ wholesale prices on the distribution of their products in retail stores. We show that wholesale price discounts are most effective with retailers who have allocated large amounts of shelf space to a category. On the other hand retailers with limited space allocated for a category might not change their assortment decisions even with large wholesale discounts. Another tactic that manufacturers can use to increase distribution is to introduce tying contracts, where retailers must store all or none of the UPCs they offer. We estimate that these contracts negatively impact retailers and small manufacturers. We find that these contracts are most damaging in the case of retailers with limited category space. Retailers can lose a majority of their profits because they offer restricted assortments and small manufacturers have minimal distribution.

An important finding from our empirical estimation is that not accounting for retail assortment decisions results in substantially different demand and supply estimates. This difference is particularly noticeable for the price estimate in the demand system, where without modeling assortment decisions, one would estimate that consumers are less price elastic. This in turn would cause a researcher to infer that retail margins are higher, in some cases greater than 100%. We believe that this provides evidence that researchers should consider assortment decisions when modeling retail demand and supply.
To ground our model in managerial practice, we worked with a large retail chain to understand their decision making process and believe this is consistent across retailers\(^1\). Assortments are based on decisions made at different levels of the organization space (Kok and Fisher [2007]). First, a retail chain headquarters (national or regional) decides which products to store in its warehouse. More explicitly, here a category “buyer” negotiates with manufacturers and decides which products to store in the retail warehouse. Second, an individual store category manager, decides which products to store on the store shelf. These decisions are important, as each retail store faces a different “localized” market, characterized by (a) demand depending on the preferences in its neighborhood, (b) costs of space and (c) competition (Rigby and Vishwanath [2006]). As an example, consider the Cookies category. Jewel Osco stores in a higher income neighborhood in Chicago stock more biscotti and macaroons, while Jewel Osco stores in a low income neighborhood stock more value products. In this paper, we consider decisions made by a local retail store category manager who observes the products stored in the chain’s warehouse and decides which of these products to sell and the price to charge.

In our empirical application, we use a unique dataset that allows us to consider local assortment decisions for retail stores. Our data have two unique features that allow us to study this problem empirically. First, we observe detailed attribute information for each UPC in each category. The advantage of these attributes is that we can distinguish one UPC from another and therefore make inferences based on which UPCs are sold. Without this detailed information a researcher would incorrectly infer that assortment decisions are based on random shocks (Richards and Hamilton [2006]). For example consider the Cookie example studied above: without detailed attribute information about the base of the cookie we could not distinguish biscotti and macaroons from other cookies. Without observable products characteristics to distinguish assortment decisions across stores, these differences would be treated as random shocks. Second, we observe a set of retail stores for each retail chain, this allows us to consider local warehouse decisions made by the chains. Without these data, a researcher would incorrectly assume that every store could offer any UPC from any manufacturer.

This paper builds on two main classes of literature: estimation of demand with endogenous product choices and pricing (e.g. Draganska et al. [2009]), and the retail assortment marketing and operations literature (e.g., Kok et al. [2006] and McAlister and Pessemier [1982]). We extend the endogenous product choice literature by introducing an approach that does not simplifies the combinatorial assortment problem. To the operations literature we add a structural view to the empirical modeling. These advances allow us to model the current assortment decisions made by retailers and consider how these are affected by changes in market conditions.

There has been a recent increase in the area of endogenous product choices in competitive markets in the marketing and industrial organization literature. Early papers in this area by Mazzeo [2002] and Seim [2006] studied endogenous product choice decisions by firms entering

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\(^1\)This process is consistent with the empirical finding in Hwang et al. [Forthcoming] find that “supermarkets owned by the same chain carry similar assortments”
markets. For example, Mazzeo [2002] considers the decisions of hotel chains to enter markets and choose the quality level. In marketing\(^2\), this literature considers the entry decisions of retailers in markets (Zhu and Singh [2007]). The paper closest to our model is Draganska et al. [2009], where the authors consider models with endogenous product assortments and pricing. In this paper, the authors study the endogenous decisions of ice cream manufacturers to sell different varieties of vanilla ice cream in different markets. To see the importance of this model, notice that literature on estimating demand and supply (e.g., Berry et al. [1995] and Nevo [2001]), looks at the demand from products sold and the prices of products in the market. Any change in product offering is considered to be exogenous (Nevo [2001]) and therefore explicitly uncorrelated with demand and supply. Considering endogenous assortment decisions essentially relaxes this assumption by modeling the decisions of firms to select certain products to sell in their stores. A drawback of the model proposed in Draganska et al. [2009] is that they need to evaluate every possible assortment to estimate their model. In some categories with 600 UPCs to choose from this leads to \(2^{600}\) evaluations which is infeasible to solve. In addition, unlike most of the discrete choice literature, Draganska et al. [2009] assume that there are no unobserved (to the researcher) demand shocks that impact demand.

Notice that while most of the multi-product firm choice literature has considered decisions made by manufacturers, we look at decisions made by retailers. While the problems are conceptually similar, considering a retailer as the “decision maker” allows us to simplify our model in two key ways. First, the set of available products for a retailer is restricted to the set of products available in the warehouse, whereas a manufacturer could produce a new product that is not currently available. Therefore, as researchers, we have more information about the consideration set for the retailer. Second, manufacturers face fixed costs to produce a good and this cost is heterogeneous across products. On the other hand, a retailer does not face an explicit cost of production, the fixed cost they face is a stocking cost which is homogeneous across products in a category. This difference allows us to simplify the optimization function and allows us to derive our analytical result.

The operations management literature considers the retail assortment problem coupled with an inventory problem, that is, they consider the number units of each UPC a retailer should stock (see Kok et al. [2006] for survey of the operations literature). The theoretical literature in the operations management, started with VanRyzin and Mahajan [1999], where the interest was in modeling the supply side of the retailing industry. VanRyzin and Mahajan [1999] consider a logit demand system, with exogenous pricing. The main analytical result of this paper is that a retailer will add SKUs in the order of expected contribution (i.e., add highest selling SKU first, 2nd highest selling SKU next, and so on) until it is not profitable to add any more SKUs. The advantage of this result

\footnote{Another branch of this literature considers product lines and firm pricing decisions (e.g., Draganska and Jain [2005], Richards and Hamilton [2006]). In this literature, it is assumed that consumers have some utility for product lines as a function of the number of products in the product line. In our paper, we assume consumer preferences for each individual UPC. We therefore derive a preference for product lines that is not based only on the number of UPCs but also on which UPCs are included in the assortment. We can extend this literature due to on a key advantage of our data which contains detailed attribute information for each UPC. This allows us to consider each UPC as a different entity with a different set of attributes.}
is that a retailer need not calculate the profitability of all combinations of the the SKUs to find the optimal assortment, instead the problem can be solved by a simple ranking. Maddah and Bish [2007] extend the VanRyzin and Mahajan [1999] model to include a joint decision of assortment and pricing. They find the main result, that optimal assortments can be found by ranking the SKUs, holds if the marginal costs are non-increasing (i.e., the most popular product does not cost more that the least popular product). In particular, in a setting where the marginal costs are constant, they find that ranking heuristic can be used by monopolistic retailers. This result can be a bit problematic as the main component of the retailer’s marginal cost is the manufacturer’s wholesale price. In most economic settings the manufacturer would charge a higher wholesale price for more popular products which would not satisfy the condition in their result. We extend the results of these papers and show that we can derive a different condition that depends on value and costs and that simplifies the optimal assortment problem. Additionally, we extend this framework to account for oligopoly markets.

Overall the operations research literature considers the retail assortment problem as one where the retailer chooses UPCs while considering inventory decisions and modeling the probability of stock-outs. In our paper we model retailer pricing and assortment decisions in competitive markets simultaneously with demand for the category. While the theory is structured similarly, there is an important conceptual difference in the empirical outlook. The empirical operations literature estimates demand and then optimizes the retail assortment decisions. But in this paper we follow the Industrial Organization view (Bresnahan [1987]), where we believe the data we observe are from equilibrium decisions made by retailers. Therefore, when modeling demand, we explicitly account for the endogenous price and assortment. Importantly we find that, without considering endogenous assortment decisions, a researcher would infer incorrect demand estimates. This suggests that the operations research approach will optimize the assortment decision with “biased” inputs.

The product assortment literature in marketing focuses on consumers responses to product assortments. In a recent meta analysis, Pan and Zinkhan [2006] (see also Briesch et al. [2009]) report that all 14 studies that have measured store patronage report that assortment selection increases the propensity of a consumer to visit the store. These studies suggest that assortment decisions can have significant implications for overall retail store patronage. When considering assortments, researchers consider the variety offered by this assortment\(^3\). This idea of retail variety started with Baumol and Ide [1956] and the marketing literature has emphasized the variety of assortments (see McAlister and Pessemier [1982] for a review) are important in consumer decision making. In empirical applications, researchers (Hoch et al. [1999], Hoch et al. [2002]) measure the

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\(^3\)The experimental marketing literature has specifically consider larger assortments might not be optimal for retailers (e.g., McAlister and Pessemier [1982], Chernev [2003], Chernev [2006]). Here researchers argue that giving too many choices might reduce consumers purchases as this makes purchase decision harder. There is some evidence from an on-line grocery retailer described in Boatwright and Nunes [2001], where the researchers show that the overall category purchases increased with smaller assortments. Additionally Sloot et al. [2006] show that 25% experimental reductions in assortment sizes did not result in a significant long term decrease in demand. In our paper, we focus only on the first order effect of retail assortments and do not model the behavioral process that might lead to difficulty in choice from large assortments. We do believe this is an area for further empirical research.
variety in an assortment using the (psychological) distance between items in the assortment. In addition to attribute based variety, these papers also show that the organization of the store shelf does indeed affect the perception of variety in an assortment. In our data we do not observe shelf allocation and therefore are restricted to attribute based variety. We view the assortment variety as an important metric, specifically as it influences demand for all products. In our paper we account for variety in our demand system to understand underlying consumer preference. We feel this notion of variety is very important and for this reason we explicitly model the specific UPCs and just not the number of UPCs a retailer stores.

The remainder of the paper is structured as follows: in section 2 we discuss the analytical model and empirical specification, in section 3 we describe the data used in the paper. Section 4 covers the results of the model, in section 5 we describe the counterfactual analysis, and section 6 concludes.

2 Model

In this section we will describe the assumed decision making process used by the retailer, and the analytical results we can derive from this to simplify the empirical analysis. In this paper we use a Bayesian econometric setup to estimate the model. The details of the empirical estimation will be shown at the end of this section.

Our model is formulated from the perspective of a grocery category manager for a large retail store. In every decision cycle, the manager observes the products that are stored in the chain’s warehouse. The manager observes the marginal costs for each product in the category, this can be thought of as the wholesale price charged by the manufacturer plus any additional economic cost associated with selling the product. Additionally the manager is given a constrained space in the store where she can store only a certain number of products. Given these inputs, the manager has two sets of decisions: first she must decide which products to sell in her store, and second she must decide the prices to charge. We will start with a model for a monopolistic retailer and then extend this to allow for retail competition.

In economic terms, the retailer manager’s problem at any time t, is

$$\max_{\Theta_t \subseteq \Omega_t} \max_{P_t} \sum_{j \in \Theta_t} \pi_j(\Theta_t, C_t, P_t)$$

subject to

$$|\Theta_t| \leq n_t$$

where $$\Omega_t$$ represents all available products in time period t, $$\Theta_t$$ is the chosen subset of products for the retailer, $$C_t$$ represents the cost of selling the products and $$P_t$$ represents the selected price for each product in the assortment. The profit depends on the profit for each product sold, which is represented by $$\pi_j$$. $$n_t$$ represents the space constraint for the retailer. Notice here we have assumed the space constraint for the retailer is a constraint on the number of different products sold. In a retail setting this is normally a constraint on the number of facings available for the category.
However, since we do not observe data on facings we need to make a simplifying assumption here. Under this formulation we assume that retailers first decide which products to store and then allocate shelf space to these products.

As standard in the literature (e.g., Nevo [2001]) we will assume that marginal cost is constant. This assumption is consistent with constant wholesale prices, that do not vary with the number of units purchased. So, we write the profit function for product \( j \) as follows

\[
\pi_j(\Theta_t, C_t, P_t) = M_t(p_{j,t} - c_{j,t})s_{j,t}(\Theta_t, P_t)
\]

For a given time period \( t \): \( M_t \) is the size of the available market, \( p_{j,t} \) and \( c_{j,t} \) are the price and marginal cost for product \( j \), and \( s_{j,t}(\Theta_t, P_t) \) represents the share captured by product \( j \) when the retailer stores \( \Theta_t \) products and charges a price \( P_t \). So, the retailers problem can be considered as

\[
\max_{\Theta_t \subseteq \Omega_t} \max_{P_t} \sum_{j \in \Theta_t} M_t(p_{j,t} - c_{j,t})s_{j,t}(\Theta_t, P_t) \tag{2}
\]

subject to

\[|\Theta_t| \leq n_t\]

This is a complex problem for the retailer as we have the retailer selecting both the optimal assortment and the optimal prices. The optimal assortment problem is a difficult combinatorial problem where the retailer selects a subset of products from a large available set. Although the optimal pricing problem is one that has been studied extensively, this remains a complex problem, as the optimal price for all products needs to be found simultaneously.

Prior research (Draganska et al. [2009]) has suggested algorithmic solutions to this problem. These require finding all possible subsets of the choice set (power set) and then calculating the optimal price in each case. To see the complexity of this, consider a case where we only have three available products in \( \Omega_t \) say, \( x, y, z \). Now we would need to calculate the optimal profits for each set in \( \mathcal{P}(S) = \{\phi, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}\} \). In general the number of calculations is \( 2^{|\Omega_t|} \). Therefore while these methods are feasible in cases with a small number of available options, they become less so as the number of available options in large. In most grocery retail categories, there are between 60 and 600 options, this would require an infeasible 1.15e18 to 4.15e180 calculation. To overcome these problems, in this paper we provide a simpler analytical solution by making specific assumptions about the nature of demand in the market. In particular, we assume a logit demand function, which has been used extensively in the literature. In the technical appendix we show that our results extend to nested-logit demand systems\(^4\). The optimal assortment will allow retailers to follow a simple algorithm for ranking a product and then decide on the optimal assortment based on this ranking.

In the next few subsections we will define specific parametric assumptions to describe the nature of the industry and then derive our main analytical results.

\( ^4 \)Please contact the author for the technical appendix
2.1 Demand model

We will assume a logit demand system in this paper (an extension to a nested logit demand system is available in the technical appendix\(^5\)). We assume that each consumer in a given market will have the following utility function

\[ u_{i,j,t} = \alpha X_{j,t} - \beta p_{j,t} + \xi_{j,t} + \epsilon_{i,j,t} \]

where \(i\) represents consumers, \(j\) represents product and \(t\) represents the time period. \(X_{j,t}\) represents the product characteristics for product \(j\). \(\alpha\) represents consumers' preferences for the product characteristics. \(p_{j,t}\) is the price charged by the consumer at time \(t\) and \(\beta\) represents the marginal utility of money. There are random utility shocks \((\xi_{j,t})\) that impact all customers in the market; these demand shocks are observed by the all agents, though unobserved by the researcher (Berry [1994]). \(\epsilon_{i,j,t}\) represents the idiosyncratic fit, per the logit assumption these are assumed to be from an i.i.d. type I extreme value distribution. Additionally consumers have a utility \(u_{i,0,t} = \epsilon_{i,0,t}\) that represents their utility from selecting out of the market.

In the model presented here, we do not model heterogeneity of consumer tastes for attributes. This is a simplifying assumption and is made to focus our attention on understanding the local assortment decisions of retailers. However, note that to study local assortments we must allow demand preference for local markets to be different. We will discuss this further in the empirical model section.

In this demand system, consumers select the product that maximizes their utility. Or, the decision for a consumer in time \(t\) is given by

\[ d_{i,t} = \arg \max_{j \in [0, \Theta_t]} u_{i,j,t} \]

With the logit assumption we can write this as a probabilistic choice, as below

\[ \varphi(d_{i,t} = j) = \varphi(u_{i,j,t} > u_{i,-j,t}) = \frac{e^{\alpha X_{j,t} - \beta p_{j,t} + \xi_{j,t}}}{1 + \sum_{k \in \Theta_t} e^{\alpha X_{k,t} - \beta p_{k,t} + \xi_{k,t}}} \]

where \(-j\) refers to all products other than \(j\).

Prior IO models (Berry [1994], Nevo [2001]) simplify this by defining this define \(\delta_{j,t} = \alpha X_{j,t} + \xi_{j,t}\). Integrating over customers and taking logs we get the following well established (Berry [1994]) relationship from our demand model:

\[ \log(s_{j,t}) - \log(s_{0,t}) = \delta_{j,t} - \beta P_{j,t} \]

\(^5\)Please contact the author for the technical appendix
2.2 Supply model

In this paper, there are two parts to the supply model: assortments and pricing. In this section we will discuss both of these and derive the analytical results to construct an empirical model for studying. We will start with considering the retailer pricing problem and then continue on to studying the retailer assortment problem.

In this section, we first look at a monopoly setting and then extend it to allow for competition between retailers.

2.2.1 Retailer pricing

We begin with the simplified model where the retailer has decided the products in the assortments \((\Theta_t)\) and is deciding the optimal price. The retailer will solve the problem of the form

\[
\max_{P_t} \sum_{j \in \Theta} M_t(p_{j,t} - c_{j,t}) s_{j,t}(\Theta_t, P_t)
\]

This is a well-studied problem and the solution from the first order condition for all \(j\) must satisfy

\[
s_{j,t}(\Theta_t, P_t) + (p_{j,t} - c_{j,t}) \frac{\partial s_{j,t}(\cdot)}{\partial p_{j,t}} + \sum_{k \neq j} (p_{k,t} - c_{k,t}) \frac{\partial s_{k,t}(\cdot)}{\partial p_{j,t}} = 0
\]

A result from prior research (Anderson de Palma 1992 and Nevo and Rossi [2008]) simplifies this equation further to describe the optimal price as

\[
p_{j,t} - c_{j,t} = \left( \frac{1}{\beta} \right) \left( \frac{1}{1 - \sum_{k \in \Theta} s_{k,t}(\Theta_t, P_t)} \right)
\]

For the interested reader we have re-derived this result in the appendix of this paper.

Interestingly, with this relationship the optimal price involves charging a constant margin across all products in the retailers assortment; this is because the right hand side of equation 5 does not involve any terms specific to \(j\). Therefore we can define \(m(\Theta_t)\) as the optimal markup given the assortment \(\Theta_t\). This simplification helps reduce the complexity of the modeling, as now the optimal pricing is reduced to a one dimensional parameter.

However, we need to expand this expression further as we currently have the price on both the left and right hand sides of equation 5 (note that price is in the share calculation).

**Claim 1.** The optimal markup in this setting can be simplified as below:

\[
\log(\beta m(\Theta_t) - 1) + (\beta m(\Theta_t)) = \log(\sum_{k \in \Theta_t} e^{\delta_{k,t} - \beta c_{k,t}})
\]

**Proof.** Proof in the appendix

This relationship will prove useful when we consider the assortment selection section of the supply model.
Next, we consider the optimal profits of an assortment $\Theta_t$ given retailers use the optimal pricing described above.

**Claim 2.** The optimal profit can we written as:

$$\max_{\Theta_t} \sum_{j \in \Theta_t} M_t(p_{j,t} - c_{j,t})s_{j,t}(\Theta_t, P_t) = M_t(m(\Theta_t) - \frac{1}{\beta})$$

(7)

**Proof.** Proof in the appendix

### 2.2.2 Assortment decision

In our supply problem we assume the retailer solves the following problem:

$$\max_{\Theta_t \subseteq \Omega_t} M_t \max_{P_t} \sum_{j \in \Theta_t} (p_{j,t} - c_{j,t})s(\Theta_t, P_t)$$

subject to

$$|\Theta_t| \leq n_t$$

**Claim 3.** The capacity constraint will always be satisfied

**Proof.** Proof in the appendix

With this result we can add a Lagrange multiplier (called $F_t$), given the constraint $|\Theta_t| \leq n_t$ is satisfied. Therefore we can rewrite the retailer’s problem as;

$$\max_{\Theta_t \subseteq \Omega_t} \max_{P_t} \sum_{j \in \Theta_t} [M_t(p_{j,t} - c_{j,t})s_{j,t}(\Theta_t, P_t) - F_t]$$

where $F_t$ is the shadow price of adding a product (or the fixed cost of space). As this is a Lagrange multiplier, we must have $F_t > 0$. From an economic perspective, the marginal benefit of adding a product to the assortment must be at least $F_t$.

Substituting in the results from the pricing section earlier, this problem simplifies to:

$$\max_{\Theta_t \subseteq \Omega_t} M_t(m(\Theta_t) - \frac{1}{\beta}) - F_t$$

with

$$\log(\beta m(\Theta_t) - 1) + (\beta m(\Theta_t)) = \log(\sum_{k \in \Theta_t} e^{\delta_{k,t}} - \beta c_{k,t})$$

The advantage of the above representation is that this equation only considers the maximization over the space of $\Theta_t$. Moreover, $\Theta_t$ only enters though $m(\Theta_t)$, therefore to simplify this further we need to understand the properties of $m(\Theta_t)$.

**Proposition 4.** The equation

$$\log(\beta m(\Theta_t) - 1) + (\beta m(\Theta_t)) = \log(\sum_{k \in \Theta_t} e^{\delta_{k,t}} - \beta c_{k,t})$$
is invertible (in \(m(.)\)). Moreover, the solution is strictly increases with \(\log(\sum_{k \in \Theta} e^{\delta_{k,t} - c_{k,t} - \beta(c_k)})\).

Proof. proof in the appendix

To extend this result further, notice that the profit for an assortment are an increasing function of \(m(.)\). By the claim above we must have that these profits are an increasing function of \(\sum_{k \in \Theta} e^{\delta_{k,t} - c_{k,t}}\) (notice we remove the log as it is a strictly increasing function). This leads us to our next claim to get the optimal assortment

**Claim 5.** When deciding the optimal assortment, the retailer can rank product based on their \(\delta_{k,t} - c_{k,t}\) and pick the top \(n_t\) products to include in the market.

Proof. In the appendix

This solves the normative problem for a monopolistic retailer and we will extend this problem to include competition. In relation to prior literature, this result is most similar to that of Maddah and Bish [2007]. Their main result using the notation of this paper is that products can be ranked using \(\delta_{.,t}\) if the \(c_{.,t}\) have the (weakly) reverse ranking. Thus they get the rank ordering works only if products with higher \(\delta_{.,t}\) have (weakly) lower \(c_{.,t}\). The result derived in our paper clearly satisfies this condition as a special case.

### 2.2.3 Extension from monopoly to oligopoly

We consider a full information simultaneous move oligopoly model where in an pure strategy Nash equilibrium each firm plays a best response to the other firms assortment decision and pricing decisions. The retailers’ problem (best response) can now be written as the solution to

\[
\max_{\Theta_t \subseteq \Omega_t} M_t \max_{P_t} \sum_{j \in \Theta_t} (p_{j,t} - c_{j,t}) s_{j,t}(\Theta_t, P_t, \Theta_t^-, P_t^-)
\]

subject to

\(|\Theta_t| \leq n_t\)

where \(\Theta_t^-\) refers to the assortment decisions of all other retailers and \(P_t^-\) refers to the pricing decisions of all other retailers in the market.

In this section we will update the claims from the monopoly model and show that similar claims hold in an oligopoly setting. First, for the pricing model, notice that the optimal margin for the retailer does still satisfy\(^6\)

\[
p_{j,t} - c_{j,t} = \left(\frac{1}{\beta}\right) \left(\frac{1}{1 - \sum_{k \in \Theta_t} s_{k,t}(\Theta_t, P_t, \Theta_t^-, P_t^-)}\right)
\]

In this setting the difference between the monopoly and oligopoly pricing problem is that the share calculation now also depends on products offered by competitive firms as well. Similar to the

\(^6\)Note we consider a full information not a partial information game
monopoly case we can rewrite this the optimal markup to satisfy the following:

\[
\log(\beta m(\Theta_t, \Theta_t^-, P_t^-) - 1) + (\beta m(\Theta_t, \Theta_t^-, P_t^-)) = \log\left(\sum_{k \in \Theta_t} e^{\delta_{k,t}^- - \beta c_{k,t}}\right) - \log\left(1 + \sum_{k \in \Theta_t^-} e^{\delta_{k,t}^- - \beta p_{k,t}}\right)
\]  

(8)

This equation differs from the monopoly case in two important ways. First, the markup \(m(.)\) now is written as a best response to the competitors’ actions and therefore depends on the assortment and pricing decisions of the competitors. Second, the right hand side of equation 8 has an additional term that captures the effect of the competition. Notice, the difference between the first and second term on the right hand side of this equation: the first only depends on the cost \(c\) where as the second depends on the price \(p\). This is important because we are solving for a best response and not an equilibrium here, therefore we can have the other firms’ decisions (prices and assortments) on the right hand side of the equation. Importantly, we can use the result in proposition 4, as there is no difference between the left hand side of the equation above in the monopoly or oligopoly cases. However, we need to reprove the claim for optimal profits with a given assortments below.

**Claim 6.** In competitive markets, we claim that the retailer’s profit conditional on assortment is given by:

\[
\max_{P_t^-} \sum_{j \in \Theta_t} M_t(p_{j,t} - c_{j,t})s_{j,t}(\Theta_t, P_t, \Theta_t^-, P_t^-) = M_t(m(\Theta_t, \Theta_t^-, P_t^-) - \frac{1}{\beta})
\]

**Proof.** In the appendix

This claim is important as it shows that the best response (profit) function for the retailer is strictly increasing in \(m(\Theta_t, \Theta_t^-, P_t^-)\). Therefore exactly as in the case of the monopoly model (claim 5), we must have the optimal assortment is defined by selecting the \(n_t\) products with the highest \(\delta_{j,t}^- - \beta c_{j,t}\). Notice that the competition effects assortment decision of the retailer in the following way: the space allocated to each category will depend on the level of competition. In an full information simultaneous move game we consider this to a reasonable prediction for a pure strategy equilibrium, as we would expect two identical firms in such a game to have identical assortments. However, if this were a sequential game, we would not expect this to be a stable equilibrium. To describe the Nash Equilibrium in our counterfactual settings, we consider the best response curves for each player in the market and consider explicitly where these best response curves intersect.

### 2.3 Empirical specification

In this section we will describe the methodology for the empirical analysis that translates this analytical model into one that can be estimated with data. In this paper we set up a hierarchical Bayesian model. In our estimation we will estimate separate coefficients for each market and time

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7Note that in our empirical setting we consider grocery stores from large retail chains.
period, this include demand and supply coefficients. As mentioned before this is important as we want to capture local market conditions that drive retail assortment decisions. In the remainder of this section every parameter is market specific. The hierarchy allows us to integrate these local parameters. Detail of the Gibbs sampler are given in the appendix of this paper.

2.4 Local market

In this paper we need to construct an estimator that allows for local market variation from from market/quarter to another. As stated in the introduction, “localization” is important, especially given the restrictions from our logit demand system. Here we introduce an additional index \( m \), to represent a market. A market is defined as a local physical market with multiple stores in a specific quarter.

The remainder of this section specifies demand, marginal cost and assortment models consistent with the analytical derivations so far.

2.4.1 Demand

As described earlier, we will consider a logit demand model in this paper. Here we will assume that the consumers have a utility of the form

\[
 u_{i,j,m} = \alpha_m X_{j,m} - \beta_m p_{j,m} + \xi_{j,m} + \epsilon_{i,j,m}
\]

where \( X_{j,m} \) is the demand characteristics, these include all product attribute information. \( p_{j,m} \) is the price charged by the retailer. In addition to the common assumptions we will assume that the random shocks \( \xi_{j,m} \) are distributed from the Normal distribution with mean 0 and variance \( \sigma_{\xi_m} \). This assumption is exactly similar to the ones made in papers that consider other structural Bayesian approaches to modeling demand (e.g., Yang et al. [2003], Jiang et al. [2007]). This however, is unlike the Berry et al. [1995] and Berry [1994] assumptions, where the researchers assume only that there are some variables \( Z \), that are uncorrelated with the demand shocks (\( \xi \)). However, Jiang et al. [2007] show that the normal assumption is not restrictive and estimate this demand system correctly.

To estimate demand with aggregate data we will use the following equation (for all products \( j \in \Theta_m \), this includes all retailers)

\[
 \log (s_{j,m}) - \log (s_{0,m}) = \alpha_m X_{j,m} - \beta_m p_{j,m} + \xi_{j,m}
\]

(9)

2.4.2 Marginal cost

We parametrize marginal cost as (note this is derived from the optimal markup model)

\[
 c_{j,m,r} = \gamma_m Z_{j,m} + \nu_{j,m} = p_{j,m,r} - \left( \frac{1}{\beta_m} \right) \left( \frac{1}{1 - \sum_{k \in \Theta_{m,r}} s_{k,m,r}(\Theta_{m,r}, \Theta_{m,-r}, P_{m,-r})} \right)
\]

(10)
where the $Z_{j,m}$ variables are possible marginal cost shifters, here we include all product attributes in this equation. $\gamma_m$ describes how these impact marginal cost. The $\nu_{j,m}$ term is a cost shock for product $j$ in market $m$. We assume $\nu_{j,m} \sim N(0, \sigma_{\nu_m})$; this parametrization allows us to estimate the effect of marginal cost shifters. Also the simultaneous modeling of equation 9 and 10 allows us control for endogenous prices.

### 2.4.3 Assortments

In this paper we have shown that the optimal assortment for a retailer consists of selecting the $n_{m,r}$ products with the highest $\delta_{j,m,r} - \beta c_{j,m,r}$. Therefore we must have the following relationship for all products in the choice set $\Omega_{m,r}$:

$$\min_{j \in \Theta_t} \delta_{j,m,r} - \beta m c_{j,m,r} > \max_{k \notin \Theta_t} \delta_{k,m,r} - \beta m c_{k,m,r}$$

(11)

where $\delta_{j,m,r} = \alpha_m X_{j,m,r} + \xi_{j,m,r}$. Notice that in equation 9 conditional on $\beta_m$ we can point identify $\delta_{j,m,r}$ for all $j \in \Theta_{m,r}$ directly from the data (without any error). Similarly in equation 10, conditional on $\beta_m$ we can point identify $c_{j,m,r}$, for all $j \in \Theta_{m,r}$ directly from the data. Therefore conditional on $\beta_m$ we can treat the left hand side of equation 11 as data. Our restriction for all products $k$ not in the market is therefore summarized by

$$\delta_{k,m,r} - \beta m c_{k,m,r} < \min_{j \in \Theta_t} \delta_{j,m,r} - \beta c_{j,m,r}$$

$$\Rightarrow \alpha X_{k,m,r} + \xi_{k,m,r} - \beta m \gamma_m Z_{k,m,r} - \beta m \nu_{k,m,r} < \min_{j \in \Theta_{m,r}} \delta_{j,m,r} - \beta m c_{j,m,r}$$

$$\Rightarrow \xi_{k,m,r} - \beta m \nu_{k,m,r} < \left( \min_{j \in \Theta_{m,r}} \delta_{j,m,r} - \beta c_{j,m,r} \right) - \alpha m X_{k,m,r} + \beta m \gamma_m Z_{k,m,r}$$

(12)

Now notice that equation 12 can be written as a CDF as we know the joint distribution of $\xi_{k,m,r} - \beta m \nu_{k,m,r}$ is $N(0, \sigma_{\xi_m} + \beta^2_m \sigma_{\nu_m})$. Therefore conditional on model parameters we write down the likelihood of those products that are not in the market.

To estimate $F_{m,r}$ we assume that the last product added to the assortment satisfies the profit constraint exactly. In other words, the marginal profit of adding the last product is exactly equal to the $F_{m,r}$. Thus, the marginal profit of adding the last product is given by:

$$\min_{j \notin \Theta_{m,r}} M_{m,r}[m(\Theta_{m,r}, \Theta_{m,-r}, P_{m,-r}) - m(\Theta_{m,r} - j, \Theta_{m,-r}, P_{m,-r})]$$

where $\Theta_{m,r} - j$ refers to the set of product $\Theta_{m,r}$ without product $j$. Notice that our estimation procedure guarantees that for all $k \notin \Theta_{m,r}$, the marginal profit of adding product $k$ to the assortment $\Theta_{m,r}$ is less that $F_{m,r}$. Also note that we estimate a $F_{m,r}$ coefficient for every store in every market. The advantage of the Bayesian setup here is that as we traverse the posterior distribution of the demand and supply parameters, with our Gibbs sampler, we can trace out the entire posterior distribution for $F_{m,r}$.

Details of the Gibbs sampler used for the estimation are available in the appendix of this paper. The key parameterization in the Bayesian analysis is to consider two model parameters $\delta$ and $c$ to be augmented parameters (Tanner and Wong [1987]). The advantage here is that only need
to consider the selection model when drawing $\beta_m$. Also notice that with this simplification the selection model here is similar to a Bayesian probit model (Rossi et al. [2005]).

Finally, to estimate this model we impose a hierarchical structure across all markets for the parameters as follow:

$$\alpha_m \sim N(\bar{\alpha}, \Sigma_\alpha) \quad (13)$$

$$\log(\beta_m) \sim N(\bar{\beta}, \sigma_\beta) \quad (14)$$

$$\gamma_m \sim N(\bar{\gamma}, \Sigma_\gamma) \quad (15)$$

$$F_{m,r} \sim N(F_m + \bar{F}_t, \sigma_F) \quad (16)$$

Notice that in the $\beta$ shrinkage model, we force the distribution to be log normal, this is done to enforce that beta (the price coefficient) is always positive (notice that in the utility we assume the term to be $-\beta$ times Price). Additionally we add fixed effects for markets and quarters in the fixed cost hierarchy to understand the variation in the fixed costs across geographical markets and time period.

3 Data

The data used for this project are IRI movement data across 6 cities for 10 categories. The data include store level movement data, including price and units sold of every UPC each week. The key advantage of these data are that we have detailed attribute descriptions for each UPC in each category. Therefore, for each UPC we observe, detailed product attribute information that allows us to distinguish one UPCs from another. For example, in the canned tuna category, for each UPC we observe the brand, size, type of tuna, cut, packaging, form, color, and the storage liquid (water versus purified water versus oil). These detailed attributes allow us to specify a detailed demand system for consumer preference over these attributes. Additionally we can better understand the supply side of the market by better estimating variation in marginal cost across these parameters. Without these data many UPCs would be indistinguishable, and therefore difficult for the researcher to understand why some UPCs are offered and others are not. These detailed attribute variables are mainly incomplete for private label brands and therefore in this analysis we limit our focus to national brand only.

In the data we observe a sample of 368 stores in 6 different markets, the markets are: Illinois (around Chicago), Massachusetts (around Boston), Washington (around Seattle), California (north and south), Georgia (around Atlanta), and Colorado (around Denver). In each of these markets we observe the weekly store data for each store in the largest grocery chains in the area. We observe these data for 56 weeks from 31st January 2006 till 25th February 2007. We will discuss these markets in further detail when defining our local markets.

8In our data we also observe data for Texas, however were unable to match the address of stores for this market and therefore drop Texas from our analysis.
We observe a wide range of categories: analgesics, canned tuna, coffee, cookies, frozen dinners, ice cream, orange juice (refrigerated), mustard, paper towels and toothpaste. A nice feature of this set of categories is that we observe both shelf-stable categories (e.g., analgesics, tuna, coffee), and refrigerated categories (e.g., orange juice, frozen dinners). In addition, we observe both categories that take up relatively less shelf space (e.g., tuna, mustard) and those that take up more (e.g., paper towels).

In the remainder of this section we will discuss our assumptions for making local markets to study local market competition and then describe each of the categories we study in this paper.

3.1 Local markets

As discussed in the introduction, retail assortments are formed based on two levels of decision-making. First, the headquarters of the chain decides on the products to store in the warehouse and second, the individual retailers decide on the products to store on the store shelves. Our objective here is to study the second of these decisions. However, since we don’t observe the warehouse decision directly we assume the warehouse decision is the union of all products sold at any of the chain’s stores. For example, we assume that the products in Jewel Osco’s warehouse in Chicago is a union of all products sold in the 18 local Jewel Osco stores. The assumptions here are: (1) the warehouse will not store a product not sold in any store, and (2) our sample of stores is wide enough to capture all the variation in the stores. To defend the second of these assumptions, our sample of stores from a chain are a random selection of stores from all local neighborhoods (e.g., the 18 Jewel Osco stores we observe in Chicago cover 5 counties and span 80 miles from North to South and 40 miles from East to West). We believe that, given this variation, we can approximate the products stored in local warehouses for large chains. Restricting our attention to large chains leads us to study the following chains in each of our geographic areas:

- Chicagoland area: Jewel Osco and Dominick’s Finer Foods
- Washington State: Albertsons, Safeway, Quality Food Center, and Fred Meyer
- Massachusetts: Stop & Shop, Shaws & Star, and Hannaford
- Colorado: Safeway, Albertsons, and King Sooper Inc.
- Georgia: Publix Supermarket, and Kroger
- California: Safeway and Albertsons

In this paper we study local market competition between grocery stores in large retail chains. To this end we needed to define local markets where retail stores compete. We mapped all our

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9Note that we use a conservative definition of local markets for two main reasons. First, we only observe a sample of stores; therefore if we observe a Jewel Osco in Chicago without a Dominick’s nearby, this could be because there is not a Dominick’s or there is one and it is not in our sample. In most cases we determined it was the latter; therefore, viewing the Jewel Osco store as a monopolist would create a natural bias. Second, in cases were we do observe a Dominick’s near a Jewel Osco, we observe the entire set of competitors and therefore can define competition cleanly.
368 stores using a mapping software called Mappoint. We then created local markets using the US census tract information for competitive retail stores that were close to one another (within 12/15 minutes driving time) and were not separated by a national highway or geographical factors (e.g., rivers, mountains, etc.). The latter two observations are used to ensure that we make conservative definitions of local markets: for example, even if two grocery stores are a 12/15 minutes drive, if they are separated by a bridge consumers are unlikely to cross the bridge for groceries, therefore we do not consider this to be a local market. Based on these definition we find 34 local markets in all (6 in Illinois, 6 in Washington, 6 in Massachusetts, 4 in Colorado, 5 in Georgia, and 7 in California). Examples of local markets are shown in the appendix. Due to our conservative definition of local markets, 31 of our local markets are duopoly markets and 3 markets (in California) are three-firm markets. To determine the market size, we look at the total population in the the census tracts around the stores in the market. This results in areas with radii of about 2.0 miles for stores in the city (where census tracts are smaller) and about 5.0 to 6.0 miles for stores in the suburbs (where census tracts are larger). We use the total population in each of these local markets times the average consumption of each category to define market size for our later analysis.

We recognize that each retail chain might make these decisions at different time-periods though we need to define a uniform time period for all stores. Most stores make their assortment decisions every quarter. Since we observe one year of data, for purposes of the analysis, we divide it into 4 quarters. We also need to define when a product is sold. Again, to be conservative with this definition, we define a product as sold only if we observe multiple weeks of sales for the UPC in a quarter. We run two forms of robustness checks to ensure that this does not affect our results: first, we consider different definitions of quarters, and second we consider different definitions of products sold. In both cases we do not find similar results for these situations to the ones presented in this paper.

3.2 Categories

In this subsection we will describe the 10 categories in this analysis and provide some descriptive statistics for these categories. A key advantage of these data is that we observe detailed product attribute information for each UPC. This allows us to construct a detailed attribute based demand system. However, in our data, the attribute descriptions are incomplete for private label products, and we restrict our attention to only non-private label (national) brands.

The reason we choose 10 categories is to create generalizable results across these categories to understand the differences in retailer decisions across categories. Table 1 below provides a summary of the total number of UPCs offered in the warehouse and the number chosen by stores. The numbers in the table represent averages across markets and quarters. As we can see there is quite a bit of variation in the number of choices in these categories, from 35 to 625 products. Notice, across the board that the size of the assortment for the retailers is quite large. In analgesics, retailers on average sell 87 distinct UPCs and in toothpaste they sell on average 153 distinct UPCs. Clearly models that consider all combinations to estimate optimal assortments will not be able to
model these decisions.

[Table 1 about here.]

A key feature if our data are that we observe detailed attribute information for each of our 10 retail categories. This information is important for our demand and marginal cost estimates. Details of each categories attribute information can be found in the technical appendix\textsuperscript{10}.

### 3.3 Assortment summaries

In this paper, we assume that retailers make retail assortment decisions every quarter (13-14 weeks), more over that these decisions are made at the individual store level. To support some of these assumptions, we provide some summary statistics below to show that assortment decisions are indeed made by stores and not chains and that these decisions do vary over time.

Table 2 below shows the variation of assortment decisions within stores in the same chain. For this we consider two categories, analgesics and coffee\textsuperscript{11}, and show the number of common UPCs sold in these stores. This table can be read as follows, the 9 in the second row represent the fact that of all the UPCs sold in all Dominick’s Finer Food (DFF) stores in quarter 1, 9 are sold in only 1 of the 6 stores in competitive markets. In this table we can see that while 51 UPCs are sold in all 6 stores, there are 61 UPCs\textsuperscript{12} are not sold in all 6 stores, therefore 54%\textsuperscript{13} are not sold in 5 DFF stores. Across the board we find that more that 50% of UPCs are not sold in all stores. This supports the belief that stores make individual decisions.

[Table 2 about here.]

To consider a more specific example to see the difference in stores in a chain, we consider the case of biscotti UPCs in the cookies category. In our 6 markets in Chicago, 4 are suburban markets, one north side of the city (Lincoln Park) and the final one is on the south side of the city. The first five of these markets are in affluent neighborhoods, the median household income in three suburban neighborhoods and the north Chicago neighborhood is between $60,000 and $74,999, and is between $75,000 and $99,999 in the other suburban neighborhood. In contract, the south side market has a much lower median household income, $25,000 and $29,999. We find that the Jewel Osco stores in the affluent markets store 62% of the biscotti UPCs in the warehouse, whereas the south side Jewel Osco store only sells 19% of these UPCs. We find the same trend in other speciality cookies. For example, the affluent neighborhood stores store 37% of the macaroon cookies, while the south side store sells only 14% of these UPCs. On the flip side, the store on the south side of Chicago stores more cookies (80% of offered UPCs) from McKee Food Corporation, a company known for its value\textsuperscript{14}, while the stores in affluent neighbourhoods store only 24% of these UPCs.

\textsuperscript{10}Please contact the author for the technical appendix

\textsuperscript{11}The trend displayed by both these categories holds for all categories in our data

\textsuperscript{12}61 is calculated as 9 sold in 1 store + 11 sold in 2 stores + 8 sold in 3 stores + 12 sold in 4 stores + 21 sold in 5 stores

\textsuperscript{13}54% is calculated as 61/(61+51)

\textsuperscript{14}McKee Food Corporation is the only manufacturer that claimed value as its brand promise on the website
To show that retail stores do make this decision differently in every quarter, consider Table 3 below. This table shows the total number of unique UPCs sold in each retail store for four chains in our data, in the analgesics category. To read this table consider the first row. It shows that in the first Jewel Osco store, there are a total of 150 analgesics UPCs sold across all quarters, of these 55% (or 82 UPCs) are sold in all four quarters, while the remaining UPCs 45% (or 68 UPCs) are not sold in at least one quarter. Based on this table we infer that on average about 61% of retail store decisions do not vary across quarter, this ranges from 40% to 77% in individual stores.

Note that this percentage does vary by category, one category in which we were surprised to see less variation is canned tuna. Canned tuna does have an increase in demand during the Lent months (in Quarter 1 of our data), however the number of UPCs stored by retailers does not vary too much across time. The percentage of of UPCs common in all quarters for a store in this category is as high as 88% (for one Jewel Osco store in Chicago).

4 Empirical results

In this section we will first present the estimates from all 10 categories, we will then summarize these estimates and key learnings. Estimates for each category are provided in the technical appendix.

4.1 Summary across all categories

Table 4 provides a store level demand estimates across the 10 categories we study. First notice that we an estimate price elasticity in between $-1.63$ and $-3.92$, this range is consistent with prior research. Consistent with common intuition we find the highest price elasticity in categories with the lowest differentiation (e.g. Paper Towels) and the lowest price elasticity in categories with strong preferences (e.g, Toothpaste and Analgesics). Second, across all 10 categories we find a very consistent estimate for the impact of display advertising. Third, for most categories we find that consumers prefer smaller sizes to larger sizes, interestingly this is only reversed in the frozen dinner. Fourth, to show the importance of product attributes we report the most preferred packaging, product form or flavor and manufacturer.

Table 5 provides a store level marginal cost estimates across the 10 categories we study. Standard for markups in retail are in the 30% to 50% range, our estimates are in that range for most categories. Consistent with expections, we find that promotions are associated with a negative cost to the retailer, suggesting that they are partially funded by manufacturers. Interstingly we find

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15While results are shown only for the analgesics category, all other categories follow a similar trend
16Please contact the author for the technical appendix
that the most expensive packaging for the retailer does not match the most preferred packaging for consumers. The best example of this is the sqround (rectangular with round corners), considered an innovation in ice cream packaging is more expensive for retailers though we estimate that consumers prefer the standard round packaging.

The average estimated fixed costs are presented in table 6 below. We notice that for seven of the categories, we observe small estimated fixed costs. Six of these categories, namely Tuna, Coffee, Analgesics, Toothpaste, Mustard, and Cookies, are shelf-stable categories with small package sizes. In particular, the UPCs that represent the lowest selling UPCs are small package sizes. We find that the shelf-stable category with larger package sizes (paper towels) and refrigerated categories (orange juice and frozen dinner) have higher fixed costs. These are higher by an order of magnitude of about 6. This is an intuitive finding, as larger UPCs take up more store shelf space and refrigerated shelf-space is more expensive. The only category that does not follow this intuition is ice cream. We expected ice cream to have a higher fixed costs\footnote{Notice that in the ice cream results section in the appendix, we confirmed that this is an attribute of the data and not of the model imposed on the data}. One reason could be that the retailers views ice cream as a “loss leader” to drive store traffic, and therefore are willing to over-stock ice cream, as it has positive externalities to other categories. Additionally in this table we include the average assortment size and observe that this does not describes the differences in fixed costs across categories. Consider frozen dinners and mustard, they both have similar assortment sizes and price elasticity, and yet have vastly different fixed costs.

Another view of the cross-category effects is to consider the correlation of the fixed cost\footnote{The fixed cost for a store is the average across the quarters} for a store across the 10 categories. Here we find that, in general, the correlations are positive and significant, suggesting there is some underlying store characteristic that impacts all categories. This could be the size of the store that impacts all categories. Interestingly we find that ice cream, coffee, and tuna, are correlated with the most other categories. This could suggests that the assortment sizes for these 3 categories are driven more by store size than the other categories. Also, cookies and analgesics are least correlated with other categories.

4.2 Importance of local markets

In the results section for each category we have shown only the mean posterior estimates of the overall distribution. One advantage of our methodology is that we have posterior estimates for each market and each quarter. For an example of these preference, consider the example from the

\footnote{Notice that in the ice cream results section in the appendix, we confirmed that this is an attribute of the data and not of the model imposed on the data}

\footnote{The fixed cost for a store is the average across the quarters}
data section, where we had observed that for five markets in the Chicagoland area are located in high income neighborhood and the sixth market was in a low income one. We notice that the Jewel Osco store in the less affluent neighborhood sell fewer UPCs of biscotti in the cookies category. We hypothesized that this was due to the inherent preferences for biscotti in this market. Consistent with this hypothesis we estimate the preference for biscotti in a suburban neighborhood is 1.48, while the estimate for the store in the low income area is 0.37. On the flip side we noticed that the less affluent store stored more products from the value manufacturer McKee. In our estimates the mean demand preference for this manufacturer in the less affluent market is 0.62, while the mean preference for McKee in the other five markets is -3.16, -1.27, -3.12, -3.36, and -1.51 respectively. These data and estimate provide some intuition for the fact that local demand preferences play an important role in a retailer’s assortment selection.

The advantage of considering local markets in our analysis is that we capture the local demand preferences, costs and competition differently. When we consider our counterfactual results in the next section we will use these local market estimates to see the impact of changes in markets. From a managerial point of view this allows us to give advice to retailers and manufacturers at the local retail store level.

4.3 Importance of modeling assortment decisions

We estimated our model for three categories without accounting for the assortment decisions made by retailers. In this model we model logit demand, and account for endogenous pricing by simultaneously modeling marginal cost. The only divergence from the complete model is that we do not consider any information from the assortment decisions in our estimation. The three categories where we run our analysis are: paper towels, ice cream and analgesics.

For paper towels, without considering assortment decisions, the average price elasticity estimated as -1.74. This is about half of the -3.93 estimated elasticity in the full model. This estimate implies the profit margins for a retailer in the paper towel category are about 68% which is almost double the estimated margin that of the full model. Therefore we believe our full model does provide better estimates of price elasticity and retail markups. In addition to the price estimate, not considering assortments does impact the other demand and supply estimates.

For the ice cream category we find find a similar result, where the price elasticity is estimated as -1.36, as opposed to -3.17 with the full model. The model without consider assortments to be endogenous results in an estimated profit margin of 76%, which is almost double that of the full model. Again we believe the estimates of the full model are more appropriate in this case.

The third category we choose is a category where our full model estimated a “low” price elasticity and high margin: analgesics. A restricted model estimates the price elasticity -0.90. With the estimated price elasticity less than -1, the estimated profit margins are greater than 100%. Here the estimated profit margins are about 140% implying that retailers are subsidized to sell products. This seems very unlikely and the estimates from the full model are clearly more appropriate in this case.
To understand the managerial impact of not modeling assortment decisions in this setting we consider the decisions retailers would make with the biased estimates. Here we would recommend that retailers in general should increase prices, as consumers are not as sensitive to changes in price. Given the magnitude of the difference between the estimates we believe that modeling assortment decisions is very important to accurately capture consumer preferences.

4.4 Summary of the empirical results

Overall, the estimates of our models across all 10 categories show the following key results. First, with regard to the demand and supply parameters, we believe our model does provide believable estimates, suggesting that we are correctly capturing the key features of these categories. We stress this because the demand and supply are the key inputs to the assortment problem. Second, we find that estimated price elasticity and profit margins are consistent with what is observed in the retailing industry. For seven of our ten estimated categories, we find that price markups are around the 30% to 50% range. Moreover we find that incorrectly estimating demand, without explicitly accounting for assortment decisions, tends to bias price elasticity upwards (less negative). This leads to very high price markups, which can be greater than 100% in some categories. Third, when looking at retailer assortment decisions we show with back of the envelope calculations, that our fixed cost estimates are close to the estimates of retailing monthly rents for space. This suggests that retailers keep adding products to the category until the last product justifies the space needed to store the product. In our counterfactual analysis we will consider the effect of reducing the assortment sizes on retailer profits. Fourth, across all categories, we find that fixed costs depend on two main category characteristics: the size of products in the category, and refrigeration requirements. Our estimates suggest that six categories (Tuna, Analgesics, Mustard, Coffee, Cookies, and Toothpaste) with similar shelf-stable UPCs sizes have similar estimated fixed costs. We find that the paper towels category with larger shelf-stable UPCs have higher fixed costs. And so do the categories (orange juice and frozen dinners) that require refrigeration.\(^\text{19}\)

In the next section we will discuss how we can use these estimates to make suggestions for improving retail assortment decisions.

5 Counterfactual predictions

5.1 Changing current assortments

The analytical model described in this paper allows us to construct optimal assortments to provide normative prediction for retailers. In this section we will show an example of the use of this model for managerial decision making. Here we consider the decision for the Tuna category manager at a Jewel Osco store in Chicago. The category manager currently stores 63 UPCs of tuna on the store shelf and wants to understand the impact of changing this decision on category sales.\(^\text{19}\)

\(^{19}\)The only category where we would have expected higher fixed costs is ice cream, in the technical appendix we show that our estimates for ice cream are driven by the raw data.
To analyze this situation we define the marginal profit of each assortment size as follows. The marginal profit of an assortment of size $n$ is the total profit from the optimal $n$ UPC assortment with optimal pricing minus the total profit from the optimal assortment (with re-optimizing prices) of size $n - 1$. In general there are three effects that we consider when calculating this: first, the customers who now purchase from the category with the expanded assortment, second, the customers who change their purchase decision, and third, the ability of retailers to change prices with large assortments. Notice that by definition in our setup the marginal profit number is always positive.

The figure below (Figure 1) shows the marginal profit for each UPC added in this category. Here we include two lines: the first line (in blue) is the raw data for total revenue from each UPC (sorted from 1 to 63), and the second line (in green) is the estimated marginal profit for each UPC added to the assortment. In this chart we notice that the marginal profit curve is always below the actual revenue, this is by definition as profit accounts for costs. Observe that the marginal profit curve is more concentrated than the revenue curve, in that the first 30% (10 UPCs) can generate 78% of the total profit, while it only accounts for 66% of the revenue. To understand this result, remember that if the retailer sold only 10 UPCs, they can reoptimize the price to charge for these UPCs. This optimal price will be lower than the current price charged by the retailer and so will result in more demand. Notice that the marginal profit curve has a long tail, where 42 of the 63 UPCs have a marginal profit of less than $100 per quarter.

A retailer can use this chart to understand the effect of changes in their assortment on expected profits. In particular, if the retailer takes the view that a UPC must have at least $50 in marginal revenue to be included in the assortment, we can advice which UPCs should be kept and estimate the impact on profit. In this case we observe that the retailer will store 29 UPCs (46%) and can expect to capture 88% of total profits from the category. A question often raised in marketing is understanding the impact of UPC reductions. Here, consider a situation where the category manager wants to understand the effect of reducing the tuna assortment size by 25%. We find that with reoptimizing the prices for the assortment, a reduced assortment can capture 97% of the total category revenue. Overall this does suggest that if a retailer has additional (more profitable) requirements for this store space, she could reduce some UPCs in the tuna category. This result is consistent with the long term\(^\text{20}\) assortment reduction impact on revenue that Sloot et al. [2006] find in an experimental setting. We feel this formulation can be used as a tool to guide retail assortment decisions for retailers in changing environments.

![Figure 1 about here.](image)

To show the importance of the counterfactual prediction, we consider the situation where the retailer just uses the total profit for a given product (rather than the total revenue shown in Figure 1). Here we consider the total profit to be the profit of each UPC given the current assortment.

\[^{20}\text{We compare our results to the long term impact as our results study the steady state equilibrium outcome of the market after a change in assortment size.}\]
Conceptually, there are two key differences between using the marginal profit and actual profit: first, the actual profits do not consider the number of sold units that are not incremental to the category and second, the actual profit does not include the ability for the retailer to change price. Consider a simple example where the current assortment included 2 products A and B, priced at $4 and $5. Currently, we find 100 consumers purchase A and 50 consumers purchase B. Now if B were not in the assortment, some of these 50 consumers would purchase A instead (first effect described above) and the retailer would reduce the price for A to get more customers to purchase A (second effect described above).

The effect of not considering price changes is apparent when assortment sizes are small. Figure H plots the marginal profit and the actual profit. Here we observe the actual profit underestimates (by as much at 30\%) the true marginal profit of UPCs. While Figure 3 shows that for the last few UPCs added to the assortment, the effect of not considering substitution is that the actual profit overestimates the true marginal profit by about 25\%.

To check the robustness of the effect of reducing assortment sizes across category, we consider the paper towel category for the Dominick’s Finer Food store in a Chicago market. The marginal profit curve for this category is shown in Figure 4 below. This chart shows that the marginal profit curve appears more gradual for the paper towel category versus the tuna category. However, consistent with the previous result, here we estimate that reducing the assortment from 28 UPCs to 21 UPCs (a 75\% reduction), will result in only a 3\% loss of total revenue. This change would imply a fixed cost of $150 per quarter for a product to be included in the assortment.

5.2 Importance of competition

To study the importance of competition in this industry, we consider the following thought experiment. We examine a retail market with two competing retailers and then construct a counterfactual setting where both these retailers are optimizing jointly (as a monopolist). The difference between these two situations allows us to understand the importance of competition on assortment and pricing decisions.

For this analysis, we consider two retail stores in a market in Washington. The chosen stores are the Quality Food Center and Fred Meyer. We consider a case where both stores have the same fixed cost. A uniform fixed cost allows us to isolate the effect of competition from effects of different fixed costs. We look at three different levels of fixed costs to understand the importance of competition: First, we consider a low fixed cost of $5, then a medium fixed cost of $50 and finally a high fixed cost of $150. We consider the Coffee category as an example category.
In this market, the Quality Food Center can select their Coffee assortment from 275 UPCs available in the warehouse and Fred Meyer can select from 276 UPCs available in their warehouse. To isolate the effect of competition we do not change the consideration sets for the stores in our counterfactual simulation. In the case of duopoly decisions, we find the competitive Nash equilibrium from the intersection of best response curves (in assortment and prices). Table 8 shows the results for this simulation.

[Table 8 about here.]

Consider the case of $50 fixed costs, in a competitive setting Quality Food Center and Fred Meyer will hold 122 and 111 UPCs respectively in their assortments. Further the two stores will charge markups of $5.17 and $6.06 respectively. We find that if these two stores were maximizing profits jointly we would find that the two stores will hold 97 and 108 UPCs respectively and will charge a markup of $6.74. This represents a 25 UPC reduction from the assortment in the Quality Food Center store and a 3 UPC reduction in the Fred Meyer store. Note that the UPCs stored in the joint optimization case are also stored in the individual store model. The intuition for this result is that the monopolist will charge a higher price when optimizing across the two stores, though now small UPCs in the Quality Food Center store will not have enough demand (therefore profit) to justify the shelf space. Also that reducing the price on these UPCs will lead some customers to switch from higher margin product to lower margin product and therefore will lead to a reduction in total profits.

Consider the effect at the different levels of fixed costs for the two stores. At a low fixed cost ($5) we see only a effect change in the assortment decisions for the two stores. In particular, we find that only two fewer UPCs are sold in the Quality Food Center store in a setting without competition. The intuition here is that at a low fixed cost, the demand threshold for storing a product is low and therefore even with higher prices most UPCs meet this low value. At a high fixed cost ($150) we can see that there is a smaller effect of competition on assortment decisions (as compared to when the fixed cost is $50). This is due to the fact that the smaller the assortment, the more assortment decisions depend on demand. Further we find that at very high fixed costs ($1,000) we simulate that competition does not change assortment decisions. Interestingly, this suggests that the effect of competition on the number of available options is non-monotonic in fixed costs (or allocatable space). On the other hand, considering markups, we notice that the larger the fixed cost the smaller the markup increase without competition.

This simulation also has implications for the study of mergers in the retail industry. We show a merger can reduce the total number of choices available to customers and increase the prices for these products. Both these events reduce consumer welfare for the market. Therefore this analysis suggests that when studying mergers between competing firms, researchers should consider not only consider price implications but also assortment implications.
5.3 Changing manufacturer prices

In our setup we explicitly model the marginal costs for a retailer. These costs are mainly made up of a manufacturer’s wholesale prices. The question we ask in this exercise is: how do changes in wholesale prices effect retailers’ assortment decisions? To study this we consider the toothpaste category in an Albertson’s store in San Francisco. As before, we consider the changes in assortment decisions at various levels of fixed costs (or space constraints). Here we consider Colgate-Pamolive as the focal manufacturer, for whom we vary wholesale prices. We will report our results here in two tables: the first shows the case in which the retailer has space cost and can increase the total number of UPCs stored in the category as long as they justify the cost; the second is where the retailer has a fixed space and needs to pick UPCs to fill this space. The difference between these two cases is that, in the first, an additional UPC can be added to the assortment if it provides sufficient profit, whereas in the second we effectively increase the threshold for adding products to the assortment.

The results of the first simulation are shown in Table 9 below. This table should be read as follows: the rows represent the different levels of fixed costs for the store, and columns represent the wholesale price discounts. For each combination of the fixed costs and discounts, the numbers in the cells are (a) the total number of UPCs Colgate UPCs stored and (b) the total number of other manufacturer UPCs stored in the store. Moving across the columns in a row shows us the effect of wholesale price discounts at each fixed cost level. Here we observe that if the fixed costs are very low (50c), then Colgate can increase their distribution in the store by offering wholesale price discounts. As a matter of fact, a 40% discount increases distribution by 43%. Interestingly most of this increase comes from category expansion and not from substiution. However, we do notice some substitution: for example, in the 40% Colgate discounts, we observe that the retailer does remove one competitor UPC from the assortment. From the second and third rows of this table we notice that under higher fixed costs we get the same effects though we observe fewer Colgate UPCs added to the assortment. Even at $100 fixed costs, with large wholesale discounts, the retailer does include more Colgate UPCs in the assortment without substituting other manufacturers’ UPCs. However at $150 fixed costs, even with a 40% discount, the retailer does not include any additional Colgate UPCs to the assortment. This is because at this cost level, the UPCs in the assortment are selected based on their consumer demand. In summary, this analysis suggests that with a fixed cost of space, wholesale discounts have the largest impact on distribution in large retail stores (where the fixed cost is low) and the smallest impact in small retail stores (where the fixed cost is high). Additionally most of the increase in distribution is from category expansion rather than substitution of other manufacturers.

[Table 9 about here.]

In the second analysis, we consider the total number of UPCs that the retailer can store to be constant (at current) across fixed cost groups. The results from this analysis are presented in table 10 below. To clarify the distinction of this analysis from the previous analysis, under the current
pricing with $0.50 fixed cost we observe that the retailer stores 186 UPCs. We keep this size fixed when studying the effects of reducing wholesale prices. The major difference in this analysis is in the first row, where we estimate that the retailer will now substitute non-Colgate UPCs to add Colgate UPCs when offered wholesale discounts. This shows that with fixed retail space, wholesale prices can have a large impact assortment decisions in stores with large category space allocated.

[Table 10 about here.]

5.4 Manufacturer tying contracts

A question in retailing is understanding the effect of manufacturers tying contracts. A tying contract is one where a manufacturer makes an offer to the retailer to store some group of UPCs or no UPCs at all. For example, Dannon could offer retailers to store a set of flavors of yogurt in their stores. To study the implications of such contracts we take an extreme version of the contract and study the optimal retailer decisions. In this case we assume that all manufacturers offer contracts where the retailer must store all UPCs offered or none of the UPCs offered by that manufacturer. Our purpose in this section is to understand the implications of these contracts for retailers, manufacturers and policy makers. Here we look at a Dominick’s Finer Food (DFF) store in Chicago and study the analgesics category. We choose the analgesics category, as there are only a few manufacturers in this category. To create the optimal assortments with tying contracts we need to consider the full combinatorial problem and cannot use the simplification derived in this paper. The DFF warehouse holds products from Bayer, GlaxoSmithKline, Insight, Johnson and Johnson, Little Drug Store, Little Necessities, Novartis, Polymedica, Upsher-Smith, and Wyeth Labs. We will study the effect of tying contract at 3 levels of fixed cost: low ($10), high ($75), and very high ($150).

The results of this simulation are shown in Table 11 below. This table has the results of the simulation at each of the three different fixed cost levels. The results include the optimal assortment decision in the presence or absence of tying contracts. Considering the low fixed cost ($10), we see that the retailer will just store additional products in their assortments. In this case, tying contracts lead the retailers to be “over assorted”. The intuition behind this is that the cost of space is lower than the missed opportunity of not selling a specific UPC. This has a small effect on retailer profits, we estimate only about 3% lowers profits. This helps all manufacturers as all have their UPCs stored on the shelf. With a high fixed cost ($75), a retailer will now only store products of the largest retailers, here Bayer and Wyeth (and GSK with only three UPCs). In this assortment, the retailer carries the top selling products (these are manufactured by Bayer and Wyeth), however tying contracts force her to also carry all other products offered by these manufacturer. Optimally (without tying contract) the retailer would store some products from the larger manufacturers and some from the smaller manufacturers. This has a large effect on retailer profits, the expected retailer profit are reduced by 53% with tying contract. Therefore tying contracts in this setting affect retailers and small manufacturers adversely. This effect is further magnified with very high fixed costs ($150), where in the presence of tying contracts the
retailer will only store Wyeth UPCs. Note that in this market Wyeth (Aleeve) manufacturers the
top two UPCs in the store. In this case, the retailer’s profits are heavily impacted (reduced about
77%) as the retailer can no longer satisfy consumer demand for other brands (e.g. Advil). Small
manufacturers in this setting do not get any retail distribution for their products.

[Table 11 about here.]

In summary, tying contracts can have one of two effects on retail assortments. First, if fixed
storage costs are low, (high category space) then tying contracts cause a retailer to “over-assort”.
Specifically, they store low selling UPCs of all manufacturers just to satisfy these contracts. In this
setting, the affect on a retailer’s profits is small. Second, when the fixed storage costs are high
(or category space is limited), tying contracts cause the retailer to only store products from large
manufacturers. Therefore a retailer does not satisfy consumer demand for smaller manufacturers’
products. We find this can lead to big profit losses for the retailer. In addition these contracts hurt
small manufacturers as they do not get distribution in retail stores.

6 Summary and conclusions

In this paper we consider the assortment decisions made by retailer category managers. The paper
presents an analytical framework that derives optimal assortment decisions. The key analytical
result is that UPCs can be ranked based on a combination of demand, price sensitivity and marginal
cost. Based on this analytical result we derive an empirical methodology that allows us to study
local retail assortment decisions.

In the empirical analysis, we study assortment decisions for retailers across 119 competitive
markets and 10 categories. Studying the assortment problem across multiple categories allows us
to make generalizations about retailers’ assortment decisions. In our model we estimate a fixed cost
of introducing a product on the retail shelf. This fixed cost represents a minimum threshold profit
that an introduced product must add to category profits. The estimated fixed costs in categories
with small UPCs are about $2 per to $5 UPC per quarter. Back of the envelope calculations suggest
this amounts to about $7 to $20 per square foot of retail space. This estimate is consistent with
trade reports for monthly costs of retail space. This suggests that retailers store products in the
assortment until the profits they add to the category cover the cost of storage. We find that the
category with larger UPCs, paper towels, does have higher fixed costs. And refrigerated categories,
frozen dinners and orange juice, have higher fixed costs. Both these trends are intuitive as they
suggest that the cost of retail space is tied to the size of the UPC and the refrigeration requirements
for the category.

With our analytical model we can predict the impact of changes in assortment sizes on cat-
egory profits. In particular, we describe which UPCs a retailer should store and the prices the
retailer should charge for smaller assortments. In our data, we estimate that a 25% reduction in
total assortment size results in about a 3% reduction in total profits. The prediction that a 25%
assortment reduction does have a large impact on long run profits is consistent with experimental studies from Sloot et al. [2006]. More broadly, the methodology used in this paper can be a powerful tool that retailers can use to decide their assortments across multiple categories. Our model considers competition between retailers in their assortment and pricing decisions. We estimate that competition between retailers can lead to an increase in the total number of UPCs available to consumers and a decrease in prices. Moreover, the effect of competition depends on the space allocated to the category in the retail stores. The effect of competition on assortment decisions varies non-monotonically with space allocated to the category. This can be an important result for policy makers as mergers can have two negative influences on consumer welfare: increase prices and reduce selection.

Understanding retailer assortment decisions is also important for manufacturers in order to increase distribution for their products. Consider wholesale price discounts for a manufacturer; these reduce costs (and increase profits) for a retailer and therefore can impact assortment decisions. We show that such discounts are most effective with retailers who have large spaces allocated to a category. On the other hand retailers with limited space might not change their assortment decisions even with large wholesale discounts. The intuition behind this result is, for retailers with limited space assortment decisions are based on demand. Another tactic that manufacturers can use to increase distribution is to introduce tying contracts, where retailers must store all or none of the UPCs offered. We estimate that these contracts negatively impact both retailers and small manufacturers. We find that these contracts are most damaging when considering retailers with limited category space. These retailers can lose a majority of their profits because they offer restricted assortments. They are also damaging to small manufacturers who have minimal distribution for their products. Consider the example of the analgesics category, with tying contracts a retailer with limited shelf space will not store Advil and Tylenol and therefore will lose consumer demand from these products and manufacturers of these products will not get retail distribution.

An important finding from our empirical estimation is that we show that not accounting for retail assortment decisions will bias demand and supply estimates. This bias is particularly noticeable for the price estimate in the demand system, where we observe that price elasticity is biased upward (or smaller negative numbers) without modeling assortment decisions. This in turn would cause a researcher to infer retail margins are biased upwards, is some cases more than 100%.

In this paper we take a simplified view of consumer demand, where we assume a consumer has a utility for each UPC in each category. There are two refinements to this model that we could consider, first, we could include some cost of making a decision from the assortment in the consumers utility function. There is evidence from the behavioral literature (e.g., Chernev [2003]) that consumers do find it harder to make decisions when faced with large assortments. Adding this term would imply that total category demand is not increasing (with fixed prices) in assortment size. Second, we could consider category shoppers (Cachon and Kok [2006]), where consumers make the store decision based on multiple product categories. Understanding cross category assortments would allow us to consider the optimal use of retail store space for a retailer.
Our current demand system models competition between similar retail or grocery stores. Another possible advancement of this research is to consider a case where retailers are in different price tiers. Here we could consider competition between grocery stores, big box retailers (e.g., Walmart) and specialty retailers (e.g., Wholefoods). The key difference in a model for this set up is that consumers’ utility from purchasing in these stores should have different formations. Another area for further research is considering the dynamics of assortment decisions. Here a retailer could be uncertain of demand for a product and might place the product on the product shelf to learn about unobservable consumer preferences.
References


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\section{A Optimal pricing}

\textbf{Claim 7.} The optimal pricing in the logit is derived as

\[ p_{j,t} - c_{j,t} = \frac{1}{\beta} \frac{1}{1 - \sum_{k \in \Theta_t} s_{k,t}} \frac{1}{s_{k,t}(\Theta_t, P_t)} \]  

(17)

\textit{Proof.} Remember the first order conditions provided the following

\[ s_{j,t} + (p_{j,t} - c_{j,t}) \frac{\partial s_{j,t}}{\partial p_{j,t}} + \sum_{k \neq j} (p_{k,t} - c_{k,t}) \frac{\partial s_{k,t}}{\partial p_{j,t}} = 0 \]

This simplifies as follows

\[ s_{j} + (p_{j,t} - c_{j,t})(-\beta s_{j,t}(1 - s_{j,t})) + \sum_{k \neq j} (p_{k,t} - c_{k,t})(\beta s_{j,t}s_{k,t}) = 0 \]

\[ s_{j} - (p_{j,t} - c_{j,t})\beta s_{j,t} + (p_{j,t} - c_{j,t})(\beta s_{j,t}s_{j,t}) + \sum_{k \neq j} (p_{k,t} - c_{k,t})(\beta s_{j,t}s_{k,t}) = 0 \]

\[ s_{j} - (p_{j,t} - c_{j,t})\beta s_{j,t} + \sum_{k} (p_{k,t} - c_{k,t})(\beta s_{j,t}s_{k,t}) = 0 \]

\[ 1 - (p_{j,t} - c_{j,t})\beta + \sum_{k} (p_{k,t} - c_{k,t})(\beta s_{k,t}) = 0 \]

\[ (p_{j,t} - c_{j,t})\beta = 1 + \beta \sum_{k} (p_{k,t} - c_{k,t})s_{k,t} \]

Now since there is no dependence of \( j \) on the RHS of the above equation, and the above equation is true for all \( j \) then we must have that \( p_{j,t} - c_{j,t} = p_{k,t} - c_{k,t} \) for all \( j, k \). Therefore we must have

\[ (p_{j,t} - c_{j,t})\beta(1 - \sum_{k} s_{k,t}) = 1 \]

\[ (p_{j,t} - c_{j,t}) = \left( \frac{1}{\beta} \right) \left( \frac{1}{1 - \sum_{k} s_{k,t}} \right) \]

\[ \square \]

\section{B Proof of claim 1}

\textbf{Claim:} The optimal markup in this setting can be simplified as below:

\[ \log(\beta m(\Theta_t) - 1) + (\beta m(\Theta_t)) = \log(\sum_{k \in \Theta_t} e^{\delta_{k,t} - \beta c_{k,t}}) \]
Proof. From equation 5, we have

\[ m(\Theta_t) = \left( \frac{1}{\beta} \right) \left( \frac{1}{1 - \sum_{k \in \Theta_t} s_{k,t}(\Theta_t, P_t)} \right) \]

\[ \Rightarrow \beta m(\Theta_t) = \left( \frac{1 + \sum_{l \in \Theta_t} e^{\delta_{k,t} - \beta m(\Theta_t) - \beta c_{k,t}}}{1} \right) \]

\[ \Rightarrow \beta m(\Theta_t) = 1 + \left( \frac{\sum_{l \in \Theta_t} e^{\delta_{k,t} - \beta c_{k,t}}}{e^{\beta m(\Theta_t)}} \right) \]

\[ \Rightarrow e^{\beta m(\Theta_t)}(\beta m(\Theta_t) - 1) = \sum_{l \in \Theta_t} e^{\delta_{k,t} - \beta c_{k,t}} \]

\[ \Rightarrow \beta m(\Theta_t) + \log(\beta m(\Theta_t) - 1) = \log(\sum_{l \in \Theta_t} e^{\delta_{k,t} - \beta c_{k,t}}) \]

\[ \square \]

C Proof of claim 2

The optimal profit can we written as:

\[ \max_{P_t} \sum_{j \in \Theta_t} M_t(p_{j,t} - c_{j,t})s_{j,t}(\Theta_t, P_t) = M_t(m(\Theta_t) - \frac{1}{\beta}) \]

Proof.

\[ \max_{P_t} \sum_{j \in \Theta_t} M_t(p_{j,t} - c_{j,t})s_{j}(\Theta_t, P_t) = \sum_{j \in \Theta_t} M_t(m(\Theta_t))s_{j,t}(\Theta_t) \]

\[ = M_t(m(\Theta_t)) \sum_{j \in \Theta_t} s_{j,t}(\Theta_t) \]

\[ = M_t \frac{1}{\beta} \left( 1 - \sum_{j \in \Theta_t} s_{j,t}(\Theta_t) \right) \sum_{j \in \Theta_t} s_{j,t}(\Theta_t) \]

\[ = M_t \frac{1}{\beta} \sum_{j \in \Theta_t} e^{\delta_{j,t} - \beta (c_{j,t} + m(\Theta_t))} \]

\[ = M_t \frac{1}{\beta} \sum_{j \in \Theta_t} e^{\delta_{j,t} - \beta c_{j,t}} \frac{1}{e^{\beta m(\Theta_t)}} \]

\[ = M_t \frac{1}{\beta} \frac{e^{\beta m(\Theta)}(\beta m(\Theta) - 1)}{1} \]

\[ = M_t(\beta m(\Theta) - \frac{1}{\beta}) \]
This derivation uses equation 5 going from line 2 to line 3 and uses equation 6 when going from line 3 to line 4.

D Proof of claim 3

Claim: The capacity constraint will always be satisfied

Proof. The above is true in general for any demand system that (1) assumes no products has a zero probability of purchase and (2) assumes the share of any product (including the outside option) does not increase if we add a new product to the assortment. This second condition is added to ensure we do not have a consumer leaving the market as the number of products increase.

Assume that the optimal assortment $\Theta_t$ has fewer than $n_t$ products. Consider adding a product $l$ ($l \in \Omega_t$ and $l \notin \Theta_t$). Now add product $l$ to the assortment now adding a new product and setting the price for product $l$ to be $c_{l,t} + \max_{j \in \Theta_t}(p_{j,t} - c_{j,t}) + \epsilon$, with $\epsilon > 0$. Therefore the profit margin of product $l$ is higher than any other product in the market. Under assumption 1, no product is a strictly dominated, so the share of product $l$ is strictly positive (non-zero). Now, there are two forms of substitution that might happen with the introduction of product $l$. First, consumers may switch from purchasing a product in $\Theta_t$ to purchasing product $l$. This substitution is strictly profitable for the firm, as the profit margin for product $l$ is higher than for any other product in $\Theta_t$. Second, consumers may switch from not purchasing to purchasing product $l$, this is clearly profitable for the firm. Therefore $\Theta_t$ could not have been the optimal assortment.

The two conditions required are clearly true for the logit model. With the idiosyncratic error term with infinite support (under the type-1 extreme value distribution), the probability of purchase of any product in always non-zero. And second, clearly the introduction of a new product can only reduce the share of other products in the market. Notice that these two conditions are satisfied by almost all of the standard demand models (e.g., probit).

E Proof of proposition 4

Define a function $f(m) \equiv \log(\beta m - 1) + (\beta m)$ defined from $\left(\frac{1}{\beta}, \infty\right)$ to $R$.

We will show this in three steps: first we will show that the inverse exists, second we will show the inverse is well-behaved and third we will show the inverse is unique.

To observe the existence of the inverse, notice that

- As $m \to \frac{1}{\beta}$, we must have $f(m) \to -\infty$. This is because $\log(\beta m - 1) \to -\infty$ and $\beta m$ is finite (tends to 1).

- $m \to \infty$, we must have $f \to \infty$. Here both $\beta m \to \infty$ and $\log(\beta m - 1) \to \infty$. Though the former tends to $\infty$ faster.

Taken together, these suggests that there exists an inverse function $g(.)$ such that $f(m) = k \Rightarrow g(k) = m$.  

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In more formal terms, we need to ensure that for every finite $k$, the function $f(.)$ has a finite inverse. To prove this, consider the function $h(m) \equiv f(m) - k$. We need to show that $\exists$ a unique finite $m$ with $h(m) = 0$, moreover this is true $\forall$ finite $k$ and $\beta$.

To show this, first define $A = \frac{1}{\beta} \left( 1 + \min(e^{k-3}, 1) \right)$. So we have $h(A) = \log(\beta A - 1) + \beta A - k = \log(\min(e^{k-3}, 1)) + 1 + \min(e^{k-3}, 1) - k \leq \log(e^{k-3}) + 1 + 1 - k = -1 < 0$. Therefore $h(A) < 0$.

Second, define $B = \frac{1}{\beta} (1 + \max(1, k))$. So we have $h(B) = \log(\beta B - 1) + \beta B - k = \log(\max(1, k)) + 1 + \max(1, k) - k \geq \log(1) + 1 + k - k = 1 > 0$. Thus, $h(B) > 0$.

The function $h(.)$ is clearly continuous; therefore, by the implicit function theorem we have the required inversion.

To complete the proof we need to show uniqueness. First, consider the fact that $\frac{\partial f}{\partial m} = \frac{\beta}{\beta m - 1} + \beta > 0$ as the function is defined for $m > \frac{1}{\beta}$. Therefore the function $f(.)$ is strictly increasing in $m$; thus $h(.)$ is strictly increasing in $m$. For all $m < A$, by construction we must have $h(m) < 0$ as $h(A) < 0$ and $\forall \ m < A, \ h(m) < h(A)$. Similarly, for all $m > B$, by construction we must have $h(m) > 0$. Therefore we cannot have $h(m) = 0$ for any $m \notin [A, B]$ Finally, since $h(.)$ is increasing in the interval $[A, B]$ we must have there only one solution.

Moreover since $f(.)$ is strictly increasing we must have the inverse $g(.)$ is strictly increasing.

This proves this claim that there is a unique $m(.)$ that satisfies equation 6 and moreover the higher the value of $log(\sum_{k \in \Theta} e^{\delta_{k,t} - \beta c_{k,t}})$ the higher the $m(.)$ that satisfies this equation.

### F Proof of claim 5

**Proof.** The importance of the previous claim is that we can now know that the solution to the problem

$$\max_{\Theta_t \subseteq \Omega_t} M_t(m(\Theta_t) - \frac{1}{\beta})$$

Subject to

$$|\Theta_t| \leq n_t$$

is the same as the solution to the problem

$$\max_{\Theta_t \subseteq \Omega_t} \sum_{k \in \Theta_t} e^{\delta_{k,t} - \beta c_{k,t}}$$

Subject to

$$|\Theta_t| \leq n_t$$

as profits are a strictly increasing function of $\sum_{k \in \Theta_t} e^{\delta_{k,t} - \beta c_{k,t}}$.

Now, suppose that picking the $n_t$ products with the highest $\delta_{k,t} - \beta c_{k,t}$ was not the optimal solution. It must be the case that $\exists \ j \in \Theta_t$ and $l \notin \Theta_t$ with $\delta_{l,t} - \beta c_{l,t} > \delta_{j,t} - \beta c_{j,t}$. However, we
have
\[
\sum_{k \in \Theta_t, k \neq j} e^{\delta_{k,t} - \beta c_{k,t}} + e^{\delta_{j,t} - \beta c_{j,t}} > \sum_{k \in \Theta_t, k \neq j} e^{\delta_{k,t} - \beta c_{k,t}} + e^{\delta_{j,t} - \beta c_{j,t}}
\]
\[
= \sum_{k \in \Theta_t} e^{\delta_{k,t} - \beta c_{k,t}}
\]
which contradicts the optimality of \(\Theta_t\).

\[\square\]

**G  Proof of claim 6**

**Proof.**

\[
\max_{P_t} \sum_{j \in \Theta_t} M_t(p_{j,t} - c_{j,t})s_j(\Theta_t, P_t, \Theta_t^-, P_t^-) = \sum_{j \in \Theta_t} M_t(m(\Theta_t, \Theta_t^-, P_t^-))s_j(\Theta_t, \Theta_t^-, P_t^-)
\]
\[
= M_t(m(\Theta_t, \Theta_t^-, P_t^-)) \sum_{j \in \Theta_t} s_j(t(\Theta_t, \Theta_t^-, P_t^-)
\]
\[
= M_t \frac{1}{\beta} \left( \frac{1}{1 - \sum_{j \in \Theta_t} s_j(t(\Theta_t, \Theta_t^-, P_t^-)) \sum_{j \in \Theta_t} s_j(t(\Theta_t, \Theta_t^-, P_t^-))} \right)
\]
\[
= M_t \frac{1}{\beta} \left( \frac{\sum_{j \in \Theta_t} e^{\delta_{j,t} - \beta c_{j,t}}}{1 + \sum_{k \in \Theta_t} e^{\delta_{k,t} - \beta c_{k,t}}} \right) \left( \frac{1}{1 + \sum_{k \in \Theta_t} e^{\delta_{k,t} - \beta c_{k,t}}} \right)
\]
\[
= M_t \frac{1}{\beta} \left( \frac{\sum_{j \in \Theta_t} e^{\delta_{j,t} - \beta c_{j,t}}}{1 + \sum_{k \in \Theta_t} e^{\delta_{k,t} - \beta c_{k,t}}} \right) \left( \frac{1}{e^{\beta m(\Theta_t, \Theta_t^-, P_t^-)} - 1} \right)
\]
\[
= M_t \frac{e^{\beta m(\Theta_t, \Theta_t^-, P_t^-)}(\beta m(\Theta_t, \Theta_t^-, P_t^-) - 1)}{e^{\beta m(\Theta_t, \Theta_t^-, P_t^-)} - 1}
\]
\[
= M_t \left( \frac{\beta m(\Theta_t, \Theta_t^-, P_t^-) - 1}{\beta} \right)
\]
\[\square\]

**H  Gibbs sampler**

In this estimation we will consider two augmented parameters, for each product in the consideration set, they are \(\delta_{j,t}\) and \(c_{j,t}\). For all products in the assortment can do the following:
Calculate base markup is given by
\[
m(\Theta_t) = \left(\frac{1}{\beta}\right) \left(\frac{1}{1 - \sum_{l \in \Theta_t} s_{l,t}}\right)
\]

Calculate the costs
\[
c_{j,t} = p_{j,t} - m(\Theta_t)
\]

Calculate the \(\delta_{j,t}\)
\[
\delta_{j,t} = \log(s_{j,t}) - \log(s_{0,t}) + \beta p_{j,t}
\]

Calculate the counter factual markup \((m(\Theta_t - j))\) by inverting the relation
\[
\log(\beta m(\Theta_t - j) - 1) + (\beta m(\Theta_t - j)) = \log(\sum_{k \in \Theta_t} e^{\delta_{k,t} - \beta c_{k,t}}) - \log(1 + \sum_{k \in \Theta_t} e^{\delta_{k,t} - \beta c_{k,t}})
\]

This in turn allows us to calculate the minimum profit of each added product \((F)\)

Posterior draws:
- draw \(\beta|\text{Prior, } \alpha, \gamma, \delta_k(\forall k \notin \Theta_t), \sigma_\xi, c_{k,t}(\forall k \notin \Theta_t)\)
  - Method: Metropolis
  - Prior given by \(\log(\beta) \sim N(\bar{\beta}, A^{-1})\)
  - Likelihood for products in the market given by two equations, first from demand we have
    \[
P(\delta_{j,t} = \log(s_{j,t}) - \log(s_{0,t}) + \beta p_{j,t})
    \delta_{j,t} \sim N(X_{j,t} \alpha, \sigma_\xi^2)
    \]
    Second, from marginal cost we have
    \[
P(c_{j,t} = p_{j,t} - m(\Theta_t))
    c_{j,t} \sim N(\mu Z_{j,t}, \sigma_\mu^2)
    \]
    Additionally we know have information from the inclusion equations for products in/out of the assortment. \(\forall k \notin \Theta_t\)
    \[
    \delta_{k,t} - \beta * c_{k,t} < \min_{j \in \Theta_t} \delta_{j,t} - \beta * c_{j,t}
    \]
    This acts like a CDF as we know \(\delta_{k,t} - \beta * c_{k,t} \sim N(X_{j,t} \alpha - \beta \mu Z_{j,t}, \sigma_\xi^2 + \beta^2 \sigma_\mu^2)\)

39
– Note that here we also estimate \( \delta_{j,t}(\forall k \notin \Theta_t) \) and \( c_{j,t}(\forall k \notin \Theta_t) \)

- draw \( \delta_{k,t}, c_{k,t}(\forall k \notin \Theta_t) | (\alpha, \beta), \sigma_\xi, \sigma_\mu, \delta_{j,t}, c_{j,t} \)
  - This is like drawing the latent utility in a probit model (see Tanner and Wong [1987] and Rossi et al. [2005])
  - Method: Giibs
  - “Prior” given by \( \delta_{k,t} \sim N(X_{j,t}, \alpha, \sigma_\xi^2), c_{k,t} \sim N(\gamma Z_{j,t}, \sigma_\mu^2) \)
  - The posterior draw is based on fact that

\[
\delta_{k,t} - \beta * c_{k,t} < \min_{j \in \Theta_t} \delta_{j,t} - \beta * c_{j,t}
\]

- draw \( \gamma, \sigma_\mu^2 | c_{.,t} \)
  - Method: Gibbs - linear regression (see Rossi et al. [2005])

- draw \( \alpha, \sigma_\xi^2 | \delta_{.,t} \)
  - Method: Gibbs - linear regression (see Rossi et al. [2005])

- Once we have estimated the parameters for each market we estimate the hierarchical parameters (see Rossi et al. [2005])
Figure 1: Chart represents the marginal profit and the total (current) revenue for each of the 63 UPCs currently stored in the Tuna category at a Jewel Osco store.
Figure 2: Marginal profit for each product added in Tuna category in Jewel Osco Chicago first 9 UPCs
Figure 3: Marginal profit for each product added in Tuna category in Jewel Osco Chicago last 12 UPCs
Figure 4: Chart represents the marginal profit for each of the 28 UPCs currently stored in the Paper towel category at a DFF (Chicago) store.
<table>
<thead>
<tr>
<th>Attribute</th>
<th>Products sold</th>
<th>Products available</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analgesics</td>
<td>87.33</td>
<td>133.62</td>
</tr>
<tr>
<td>Canned Tuna</td>
<td>52.39</td>
<td>65.41</td>
</tr>
<tr>
<td>Coffee</td>
<td>189.97</td>
<td>300.69</td>
</tr>
<tr>
<td>373</td>
<td>09.625</td>
<td>88.</td>
</tr>
<tr>
<td>Ice Cream</td>
<td>208.05</td>
<td>253.39</td>
</tr>
<tr>
<td>Frozen Dinner</td>
<td>46.13</td>
<td>57.50</td>
</tr>
<tr>
<td>Mustard</td>
<td>48.05</td>
<td>74.49</td>
</tr>
<tr>
<td>Orange Juice</td>
<td>48.43</td>
<td>52.57</td>
</tr>
<tr>
<td>Paper Towels</td>
<td>27.30</td>
<td>35.52</td>
</tr>
<tr>
<td>Toothpaste</td>
<td>153.20</td>
<td>217.20</td>
</tr>
</tbody>
</table>

Table 1: Products offered in each category
<table>
<thead>
<tr>
<th>Number of stores</th>
<th>Quarter</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Analgesics category</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DFF, Chicago (6 stores)</td>
<td>0</td>
<td>6</td>
<td>13</td>
<td>15</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>9</td>
<td>8</td>
<td>11</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>11</td>
<td>5</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8</td>
<td>9</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>12</td>
<td>15</td>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>21</td>
<td>16</td>
<td>13</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>51</td>
<td>50</td>
<td>43</td>
<td>45</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>118</td>
<td>116</td>
<td>112</td>
<td>94</td>
</tr>
<tr>
<td>% not in all</td>
<td></td>
<td>54%</td>
<td>51%</td>
<td>56%</td>
<td>46%</td>
</tr>
<tr>
<td><strong>Alberstons, Seattle (4 stores)</strong></td>
<td>0</td>
<td>23</td>
<td>26</td>
<td>39</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>28</td>
<td>32</td>
<td>33</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>24</td>
<td>24</td>
<td>49</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>45</td>
<td>36</td>
<td>33</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>24</td>
<td>30</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>144</td>
<td>148</td>
<td>154</td>
<td>138</td>
</tr>
<tr>
<td>% not in all</td>
<td></td>
<td>80%</td>
<td>75%</td>
<td>71%</td>
<td>69%</td>
</tr>
<tr>
<td><strong>Shop and shop, MA (5 stores)</strong></td>
<td>0</td>
<td>13</td>
<td>12</td>
<td>13</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>22</td>
<td>20</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>18</td>
<td>16</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>10</td>
<td>13</td>
<td>5</td>
<td>12</td>
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<td>22</td>
<td>22</td>
<td>19</td>
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<td></td>
<td>5</td>
<td>31</td>
<td>31</td>
<td>39</td>
<td>32</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>114</td>
<td>114</td>
<td>114</td>
<td>111</td>
</tr>
<tr>
<td>% not in all</td>
<td></td>
<td>69%</td>
<td>70%</td>
<td>61%</td>
<td>67%</td>
</tr>
<tr>
<td><strong>Coffee category</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jewel Osco, Chicago (6 stores)</td>
<td>0</td>
<td>65</td>
<td>51</td>
<td>66</td>
<td>52</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>36</td>
<td>39</td>
<td>56</td>
<td>49</td>
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<td></td>
<td>2</td>
<td>8</td>
<td>6</td>
<td>14</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>8</td>
<td>11</td>
<td>18</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>25</td>
<td>28</td>
<td>20</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>50</td>
<td>70</td>
<td>53</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>146</td>
<td>115</td>
<td>145</td>
<td>146</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>175</td>
<td>177</td>
<td>172</td>
<td>160</td>
</tr>
<tr>
<td>% not in all</td>
<td></td>
<td>47%</td>
<td>57%</td>
<td>53%</td>
<td>49%</td>
</tr>
</tbody>
</table>

Table 2: Variation in assortment across stores within a chain. Numbers represent the number of common products sold, therefore the 6 in the first row of the data represents the fact that 6 UPCs held in any DFF store in Chicago in quarter 1 are not sold in any of the 6 stores we consider. The 9 in the second row, represents the fact that 9 UPCs are sold in only one of the DFF stores.
<table>
<thead>
<tr>
<th>Store</th>
<th>Number of UPCs sold</th>
<th>% of UPCs same in all quarters</th>
<th>% of UPCs different across quarters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jewel Osco, Chicago</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Store 1</td>
<td>150</td>
<td>55%</td>
<td>45%</td>
</tr>
<tr>
<td>Store 2</td>
<td>179</td>
<td>60%</td>
<td>40%</td>
</tr>
<tr>
<td>Store 3</td>
<td>166</td>
<td>73%</td>
<td>27%</td>
</tr>
<tr>
<td>Store 4</td>
<td>173</td>
<td>73%</td>
<td>27%</td>
</tr>
<tr>
<td>Store 5</td>
<td>185</td>
<td>66%</td>
<td>34%</td>
</tr>
<tr>
<td>Store 6</td>
<td>173</td>
<td>64%</td>
<td>36%</td>
</tr>
<tr>
<td>DFF, Chicago</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Store 1</td>
<td>110</td>
<td>65%</td>
<td>35%</td>
</tr>
<tr>
<td>Store 2</td>
<td>97</td>
<td>53%</td>
<td>47%</td>
</tr>
<tr>
<td>Store 3</td>
<td>107</td>
<td>59%</td>
<td>41%</td>
</tr>
<tr>
<td>Store 4</td>
<td>104</td>
<td>62%</td>
<td>38%</td>
</tr>
<tr>
<td>Store 5</td>
<td>107</td>
<td>63%</td>
<td>37%</td>
</tr>
<tr>
<td>Store 6</td>
<td>104</td>
<td>40%</td>
<td>60%</td>
</tr>
<tr>
<td>Alberstons, Seattle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Store 1</td>
<td>58</td>
<td>43%</td>
<td>57%</td>
</tr>
<tr>
<td>Store 2</td>
<td>109</td>
<td>67%</td>
<td>33%</td>
</tr>
<tr>
<td>Store 3</td>
<td>117</td>
<td>60%</td>
<td>40%</td>
</tr>
<tr>
<td>Store 4</td>
<td>134</td>
<td>57%</td>
<td>43%</td>
</tr>
<tr>
<td>Shop and shop, MA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Store 1</td>
<td>113</td>
<td>77%</td>
<td>23%</td>
</tr>
<tr>
<td>Store 2</td>
<td>85</td>
<td>68%</td>
<td>32%</td>
</tr>
<tr>
<td>Store 3</td>
<td>72</td>
<td>57%</td>
<td>43%</td>
</tr>
<tr>
<td>Store 4</td>
<td>98</td>
<td>70%</td>
<td>30%</td>
</tr>
<tr>
<td>Store 5</td>
<td>72</td>
<td>63%</td>
<td>38%</td>
</tr>
</tbody>
</table>

Table 3: Variation in assortment across quarters within a chain
<table>
<thead>
<tr>
<th>Category</th>
<th>Price Elasticity</th>
<th>Display Size</th>
<th>Packaging Type</th>
<th>Manuf</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analgesics</td>
<td>-1.63*</td>
<td>0.27*</td>
<td>Plastic Box</td>
<td>Wyeth</td>
</tr>
<tr>
<td>Canned Tuna</td>
<td>-3.31*</td>
<td>0.30*</td>
<td>Vac. pouch</td>
<td>Bumble Bee</td>
</tr>
<tr>
<td>Coffee</td>
<td>-3.30*</td>
<td>0.34*</td>
<td>Box</td>
<td>Sara Lee</td>
</tr>
<tr>
<td>Cookies</td>
<td>-2.82*</td>
<td>0.30*</td>
<td>Tin box</td>
<td>Campbell</td>
</tr>
<tr>
<td>Frozen Dinner</td>
<td>-2.02*</td>
<td>0.35*</td>
<td>non sig</td>
<td>Perdue</td>
</tr>
<tr>
<td>Ice Cream</td>
<td>-3.18*</td>
<td>0.35*</td>
<td>Round</td>
<td>Ben &amp; Jerry</td>
</tr>
<tr>
<td>Mustard</td>
<td>-2.36*</td>
<td>0.30*</td>
<td>non sig</td>
<td>Altra, Unilever</td>
</tr>
<tr>
<td>Orange Juice</td>
<td>-1.71*</td>
<td>0.20*</td>
<td>Bottle</td>
<td>Tropicana</td>
</tr>
<tr>
<td>Paper Towels</td>
<td>-3.92*</td>
<td>0.23*</td>
<td>Mega</td>
<td>P &amp; G</td>
</tr>
<tr>
<td>Toothpaste</td>
<td>-1.92*</td>
<td>0.30*</td>
<td>non sig</td>
<td>J &amp; J</td>
</tr>
</tbody>
</table>

Table 4: Summary of demand estimates across categories.
<table>
<thead>
<tr>
<th>Category</th>
<th>Percentage</th>
<th>Markup</th>
<th>Display</th>
<th>Size</th>
<th>Packaging</th>
<th>Type</th>
<th>Manuf</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analgesics</td>
<td>77%</td>
<td>non sig</td>
<td>non sig</td>
<td>non sig</td>
<td>Plastic Box</td>
<td>non sig</td>
<td>non sig</td>
</tr>
<tr>
<td>Canned Tuna</td>
<td>36%</td>
<td>-0.21*</td>
<td>neg</td>
<td></td>
<td>Vacuum pouch</td>
<td>Albacore</td>
<td>Bumble Bee</td>
</tr>
<tr>
<td>Coffee</td>
<td>41%</td>
<td>-0.30*</td>
<td>neg</td>
<td></td>
<td>Box</td>
<td>Coffee Subsitute</td>
<td>P &amp; G</td>
</tr>
<tr>
<td>Cookies</td>
<td>45%</td>
<td>0.30*</td>
<td>neg</td>
<td></td>
<td>Tin box</td>
<td>Biscotti</td>
<td>Campbell</td>
</tr>
<tr>
<td>Frozen Dinner</td>
<td>56%</td>
<td>non sig</td>
<td>neg</td>
<td></td>
<td>Vacuum pack</td>
<td>Seasonal</td>
<td>Perdue</td>
</tr>
<tr>
<td>Ice Cream</td>
<td>39%</td>
<td>non sig</td>
<td>neg</td>
<td></td>
<td>Sqround</td>
<td>non sig</td>
<td>Haagendaz</td>
</tr>
<tr>
<td>Mustard</td>
<td>50%</td>
<td>-0.19*</td>
<td>neg</td>
<td></td>
<td>Glass</td>
<td>non sig</td>
<td>Uniliver</td>
</tr>
<tr>
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<td>78%</td>
<td>non sig</td>
<td>non sig</td>
<td>non sig</td>
<td>non sig</td>
<td>non sig</td>
<td>non sig</td>
</tr>
<tr>
<td>Paper Towels</td>
<td>32%</td>
<td>non sig</td>
<td>neg</td>
<td></td>
<td>non sig</td>
<td>Max Strength</td>
<td>Kimberly</td>
</tr>
<tr>
<td>Toothpaste</td>
<td>68%</td>
<td>-0.70*</td>
<td>neg</td>
<td></td>
<td>Tube</td>
<td>Mint</td>
<td>Gerber</td>
</tr>
</tbody>
</table>

Table 5: Summary of marginal cost estimates
<table>
<thead>
<tr>
<th>CATEGORY</th>
<th>STORAGE</th>
<th>FIXED COST</th>
<th>ASSORTMENT SIZE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analgesics</td>
<td>Shelf (small UPC)</td>
<td>$2.67</td>
<td>87</td>
</tr>
<tr>
<td>Canned Tuna</td>
<td>Shelf (small UPC)</td>
<td>$4.83</td>
<td>87</td>
</tr>
<tr>
<td>Coffee</td>
<td>Shelf (small UPC)</td>
<td>$3.38</td>
<td>190</td>
</tr>
<tr>
<td>Cookies</td>
<td>Shelf (small UPC)</td>
<td>$1.30</td>
<td>373</td>
</tr>
<tr>
<td>Frozen Dinner</td>
<td>Refrigerated</td>
<td>$10.03</td>
<td>46</td>
</tr>
<tr>
<td>Ice cream</td>
<td>Refrigerated</td>
<td>$2.68</td>
<td>209</td>
</tr>
<tr>
<td>Mustard</td>
<td>Shelf (small UPC)</td>
<td>$1.89</td>
<td>48</td>
</tr>
<tr>
<td>Orange Juice</td>
<td>Refrigerated</td>
<td>$14.09</td>
<td>48</td>
</tr>
<tr>
<td>Paper towels</td>
<td>Shelf (large UPC)</td>
<td>$13.59</td>
<td>27</td>
</tr>
<tr>
<td>Toothpaste</td>
<td>Shelf (small UPC)</td>
<td>$2.50</td>
<td>153</td>
</tr>
</tbody>
</table>

Table 6: Summary of fixed cost estimates
<table>
<thead>
<tr>
<th>Category</th>
<th>Ice Cream</th>
<th>Coffee</th>
<th>Tuna</th>
<th>Dinner</th>
<th>Orange juice</th>
<th>Toothpaste</th>
<th>Paper towels</th>
<th>Mustard</th>
<th>Cookie</th>
<th>Analgesics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice cream</td>
<td>1.00*</td>
<td>0.36*</td>
<td>0.37*</td>
<td>0.34*</td>
<td>0.29*</td>
<td>0.26*</td>
<td>0.36*</td>
<td>0.34*</td>
<td>0.19</td>
<td>0.22</td>
</tr>
<tr>
<td>Coffee</td>
<td>0.36*</td>
<td>1.00*</td>
<td>0.40*</td>
<td>0.37*</td>
<td>0.21</td>
<td>0.28*</td>
<td>0.24*</td>
<td>0.09</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Tuna</td>
<td>0.37*</td>
<td>0.40*</td>
<td>1.00*</td>
<td>0.48*</td>
<td>0.39*</td>
<td>0.23</td>
<td>0.30*</td>
<td>0.25*</td>
<td>0.23</td>
<td>0.09</td>
</tr>
<tr>
<td>Frozen</td>
<td>0.34*</td>
<td>0.37*</td>
<td>0.48*</td>
<td>1.00*</td>
<td>-0.04</td>
<td>0.39*</td>
<td>0.11</td>
<td>0.42*</td>
<td>0.23</td>
<td>0.10</td>
</tr>
<tr>
<td>Orange Juice</td>
<td>0.29*</td>
<td>0.21</td>
<td>0.39*</td>
<td>-0.04</td>
<td>1.00*</td>
<td>0.19</td>
<td>0.52*</td>
<td>-0.07</td>
<td>0.04</td>
<td>0.24*</td>
</tr>
<tr>
<td>Toothpaste</td>
<td>0.26*</td>
<td>0.28*</td>
<td>0.23</td>
<td>0.39*</td>
<td>0.19</td>
<td>1.00*</td>
<td>-0.07</td>
<td>0.12</td>
<td>0.28*</td>
<td>0.13</td>
</tr>
<tr>
<td>Paper towels</td>
<td>0.36*</td>
<td>0.24*</td>
<td>0.30*</td>
<td>0.11</td>
<td>0.52*</td>
<td>-0.07</td>
<td>1.00*</td>
<td>0.07</td>
<td>0.02</td>
<td>0.08</td>
</tr>
<tr>
<td>Mustard</td>
<td>0.34*</td>
<td>0.09</td>
<td>0.25*</td>
<td>0.42*</td>
<td>-0.07</td>
<td>0.12</td>
<td>0.07</td>
<td>1.00*</td>
<td>0.15</td>
<td>0.00</td>
</tr>
<tr>
<td>Cookies</td>
<td>0.19</td>
<td>0.10</td>
<td>0.23</td>
<td>0.23</td>
<td>0.04</td>
<td>0.28*</td>
<td>0.02</td>
<td>0.15</td>
<td>1.00*</td>
<td>0.17</td>
</tr>
<tr>
<td>Analgesics</td>
<td>0.22</td>
<td>0.10</td>
<td>0.09</td>
<td>0.10</td>
<td>0.24*</td>
<td>0.13</td>
<td>0.08</td>
<td>0.00</td>
<td>0.17</td>
<td>1.00*</td>
</tr>
</tbody>
</table>

Table 7: Correlations of fixed costs
<table>
<thead>
<tr>
<th>Fixed Cost</th>
<th>Variable</th>
<th>Duopoly Store 1</th>
<th>Duopoly Store 2</th>
<th>Monopoly Store 1</th>
<th>Monopoly Store 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5</td>
<td>Number of UPCs sold</td>
<td>208</td>
<td>187</td>
<td>206</td>
<td>187</td>
</tr>
<tr>
<td></td>
<td>Markup</td>
<td>$5.23</td>
<td>$6.10</td>
<td>$6.91</td>
<td></td>
</tr>
<tr>
<td>$50</td>
<td>Number of UPCs sold</td>
<td>122</td>
<td>111</td>
<td>97</td>
<td>108</td>
</tr>
<tr>
<td></td>
<td>Markup</td>
<td>$5.17</td>
<td>$6.06</td>
<td>$6.74</td>
<td></td>
</tr>
<tr>
<td>$150</td>
<td>Number of UPCs sold</td>
<td>45</td>
<td>47</td>
<td>31</td>
<td>47</td>
</tr>
<tr>
<td></td>
<td>Markup</td>
<td>$4.97</td>
<td>$5.93</td>
<td>$6.37</td>
<td></td>
</tr>
<tr>
<td>$1,000</td>
<td>Number of UPCs sold</td>
<td>1</td>
<td>12</td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Markup</td>
<td>$4.54</td>
<td>$5.62</td>
<td>$5.68</td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Understanding the effect of competition on assortment decisions in coffee
<table>
<thead>
<tr>
<th>Fixed Cost</th>
<th>UPCs by</th>
<th>Colgate wholesale discounts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Colgate</td>
<td>10%</td>
</tr>
<tr>
<td>$0.50</td>
<td>Colgate</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>Other manuf</td>
<td>142</td>
</tr>
<tr>
<td>$8.00</td>
<td>Colgate</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>Other manuf</td>
<td>57</td>
</tr>
<tr>
<td>$30.00</td>
<td>Colgate</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>Other manuf</td>
<td>32</td>
</tr>
<tr>
<td>$100.00</td>
<td>Colgate</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Other manuf</td>
<td>12</td>
</tr>
<tr>
<td>$150.00</td>
<td>Colgate</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Other manuf</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 9: Understanding the effect of wholesale discounts, with cost constraint and no space constraint, table contains the number of UPCs stored
<table>
<thead>
<tr>
<th>Fixed Cost</th>
<th>UPCs by</th>
<th>Colgate wholesale discounts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Current</td>
<td>10%</td>
</tr>
<tr>
<td>$0.50</td>
<td>Colgate</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>Other manuf</td>
<td>142</td>
</tr>
<tr>
<td>$8.00</td>
<td>Colgate</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>Other manuf</td>
<td>57</td>
</tr>
<tr>
<td>$30.00</td>
<td>Colgate</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>Other manuf</td>
<td>32</td>
</tr>
<tr>
<td>$100.00</td>
<td>Colgate</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Other manuf</td>
<td>12</td>
</tr>
<tr>
<td>$150.00</td>
<td>Colgate</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Other manuf</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 10: Understanding the effect of wholesale discounts, with cost constraint and with space constraint, table contains the number of UPCs stored.
<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>UPC offered</th>
<th>Fixed cost $10</th>
<th>Fixed cost $75</th>
<th>Fixed cost $150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bayer</td>
<td>28</td>
<td>82% √</td>
<td>43% √</td>
<td>25%</td>
</tr>
<tr>
<td>Glaxo Smith-Kline</td>
<td>3</td>
<td>100% √</td>
<td>33% √</td>
<td>33%</td>
</tr>
<tr>
<td>Insight</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Johnson and Johnson</td>
<td>43</td>
<td>72% √</td>
<td>35%</td>
<td>14%</td>
</tr>
<tr>
<td>Little drug store</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Little necessities</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Novartis</td>
<td>22</td>
<td>77% √</td>
<td>41%</td>
<td>9%</td>
</tr>
<tr>
<td>Polymedica</td>
<td>1</td>
<td>100% √</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Upsher-Smith</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wyeth Labs</td>
<td>15</td>
<td>53% √</td>
<td>53% √</td>
<td>20% √</td>
</tr>
<tr>
<td>Total UPCs</td>
<td>118</td>
<td>83 112</td>
<td>45 46</td>
<td>19 15</td>
</tr>
<tr>
<td>Total profits</td>
<td>$9,904</td>
<td>$9,637</td>
<td>$5,828</td>
<td>$2,731</td>
</tr>
<tr>
<td>% of optimal profits</td>
<td>97%</td>
<td>47%</td>
<td>23%</td>
<td></td>
</tr>
</tbody>
</table>

Table 11: Understanding the effect of tying contracts