The Cost of Debt

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Abstract

This paper proposes a practical way of estimating the cost of risky debt for use in the cost of capital. The cost of debt is different from both the promised yield and the risk-free rate, which are sometimes used for this purpose, because of the expected probability of default. The Merton (1974) model of risky debt is employed to decompose the promised yield spread into expected default and return premium components. The advantage of the proposed approach is that all inputs are easily observable. The parameters of the Merton model implied by these inputs are used to compute the expected return on debt. It is argued that, although Merton's framework is simple and stylised, it can be used to estimate the expected return as a fraction of the observed promised market yield in a way consistent with equilibrium. The cost of debt is computed for parameter values that are typical for high grade and low grade debt. It is found that, while using the promised yield as the cost of debt may be adequate for high grade debt, it is likely to cause significant errors for high-yield bonds. In such cases the approach proposed in this paper can be used to adjust the WACC for the probability of default on the firm's debt.

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1 Introduction

This paper proposes an easily implementable analytical method of estimating the expected return on debt, which is one of the most important inputs to the average weighted cost of capital (WACC). The WACC is the required return on the operating assets of a firm. It is used in valuation, capital budgeting, goal-setting, performance measurement and regulation. Its value is one of the most important issues in corporate finance. Yet little attention has thus far been focused on estimating one of its key inputs - the cost of debt. It is shown below that employing existing methods may result in significant errors in estimating the WACC.

The WACC cannot be observed and so must be estimated. The standard estimation method is to take a weighted average of the estimated expected returns on debt and equity:

$$WACC = (1 - p_E)r_D + p_E r_E \tag{1}$$

where: p_E is the market-value proportion of the equity of the firm, r_D is the required return or the 'cost' of debt, and r_E is the required return or 'cost' of equity (Brealey and Myers (2000) and Bruner et al (1998)).[1] The expected return on equity is formed by:

$$r_E = r + \Pi_E \tag{2}$$

where r is the risk-free rate, and Π_E is the equity risk premium. The cost of equity is typically estimated using the CAPM, APT or variants of the dividend growth model. Its estimation has been the subject of extensive debate (see, for example, Bruner et al (1998) and Welch (2000)).

By contrast, little attention has been focused on estimating the cost of debt in the context of WACC estimation.[2] The most common way of estimating the cost of debt is to use the promised yield on newly-issued debt of the firm (see, for instance, Erhardt (1994)). However, this is not correct. The expected return on debt should allow for the probability of default whereas the promised yield does not. If the

[1] Taxes are ignored throughout.

[2] Although models of credit risk have been widely studied (for good examples see Jarrow, Lando and Turnbull (1997) and Duffie and Singleton (1999)), none is in a form that can be easily used to obtain the cost of debt for WACC estimation.

promised yield is used for the cost of debt then the WACC will be too high. In extreme cases, use of the promised yield as the cost of debt could lead to the nonsensical result that the cost of debt exceeds the cost of equity. As Kaplan and Stein (1990) say "Because of default risk *expected returns* [on corporate debt] are undoubtedly lower than promised returns" (page 221).

An alternative to using the promised yields as the cost of debt that is sometimes advocated is to assume that the debt has zero risk premium.[3] However, this is also impossible, as there must be some chance of default if the promised yield spread is positive. The debt risk premium must therefore be greater than zero unless the default risk is entirely diversifiable, which is unlikely.

The problem with obtaining the expected return on risky debt arises because the spread between the promised yield and a riskless interest rate with the same maturity, liquidity and tax characteristics consists of two parts. The first is the part that reflects the chance of default. The second is the expected return premium:

$$\text{Promised yield spread} = \text{Expected default effect} + \text{Expected return premium}$$

The expected return premium is the part of the yield spread that should be included in the cost of debt. So the bias in the WACC resulting from using the promised yield rather than the expected return depends on the proportion of the promised yield spread that is an expected return premium. If the whole spread were an expected return premium then it would be correct to use the promised yield. However, this is impossible as there must be some chance of default for the debt to be risky. Some part of the spread must be due to expected default. Thus, the true cost of debt must lie somewhere between the two extremes of the promised yield and the riskless rate. We can describe where in this range it falls by the proportion of the promised yield spread that is an expected return premium.

The difference between the promised yield and the expected return on debt can have a material impact on the WACC. For example, consider a highly-leveraged transaction with an equity ratio $p_E = 30\%$ Suppose that the riskless real rate is 3%, the risk-premium on the equity of the firm is 6%, and the promised real

[3] This is often implemented by assuming that the debt has zero beta and using the CAPM.

return on the debt is 7%. The conventional WACC calculation would give:

$$WACC(1) = 0.7 \times 7\% + 0.3 \times (3\% + 6\%) = 7.6\%$$

If half of the debt premium is simply compensation for the probability of default, the true expected return on the debt is 5%. The true WACC is then:

$$WACC(2) = 0.7 \times 5\% + 0.3 \times (3\% + 6\%) = 6.2\%$$

With a real WACC of 7.6% the multiplier for a real perpetuity growing at 3% is 22 times. With a WACC of 6.2% it is 31 times. Thus, the difference in the cost of debt estimates can have a material effect on valuation. Table 1 shows that there is a simple linear relationship between the proportion of the spread which is an expected return premium and must be included in the return on debt, and the WACC.

INSERT TABLE 1 HERE

The errors in the WACC which arise from using the conventional cost of debt estimates are most significant when the debt is risky. However, this is also when the problem of estimating the expected return on debt is greatest. As Brealey and Myers (2000) say: "This is the bad news: There is no easy or tractable way of estimating the rate of return on most junk debt issues" (page 548).

Several approaches to estimating the expected return on debt might be taken when the promised yield is not used as the cost of debt:

1. Empirically estimate the debt beta and use the CAPM to estimate the expected return on debt.
2. Empirically estimate the frequency of defaults and the size of write-downs and adjust the promised yield to give the expected return.
3. Use a model to impute the required rate of return on debt from other inputs to the WACC.

The first approach is often infeasible because the relevant data are not available. Also, debt betas vary over time because of changes in interest rates and different debt maturities. Moreover, the market risk premium is difficult to estimate. So the approach is hard to implement. The second approach is problematic because ex-post default frequencies may be very different from ex-ante probabilities (see Asquith et al (1989) and Waldman et al (1998)).

The third approach is pursued here. The purpose of the paper is to propose a practical way of estimating the cost of risky debt from the standard inputs to the WACC and other observed firm characteristics. The model employed is the Merton (1974) model of risky debt. This is used to decompose the promised yield spread into the part that is compensation for expected default and the part that is an expected return premium.

2 The cost of debt implied by the Merton model

The Merton model is the simplest equilibrium model of the relationship between corporate interest rates and the inputs to the WACC. It assumes that the value of the firm's assets follows a geometric Brownian motion:

$$\frac{dV}{V} = \mu dt + \sigma dW_t \tag{3}$$

where V is the value of the firm's assets, μ and σ are constants, and W_t is a standard Wiener process.

The Merton model further assumes that the firm has a single class of zero coupon risky debt of maturity T. Other assumptions include a constant interest rate and a simple bankruptcy procedure; namely, if at maturity the value of the assets is lower than the liability, the assets are handed over to the bondholders without costs or violation of priority rules. The simplicity of the model has led to difficulties in using it to explain the relationship between the absolute level of debt spreads, capital structure and asset volatility. In the context of this paper, however, we are not interested in the absolute level of the spread. We simply wish to divide the *observed market spread* between the part that represents expected default and the part that is the expected return premium. If the Merton model picks up at least first-order effects relevant to splitting the spread on risky debt, it can be used to estimate the expected return *relative* to the promised yield. For this purpose the model has the merit of being an equilibrium model and being relatively simple.

We test below whether the split of the spread that it gives is robust.

Neither the expected return on assets μ nor the asset volatility σ can be directly observed. Moreover, in the presence of multiple issues of debt and debt with coupons and call provisions the value of debt maturity T in the Merton model is not well defined. The basic idea of this paper is to find the values for μ, σ and T that reconcile the Merton model with the observed market debt prices given firm's capital structure and other observed characteristics. Once the parameters of the asset distribution implied by the debt prices are found, one can compute the expected return on debt which is consistent with this distribution and the return on equity estimated from (2).

Merton applies the Black-Scholes option pricing analysis to value equity as a call option on firm's assets. Merton's formula can be written in a form that gives a relationship between the proportion of equity, p_E, the maturity of the debt, T, the volatility of the assets of the firm, σ, and the promised yield spread, s_D (see Appendix 1):

$$p_E = N(d_1) - (1 - p_E)e^{s_D T} N(d_2) \tag{4}$$

where $N(\cdot)$ is the cumulative normal distribution function and

$$d_1 = [-\ln(1 - p_E) - (s_D - \sigma^2/2)T]/\sigma\sqrt{T} \tag{5}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{6}$$

Equation (4) includes two unknowns – σ and T. However, another implication of the Merton model is that the equity volatility[4] σ_E satisfies:

$$\sigma_E = \sigma N(d_1)/p_E \tag{7}$$

We now have three observable inputs: p_E, s_D and σ_E, and two unknowns: σ and T. We solve equations (4) and (7) simultaneously to find values of σ and T that are consistent with the observed values of p_E, s_D and σ_E.[5] Thus, σ is computed as the implied volatility of the firm's assets when the equity is viewed as

[4]In contrast to the asset volatility, the short-term equity volatility is easily observable either from option implied volatilities or from analysis of historical returns data.

[5]The system of equations is well-behaved, and we generally had no difficulties solving it applying standard software such as Mathematica. To assure a starting point for which standard numerical methods quickly yield a solution, one can solve equations (4) and (7) separately for σ for a few fixed values of T (or vice versa). This procedure always converged for any reasonable

a call option. The parameter T reflects not only the actual maturities of different debt issues in complex capital structures, but also the presence of distress costs, strategic behaviour by equity holders, and other complications not included in the Merton model but reflected in the observed spread s_D.

Once the values of σ and T are known, they are combined with the estimate of the expected return on equity given by (2) to calculate the expected return on assets and debt as follows. Since equity in this model is a call option on the assets and therefore has the same underlying source of risk as the assets, the risk premia on assets π and equity π_E are related as:[6]

$$\frac{\pi}{\pi_E} \equiv \frac{\mu - r}{\mu_E - r} = \frac{\sigma}{\sigma_E} \tag{8}$$

Substituting (7) for σ_E yields:

$$\pi = \pi_E p_E / N(d_1) \tag{9}$$

Now the expected return premium over the maturity period[7] can be calculated as (see Appendix 2):

$$r_D - r = s_D + \frac{1}{T}\ln\left[N(d_2 + \pi_E\sqrt{T}/\sigma_E) + \frac{\exp\{[\pi_E p_E/N(d_1) - s_D]T\}}{1 - p_E} N(-d_1 - \pi_E\sqrt{T}/\sigma_E)\right] \tag{10}$$

The second term of (10) is negative, as the expected return on debt is lower than the promised yield.

3 Numerical results

Table 2 shows the breakdown of the promised yield spread between the expected return premium and the default risk for representative values of p_E, s_D, π_E, and σ_E. The values of p_E and s_D that are used are similar to those in Kaplan and Stein (1990). The equity risk premium is set at values that are commonly

starting points. The intersection of the to solution curves $\sigma(T)$ from equations (4) and (7) can then be used as a starting point for the system of these equations.

[6] π_E is the instantaneous risk premium on equity, which is different from Π_E, the premium over a discrete horizon.

[7] Note that, unlike the return on assets and equity, the calculated return on debt is an annualised compounded return rather than an instantaneous return.

used. The volatility of equity is set at a typical level.

INSERT TABLE 2 HERE

Panel A of Table 2 shows the results for values for a typical high grade debt firm. The first row shows the base case with a proportion of equity of 70% and equity volatility of 30% per annum. The equity risk premium is 6% per annum and the debt spread is 1% per annum. The expected return premium is 84 basis points, so that most of the promised yield spread (84%) is an expected return premium. As a result, the error from using the proposed yield as the cost of debt would be low in this case.

Panel B shows the results for high leverage firm, with a proportion of equity of 30%. The volatility of equity is higher, at 50% per annum, and the debt spread is higher, at 4%. For the base case, less than half of the debt spread is an expected return premium. Although the promised yield spread is four times that in Panel A, the expected return premium is less than doubled. In this case, most of the increase in the promised yield spread between a typical leverage and a high leverage firm is caused by an increased chance of default. The proportion of the yield spread that represents an expected return premium is only forty percent. As a result, the error in using the promised yield on debt as the cost of debt would be substantial for a highly leveraged firm with these parameter values.

4 Robustness of the approach

The proportion of the spread that is an expected return is not very sensitive to any input other than σ_E. In particular, it can be seen from (10) it does not depend on the riskless interest rate. From Table 2, it is relatively insensitive to realistic levels of variation in p_E, s_D and π_E. The insensitivity to p_E and π_E is important as these are variables that cannot be precisely estimated. It is sensitive to σ_E, but second moments such as σ_E can be estimated quite accurately for equity returns. Thus, the procedure has the merit of giving a result that is very sensitive only to a parameter that can be observed relatively accurately. We also tested the effect of assuming that the equity premium estimate applies to the maturity period of the debt rather than being an instantaneous premium. This makes little difference to the results.

5 Conclusion

The cost of equity and the cost of debt are two key inputs to the weighted average cost of capital. While the former has been the subject of extensive debate, little attention has been focused on the latter. The two common ways of estimating the cost of debt are to use the promised yield or the riskless rate. Both yield biased results, and the errors can be material. Other estimation methods, such as using the CAPM or adjusting the promised yield for the expected frequency of default, are hard to implement.

This paper proposes a practical way of estimating the true cost of debt. This involves splitting the promised yield spread into the part due to expected default and the part which is an expected return premium. The inputs required are the standard inputs to the WACC and the volatility of equity, all of which are easily observable. The proposed method uses these inputs to impute the parameters of the Merton risky debt model and to compute the expected return on debt. The idea is that, although the Merton model is a stylised version of real debt structures, it should pick up the first order effects that are relevant to the cost of debt. Thus it can be used to estimate the expected return relative to the promised yield.

The cost of debt is analysed for parameter values that are typical for a firm with high grade debt and a firm with low grade debt. In the high grade case, most of the promised yield spread is an expected return premium. In contrast, for a high leverage firm with low grade debt, most of the promised yield spread is expected default. The standard approach of using the promised yield as the cost of debt may be adequate for firms with high grade debt. For firms with low grade debt, it is likely to cause significant errors. In these cases the approach proposed in this paper can be used to adjust the WACC for the probability of default on the firm's debt.

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Appendix 1: The Merton Model

The standard form of the Merton model is:

$$E = VN(d_1) - Fe^{-rT}N(d_2) \tag{A1.1}$$

$$d_1 = [\ln(V/F) + (r + \sigma^2/2)T]/\sigma\sqrt{T} \tag{A1.2}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{A1.3}$$

$$B = V - E \tag{A1.4}$$

where V is the value of the assets of the firm, E is the value of the equity, B is the value of the debt and F is the promised debt payment. The other variables are as in the main text.

The continuously compounded promised yield on debt is defined by:

$$y_D = \frac{1}{T}\ln[F/B] \tag{A1.5}$$

The continuously compounded spread is defined by:

$$s_D = y_D - r \tag{A1.6}$$

Substitution of (A1.4)-(A1.6) in (A1.1)-(A1.3) gives equations (4)-(6) of the main text.

Appendix 2: Derivation of the Expected Return on Debt

It follows from (3) that the asset returns are log-normally distributed:

$$\ln(V_T/V) \sim N\Big((\mu - \sigma^2/2)T,\ \sigma^2 T\Big) \tag{A2.1}$$

where V_T is the value of the firm's assets at date T. Therefore,

$$V_T = Ve^{\mu T + \sigma\sqrt{T}Z} \tag{A2.2}$$

where Z is a standard normal variable.

The face value F of the debt is

$$F = (1 - p_E)Ve^{y_D T} = (1 - p_E)Ve^{(s_D + r)T} \tag{A2.3}$$

With the assumptions of the Merton model, the compound return on debt over T is determined by:

$$(1 - p_E)Ve^{r_D T} = \int_{-\infty}^{V_T = F} V_T\ dN(Z) + \int_{V_T = F}^{\infty} F\ dN(Z) \tag{A2.4}$$

Evaluating the integrals, we obtain:

$$(1 - p_E)Ve^{r_D T} = Ve^{\mu T} N(-K_1) + F\ N(K_2) \tag{A2.5}$$

where F is given by (A2.3) and:

$$K_1 = \left[\ln\left[e^{\mu T}/F\right] + \sigma^2 T/2\right]/\sigma\sqrt{T} \tag{A2.6}$$

$$K_2 = K_1 - \sigma\sqrt{T} \tag{A2.7}$$

Rearranging terms in (A2.5) and noting that (see (5), (6) and (8)):

$$K_1 = d_1 + (\mu - r)\sqrt{T}/\sigma = d_1 + \pi_E\sqrt{T}/\sigma_E \tag{A2.8}$$

$$K_2 = d_2 + (\mu - r)\sqrt{T}/\sigma = d_2 + \pi_E\sqrt{T}/\sigma_E \tag{A2.9}$$

we obtain Equation (10) of the main text.

Table 1: The Relationship between the Proportion of the Promised Yield Spread that is a Risk Premium, and the WACC

Proportion of promised debt yield spread that is a risk premium	Debt Risk Premium	True Cost of Debt	True WACC
1.00	4%	7%	7.6%
0.75	3%	6%	6.9%
0.50	2%	5%	6.2%
0.25	1%	4%	5.5%
0.00	0%	3%	4.8%

In this example, the real riskless rate is 3%, the promised yield on debt is 7%, the proportion of equity is 30%, and the equity risk premium is 6%.

Table 2: The relationship between promised yield and the expected excess return on debt

p_E	s_D (%)	π_E (%)	σ_E	$r_D - r$ (%)	α (%)
Panel A: Typical high grade debt firm					
0.7	1.0	6.0	0.3	0.84	83.6
0.6				0.81	81.3
0.8				0.87	86.6
	0.5			0.41	81.0
	1.5			1.28	85.5
		5.0		0.77	76.6
		7.0		0.89	88.9
			0.2	NA	NA
			0.4	0.58	58.5
Panel B: High leverage firm					
0.3	4.0	6.0	0.5	1.52	38.1
0.2				1.48	37.1
0.4				1.56	39.1
	3.0			1.12	37.3
	5.0			1.93	38.7
		5.0		1.30	32.5
		7.0		1.73	43.3
			0.4	2.27	56.7
			0.6	1.08	27.0

The table shows the relationship between promised yields and expected returns on debt from the Merton model. The inputs are the proportion of equity, p_E, the promised spread on debt, s_D, the volatility of equity, σ_E, and the equity risk premium, π_E. The outputs are the expected return premium on debt, $r_D - r$, and the proportion of the promised debt spread that represents an expected return premium, $\alpha = (r_D - r)/s_D$. NA means that there was no solution was found that gave these values.