Venture Capital Investment

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1. Introduction

The modern theory of finance has been relatively successful at explaining the prices of securities that are traded in well-functioning markets and providing investment rules for the companies that issue those securities. Venture capital investment is not the type of asset for which the theory has direct application. By venture capital investment we mean investment in new, small, and risky companies.

The problems that arise in venture capital investment are threefold. The diffuse nature of the information available for an external investor to evaluate the company means that investors may be in an inferior position to judge the true value of the investment compared to entrepreneurs who are seeking finance. This asymmetric information induces investment behavior that is not present to nearly the same degree for large, marketable companies [4–6]. It makes the venture capitalists’ claims on the portfolio companies whose securities they hold very difficult to market. Venture capitalists must take the returns to their investments as cash flows from the portfolio firms, or wait until they have proven their worth, so that the asymmetric information disappears and financial claims on the portfolio firms become marketable.

The second special feature of venture capital investment is the long time horizon over which the investment must be held before it can be liquidated (usually at least five years)[1]. Of course, many portfolio firms will never reach the stage where they can be sold, but the anticipated returns for venture capitalists rest largely on the expectation that some of their investments will reach that stage.

The investment required to carry the portfolio firm through this long holding period takes place in several stages [1]. This is a further complication to the evaluation of venture capital projects. The initial investment is made, not with the purpose of carrying the project through all stages whatever happens, but to purchase more information. In light of this new information, the project will either be continued through its next stage or abandoned. This multiperiod contingent decision process makes venture capital investment intractable for the conventional single-period tools of modern finance theory.

This reading takes those three distinguishing features of venture capital investment (diffuse information and nonmarketability, long horizon, and sequence of investment decisions) and builds around them a model of the venture capital investment process. The model is then used to examine optimal strategies for venture capital investment, the interpretation of empirical results pertaining to venture capital, and public policy toward venture capital.

The model and the assumptions underlying it are set out in Section II. In Section III, optimal investment criteria are derived. Section IV gives an interpretation of empirically observed phenomena in venture capital investment, with some numerical examples. In Section V, implications for public policy are discussed. A summary is given in the final section.

II. A Model of the Venture Capital Investment Process

Prices of equity shares in marketable companies are observed to be approximately log-normally distributed. Claims on the portfolio firm, in which the venture capitalist is invested, could, hypothetically, be made marketable at any time. For instance, the firm could be merged with an already marketable firm. At time $t$, this would give rise to a market value of the portfolio firm, denoted by $X(t)$. By analogy with price distributions of marketable securities, the log of $X(t)$ should follow a Wiener process.

The venture capitalist cannot, of course, observe the value $X(t)$. What the venture capitalist does observe are various operating data for the portfolio firm. The flow of information of this sort in the time period $(t, t+1)$ is denoted by a vector $Z(t)$. Components of $Z$ might be, for instance, sales growth and earnings changes during the period. These are assumed to be multinormally distributed with the change in the log of the firm value, $Y(t)$, where

$$Y(t) = \log X(t) - \log X(t-1)$$  \hspace{1cm} (1)

If this multinormal distribution is stable, the venture capitalist can use the
fundamental information to make inferences on the changes that have occurred in the value of the portfolio firm. The information also sheds light on the parameters of the process generating the fundamental information. Thus observations of high sales growth imply both increasing market value and a trend rate of sales growth that is high. In this way, the venture capitalist's beliefs about the present market value of the claim, and the future course of this value, are affected.

In particular, consider the case with a fixed horizon, $T$, at which, if all has gone well, the venture capitalist expects in claims on the portfolio firm to become marketable so that the investment can be cashed in. Then, the problem is that of predicting $X(T)$. In this case, it can be shown that the venture capitalist's beliefs at time $t$ can be summarized by a variable $L(t)$, which represents the expectation of the log payoff, $\log X(T)$ [1]. Quantity $L(t)$ performs a normal process with zero drift and decreasing variance rate as time passes. The zero drift is a result of the fact that

$$E_t[L(t+1)|L(t)] = L(t)$$

(2)

where $L(t) = E_t \log X(T)$

$E_t(\cdot)$ = expectation at time $t$

The decreasing variance rate results from observations of the fundamental variables, $Z(t)$, yielding incrementally smaller amounts of information on the mean of $Z$ as time passes.

At the horizon, the venture capitalist has a subjective distribution for $Y$ and $Z$, and observes $X(T)$, the value of the firm. The process from then on may be considered to follow the analysis of Winkler [9].

The input by the venture capitalist to the project does not usually consist of just a single investment of funds at the time the venture capitalist enters the project (time zero). At various times during involvement with the project, the venture capitalist will be called upon to provide further resources, either in the form of further financing or managerial assistance. These inputs to the project are denoted by

$I(0), I(1), \ldots, I(t), \ldots, I(T)$

where $I(t)$ is the input at time $t$. At each decision point, the venture capitalist may either continue to support the project or take a salvage value, $S(t)$.

In summary, the assumptions of the model are:

1. The investment consists of a series of fixed inputs, where the input at time $t$ is denoted by $I(t)$.
2. The payoff to the investment is the market value of the company at time $T$, $X(T)$.
3. The venture capitalist forms beliefs about the expected log payoff, $E_t \log X(T)$ = $L(t)$, by observing fundamental data.

4. When required at any time to make a further investment to support the project, the venture capitalist may do so, or may take a salvage value, $S(t)$.

The beliefs of the venture capitalist, described by $L(t)$, will perform a normal process with independent increments and a decreasing variance rate. The independent increments result from the independence of "new" information flows, and the decreasing variance is a result of the decreasing marginal information value of new observations of fundamental variables.

III. Optimal Decisions

The relevant state variable for the venture capitalist is the level of expectation about the final payoff to the project. This is summarized in the value of $L(t)$, the expected log payoff. At any decision point, the venture capitalist will continue to support the project if the marginal utility of doing so is greater than the marginal utility of abandonment. In general,

$$U^*[L(t)] = \max \left\{ u[S(t)], C[L(t)] \right\}$$

(3)

where $C[L(t)] = u[-L(t)] + \int u^*[L(t+1)]f[L(t+1) - L(t)] \, dL(t+1)$

(4)

where $u^*[L(t)] = \text{utility of an optimal policy pursued from state } L(t) \text{ onward}$

$u(\cdot) = \text{utility of the venture capitalist for wealth increments}$

$a(\cdot) = \text{time preference parameter in the utility function of the venture capitalist, which is assumed to be separable}$

$f(\cdot) = \text{normal density function, with variance equal to the variance of } L(t+1) - L(t)$

Equations (3) and (4) hold for $0 < t < T$, and the boundary value is given by

$$U^*[L(T)] = \max \left\{ u[S(T)], u[-C(T)] + u \exp LT(t) \right\}$$

(5)

The optimal policy consists of a series of cutoff values for expectations, below which it pays to abandon the project, and above to continue. This result depends only on the fact that the utility function is monotonic increasing, so that higher values of $L(t)$ give higher utility for continuation, $C[L(t)]$. The optimal policy is

Continue at time $t$ if $L(t) > L^*_t$.
Abandon at time $t$ if $L(t) < L^*_t$.

The actual values of the cutoff expectations, $L^*_t$, will, clearly, be determined by
the specific attributes of the project.

The value of the asset does not have an obvious meaning, since it is not traded, other than the monetary value of the utility increment to the venture capitalist. This value will be between two limits: the lower limit is the value of the same asset with no possible of abandonment; the upper limit is the value of a call option on the final payoff with an exercise price equal to the future value at time $T$ of the stream of inputs to the project. These limits must hold, since increased choice options increase the value of an asset, and the choices open to the venture capitalist lie between these two extremes. The degree to which the asset behaves like one or the other of the extremes depends on the timing of information flows relative to investment inputs. If most of the information takes place before significant information is received, the choice options have little value; if the other way around, the choice options have value, and the asset is similar to a pure call option.

The asset with no abandonment options plays the role of the underlying asset in option theory. This asset has inputs of $I(t)$ and payoff $X(T)$, with no choice except whether to invest at time zero. Its utility to the venture capitalist in state $L(t)$ will be denoted by $U[L(t)]$. The Appendix shows the relationship between the utility of this underlying asset and the utility of the venture capital asset.

The utility of one venture capital asset bears the same relationship to the utility of the underlying asset as the value of a call option bears to the value of its underlying stock. In both cases, the latter is increasing convex in the former, and the difference is decreasing. If there are traded assets that offer perfect substitutes for the payoffs to the venture capital project, the venture capitalist could sell the claim by selling these substitute securities, and the venture capital asset would effectively be marketable. Only then would arbitrage ensure the convex relationship between the value of the venture capital asset and the value of the underlying asset. The unique nature of venture capital investment makes it highly unlikely that marketable perfect substitutes can be found, so its value is not necessarily convex in the value of the underlying asset.

The difference between the value of the venture capital asset, $V^*[L(t)]$, and the value of the underlying asset, $V[L(t)]$, decreases as expectations rise. This indicates that as expectations rise, the value of the portfolio firm becomes closer to that of a going concern, whose prolonged existence is not in doubt.

The cutoff states representing an optimal policy give the values of $L(t)$, the expectation about the success of the project, for which the value of continuation is equal to the value of abandonment.

$$C[L(t)] = u[S(t)]$$

The value of the underlying asset is less than the value of the venture capital asset, so the cutoff criterion will correspond to a state where

$$V[L(t)] < S(t)$$

where $V$ = the value of the underlying asset.

Thus, if venture capital projects are evaluated on the full cash flow stream of inputs and the unconditional expected payoff, the acceptance criterion will appear to be a negative net present value. Alternatively, if an internal rate of return criterion is used on the cash flow stream, that is,

$$-I(0), \ldots, -I(T), E_0[X(T)]$$

the cutoff rate of return will appear to be low whenever the abandonment opportunity is contributing significantly to the value of the venture capital asset.

If the model is correct, it is clear that returns to venture capital investments will fall into two groups. Large losses will be generated by the group that is abandoned, and positive returns by the group that completes the venture capital phase and is sold by the venture capitalist. The expected distribution of payoffs from the successful projects will be affected by the decision process.

The decision process will prevent low payoffs from being realized, since, if the project is unsuccessful, it will be abandoned before it reaches maturity. Terminal states with extremely low payoffs will be almost entirely eliminated from the conditional payoff distribution. The ratio of the conditional to the unconditional payoff probability will increase with the level of the payoff. In fact, the conditional payoff distribution, $q[X(T)]$, can be characterized by a function $A[X(T)]$, which describes the proportion of the unconditional payoff distribution, $p[X(T)]$, that will be realized for a particular value of $X(T)$, given the optimal policy.

$$A[X(T)] = q[X(T)]/p[X(T)]$$

Figure 1 gives the function $A(\cdot)$ for three different assets. Curve a is that function for the underlying asset which has no possibility of abandonment, so its conditional and unconditional payoff probabilities are equal. Curve c is for a call option on the final payoff. The truncation of the probability distribution in this case is perfectly efficient, since decisions are made at intermediate time points, when information about the value of $X(T)$ that will occur is incomplete.

That $A(\cdot)$ is increasing means that the conditional distribution of log $X(T)$ is positively skewed. The impact of earlier resolution of uncertainty is to make the truncation more efficient, and so case b becomes more like the option in case c. Increased investment inputs move the curve to the right.

The combination of a highly skewed final payoff for successful projects with substantial losses for abandoned projects means that one should expect to observe bimodal return distributions for venture capital projects, a low mode for unsuccessful ones, and a high mode with a long right tail for those that achieve marketability.

The impact of risk on the value of an asset is central to the theory of finance. Risk is usually measured by the standard deviation of returns, but here it is open to several interpretations. If risk is interpreted as the dispersion of the unconditional payoff distribution of $X(T)$, then increasing risk (holding expected payoff constant) has two effects. Higher variability decreases the value $V[L(t)]$ of the underlying
asset if the venture capitalist is risk-averse. It also, however, increases the value of
the abandonment option. The intermediate decisions enable the venture capitalist
to avoid continuing investment in unprofitable projects and gives an extra chance of
great success. Thus, even if solely interested in the mean and variance of returns,
the venture capitalist will not necessarily react adversely to an increase in the under-
lying riskiness of the project.

IV. Empirical Issues and Numerical Results

The most commonly observed feature of returns to venture capital investment
in the United States is that they are, on average, not much greater than returns
earned from riskless investments [1, 2, 7]. A common interpretation of this result
is that the venture capital market is competitive. Although this may seem a reason-
able conclusion, in the context of mean-variance analysis, the issue becomes more
complex if one moves outside this paradigm.

As shown previously, the distribution of returns to venture capital investments
is likely to be bimodal and skewed to the right. Figure 2 presents empirical evidence
that this is the case. It gives the values of the cash flows from sixteen venture cap-
tal projects. The data were collected for a pilot study by the National Science
Foundation in 1977. The data for each project consisted of the investment stream
\( I(t) \) and the payoff \( X(T) \).

Since several of the projects had no positive cash flows, the use of internal rate
of return was inappropriate, so the data were analyzed by computing the net
present value (NPV) of the project at a rate of 5 percent. This was then standard-
ized by dividing by the present value of the input stream \( PV(I) \) computed at the
same rate.

The results confirm the basic features of the model given. A group of projects
give negative net present values. For the rest, the distribution is skewed heavily to
the right. The usual justification for mean-variance analysis—that asset returns are
normally distributed—may not be used here. Faced with this complex bimodal
skewed return distribution, the low average return is conclusive evidence for a com-
petitive venture capital market only if the effects of risk aversion and other factors
such as skewness preference are assumed to cancel each other out.

Since the empirical results confirm some of the basic features of the model, it
is reasonable to investigate numerically the factors affecting the value of a venture
capital project. The fundamental variables affecting the value are:

1. Expected log payoff, \( E[I(t)] \)
2. Variability of payoffs
3. Present value of the input stream
4. Pattern of uncertainty resolution relative to the pattern of inputs through time
5. Salvage values  
6. Life of the project.

An increase in the expected payoff, a decrease in the present value of the input stream, an increase in salvage value, or earlier resolution of uncertainty relative to the input stream— all unambiguously increase the value of the project. The impact of uncertainty is more ambiguous.

Increasing uncertainty will increase the value of the project if expectations are low, so the asset behaves more like a pure call option. If the degree of risk aversion of the venture capitalist is low, then an increase in variability will almost always increase the value of the venture capital asset.

V. Public Policy

The issue of whether to subsidize investment in small companies is beyond the scope of this reading. Subsidies may be justified on the grounds of externalities to venture capital investment, such as imperfections in the supply of funds or the relative burden of compliance with government regulations for small as opposed to large companies.

This section does, however, examine, in the light of the preceding model, various types of government schemes for subsidizing venture capital investment. Any policy of subsidy will have the effect of lowering the cutoff states for the venture capitalist \(L(t)\), so the venture capitalist will be prepared to continue the project in states when he would otherwise have abandoned it. To avoid the question of distribution effects, schemes that give the same optimal policy for the venture capitalist will be compared. The criterion for judging the superior subsidy scheme will be the minimum cost for a given policy objective.

First, consider a system of subsidies where the payoff to the venture capitalist is subsidized, for instance, by tax concessions. For simplicity, assume that this subsidy is proportional to the payoff \(X(T)\) so that the venture capitalist receives \((1 + S) \times X(T)\). An alternative method of subsidy would be to contribute part of the inputs \(I(t)\) so that the venture capitalist contributes only \((1 - r)I(t)\). The relative costs of these two schemes are:

\[
\text{Subsidize payoffs: } PV(SX) \\
\text{Subsidize inputs: } PV(rI)
\]

where \(PV(\quad)\) = present-value operator applied to the subsidies that actually get paid.

For the schemes to give the same optimal investment policies, the cutoff stakes must be such that

\[
PV(SX) = PV(rI).
\]

As expectations rise, the value of the project rises, so the value of payoffs must rise faster than the value of inputs. Therefore, for a profitable project,

\[
PV(SX) > PV(rI).
\]

Thus, for projects that are accepted, the cost of the payoff subsidy scheme must be higher than the cost of subsidizing inputs.

Apart from the general subsidy schemes designed to stimulate investment in small companies, schemes can be designed to correct specific suboptimal behavior on the part of venture capital investors. A common complaint against venture capital suppliers is that they are deterred from investing in situations where there is a large chance of failure because abandonment carries nonfinancial costs. In this case, the abandonment value \(S(t)\) could be raised by subsidy, but there is an alternative, more conventional approach. A loan program that supplies funds at the start of the project and takes a fixed amount from the payoff lowers the cutoff stakes for the venture capitalist at times after period zero (the initiation of the project). Thus the perceived costs of abandonment can be negated by such a scheme. The loan lowers the venture capitalist's initial investment in the project so that the choice of which projects to invest in can remain unchanged. As long as the perceived cost of abandonment is not a real cost, this can be accomplished at no net cost to the subsidizing agency since the net present value of the project, apart from the nonfinancial perceived costs, will be raised, and the loan can be made to have a net present value of zero.

VI. Summary and Conclusions

The model of venture capital investment presented can explain well various observed aspects of venture capital behavior. The main focus of the model is on the multiperiod nature of venture capital investment and the possibility of abandoning the project along the way.

Optimal investment policies consist of a cutoff level for expectations at each stage of the venture capital process. Below these levels of expectations about the ultimate payoff from the project, it will not pay the venture capitalist to continue with the project. This sequential decision process makes a venture capital asset behave like a hybrid of a pure call option and a more conventional asset with no discretionary element. When expectations are high, the asset behaves like a going concern; when low, like a call option or pure growth opportunity.

Payoffs to venture capital investments fall into two groups. Negative returns are achieved by projects that are abandoned, positive returns with heavy skewness by those that eventually become marketable. Interpretation of data on venture capital returns is hazardous, and conventional mean-variance analysis can be ex-
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expected to give strange results when applied in this area.

Government schemes to aid venture capital investment must be determined by
judgments of external benefits to small businesses or by other policy judgments. In
general, however, it will be cheaper to subsidize the investment in venture capital
projects than to grant tax concessions on payoffs. Also, specific distortions need
specific remedies. For instance, the unwillingness of venture capitalists to abandon
unprofitable projects can be corrected by a zero-cost government loan scheme.

APPENDIX

1. If states are ordered by some scalar statistic $K(t)$, and the distribution of
$K(t+1)$ given $K(t)$ is first-order stochastic dominant (FSD) over that given any
$K(t)$ less than $K(t)$, then

$$U^*[L(t)]$$

is nondecreasing in $K(t)$.

Proof: The proposition is true for $t = T$ since

$$U^*[L(T)] = \max \{u[S(T)], U[L(T)]\}.$$ 

Assume that it is true for $t + 1$. Then,

$$G[K(t)] = \int [K(t+1) | K(t)] / dK(t+1)$$

is nondecreasing in $K(t)$ by FSD.

$$U^*[K(t)] = \max \{u[S(t)], u[-i(t)] + aG[K(t)]\}$$

which is nondecreasing in $K(t)$. Thus the proposition is true by induction.

The value $L(t)$ satisfies the conditions on $K(t)$, and $U[L(t)]$ is monotone in
creasing in $L(t)$, so $U^*[L(t)]$ is monotone nondecreasing in $L(t)$ and $U[L(t)]$.

2. $U^*[L(t)] - U[L(t)]$ is nonincreasing in $L(t)$.

Proof:

$$U^*[L(t)] - U[L(t)] = \max(U1, U2)$$

where $U1 = -u[-i(t)] - \int U[L(t+1)] f[L(t+1) | L(t)] dL(t+1)$

$U2 = \int [U^*[L(t+1)] - U[L(t+1)] f[L(t+1) | L(t)] dL(t+1).$

The proposition holds for $t = T$. Then, by FSD, $U2$ is nonincreasing in $L(t)$, and
$U1$ is nonincreasing in $L(t)$ from the previous proof, so the proposition is true
by induction.

3. $U^*[L(t)]$ is convex in $U[L(t)]$.

Proof:

$$U^*[L(T)] = \max \{u[S(T)], U[L(T)]\}$$

so the proposition holds for $t = T$. Assume that it holds for $t + 1$. Then, for all $y$,

$$L(t) + y = L(t + 1)$$

$$L^*(t) + y = L^*(t + 1)$$

$$U[L^*(t + 1)] = kU[L(t+1)] + (1 - k) U[L^*(t + 1)]$$

$$U^*[L^*(t + 1)] \leq kU^*[L(t+1)] + (1 - k) U^*[L^*(t + 1)].$$
Thus
\[ f(y) dy = k \int U[L(t+1)] f(y) dy \]
implies that
\[ f(y) dy = k \int U[ L(t+1)] f(y) dy. \]
Since \( f[L(t+1) | L(t)] \) is a function only of \( y \),
\[ G[L(t)] = u(-k) + k \int U[L(t+1)] f(y) dy \]
is convex in \( U[L(t)] \). Therefore,
\[ U^{*}[L(t)] = \max \{ u(S(t)), G[L(t)] \} \]
is convex in \( U[L(t)] \), and the proof is complete by induction.

ENDNOTES

1. This assumption is similar to that in Turnbull (1974).
2. The assumptions that the investments and salvage values are independent of the state variable, \( L(t) \), are not entirely realistic. To the extent that they do vary with the success of the project, \( I(t) \) and \( S(t) \) may be viewed as representing the fixed elements of the inputs and salvage values, with the variable portion being incorporated as an adjustment to \( L(t) \).
3. In general, the utility for wealth increments will depend upon the wealth level of the venture capitalist. For notational simplicity, this is not shown explicitly as an argument of the utility function.
4. This result does not conflict with the option theory result that increasing variability of returns increases the value of a call option. In that case, the value of the underlying asset is held constant; here, the expected terminal payoff is being held constant.
5. This is the internal rate of return on the aggregated cash flows from the projects.
6. This may be caused by their fiduciary role.

REFERENCES