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6 GROWTH OPPORTUNITIES AND REAL INVESTMENT DECISIONS

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Several authors, especially Myers [4], have noted that part of the value of any company is attributable to its opportunities to make investments, at future dates, that have positive net present values. A generic term for such opportunities is *growth opportunities.*

In his paper, Myers investigates the impact of such growth opportunities on the optimal capital structure of a firm. Growth opportunities will affect real investment decisions, as well as financing decisions, of firms. This paper shows two ways in which capital budgeting procedures must be adjusted to take account of the options implicit in future opportunities for growth.

The first adjustment concerns the choice of discount rate for risky capital projects. Franks and Broyles [2] have suggested a procedure for employing the Capital Asset Pricing Model (CAPM) in the choice of discount rates for investments in real assets. The procedure involves determining the systematic risk (beta) for the average project in a company (or division) by measuring the beta of the firm (or a collection of similar firms in the same market) adjusted for financial leverage. A further adjustment for an individual project's operational leverage establishes the beta for the project. The project's beta is then used in the CAPM to determine an appropriate discount rate for the project. However, we will show that a further adjustment to estimates of beta based on market prices is required, since market values of a company reflect the value of growth opportunities, as well as the value of existing assets already in place. The appropriate beta for capital budgeting is that of the asset once it has been created (since it is the cash-flow stream of the asset in place that is being discounted). This can be derived by adjusting the beta estimated from market value by a factor that reflects the extent of the growth opportunities facing the firm.

The second adjustment that must be made affects the acceptance criterion. Normally, it is advocated that companies accept projects with net present values greater than zero. However, net present values, as conventionally computed, do not reflect the value of the investment as a growth opportunity even if investment is postponed. The opportunity cost of postponement can substantially affect optimal real investment decisions by companies.

The topics covered in this paper are: assumptions and growth opportunity valuation, adjusting betas for growth opportunities, the timing of investment decisions, and the impact of competition in product markets.

The valuation model used is the Black-Scholes option valuation model. To achieve the arbitrage underlying this model requires the existence of assets that can hedge the option. The conditions necessary for this are set out fully in the next section.

ASSUMPTIONS AND GROWTH OPPORTUNITY VALUATION

A growth opportunity is the ability to make a real investment, at some future date, that may turn out to have a positive net present value at that time. There are three crucial elements in this opportunity: the time at which the investment should be made, the investment required to create the new asset at that date, and the value of the asset, once created.

Assumption A1: A growth opportunity consists of the option to invest, at time T, an amount $I(T)$ that will create an asset of value $A(T)$.

The impact of this opportunity (at some time prior to T) on the value of the company concerned will be determined by expectations about asset value ($A(T)$) and the investment required ($I(T)$), both of which may be random when viewed from time t ($<T$). To value growth opportunities as individual entities, we need the condition that gives value additivity. This condition will also prove useful in setting up a Black-Scholes hedge.

Assumption A2: The payoff contingent state space of $(A(T) - I(T))$ at time T is spanned by securities in the market.

This condition implies that the company can value its growth opportunity independently of other assets that it possesses. It also means that there are securities in the market that can be combined to give a portfolio at time t $(<T)$ that will have the same value $A(t)$ as the underlying real asset.

The value of this portfolio plays the same role as the underlying stock in the Black-Scholes call option valuation formula. A riskless hedge can be set up between this portfolio and the growth opportunity and, by varying the amount of the portfolio sold against the growth opportunity, the riskless hedge may be maintained. Another condition necessary to the Black-Scholes valuation formula is that the exercise price of the option be fixed. The equivalent of the exercise price here is the investment required to create the new asset $(I(T))$. This is likely to vary with the value of the growth opportunity, and we make a simple linear assumption:

Assumption A3:

$$I(T) = \bar{I}(T) + (1 - \alpha)\,A(T) \qquad 1 > \alpha > 0.$$

This implies that there is a fixed component of the investment required to create the new asset, as well as a variable component that increases with the value of the asset, but not as quickly.[1]

The final assumption necessary concerns the return dynamics of the value of the underlying asset, $A(t)$. There is ample evidence that returns on equity securities are approximately lognormally distributed. This should be true *a fortiori* here since we are eliminating the option element, which would induce nonlognormality in the value of the growth opportunity, and just looking at the value of the underlying asset, $A(t)$.

Assumption A4: $A(t)$ follows an Itô process.

The optimal decision at time T will be to undertake the investment if it has a positive net present value:

$$\begin{aligned} \text{Accept at } T \text{ if} \quad A(T) &> I(T) \\ A(T) &> \bar{I}(T) + (1 - \alpha)\,A(T) \\ \alpha A(T) &> \bar{I}(T). \end{aligned}$$

This growth opportunity is thus a call option on the asset $\alpha A(t)$, with an exercise price of $\bar{I}(T)$. Given assumptions A1-A4, we may price the growth opportunity by using the Black-Scholes formula:

$$G(t) = \alpha A(t)\,N(d_1) - \bar{I}(T)e^{-r\tau}N(d_2), \tag{1}$$

where

$G(t)$ is the value at time t of the growth opportunity;
$N(\cdot)$ is the normal distribution function;
r is the continuous risk-free rate;
σ^2 is the variance rate of the return of $A(t)$;

and

$$\tau = T - t$$

$$d_1 = \frac{\log \alpha A(t) / \bar{I}(T) + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}.$$

The response of this value to the various parameters has been adequately discussed in Black and Scholes [1] and Galai and Masulis [3].

THE VALUE OF THE COMPANY AND BETAS FOR USE IN CAPITAL BUDGETING

So far we have discussed the valuation of a single growth opportunity. A company consists of many assets in place and many growth opportunities. Given our value additivity assumption, the total value of the company will be

$$V(t) = \sum_{i=1}^{n} E(t, i) + \sum_{T=t+1}^{\infty} \sum_{j=i}^{M(T)} G(t, T, j), \tag{2}$$

where

$V(t)$ is the value of the company at time t;
n is the number of assets in place at time t;
$M(T)$ is the number of growth opportunities for time T;
$E(t, i)$ is the value at time t of the ith existing asset;
$G(t, T, j)$ is the value at time t of the jth growth opportunity for time T.

The usually advocated method of computing a discount rate for capital budgeting involves estimating a beta for the new asset based upon market returns for assets of similar systematic risk. This is done by estimating the betas of companies in the industry of the new project.

From (2) the beta of a company is given by

$$\beta(V)V(t) = \sum_{i=1}^{n} \beta(E, t, i,)E(t, i) + \sum_{T=t+1}^{\infty} \sum_{j=1}^{M(T)} \beta(G, t, T, j)\, G(t, T, j), \tag{3}$$

where

$\beta(E, t, i)$ is the beta of the ith existing asset;
$\beta(G, t, T, j)$ is the beta at time t of the (Tj)th growth opportunity.

From (1) it is possible to show that[2]

$$\beta(G)G(t) = \alpha A(t)\, N(d_1)\, \beta(A). \tag{4}$$

Here the subscripts have been dropped for simplicity of presentation, but this expression applies to each growth opportunity separately. Using (1), we can eliminate the value of the underlying asset from (4) :

$$\beta(G)G(t) = \{G(t) + \bar{I}(T)e^{-r(\tau)}\, N(d_2)\}\ \beta(A). \tag{5}$$

It is well known that the beta of a company is only suitable for use in capital budgeting if the risk of the project can be related to the risk of the company. Clearly, this does not imply that the beta of all growth opportunities is the same as the mean beta of existing assets, since the former vary over time as the growth option matures.

Even if there are no actual companies with all assets of the same risk, the use of such a company as an example will give an expression for the bias induced by growth opportunities.

Assumption A5:

$$\bar{\beta} = \beta(E, t, i) = \beta(E, t, j) = \beta(A, t, T, i) = \beta(A, t, T, j) \qquad \text{all } i, j, \tag{6}$$

where

$$\beta(A, t, T, j) = \beta(A(t)) \text{ for growth opportunity } Tj.$$

Using (3), (5), and (6) gives

$$\beta(V)V(t) = \sum_{i=1}^{n} \bar{\beta}E(t, i) + \sum_{T=t+1}^{\infty} \sum_{j=1}^{M(T)} \bar{\beta}G(t, T, j)$$
$$+ \sum_{T=t+1}^{\infty} \sum_{j=1}^{M(T)} \bar{\beta}PV_t(\bar{I}, T, j), \tag{7}$$

where

$$PV_t(\bar{I}, T, j) = \bar{I}(T, j)e^{-r(T-t)}N(d_2(T, j)).$$

Using (2) gives

$$\beta(V)V(t) = \bar{\beta}V(t) + \bar{\beta}PV_t(\bar{I}), \tag{8}$$

where

$$PV_t(\bar{I}) = \sum_{T=t+1}^{\infty} \sum_{j=1}^{M(T)} PV_t(\bar{I}, T, j).$$

Suppose that $\beta(V)$ is estimated from market returns. The correct beta to use in capital budgeting for this company is $\bar{\beta}$, since this reflects the risk of the cash flows generated by an asset once in place; it is these cash flows that are being discounted to give a present value. Equation (8) gives the adjustment that must be applied to the market estimate of beta for use in capital budgeting.

$$\bar{\beta} = \beta(V)\left\{\frac{V(t)}{V(t) + PV_t(\bar{I})}\right\}. \tag{9}$$

This adjustment clearly reduces the beta. The reason is simple: Market-based betas contain the extra volatility of growth opportunities. If the decision being made concerns the creation of a new asset, this extra volatility will not be present in the cash-flow stream from the asset, once created.

Although the adjustment given in (9) is simple, it contains one term, $PV_t(\bar{I})$, that has not yet been given an intuitive interpretation. The easiest case is where there is no variable element in the investment required to undertake the growth opportunity. In this case, alpha is unity and $PV_t(\bar{I})$ is the present value at time t of all future investment in growth opportunities. The magnitude of this adjustment can be seen by considering two extreme cases:

1. No growth opportunities
2. No existing assets

In case 1, there is no adjustment necessary, and the market beta is equal to the capital budgeting beta. In case 2, the adjustment factor necessary to convert the market beta into a capital budgeting beta falls as the value of the growth opportunities falls relative to the investment required. This adjustment factor can never fall to zero as long as the company has any value.

A typical growth company might spend 5 percent of its current value on growth each year, over and above the investment required to maintain its existing assets.[3] If this investment in growth is expected to continue at the same level in perpetuity, and we discount it at a rate of, say, 10 percent, we get

$$PV_t(\bar{I}) = 0.50V(t) \tag{10}$$

and

$$\bar{\beta} = 0.33\beta(V).$$

Thus, even with these conservative assumptions about growth, we get a sizable adjustment to the market beta to give a beta for use in capital budgeting.

Where the investment in the growth opportunities varies with the value of those opportunities ($\alpha < 1$), the adjustment factor depends only on the present value of the fixed element in the investment. In this case, the minimal level of expenditure on a particular growth opportunity occurs when

$$\bar{I} + (1 - \alpha)A = A \tag{11}$$

so that

$$\bar{I} + (1 - \alpha)A = I_m = A, \tag{12}$$

where I_m is the minimum amount that would be spent on this opportunity if it proved to be profitable. Rearranging (12) gives

$$\bar{I} = \alpha I_m . \tag{13}$$

Thus $PV_t(\bar{I})$ may be reinterpreted as the present value of the minimal profitable levels of investment on growth opportunities adjusted downward to the extent that the amounts of these investments are variable.

For example, consider an all-equity financed company that is generating a perpetual income stream from current assets of \$10 million p.a. The beta of this income stream is 0.5, and the market value of existing assets is \$100 million. The company will, one year from now, double its capacity at a cost of \$80 million if the investment proves profitable at that time. The revenue stream from the new capacity will be identical to that from the old. The variance rate of returns on existing capacity is .1 per year on a continuous basis. The risk-free rate is 5 percent.

The value of the growth opportunity is

$$G = 100N(d_1) = 80e^{-.05}N(d_2)$$

$$d_1 = \frac{\log(100/80) + (.05 + .5 \times .1)}{\sqrt{.1}} = 1.022$$

$$d_2 = d_1 - \sqrt{.1} = .706$$

$$\frac{\delta G(t,T)}{\delta T}=\frac{\delta A(t,T)}{\delta T}N(d_1^T)-\frac{\delta I(T)}{\delta T}e^{-rt}N(d_2^T)$$
$$+I(T)e^{-rt}[f(d_2^T)\sigma/2\sqrt{\tau}+rN(d_2^T)]. \quad (15)$$

An optimal fixed exercise date is given by setting (15) equal to zero, but this may not be an optimal overall policy since a variable exercise date must have at least as great a value.

If the investment is currently profitable, then $A(t, t)$ is greater than $I(t)$, and the investment will have more value as a growth opportunity. However,

$$\left.\frac{\delta G(t,T)}{\delta T}\right|_{T=t}>0; \quad (16)$$

then substituting (for a currently profitable investment) the values

$$d_1^t=d_2^t=\infty$$

so that

$$f(d_1^t)=0$$
$$N(d_1^t)=N(d_2^t)=1,$$

the condition (16) gives

$$\left\{\frac{\delta A(t,T)}{\delta T}-\frac{\delta I(T)}{\delta T}+I(T)r\right\}_{T=t}>0 \quad (17)$$

$$-\left.\frac{\delta \mathrm{NPV}(t,T)}{\delta T}\right|_{T=t}<I(t)r, \quad (18)$$

where NPV(t, T) is the net present value at time t of the unconditional payoffs at time T.

The speed at which the net present value of the project is eroded depends upon the reactions of competitors. Equation (18) states that a sufficient condition for postponing a profitable investment is that the rate of erosion of net present value should be less than the interest that can be earned on the amount of the investment. Since (18) is based on a fixed investment date and considers only an incremental postponement of the project, it is not a necessary condition for postponement. The necessary condition is that the value of the investment as a growth opportunity be greater than its value as an asset in place.

The conditions for the asset to have more value as a growth opportunity are:

1. The rate of erosion of competitive advantage is low.
2. The investment required is high.

3. The interest rate is high.
4. The uncertainty about the project is high.

In particular, the fourth of these, a high degree of uncertainty about the project, will mean that the postponement criterion (18) is too stringent. With much uncertainty, the asset as a growth opportunity has a high value, so that profitable projects whose net present values are eroding faster than the interest on the investment required may still be optimally postponed.

COMPETITIVE ASSUMPTIONS

In the previous section, the way in which competitive advantage (positive NPV) is eroded through time was seen to play a crucial part in the investment decision. This section shows the impact of a specific realistic assumption about this process on optimal investment decisions. To do this, we will once again consider an individual asset, which may be bought at time t or retained as a growth opportunity until time T.

Investing in the project now gives a value of

$$E(t) - I(t),$$

where $E(t) = A(t, t)$, the value of the asset in place at time t.

Retaining the investment as a growth opportunity has the value of

$$G(t, T) = A(t, T)N(d_1) - I(T)e^{-rt}N(d_2),$$

where

$A(t, T)$ is the value at time T of the unconditional asset value at time T;
$I(T)$ is the investment required at time T.

If we plot the value of the growth opportunity against the value of its underlying asset ($A(t, T)$), we get the line (GG) in Figure 1. This is the value of the asset as a growth opportunity.

The line (UU) gives the net present value at time t of the asset created unconditionally at time T. This value is equal to

$$U(t, T) = A(t, T) - I(T)e^{-rt}. \tag{19}$$

If competition erodes net present value, this will be less than the corresponding net present value of the asset created now:

$$\text{NPV}(t) = E(t) - I(t). \tag{20}$$

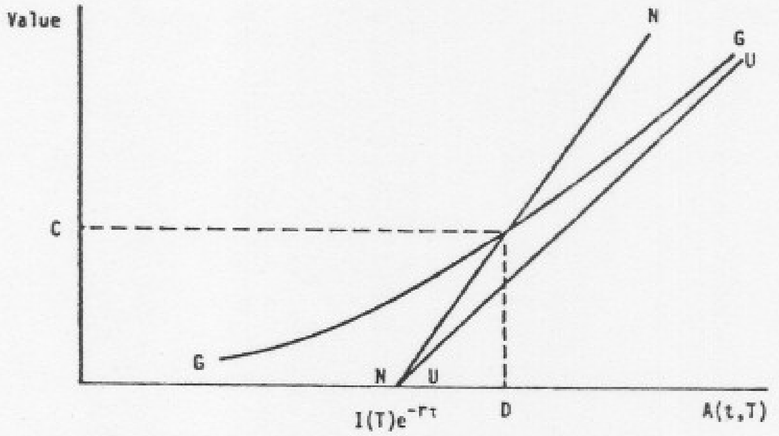

Figure 1. Asset Values

Furthermore, the higher the current net present value, the more it will be eroded between now and time T, so that the difference between NPV(t) and $U(t)$ will increase with the level of NPV(t). Finally, if NPV(t) is zero, so that the project is only marginal, we expect no erosion of net present value over time.

These assumptions about competitive forces are depicted by the line (NN) in Figure 1. The comparison that is relevant to capital budgeting decisions is that between the net present value of the project created now (NN) and the value of the project as a growth opportunity (GG). Above a net present value of C, the investment should be made immediately; below that level, it should be postponed. There is thus a single cutoff criterion for the investment, given these reasonable assumptions about competition, but that cutoff rate is at a net present value greater than zero.[5]

CONCLUSIONS

We have shown that there are two significant biases introduced into capital budgeting decisions by ignoring growth opportunities. Market-measured betas are biased upward for growth companies. A simple adjustment can be made to give, in principle, a correct beta for use in capital budgeting. Accepting all projects with positive net present value ignores the opportunity to keep the asset as a growth option. Under a reasonable set of assumptions about competitive forces, there is a single optimal cutoff point for investment in a project, but it is at a level of NPV greater than zero.

NOTES

1. A less restrictive assumption would be to make z stochastic, so that

$$I(T) = \bar{I}(T) + (1 - \tilde{z})A(T).$$

Then the acceptance criterion is

$$\tilde{z}A(T) > \bar{I}(T),$$

and the underlying asset must be interpreted as the value of a claim on the payoff $zA(T)$. The adjustment to beta remains the same as long as $\tilde{z}$ is independent of the return to the market portfolio. The variance σ^2 in equation (1) would then include the variance of $\tilde{z}$, as well as the variance of $A(t)$.

2. See, for example, Galai and Masulis [3].

3. To the extent that maintenance and replacement of existing assets is optional, this investment should also be included in $PV_t(\bar{I})$.

4. This result uses the fact that

$$A(t, T)f(d_1^T) = I(T)e^{-rt}f(d_2^T).$$

5. Figure 1 refers to the choice between investing at time t and investing at some specific future date, T. The argument may be generalized to allow for exercise of the growth opportunity at any optimal future date, and the conclusions are the same.

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