Arbitrage, Hedging, and Financial Innovation

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I consider the costs and benefits of introducing a new security in a standard framework where uninformed traders with hedging needs interact with risk-averse informed traders. Opening a new market may make everybody worse off, even when the new security is traded in equilibrium. This article emphasizes cross-market links between hedging and speculative demands: risk-averse arbitrageurs can use the new market to hedge their positions in the preexisting security, which can affect liquidity in the old market. More generally, the availability of such hedging opportunities will influence the strategies to which traders will direct resources.

This article has two purposes. The first is to analyze the economic consequences of introducing a new security. Welfare analysis of the standard finance models of trading with informational asymmetry has been rare, and has not previously been used to study the value of financial innovation.

The second purpose is to explore the links between speculative and hedging demands for securities. A trader who believes a security to be mispriced will not normally be able to predict its value perfectly: in other words, trading in the security will expose him to risk. However, by simultaneously trading in other securities he may be able

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to hedge these risks, at least partially and sometimes even perfectly to create a riskless arbitrage. Although strictly speaking "arbitrage" refers to an entirely riskless trade, I use the term in this article in the broader sense common among practitioners, that is, to describe speculative trading with a high expected value and a risk that may sometimes be partially hedged but generally cannot be eliminated entirely. Hence a large volume of hedging demand for securities may actually emanate from arbitrage activity. In this article, I use this link between hedging and arbitrage to analyze the welfare effects of introducing a new security.

The general problem of understanding the benefits and costs of financial innovation is complex, but some very basic questions in this area remain unanswered: Does opening a market tend to increase economic welfare? If the market in a new security attracts many traders does this indicate that the market is beneficial? The First Welfare Theorem of general equilibrium theory shows that an economy with complete markets is Pareto efficient. Hence, introducing new markets to an economy with incomplete markets will make at least one person better off, so long as sufficient new markets are introduced to make the set of markets complete. The statement that at least one agent is better off is of course very weak, but it has been intensively researched since it is the strongest statement that can be hoped for in general.

In reality, markets could never be complete in the literal sense required to apply this result. Yet introducing some new markets without opening all conceivable markets can in principle make everybody worse off, and it is simple to generate examples when there are externalities. Although these examples of externality do not apply to financial markets, Hart (1975) and Kemp and Sinn (1993) give examples of how opening financial markets could make everybody worse off. Cass and Citanna (1998) and Elul (1995) show that these examples are generic when there are several consumption goods, and arise because introducing a new market changes the relative prices of different consumption goods across different states of nature — an effect that seems intuitively negligible from the perspective of standard finance theory. Conversely, if there is only a single consumption good ("money"), introducing a new market must make at least one agent better off. The above models all use the full-information Walrasian model.

However, the standard approach in finance theory, following Kyle (1985) and Glosten and Milgrom (1985), is to use a model with a single consumption good (money) but with asymmetric information. This article uses this framework to consider the welfare effect of adding a new security. Although, with the market for the new security open, markets are not complete in the Arrow–Debreu sense, they are complete enough to allow agents to construct a perfect hedge against their initial risk exposure.

The basic idea explored in this article is of cross-market links between hedging and speculative demands. A risk-averse trader who believes an asset to be mispriced will typically be able to reduce the risk of speculating on his belief by hedging with other assets. The availability of such hedging instruments will in turn determine which types of speculative activity are of low risk, and this will influence the types of speculative strategies to which traders will direct resources. Although the links between arbitrage and hedging seem important, most research in the literature has focused on riskless arbitrage. Exceptions include Working (1953) and Oh (1994), who emphasized that arbitrage and hedging are in practice inextricably linked. Both of these articles are about spread trading in futures markets, where the link is particularly clear.

In practice, even the least risky "arbitrages" involve an element of risk. The following examples include both cases of virtually risk-free arbitrages, and of risky speculative positions where the risk may be reduced but not eliminated. (i) In stock index arbitrage, an investment bank buys individual stocks and sells short an index futures contract. To reduce transaction costs and execution problems, the stock portfolio consists of a much smaller number of stocks than the whole index, leaving a residual tracking risk. (ii) A bank may hold a speculative position in the debt of an emerging-market economy (denominated in US$) because it believes the probability of default to be implicitly overestimated by the current market prices. This speculative position exposes it to movements in U.S. interest rates, and it can hedge the exposure using interest rate futures. (iii) Stock pickers buy individual stocks they believe to be underpriced relative to the market in general, and hedge exposure to overall market movements by short-selling index futures. (iv) A bank taking a short position in the interest rate swap spread (the spread between the swap rate and the Treasury bond rate) holds a swap portfolio that is short the fixed side of the swap and long the floating side, and hedges it with a long position in bond futures, generating a hedging demand for bond futures that arises as a by-product of the development of the swap market. (v) Introduction of a new commodity futures contract with different delivery location to an existing contract may boost volume in both contracts as traders are induced to trade the location spread. (vi) A typical "arbitrage" position in the government bond market consists of a combination of long and short positions in different bonds designed to isolate the risk factor which the trader believes to be mispriced while minimizing exposure to other risks. (vii) Finally, one of the main trading strategies that the management of Barings believed Nick Leeson to be engaged in was a nearly risk-free arbitrage combining a long position in one contract, Nikkei 225 stock index futures traded on the Osaka Securities Exchange, with an offsetting short position in an almost identical contract traded on SIMEX.

This article shows that opening a financial market in a new security can in principle make everybody worse off. This can happen even though the new security trades successfully (generates high trading volume and liquidity). The intuition for the result is straightforward: risk-averse speculators can use hedging in the new market to eliminate the risk of their positions in the preexisting market. This causes a greater incidence of arbitrage activity in the old market, leading some traders with pure hedging motives in that market to withdraw. Liquidity in the old market is reduced, as manifested by a greater bid-ask spread for trades. This illiquidity, a form of the "lemons" or adverse selection effect, can harm both arbitrageurs and hedgers.

This liquidity effect is similar to the "destructive interference" between different securities in Bhattacharya, Reny, and Spiegel (1995), but it is weaker because here the market does not close completely. In the context of a single security market, Glosten (1989), Bhattacharya and Spiegel (1991), and Spiegel and Subrahmanyam (1992) have studied market breakdown due to an excessive incidence of informed trading. An interesting contrast to the "adverse selection" effect studied here is provided by Ohashi's (1994) example of damaging security innovation, based on the Hirshleifer effect whereby premature revelation of information prevents insurance [Hirshleifer (1971)].

The model is simple and abstracts from many potentially important effects of financial innovation, for example, the possible improvement in production and investment decisions caused by more efficient prices. As usual in the literature on financial innovation, this article studies an exchange economy with no production [see Allen and Gale (1994) and Duffie and Rahi (1995) for surveys of the literature].

The rest of the article shows the result in the context of a formal model. Section 1 describes the model. Section 2 describes equilibrium in the preexisting market. Section 3 describes equilibrium when the new market opens. The possibility of Pareto-inferior innovation is demonstrated in Section 4. Section 5 presents brief concluding remarks.

1. The Model

I consider a model with two periods. In the first period, agents trade securities. In the second period, securities' payoffs are realized and agents consume. The securities may be thought of as derivative contracts, or alternatively as stocks that pay a liquidating dividend, or as zero-coupon bonds. The interpretation as derivative contracts will perhaps seem most natural to the reader.

1.1 Securities

The first security has an uncertain payoff $\tilde{x}$ that can be decomposed into two independent components: $\tilde{x} = \tilde{y} + \tilde{z}$. The second security has payoff $\tilde{y}$. The random variables $\tilde{y}$ and $\tilde{z}$ are each equally likely to take the values $+1$ and -1, hence $\tilde{x}$ takes the value 0 with probability $\frac{1}{2}$, and the values -2 and $+2$ each with probability $\frac{1}{4}$. (The assumption that the expected values of the securities are of mean zero entails no loss of generality, since one could interpret the variables treated here as deviations from the mean, and add on the mean as a constant in all expressions for valuations and prices.)

1.2 Agents

There are two main types of agents. The first type (hedgers) are risk averse and have risky endowments which they can hedge by trading suitable quantities of securities. The second type of agent (an arbitrageur) may also be risk averse but is not endowed with risk initially. He has private information about the value of the first security, $\tilde{x}$, although he cannot predict its value perfectly: he learns $\tilde{z}$. Taking a position in the first security therefore exposes him to risk, which he may choose to hedge using the other security, $\tilde{y}$, if it is traded. The relative frequencies in the market of the arbitrageur and the hedgers affect the liquidity of the market. It will be convenient to think of a given order as a sample from a large population of hedgers and arbitrageurs. The frequency of arbitrageurs is denoted π, the frequency of hedgers is $(1 - \pi)$.

1.3 Trading and price formation

Prices are set by a risk-neutral market maker. He observes one order at a time, infers (in equilibrium) the relative likelihood that the order emanates from an informed trader or a hedger, sets price equal to conditional expected value, and meets the order from inventory [as in Glosten and Milgrom (1985)]. In case both markets are open, the market makers for each security are able to communicate.

1.4 Hedgers

Hedgers have a concave utility function U and a risky endowment of $\pm\tilde{z}$. They first find out whether their endowment is $\tilde{z}$ or $-\tilde{z}$, then they are able to trade, and then $\tilde{z}$ is realized. In case the endowment is $\tilde{z}$, they have a negative hedging need for the first security: they will hedge their endowment risk by selling short one unit of the security. Conversely, if the endowment is $-\tilde{z}$, they have a positive hedging need and can hedge with a long position; positive and negative hedging needs are equiprobable.

Hedgers are of two types: they may either be very risk averse or less risk averse. With probability δ the hedgers (if present) are very risk averse, and with probability $(1 - \delta)$ they are less risk averse. For the very risk-averse type, the hedgers' utility function is $U(w) = w$ for $w \leq 0$, and $U(w) = \alpha w$ for $w > 0$, where $\alpha \in (0, 1)$. This may also be written as $U(w) = \min\{w, \alpha w\}$, a more compact notation that will be used below. For technical reasons the hedgers should be thought of as a unit mass of a continuum of infinitesimal agents with identical endowments [see Dow and Gorton (1997)]. The less risk-averse type has a similar utility function, but with a different parameter: $U(w) = \min\{w, \beta w\}$ where $\beta \in (\alpha, 1)$.

Table 1
Assumptions on random variables, information structure, and hedging needs

Random variables	$\tilde{x} = \tilde{y} + \tilde{z}$ $\tilde{y}, \tilde{z}$ iid
Markets	First security has payoff $\tilde{x}$ Second security has payoff $\tilde{y}$
Arbitrageur's information	Learns $\tilde{z}$
Hedger's endowment	$\pm\tilde{z}$

1.5 Arbitrageurs

Arbitrageurs know the realization of $\tilde{z}$ in advance of the trading round. An arbitrageur should be interpreted as a financial institution (bank, investment bank, hedge fund, etc.) whose risk aversion in a given market at any time depends on the positions it has taken in other markets, in relation to its capital. This may be represented by $U(w) = \min\{w, \gamma w\}$ [for $\gamma \in (0, 1)$], with an initial endowment of 0 with probability $1 - \epsilon$ and 100 with probability ϵ. (The number 100 was chosen simply because it is a large number in relation to the amounts of money that can be made or lost by speculating in the assets considered here; hence, when the initial wealth is 100, there is no possibility for the agent's wealth to reach the risk-averse region at 0.)

Effectively, therefore, an arbitrageur acts as if his risk aversion were variable (and his endowment constant), that is, as if with probability ϵ he were risk neutral, and with probability $(1-\epsilon)$ he were risk averse with utility function $U(w) = \min\{w, \gamma w\}$. For convenience, I shall refer to a "risk-neutral arbitrageur" or a "risk-averse arbitrageur" rather than an "arbitrageur whose initial wealth realization is 100" or an "arbitrageur whose initial wealth realization is 0."

The assumptions on the random variables, the information structure and the hedging needs are summarized in Table 1.

1.6 Remarks on the assumptions

1.6.1 Functional forms. The functional forms of the utility functions and the random variables have been chosen to be the simplest possible, but the results are not dependent on these specific functional forms. A previous version of this article (available on request from the author) showed the same results using CARA utility and normal random variables, using essentially a two-asset version of Spiegel and Subrahmanyam (1992) with a risk-averse informed trader. It is not included here since the equilibrium in terms of all the exogenous parameters has no closed-form solution and can only be performed numerically. It is interesting to note, however, that the CARA/normal version of the model does not need two different types of hedgers, nor the random endowment for the arbitrageur, since with these functional forms agents respond continuously to changes.

1.6.2 Price formation process. For the purposes of this article, the details of the trading process are not particularly important. What is needed is a trading mechanism that allows only partial revelation of private information (this implies that privately informed traders will profit to some extent from their information). The mechanism used here, proposed by Glosten and Milgrom (1985), is often used in the literature: recent examples include Bhattacharya and Krishnan (forthcoming), Biais and Hillion (1994), and Madhavan (1995). An earlier version of this article (available on request from the author) used the price formation mechanism of Kyle (1985) to derive the same results. The advantage of the Glosten–Milgrom process is that agents do not face execution risk in their orders, that is, when they place an order they know exactly at what price it will clear.

1.6.3 Information flow between the two markets. The assumption that the market makers for different securities can communicate is not important for the results. In fact, it is easy to verify that in this model there is no information gained (in equilibrium) from observing the orders in the other market, so that it makes no difference whether communication is possible.

2. Equilibrium with One Market

I start by considering the case where only the first market is open: I assume that the second market is exogenously closed, perhaps by regulatory fiat. Since similar models have been studied in the literature following Glosten and Milgrom (1985) and Kyle (1985), this is a standard problem. In equilibrium the hedgers will trade an amount $\pm t$, to be determined below. It follows that arbitrageurs will also trade $\pm t$ to avoid detection (if they do choose to trade). Any other quantity would reveal the information about the asset value to the market. There is a possibility that an arbitrageur, when risk averse, will choose not to trade at all, because speculation forces him to bear the $\tilde{y}$-risk. Alternatively, both risk-averse and risk-neutral arbitrageurs may trade; which type of equilibrium occurs will depend on the values of the exogenous parameters. The focus of this article is on the case where the exogenous parameters (particularly the cautiousness of the risk-averse arbitrageur) are such that in equilibrium the arbitrageur trades only if he is risk neutral, while both types of hedger trade.

Since risk-averse arbitrageurs [a fraction ($\pi(1-\epsilon)$ of traders] are inactive, a fraction $\pi\epsilon$ of traders are (risk neutral) arbitrageurs, and a fraction $(1-\pi)$ of traders are hedgers, it follows that the conditional probability of an order coming from an informed trader is $\pi\epsilon/[1-\pi(1-\epsilon)]$. Define p to be the price on a buy order. Since the expected value of the asset is one if an informed agent buys and zero if an uninformed hedger buys, we have

$$p = \pi\epsilon/[1 - \pi(1 - \epsilon)].$$

Similarly the price on a sell order is $-p$.

Next I verify the condition for an arbitrageur to choose not to trade when he is risk averse. Recall that his utility function is $\min\{w, \gamma w\}$, with a random initial endowment, and he is effectively risk averse when the realization of his endowment is zero which occurs with probability $(1 - \epsilon)$. Suppose that he learns that the realization of $\tilde{z}$ will be one. If he buys the asset, his wealth will be

$$t(\tilde{x} - p),$$

that is, it is equally likely to be $t(2 - p)$ or $-tp$ (in case $\tilde{y} = 1$ or -1, respectively). Similarly if he learns that $\tilde{z} = -1$ and sells the asset at price $-p$, his wealth is $t(-p - \tilde{x})$, that is, it is equally likely to be $t(-p + 2)$ or $-tp$. So his expected utility (given initial wealth zero) is

$$\tfrac{1}{2}t[\gamma(2 - p) - p].$$

He will choose not to trade whenever this is negative, that is, when $\gamma < p/(2 - p)$. Alternatively the arbitrageur may act as if he is risk neutral (i.e., his endowment realization is 100), which occurs with probability ϵ. Then his expected trading profits are $t(1 - p)$. His wealth is equally likely to be $100 + (2 - p)$ or $100 - p$, and his expected utility (given initial wealth of 100) is

$$\tfrac{1}{2}\gamma[100 + t(2 - p) + 100 - tp] = \gamma[100 + t(1 - p)].$$

In the case where he chooses not to trade if risk averse [i.e., $\gamma < p/(2 - p)$], his expected utility (averaging across the different initial endowments) is

$$(1 - \epsilon)0 + \epsilon\gamma[100 + t(1 - p)] = \epsilon\gamma[100 + t(1 - p)].$$

Next consider the hedgers' behavior. Recall that the hedgers' initial endowment is $\pm\tilde{z}$, so by trading $\tilde{x}$ they will not be perfectly hedged; they will still be exposed to $\tilde{y}$. For the hedgers with a positive hedging need who buy t units of the security at price p, their wealth is

$$t(\tilde{x} - p) - \tilde{z} = t(\tilde{y} + \tilde{z} - p) - \tilde{z}.$$

As shown in Appendix A, the expected utility of the (more risk averse) hedgers when they buy a quantity t is

$$\tfrac{1}{4}[\min\{t(2 - p) - 1,\ \alpha[t(2 - p) - 1]\} + \alpha(1 - tp) + (-1 - tp) + \min\{1 - t(2 + p),\ \alpha[1 - t(2 + p)]\}].$$

Their optimal hedging policy is to buy a quantity

$$t = 1/(2 + p)$$

if $\alpha < (1 - p)/(1 + p)$, and not to trade at all otherwise. Symmetrically, hedgers with a negative hedging need will sell $t = 1/(2 + p)$ as long as $\alpha < (1 - p)/(1 + p)$. Their resulting expected utility is

$$\tfrac{1}{2}(\alpha - 1 - 2p)/(2 + p).$$

Again, Appendix A gives details of the computations for the above expressions. For the less risk-averse hedgers, similar expressions hold with α replaced by β, that is, they trade a quantity $t = 1/(2 + p)$ as long as $\beta < (1-p)/(1+p)$, obtaining an expected utility of $\frac{1}{2}(\beta-1-2p)/(2+p)$.

To summarize the above derivation, the article will consider situations where (with only the first market open) both types of hedgers trade, but an arbitrageur trades only when risk neutral. This imposes the following restrictions on the exogenous parameters: $\beta < (1 - p)/(1 + p)$ and $\gamma < p/(2 - p)$, where $p = \pi\epsilon/(1 - \pi(1 - \epsilon))$.

3. Equilibrium with Both Markets Open

Now consider the case where both markets are open. An arbitrageur who takes a position in $\tilde{x}$ will be able to hedge his risk entirely in the new market, the market for $\tilde{y}$, and perform a riskless arbitrage. Hence he will always trade, even if risk averse. This increased arbitrage activity makes the equilibrium price less favorable to the hedgers, and the less risk-averse hedgers may drop out of the market. The article focuses on the case where the exogenous parameters are such that this is indeed what happens.

Since the less risk-averse hedgers are inactive [a fraction $(1 - \pi)(1 - \delta)$ of traders], while the population of traders contains a fraction π of informed traders, and a fraction $(1 - \pi)\delta$ of more risk-averse hedgers, the price on a buy order, which is equal to the conditional probability of an order coming from an informed trader, is

$$p' = \pi/[1 - (1 - \pi)(1 - \delta)].$$

Similarly the price on a sell order is $-p'$. The order size is denoted t'. Note that in the market for $\tilde{y}$ (the new market), on the other hand, the price is always zero since there is no private information.

If an arbitrageur buys t' of asset $\tilde{x}$ and sells an equal amount of asset $\tilde{y}$, his trading profits will be $t'(\tilde{x} - p') - t'\tilde{y} = t'(\tilde{z} - p')$. So when he learns $\tilde{z} = 1$, he buys $\tilde{x}$, sells $\tilde{y}$, and makes a sure profit of $t'(1 - p')$. Similarly when he learns $\tilde{z} = -1$, he sells $\tilde{x}$, buys $\tilde{y}$, and makes the same profit. His expected utility (averaging across the different initial endowments) is therefore

$$\gamma[t'(1-p')+100\epsilon].$$

Next consider the hedgers' behavior. They can hedge their endowment risk perfectly by taking appropriate positions in $\tilde{x}$ and $\tilde{y}$, but they may choose not to do so because of the cost of hedging (reflected in a high p'). For a hedger with a positive hedging need who buys t' units of $\tilde{x}$ and sells t' units of $\tilde{y}$, terminal wealth is $t'(\tilde{x}-p')-t'\tilde{y}-\tilde{z}=t'(\tilde{z}-p')-\tilde{z}$. As shown in Appendix B, this results in expected utility (for the more risk-averse hedger) of

$$\tfrac{1}{2}[\min\{1-t'(1+p'),\ \alpha(1-t'(1+p'))\}]+\tfrac{1}{2}[t'(1-p')-1],$$

which is maximized at $t'=1/(1+p')$ so long as $\alpha<(1-p')/(1+p')$. The case of a negative hedging need is symmetric. The discussion here has not derived the result that hedgers take equal and opposite positions of the same size in $\tilde{y}$ and $\tilde{z}$, but that is straightforward to show. Similarly, the less risk-averse hedgers choose not to trade if $\beta>(1-p')/(1+p')$. If they do hedge, the hedgers' maximal expected utility is $-p'/(1+p')$. Details are in Appendix B. Note that the quantity traded by the hedgers (if they do trade) is larger when both markets are open: $t'=1/(1+p')>\frac{1}{2}>1/(2+p)=t$. The reason is simply that with both markets open, the security is a better hedge (has a higher delta).

To summarize the above derivation, the article will consider situations where (with both markets open) the arbitrageur always trades, but only the more risk-averse hedgers trade. This imposes the following restrictions on the exogenous parameters: $\alpha<(1-p')/(1+p')<\beta$ where $p'=\pi/[1-(1-\pi)(1-\delta)]$.

4. Opening a New Market

The implications of the above analysis can now be drawn together into the following statement:

Theorem. *It is possible that opening a new market may make everybody worse off.*

Proof. I will show that there are values of the exogenous parameters, α, β, γ, δ, ϵ, and π, where all agents are made worse off. To start with, consider the restrictions derived in Sections 2 and 3, that is, $\beta<(1-p)/(1+p)$ and $\gamma<p/(2-p)$, where $p=\pi\epsilon/(1-\pi(1-\epsilon))$, and $\alpha<(1-p')/(1+p')<\beta$, where $p'=\pi/[1-(1-\pi)(1-\delta)]$. As was shown in Sections 2 and 3, they imply that when the new market opens the less risk-averse hedgers will cease to trade, while the risk-averse arbitrageurs, previously inactive, will now be willing to take speculative positions. Clearly, the conditions on α, β, and γ can always be satisfied for given values of δ, ϵ, and π by choosing α and γ sufficiently small, and β in the interval $[(1-p')/(1+p'),\ (1-p)/(1+p)]$.

It is clear that the less risk-averse hedgers are worse off, by revealed preference. For the more risk-averse hedgers, the effect is ambiguous since although they now face a wider bid-ask spread in the market for $\tilde{x}$, they are also now able to trade in $\tilde{y}$, which gives them a better hedge for their initial risk exposure. For them to be worse off, we require

$$-p'/(1+p') < \tfrac{1}{2}(\alpha - 1 - 2p)/(2+p).$$

As shown in Appendix C, substituting for p and p' in terms of the exogenous parameters, this is implied by

$$2\pi[2(1-\pi) + 3\pi\epsilon] > (1-\pi+3\pi\epsilon)[2\pi + \delta(1-\pi)]. \tag{1}$$

An arbitrageur now faces a wider spread in the market for $\tilde{x}$, but is able to construct a riskless arbitrage by trading in $\tilde{y}$ also. For him to be worse off we need

$$\gamma(1-p')/(1+p') < \epsilon\gamma(1-p)/(2+p).$$

Substituting for p and p', this becomes

$$\delta/[2\pi + \delta(1-\pi)] < \epsilon/[2 - 2\pi + 3\pi\epsilon], \tag{2}$$

as shown in Appendix C. I will show that Inequalities (1) and (2) may be satisfied simultaneously by setting $\epsilon = k\delta$ for fixed k and considering the limit for small δ (hence small ϵ also). In the limit, Inequality (1) becomes

$$4\pi(1-\pi) > (1-\pi)2\pi,$$

which clearly holds. Setting $\epsilon = k\delta$ in Inequality (2) gives $\delta/[2\pi + \delta(1-\pi)] < k\delta/[2 - 2\pi + 3\pi k\delta]$, that is,

$$k(2\pi + \delta(1-\pi)) > 2(1-\pi) + 3k\pi\delta.$$

In the limit, this becomes $2k\pi > 2(1-\pi)$, that is, it will hold if we set $\pi > 1/(k+1)$. For example, $k = \frac{1}{2}$ and $\pi = \frac{3}{4}$. ■

The intuition behind the parameter values in the example is straightforward. For small γ, the arbitrageur is very risk averse at the lower initial wealth level (0) so is unwilling to trade except when arbitrage is riskless. Also, small α means that the very risk-averse hedger will be willing to hedge with both markets open, even if the market (for $\tilde{x}$) is highly illiquid because of the high incidence of informed trading. β has to be chosen so that the

Table 2
Conditions for the new security to make each type of agent better or worse off

Agent	Worse off if
Hedger	$-p'/((1-p') < \frac{1}{2}(\alpha - 1 - 2p)/(2+p)$ and $\alpha < (1-p')/(1+p')$, or $\alpha > (1-p')/(1+p')$
Arbitrageur	$\epsilon\gamma(1-p)/(2+p) > \gamma(1-p')/(1+p')$ and $\gamma < p/(2-p)$, or $\gamma(1-p)/(2+p) + \frac{1}{2}p(1-\epsilon)(\gamma-1)/(2+p) > \gamma(1-p')/(1+p')$ and $\gamma > p/(2-p)$

The prices are given by $p = \pi\epsilon/[1-\pi(1-\epsilon)]$ (before the new security is introduced) and $p' = \pi/[1-(1-\pi)(1-\delta)]$ (with the new security). The table gives conditions for the hedger with utility $\min\{w, \alpha w\}$; similar expressions with β substituted for α hold for the hedger with utility $\{w, \beta w\}$.

less risk-averse hedgers are risk averse enough that they want to trade when the first market is quite liquid (and the second market is closed), but not so risk averse that they continue wanting to hedge when the first market becomes less liquid (as a result of the second market opening). π large enough ensures there is enough arbitrage activity to make the effects studied here quite strong. When ϵ becomes small, the arbitrageur is more likely to be risk averse: hence opening the new market causes a big increase in arbitrage activity in the market for $\tilde{x}$. When δ becomes small, most hedgers are less risk averse and will cease trading when the new market is open. Both the last two effects (small δ and ϵ) cause an increased bid-ask spread in the market for $\tilde{x}$ as a result of the new market opening, and this tends to damage both arbitrageurs and the more risk-averse hedgers who continue to trade.

More generally, we can compute the conditions under which, depending on the parameters of the model, each type of agent will be better off or worse off. For both hedgers and the speculator, the result depends on the trade-off between two opposing effects: they can hedge better, but the bid-ask spread is wider. The arbitrageur must also take into account the fact the quantity traded by the hedgers is bigger in the case where both markets are open (i.e. as noted above in Section 3, $t < t'$). The conditions that make each type of agent better or worse off are summarized in Table 2.

5. Concluding Remarks

The model studied here is very simple. It focuses on cross-market liquidity effects, and ignores many other effects such as the implications of more efficient security prices for allocative efficiency, individual incentives for futures exchanges or other financial innovators that may differ from social incentives, etc. Subject to these provisos, however, one can venture the following observations: introducing a new security in this type of model will increase the incidence of arbitrage activity by making informed trading less risky. For hedgers, there is a trade-off between the increased illiquidity that may result and the greater flexibility to design appropriate hedges with a wider range of securities. Only if the former effect is very strong will they reduce their trades to the extent that even arbitrageurs are worse off — a possibility that is demonstrated in this article.

One limitation of the model is that it does not take account of the effect of security prices on production and investment decisions in the economy: generally, more efficient security prices should convey better information to producers and hence lead to better production decisions [as noted in Hirshleifer (1971)]. This effect is also ignored by most of the recent literature on market microstructure and on financial innovation, for example, Kyle (1985), Glosten and Milgrom (1985), Allen and Gale (1994), and the articles in the 1995 *Journal of Economic Theory* symposium issue on financial innovation [including Duffie and Rahi (1995), Elul (1995), and Bhattacharya, Reny, and Spiegel (1995)]. Dow and Rahi (1996) model the effect on improved investment decisions in a model of share price formation. However, they show that depending on the parameters of the economy the beneficial effect on allocative efficiency may be either arbitrarily large or arbitrarily small and, since this effect does not interact with the effects studied in this article, this provides some justification for omitting it from the formal models. Note, however, that Newbery and Stiglitz (1984) demonstrate that in principle, in an incomplete markets setting, introducing a new market may actually reduce productive efficiency.

There are many interesting questions for further research on financial innovation. One of them concerns the order of opening of securities markets. In the model presented here, the order of opening of markets makes a difference in that the market for $\tilde{y}$ is of no value, and would generate no trading, without the market for $\tilde{x}$. If the market for $\tilde{y}$ were open first, opening the market for $\tilde{x}$ would benefit everybody — though not as much, of course, as simultaneously closing the market for $\tilde{y}$. In that sense, the model in this article exhibits the property that the order of opening of markets makes a difference. However, a more satisfactory, more dynamic treatment of this question would capture the fact that a market may become established, and liquidity develop over time, as a result of people trading the security.

Finally, a more general implication of the model in this article is that a large part of trading volume may be due to speculators' hedging needs. This is of interest because of the general inability of finance theory to explain high trading volume [see Dow and Gorton (1997)]. Effects like the ones studied in this article can arise under a wide variety of conditions, and in liquid markets a speculator may take large hedging positions in several different markets.

Appendix A

Hedger's optimization problem when only one market is open. As derived in the main text, the wealth of the (more risk averse) hedgers with a positive hedging need who buy t units of the security at price p is $t(\tilde{x} - p) - \tilde{z} =$

$t(\tilde{y} + \tilde{z} - p) - \tilde{z}$. Depending on the realized values of $\tilde{y}$ and $\tilde{z}$, there are four possible values of wealth:

(i) $\tilde{y} = 1, \tilde{z} = 1$ (which implies $\tilde{x} = 2$).
Wealth: $t(2 - p) - 1$
Utility: $\min\{t(2 - p) - 1, \alpha[t(2 - p) - 1]\}$.

(ii) $\tilde{y} = 1, \tilde{z} = -1$ (which implies $\tilde{x} = 0$).
Wealth: $t(0 - p) + 1$
Utility: $\alpha[1 - tp]$.

(iii) $\tilde{y} = -1, \tilde{z} = 1$ (which implies $\tilde{x} = 0$).
Wealth: $t(0 - p) - 1$
Utility: $-1 - tp$.

(iv) $\tilde{y} = -1, \tilde{z} = -1$ (which implies $\tilde{x} = -2$).
Wealth: $t(-2 - p) + 1$
Utility: $\min\{1 - t(2 + p),\ \alpha[1 - t(2 + p)]\}$.

The expected utility of the hedger is therefore

$$\tfrac{1}{4}[\min\{t(2 - p) - 1,\ \alpha[t(2 - p) - 1]\} + \alpha(1 - tp) + (-1 - tp) \\ + \min\{1 - t(2 + p),\ \alpha[1 - t(2 + p)]\}].$$

Note that of the four equiprobable possible values for wealth: $t(2-p)-1$, $1-tp$, $-1-tp$, $1-t(2+p)$, the first is positive for $t > 1/(2-p)$, while the last is positive for $t < 1/(2+p)$ (the second is always positive and the third always negative). Expected utility is therefore linear in t on each of the three intervals $[0, 1/(2+p)]$, $[1/(2+p), 1/(2-p)]$ and $[1/(2-p), 1]$. If follows that one of the four values 0, $1/(2+p)$, $1/(2-p)$, 1 must be optimal for t.

On the interval $t \in [0, 1/(2+p)]$, expected utility is

$$\tfrac{1}{4}[t(2 - p) - 1 + \alpha(1 - tp) - (1 + tp) + \alpha(1 - t(2 + p))],$$

whose derivative with respect to t is $\frac{1}{2}(1-p-\alpha(1+p))$. This is positive iff $\alpha < (1-p)/(1+p)$. Similarly, on the interval $t \in [1/(2+p), 1(2-p)]$, expected utility is

$$\tfrac{1}{4}[t(2 - p) - 1 + \alpha(1 - tp) - (1 + tp) + (1 - t(2 + p))],$$

whose derivative with respect to t is $\frac{1}{4}(-3p - \alpha p) < 0$. Finally, on the interval $t \in [1(2-p), 1]$, expected utility is

$$\tfrac{1}{4}[\alpha(t(2-p)-1) + \alpha(1-tp) - (1+tp) + (1-t(2+p))],$$

whose derivative with respect to t is $\frac{1}{2}[\alpha(1-p)-(1+p)] < 0$. This shows the hedgers will never trade more than $1/(2+p)$.

Appendix B

Hedger's optimization problem when both markets are open. As derived in the main text, the wealth of the (more risk averse) hedgers with a positive hedging need who trade t' units of the securities is $t'(\tilde{z} - p') - \tilde{z}$. Depending on the realization of $\tilde{z}$, wealth is equally likely to be either $[t'(-1-p')+1]$ or $t'(1-p')-1]$. Note that the latter is always negative, while the former is positive if $t' < 1/(1+p')$. Hence expected utility is

$$\tfrac{1}{2}[\min\{1 - t'(1+p'),\ \alpha(1-t'(1+p'))\}] + \tfrac{1}{2}[t'(1-p') - 1].$$

On the interval $t \in [0, 1/(1+p')]$ this is

$$\tfrac{1}{2}\alpha[1 - t'(1+p')] + \tfrac{1}{2}[t'(1-p') - 1],$$

which is increasing in t' if the derivative $\frac{1}{2}\alpha(1+p') + \frac{1}{2}(1-p')$ is positive, that is, $\alpha < (1-p')/(1+p')$. On the interval $t \in [1/(1+p')$ expected utility is

$$\tfrac{1}{2}[1 - t'(1+p')] + \tfrac{1}{2}[t'(1-p') - 1] = -t'p',$$

so the hedgers will never trade more than $1/(1+p')$. Finally note that expected utility at $t' = 1/(1+p')$ is $-p'/(1+p')$.

Appendix C

Details for proof of theorem. Consider the inequality in the text:

$$-p'/(1+p') < \tfrac{1}{2}(\alpha - 1 - 2p)/(2+p).$$

Since the right-hand side exceeds $-\frac{1}{2}(1+2p)/(2+p)$, a sufficient condition for the inequality is given by $-p'/(1+p') < -\frac{1}{2}(1+2p)/(2+p)$, or substituting $p = \pi\epsilon/(1-\pi(1-\epsilon))$ and $p' = \pi/[1-(1-\pi)(1-\delta)]$:

$$-\pi/[1-(1-\pi)(1-\delta)+\pi] < -\tfrac{1}{2}(1-\pi(1-\epsilon)+2\pi\epsilon)/[2-2\pi(1-\epsilon)+\pi\epsilon]$$

or $2\pi[2(1-\pi) + 3\pi\epsilon] > (1-\pi+3\pi\epsilon)[2\pi + \delta(1-\pi)]$, which is Inequality (1). Next consider

$$\gamma(1-p')/(1+p') < \epsilon\gamma(1-p)/(2+p).$$

Substituting for p and p' we obtain

$$\begin{aligned}&[1-(1-\pi)(1-\delta)-\pi]/[1-(1-\pi)(1-\delta)+\pi]\\&\quad<\epsilon(1-\pi(1-\epsilon)-\pi\epsilon)/[2-2\pi(1-\epsilon)+\pi\epsilon]\end{aligned}$$

or $\delta(1-\pi)/[2\pi+\delta(1-\pi)] < \epsilon(1-\pi)/[2-2\pi+3\pi\epsilon]$. Cancelling $(1-\pi)$, we obtain Inequality (2).

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