Dynamic Competition, Innovation and Strategic Financing

by

Matthew Spiegel†

and

Heather Tookes‡

Yale School of Management

P.O. Box 208200

New Haven, CT 06540-8200

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† Email: matthew.spiegel@yale.edu, phone (203) 432-6017, fax (203) 436-0630, http://som.yale.edu/~spiegel.

‡ Email: heather.tookes@yale.edu, phone (203) 436-0785, fax (203) 436-0630, http://www.som.yale.edu/faculty/het7/.

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ABSTRACT

We model the interactions among innovation, financing and product market competition. We provide closed form solutions for the values of competing firms in the context of a dynamic duopoly, in which one competitor faces an opportunity to adopt a new technology. If adopted, the firm must also determine whether it will obtain public or private financing. Our results allow us to relate current firm and industry characteristics to these decision variables. In particular, larger, more profitable firms with small rivals have greatest incentives to innovate. The private versus public financing decision depends mainly on the magnitude of the technological improvement and length of the period during which private financing extends the innovator's product market advantage.

How important are the interactions among innovation, product market competition and the going public decision?[1] This paper presents a tractable framework for examining financial decision making in dynamic competitive environments. In particular, it derives explicit, closed form solutions for a dynamic duopoly in which a firm decides whether to adopt an innovation. If adopted, the firm also must determine whether it will obtain public or private financing. The model allows one to relate current firm and industry characteristics to these decision variables in ways that are empirically measurable.

Consider the biotechnology industry, in which firms commonly wait until products have made it to late development stages before going public. Private financing is one way that firms' activities can remain hidden in the early stages of development, when the value of competitive secrets is high. This can be a particular advantage in intense R&D races. Further, dominant incumbents (e.g., Amgen and Genetech in the example of the biotech industry) may decrease the likelihood of a new firm's success, if information regarding the new technology is released too soon. The same forces also appear in more mature markets. For example, in the package delivery market, Federal Express listed publicly on the NYSE in 1978, while UPS did not undertake its IPO until 1999 (in part, for currency to aid in its acquisition strategy). During the years preceding the IPO, there was significant industry investment in technological infrastructure (particularly logistics). We demonstrate how private status during this development stage can significantly impact value. Later, when the nature of the investment opportunities changes, public financing can become more attractive. The general importance of competitive structure

[1] In the results of their survey of CFOs, Brau and Fawcett (2006) report that "Disclosing information to competitors" and "SEC reporting requirements" ranked fourth and fifth, respectively, of 11 factors contributing to the firms' decisions to undertake an IPO. Their sample consists of 336 firms: 37 with withdrawn IPOs; 87 with successful IPOs and 212 firms that were large enough to go public but did not undertake an IPO. Not surprisingly, these factors were most important to those firms actually undertaking the IPO as well as those that had not tried an IPO. They were less important to firms with withdrawn IPOs.

in financing decisions is an empirical question, and an important advantage of this paper is that it generates numerous testable hypotheses that can easily be matched with available data. Directly testable dynamic models are relatively rare in the capital structure literature although Leland (1994), Leland (1998), Goldstein, Ju and Leland (2001) are notable exceptions.[2]

Potential issuers face a tradeoff between releasing information and the promised return when deciding between a public or private financing. Public issuance in the United States involves the release of information that is potentially valuable to competitors and thus may hurt the future product market performance of the issuer.[3] The Sarbanes-Oxley Act (notably, Section 409) also mandates "real-time" disclosure of material information, which implies significant post-issuance disclosure. On the other hand, private financing involves a limited number of investors who may require higher returns due to, for example, the relative illiquidity of their investment. Indeed, there is a well documented discount for private securities (see e.g., Hertzel and Smith (1993) for an analysis of private placements).[4]

To address these issues, we solve a differential game based upon a variant of the Lanchester (1916) "battle" model. In our application two firms compete against each other for market share by spending funds to acquire each others customers. The adaptation we develop

[2] For example, in an opinion article, one empiricist writes: "Can we build a good model of, e.g., optimal corporate leverage decisions based on observable firm characteristics, and measure the effects of moving towards/away from this optimum?...I dream of models whose predictions are more quantitative than qualitative; models which are of direct use to empiricists." (Welch, 2001, p. 11).

[3] The competitive effects of information-sharing are well-known in the industrial organization literature. Vives (1990) provides a survey. In general, sharing of cost information occurs in equilibrium under Cournot competition and does not occur under Bertrand (the results are the reverse for common value demand information). See also Li (1985) and Gal-Or (1985). There is a related accounting literature on disclosure in imperfectly competitive markets e.g., Darrough and Stoughton (1990); Wagenhofer (1990); Masako (1993)). With the exception of Bhattacharya and Ritter (1983), in which a firm with private information takes into account the impact of disclosure on the ability to raise funds in financial markets and the probability of the success of a rival firm in an R&D game, these issues have received little attention in the finance literature. Note, also, that these papers do not model dynamic competitive interactions.

[4] We do not distinguish between issuance of debt versus equity. We assume all-equity firms and focus on differences in disclosure requirements across security types.

provides a simple, yet flexible structure for examining the dynamic interactions among product market competition, innovation and public versus private financing.

In the model, one firm has an opportunity to invest in a new value enhancing technology. Assuming the firm decides to innovate it then chooses between financing development costs with public or privately placed securities. The main benefit of private financing is that it allows the firm to extend the time during which it can hide its technological progress. This in turn extends the time it retains its competitive advantage over its rival which eventually catches up.[5]

The model's duopoly setting allows it to produce predictions regarding the competitive environment's impact on innovation and financing decisions; insights that are obviously impossible to derive in a single firm model. For example, the paper shows that increasing a rival's ability to attract customers can encourage innovators to use public financing. Many other questions like these can be addressed as well. Also, note that actions by one firm in an industry have value implications for its rivals. Thus, the model also provides testable predictions regarding the impact of technological innovations and financing decisions by one firm on the value of another.

Our results also indicate that the relative size of the firms in an industry can play an important role in the innovation decision. In particular, larger, more profitable firms with small rivals have the greatest incentives to innovate. Intuitively, this is because small, less profitable firms are less able to withstand the aggressive competitive behavior by rivals that their own innovation triggers. The positive relationship between firm size and the number of innovations predicted by the model has been documented empirically (see e.g., Acs and Audretsch (1988), Henderson and Cockburn (1996), and Nahm (2001)).

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[5]Maksimovic and Pichler (2001) also model the advantage of privately placed securities as reducing the information available to competitors.

Two extensions to the model examine how the innovation and financing decisions vary with the type of innovation. In the first, both firms have simultaneous access to a technology that, if adopted at some cost, will improve their ability to draw customers from their rival but does not increase industry demand (e.g. an improved washing machine). Alternatively, the firms can wait and adopt the technology at a reduced or no additional cost (for example when a manufacturing plant needs to be replaced).[6] If the parameters are such that both firms decide to immediately adopt the new technology then both firms see their values fall. This happens because the firms spend resources to bring the technology on line, but given that both adopt it there are in net no competitive benefits. However, even if only one firm initially adopts the new technology there remain parameters under which both firms see their values decline. These results derive directly from the competitive environment and would be difficult to mimic in a single firm setting.

The second extension examines a setting in which the currently private firm can secretly innovate in a way that its profits increase per unit sold. Such innovations can end up increasing the amount both firms in the industry spend to acquire market share. Of course, both would prefer to spend less, but cannot credibly commit to doing so in the full information game. We consider a pooling equilibrium in which the innovating firm secures private financing and chooses the equilibrium spending of a less profitable firm (i.e., firm without the new technology). Here, private financing allows this technologically superior firm to spend less on acquiring market share. This in turn reduces the rival's market share spending leading to overall increased instantaneous earnings. Note, counter intuitively, it pays for the rival to adopt a "hear no evil, see no evil" strategy since finding out the truth would actually reduce its profits.

[6] Here public financing is always chosen by early adopters since the innovation is common knowledge.

Within the existing literature the paper that comes closest to this one is Maksimovic and Pichler (2001). They examine the public-private financing decision within an industry that produces a homogenous good over two periods and for which there is costly entry and exit. In contrast, this paper examines a duopoly competing in a heterogenous goods industry over an infinite horizon. Other contrasts between the papers are discussed later on in the text.

The paper is organized as follows. Section I presents the basic model. Section II examines the solution in the infinite horizon case. Section III looks at the case where a firm can gain a competitive advantage for a finite time via an innovation that improves its ability to attract customers. Section IV discusses the conditions that make private financing desirable. Section V considers two extensions to the model: In one both firms have an opportunity to adopt an innovation, in the other just one. Section VI examines the relationship between this paper and the prior literature. Section VII concludes. Finally, the Appendix contains details regarding the derivation of the model's equilibrium.

I. Model

A. Players, Timing, Dynamics and Strategies

The Lanchester (1916) battle model was originally designed to study military strategy. Since then variants have been widely used in the marketing literature to examine advertising strategies (see e.g., Erickson (1992); Erickson (1997); Fruchter and Kalish (1997); for a review, see Dockner, Jørgensen, Van Long, and Sorger (2000)).[7] Here it is adapted to produce a differential game within which to explore competition among duopolists over new innovations, and financing choices.

[7] Although, to our knowledge, not in the form presented here.

Consider two firms battling for market share. Call the firm that faces the innovation and financing decision "Firm 1." The rival is "Firm 2." Let $u_i(t)$ be the dollars spent by firm $i \in \{1,2\}$ on gaining market share at instant t. Let s_i denote the effectiveness of spending. Note that spending to acquire a competitor's customers ($u_i(t)$) can imply a wide range of activities including advertising, new product design, store openings and R&D.

The market share of Firm 1 at time *t* is denoted $m(t)$. Firm 2's market share is then $1 - m(t)$. Time is continuous and there is a finite starting point at $t = 0$. Given initial the condition $m(0)$, m evolves as follows:

$$dm = \frac{\phi[(1-m)s_1u_1 - ms_2u_2]}{s_1u_1 + s_2u_2}dt \qquad (1)$$

where ϕ represents the speed with which consumers react to each firm's entreaties. Intuitively, (1) says that the variation in Firm 1's market share is simply the difference between what it gains from Firm 2's market share and what it loses to Firm 2.[8] The market share of Firm 1 increases with its own spending and effectiveness (u_1 and s_1, respectively) and decreases with spending and effectiveness of the competitor's spending. Note that high current $m(t)$ gives Firm 1 "more to lose" to Firm 2.[9]

Firms are assumed to be risk neutral profit maximizers: choosing spending to maximize their value. Instantaneous profits are assumed to be proportional to market share. Let α_i denote the revenue generating ability of firm *i* per unit of market share. Profits π equal revenues minus both spending on market share competition and a fixed operating cost f_i:

[8] The model can be modified to include a stochastic *dm*. The results are unchanged since the firms are risk neutral profit maximizers.

[9] In the marketing literature researchers tend to use as the law of motion either $dm/dt = u_1(1-m)-u_2m$ or $dm/dt = u_1\sqrt{1-m} - u_2\sqrt{m}$ (Dockner et al. (2000)). One advantage of using (1) instead is that it is unit free. This makes it easier to take to the data. In addition, it eliminates the problem that if one changes the unit of currency then one also changes the rate at which *m* changes over time.

$$\pi_1(t) = e^{gt}\left(\alpha_1 m(t) - u_1(t) - f_1\right)$$
$$\pi_2(t) = e^{gt}\left(\alpha_2(1 - m(t)) - u_1(t) - f_2\right) \quad (2)$$

The term g represents the industry's rate of growth. It is assumed that as the industry grows larger profits and costs grow proportionately. Firm 1 is currently financially constrained and has secured financing sufficient only to finance the current equilibrium path.

To help streamline the exposition details regarding the derivation the model's equilibrium conditions, for a general version of the model, can be found in the Appendix. Each of the following sections then employs that general solution to discuss the interactions between firms and their financial structure in various special cases. Thus, in the main body of the paper equilibrium conditions are simply stated without proof except for occasional references back to the Appendix.

B. The Equilibrium Value Functions

Let r denote the instantaneous discount rate. Assume $r>g$ and let $\delta=r-g$. Assume neither firm ever exits. The Appendix shows that each firm's value function V_i at time t (i.e., the present discounted value of each firm's profit stream conditional on the equilibrium strategies) can be written as:

$$V_1(m, T-t) = a_1(T-t) + b_1(T-t)m \quad (3)$$

and

$$V_2(m, T-t) = a_2(T-t) + b_2(T-t)m \quad (4)$$

within the scenarios considered in this paper. The terms a_i and b_i are functions of time and as shown in the Appendix equal:

$$a_1(t) = \delta^{-1}\left[\frac{\phi\alpha_1^3 s_1^2}{(\phi+\delta)(\alpha_1 s_1+\alpha_2 s_2)^2} - f_1\right] + C_1 e^{\delta t}, \tag{5}$$

$$a_2(t) = \delta^{-1}\left[\frac{\phi\alpha_2^3 s_2^2}{(\phi+\delta)(\alpha_1 s_1+\alpha_2 s_2)^2} + \frac{\delta\alpha_2}{\phi+\delta} - f_2\right] + C_2 e^{\delta t}, \tag{6}$$

$$b_1(t) = k_1 e^{-(\phi+\delta)(T-t)} + \alpha_1(\phi+\delta)^{-1}, \tag{7}$$

and

$$b_2(t) = k_2 e^{-(\phi+\delta)(T-t)} + \alpha_1(\phi+\delta)^{-1} \tag{8}$$

where the constants C_i and k_i depend upon a particular problem's boundary value conditions (i.e., the value of the V_i terms at some terminal date T).[10]

II. Full Information, Infinite Horizon Equilibrium

The simplest version of the model involves two firms that do not innovate, do not need outside financing, and compete over an infinite horizon. Since the game lasts forever the solutions to the model must be time independent. Thus, in equations (5) through (8) the C_i and k_i must all equal zero.

Since $V_{i,m}$ equals b_i one can now plug equations (7) and (8) (with k_i equal to zero) into (51) and (52) to find each firm's equilibrium spending on customer acquisition of:

$$u_1^* = \frac{\phi\alpha_1^2\alpha_2 s_1 s_2}{(\phi+\delta)(\alpha_1 s_1+\alpha_2 s_2)^2} \tag{9}$$

and

$$u_2^* = \frac{\phi\alpha_1\alpha_2^2 s_1 s_2}{(\phi+\delta)(\alpha_1 s_1+\alpha_2 s_2)^2} \tag{10}$$

[10] There are, of course, boundary conditions under which the solutions given above will not hold. However, for the problems considered in this paper equations (5) through (8) fully characterize each firm's equilibrium value function.

(see the Appendix for their derivation). The optimal controls, u_i^*, are dependent on the discount rate net of growth ($\delta = r - g$), each firm's revenue generating ability (α_i), their spending efficiency (s_i), and the speed with which consumers react to their attempts to gain market share (ϕ). Further, the marginal value of market share for a given firm (b_i) depends only on its own firm revenue generation ability α_i.

To gain further insight into the impact of the model's parameters on equilibrium behavior consider what happens in the steady state: $dm = 0$. From (1) if dm equals zero, then the steady state market share m^* equals,

$$m^* = \frac{\alpha_1 s_1}{\alpha_1 s_1 + \alpha_2 s_2}. \tag{11}$$

Clearly, both increased revenue generating ability and the efficiency of spending increase Firm 2's equilibrium market share. Overall industry concentration (sum of squared market shares) increases in $|\alpha_1 s_1 - \alpha_2 s_2|$. Thus one can think of the product of $\alpha_i s_i$ as a firm's competitive ability and the difference $\alpha_i s_i - \alpha_j s_j$ as i's competitive advantage relative to its rival. Note, that ϕ drops out of (11). In the long run it is irrelevant how long consumers take react to each firm's attempts to acquire market share so long as they react at all.

Returning to the paper's main objective one can write the explicit value functions for the current case as:

$$V_1(m) = \delta^{-1}\left\{(\phi+\delta)^{-1}\left\{\frac{\phi\alpha_1^3 s_1^2}{(\alpha_1 s_1 + \alpha_2 s_2)^2}\right\} - f_1\right\} + (\phi+\delta)^{-1}\alpha_1 m, \tag{12}$$

and

$$V_2(m) = \delta^{-1}\left\{\alpha_2 + (\phi+\delta)^{-1}\frac{\phi\alpha_2^3 s_2^2}{(\alpha_1 s_1 + \alpha_2 s_2)^2} - \frac{\alpha_2}{\phi+\delta} - f_2\right\} - (\phi+\delta)^{-1}\alpha_2 m \qquad (13)$$

for each firm. As one expects, the value of Firm 1 increases in its own market share (m) while the value of Firm 2 decreases in m . These relationships are proportional to the firms' revenue generating abilities α. Values of both firms are decreasing in δ , and both the competitor's revenue generating ability (α) and spending effectiveness (s).

Oddly, if m is at its steady state value then decreasing consumer responsiveness (ϕ) increases the value of both firms. The reason for this can be found in the equilibrium values of u_i and the fact that the steady state value of m does not depend on ϕ. (For the latter see equation (11).) From (9) and (10) both firms will reduce their spending on market share competition if consumers become less reactive. Thus, both firms benefit from ϕ's reduction since they then earn the same steady state revenue stream while wasting fewer resources trying to lure away each other's customers. Formally then, $\partial V_i / \partial\phi\big|_{m=m^*}$ >0 for both firms.

The next section now proceeds to build upon the basic framework developed here in order to examine the interactions among innovation, competition and financing.[11]

A. Financing and Innovation

The innovation game analyzed here assumes that at time 0 Firm 1 observes an opportunity to improve its operations. This new technology allows it to increase spending effectiveness from s_1 to s_1^* at time T_1. To develop the technology Firm 1 must first incur a cost of S. Firm 1 is currently private. Finally, to make the public/private financing question interesting, assume that

[11]The basic model can be extended to examine competition among N firms. The resulting value functions are qualitatively similar to the two-firm case.

Firm 1 is financially constrained. Thus, it only has enough capital to sustain equilibrium spending in the current competitive environment. If Firm 1 decides to innovate, it must secure financing of S for the project.

Thus, at time zero Firm 1 must choose among the following strategies:[12] (i) innovate and finance publicly, (ii) innovate and finance privately, or (iii) do not innovate. If Firm 1 decides not to innovate spending effectiveness remains at S=$\{s_1, s_2\}$ forever. Firm values are then given by (12) and (13). Adopting the technology means that, after a period of development from T_0 to T_1, Firm 1 enjoys a first mover advantage in the product market. This competitive advantage does not last forever; Firm 2 eventually copies the technology and increases its spending effectiveness s $= s_2^* > s_2$. The paper assumes that the relative spending effectiveness remains constant whenever both firms employ the same underlying technology. That is, $s_1 / s_2 = s_1^* / s_2^*$. Under this assumption competition eventually drives each firm's profitability back to pre new technology levels. However, markets experience real efficiency improvements via the adoption of the innovation.[13]

The financing decision (public versus private) has important value implications. Public financing is cheaper than private financing (i.e., there is a private market discount). This is due to the smaller pool of investors in the private markets as well as the relative illiquidity of these markets. On the other hand, if Firm 1 decides to finance the project via a public offering,

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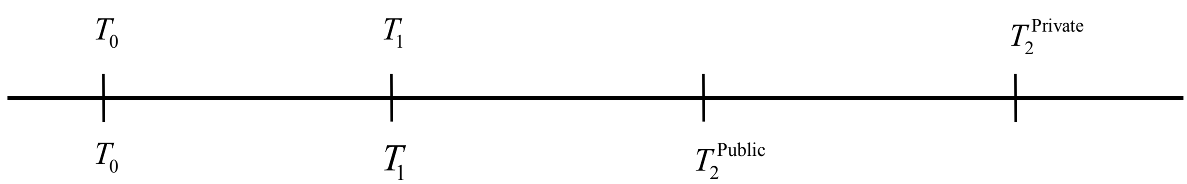
[12] We are assuming that the opportunity to innovate expires immediately if it is not taken public. This might be expected in actual product markets. For example, when technologies can be patented, a firm's decision to abandon a potential development may open opportunities for other firms to profit from development. For a model that assumes that the option to go public at a later date is valuable, see e.g., Benninga, Helmantel and Sarig (2005).

[13] It is easy to relax this assumption in our framework; however, we believe that $s_1 / s_2 = s_1^* / s_2^*$ is more realistic since it captures the idea that, in competitive markets, innovations are eventually adopted by all firms in an industry.

regulatory disclosure requirements force him to reveal investment behavior and financial data that increases the speed at which Firm 2 is able to successfully copy and adopt the technology.[14]

To capture the above conditions assume that if Firm 1 is privately financed Firm 2 will not be able to duplicate the technology until date T_2^{Private}. Conversely, if Firm 1 instead finances publicly then the required disclosures allow Firm 2 to begin the process of copying the technology somewhat earlier. In this case, Firm 2 successfully adopts the innovation by T_2^{Public} where the relative dates satisfy $T_2^{\text{Private}} > T_2^{\text{Pulic}} > T_1$. Upon Firm 2's successful adoption of the new technology s_2 increase to s_2^*.

Figure 1: Innovation and Financing: Timeline



T_0: T=0 Firm 1 decides whether to adopt innovation s_1^*. If innovation occurs, he decides how to finance it (publicly or privately). Development stage begins and S={s_1, s_2}. If no innovation, firms play full information, infinite horizon differential game with S={s_1, s_2}.

T_1: Firm 1's innovation becomes effective. S={s_1^*, s_2}.

T_2: Firm 2 successfully copies and adopts the innovation. S={s_1^*, s_2^*}.

Continuing with the example of a biotechnology firm from the introduction, consider Coley Pharmaceutical's recent IPO. (Lähteenmäk and Lawrence (2006) report that this was the sector's largest U.S. IPO in 2005). At time zero, Coley observes the opportunity for a breakthrough in cancer immune therapy; if financed, it takes until T_1 to bring the technology to

[14] We assume that copying is costless. It might be more realistic to assume that the cost of copying is positive (though much less than S). As long as the copying cost is sufficiently small that Firm 2 finds it worthwhile to copy, results and intuition regarding Firm 1's innovation and financing decision are identical to the costless copying case.

market. If privately financed, it will retain a competitive advantage until date T_2^{Private}. If publicly financed, rivals can observe Coley's investment patterns and perhaps glean other information from its mandated disclosures and "road show" documents to begin early development of the technology. This then leads to the erosion of Coley's competitive advantage on date T_2^{Public}. In real life, Coley initially financed the development privately and waited until T_1 (market stage when there was no longer any information to hide) to undertake the public offering.[15] Indeed, this is consistent with general empirical evidence suggesting the importance of private information in the entire biotech sector. Guo, Lev and Zhou (2004) create an index of disclosure by biotech IPO firms in their prospectuses. They find a negative relationship between the amount of disclosed information and common proxies for information asymmetry.

III. Value and Incentives to Innovate: Public Financing

To determine Firm 1's optimal decision rule one needs to compare its value under each of the three possible scenarios (i.e., do not innovate; innovate, finance publicly; innovate, finance privately). As usual, the solutions to each game are obtained by working backwards from the date in time when both firms successfully adopt the innovation. At T_2, by assumption, the firms compete against each other with parameters S={s_1^*, s_2^*} forever. Recall that the steady state firm values are given by Equations (12) and (13). Within the current setting one simply replaces s_1, s_2 with s_1^*, s_2^* in these two equations to distinguish between the pre- and post-innovation states.

[15] Of the Top 10 IPOs of biotechnology firms of 2005 reported by Lähteenmäk and Lawrence (2006): 1 had a product at the market stage; 5 had products in Phase 3 development; and 4 firms had products in Phase 2. Importantly, none of these IPO firms had products in Phase I development, pre-clinical testing or discovery. In our context, an important observation is that the industry has two large incumbents: Amgen and Genetech. Our model provides the testable implication that their relative dominance plays an important role in the going public decision.

While at time T_2 the steady state equations hold, prior to then one needs to solve the time dependent system based on the appropriate boundary value conditions. At date T_2 one knows that (12) and (13) with s_1, s_2 replaced with s_1^*, s_2^* must hold. These then make up the boundary value conditions for the value functions during the period T_1 to T_2:

$$a_1(T_2) = \delta^{-1}\left\{(\phi+\delta)^{-1}\left\{\frac{\phi\alpha_1^3 s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2}\right\} - f_1\right\}; \quad b_1(T_2) = \alpha_1(\phi+\delta)^{-1}$$
$$a_2(T_2) = \delta^{-1}\left\{\alpha_2 + (\phi+\delta)^{-1}\frac{\phi\alpha_2^3 s_2^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{\alpha_2}{\phi+\delta} - f_2\right\}; \quad b_2(T_2) = -\alpha_2(\phi+\delta)^{-1}. \tag{14}$$

Given the above boundary value conditions, one can solve for the constants C_i and k_i in (5) through (8) (see the Appendix for details) to yield the equilibrium value functions of:

$$V_1(T_1) = \frac{\alpha_1}{\delta(\phi+\delta)}\frac{\phi\alpha_1^2 s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} + e^{-\delta(T_2-T_1)}\frac{\phi\alpha_1}{\delta(\phi+\delta)}\left[\frac{\alpha_1^2 s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{\alpha_1^2 s_1^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2}\right] - \frac{f_1}{\delta} + \frac{\alpha_1}{\phi+\delta}m(T_1) \tag{15}$$

and

$$V_2(T_1) = \frac{\alpha_2}{\delta(\phi+\delta)}\frac{\phi\alpha_2^3 s_2^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} + e^{-\delta(T_2-T_1)}\frac{\phi\alpha_2}{\delta(\phi+\delta)}\left[\frac{\alpha_2^2 s_2^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{\alpha_2^2 s_2^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2}\right] - \frac{f_2}{\delta} + \frac{\alpha_2}{\phi+\delta}\left(1 - m(T_1)\right). \tag{16}$$

Note that, except for the second terms in each equation, the value functions are of the same form as in the infinite horizon base case. These additional terms can be interpreted as the "first mover advantage" and "second mover disadvantage." Importantly, the marginal value of market share (and the optimal control u_i) is time independent.

Finally, we solve backwards once more to obtain the value functions at t=0. Recall that at the game's start $S=\{s_1,s_2\}$ and remains so during the initial technology development interval

[0,T_1]. The value functions at t=0 under public financing are (see the Appendix for additional details):

$$V_1^{Public}(T_0) = \frac{\alpha_1}{\delta(\phi+\delta)} \frac{\phi\alpha_1^2 s_1^2}{(\alpha_1 s_1 + \alpha_2 s_2)^2}$$
$$+ e^{-\delta T_1} \frac{\phi\alpha_1}{\delta(\phi+\delta)} \left[\frac{\alpha_1^2 s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} - \frac{\alpha_1^2 s_1^2}{(\alpha_1 s_1 + \alpha_2 s_2)^2} \right] \tag{17}$$
$$+ e^{-\delta T_2^{Public}} \frac{\phi\alpha_1}{\delta(\phi+\delta)} \left[\frac{\alpha_1^2 s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{\alpha_1^2 s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} \right] - \frac{f_1}{\delta} + \frac{\alpha_1}{\phi+\delta} m(0)$$

and

$$V_2^{Public}(T_0) = \frac{\alpha_2}{\delta(\phi+\delta)} \frac{\phi\alpha_2^2 s_2^2}{(\alpha_1 s_1 + \alpha_2 s_2)^2}$$
$$+ e^{-\delta T_1} \frac{\phi\alpha_2}{\delta(\phi+\delta)} \left[\frac{\alpha_2^2 s_2^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} - \frac{\alpha_2^2 s_2^2}{(\alpha_1 s_1 + \alpha_2 s_2)^2} \right] \tag{18}$$
$$+ e^{-\delta T_2^{Public}} \frac{\phi\alpha_2}{\delta(\phi+\delta)} \left[\frac{\alpha_2^2 s_2^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{\alpha_2^2 s_2^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} \right] - \frac{f_2}{\delta} + \frac{\alpha_2}{\phi+\delta} \left(1 - m(0)\right)$$

for Firms 1 and 2 respectively.

While equations (17) and (18) look rather daunting they are in fact quite simple. The first term on the right represents the present value of each firm's profits until Firm 1's new technology comes on line. The second term equals the present value of each firm's profits during the period when Firm 1 has a technological advantage. The third term represents the final time period when Firm 2 catches up technologically. The final terms adjust the value function for the firm's fixed operating costs and current market share. Intuitively, one can think of the term $\alpha_i s_i/(\alpha_1 s_1+\alpha_2 s_2)$ as representing a firm's relative competitive strength ($\alpha_i s_i$) exclusive of the impact of its current market share. As time progresses the s_i terms change in each fraction to track each firm's current technological level. The different profit levels in different time periods are then discounted using the appropriate discount values ($e^{-\delta t}$) for the time period in question.

The observation that competitive environments greatly reduce the potential value of technological improvements routinely appears in the statements made by high-tech company executives. Their common complaint is that development of Microsoft compatible software is hindered by competition with Microsoft itself. The scenario typically outlined is that of a firm which produces an innovation and succeeds in acquiring customers. If this happens the claim is that Microsoft then copies the innovation (by building it into its existing products) and thereby steals away the innovator's market share. For example, a 2001 article on CNET News.com starts with,

> The battle over today's instant messenger market is vintage Microsoft, whose strategy enemies call "the three E's" in a parody of the company's marketing mantra: Embrace a rival's technology, extend it to work best with Windows, and extinguish the competition.
>
> Hu (2001)

This is the kind of industry dynamics captured in the value functions (17) and (18). Initially, a Microsoft rival creates an innovation at time T_0 that allows it to better capture market share. At time T_1 the innovation enters the firm's production function thus increasing s_1. Eventually, though, Microsoft discovers a way to incorporate the innovation into its own product line at date T_2 which then increases s_2 and eliminates the innovator's competitive advantage. Compared to a single firm setting, where T_2 is effectively set to infinity, the above scenario can greatly reduce if not eliminate innovation's value to its discoverer.

Given the above intuition, equations (17) and (18) can be viewed as the sum of the value functions in the base case plus the first mover advantage and second mover disadvantage respectively.

$$\begin{aligned} First\_Adv_i &= e^{-\delta T_1}(\delta(\phi+\delta))^{-1}\phi\alpha_i\left[\frac{\alpha_i^2 s_i^{*2}}{(\alpha_i s_i^* + \alpha_j s_j)^2} - \frac{\alpha_i^2 s_i^2}{(\alpha_i s_i + \alpha_j s_j)^2}\right] \\ &+ e^{-\delta T_2^{Public}}(\delta(\phi+\delta))^{-1}\phi\alpha_i\left[\frac{\alpha_i^2 s_i^{*2}}{(\alpha_i s_i^* + \alpha_j s_j^*)^2} - \frac{\alpha_i^2 s_i^{*2}}{(\alpha_i s_i^* + \alpha_j s_j)^2}\right] \end{aligned} \qquad (19)$$

and

$$Second\_Disadv_i = e^{-\delta T_1}(\delta(\phi+\delta))^{-1}\phi\alpha_i\left[\frac{\alpha_i^2 s_i^2}{(\alpha_j s_j^* + \alpha_i s_i)^2} - \frac{\alpha_i^2 s_i^2}{(\alpha_j s_j + \alpha_i s_i)^2}\right]$$
$$+e^{-\delta T_2^{Public}}(\delta(\phi+\delta))^{-1}\phi\alpha_i\left[\frac{\alpha_i^2 s_i^{*2}}{(\alpha_j s_j^* + \alpha_i s_i^*)^2} - \frac{\alpha_i^2 s_i^2}{(\alpha_j s_j^* + \alpha_i s_i)^2}\right]. \quad (20)$$

The analysis will return to equations (19) and (20) later on when it considers the case of simultaneous opportunities to innovate.

Notice that the solutions given by (17) and (18) assume that the magnitude of the innovation is common knowledge: only the information necessary for successful adoption is hidden from Firm 2. It might be more natural to assume that Firm 2 does not know whether technology is being developed. Note, however, that the optimal controls are a function only of a firm's current value of *s*. Thus, adding uncertainty regarding the rival *j*'s future value of *s* does not change *i*'s current spending on customer acquisition u_i. The only difference between the common knowledge and asymmetric information case would be that Firm 2's value function associated with $t<T_1$ (the second and third terms in (22)) would have expectations over the distributions of s_j. Thus, for the purposes of drawing out the model's implications, the common knowledge assumption just reduces the notational complexity.

A. Comparative Statics: Incentives to Innovate

Before examining the question of how an innovation should be financed the first question to be addressed is how competitive forces impact the decision to innovate at all. This section thus examines how the incentives to innovate vary with the size of the innovation and with the size/profitability of Firm 1 and Firm 2.

The main results presented so far demonstrate how competitive pressures in real markets can significantly reduce the value of innovations that might appear worthwhile if considered in isolation. The comparative statics analysis allows us to shed additional light on the issue of where (within industries) innovations can be expected to occur. To make it clear whether or not Firm 1 has a competitive advantage within each equation let $\psi_1 = s_1 / s_2 = s_1^* / s_2^*$ and $\psi_2 = s_1^* / s_2$.
Then one can rewrite the value function of Firm 1 as:

$$\begin{aligned} V_1(T_0) = & \frac{\phi\alpha_1^3\psi_1^2}{\delta(\phi+\delta)(\alpha_1\psi_1+\alpha_2)^2} + \\ & e^{-\delta T_1}\frac{\phi\alpha_1}{\delta(\phi+\delta)}\left[\frac{\alpha_1^2\psi_2^2}{(\alpha_1\psi_2+\alpha_2)^2} - \frac{\alpha_1^2\psi_1^2}{(\alpha_1\psi_1+\alpha_2)^2}\right] + \\ & e^{-\delta T_2}\frac{\phi\alpha_1}{\delta(\phi+\delta)}\left[\frac{\alpha_1^2\psi_1^2}{(\alpha_1\psi_1+\alpha_2)^2} - \frac{\alpha_1^2\psi_2^2}{(\alpha_1\psi_2+\alpha_2)^2}\right] - \frac{f_1}{\delta} + \frac{\alpha_1}{\phi+\delta}m(0). \end{aligned} \tag{21}$$

The derivative of Firm 1's value function with respect to the magnitude of the competitive advantage during period T_1 to T_2 is thus:

$$\frac{dV_1(0)}{d\psi_2} = \left[\frac{\psi_2}{(\alpha_1\psi_2+\alpha_2)^2} - \frac{\alpha_1\psi_2^2}{(\alpha_1\psi_2+\alpha_2)^3}\right]\left[\frac{2\phi\alpha_1^3(e^{-\delta T_1} - e^{-\delta T_2})}{\delta(\phi+\delta)}\right]. \tag{22}$$

This is clearly positive (as would be expected). What is of greater interest is the question of how the incentive to innovate varies with Firm 1's characteristics, in particular, Firm 1's revenue generating ability (α_1):

$$\begin{aligned} \frac{d^2V_1(0)}{d\psi_2 d\alpha_1} = & \left[\frac{6\alpha_1^2\psi_2}{(\alpha_1\psi_2+\alpha_2)^2} - \frac{4\alpha_1^3\psi_2^2}{(\alpha_1\psi_2+\alpha_2)^3} - \frac{8\alpha_1^3\psi_2^2}{(\alpha_1\psi_2+\alpha_2)^3} + \frac{6\alpha_1^4\psi_2^3}{(\alpha_1\psi_2+\alpha_2)^4}\right] \times \\ & \left[(\delta(\phi+\delta))^{-1}\phi\alpha_1^3(e^{-\delta T_1} - e^{-\delta T_2})\right]. \end{aligned} \tag{23}$$

This is always greater than zero and implies that larger, more profitable firms (recall that the equilibrium size of Firm 1 is $m^* = \alpha_1 s_1 / \left(\alpha_1 s_1 + \alpha_2 s_2\right)$), are more likely to adopt new technologies.[16]

Now consider the incentive to innovate as a function of the rival firm's characteristics:

$$\frac{d^2 V_1(0)}{d\psi_2 d\alpha_2} = \left[\frac{-4\psi_2}{(\alpha_1\psi_2+\alpha_2)^3} + \frac{6\alpha_1\psi_2^2}{(\alpha_1\psi_2+\alpha_2)^4}\right]\left[\frac{\phi\alpha_1^3(e^{-\delta T_1}-e^{-\delta T_2})}{\delta(\phi+\delta)}\right]. \qquad (24)$$

Equation (24) is greater than zero when $3\,\alpha_1\psi_2 - 2\alpha_2 > 0 \quad \Rightarrow \quad \alpha_1 s_1^* - 2\alpha_2 s_2 > 0$. Therefore, for small firms, an increase in the competitors' profitability and size makes the innovation less valuable. This is because an increase in Firm 2's α or s increases its marginal value of market share and makes Firm 2 more aggressive. This can impose a cost that is greater than the potential gains from adopting the technology.

The discount rate net of growth ($\delta = r - g$) is also important. From (22), observe that

$$\frac{d^2 V_1(0)}{d\psi_2 d\delta} = 2\phi\alpha_1^3\left[\frac{\psi_2}{(\alpha_1\psi_2+\alpha_2)^2} - \frac{\alpha_1\psi_2^2}{(\alpha_1\psi_2+\alpha_2)^3}\right]\times \left[\frac{\left(T_2 e^{-\delta T_2} - T_1 e^{-\delta T_1}\right)\delta\left(\phi+\delta\right) - \left(\phi+2\delta\right)\left(e^{-\delta T_1}-e^{-\delta T_2}\right)}{\phi^2\left(\phi+\delta\right)^2}\right] < 0. \qquad (25)$$

[16] The relationship between firm size and innovative activity has long been the subject of academic debate. See Kamien and Schwartz (1975) for a survey of early work (static models) and a discussion of the Schumpterian hypothesis that product market rivalry will impact innovation incentives. Acs and Audretsch (1987) find that large firms have the innovative advantage in industries that are capital-intensive with high concentration and advertising expenditure. They conclude that large firms have advantage in imperfectly competitive industries, while small firms have greater advantage in imperfectly competitive industries. More recently, Hall (2005) also traces some of the basic ideas that we investigate, namely that innovations can be copied by rival firms thereby decreasing incentives to invest, to Schumpeter. We not only explicitly model this possibility, but we do so within the framework of a dynamic, continuous time model.

Thus, incentives to adopt new technologies are higher when δ is lower.[17] A similar exercise shows that increasing consumer responsiveness increases the value of any new innovation; differentiating (22) with respect to ϕ yields $\partial^2 V_1(0)/\partial\psi\partial\phi > 0$.

The comparative static results are summarized in the table below.

Table 1: Change in the Value to the Innovator from an Innovation

Derivative	Economic Interpretation	Sign	Condition
$d^2V_1(0)/d\psi_2 d\alpha_1$	The impact of an increase in the innovator's profitability on the value of any innovation.	+	All firms.
$d^2V_1(0)/d\psi_2 d\alpha_2$	The impact of an increase in the rival's profitability on the value of any innovation.	-	Small firms.
$d^2V_1(0)/d\psi_2 d\alpha_2$	The impact of an increase in the rival's profitability on the value of any innovation.	+	Large firms.
$d^2V_1(0)/d\psi_2 d\delta$	An increase in the discount rate reduces the value of any innovation.	-	All firms.
$\partial^2 V_1(0)/\partial\psi\partial\phi$	An increase in consumer responsiveness increases the value of any innovation.	+	All firms.

Maksimovic and Pichler (2001) examine closely related issues. They consider financing choice in a growing industry and the potential for herding by potential entrants. The cost of disclosure is letting rivals know how lucrative the business is. Their main result is that private financing occurs when start-up costs are high and when there is a high probability of displacement by a superior rival; public financing occurs when the technology is not costly and when the probability of displacement is low. Our focus differs in that we study the financing choice in a model with dynamic competitive interactions and a finite interval over which a

[17] To prove that the inequality holds, note that for $T_1=T_2$ the second derivative equals zero. Next, differentiate (25) with respect to T_2 to find that it is decreasing in T_2 and thus for $T_2>T_1$ the inequality must hold.

competitive advantage can be maintained. We also explicitly examine the potential value implications of the existence of a strong rival for a growing firm in an industry with heterogeneous goods.

An advantage of the model developed here is that it can easily be fit to data that is readily available. Besides offering a potentially rich set of cross sectional predictions for future testing it also allows for the investigation of competitive dynamics along the equilibrium path. This too is well suited for real data. Because competition evolves through time one can use the model to make better use of the available panel datasets chronicling stock returns and corporate accounting statements. One possible mapping between the model's parameter values and variables available on CRSP and COMPUSTAT can be found in the Appendix's Table 5.

IV. Private versus Public Financing

We now turn to a primary question underlying our analysis: given the competitive structure and new technology presented above, how (if at all) will the potential innovator finance the investment? In this case the value functions at time zero are analogous to those under public financing and given by equations (17) and (18). The only difference is that the T_2^{Public} terms are replaced by T_2^{Private}. Label the value functions with this change V_i^{Private}. While the forms of the value functions are identical to the public financing case keep in mind that $T_2^{\text{Private}} > T_2^{\text{Public}}$ and thus Firm 1 enjoys first mover advantages for a longer time period under private financing.

In concurrence with the empirical literature the model assumes that private financing is more costly than public financing (see e.g., Hertzel and Smith (1993) for evidence of this in the context of private placements). The simplest way to capture the increased cost of private financing is to build it directly into the technology's implementation cost. Thus, if the new technology costs S to implement under public financing assume that its cost increases to $S(1+D)$ under private financing. Here, D is the private market discount.

Thus far, the value functions under external financing have been presented net of the investment costs. These need to be added back in order to evaluate Firm 1's choice of whether to innovate. Firm 1 finances the technology when:

$$Max\left\{V_1^{\text{Private}}(0)-S(1-D),V_1^{\text{Public}}(0)-S\right\}>V_1^{\text{No Innovation}}(0). \tag{26}$$

Clearly, a high cost of adopting the technology decreases incentives to innovate. After referring back to (21), inequality (26) also shows that incentives are increasing in both the time over which Firm 1 is able to enjoy a first mover advantage and the magnitude of the innovation ψ_2.

Some algebra shows that if Firm 1 innovates private financing is preferred when:

$$\left[e^{-\delta T_2^{\text{Public}}}-e^{-\delta T_2^{\text{Private}}}\right]\frac{\phi\alpha_1^3}{\delta(\phi+\delta)}\left[\frac{s_1^{*2}}{(\alpha_1 s_1^*+\alpha_2 s_2)^2}-\frac{s_1^{*2}}{(\alpha_1 s_1^*+\alpha_2 s_2^*)^2}\right]-SD>0. \tag{27}$$

A. Comparative Statics: Private versus Public Financing

This section examines how the incentives to finance publicly vary with firm characteristics. As in Section III.A, let $\psi_1 = s_1/s_2 = s_1^*/s_2^*$ and $\psi_2 = s_1^*/s_2$. Then one can rewrite the value of private financing as:

$$V_1^{\text{Private}}-V_1^{\text{Public}}=\left[e^{-\delta T_2^{\text{Public}}}-e^{-\delta T_2^{\text{Private}}}\right]\frac{\phi\alpha_1^3}{\delta(\phi+\delta)}\left[\frac{\psi_2^2}{(\alpha_1\psi_2+\alpha_2)^2}-\frac{\psi_1^2}{(\alpha_1\psi_1+\alpha_2)^2}\right]-SD \tag{28}$$

We are interested in the cross-sectional relationships among the available financing choices and firm characteristics. First consider Firm 1's revenue generating ability, α_1. To simplify the notation, let $q=\left[e^{-\delta T_2^{\text{Public}}}-e^{-\delta T_2^{\text{Private}}}\right](\delta(\phi+\delta))^{-1}\phi$. Then:

$$\frac{\partial(V_1^{\text{Private}}-V_1^{\text{Public}})}{\partial\alpha_1}=q\alpha_1^2\left[\frac{\psi_2^2(\alpha_1\psi_2+3\alpha_2)}{(\alpha_1\psi_2+\alpha_2)^3}-\frac{\psi_1^2(\alpha_1\psi_1+3\alpha_2)}{(\alpha_1\psi_1+\alpha_2)^3}\right]. \tag{29}$$

As shown in the Appendix, (29) is positive for all $\psi_2>\psi_1$. Therefore, the incentive to secure private financing is increasing in Firm 1's revenue generating ability (α_1). Intuitively, this occurs because high α firms are in the best position to use any technological advance to aggressively pursue market share. Equation (29)'s prediction that the most lucrative projects should be financed privately is consistent with findings of operational underperformance of IPO firms in the years following issuance (e.g., Loughran and Ritter (1995)).[18] That is, initially firms finance their very high ψ_2 projects privately and then go to the public markets when only more modest ψ_2 innovations remain.

Now consider the private financing choice as a function of the rival's revenue generating ability (α_2):

$$\frac{\partial(V_1^{\text{Private}}-V_1^{\text{Public}})}{\partial\alpha_2}=2q\alpha_1^3\left[\frac{\psi_1^2}{(\alpha_1\psi_1+\alpha_2)^3}-\frac{\psi_2^2}{(\alpha_1\psi_2+\alpha_2)^3}\right]. \tag{30}$$

The sign of equation (30) depends on the relative values of α_1 and α_2. Incentives are decreasing in α_2 when $\alpha_2 > \alpha_1\psi/2$, and increasing in α_2 when α_2 is small (i.e., $\alpha_2 < \alpha_1\psi/2$). This is because profitable rivals (i.e., high α_2) will spend more aggressively during the period of Firm 1's first mover advantage, making it optimal for Firm 1 to spend aggressively during this period as well. All else equal, in this range, increases in α_2 will decrease the value of extending the period of competitive advantage to T_2^{Private} through private financing. On the other hand, when α_2 is small, the benefits from a extending the period of first-mover advantage outweigh the costs of higher equilibrium spending.

[18] Chemmanur, He and Nandy (2006) report that total factor productivity of IPO firms increases prior to the IPO and then decreases in the years following the IPO. This is consistent with the prediction that the most lucrative projects are financed privately.

There are several additional observations. First, Firm 1's incentives to finance privately increase in the technological advantage the innovation provides (i.e., in ψ_2). Second, as one might then expect, the opposite is true of ψ_1. An increase in Firm 1's initial (and final) spending effectiveness relative to Firm 2 decreases Firm 1's incentive to issue private securities. From equation (28), observe that the incentives to remain private decrease when real interest rates are high. This provides an explanation for expected IPO patterns based on economic fundamentals that is distinct from the market timing arguments in Baker and Wurgler (2000).

Third, the incentive to remain private increases as the time gained from delaying Firm 2's adoption increases (this can also be interpreted as an increased incentive to avoid disclosure requirements).[19] Fourth, increasing consumer responsiveness (ϕ) increases the incentive to finance privately. Fifth, the private market discount decreases Firm 1's incentives to remain private as it clearly makes such financing more costly.

[19] An obvious implication is that, if the private market discount is sufficiently high (making private financing prohibitive), disclosure requirements can inhibit innovation.

Table 2: Change in the Value of Remaining Private ($V_1^{\text{Private}} - V_1^{\text{Public}}$) when Financing an Innovation

Derivative w.r.t.	Economic Interpretation	Sign	Condition
α_1	The impact of an increase in the innovator's profitability.	+	All firms.
α_2	The impact of an increase in the rival's profitability.	-	Small innovator.
α_2	The impact of an increase in the rival's profitability.	+	Large innovator.
ψ_1	The relative ability of the innovator to take market share relative to its rival.	+	All firms.
ψ_2	The innovation's improvement in the innovator's ability to increase its market share during the period it has a technological advantage, T_1 to T_2.	-	All firms.
ϕ	Consumer responsiveness to corporate spending seeking to increase market share.	+	All firms.
δ	An increase in the real interest rate.	-	All firms.

V. Extensions

The main model focuses on a particular type of innovation: one that provides Firm 1 with an opportunity for first mover profits. While we believe that this setting is most appropriate when considering a firm's incentive to secure financing for R&D investment, it is also the case that some technology shocks do not involve first mover opportunities. In fact, industry shocks are often characterized by common shifts in investment opportunities. This might occur, for example, if an upstream firm develops a low cost input to production that can be used after incurring a one time switching cost. It is also possible that firms face firm specific opportunities (innovations that cannot be copied). The structure of our basic model is sufficiently pliable to examine these possibilities. This section considers two extensions: the impact of industry wide and firm specific shocks on innovation and financing.

A. Industry Wide Opportunity

Consider an innovation that appears at time zero in which both firms simultaneously have the opportunity to develop the technology s_i^*. If both firms innovate, they pay *S* now to finance the innovation. Assume that the actual development occurs privately; if a firm does not pay *S*, it takes until T_2^{Public} to successfully copy the innovator's product at which time it receives it for free. Because general information regarding the technology is public, there is no benefit to private financing. If only one firm innovates, then the first mover advantage game (solved in the main model) is played. The payoffs to adopting the technology are given in the table below.

Table 3

		Firm 2	
		Innovate	**Do Not Innovate**
Firm 1	**Innovate**	V_1-S, V_2-S	V_1 - S + $First\_Adv_1$, V_2 + $Second\_Disadv_2$
	Do Not Innovate	V_1 + $Second\_Disadv_1$ V_2 - S + $First\_Adv_2$	V_1, V_2

Where:

- V_1 and V_2 are the value functions defined in (12) and (13).
- First and Second Mover Advantage are from Equations (19) and (20). Rearranging (and using $s_1 / s_2 = s_1^* / s_2^*,\ s_i^* > s_i$) produces:

$$First\_Adv_i = \left(e^{-\delta T_1} - e^{-\delta T_2}\right)\frac{\phi\alpha_i}{\delta(\phi+\delta)}\left[\frac{\alpha_i^2 s_i^{*2}}{(\alpha_i s_i^* + \alpha_j s_j)^2} - \frac{\alpha_i^2 s_i^{*2}}{(\alpha_i s_i^* + \alpha_j s_j^*)^2}\right] > 0 \qquad (31)$$

and

$$Second\_Disadv_i = \left(e^{-\delta T_1} - e^{-\delta T_2}\right)\frac{\phi\alpha_i}{\delta(\phi+\delta)}\left[\frac{\alpha_i^2 s_i^2}{(\alpha_j s_j^* + \alpha_i s_i)^2} - \frac{\alpha_i^2 s_i^2}{(\alpha_j s_j + \alpha_i s_i)^2}\right] < 0. \quad (32)$$

Assume that S is sufficiently low that at least one firm will find innovation profitable.[20] Also assume that there is no way for the firms to coordinate and share the cost S. From the table, it is clear that incentives to innovate are increasing in: firm size/revenue-generating ability; time horizon of the first mover advantage; and the magnitude of the innovation. As long as S is sufficiently low (i.e., $FirstMove\ Adv_i > S$ for $i \in \{1,2\}$), both firms will choose to innovate. In that case, an IPO will occur; however (interestingly), it will accompany value *decreases* for both firms due to the initial required investment. This is due to the fact that both firms spend money on the innovation but, in the end, do not obtain a competitive advantage. As noted in the introduction results like these are very difficult to reproduce in a single firm model. In such models firms only take actions that increase their present value. However, the duopoly model makes clear that sometimes firms are forced to pick between options all of which result in a value reduction.

In line with the modeling assumptions used in Section III if just one of the firms chooses to innovate at date zero then the other can costlessly adopt the technology at date T_2. (It is easy to show that it is never optimal to adopt the technology between dates 0 and T_2.) Therefore, each firm trades off the cost of developing the innovation with the second mover disadvantage. This setup gives some insight as to where and when innovation is most likely to occur (i.e., in larger, more profitable firms, when technology shocks are large).

While the simultaneous adoption of the technology by the two firms leads to a reduction in both their values, the same is not always true when just one adopts the technology. When

[20] This ignores the trivial case in which no innovation occurs and V_1 and V_2 are the value functions defined in (12) and (13).

there is just one initial adopter its value can go either up or down depending on the problem's parameters. The exact change in value is given by:

$$\Delta V_1(0) =$$
$$\frac{\phi\alpha_1^3}{\delta(\phi+\delta)}\left\{e^{-\delta T_1}\left[\frac{\psi_2^2}{\left(\alpha_1\psi_2^2-\alpha_2\right)^2}-\frac{\psi_1^2}{\left(\alpha_1\psi_1^2-\alpha_2\right)^2}\right]+e^{-\delta T_2}\left[\frac{\psi_1^2}{\left(\alpha_1\psi_1^2-\alpha_2\right)^2}-\frac{\psi_2^2}{\left(\alpha_1\psi_2^2-\alpha_2\right)^2}\right]\right\} \quad (33)$$
$$-S,$$

where $\Delta V_1(0)$ represents the change in the firm's value at date zero when the innovation's existence becomes known given just Firm 1 adopts it. As one expects, the greater the competitive advantage afforded by the innovation (ψ_1), the earlier it comes on line (T_1), and the longer until the second firm acquires it (T_2) the more likely it is that the firm sees a value increase upon the technology's announced existence ($\Delta V_1(0)>0$). The more interesting comparative statics have to do with the second derivatives of $\Delta V_1(0)$ (the influence of industry structure on the value of the innovation to the early adopter). But, with a single initial adopter these derivatives are identical to those found in Table 1 and thus are not repeated here.

One can also examine the case of an industry wide shock to a variable that does not impact strategic spending u_i^*. It is easy to see how investment in this technology would increase the value of all firms in the industry. If all firms in the industry face a common opportunity to immediately reduce fixed costs from f_i *to* $(1-\kappa)f_i$ where $\kappa\in\{0,1\}$, the value functions become:

$$V_1(m)=\delta^{-1}\left\{(\phi+\delta)^{-1}\left\{\frac{\phi\alpha_1^3 s_1^{*2}}{(\alpha_1 s_1^*+\alpha_2 s_2^*)^2}\right\}-f_1(1-\kappa)\right\}+(\phi+\delta)^{-1}\alpha_1 m \quad (34)$$

and

$$V_2(m)=\delta^{-1}\left\{\alpha_2+(\phi+\delta)^{-1}\frac{\phi\alpha_2^3 s_2^{*2}}{(\alpha_1 s_1^*+\alpha_2 s_2^*)^2}-\frac{\alpha_2}{\phi+\delta}-f_2(1-\kappa)\right\}-(\phi+\delta)^{-1}\alpha_2 m \quad (35)$$

respectively for firms 1 and 2.

As long as the innovation κ is large enough (i.e., $\delta^{-1} f_i \kappa > S$), then both firms will innovate. As in the case above, private financing is ruled out since there is no competitive cost to transparency. Here, Firm 1's decision to undertake the IPO is associated with value *increases* for both firms. This is consistent with public equity issuance near high industry valuations but is again distinct from market timing explanations.

To summarize, the value implications of the industry wide shock examples depend on whether or not the shock involves a parameter in the optimal control function (e.g., a change in an important input price versus a new investment technology). Because there are no competitive costs associated with transparency industry wide innovations are financed publicly.

Table 4: Impact of an Industry Wide Innovation on Firm Value

Innovation Type	Impact on Firm Value	Condition
Increase in a firm's ability to acquire market share.	Both firms decline in value upon adopting the innovation.	Both adopt at time zero.
Increase in a firm's ability to acquire market share.	Early adopter may see a rise in value. Late adopter sees an immediate decline in value. Ultimately both firms may see a value decline.	One firm adopts at time zero, the other at time T_2.
Reduction in a firm's fixed operating costs.	Adopters see a value increase.	All firms.

B. Firm Specific Opportunity in α_i: Hear no Evil, See no Evil.

The analysis now turns to firm specific opportunities to innovate. In this case, Firm 1's new technology is inappropriate for use by Firm 2. If the innovation involves changes to s then the main model can easily be modified to handle the problem: simply let $T_2 \to \infty$ in (27). Since changing either s_1 or s_2 alters the law of motion governing market share, Firm 2 can immediately deduce Firm 1's value of s by simply watching m change over any one instant. There can be no hidden information under this scenario.

While changes in *s* can be immediately deduced by either firm there exist other variables whose value a firm can potentially keep "hidden." One such variable is α; thus, this section of the paper concentrates on the case where Firm 1 has an opportunity to increase its value. As will be seen this case shows that private financing can act as an information shield that benefits both firms in the industry.

Assume that Firm 1 has an opportunity to immediately raise α_1 from $\alpha_{1,L}$ to $\alpha_{1,H}$. The cost of the technology is *S*. As before, Firm 1 can finance the innovation publicly or privately. If it is publicly financed, all information regarding fundamentals (α_1) become common knowledge. If the technology is privately financed, Firm 1 can choose to "hide" and operate as though it were a low type firm (i.e., it pretends no innovation has occurred). At some finite time *T*, the true α_1 becomes common knowledge (e.g., through taxes or some other public signal revealed to the market). If the high type deviates from the pooling equilibrium before *T* the firms return to the full information game. An interesting aspect of the pooling equilibrium, as will be shown, is that Firm 2 often *prefers* to be uninformed. (Hence, the section's title.) This is because lack of information about Firm 1's marginal value of market share commits Firm 2 to less aggressive product market behavior.[21]

Before solving for the conditions under which Firm 1 "hides," it is useful to point out that the potential incentive to hide comes from the fact that optimal spending (u_1^*) is increasing in α_1. The high type firm would like to credibly commit to spend less on gaining market share (as long as Firm 2 also does so), but cannot do this when there is full information. However, if the

[21] In the case of UPS's IPO mentioned in the introduction, this framework offers a potential explanation for the observation in the popular press of intense competition in the package delivery market over the past several years. Martin, Neil A., 2003, "New Ground War: The package-delivery business is turning brutal, hurting both FedEx and UPS," Barron's 4/21/2003.

conditions for a pooling equilibrium exist then the lack of complete information can make, in equilibrium, lower spending individually rational.

In this example (for simplicity) hold spending effectiveness constant across firms (s_1=s_2=1) and set the consumer responsiveness parameter (ϕ) to one. In a pooling equilibrium the "high type" firm chooses private financing to avoid having to reveal so much information to the market that Firm 2 becomes aware of the change in α_1. Firm 1 can then continue to hide its new status by spending less on obtaining market share. Pooling will be profitable if the gains from keeping u_i low outweigh the opportunity cost of a more aggressive campaign. Naturally, the cost is that Firm 1's market share increases at a slower rate. There are two relevant time periods:

(1) t=0 when, if the investment is undertaken, $\alpha_{1,L}$ increases to $\alpha_{1,H}$; and

(2) t=T, when the firms return to the full information game and play $\{\alpha_{1,H}, \alpha_2\}$ forever.

Consider now the value, $V_1(0)$ of Firm 1 when it decides to privately finance in order to hide its true revenue generating capacity.[22] It continues to do so through period T, at which point it is no longer possible to hide. Profits from pooling until T are given by:

$$\pi = \int_0^T (\alpha_{1,H} m(t) - u_{1,L} - f_1) e^{-\delta t} dt + e^{-\delta T} V_1\left(T \middle| \alpha_{1,H}\right). \quad (36)$$

Firm 1 chooses to "hide" as the low type if firm value from taking that strategy exceeds firm value when he reveals his true type:

$$a_1(\alpha_{1,H}) + b_1(\alpha_{1,H}) m(0) < \int_0^T (\alpha_{1,H} m(t) - u_{1,L} - f_1) e^{-\delta t} dt + e^{-\delta T} V_1\left(m(T) \middle| \alpha_{1,H}\right). \quad (37)$$

[22] Conditions will hold for t<T.

Parameters a_1 and b_1 are identical to those in the full information game. To obtain the value of the right hand side of (37), note that pooling implies:

$$\int_0^T \alpha_{1,H} m(t) e^{-\delta t} dt = \alpha_{1,H} e^{-\delta t} (1+\delta)^{-1} \left[-m(t) - \frac{u_{1,L}}{\delta(u_{1,L}+u_{2,L})} \right]. \tag{38}$$

Therefore, Firm 1 chooses to hide when:

$$a_1(\alpha_{1,H}) + b_1(\alpha_{1,H}) m(0) < \alpha_1^H (1+\delta)^{-1} \left[m(0) + \frac{u_{1,L}}{\delta(u_{1,L}+u_{2,L})} - e^{-\delta T} m(T) - \frac{e^{-\delta T} u_{1,L}}{\delta(u_{1,L}+u_{2,L})} \right] \tag{39}$$
$$+ \frac{u_{1,L}+f_1}{\delta}(e^{-\delta T}-1) + e^{-\delta T}\left[a_1(\alpha_{1,H})) + b_1(\alpha_{1,H}) m(T) \right].$$

Rearranging and substituting for a_1 and b_1, given by (5) and (7) with C_1=k_1=0 gives the conditions for pooling in this example:

$$0 < \frac{\alpha_{1,L}}{(\alpha_{1,L}+\alpha_2)} - \frac{\alpha_{1,L}^2 \alpha_2}{(\alpha_{1,L}+\alpha_2)^2 \alpha_{1,L}} - \frac{\alpha_{1,H}^2}{(\alpha_{1,H}+\alpha_2)^2}. \tag{40}$$

Inequality (40) holds when $\alpha_{1,L}$ is small relative to $\alpha_{1,H}$ and α_2 is large relative to both parameters.[23] Note that the conditions for the pooling equilibrium hold for any time *t*. As long as the parameters in the model are such that pooling is preferred, Firm 1 will choose to secretly profit.[24]

So far the analysis has ignored the low type's incentives. However, that is because in the pooling equilibrium the high type firm acts as if it only obtains the low type's profit per unit of market share. Because both the high and low types spend $u_{1,L}^*$ in the pooling equilibrium the

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[23] When $\alpha_{1,L} = \alpha_{1,H}$, Equation (40) = 0. The partial derivative of the right-hand-side of (40) with respect to $\alpha_{1,H}$ equals $\frac{\alpha_{1,L}^2}{(\alpha_{1,L}+\alpha_2)^2} - \frac{2\alpha_{1,H}^3}{(\alpha_{1,H}+\alpha_2)^3}$. This is positive for sufficiently high α_2.

[24] Note, that while the formulation of the solution ignores the potential option value of deviating from pooling before *T* it is irrelevant. Equation (40) does not contain *T* and thus the option value of deviating early is zero. That is if it pays to pool at date 0 it pays to pool at all dates up to *T*.

optimal response of Firm 2 remains $u^*_{2,L}$. Thus, there is no incentive for the low type to deviate from its optimal strategy $u^*_{1,L}$ derived in the full information game.

In the pooling equilibrium Firm 1 sometimes want to "hide," but Firm 2 prefers this equilibrium as well. In fact, the range of parameters over which Firm 2 prefers pooling is even larger than the range over which Firm 1 chooses to pool. This is because pooling by Firm 1 allows Firm 2 to maintain a larger market share while spending less money.

To prove that if Firm 1 pools then Firm 2 prefers to remain ignorant repeat the same process that led to (40), but now solve for the conditions under which Firm 2 prefers pooling:

$$a_2(\alpha_{1,H})+b_2(\alpha_{1,H})m(0)<\int_0^T(\alpha_2(1-m(t))-u_{2,L}-f_2)e^{-\delta t}dt+e^{-\delta T}V_2(\alpha_{1,H},m(T)). \quad (41)$$

Again, the parameters a_2 and b_2 take on the same value as they do in the full information game. To obtain the value of the right hand side, note that pooling implies:

$$-\int_0^T \alpha_2 m(t)e^{-\delta t}dt=-\alpha_2 e^{-\delta t}(1+\delta)^{-1}\left[-m(t)-\frac{u_{1,L}}{\delta(u_{1,L}+u_{2,L})}\right]. \quad (42)$$

Therefore, Firm 2 prefers for Firm 1 to remain private and hide when:

$$\begin{aligned}a_2(\alpha_{1,H})+b_2(\alpha_{1,H})m(0)<-\alpha_1^H(1+\delta)^{-1}\left[m(0)+\frac{u_{1,L}}{\delta(u_{1,L}+u_2)}-e^{-\delta T}m(T)-\frac{e^{-\delta T}u_{1,L}}{\delta(u_{1,L}+u_2)}\right] \\ +\frac{u_{1,L}+f_1-\alpha_2}{\delta}(e^{-\delta T}-1)+e^{-\delta T}\left[a_2(\alpha_{1,H}))+b_2(\alpha_{1,H})m(T)\right].\end{aligned} \quad (43)$$

This in turn implies that:

$$0<\frac{-\alpha_1^L}{(\alpha_1^L+\alpha_2)}-\frac{\alpha_1^L\alpha_2}{(\alpha_1^L+\alpha_2)^2}+\frac{\alpha_1^{H2}+2\alpha_1^H\alpha_2}{(\alpha_1^H+\alpha_2)^2}. \quad (44)$$

The rival firm is able to spend less to obtain market share under pooling, and (44) holds for all $\alpha_2 > 0$.[25]

Given U.S. regulations regarding disclosure for public firms, it can be very difficult for them to withhold information from rivals while complying with the requirement that they keep investors informed. The results in this example show that private financing may provide a mechanism through which firms can commit to less aggressive spending. The intuition behind this result can be compared to the capacity pre-commitment in Gelman and Salop (1983) in that by pre-committing (via private financing) to withhold information, an equilibrium outcome is one in which both firms spend less on market share gains. It is also analogous to the "Fat Cat Effect" in Fudenberg and Tirole (1984) in that spending on market share is a strategic complement and private financing provides a commitment device for less aggressive behavior.[26]

VI. Relationship to the Prior Literature

IPO activity has been extensively studied in both the theoretical and empirical literature. In their survey paper, Ritter and Welch (2002) provide several reasons for IPOs, including a role for product market competition (such as gaining a first-mover advantage via being the first firm in an industry to have an IPO). Our paper adds to this general literature by bring attention to the possibility that dynamic interactions between firms competing within a single industry might influence when and if a firm goes public.

This is, of course, not the first paper to posit a relationship between the product and financial markets. However, the previous literature has tended to focus on the strategic use of

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[25] When $\alpha_{1,L} = \alpha_{1,H}$, Equation (44) = 0. The partial derivative of the right-hand-side of (44) with respect to $\alpha_{1,H}$ equals $2\alpha_2^2 / \left(\alpha_{1,H} + \alpha_2\right)^3$, which is always positive for $\alpha_2 > 0$.

[26] Spulber (1995) also shows how, in Bertrand competition, not knowing rivals' costs implies equilibrium prices that are above marginal costs (i.e., information asymmetry softens product market competition).

debt in a firms' capital structure to obtain a competitive advantage. This happens because the debt distorts the firm's incentives away from future profit maximization. In an attempt to exploit the then outstanding debt firms may behave more aggressively leading them to generate higher profits. Although, in equilibrium these profits are often not realized as every firm in the industry takes out debt. In the end, industry profits can actually be lower than if firms were unable to lever up at all. Examples along these lines include Brander and Lewis (1986), Maksimovic (1988), and Bolton and Scharfstein (1990) (further details, examples, and discussion can be found in the survey by Maksimovic (1990)). On the empirical side of the literature, Chevalier (1995) provides evidence on the interaction between leverage and corporate behavior based on a sample of supermarkets following LBOs. Her analysis suggests that, contrary to the limited liability effect of debt effect that is predicted by much of the theoretical literature (e.g., Brander and Lewis (1986); Maksimovic (1988)), that instead debt "softens" product market competition. In contrast to this literature our paper's focus has been on the public-private decision rather than the leverage decision.

With the exception of Maksimovic and Pichler (2001) in the theoretical literature and recent empirical work by Chemmanur, He and Nandy (2006), little has been done to improve our understanding of the potential strategic role played by the private versus public financing decision. Given the size of the private equity (and debt) markets it is important to identify the factors that encourage a firm to use this source of financing rather than the public markets. Indeed, over the past decade or so the private equity market has grown significantly. It is reported to have peaked at $160 billion in 2000, up from $10 billion in 1991 was $40 billion in 2003. (See e.g., "The New Kings of Capitalism," The Economist, 11/27/2004, 373 (8403), 3-5.)

VII. Conclusions

The paper's main goal has been to answer the following questions: First, what are the characteristics of firms that benefit most from innovation and from private financing? Second, how important are industry structure, rival characteristics, and the nature of the innovation? In the context of a dynamic duopoly, we provide closed form solutions for the values of two competing firms, in a setting in which one firm faces an opportunity to innovate. If the technology is adopted, the firm must also determine whether it will obtain public or private financing. Our results relate current firm and industry characteristics to these decision variables. In particular, larger, more profitable firms with small rivals have the greatest incentives to innovate. The private versus public financing decision depends mainly on the magnitude of the technological improvement and length of the period during which private financing extends the innovator's product market advantage. The results from our model suggest that future empirical work examining financing patterns should also explicitly consider competitive dynamics.

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IX. Appendix

A. The General Model and its Solution

This appendix contains the solution to the most general version of the model discussed in this paper. For the problem described in Section I the value functions for the firms at time 0 are:

$$V_1 = \int_0^T (\alpha_1 m(t) - u_1 - f_1)e^{-\delta t}dt + B_1 \tag{45}$$

and

$$V_2 = \int_0^T (\alpha_2(1 - m(t)) - u_2 - f_2)e^{-\delta t}dt + B_2 \tag{46}$$

respectively. Here T is a terminal date on which the game ends, and B_i the present value of each firm's terminal value. Note, because the game ends at date T the value functions (V_i) depend both on m and the time remaining until date T.

Following standard practice in the literature on differential games the analysis seeks a Nash equilibrium in which the players use Markovian strategies (see Dockner, Jørgensen, Van Long, and Sorger (2000)). For each firm the instantaneous value functions given by (2) imply that in a Markovian Nash equilibrium the following Hamilton-Jacobi-Bellman (HJB) equations must hold:

$$0 = \max_{u_1} \quad \alpha_1 m - u_1 - f_1 + V_{1,m}\left\{\frac{\phi[(1-m)s_1u_1 - ms_2u_2]}{u_1s_1 + u_2s_2}\right\} + V_{1,t} - \delta V_1 \tag{47}$$

and

$$0 = \max_{u_2} \quad \alpha_2(1-m) - u_2 - f_2 + V_{2,m}\left\{\frac{\phi[(1-m)s_1u_1 - ms_2u_2]}{u_1s_1 + u_2s_2}\right\} + V_{2,t} - \delta V_2 \tag{48}$$

where $V_{i,m} = \partial V_i / \partial m$ and $V_{i,t} = \partial V_i / \partial t$ subject to the terminal condition that $V_i(T)$ equals $B_i(T)$.

The first order condition for Firm 1 is:

$$V_{1,m}\phi u_2 s_2 s_1 = (u_1 s_1 + u_2 s_2)^2 \tag{49}$$

and the correspondingly for Firm 2:

$$-V_{2,m}\phi u_1 s_1 s_2 = (u_1 s_1 + u_2 s_2)^2. \tag{50}$$

Equations (49) and (50) yield equilibrium spending of u_1^* and u_2^*:

$$u_1^* = -\frac{V_{1,m}^2 V_{2,m}\phi s_1 s_2}{(V_{1,m}s_1 - V_{2,m}s_2)^2} \tag{51}$$

and

$$u_2^* = \frac{V_{2,m}^2 V_{1,m}\phi s_1 s_2}{(V_{1,m}s_1 - V_{2,m}s_2)^2}. \tag{52}$$

Plugging the solutions for u_1^* and u_2^* into the HJB equations (47) and (48) yield the two differential equations (after some extensive algebra):

$$0 = \alpha_1 m - f_1 + \frac{V_{1,m}^3 \phi s_1^2}{(V_{1,m}s_1 - V_{2,m}s_2)^2} - V_{1,m}\phi m - V_{1,t} - \delta V_1 \tag{53}$$

and

$$0 = \alpha_2(1-m) - f_2 - \frac{V_{2,m}^3 \phi s_2^2}{(V_{1,m}s_1 - V_{2,m}s_2)^2} + V_{2,m}\phi(1-m) - V_{2,t} - \delta V_2 \tag{54}$$

that need to be solved.

The solutions for the value functions in (53) and (54) are determined by guessing and verifying that they take on the time dependent forms given by (3) and (4) at any date *t*. Those functional forms then imply that the derivatives with respect to *m* and *t* of the value functions equal:

$$V_{1,m} = -b_1, \qquad V_{1,t} = -a_1' - b_1' m \tag{55}$$

for firm 1 and

$$V_{2,m} = -b_2, \qquad V_{2,t} = -a_2' - b_2' m \tag{56}$$

for firm 2, where a' and b' represent each parameter's derivative with respect to time.

Plugging equations (55) and (56) into (53) and (54) yields a set of differential equations that need to be solved for the a_i and b_i terms subject to the boundary conditions $B_i(T)$. However, since the equalities have to remain true for all m there are in fact four equations that must hold, one for each a_i term and one for each b_i term. After some algebra this implies that solutions need to be found for the following four ordinary differential equations (ODE). For a_1 the ODE is

$$0 = -f_1 + \frac{\phi \alpha_1^3 s_1^2}{(\phi + \delta)(\alpha_1 s_1 + \alpha_2 s_2)^2} + a_1' - \delta a_1 \tag{57}$$

while for a_2 it is

$$0 = \alpha_2 - f_2 + \frac{\phi \alpha_2^3 s_2^2}{(\phi + \delta)(\alpha_1 s_1 + \alpha_2 s_2)^2} - \frac{\alpha_2}{\phi + \delta} + a_2' + \delta a_2. \tag{58}$$

Solving for the a_i in the above two equations yields (5) and (6). Next, collect the terms multiplying m to yield for b_1 the ODE:

$$\alpha_1 - \phi b_1 - b_1' - \delta b_1 = 0 \tag{59}$$

and for b_2:

$$-\alpha_2 - \phi b_2 + b_2' - \delta b_2 = 0. \tag{60}$$

Solving these last two equations for b_i produces equations (7) and (8).

Given equations (5) through (8) a particular problem's boundary conditions then determine the C_i and k_i terms and thus provide a full characterization of the economy's equilibrium behavior. The main body of the text presents various scenarios, their boundary value conditions, and the solutions they impose on the C_i and k_i terms. There can also be found the paper's analysis of the economy's overall behavior.

B. Solving for $a_i(t)$ and $b_i(t)$

1. Time Independent Case

In the time independent case the values a_i and b_i do not depend on t. Simple inspection of (5) through (8) then implies that the C_i and k_i terms must equal zero.

2. Time Dependent Case: Value Functions for $t \in [T_1, T_2)$

The solutions for the b_i given by (57), and (58) subject to the boundary value condition (14) can be found by inspection. Simply note that if the k_i terms equal zero then at date T_2 the b_i terms will satisfy (14).

To solve for the a_i terms one needs find C_i such that

$$\delta^{-1}\left[\frac{\phi\alpha_1^3 s_1^{*2}}{(\phi+\delta)(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - f_1\right] = \delta^{-1}\left[\frac{\phi\alpha_1^3 s_1^{*2}}{(\phi+\delta)(\alpha_1 s_1^* + \alpha_2 s_2)^2} - f_1\right] + C_1 e^{\delta T_2} \tag{61}$$

and

$$\delta^{-1}\left[(\phi+\delta)^{-1}\frac{\phi\alpha_2^3 s_2^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} + \frac{\delta\alpha_2}{\phi+\delta} - f_2\right] = \delta^{-1}\left[(\phi+\delta)^{-1}\frac{\phi\alpha_2^3 s_2^{2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} + \frac{\delta\alpha_2}{\phi+\delta} - f_2\right] + C_2 e^{\delta T_2} \tag{62}$$

hold. These yield values $C_{i,T2}$ (subscripts on the constants represent solutions to the value functions starting at date T_1 with boundary values given at T_2):

$$C_{1,T_2} = e^{-\delta T_2}(\delta(\phi+\delta))^{-1}\phi\alpha_1^3\left[\frac{s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2}\right] \tag{63}$$

and

$$C_{2,T_2} = e^{-\delta T_2}(\delta(\phi+\delta))^{-1}\phi\alpha_2^3\left[\frac{s_2^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{s_2^{2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2}\right]. \tag{64}$$

3. Time Dependent Case: Value Functions for t∈[0, T_1)

From the solution in the previous section, the boundary value conditions for the b_i are $b_1(T_1) = \alpha_1(\phi+\delta)^{-1}$ and $b_2(T_1) = -\alpha_2(\phi+\delta)^{-1}$ respectively. Plugging these into (59) and (60) yields the result that the k_i terms must equal zero. Thus, $b_1(t) = \alpha_1(\phi+\delta)^{-1}$ and $b_2(t) = -\alpha_2(\phi+\delta)^{-1}$ for all t.

The $a_i(t)$ terms must satisfy equations (5) and (6) subject to the boundary value conditions that $a_i(T_1)$ equals (5) and (6) with the C_i terms taken from (63) and (64) . Given this, the $C_{i,T1}$ must satisfy:

$$\begin{aligned}
&\delta^{-1}\left[\frac{\phi\alpha_1^3 s_1^{*2}}{(\phi+\delta)(\alpha_1 s_1^* + \alpha_2 s_2)^2} - f_1\right] + \\
&e^{-\delta(T_2-T_1)}(\delta(\phi+\delta))^{-1}\phi\alpha_1^3\left[\frac{s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2}\right] \\
&= \delta^{-1}\left[\frac{\phi\alpha_1^3 s_1^2}{(\phi+\delta)(\alpha_1 s_1 + \alpha_2 s_2)^2} - f_1\right] + C_{1,T1}e^{\delta T_1}
\end{aligned} \tag{65}$$

and

$$\begin{aligned}
&\delta^{-1}\left[(\phi+\delta)^{-1}\frac{\phi\alpha_2^3 s_2^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} + \frac{\delta\alpha_2}{\phi+\delta} - f_2\right] + \\
&e^{-\delta(T_2-T_1)}(\delta(\phi+\delta))^{-1}\phi\alpha_2^3\left[\frac{s_2^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{s_2^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2}\right] \\
&= \delta^{-1}\left[\frac{\phi\alpha_2^3 s_2^2}{(\phi+\delta)(\alpha_1 s_1 + \alpha_2 s_2)^2} - f_2\right] + C_{2,T1}e^{\delta T_1}.
\end{aligned} \tag{66}$$

Solving for $C_{1,T1}$ and $C_{2,T1}$ produces:

$$\begin{aligned}
C_{1,T_1} &= (\delta(\phi+\delta))^{-1}e^{-\delta T_1}\phi\alpha_1^3\left[\frac{s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} - \frac{s_1^2}{(\alpha_1 s_1 + \alpha_2 s_2)^2}\right] \\
&+ (\delta(\phi+\delta))^{-1}e^{-\delta T_2}\phi\alpha_1^3\left[\frac{s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2}\right]
\end{aligned} \tag{67}$$

and

$$C_{2,T_1} = (\delta(\phi+\delta))^{-1} e^{-\delta T_1} \phi \alpha_2^3 \left[\frac{s_2^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} - \frac{s_2^2}{(\alpha_1 s_1 + \alpha_2 s_2)^2} \right]$$
$$+ e^{-\delta T_2} (\delta(\phi+\delta))^{-1} \phi \alpha_2^3 \left[\frac{s_2^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{s_2^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} \right]. \quad (68)$$

C. Comparative Statics: Private versus Public Financing

1. $\partial\left(V_1^{\text{Private}} - V_1^{\text{Public}}\right) / \partial \alpha_1$

Equation (29) characterizes Firm 1's incentives to finance publicly as we vary its revenue-generating ability: $\frac{\partial (V_1^{\text{Private}} - V_1^{\text{Public}})}{\partial \alpha_1} = q\alpha_1^2 \left[\frac{\psi_2^2(\alpha_1\psi_2 + 3\alpha_2)}{(\alpha_1\psi_2 + \alpha_2)^3} - \frac{\psi_1^2(\alpha_1\psi_1 + 3\alpha_2)}{(\alpha_1\psi_1 + \alpha_2)^3} \right]$.

To sign this, note that at $\psi_1 = \psi_2$, (29) equals 0. Since $\frac{\partial \left(\frac{\alpha_1 \psi^3 + 3\alpha_2 \psi^2}{(\alpha_1 \psi + \alpha_2)^3} \right)}{\partial \psi} = (\alpha_1 \psi_2 + \alpha_2)^{-4} 6 \alpha_2^2 \psi$ is clearly positive, then for all $\psi_2 > \psi_1$, $\frac{\psi_2^2(\alpha_1 \psi_2 + 3\alpha_2)}{(\alpha_1 \psi_2 + \alpha_2)^3} > \frac{\psi_1^2(\alpha_1 \psi_1 + 3\alpha_2)}{(\alpha_1 \psi_1 + \alpha_2)^3}$. Thus, (29) is positive and the incentive to secure private financing for an innovation is increasing in Firm 1's revenue generating ability (α_1).

2. $\partial\left(V_1^{\text{Private}} - V_1^{\text{Public}}\right) / \partial \alpha_2$

Equation (30) shows how Firm 1's incentives to finance publicly vary with the profitability of Firm 2: $\frac{\partial (V_1^{\text{Private}} - V_1^{\text{Public}})}{\partial \alpha_2} = 2q\alpha_1^3 \left[\frac{\psi_1^2}{(\alpha_1\psi_1 + \alpha_2)^3} - \frac{\psi_2^2}{(\alpha_1\psi_2 + \alpha_2)^3} \right]$. Observe that at $\alpha_2 = 0$, Equation (30) is greater than zero since $\psi_2 > \psi_1 \Rightarrow \frac{1}{\psi_1} - \frac{1}{\psi_2} > 0$. Also note that

$$\frac{\partial\left(\frac{\psi^2}{(\alpha_1\psi+\alpha_2)^3}\right)}{\partial\psi}=(\alpha_1\psi_2+\alpha_2)^{-4}\left(2\alpha_2\psi-\alpha_1\psi^2\right).$$ This means that when $\alpha_2 > (\alpha_1\psi/2)$, Equation (30) is negative. Firm 1's incentives to obtain private financing are decreasing in α_2 when α_2 is large relative to α_1. The opposite is true for small α_2.

3. $\partial\left(V_1^{\text{Private}}-V_1^{\text{Public}}\right)/\partial\psi$

The impact of the size of Firm 1's relative spending advantage (ψ parameter) on incentives to finance publicly is also of interest. Since $\frac{\partial\left(\frac{\psi}{(\alpha_1\psi+\alpha_2)}\right)}{\partial\psi}=(\alpha_1\psi+\alpha_2)^{-2}\alpha_2>0$, Equation (28) implies that increasing Firm 1's spending advantage due to the technology (high ψ_2) will increase incentives to finance publicly. Conversely, increasing Firm 1's current spending advantage (ψ_1) decreases its incentive to finance innovations with privately issued securities.

4. $\partial\left(V_1^{\text{Private}}-V_1^{\text{Public}}\right)/\partial\phi$

Since $\partial q/\partial\phi>0$ and since ϕ does not enter (28) outside of q one has $\partial\left(V_1^{\text{Private}}-V_1^{\text{Public}}\right)/\partial\phi>0$.

Table 5: **Possible Empirical Proxies for the Model's Parameters**

Parameter	Description	Possible Empirical Proxies
m	Market share	Share of total industry: ▪ Sales ▪ Assets ▪ Market Value Equity + Book Vale of Total Debt
u	Spending to gain market share	▪ Advertising ▪ R&D
α	Revenue-generating ability	▪ Sales ▪ Operating Profit
s	Effectiveness of spending	▪ Estimation based on the discrete time version of equation (1): $m_{t+1} - m_t = \frac{\phi u_1 s_1}{u_1 s_1 + u_2 s_2} - \phi m_t$
f	Costs of Operations	▪ Operating Expenses (net of proxy for market share spending)
δ	r-g: discount rate minus industry growth rate	▪ Interest Rates ▪ Industry Growth Rate
T_1, T_2		▪ Patent protection periods ▪ R&D expenditures
Other Variables of Interest		
	Opportunities to Innovate	▪ Number of Patents ▪ R&D Expenditures
$\alpha_1 - \alpha_2$	Relative competitive advantage	▪ Difference in market shares ▪ Industry HHI