Currency Option Prices

WITH THE GROWTH of the currency option market has come new jargon: 'model prices', 'deltas', 'volatility', 'time premium'. All these terms have been borrowed from the share option market, and corporate treasurers are confronted with a language that appears to be quite unrelated to traditional financial management. The purpose of this article is to explain the thinking that lies behind option pricing models and the way that these complicated ideas all arise from a particular characteristic of options.

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Two characteristics of options

The aspect of options that is easiest to understand, and makes them useful for controlling exchange exposure is:

If held to maturity, an option provides insurance.

In exchange for the immediate payment of the option price, the purchaser receives an option which protects him if the spot rate ends up above the exercise price of the option, with no penalty if the spot rate is below the exercise price. This is like being able to take forward cover after you know whether it is valuable or not. For instance, an exporter who will receive $100,000 six months from now can guarantee a minimum rate of exchange by buying a six-month call on sterling. If the spot rate is $1.10, and the exercise price on the call is also $1.10, the price of the call might be about 4 cents per dollar. At the maturity date of the option, if the spot dollar rate is above $1.10, the option is exercised, and the $100,000 is converted to sterling at $1.10, rather than at the spot rate. This guarantees that the net proceeds of the trade will be at least £90,909 minus the £3,636 paid for the call, giving an effective maximum on the net $/£ exchange rate of $1.1458.

Issuers of options charge for the valuable privilege they are granting to the purchasers. How do they set the price? They use a model that computes a 'fair value' for the option. This model is based upon a characteristic of options that banks can use to hedge the exposure arising from writing options:

Over a short period of time, such as a day, a currency option gives the same return as a particular highly geared spot purchase.

In the example above, at the time the call is written, sterling is trading at $1.10 and the value of the call is 4 cents. If sterling rises to $1.12, the call might be worth 5 cents, and if sterling falls to $1.08, the call might be worth 3 cents. If the bank writes the option and does nothing else, it has, effectively, a short position in sterling. How big is its effective short position? For every 2 cents move in the spot price, there is a 1 cent move in the option value, so the call is behaving like a spot position equal to half the currency on which the call is written. The ratio of the move in the call value to the move in the spot rate is known as the 'hedge ratio' or 'delta' of the option. In this case, the delta is 0.5.

The importance of the hedge ratio is that it tells the writer of the option how much spot currency to hold to eliminate the risk of the option. In this case, if the writer of the option had held sterling of an amount equal to £45,454 (half the face value of the call), he would not have been exposed to the change in the value of the currency option. Over the day, if the spot rate rose to $1.12, the call value would have risen in value by $1,000, and the spot sterling position also changed by the same amount, offsetting the loss on the written call.

There is an apparent contradiction between the equivalence of the option to a geared spot position, and the unique insurance characteristic of the option. The link lies in understanding the difference between the time horizons involved. The insurance characteristic of options applies if they are held to maturity with no intervening adjustment of the position. This is the usual perspective of corporate treasurers. The equivalence to a geared spot purchase applies over a short period of time such as a day. As the spot price moves, the delta changes, and the spot position that is equivalent to the option changes correspondingly. To construct a spot position that is equivalent to the option over its entire life, the amount of spot currency held to hedge the option must change in response to the spot rate. As a result:

If a call is written and a spot position of delta is taken, and delta is revised frequently enough, the net position will be riskless over the entire life of the option.

This forms the basis of hedging for institutions that write options. By following a delta strategy in the spot market, they can eliminate the risk of the option position. Indeed, if they use forward contracts to perform the hedging strategy, they need not invest funds in the position that hedges the exposure of the written option.

... can be replicated by holding the delta in the spot market and changing the amount held spot whenever the delta changes as the spot rate changes.**

The cost of this replication strategy is the model price of the option. The model price depends upon: the exercise price of the option, the ... of the currency over the life of the option. The model price of the option *does not* depend upon whether the holder is bullish or bearish about the currency concerned. To understand why this is the case, think of an option as a more complicated version of a forward contract. We know that the price of a forward contract depends only upon the no-arbitrage condition known as interest-rate parity. If the forward rate deviates from this (other than in the bounds set by transaction costs), the deviation will be eliminated by arbitageurs who borrow one currency, lend the other, sell forward, and make riskless arbitrage gains.

Similar techniques can be used to arbitrage deviations of option prices from model prices:

- **If the price of a call is above its model price, write calls, hold a spot position equal to the delta, and revise the spot position whenever the delta changes.**
- **If the price of a call is above its model price, and you want to buy the call, 'replicate' the call by holding spot the delta and revising this position whenever the delta changes.**

The possibility of these arbitrage transactions is what should keep actual prices close to model prices. Currency option prices are not as close to model prices, however, as actual forward rates are to the interest-parity forward rates. Why?

In fact, the 'arbitrage' transactions that could guarantee that market prices are equal to model prices suffer from two limitations:

1 The arbitrage transactions require a lot of trading.

A typical hedging strategy for a one-year option on $1 ... over the life of the option. The transaction costs of this trading may be quite high and, therefore, limit the ability of corporate treasurers to replicate their own options.

2 The arbitrage or replication is only as good as the forecast of currency volatility on which it is based.

The price of the option and the delta are based upon a forecast of the volatility of the currency over the life of the option. If this forecast is wrong, the model price will

Volatility

This example shows the importance of volatility to the option market. What does 'volatility' actually mean? Options as insurance, and options as geared spot positions.

Consider first the option held for its insurance protection. The value of this depends upon the chance that the spot price will be above or below the exercise price of the option at the maturity date. This probability depends upon the uncertainty about the spot rate at the time the option matures. The standard deviation of the distribution of possible levels of the spot rate at the maturity date describes this uncertainty.

Another way of thinking about this volatility is to take the view of a writer of options who hedges them by holding the delta in the spot market. He will adjust this delta periodically, for instance once a day. Over the day that the delta is held constant, the net value of the hedge position responds to the size of the spot price move, but not its direction. Over the entire life of the option, it is the average size of the daily price moves that matters to the hedger. This can be summarised by the standard deviation of the daily price moves.

A concern with volatility, therefore, arises quite naturally out of a concern either with the probability that ... maturity date of the option, or a concern with the average size of price movements on the days over which a hedge is held. Once again, there is an apparent difference between the perspective of treasurers who hold the option position open to maturity, and banks, who continually hedge their exposure on written options by holding spot currency. In fact there is a simple relationship between volatility over a short period and volatility over a long period:

The standard deviation over T days is equal to the square root of T times the daily standard deviation.

For instance, the standard deviation over 3 months is equal to approximately eight times the standard deviation over one day, since there are approximately 63 trading days in a quarter, and the square root of 63 is about eight.

Table 1 gives standard deviations over different time periods. All of these are equivalent to a standard deviation of 10% per annum. Because there is this equivalence, the market has adopted the ... always quoting volatilities on an annualised basis, for much the same reasons that interest rates are always quoted on an annual basis.

Table 1: Volatility over different periods					
Period	1 day	3 months	6 months	9 months	1 year
Standard deviation	0.6%	5.0%	7.1%	8.7%	10.0%

Note from Table 1 that volatility does not increase one-for-one with time. Over 3 months there is half as much volatility as over one year. Table 2 is a simple illustration of why this is so. Suppose a currency rate can move up or down by 1% or stay the same over a single day. The standard deviation is 0.82%. Over two days the standard deviation is 1.15%, which is equal to the square root of two times 0.82%. The two-day moves are not twice as scattered as the one-day moves, since sometimes when the currency moves up on the first day, it moves back down on the second day and vice-versa. This offsets part of the volatility contributed by the first day, resulting in the two-day standard deviation being less than twice the one-day standard deviation. (Of course, if both are annualised, they will give the same number).

Table 2: Volatility over one and two days.

One day		Two days	
Move	Chance	Move	Chance
		+2%	1/9
+1%	1/3	+1%	2/9
0	1/3	0	3/9
−1%	1/3	−1%	2/9
		−2%	1/9
SD	0.82%	SD	1.15%

Time premium decay

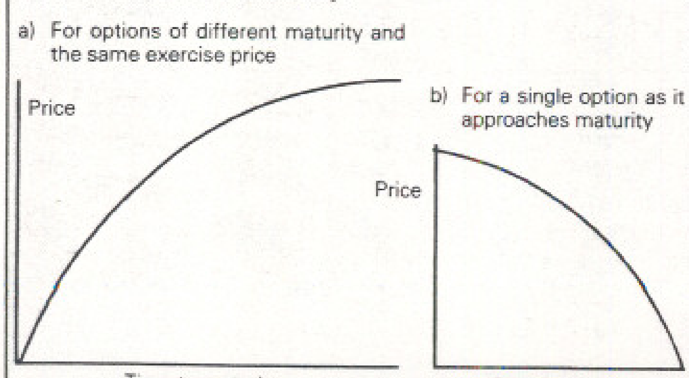
Now that we have the relationship between volatility and length of time, it is quite easy to understand another feature of options known as 'time premium decay'. This can be stated in two ways:

As you increase the maturity of an option, its price does not increase proportionately to the maturity increase.

As an option approaches maturity, its value falls faster than when it is a long way away from maturity.

An option with four weeks to maturity has twice as much volatility over its remaining life as one with one week to maturity. An option with eight weeks to maturity has only 40% more currency volatility over its life than one with four weeks to maturity. The pattern of prices one sees, is, therefore like Figure 1, with a rapid fall-off close to maturity, and a much slower fall-off away from maturity.

Figure 1: The pattern of time decay.



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Putting it all together: 'model prices' vs 'supply and demand'

I said above that the 'model' price of an option is set by a no-arbitrage restriction similar to the interest-parity condition on forward rates. If the arbitrage underlying the model price were as cheap and easy as the arbitrage underlying the interest-parity condition, we would see currency option prices close to model prices and buy/sell spreads as small as those in the short-term forward markets.

What we actually observe is market option prices that deviate (sometimes substantially) from model prices, and large buy/sell spreads. The price premia and spreads are compensation for the writers of options who must themselves perform the hedging transactions in the spot market. In doing so, they must pay transaction costs and suffer the risk that the volatility forecast on which they base their hedging is wrong.

Figure 2: The structure of the market

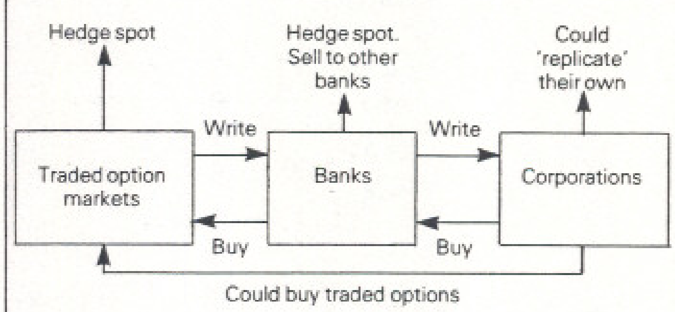

Figure 2 presents a schematic view of the market place. An important feature of the marketplace is:

Most options are hedged by someone.

Someone, somewhere, whether it is the bank that writes the option, or another bank, or a market-maker in the traded option market, performs the delta hedging strategy that offsets the option. In principle, the corporation could perform these trades itself and 'replicate' the option. There are various reasons why this is not current market practice. Low transactions costs and economies of scale mean that it is transactionally more efficient for intermediaries to perform the hedging transaction that 'synthesise' the option, and write the option to the corporation. Nevertheless, potential competition from corporations who synthesise their own options and from substitute intermediaries who are prepared to write options for small spreads off the model price must limit the deviation of market prices from model prices. Thus:

The spread of the market price over the model price is limited by the possibility of corporations and substitute market makers synthesising options.

The actual price paid by a corporation is equal to:

Model price
plus transaction cost of hedging by bank
plus premium for risk of volatility changing
plus pure profit spread

The cost of substitution is:

Model price
plus transaction cost of replication by the corporation
plus premium for risk of volatility changing

Thus, the profit spread that the bank can charge is limited by its competitive advantage in performing spot currency transactions and forecasting and managing exposure to volatility changes.

As corporations become more knowledgeable about currency options, and competition among intermediaries increases, deviations of market prices from model prices will grow smaller. Eventually, we can expect to see all prices set close to model prices, because the models represent arbitrage and substitution

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possibilities that will dominate the market place and drive out products that are priced far away from the model prices. ■

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