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Interest Rates

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Interest Rates and the Fixed-Income Market

Different **terms** for borrowing and lending have different interest rates

> e.g. rates for a long-term, fixed-rate loan may be higher than short-term rates.

Interest rates also differ for different kinds of borrowers because of different **default** risks

We will look at **risk-free** interest rates, i.e. returns on bonds issued by the government

(insert table of bond prices and yields from newspaper)

Objective of the lecture

1) To understand the relationship between short-term and long-term interest rates

2) To understand the difference between real and nominal interest rates in the bond market

3) To learn some institutional details about the fixed-income market

Some terminology

"**fixed-income**" means securities that pay a pre-specified return (principal and coupon payment), in other words, bonds.

In contrast, shares (equity) entitle the owner to a share of the business, and to dividends which vary depending on profits.

Stocks (UK) = Government Bonds

Stocks (US) = Shares

Gilts (UK) = Government Bonds

Treasury Bills (US, UK) = Short-term (a few months up to a year) government bonds

Zero-coupon yields

The **simplest** type of government bond is a zero-coupon bond

> (e.g. US Treasury bond "strips," which were later followed by original issue zero-coupon bonds)

For example, the cash flows on a bond with £1 face value, selling at a market price p, and maturing in four years, are:

t =	0	1	2	3	4
Cash Flow	-p	0	0	0	1

The **yield** is given by:

$p(1+r)^4 = 1$

so, $r = (1/p)^{1/4} - 1$

(r is the IRR on the cash flows.)

Yield Curve

(or "term structure")

The yield curve shows how different bonds of different maturities have different yields r_1, r_2, r_3, ...

What does the yield curve tell us?

(insert recent yield curve from newspaper)

Rolling over a short-term investment vs. holding long-term bonds

To invest £1 for two years I could proceed in two different ways:

1) By buying a two-year bond I get:

$(1+r_2)^2$

2) By buying a one-year bond then reinvesting the proceeds after one-year I get:

$(1+r_1)(1+{}_1r_2)$

where ${}_1r_2$ means the rate I will get at time 1 for another short-term investment, up to time 2.

> In other words this is next year's short-term rate, and I won't find out what it is until next year when I come to reinvest. This second strategy is therefore risky for me.

By arbitrage-type arguments (and ignoring this reinvestment risk) I should get the same expected return from both investment strategies. This gives us the **expectations hypothesis** of the yield curve.

Expectations Hypothesis

If I get the same **expected** return from both strategies:

$1 + E[{}_1r_2] = (1+r_2)^2 / (1+r_1)$

This is the **expectations hypothesis** of the yield curve.

It says that the yield curve tells us the **expected future interest rates.**

In **reality** the expectations hypothesis needs to be modified -- it generally seems to give an **overestimate** of rates in the future.

There are various theories that try to modify the expectations hypothesis by modelling risk.

But the expectations hypothesis is a good enough approximation for many purposes

Example of the Expectations Hypothesis

If r_1=4% and r_2=5%,

the expectations hypothesis predicts

$$1 + E[{}_1r_2] = (1+r_2)^2 / (1+r_1)$$

$$= 1.05^2/1.04 = 1.06$$

Roughly, we can see this from the approximation

$$E[{}_1r_2] = 2r_2 - r_1 = 10\% - 4\% = 6\%$$

In other words,

the two-year rate is roughly

an average of the one-year rate and next year's expected one-year rate:

5% is the average of 4% and 6%.

(refer back to graph of term structure from newspaper)

Bonds with Coupons

Most bonds are **not** zero-coupon bonds.

They have regular **coupon** payments.

e.g. a bond with a 7% coupon and a four-year maturity, and a market price p for each £1 of face value, has cash flows:

t =	0	1	2	3	4
Cash Flows					
	-p	0.07	0.07	0.07	1.07

The **yield** on the bond is just the **IRR** of the cash flows (generally the yield will be somewhere in between the different r's for the different terms).

(also called the "redemption yield")

If we know the different interest rates from the zero-coupon yield curve, we can calculate what the market price should be:

$$p = 0.07/(1+r_1) + 0.07/(1+r_2)^2 +$$

$$0.07/(1+r_3)^3 + 1.07/(1+r_4)^4$$

(in **practice**, the zero-coupon yield curve usually ends up being estimated by **inference** from the prices of various bonds with coupons).

Duration

(advanced topic for reference only)

You will often hear people refer to the **duration** of a bond. The **duration** of a zero-coupon bond is just the time until it matures.

For a **coupon bond** the **duration** is a weighted average of the times when it pays coupons up until it matures. The different times when a payment occurs (e.g. 1 year, two years, three years, ... up to maturity) are weighted by the payment's share in the PV of the bond.

The prices of **longer-term** bonds are **more** sensitive to interest rate changes.

Duration can be used to measure this, and to help investors **hedge** against the **risk** that interest rates will change.

> (it works well when short-term and long-term rates move by the same amount)

Index-linked bonds

Some governments (and even a few companies) have issued **index-linked** bonds. E.g. UK, US, Israel, Australia.

These are extremely convenient when setting discount rates in corporate finance.

E.g. at the moment for the UK yields on nominal bonds are about 4.5% and yields on index-linked bonds are 1.7%.

This is consistent with expected inflation of 2.8%.

(possible investment implications)