Tax-adjusted discount rates with investor taxes and risky debt

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October 2004

Abstract

This paper derives tax-adjusted discount rate formulas with Miles-Ezzell leverage policy, investor taxes, and risky debt. The formulas assuming riskless debt have been derived by Taggart (1991). The paper shows that the error from assuming riskless debt is material at reasonable parameter values. The expression for the asset beta is also given.

JEL codes: G31, g32

Keywords: Capital structure, valuation, risky debt, cost of capital.

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1 Introduction

The Miles-Ezzell (1980) formula for tax-adjusted discount rates is extended by Taggart (1991) to include the effect of personal taxes. The Taggart formulas, rather than the more familiar Miles-Ezzell (ME) formulas, should be used if investor taxes reduce the tax advantage to corporate borrowing. For the U.S. there is some evidence that this may be the case. For instance, Fama and French (1998) fail to find any increase in firm value associated with debt tax savings. On the other hand, Kemsley and Nissim (2002) use a different estimation technique and find that the net tax advantage to debt similar to the corporate tax rate. Thus, while the U.S. evidence is inconclusive, it certainly admits the possibility that investor taxes make the net tax advantage to debt less than the full corporate tax rate. The changes to the U.S. tax code introduced in 2003 also favors dividend income over interest income for most investors. In other countries, there are even stronger reasons to believe that investor taxes offset part of the corporate tax advantage to debt. In particular, countries that have imputation taxes have such an effect built directly into their tax systems (see Rajan and Zingales (1995)).

The formulas developed by Taggart assume that corporate debt is riskless. In practice, the cost of corporate debt includes a default spread. This component of the cost of capital can be a significant part of the cost of capital for some firms, especially with the lower equity risk premia that are now often used, and the high leverage of some firms. Therefore, the inclusion of the debt risk premium in the formula is necessary for realistic valuation. This paper extends the Taggart analysis by including risky debt. It derives the tax-adjusted discount rate formulas for companies that follow the ME leverage policy when both investor taxes and risky debt are included.

2 The tax-adjusted discount rate

The ME formula applies to any profile of cash flows as long as the company maintains constant market value leverage. It gives a relationship between the leveraged discount rate, $R_L$, and the unleveraged rate, $R_A$. We derive the formula for a firm with expected pre-tax cash flows $C_t$, at dates $t = 1, \ldots, T$. Between these dates, leverage remains fixed. After each cash flow, leverage is reset to be a constant proportion, $L$, of the leveraged value of the firm. The required rate of return

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1 Graham (2000) finds that the net tax advantage to debt is less than the full corporate tax rate because of non-debt tax shields. The valuation consequences of this are more complicated, because it implies that the tax rate that should be used in valuation is state and time-dependent.
The two rates $R_L$ and $R_A$ are defined implicitly as the discount rates that give the correct unleveraged and leveraged values when the after-tax operating cash flows are discounted:

\[
V_{At} = \sum_{i=t+1}^{T} C_i (1 - T_C)/(1 + R_A)^i \quad t = 1, \ldots T
\]

(1)

\[
V_{Lt} = \sum_{i=t+1}^{T} C_i (1 - T_C)/(1 + R_L)^i \quad t = 1, \ldots T
\]

(2)

where $V_{At}$ is the unleveraged value of the firm and $V_{Lt}$ its leveraged value.

We assume that the representative investor has tax rates of $T_{PD}$ on debt returns and $T_{PE}$ on equity income and capital gains. The increase in the after-tax cash flow to this investor resulting from an incremental dollar of corporate interest is:

\[
T_S = (1 - T_{PD}) - (1 - T_C)(1 - T_{PE})
\]

(3)

This result is standard. We define the related variable, $T^*$, by:

\[
T^* = T_S/(1 - T_{PD})
\]

(4)

Thus,

\[
1 - T^* = \frac{(1 - T_C)(1 - T_{PE})}{1 - T_{PD}}
\]

(5)

Following Taggart (1991), we define the required return on riskless equity as:

\[
R_{FE} = \frac{R_F(1 - T_{PD})}{(1 - T_{PE})} = \frac{R_F(1 - T_C)}{(1 - T^*)}
\]

(6)

The first equality results from setting the after-investor-tax returns on riskless debt and riskless equity equal to each other. The second equality follows from (5). Note that, if $T_{PD}$ and $T_{PE}$ are equal, then $R_{FE} = R_F$.

Appendix 1 shows our first main result, namely that the relationship between $R_L$ and $R_A$ is:

\[
R_L = R_A - LR_D T^*(1 + R_A)R_{FE}(1 + R_F(1 - T_{PD}))/ (1 + R_{FE})R_F(1 + R_D(1 - T_{PD}))
\]

(7)

Two special cases of (7) are worth noting. If the debt is riskless, then $R_D = R_F$, giving:

\[
R_L = R_A - LR_{FE} T^*(1 + R_A) / (1 + R_{FE})
\]

(8)
This is the expression given in Taggart (1991). If the debt is riskless and there are also no investor
taxes, then $R_D = R_F$ and $T^* = T_C$, giving:

$$R_L = R_A - \frac{LR_FT_C(1 + R_A)}{1 + R_F}$$  \quad (9)

This is similar to the expression given in Brealey and Myers (2003), although they generalize it
using $T^*$ rather than $T_C$, and $R_D$ rather than $R_F$:

$$R_L = R_A - \frac{LR_DT^*(1 + R_A)}{1 + R_D}$$  \quad (10)

As we have shown above, the general form of the relationship between $R_L$ and $R_A$ is (7), rather
than this simpler expression. We discuss below the size of the errors that result from using various
approximations.

If the period between rebalancing the leverage becomes short, (7) converges to:

$$R_L = R_A - LR_D T^* \frac{1 - T_C}{1 - T^*} = R_A - LR_D T_S \frac{T_S}{1 - T_{PE}}$$  \quad (11)

Both this and the more complex version of the ME formula given by (7) are approximations.
Neither policy exactly reflects the actual leverage policies that firms follow. It is not clear which
expression is more accurate. Apart from its simplicity, (11) has one significant advantage, in that
it is consistent with the assumption that underlies the standard formula for asset betas (provided
that $T_C = T^*$), as we show below.

### 3 Implementation using the CAPM

A common approach is to use formulas for tax adjusted discount rates in conjunction with the
CAPM. For example, one first estimates the asset beta, then calculates $R_A$ using the CAPM, and
finally calculates $R_L$ using one of the formulas above. Alternatively, one calculates $R_L$ as the
weighted average cost of capital by plugging equity and debt betas into the CAPM. From that one
can infer $R_A$ and perhaps calculate new $R_L$’s for a different leverage ratios. In this section, we
derive the formula for the asset beta that is consistent with (11). We also explore the numerical
differences that result from using (7) or (11) versus the approximation formulas (8) and (10) when
calculating discount rates.

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2 See Brealey and Myers (2003) page 542.
3.1 The CAPM with personal taxes

The consensus investor will set returns so that the risk-return ratio from assets is equal on an after-investor-tax basis. This means that the standard version of the CAPM, where returns and betas are measured before investor taxes, will be affected by investor tax rates. Appendix 2 shows that the version of the CAPM that is consistent with the assumptions about tax that determine $T^*$ is:

$$R_E = R_{FE} + \beta_E P$$ (12)

where $R_E$ is the expected rate of equity return before investor taxes, $\beta_E$ is the beta of the equity, and

$$P = R_M - R_{FE}$$ (13)

$R_M$ and $R_F$ are measured in the standard way, using returns before investor taxes. Betas are also measured in the standard way, using pre-tax returns.

Note that only if $T^* = T_C$ (or, equivalently $T_{PE} = T_{PD}$) is the standard version of the CAPM, with an intercept equal to $R_F$, valid. In particular, this means that the assumption in, for instance, Modigliani and Miller (1963) that $T^* = T_C$ corresponds to the normal CAPM. In contrast, the Miller (1977) view that $T^* = 0$ corresponds to a CAPM where the intercept is $R_F(1 - T_C)$. A similar effect can be seen in the formula for the required return on assets:

$$R_A = R_{FE} + \beta_A P$$ (14)

The required return on debt follows a different version of the CAPM, because the tax treatment of debt and equity differ in all cases other than the standard MM case:

$$R_D = R_F + \beta_D P$$ (15)

Regardless of the assumption about taxes, the pre-tax CAPM holds for debt, because all debt is taxed in the same way.\(^3\)

3.2 The WACC and the asset beta

The standard WACC relationship is:

$$R_D(1 - T_C)L + R_E(1 - L) = WACC$$ (16)

\(^3\)There is another complexity with risky debt. It is not clear that the entire premium over the riskless rate is due to beta risk. We do not deal with this issue here. See Cooper and Davydenko (2004) for a discussion.
Under the ME assumptions, the WACC is equal to \( R_L \), so, using the continuous rebalancing assumption [see (11)]:

\[
R_D(1 - T_C)L + R_E(1 - L) = R_A - LT^* R_D \frac{1 - T_C}{1 - T^*} \tag{17}
\]

Substituting in \( R_E, R_A, \) and \( R_D \) from (12), (14), and (15), respectively, we have

\[
\beta_A = \beta_D [(1 - T_C)/(1 - T^*)] L + \beta_E (1 - L) \tag{18}
\]

If \( T^* = T_C \) [or, equivalently \( T_{PE} = T_{PD}, \) see (5)], this reduces to the standard asset beta equation:

\[
\beta_A = \beta_D L + \beta_E (1 - L) \tag{19}
\]

We can intuitively understand the relationships between betas in the following way. The leveraged firm’s operating assets are the same as those for the all-equity firm. But the leveraged firm generates extra value through the tax saving from interest and changes the after-tax risk of the cash flow by channeling some of it to debtholders rather than equityholders, which changes the associated tax treatment. The weighted average of the equity beta and the tax-adjusted debt beta for the leveraged firm must equal the asset beta adjusted for the effect of the tax saving:

\[
E\beta_E + D \frac{1 - T_{PD}}{1 - T_{PE}} \beta_D = \beta_A (V_L - V_{TS}) + V_{TS} \beta_{TS} \tag{20}
\]

where \( E \) is the value of the equity, \( D \) the value of the debt, \( V_L = E + D, \) \( V_{TS} \) is the value of the tax shield and \( \beta_{TS} \) is its beta. The value \( (V_L - V_{TS}) \) is the all-equity value of the firm, which has beta equal to \( \beta_A \). The adjustment \( \frac{1 - T_{PD}}{1 - T_{PE}} \) to the debt beta reflects the fact, shown in (31), that the differential tax treatment of debt and equity results in a change in beta when cash flow is switched from equity to debt, even apart from the effect on the value of the firm. With the ME assumptions and continuous debt rebalancing, \( \beta_{TS} = \beta_A \), giving (18).

### 3.3 Errors arising from using approximate formulas

To evaluate the size of possible errors arising from using approximations to the correct formulas, we examine two types of firm. These are shown in Table 1. The first firm, in Cases 1 - 3, has a typical amount of leverage and typical values of debt and equity betas. The second firm, shown in Cases 4 - 6, has higher leverage and higher debt and equity betas. Both firms have similar asset betas. The general parameters are a riskless rate of 4%, a corporate tax rate of 38%, an investor tax rate on debt of 30%, and a market risk premium of 5%. We compute \( R_L \) from the WACC.
formula. We assume that leverage is rebalanced continuously, making (11) the correct formula for $R_A$ and (18) the correct asset beta. Then we vary the level of $T^*$ and examine the errors arising from using approximations to $\beta_A$ and $R_L$.

The first approximation error, in row (a), is that resulting from using the standard asset beta formula (19), rather than (18). This gives relatively small errors when applied to our standard firm. When applied to the highly leveraged firm it gives more material errors. Errors of ten percent of the asset beta can result, depending on the level of $T^*$.

The other errors shown in the table are the errors in using various approximations to $R_L$, starting from the true value of $R_A$. These may be interpreted as indicating the size of the error in using a particular approximation to estimate $R_L$ from $R_A$, or in estimating $R_A$ from $R_L$, which will be of similar absolute magnitude. Row (b) shows the difference between assuming discrete and continuous adjustment of leverage as in (7) versus (11). This error is small, indicating that the extra complexity of the ME formulas that assume discrete adjustment of leverage is probably unnecessary. Row (c) shows the Taggart releveraging formula, that assumes riskless debt, (8) versus (11), which allows for the risk of debt. As might be expected, the error is large when debt is risky. The cost of capital can be in error by forty basis points, which is almost ten percent of the asset risk premium. Row (d) shows the approximate formula given in Brealey and Myers, (10) minus the correct formula with discrete rebalancing (7). The error is smaller than that arising from ignoring the risk of debt, because the Brealey and Myers formula includes the debt risk premium. However, there is still an error, and the simplicity of the correct expressions given by (11) and (18) suggest that it is worthwhile using these when there is any potential doubt about the accuracy of the other approximations.

4 Conclusions

We have presented tax-adjusted discount rate and asset beta formulas with Miles-Ezzell constant debt to value leverage policy, investor taxes, and risky debt. Formulas assuming riskless debt have previously been derived by Taggart (1991). If various approximations that have been suggested are used, errors of ten percent of the asset beta can result, depending on the level of leverage and investor taxes. Similarly, the level of the cost of capital can be in error by forty basis points, which is almost ten percent of the asset risk premium in our examples. The size of these potential errors, and the simplicity of the correct expressions, suggest that it is worthwhile using our formulas when
there is any potential doubt about the accuracy of the other approximations.

Appendix 1: Proof of the relationship between $R_L$ and $R_A$

We derive relationship between $R_L$ and $R_A$ by induction, starting at time $T-1$. At that time, the only cash flow remaining is $C_T$. The unleveraged value is:

$$V_{AT-1} = C_T(1 - T_C)/(1 + R_A)$$

This is the unleveraged value of the last cash flow. It includes the value to the representative investor of the tax deduction associated with the purchase price, $V_{AT-1}$.

From the leveraged firm, the representative shareholder will receive, at date $T$, a cash flow after personal taxes of $C_T(1 - T_C)(1 - T_{PE}) + I_T T_S$. The first part of this cash flow is identical to that from the unleveraged firm and so has value equal to $V_{AT-1}$, if it comes with a tax deduction of $V_{AT-1}$. The second flow has risk equal to the debt of the firm, and should be discounted at the after-tax rate appropriate to the debt of the firm, $R_D(1 - T_{PD})$. There is a third component of value. Relative to an investment in the unleveraged firm, the representative investor also gets an extra tax deduction equal to $(V_{LT-1} - V_{AT-1})$. This is riskless, and is discounted at his after-tax riskless rate. Using $I_T = LV_{LT-1} R_D$, the resulting value of the leveraged firm is:

$$V_{LT-1} = V_{AT-1} + \frac{LV_{LT-1} R_D T_S}{1 + R_D(1 - T_{PD})} + \frac{(V_{LT-1} - V_{AT-1}) T_{PE}}{1 + R_F(1 - T_{PD})}$$

The last term in this expression is the incremental saving in capital gains taxes, which are assumed to be paid every year.\(^4\) The tax basis is higher for the leveraged firm by the amount $(V_{LT-1} - V_{AT-1})$, and capital gains taxes are consequently reduced.

Using (6), we can rearrange (22) as:

$$V_{LT-1} = V_{AT-1} + \frac{LV_{LT-1} R_F R_D (1 + R_F(1 - T_{PD}))}{(1 + R_{FE}) R_F (1 + R_D(1 - T_{PD}))}$$

At time $T-1$, $R_L$ and $R_A$ are defined by:

$$(1 + R_L) = C_T(1 - T_C)/V_{LT-1}$$

$$(1 + R_A) = C_T(1 - T_C)/V_{AT-1}$$

\(^4\)There is an emerging literature that introduces realistic treatment of capital gains taxes into the capital structure literature (see Lewellen and Lewellen (2004)). The implications of their results for practical valuation are not yet clear.
Thus $V_{AT-1} = V_{LR-1}(1 + R_L)/(1 + R_A)$. Substituting this into (23), we get

$$R_L = R_A - \frac{LR_DT^*(1 + R_A)R_F(1 + R_F(1 - T_{PD}))}{(1 + R_F E)R_F(1 + R_D(1 - T_{PD}))} \quad (26)$$

We will establish the generality of this formula by induction. Consider the expected cash flow at $T - 1$, $C_{T-1}$, and the continuation value, $V_{LT-1}$, as two separate but equally levered flows. Denote their values at $T - 2$ by $w_{T-2}$ and $W_{T-2}$, respectively. The same argument as above establishes that $w_{T-2} = C_{T-1}(1 - T_C)/(1 + R_L)$, with $R_L$ given by (26). Now, we have established above that $V_{LT-1}$ is proportional to $V_{AT-1}$. Therefore, its unleveraged value at $T - 2$ is $V_{LT-1}/(1 + R_A)$. Thus, using the same argument as above, the leveraged value of $V_{LT-1}$ at $T - 2$ is $W_{T-2} = V_{LT-1}/(1 + R_L)$, with $R_L$ given by (26). This establishes that

$$V_{T-2} = \frac{C_{T-1}(1 - T_C)}{1 + R_L} + \frac{C_T(1 - T_C)}{(1 + R_L)^2} \quad (27)$$

This argument can be repeated for arbitrary $t$, which establishes the generality of (26).

**Appendix 2: Relationships between betas and returns**

The representative investor sets returns so that after-tax returns are in equilibrium. However, the CAPM is usually stated in terms of pre-tax betas and risk premia. This Appendix uses the after-tax CAPM to derive the pre-tax version that is consistent with the assumptions about the tax saving on debt.

Assuming that the market portfolio consists of only equities and not risky debt, and denoting betas that are after personal tax by primes, and pre-tax betas with no primes:

$$\beta'_E = \frac{Cov(\tilde{R}_E(1 - T_{PE}), \tilde{R}_M(1 - T_{PE}))}{Var(\tilde{R}_M(1 - T_{PE}))} = \frac{Cov(\tilde{R}_E, \tilde{R}_M)}{Var(\tilde{R}_M)} = \beta_E \quad (28)$$

similarly:

$$\beta'_A = \beta_A \quad (29)$$

and:

$$\beta'_D = \frac{Cov(\tilde{R}_D(1 - T_{PD}), \tilde{R}_M(1 - T_{PE}))}{Var(\tilde{R}_M(1 - T_{PE}))} = \frac{(1 - T_{PD})}{(1 - T_{PE})}\beta_D \quad (30)$$

Expected returns are set by the consensus investor to give equal after-tax risk premia per unit of after-tax beta:
\[ RE(1 - T_{PE}) = RF(1 - T_{PD}) + \beta_EP' \]  \hspace{1cm} (31)

\[ RA(1 - T_{PE}) = RF(1 - T_{PD}) + \beta_AP' \]  \hspace{1cm} (32)

\[ RD(1 - T_{PD}) = RF(1 - T_{PD}) + \beta_D \frac{(1 - T_{PD})}{(1 - T_{PE})}P' \]  \hspace{1cm} (33)

Substituting \( P \) for \( P' \):

\[ RE = R_{FE} + \beta_EP \]  \hspace{1cm} (34)

\[ RA = R_{FE} + \beta_AP \]  \hspace{1cm} (35)

\[ RD = RF + \beta_DP \]  \hspace{1cm} (36)
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Table 1: *Discount rates and betas resulting from applying the ME formulae*

This table shows the outputs from calculations related to the cost of capital. The values of other variables are: $R_F = 4\%$, $T_C = 38\%$, $T_{PD} = 30\%$, $P = 5\%$. Panel B: The values of the WACC, $R_A$ and $\beta_A$ are calculated using (16), (11), and (18), respectively. Panel C: a. The error arising from using the standard asset beta formula rather than the correct one, (18) – (19). b. The difference between the values of $R_L$ under discrete and continuous leverage rebalancing, (7) – (11), using $R_A$ from Panel B. c. The difference between the value of $R_L$ using the continuous version of Taggart’s formula (8), which assumes riskless debt, and the correct formula, (11), which allows for risky debt. d. $R_L$ from the approximate discrete rebalancing formula (10) less that from (7), using $R_A$ from Panel B.

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<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
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