SELF-FULFILLING LIQUIDITY DRY-UPS*

Frédéric Malherbe†

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Abstract

In a model where markets might be illiquid due to adverse selection, I show that hoarding behaviors may dry up future market liquidity. Hoarding is triggered by the anticipation of illiquidity, and actually worsens future adverse selection. I show that this feedback effect creates strategic complementarities in the extent to which agents expose themselves to maturity mismatch, and that this may lead to multiple equilibria that can be Pareto ranked according to their level of market liquidity. The paper highlights a channel by which policies that aim at raising financial institutions liquidity ratios may have unintended consequences.

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†London Business School, Regent's Park, NW1 4SA, London, United Kingdom, +44 (0)20 7000 8428 (tel), +44 (0) 20 7000 8401 (fax), fmalherbe@london.edu.

1 Introduction

What is the impact of hoarding behavior on adverse selection in financial market? It is well understood that the anticipation of future illiquidity may induce hoarding behaviors (Allen and Gale, 1994). Since liquidity can be understood as the ability to transform long-term assets into current consumption goods, adverse selection (Akerlof, 1970) is a potential cause of illiquidity. What I present here is a model in which hoarding behavior, triggered by illiquidity fears, actually worsens future adverse selection and *causes* a market breakdown. In that case, it becomes extremely costly to transform long-term assets into current consumption goods, which is the rationale for calling such self-fulfilling episode a liquidity dry-up.

The intuition why the proportion of *lemons* in the market increases in the presence of hoarding behavior is best apprehended from a buyer point of view: when sellers are already sitting on a pile of cash, it is not very credible that they trade because they need more. Why would they want to sell then? The answer is striking: the asset must be a *lemon*! This mechanism resonates with the recent financial crisis, which has seen unusual cash hoarding behavior and the persistent breakdown of markets such as those for the asset back securities. The fact that these assets have then been dubbed "toxic" strongly suggests that adverse selection has been at play, which has been the focus of several recent papers (see Tirole, 2011b; Bolton, Santos, and Scheinkman, 2011; Morris and Shin, 2010, among others). However, the fact that hoarding behavior and adverse selection may *reinforce* each other has received little attention so far.

That hoarding behaviors worsen adverse selection highlights a channel by which prudential policies that aim at raising financial institutions liquidity ratios may have unintended consequences (such policies are explicitly recommended by the Basel Committee on Banking Supervision (BIS, 2011), and the "Dodd-Frank Act" stipulates that liquidity requirement should be taken into account for setting prudential standards for systemically important financial institutions). Another implication concerns the design of public intervention, should a crisis occur. In the model, the promise of *future* public intervention ensures efficiency because it makes hoarding unattractive and prevents thus liquidity dry ups. However, once agents have decided to hoard, it is "too late" and public intervention cannot restore efficiency. The key policy insight here is that participation constraints, not only participation in the market but also in public schemes (such as TARP or PPIP for instance), are endogenous to hoarding decision. This "hoarding effect" may thus impact the efficiency of public intervention and should therefore not been overlooked.

Firstly, I formally establish the result that hoarding behavior worsens adverse selection, which generates strategic complementarities and may lead to self-fulfilling liquidity dry-ups. This is the main contribution of the paper. Then, as a secondary contribution, I introduce heterogeneity in preferences and I discuss the interactions between idiosyncratic illiquidity shocks, market liquidity, and risk sharing.

The ex-ante identical agents of the model live for three dates and desire to smooth consumption. They may invest in long-term risky projects and they also have access to a risk-less one-period storage technology. If successful, long-term projects yield a better return than storage, and conversely if they fail. Agents privately observe the quality (success or failure) of their projects at the interim date. At this point, they might want to realize a share of their long-term projects either to take advantage of their private information or to provide for current consumption needs. Because of adverse selection, the price they can get on the secondary market is determined by the average seller motive for trading. There is therefore a return-liquidity trade-off: long-term investment is on average more productive but selling shares in the secondary market might be endogenously costly because of adverse selection.[1]

One of the main points of the paper is to show that, in such a situation, balance sheet decisions (the extent to which agents expose themselves to maturity mismatch) present strong strategic complementarities. To illustrate this, let me consider two polar cases:

If agents hoard enough resources, they will *not need* to participate in the secondary market to provide for interim consumption. Therefore, pretending to sell a good asset will not credible, and assets will trade at the *lemons* price. Hence, the market will be illiquid, which justifies the initial hoarding decision.

Conversely, if agents decide to be fully invested in the long-term technology, they will *need* to participate in the secondary market to provide for interim consumption. This is true irrespective of their asset quality. Thus there will be bad *and* good assets in the market and, if the mixture is good enough, the market price

[1] This concept of adverse-selection-driven endogenous liquidity in asset markets is introduced by Eisfeldt (2004).

might be higher than the return to storage. In that case, the market will be liquid and there is no reason to hoard, which justifies the initial decision to be fully invested in the long term technology.

These strategic complementarities may lead to multiple equilibria. In this case, even if a “high liquidity” equilibrium is possible, when agents expect the interim market to be illiquid, they optimally choose to hoard. Such a response reduces ex-post market participation which worsens adverse selection and dries up market liquidity. This is how a liquidity dry-up can be self-fulfilling.

Whereas the decision to self-insure (that is, to hoard) is individually optimal in a low-liquidity world, it is socially costly in two respects. First, because it wastes resources on the storage technology (long-run investment is on average more productive), and second, because ex-ante self-insurance hinders ex-post risk sharing since it prevents agents from providing the positive pecuniary externalities associated with the sale of good assets. Hence, initial investment decisions themselves embed externalities, which are at the roots of equilibrium multiplicity in the model.

Equilibrium multiplicity is a typical outcome of games with strategic complementarities. However, uniqueness generally results when relaxing the assumption that agents share a common knowledge of the economic environment (Carlsson and van Damme, 1993; Morris and Shin, 1998; Frankel, Morris, and Pauzner, 2003). Interestingly, this needs not be the case here because investment decisions are actually strategic *substitutes* at the low-liquidity equilibrium level and only become strategic *complements* at sufficiently high levels. As a consequence, and in a metaphoric way, an agent takes an action that will improve future market liquidity only when he believes that others are truly optimistic about future liquidity conditions. In the case he is rather pessimistic, a slight improvement of his believes about others believes makes him take an action that will actually crowd out of future liquidity. This makes equilibrium multiplicity a natural outcome of the model and captures quite well the susceptibility of liquidity to regime changes.

To expose the secondary contribution of the paper, I consider heterogeneous preferences: as in the banking literature, agents face idiosyncratic illiquidity shocks.

When the realization of such shocks is private information, I find that market liquidity improves. Because *early* agents (those that are hit by the shocks) sell off their projects irrespective of their quality, it indeed reduces adverse selection. As market liquidity improves risk sharing, requiring investors to self-insure against them would decrease welfare and may even provoke a liquidity dry-up. This is an illustration of the possible unintended consequences liquidity regulation may have. Of course, one should not throw the baby with the bath water, and such regulation may still be desirable for reasons that lie outside the scope of this model (preventing fire sales externalities is one of the main rationale for those policies), but one should not overlook the insights of this model either and take for granted that liquidity buffers have positive effects only. Finally, when there exists a technology that enables them to disclose their liquidity position, *early* agents are better-off under disclosure. It isolates them from the negative externalities exerted by other *lemons* owners (those that are not hit by the shock; I dub them *normal* agents). However, *normal* agents incur a larger liquidity discount because the probability that they try to sell a lemon is correctly perceived to be higher. This may lead to a liquidity dry-up and shows that more transparency might decrease welfare. This last result illustrates the ambiguity of transparency on liquidity suggested by Holmström (2008).

Related literature

The paper is related to several strands of the financial economics literature. The main strand comprises adverse selection models of illiquidity. The second is at the intersection between *Cash-in-the-market* pricing theory and the market micro-structure literature. The third is the banking literature, and the fourth is the literature on financial crises.

Among the adverse selection models of illiquidity, a very closely related paper is Plantin (2009). It indeed presents a model where investment decision depends on liquidity anticipation and where future adverse selection is less severe when many investors invest in the long term risky project. However, the feedback effect is essentially assumed: the *learning by holding* assumption posits that the informational advantage of the seller decreases with the number of investors. My set up is thus more elaborate in that dimension since the feedback effect naturally arises through the impact of past agent decisions on their current marginal rate of intertemporal substitution, without assuming any change in the information structure.

While it builds on Eisfeldt (2004), my paper has a different focus and highlights distinct mechanisms.

Eisfeldt (2004) develops an adverse-selection-driven endogenous liquidity framework and studies the interaction between productivity and liquidity. She shows that higher productivity induces higher investment, which makes income more risky. This, in turn, makes agents more eager to share risk in the secondary market, which improves market liquidity. This risk-sharing channel is the key determinant of market liquidity in that paper but is absent in mine since all uncertainty resolves before the market opens.

The paper also relates to two recent pieces of research: Bolton, Santos, and Scheinkman (2011) and Heider, Hoerova, and Holthausen (2010). We share the result that the fear of future adverse-selection-driven illiquidity may trigger hoarding behaviors. However, we differ on the effects hoardings have on adverse selection. In particular, I do not have *cash-in-the-market* pricing effects, but they do not capture the direct effect by which hoardings worsen adverse selection in my set-up.

Bolton, Santos, and Scheinkman (2011) construct a model, with both adverse selection and *cash-in-the-market* pricing, that shows that fears of future illiquidity may inefficiently accelerate trade. The "delayed" market has two potential outcomes: high price with full participation, or low price with only *lemons* traded. The "anticipated trade" equilibrium, which has the flavor of my equilibrium with hoardings, is supported by the understanding that the *lemons* price would prevail in the "delayed" market. This is how fears of adverse selection trigger hoardings. However, hoardings do not "cause" the market breakdown. Instead, because utility is linear, what determines market participation in the "delayed" market, is essentially the market price. In my set-up, agents want to smooth consumption and the "good type" marginal rate of intertemporal substitution crucially depends on hoardings. This determines the extent to which he participates in the market and, therefore, the severity of adverse selection.

Heider, Hoerova, and Holthausen (2010) model the different adverse selection regimes that may arise in the interbank market due to counterparty risk. Their narrative focuses on a regime in which the anticipation of a market breakdown implies hoarding behavior, but the relationship is generally ambiguous. At the aggregate level, this is essentially a *cash-in-the-market* pricing story: an increase in hoardings decreases interest rate, which decreases adverse selection, but a decrease in adverse selection increases the volume of trade, which requires more *cash-in-the-market*. Which effect dominates depends on parameter values. Yet, what determines whether a given bank borrows in the market is an arbitrage between the market interest rate and the price at which it can sell assets. This price is exogenous and assumed to be higher for "safer" banks, which explains why participation may vary. In my model, both types trade at the same endogenously determined price, but the difference in their marginal utility gain to participate in the market endogenously depends on hoardings.

Chari, Shourideh, and Zetlin-Jones (2010) study a dynamic model of adverse selection where banks may decide to sell an asset at a discount today to signal their type, that is the quality of the assets they produce, in order to sell at a better price in the future. They show that such reputational mechanism may lead to multiple equilibria. The crucial assumption is that outsiders learn about the quality of sold assets and update their believes about bank types accordingly. This is thus related to Plantin (2009) in that the information structure evolves with selling decisions.

Parlour and Plantin (2008) propose a model in which banks issue loans to firms and may sell them on a secondary market. Banks might then be tempted not to monitor the firm properly and therefore sell *lemons* on the market. The authors analyze the optimal contract between firms and banks and show that the existence of a liquid secondary market might be inefficient from an ex-ante point of view.

The *cash-in-the-market* pricing theory (Allen and Gale, 1994) takes a general equilibrium approach to liquidity. The idea is the following: there is an opportunity cost to hoard cash if the alternative, a productive long-term asset, has a higher expected return. This cost should be compensated by gains in some states of the world. These gains are realized when there are "many" sellers of the long-term asset. In this case, as the demand is limited by the aggregate amount of cash hoarded (this is the general equilibrium effect), the price might drop below the fundamental value.

Cash-in-the-market pricing and similar mechanisms have been widely used in the literature to study liquidity dry-ups and related events. Papers directly building on *cash-in-the-market* pricing include Bolton, Santos, and Scheinkman (2011), already mentioned in the adverse selection section, Gale and Yorulmazer (2011), and Diamond and Rajan (2011). In the market micro-structure literature, similar effects are obtained either with an exogenously downward sloping demand for assets and arbitrageurs that face resource constraints that are functions of price (Morris and Shin, 2004; Gennotte and Leland, 1990), or with limits-of-arbitrage constraints (Shleifer and Vishny, 1997), which create an endogenously inelastic supply of interim liquidity (Gromb and Vayanos, 2002; Brunnermeier and Pedersen, 2009). In my paper, the interim supply of liquidity is perfectly elastic and cash-in-the-market pricing does therefore not occur. The mechanisms by which the market outcome may be inefficient differ completely and the two approaches seems therefore complementary.

Liquidity provision as an insurance against uncertainty about the preferred timing of consumption is a central theme of the banking literature. In Diamond and Dybvig (1983), there is no market and banks can generally provide liquidity and improve on the autarky allocation. Jacklin (1987) shows that this is not the case if there exists a secondary market because the bank demand deposit contract is no longer incentive compatible. Diamond (1997) generalizes these results with a model of exogenous limited market participation. He finds that the lower the participation in the market, the greater the role of the banking sector. In that respect, besides the addition of a *lemons problem*, the key differences of my model are that limited market participation is endogenous and investors are needed to run the initial phase of the project. Indeed, I assume that projects could not be run mutually in the first period and that there is no means by which agents could credibly commit to invest. Otherwise, agents could form a coalition in order to pool resources and diversify idiosyncratic risk away. This coalition would correspond to the bank[2] in Diamond and Dybvig (1983) and, as there is no aggregate shock to the fundamentals in my model, it could implement the first-best allocation. These assumptions are strong, as real banks do pool individual resources. However, there certainly are frictions preventing them to ex-ante pool resources among themselves and achieve full risk-sharing.

The literature on financial crises is considerably expanding due to the 2007-2009 crisis (see Brunnermeier, 2009, for a chronology of the crisis and Tirole (2011a), for a survey on the economics of liquidity). Through the insights it delivers on the strategic complementarities in maturity mismatch decisions, the paper also relates to the research on liquidity regulation (see for instance Perotti and Suarez, 2010, Farhi and Tirole, 2011, Tirole, 2011a, and Stein, 2011) and the financial crisis management literature, where a lot of atten-

[2]To avoid confusion, such a bank would correspond to a coalition of investors (or of small banks, seen as investors) in my model.

tion has recently been given to public intervention in market plagued with adverse selection (Tirole, 2011b; Philippon and Skreta, 2011; Chari, Shourideh, and Zetlin-Jones, 2010; Chiu and Koeppl, 2010; Guerrieri and Shimer, 2011a; House and Masatlioglu, 2010).

Section 2 presents the model, *section 3* studies liquidity dry-ups, *section 4* relates the model to the 2007-2009 financial crisis, *section 5* considers the impact of idiosyncratic liquidity shocks, and *section 6* concludes.

2 The model

Investors

There are three dates ($t = 0, 1, 2$) with a unique consumption good that is also the unit of account. There is a measure one of ex-ante (at $t = 0$) identical *investors* maximizing expected utility, which they derive from consumption at date 1 and 2. One can think of these investors as entrepreneurs undertaking real projects or as banks issuing loans. I call them *investors* because I model their problem as a portfolio choice. Their period utility function is $\ln C_t$, where C_t is their consumption at date t. The specific functional form is chosen for convenience only, but it needs to be concave as is explained below. At date 0, they are endowed with one unit of the consumption good, which they allocate between long-term risky investment and short-term risk-less storage.

In *section **5***, these agents will also be subject to idiosyncratic illiquidity shocks which take the form of preference shocks as is standard in the banking literature (Diamond and Dybvig, 1983). Since such shocks do not matter for the main contribution, I delay their description until then.

Technology and information

At dates 0 and 1, investors have access to a risk-free one-period storage technology that yields an exogenous rate of return, normalized to 0. At date 0, they have also access to a risky long-run technology. Such projects are undertaken at date 0 and only pay off at date 2. They succeed or fail with equal probability. In case of success, they yield a return R_H per unit invested, and a return R_L, with $0 \leq R_L < R_H$, otherwise. To make the analysis interesting, I assume $R_L < 1 < (R_H + R_L)/2$: on average, long-term projects are more productive than storage, but they yield less than storage in case of failure. If projects are stopped at date 1, they yield nothing. However, at that date, investors may issue claims to the payoff of their projects in a secondary market, which I describe later.

At the beginning of date 1, investors privately observe their projects' quality, that is, whether their projects are going to succeed or fail. Quality is common to all the projects of a given investor. One can thus think of each investor owning only one project of variable size. However, quality is independent across agents, and, assuming a law of large numbers, average quality is thus deterministic.

Starting from the assumption that the long-term technology is on average more productive than storage, the two key ingredients to obtain the feedback effect between hoarding and adverse selection is a potential *lemons problem* and that (before trade) interim marginal utility of consumption is endogenous to initial investment decision.

First, to generate the *lemons problem*, I have to restrict the set of claims agents can trade (or contracts they can write) before learning their private information. Otherwise, if markets were complete, they could achieve the first best allocation and a *lemons problem* would not obtain[3]. Concretely, as in Eisfeldt (2004), I assume that investors are needed to initiate their own projects and that they cannot commit to properly invest, which prevents them to sell the projects upfront. What I have in mind is a non-modeled underlying moral hazard problem[4]. Alternatively, one can think that they incur some dis-utility if they initiate the project on behalf of someone else. Also, storage is not observable by outsiders. Given the difficulties a firm, or a bank, would have to prove the extent to which their balance sheets are illiquid, this restriction seems however reasonable (Tirole, 2011a).

Second, the link between initial investment decision and marginal utility of consumption comes from

[3]There is a consequent literature on the role of banks for such purpose when contracts are available to pool resources ex ante. See for instance Diamond (1984). What I study here is an economy in which such pooling is not possible.

[4]Malherbe (2010) studies a model with both moral hazard and adverse selection and derives the conditions under which it is more efficient to wait until the interim date before selling the project.

agents' desire to smooth consumption across time, this is why the utility function concavity is important. Other modeling strategies, for instance an interim investment opportunity or a refinancing need[5], may yield a similar relationship.

These two ingredients give agents a reason to hoard. But hoarding also feeds back on adverse selection: while *lemon* owners trade because private information allows them to make an arbitrage, others trade according to their marginal rates of substitution, which depend on hoardings. This really is the core of the model. The assumptions about the return to storage, the probability of project success, and the specific form of the utility function are thus made for the sake of simplicity only (a generalization and a proof of the main propositions are provided in the appendix).

The time line

At date 0, investors

- Form anticipations about date-1 secondary market price;
- Choose λ the share of endowment they invest in the long-term technology, the remaining being stored.

At date 1, they

- Learn their projects quality R_j;
- Choose the number of shares (to unit projects) to sell, and how much to consume at date 1 and store until date 2;
- Take P, the unit price of a share, as given.

At date 2:

- Projects pay off and output is distributed to claimants.
- Agents consume their remaining resources and die.

[5]Which is a more standard way to introduce a liquidity need in the corporate finance literature (see for instance: Holmström and Tirole, 1998, 2011).

Secondary market

I consider a competitive secondary market, in which investors trade perfectly divisible shares of their projects. There is no other means to borrow against future income than to issue shares of ongoing projects, and issuance is limited to existing projects. Short sales are thus ruled out. In line with most of the literature[6], I assume that all trades take place at the same price[7]. Henceforth, I will call shares of high quality projects "good assets" and shares of low quality projects "bad assets" or *lemons* interchangeably.

Demand for shares and market price

There is also a measure one of "deep-pocket" agents which have resources available but do not have access to the long-term technology. They only have access to storage and to the secondary market. For simplicity, they are risk-neutral[8], and hence they are ready to buy any asset at the expected discounted value of the underlying payoffs. I assume that they have, on aggregate, enough resources to clear the market at that price.

This implies a perfectly elastic demand for shares, which is of course not realistic, at least in the short run. Nevertheless, this assumption is desirable because it neutralizes *cash-in-the-market* pricing mechanisms and allows to clearly identify the key externalities. Besides, the results are robust to *cash-in-the-market* pricing (this is shown in the appendix in an extension of the model along the lines of Bolton, Santos, and Scheinkman, 2011).

When asset quality is private information, the key variable to determine asset prices on secondary market is average quality (Akerlof, 1970). As the deep-pocket agents have access to the storage technology, a simple no-arbitrage argument suffices to establish the market unit price P of an asset, given average quality on that market:

[6]This is assumed for instance in Akerlof, 1970; Eisfeldt, 2004; Parlour and Plantin, 2008; Heider, Hoerova, and Holthausen, 2010; Bolton, Santos, and Scheinkman, 2011; Chari, Shourideh, and Zetlin-Jones, 2010. However, in a recent paper Guerrieri and Shimer (2011b) study an economy where assets of different qualities always trade at different price.

[7]It would be a natural outcome of an anonymous market in which buyers cannot infer quality from quantities because sellers can split their sales. If they could, all trades would take place at the same price in a pooling equilibrium (on the basis of which the analysis could be done), but a separating equilibrium in which different quantities are traded at different prices might also exist.

[8]Eisfeldt (2004) proposes an alternative formalization: she assumes that such agents are risk-averse and that their endowment streams and utility function are such that they want to save, for instance for precautionary saving motive. Then, she assumes perfect divisibility of claims and costless diversification. This generates the same perfectly elastic demand function.

$$P(\eta) = R_L + \eta(R_H - R_L) \tag{1}$$

Where η denotes the proportion of good assets in the market.

Endogenous market liquidity

The higher the price they get on the secondary market, the better the terms at which agents can transform long-term assets into current consumption goods. Since adverse selection depresses the price, it impairs liquidity. Thus, for it measures the proportion of trades not driven by adverse selection, η embodies market liquidity in this model.

Equilibrium definition

A triple $\gamma \equiv (P^*, \lambda^*, \eta^*)$ is an equilibrium for this economy if and only if:

$$\begin{cases} P^* = P(\eta^*) \\ \lambda^* \in \lambda(P^*) \\ \eta^* = \eta(P^*, \lambda^*) \end{cases} \tag{2}$$

That is, P^* is the unit price buyers are ready to pay for the average quality implied by η^*; λ^* is an optimal investment decision given P^*; and η^* is the proportion of good assets in the market implied by the level of investment λ^* and at price P^*.

There always exists at least one such equilibrium[9]. Uniqueness, however, depends on parameter values. In this simple version of the model, it only requires that average project return $E[R]$ is low enough, otherwise there are multiple equilibria.

[9] Kakutani's theorem ensures that there exists a price $P^{'}$ such that: $P^{'} \in \eta\left(P^{'}, \lambda(P^{'})\right) R_H + \left(1 - \eta\left(P^{'}, \lambda(P^{'})\right)\right) R_L$. Such a price pins down an equilibrium.

3 Liquidity dry-ups

In this section, I solve the model backward and I show how the feedback relationship between hoarding and adverse selection may lead to multiple equilibria that can be Pareto ranked according to their respective level of market liquidity. Then, I identify the key externality that drives the results, I show how the government can implement the second-best allocation. Finally, I discuss equilibrium multiplicity.

3.1 Equilibria

There are two types of date-1 agents and, assuming a law of large numbers, there is a measure one half of each type. I therefore consider two date-1 representative agents named after their type $j \in (H, L)$. Where H stands for the agent with good assets (or *high* quality projects) and L for the other.

The problem of the agent

Long term investment is risky: on the one hand, it pays well in the case of success but, on the other hand, it might yield a relatively low return in case of failure or early liquidation. Conversely, storage yields the same amount in each state of nature and is liquid; it can be consumed 1 to 1 at each date. There is thus a *return-liquidity* trade-off and risk-averse agents might use storage to self-insure in order to reduce consumption volatility.

Formally, at date 0, agents solve:

$$\max_{\lambda, L_j, S_j} U_0 = E_0\left[\ln(C_{1j}) + \ln(C_{2j})\right] \tag{3}$$

$$s.t. \begin{cases} C_{1j} + S_j = 1 - \lambda + L_j P \\ C_{2j} = (\lambda - L_j)R_j + S_j \\ 0 \leq L_j \leq \lambda \leq 1 \\ prob(j = H) = 0.5 \end{cases}$$

Where $E_t[.]$ is the conditional (upon information available at date t) expectation operator, λ is the share of endowment invested in the long-term technology, and the following variables are contingent on being in state of nature j: C_{tj} is consumption at date t , $S_j \geq 0$ is storage between dates 1 and 2 , L_j is the number of shares (to unit projects) issued at date 1.

The budget constraints state the following: date-1 resources consist of storage $(1-\lambda)$ from date 0 plus the revenue from sale of shares (L_j) at the market price (P). These resources can be consumed (C_{1j}) or transferred to date 2 through storage (S_j). At date 2, resources available for consumption consist of the output from long-term investment that has not been sold $(\lambda - L_j)R_j$ plus storage from date 1.

To determine optimal behavior with respect to this trade-off, I solve the problem backward.

Date-1 optimal liquidation policy

Let $L_j(P,\lambda)$ denote the optimal "liquidation" correspondence that solves the date-1 problem for agent j for each couple (P,λ). Keeping in mind that projects loose all their value if they are physically liquidated at date 1, liquidation should be understood here as "being transformed into current consumption goods", that is sold in the secondary market. Note also that I restrict my analysis to prices that are consistent with (1): $P \in [R_L, R_H]$.

Agent L knows he has *lemons* and he sells off any projects he holds if $P > R_L$. In the case $P = R_L$, optimal liquidation is undetermined. I assume for simplicity that he sells off any projects too. Accordingly:

$$L_L(P,\lambda) = \lambda \tag{4}$$

The problem of H, the agent with good assets, is given by:

$$\max_{L_H, S_H} U_H(P,\lambda) = \ln(C_{1H}) + \ln(C_{2H})$$

$$s.t.\begin{cases} C_{1H}+S_H=1-\lambda+L_HP \\ C_{2H}=(\lambda-L_H)R_H+S_H \\ 0\leq L_H\leq\lambda \end{cases}$$

First, observe that it is not optimal to have both $L_H > 0$ and $S_H > 0$. This is because such transfer of resource back and forth in time would be done at a loss. Then, from the first order conditions, I have:

$$L_H(P,\lambda)=\max\left\{0;\frac{P\lambda-1+\lambda}{2P}\right\} \tag{5}$$

The optimal liquidation of agent H is weakly increasing in λ. If λ is high enough, $L_H(P,\lambda)$ is strictly positive because the resources available at date 1 (before liquidation) are smaller than the share of wealth he wants to dedicate to consumption at that period. Conversely, if $\frac{P\lambda-1+\lambda}{2P}$ is negative, the agent would like to "create" ongoing projects. This is of course ruled out by the definition of the long-term technology. In that case, agent H does not participate in the market and $L_H(P,\lambda)=0$. Note that if λ is really small, he might roll over part of its storage to date 2.

From (4) and (5), for any given P, the proportion of *lemons* in the market $\frac{L_L(P,\lambda)}{L_H(P,\lambda)+L_H(P,\lambda)}$ increases thus when λ decreases. The intuition is the following: agent L first trades to make an arbitrage, then uses storage to smooth consumption. On the contrary, agent H trades if consumption smoothing requires to transform long-term asset holding into current consumption goods. Hence, the more he had stored, the less he is eager to trade. I have thus established:

PROPOSITION 1 (**hoarding effect**)

An increase in date-0 storage worsens date-1 adverse selection.

Remark: models with preference shocks *à la* Diamond and Dybvig, the severity of adverse selection at equilibrium is generally uniquely determined by the fraction of agents hit by the shocks, as is for instance the case in Parlour and Plantin (2008). The reason is that such modeling strategy is equivalent to assuming that *early* agents' intertemporal marginal rate of substitution is infinite. Therefore, such agents only sell when it is "assumed" that they should sell, and hoarding has no impact on the proportion of good assets in the market.

Date-0 optimal investment policy

Agents choose investment according to expected utility maximization.

PROPOSITION 2 (**self-insurance**)

Let $\lambda(P) \equiv \arg\max_{\lambda} U_0(\lambda, P)$ *be the set of solutions for a given P to the date* 0 *problem (3), then:*

$$\lambda(P) = \begin{cases} \left\{\check{\lambda}\right\} & ;P < 1 \\ \{1\} & ;P > 1 \\ \left\{\left[\frac{1}{2}, 1\right]\right\} & ;P = 1 \end{cases} \tag{6}$$

With $\check{\lambda} < \frac{1}{2}$.Proof: see the appendix.

When P is smaller than the gross return to storage, it hurts to liquidate. In that case, agent L (which ends up with lemons), wishes he had invested nothing. Agent H, however, wishes he had invested $1/2$ (which is a well known property of logarithmic utility functions). Consequently, expected utility maximization implies a choice of λ strictly lower than $1/2$. Self-insurance is thus crowding out productive investment.

Conversely, if P is to be high, investment dominates storage -even in the case of early liquidation- and makes thus the agent better-off irrespective of its project quality. They therefore choose to invest as much as they can.

If $P = 1$, investment is rather high, though undetermined over the range $\left[\frac{1}{2}, 1\right]$: whereas agent L is indifferent over the whole range of admissible values $[0, 1]$, agent H is indifferent over this specific range and strictly prefers it to any lower value. This is the reason why the optimal investment policy has not a functional form at $P = 1$.

Supply of assets and average quality

I can now evaluate the optimal liquidation functions (4) and (5) at the optimal investment level given price P (*6*):

$$L_L(P,\lambda(P)) = \lambda$$

$$L_H(P,\lambda(P)) = \begin{cases} 0 & ;P < 1 \\ \frac{1}{2} & ;P > 1 \\ \in \left[0, \frac{1}{2}\right] & ;P = 1 \end{cases}$$

And I can define $\eta(P)$, the proportion of good assets for a given P:

$$\eta(P) \equiv \frac{L_H(P,\lambda(P))}{L_L(P,\lambda(P)) + L_H(P,\lambda(P))} = \begin{cases} \eta_{illiq} = 0 & ;P < 1 \\ \eta_{liq} = \frac{1}{3} & ;P > 1 \\ \eta_1 \in \left[\eta_{illiq}, \eta_{liq}\right] & ;P = 1 \end{cases} \tag{7}$$

If the price is anticipated to be low, relative to return on storage, market participation is anticipated to be limited: there will only be *lemons* in the market. However, in the case of a high price, there is full participation and therefore there is a higher proportion of good assets in the market. The latter case implies a smaller discount and thus greater market liquidity (recall that η is a direct measure of liquidity in this model).

Equilibria and liquidity dry-ups

In this economy, the same fundamentals (R_H, R_L) might lead to multiple equilibria that primarily differ by their level of liquidity. Accordingly, I interpret equilibria with the lowest level of liquidity as liquidity dry-ups.

To find the equilibria of this economy, I combine the no-arbitrage condition of the deep-pocket agents

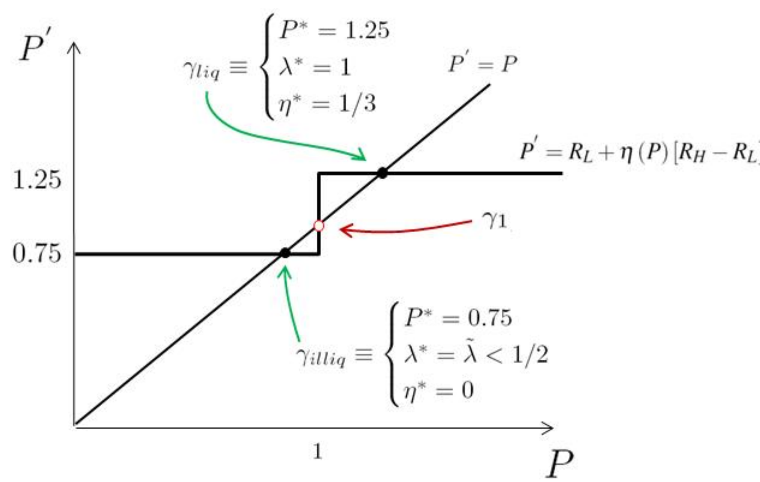
(1) with (7) to define the implied price correspondence:

$$P^{'} = R_L + \eta(P)\left[R_H - R_L\right]$$

$P^{'}$ is the market price corresponding to a proportion of good assets $\eta(P)$. Therefore, a fixed point $P^{'} = P$ pins down an equilibrium price (call it P^*) for the economy. The corresponding values of λ^* and η^* are given by (6) and (7) respectively.

Figure (1) gives an example with three equilibria for a given parameter set.

Figure 1: Multiple equilibria ($R_H = 2.25$ and $R_L = 0.75$)



In a first equilibrium, which I denote γ_{illiq}, agents anticipate a liquidity dry-up. They take as given that the price will be low ($P < 1$) and, according to *proposition 2,* they choose $\lambda(P) = \check{\lambda} < 1/2$. This in turn imply that agents with good assets will not enter the secondary market at date 1 and that the resulting liquidity will be low ($\eta^* = 0$) which make the anticipation of a low price a self-fulfilling prophecy: the liquidity has dried up.

Similar argument applies to γ_{liq}, the self-fulfilling high-liquidity equilibrium. Expecting high liquidity, which means that liquidation does not hurt ($P \geq 1$), agents invest only in the long term technology ($\lambda^* = 1$). Given that investment is high, type-H agents enter the market, and their participation increases the proportion of trade for reasons other than private information about future payoff. Hence, equilibrium liquidity and price are indeed high ($\eta^* = 1/3$ and $P^* = 1.2$).

Both equilibria are locally stable in the sense that agents best-response to any small perturbation to the equilibrium price would bring the price back to equilibrium. There is also an equilibrium (call it γ_1) which is unstable.

The example depicted in figure (1) is not an exception, and any admissible low return ($0 \leq R_L < 1$) there is a threshold for the high return (R_H) from which these fundamentals lead to multiple equilibria. I formalize this statement in the following proposition.

PROPOSITION 3 (**multiplicity**)

Under assumption 1, $\forall R_H > 3 - 2R_L$*, problem (3) has at least two distinct solutions with different level of liquidity.*

Proof: Let $\Gamma(R_L, R_H)$ denote the set of equilibria for these parameters. From (1), (7) and (6)**,** I directly get that, if $R_H > 3 - 2R_L$, the following two elements belong to $\Gamma(R_L, R_H)$:

$$\gamma_{illiq}(R_L, R_H) \equiv \left\{ P^*_{illiq} = R_L; \lambda^*_{illiq} = \check{\lambda} < 1/2; \eta^*_{illiq} = 0 \right\}$$

$$\gamma_{liq}(R_L, R_H) \equiv \left\{ P^*_{liq} = R_L + \frac{1}{3}(R_H - R_L); \lambda^*_{liq} = 1; \eta^*_{liq} = \frac{1}{3} \right\}$$

□

The low liquidity equilibrium can be interpreted as a complete market breakdown. Indeed, there are no interim gain from trade and agent L is indifferent whether to trade or not. Apart from this indeterminacy, this equilibrium is unique from a date-1 perspective, that is *given* that agents are self-insured. The high-liquidity equilibrium is also unique *given* the date-0 decision to fully invest in the long-term technology. It is a pooling equilibrium (in prices, not in quantities) in the sense that agents L pretend they are of the H type and get the same price, even though they sell larger quantities. A pooling equilibrium in price and quantities may arise under different assumptions on the market structure (for instance if buyers could observe seller quantities and use it to infer quality). In such a case, the market price would be $E[R_j] > 1$, and the market would thus be liquid too.

3.2 Welfare and externalities

This subsection studies the mechanisms by which endogenous liquidity dry-ups affects welfare. The key starting point of this exercise is that, in this economy, the market may fail to allocate resources efficiently.

PROPOSITION 4 (**market failure**)

A liquidity dry-up is a Pareto-dominated equilibrium, both with an ex-ante and an ex-post criterion.

Proof: see the appendix.

Ex-post inefficiency means that, no type ends up better-off in a liquidity dry-up than in the corresponding high-liquidity equilibrium, and at least one type is strictly worse-off. Ex-post inefficiency of course implies ex-ante inefficiency: expected utility is lower in the case of a dry-up. Here, all types are strictly worse-off in a dry-up.

The two reasons why the low-liquidity equilibrium is inefficient are he following:

- First, because some resources are not invested in the most productive technology.
- Second, because a liquid interim market provides ex-ante insurance: a pooling equilibrium implies cross-subsidies, and such ex-post transfers, from agent H to agent L, are desirable from an ex-ante risk-sharing perspective.

The reason why agent H is strictly better-off despite cross-subsidizing agent L resides in the presence of date-1 positive externalities. When an agent optimally chooses to sell a good asset, it increases average quality and all assets can be sold for a better price. The social benefit is thus higher than the individual cost. If enough of these externalities are provided in equilibrium, it becomes less costly to rely on the market for date-1 consumption good provision than to use storage. However, agents only provide such externalities if they are sufficiently invested in the long-run technology. The decision to invest embeds thus strategic complementarities.

The idea that liquidity hoarding might be wasteful is not new (see for instance Diamond, 1997, Holmström and Tirole, 1998, Caballero and Krishnamurthy, 2008, and Bolton, Santos, and Scheinkman, 2011), but the nature of the externality is novel: exposing oneself to maturity mismatch acts as a commitment to future market participation and therefore enhance ex-post risk-sharing.

3.3 The government

Implementing the second-best allocation in the presence of positive externalities is here a standard and rather simple problem (see for instance Dybvig and Spatt, 1983). I propose in this subsection a public liquidity insurance scheme that enables the government to achieve such a goal. Note that I assume that the fundamentals are such that a market failure is possible, that is: $R_H > 3 - 2R_L$ (*proposition 3*).

The public liquidity insurance

The idea for the insurance is extremely simple. The bad equilibrium is a coordination failure, which happens when investors fear to sell assets in an illiquid market. Therefore, if the government pledges to compensate them for the loss (with respect to storage) in such a case, the incentive to self-insure vanishes and the only possible outcome is the high-liquidity equilibrium. Of course, this result relies on the possibility for the government to levy a break-even lump-sum tax after observing aggregate behavior. A private agent could not do it because the tax scheme violates agent's H participation constraint. This public liquidity insurance is in fact very similar to Diamond and Dybvig (1983) demand deposit insurance with the same assumptions about the fiscal ability of the government

PROPOSITION 5 (**public insurance**)

A public liquidity insurance implements the second-best.

Proof: see the appendix.

Here is the intuition. Under this public liquidity insurance, the ex-ante trade-off between return and liquidity (with respect to storage) disappears and the date-0 first order condition for $\lambda = 1$ always holds:

$$\frac{\partial U_0}{\partial \lambda} > 0$$

Whatever the anticipated date-1 market price, $\lambda^* = 1$ maximizes expected utility. Hence, it is a dominant strategy to fully invest in the long-term technology. Under this scheme, the only one equilibrium corresponds to $\gamma_{liq}(R_L, R_H)$ and the insurance is never claimed. As in Dybvig and Spatt (1983), such an insurance is thus free.

Implementation

There are several ways to implement the public liquidity insurance. For instance, the government may pledge to buy any asset at a price of 1[10]. Of course, sellers would only claim this insurance in the case of a dry-up. To break even, the government needs to levy the following lump-sum tax:

$$\tau(P) = \begin{cases} (1-P)\sum_j \frac{L_j}{2} & ; P < 1 \\ 0 & ; P \geq 1 \end{cases}$$

Where τ is the per capita lump-sum tax needed to fund the insurance.

The net effect of such a scheme is thus a transfer from agents that liquidate little to agents that liquidate more:

$$transfer_j = \begin{cases} (1-P)\left[L_j - \frac{\sum_j L_j}{2}\right] & ; P < 1 \\ 0 & ; P \geq 1 \end{cases}$$

Feasibility and credibility

Such a transfer is always feasible. As the incentive and participation constraints are circumvented thanks to the government regalian power to raise lump-sum taxes, the only remaining issue is the one of binding

[10]The following subsidy to liquidation (and a break-even lump-sum tax) would lead to an equivalent outcome: $Subs(L) = \min\{(1-P), 0)\}$

resource constraints. However, they would never be binding. First, from an aggregate point of view, because the maximum total price the government would pay is $\lambda^* \leq 1$, which is strictly smaller than the date-1 aggregate resource in the economy $(1+\lambda^*(E[R]-1))$. Second, from an individual point of view, because a highly negative transfer to agent H would force him to liquidate part of its portfolio. This would trigger positive externalities and increase the average value of traded assets which, in turn, would decrease the size of the needed transfer and relax the government budget constraint. Consequently, agent H could never run out of resources because of this scheme[11].

Whether the government would still be willing to impose such a transfer in the (out-of-equilibrium) low-liquidity case raises the question of credibility. Feasibility does not mean that it is always ex-post desirable. One could state the problem as follows: under which condition is it credible for the government to intervene in case of a dry-up? As it implies a transfer from rich (or lucky) to poor (or unlucky) agents, an utilitarian government would for instance be willing to do so. Still, if there is a doubt about government commitment, one can no longer rule out the bad equilibrium (this could be formalized in a model with uncertainty about the government true type).

Policy implications

During financial crises, the fear of a credit crunch[12] might lead to various public interventions such as liquidity injection, bank recapitalization, asset repurchase, or even nationalization. This has recently received a rapidly growing attention in the academic literature. In particular, Tirole (2011b), Philippon and Skreta (2011), Chari, Shourideh, and Zetlin-Jones (2010), Chiu and Koeppl (2010), and House and Masatlioglu (2010) focus on public intervention in market plagued with adverse selection. These papers provide models specifically dedicated to the study if this question and are probably better suited to draw direct recommendations on the efficient design of such policies.

Also, my model is very stylized, and implementing such a public liquidity insurance scheme would raise

[11] Were agent H to liquidate its whole portfolio, the equilibrium price would be $E[R] > 1$ which is not consistent with agents selling assets to the government.

[12] See for instance Bernanke and Gertler (1989) for the magnifying effect on business fluctuations of a drop in net worth.

important issues that the model can not account for. For instance, it is well known that the promise of public intervention might induce investors to take on too much risk in the future[13]. Still, the model delivers insights that are relevant to financial regulation.

First, that hoarding behaviors worsen adverse selection highlights a channel by which prudential policies that aim at raising financial institutions liquidity ratios may have unintended consequences (see *section 5* for a concrete example). Second, as Tirole (2011b) and Philippon and Skreta (2011) point out, agents participation constraints are endogenous to public intervention. In my model, they are also endogenous to hoarding decisions, which themselves depend on anticipations of future public intervention. One example is that the public liquidity insurance is only effective ex ante. Once agents have decided to hoard, it is still feasible to implement the scheme, but it would not restore liquidity[14] and would no longer be a Pareto improvement. Hoarding effects may thus impact the efficiency of public interventions such as those considered by Tirole (2011b) and Philippon and Skreta (2011) and it would therefore be interesting to extent theirs set-ups to allow for hoarding behaviors. And third, the result that long-term investment decisions crucially depend on future market liquidity is relevant to the design of policies that aim at exiting a credit crunch.

3.4 Discussion on multiplicity

Multiplicity and lemons problem

In the textbook *lemons problem*, adverse selection leads to multiplicity when different prices can clear the *same* market.

The story is different in my model: *given date-0 investment decision*, there is only one price that clears the secondary market at date 1. However, the opportunity cost of not participating in the market at date 1 (the marginal utility of current consumption goods) is endogenously determined by date-0 investment decision. Therefore, different date-0 decisions imply different market-participant characteristics at date 1, which explains why the same fundamentals can lead, from a date-0 perspective, to multiple self-fulfilling

[13] See for instance Diamond (1984) and Freixas, Rochet, and Parigi (2004).

[14] While public interventions such as liquidity injections (through TARP for instance), eased the short-term funding of financial institution in the Fall of 2008, one might easily argue that they did not restore liquidity in the securitized markets.

date-1 equilibrium prices. The roots of multiplicity lie thus in the dynamic strategic complementarities underlying the feedback relationship between adverse selection and hoarding.

Eisfeldt (2004) and Kurlat (2009) study adverse-selection-driven illiquidity in similar contexts but do not find multiple equilibria, which deserves a brief discussion. The former, does not prove uniqueness and parameter values are selected such that the solution algorithm does not lead to multiplicity, up to numerical precision. My guess is that the increments in productivity are too large relative the parameter multiplicity region, which is therefore not identified[15]. In the latter, storage is simply ruled out by assumption and hoarding can therefore not affect adverse selection.

Multiplicity and common knowledge assumption

It is typical to find multiple equilibria in games of strategic complementarities. Still, relaxing the assumption that agents share a common knowledge of the economic environment is likely to imply equilibrium uniqueness. Interestingly, this need not be the case here because investment decisions are actually strategic *substitutes* up to the level at which they become *complements*, which precludes the use of standard global games techniques**.** This is because *global strategic complementarities* are required to apply equilibrium selection techniques based on the iterative deletion of dominated strategies (Carlsson and van Damme, 1993; Morris and Shin, 2004; Frankel, Morris, and Pauzner, 2003; Vives, 2005). Furthermore, the *single-crossing conditions* for uniqueness required by other approaches (Goldstein and Pauzner, 2005; Mason and Valentinyi, 2010; Bueno de Mesquita, 2011) are also violated[16].

To understand why, one might consider the case in which a strictly positive measure of investors choose not to self-insure. In such a case, the decision for an agent to increase investment from a low level has *negative* externalities, as long as the considered increase is not large enough to ensure future market participation (that is, as long as $\lambda < 1/2$). The intuition is the following: a self-insured agent does not participate in the

[15]This intuition finds support in figure 4 in that paper. Consider the mapping and the fixed point corresponding to the benchmark economy. It is clear that the mapping slope is greater than 1 for higher initial values than the fixed point. As an increase in productivity shifts mappings upward, this strongly suggests the existence of multiple equilibria for an economy with slightly higher productivity.

[16]See the appendix for further discussion.

market if he ends-up with good assets, and, therefore, increasing his investment only increases the quantity of potential *lemons* he would sell. Hence, it is only if he decides to rely on market liquidity provision that he may contribute to it too and thus provide positive externalities. In a metaphoric way, an agent takes an action that will improve future market liquidity only when he believes that others are truly optimistic about future liquidity conditions. In the case he is rather pessimistic, a slight improvement of his believes about others believes makes him take an action that will actually crowd out of future liquidity. This makes equilibrium multiplicity a natural outcome of the model and captures quite well the susceptibility of liquidity to regime changes.

Finally, if one were to reduce the model to a binary decision game[17] (imposing $\lambda \in \left\{\frac{1}{4}; 1\right\}$ for instance), it would then be solvable in global games. Such simplification might prove useful to the study of some specific questions, but it should not occult the fact that it is only a particular case. This is the reason why I have chosen to conduct the comparative statics exercise of *section 5* in terms of expansion or shrinkage of parameter regions consistent with the existence of high and low liquidity equilibria.

4 Liquidity dry-ups and the *2007-2009* financial crisis

During the recent crisis the functioning of many markets has been seriously impaired and adverse selection has been pinpointed as one of the important causal elements (Tirole, 2011b; Bolton, Santos, and Scheinkman, 2011; Morris and Shin, 2010). In this section, I explain how the insights delivered by the model can contribute to the joint understanding of cash hoarding behavior[18] and the persistence of the asset back securities (ABS) market breakdown that have been observed in the aftermath of the recent financial crisis.

First, note that by "aftermath", what I have in mind is a period that starts somewhere a few weeks *after* the collapse of Lehman Brothers. Hoarding behavior may have started earlier, but the sharp deleveraging

[17] See Holmström and Tirole (2011) for such a version of the model.

[18] From August 2007, UK banks have increased their liquidity buffers by 30% (Acharya and Merrouche, 2009); from September 2008, there has been a dramatic increase in the excess reserves of European banks (Heider, Hoerova, and Holthausen, 2010), and of US major deposit institutions (Keister and McAndrews (2009) however points out that a substantial part of the increase may be due to factors other than hoarding), and, despite the huge surge in supply (net of Fed open-market operations), T-Bills prices have been high during the whole period. Therefore, the demand for short term safe assets should have been very high too.

process and the fire sales spirals that occurred at the time (Brunnermeier, 2009; Adrian and Shin, 2010) have probably largely dominated the feedback hoardings may have had on adverse selection.

The story my model tells may thus be combined with a *cash-in-the-market* pricing episode, which would capture the unfolding of the crisis and the following massive deleveraging process[19]. It seems indeed reasonable to think that a severe *cash-in-the-market* pricing episode would trigger concerns about future market liquidity. Then, as I have shown, the fear of illiquidity would lead to hoarding, which would feed illiquidity as it would make more likely that future sales will be motivated by private information rather than a need for cash. The self-fulfilling prophecy might therefore help to explain why some markets, ABS markets for instance, did not recover when cash became available again (i.e. when there is enough *cash-in-the-market* again). Why would one sell an asset in an illiquid market, if he does not need cash? Most likely to get rid of a *lemon*!

Also, it is fair to say the assets traded in the model look rather like equity than ABS. However, the insight that hoarding behavior has a feedback effect on adverse selection applies to the trade of any asset whose price is information sensitive, which precisely began to be the case of ABS during the crisis.

Despite being complex, ABS used to be quite liquid before the crisis. They were also widely used as collateral in repurchase agreements[20]. As noted by Gorton and Pennacchi (1995), debt claims are, in normal times, not sensitive to the value of the underlying assets. In that case, asymmetry of information or heterogeneity in investor sophistication should not really matter. A slightly different view is that better informed or more sophisticated investor are actually able to extract a rent but that, in normal times, the gains from trade are large enough to fund this rent (Morris and Shin, 2010). However, following bad news, debt claim prices can become information sensitive, which raises adverse selection concerns. This is particularly true of Collateralized Debt Obligations backed by Mortgage Backed Securities because their payoffs are highly skewed (Coval, Jurek, and Stafford, 2009). Furthermore, this can be reinforced by their lack of transparency (Pagano and Volpin, 2008). Arguably, the burst of the US housing-market bubble hit sufficiently

[19]This combination may reconcile adverse selection stories with Uhlig (2010), which points out that adverse selection concerns are mitigated when many people are forced to sale. This insights is also consistent with the result of *section 5*.

[20]See Martin, Skeie, and Von Thadden (2011) for the impact of collateral value on the stability of these “repo” markets.

hard the value of their underlying claims so as to make their value sensitive to their specific composition (Morris and Shin, 2010), making them potentially "toxic" and laying the ground to the *lemons problem*.

5 Preference shocks and market liquidity

In this section, I consider an extension in which investors face idiosyncratic preference shocks. At date 1, they learn whether they are *early* or *normal* consumers[21], the latter deriving utility from consumption at date 1 only.

I find that the presence of such *early* agents has a positive effect on market liquidity. Adverse selection is indeed reduced because *early* agents sell off their projects irrespective of the quality. Such an effect suggests that idiosyncratic liquidity shocks need not be socially a bad thing and that policies affecting agents exposure to such shocks may have unintended adverse consequences.

Formally, in this extension, agents differ by the subjective factor β they use to discount date-2 utility: $\beta \in \{0,1\}$ with $Prob(\beta = 1) = p$ and $0 < p < 1$. The machinery of the model is basically the same, the major difference is that there are now four types of agents as of date 1. Agents indeed differ on two dimensions: project return R_j and patience β_i. From here onward, I name agents after their type ij where j still reflects projects return and $i = e, n$ accounts for *early* ($\beta_e = 0$) and *normal* ($\beta_n = 1$) agents respectively. Indices on date-1 decision variables are modified accordingly.

Equilibrium with illiquidity shocks

Investors are still ex-ante identical. They solve:

$$\max_{\lambda, L_{ij}, S_{ij}} U_0 = E_0 \left[\ln(C_{1ij}) + \beta_i \ln(C_{2ij}) \right] \tag{8}$$

[21]Most authors that use such preference shocks use the terminology *early* and *late* to distinguish between consumer types. I do not stick to that terminology because whereas the typical *late* consumers optimally chooses to consume at date 2 only, the *normal* agents of my model want to smooth consumption. Not consuming at date 1 can therefore not be optimal.

$$s.t. \begin{cases} C_{1ij} + S_{ij} = 1 - \lambda + L_{ij}P \\ C_{2ij} = (\lambda - L_{ij})R_j + S_{ij} \\ 0 \leq L_{ij} \leq \lambda \leq 1 \end{cases}$$

Where $j \in \{L, H\}$, with $Prob(j = H) = 0.5$.

While the liquidation decision of *normal* agents (*nL* and *nH*) are still given by (4) and (5), those of *early* agents (*eL* and *eH*) are now determined by their first order condition for $L_{ej} = \lambda$, which always holds:

$$\frac{P}{C_{1ej}} \geq 0 \quad ; \quad j = H, L$$

As they only care about utility of consumption at date 1, they sell off any project they hold, whatever the quality[22]:

$$L_{eH}(P, \lambda) = L_{eL}(P, \lambda) = \lambda$$

Accordingly, the proportion of good assets depends on p:

$$\eta(P) = \begin{cases} \eta_{illiq} = \frac{1-p}{2-p} & ; P < 1 \\ \eta_{liq} = \frac{2\lambda - p}{4\lambda - p} & ; P > 1 \\ \eta_1 \in \left[\eta_{illiq}, \eta_{liq}\right] & ; P = 1 \end{cases} \tag{9}$$

Equilibrium prices are still given by the fixed point $P' = P$, with:

$$P' = R_L + \eta(P)(R_H - R_L)$$

It follows that for any admissible value of p and R_L there is a range for R_H that implies multiple equilibria (see the appendix for the proof). And the two possible levels of liquidity in stable equilibria depend on p:

[22] If $P = 0$ they are indifferent, in which case I assume for simplicity that they liquidate their whole position.

$$\begin{cases} \eta^*_{illiq} = \frac{1-p}{2-p} & ;P < 1 \\ \eta^*_{liq} = \frac{2-p}{4-p} & ;P > 1 \end{cases}$$

Effect on equilibrium market liquidity

I present here a short comparative statics exercise.

PROPOSITION 6 (**market liquidity**)

For any admissible value of R_L, as *the probability* $(1-p)$ *of being hit by an illiquidity shock increases:*

1. *Equilibrium market liquidity increases:* $\frac{\partial \eta^*}{\partial(1-p)} > 0$
2. *The region for a high-liquidity equilibrium expands*
3. *The region for a low equilibrium shrinks*

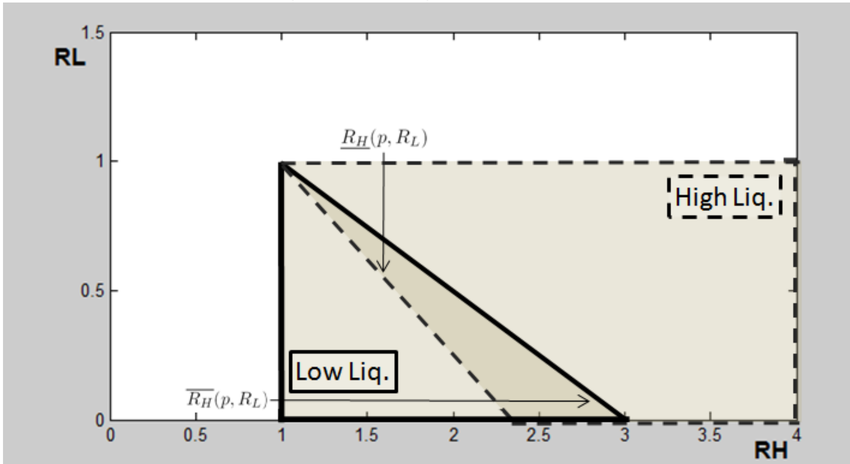
Proof: straightforward.

First, illiquidity shocks enhance market liquidity. In fact, investors hit by such shocks sell off their projects irrespective of the quality. They are in fact very similar to noise traders in the market micro-structure literature[23]. As a consequence, average quality increases with $1-p$, the proportion of *early* agents. Second, when this proportion increases, a lower R_H is needed for a high-liquidity equilibrium to exist, that is, to bring the price up to 1. Third, if $p < 1$, there is always a strictly positive measure of good assets in the market. So, if R_H is sufficiently high for the price at the low level of market liquidity to be higher than 1, it cannot be an equilibrium. Obviously, the higher the proportion of *early* agents, the lower that upper bound.

The lower bound for a high-liquidity equilibrium is $\underline{R_H}(p, R_L) \equiv R_L + \frac{4-p}{2-p}(1-R_L)$. The upper bound for a low-liquidity equilibrium is $\overline{R_H}(p, R_L) \equiv R_L + \frac{2-p}{1-p}(1-R_L)$. For a given p, they define three regions in the space of admissible parameter values for R_H and R_L. Figure 2 illustrates this for $p = 0.5$.

[23] See Kyle, 1985. See also Dow (2004) for a model where the participation of such noise traders depends on market liquidity.

Figure 2: 3 regions ($p = 0.5$)



This figure presents the three regions that define multiplicity of equilibrium. As $(1 - p)$ increases, the low-liquidity-equilibrium region (delimited by the solid lines) shrinks and the high-liquidity-equilibrium one (dashed lines) widens. The overlap is the region with multiple equilibria.

Liquidity buffer requirement

In the aftermath of the recent crisis, it has been suggested that banks should self-insure against idiosyncratic illiquidity shocks, for instance through the holding of liquidity buffers (see for instance the June 2009 Bank of England Financial Stability Report, and more recently the recommendations of the Basel Committee on Banking Supervision BIS, 2011), or that the government should use Pigovian taxation to reduce maturity mismatch (Perotti and Suarez, 2010). The main rationale being to prevent fire sales externalities.

I present here a very simple exercise that illustrates the unintended consequences such policies might have. I focus on liquidity requirement but the taxation of storage would have the same effects. To be clear, my model has nothing to say about fire sales externalities and I do certainly not intend to challenge the view that they should be avoided. Rather, what I claim is that the policy maker should not overlook the *negative* externalities of hoardings either. This is important because the conceptual frameworks that are being used to study liquidity regulation generally focus on the negative externalities, and sometimes even assume them in reduced form (as is the case in Perotti and Suarez, 2010, for instance).

I consider a liquidity hoarding requirement imposed by the government to the investors of my model:

$$1 - \lambda \geq \alpha,$$

which simply means that a fraction α of the initial endowment should be kept in liquid assets (i.e. should be stored). There is of course no rationale for such a requirement in the model but, as I am only interested here in shedding on a mechanisms by which such policy may have adverse unintended consequences, such concern is irrelevant.

The direct effect of such a policy is that, in a high liquidity equilibrium (assuming it exists), agents are less exposed to maturity mismatch, hence they sell fewer good assets:

$$L^*_{nH}(\lambda = 1-\alpha, P) = \frac{1}{2} - \alpha\frac{(P+1)}{2} < \frac{1}{2} = L^*_{nH}(\lambda = 1, P) \tag{10}$$

and adverse selection is more severe. Therefore, the parameter region consistent with a high-liquidity equilibrium shrinks, and, if a high-liquidity equilibrium exists, fewer positive externalities are provided. This leads to all agents being strictly worse off, compared to the initial high-liquidity equilibrium.

PROPOSITION 7 (**unintended consequences**)

A liquidity buffer requirement strictly reduces welfare, and may even cause a liquidity dry-up.

Proof: From (10), one can derive $\eta^*(P)$ and show that it is decreasing in α and in P. The fixed point that pins down the equilibrium, call it $P^*_{liq}(\alpha)$, should therefore be decreasing in α. If $P^*_{liq}(\alpha) < 1$, a high liquidity equilibrium no longer exist, and the requirement dries up liquidity. Then, if the high-equilibrium still exists, both agents are still strictly worse-off. Since the price is depressed, any share sold give fewer units of date-1 consumption good. The agents that liquidate everything are thus strictly worse off. Agent nH is worse off too because his initial consumption bundle is no longer available (P is lower) and the maximum he can consume at each date strictly decreases too.□

Transparency and liquidity

Introducing illiquidity shocks also permits to study the question of transparency. I show in this subsection that transparency about liquidity position has two effects. First, it decreases the liquidity discount for *early* agents, which explains why sellers on a secondary market often pretend they sell for reasons exogenous to the asset's quality[24]. Second, transparency increases the liquidity discount for the others investors: as they did not incur a liquidity shock, it is more likely that they sell because of adverse selection. As a consequence, the effect of transparency on ex-ante risk sharing is ambiguous, as suggested by Gorton (2008), and Holmström (2008).

First, note that under perfect information, which could be interpreted as full transparency, agents could achieve perfect ex-ante risk sharing. I will therefore take for granted that transparency may improve welfare, and focus here on the mechanism by which partial transparency can make agents worse off.

Concretely, I assume that there exists a technology that enables agents to credibly disclose their patience parameter β_i. What I have in mind is for instance a bank that could credibly disclose that it has been hit by a liquidity shock (a wave of unexpected deposit withdrawal for instance). In that case, buyers are able to classify sellers in two categories and they update their priors about average quality accordingly. There are thus two separated markets.

Before formalizing the result in a proposition, I show that *early* agents always choose to disclose their patience parameter. Let η_e^* and P_e^* denote equilibrium liquidity and price, conditionally on the seller to be an *early* agent. Such agents liquidate any project they hold: $L_{eH} = L_{eL} = \lambda = 1$. Thus $\eta_e^* = \frac{1}{2}$ and $P_e^* = E[R]$, which is obviously higher than the price in the other market. Therefore, as these agents get a better price, their ex-post wealth increase, they are better-off and they have no incentive to deviate (that is, not to disclose their liquidity position).

PROPOSITION 8 (**partial transparency**)

Transparency may decrease welfare, and even dry up liquidity.

[24]For instance, in their ads for second-hand cars, students often mention that they want to sell because they are graduating and moving out of town. I thank Jean Tirole for pointing out this relationship between liquidity position disclosure and market liquidity discount, and for this example.

Proof: I first construct an example where transparency dries up market liquidity: a high-liquidity equilibrium is possible without disclosure, but impossible with disclosure. Then, I give an example where a high-liquidity equilibrium still exists, but where transparency deteriorates risk sharing so that agents are ex-ante worse off.

First, let $R_L = 0$, $R_H = 2.4$ and $p = 1/2$. From (9), since, $p < (2R_H - 4)/(R_H - 1)$, the market is liquid ($P \geq 1$) when agents are fully invested in the long-term technology, and a high-liquidity equilibrium exists without disclosure. In such an equilibrium, $\lambda^* = 1$, $P^* = 15/14$, and $U_0 \simeq -0.38$. However, under disclosure the price *normal* agents can get is $R_H/3 < 1$, the price that would prevail in an economy without illiquidity shocks, and a high-liquidity equilibrium is no longer possible. Accordingly, agents decide to hoard: numerical optimization gives $\lambda^* \simeq 0.49$ and expected utility is lower: $U_0 \simeq -0.47$.

Then, let $R_L = 0$, $R_H = 5$ and $p = 1/2$. Without disclosure, there is a high liquidity equilibrium with $\lambda = 1$, $P = 15/7$, and $U_0 \simeq 0.66$. Under disclosure, the price *normal* agents can get is $R_H/3 > 1$, there exists thus a high-liquidity equilibrium, with $\lambda = 1$, but risk-sharing is impaired (*normal* agents with bad assets, which have the highest marginal utility of consumption, end-up consuming less), and ex-ante welfare decreases: $U_0 \simeq 0.55$.□

6 Conclusion

In this paper, I have shown that adverse-selection-driven market breakdowns are endogenous to past balance sheet decisions.

When they face a return-liquidity trade-off and when secondary markets for long term assets are subject to adverse selection, agent investment decisions, and in particular the extent to which they expose themselves to maturity mismatch, present strong strategic complementarities. In other words, hoarding has strong negative externalities: when agents choose hoarding as a mean to accommodate short-term needs, it becomes more likely that they subsequently trade for private information motive. There are thus proportionally more *lemons* in the market and it becomes more costly to transform long-term investment into current consumption goods, which justify the decision to hoard. This is a self-fulfilling liquidity dry-up.

This feedback relationship between hoarding and adverse selection is intuitive and helps to jointly explains the persistent market breakdowns and the unusual hoarding behavior that took place in the aftermath of the recent financial crisis.

The model delivers insights that are relevant to financial regulation. First, that hoarding behaviors worsen adverse selection highlights a channel by which prudential policies that aim at raising financial institutions liquidity ratios may have unintended consequences. Second, that participation constraints, not only participation in the market but also in public schemes, are endogenous to hoarding decision is relevant to the design of public intervention during a crisis. And third, that long-term investment decisions crucially depend on future market liquidity is relevant to the design of policies that aim at exiting a credit crunch.

A Appendix

A.1 Proofs

Proof of *Proposition 2* (self-insurance)

1) The case where $P > 1$ is trivial as storage is a dominated mean to transfer resource to date 1 and date 2. Thus $\lambda(P > 1) = 1$.

2) When $P = 1$, storage is equivalent to invest and then liquidate, and it is straightforward to show that the optimal consumption plan is $C^*_{2H} = R_H/2$ and $C^*_{1H} = C^*_{1L} = C^*_{2L} = 1/2$, which can be implemented for any $\lambda \in \left[\frac{1}{2}, 1\right]$ by setting $L_H = \lambda - 1/2$.

3) Consider now $P < 1$ and let $U^{'}_j \equiv \left[\frac{\partial U_0}{\partial \lambda} \,|\, j\right]$ be the marginal utility of λ conditionally of being in the state j.

I first have: $U^{'}_L < 0, \forall \lambda \in [0,1]$, which simply comes from the fact that, in state L, any resource invested in a project gives a negative return whether it is liquidated ($P < 1$) or is held to maturity ($R_L < 1$). Hence, when markets are illiquid, investors that realize that their project will fail wish they had invested nothing.

From the date-1 first order conditions, I have: $L_H^*(P,\lambda) = \max\left\{0; \frac{P\lambda-1+\lambda}{2P}\right\}$ and $S_H^*(P,\lambda) = \max\left\{0; \frac{1-\lambda-\lambda R_H}{2}\right\}$.

Thus, I can evaluate utility in state H:

$$U_H(\lambda, P<1) \equiv \begin{cases} 2\ln\left(\frac{1+\lambda(R_H-1)}{2}\right), & \lambda \leq \frac{1}{1+R_H} \\ \ln(1-\lambda)+\ln(\lambda R_H), & \frac{1}{1+R_H} \leq \lambda \leq \frac{1}{1+P} \\ \ln\left(\frac{1+\lambda(P-1)}{2}\right)+\ln\left(\frac{1+\lambda(P-1)}{2}\left(\frac{R_H}{P}\right)\right), & \frac{1}{1+P} \leq \lambda \end{cases} \tag{11}$$

And take the derivative with respect to λ:

$$\begin{cases} U_H^{'} > 0, & \lambda < \frac{1}{2} \\ U_H^{'} = 0, & \lambda = \frac{1}{2} \\ U_H^{'} < 0, & \lambda > \frac{1}{2} \end{cases}$$

Since $U_L^{'} < 0, \forall\lambda$, expected utility maximization implies thus that $\lambda(P<1) < 1/2.\square$

Proof of *Proposition* 4 (market failure)

Let $R_H > 3-2R_L$. By *proposition 3*, I have:

$$\left\{\gamma_{illiq}(R_L,R_H) = \left(R_L, \check{\lambda}, 0\right); \gamma_{liq}(R_L,R_H) = \left(R_L + \frac{(R_H-R_L)}{3}, 1, \frac{1}{3}\right)\right\} \in \Gamma(R_L,R_H)$$

If both agents are better-off in γ_{liq}, it ex-post Pareto dominates γ_{illiq}. Denote C_{tj}^{γ} the optimal consumption of agent j at date t in equilibrium γ.

i) Agent L is better-off

Obvious since $C_{1L}^{\gamma_{illiq}} = C_{2L}^{\gamma_{illiq}} = \frac{1-\lambda(R_L-1)}{2} < \frac{R_L+\frac{(R_H-R_L)}{3}}{2} = C_{1L}^{\gamma_{liq}} = C_{2L}^{\gamma_{liq}}$.

ii) Agent H is better-off

First, H lifetime consumption is higher in γ_{liq}, and, second, both its γ_{illiq} period consumption are attainable in γ_{liq}. Formally: I have $C_{1H}^{\gamma_{illiq}} \leq 1-\lambda^{illiq}$ and $C_{2H}^{\gamma_{illiq}} \geq \lambda^{illiq}R_H$ (both with equality if $S_H = 0$), and

$C_{1H}^{\gamma_{illiq}} + C_{2H}^{\gamma_{illiq}} = 1 - \lambda^{illiq} + \lambda^{illiq} R_H$. And also, $C_{1H}^{\gamma_{liq}} = P/2$ and $C_{2H}^{\gamma_{liq}} \geq R_H/2$. Thus: $1 - \lambda + \lambda R_H < P/2 + R_H/2$ since $\lambda^{illiq} < 1/2$, and for any λ^{illiq}, there is a L_H such that $C_{1H}^{\gamma_{illiq}} \leq L_H P$ and $C_{2H}^{\gamma_{illiq}} \leq (1 - L_H) R_H$.$\square$

Proof of *Proposition* 5 (public liquidity insurance)

Under this public scheme, date-1 budget constraints are:

$$\begin{cases} C_{1j} + S_j = 1 - \lambda + L_j \max(P, 1) - \tau \\ C_{2j} = (\lambda - L_j) R_j + S_j \end{cases}$$

i) In state L, such a subsidy will of course not decrease the willingness to liquidate: $L_L^*(P) = \lambda^*(P)$. Thus:

$$C_{1L}^* = C_{2L}^* = \frac{1 + \lambda(\max(P, 1) - 1) - \tau(P)}{2}$$

And the marginal utility of λ in that state is always positive.

2) In state H, there are two cases, depending on P:

If $P \geq 1$, the scheme has no effect and storage is still a dominated technology.

If $P < 1$, the budget constraints are:

$$\begin{cases} C_{1H} + S_H = 1 - \lambda + L_H - \tau \\ C_{2H} = (\lambda - L) R_H + S_H \end{cases}$$

And one compute the optimal L_H and S_H to adapt (11):

$$U_H(\lambda) \equiv \begin{cases} 2\ln\left(\frac{1-\tau+\lambda(R_H-1)}{2}\right), & \lambda \leq \frac{1-\tau}{1+R_H} \\ \ln(1 - \lambda - \tau) + \ln(\lambda R_H), & \frac{1-\tau}{1+R_H} \leq \lambda \leq \frac{1-\tau}{2} \\ 2\ln\left(\frac{1-\tau}{2}\right) + \ln\left(\frac{1-\tau}{2}(R_H)\right), & \frac{1-\tau}{2} \leq \lambda \end{cases}$$

And check that the marginal utility of λ in this state is always positive too.

Hence $\frac{\partial U_j(\lambda)}{\partial \lambda} \geq 0, \forall j$, and there is no incentive to hoard. Still, there is an indeterminacy as agents may choose $\lambda^* = 1/2$. This is however not really credible as it is a weakly dominated strategy, and in any case, the government can always choose an arbitrarily small ε and set the "bailout price" to $1+\varepsilon$ so that this strategy becomes strictly dominated, in which case: $\frac{\partial U_0}{\partial \lambda} > 0$.□

A.2 Robustness

Cash-in-the-market **pricing**

I extend here the model along the lines of Bolton, Santos, and Scheinkman (2011) to combine both adverse selection and *cash-in-the-market* pricing.

To get *cash-in-the-market* pricing, one needs a downward sloping interim demand curve. Instead of assuming the presence of agents with sufficiently deep pockets to clear the market at the fair Akerlovian price, I assume that those agents, which only value consumption at date 2, have limited resources at date 0 (say Y units of the consumption good on aggregate). They may allocate their endowment between storage and a long-term risk-less technology with decreasing return to scale: investing X yields $F(X)$ with $F'(X) > 0$ and $f''(X) < 0$. For the analysis to be interesting, further assume that $F'(Y) > 1$ so that it is "costly" for them to hoard cash from date 0 to date 1. When they hold assets between 1 and 2, the demand of these agents at date 1 is such that:

$$F'(Y-D) = \frac{R_L + \eta(R_H - R_L)}{P}$$

where D is the quantity of consumption good they hoard between 0 and 1.

One can easily show that in the low-liquidity equilibrium $D = 0$ and the market completely breakdowns. However, a high-liquidity equilibrium remains sustainable as long as:

$$P^* = \frac{R_L + \eta(R_H - R_L)}{F'(Y-D^*)} \geq 1 \tag{12}$$

with $D^* = \sum_j L_j$.

The intuition is the following: because of the cash-in-the-market pricing effect, the market discount rate is $F'(Y - D^*)$ instead of the return to storage. Therefore, the price in the secondary market is depressed. However, it needs not be low enough to rule out the high-liquidity equilibrium: one can always find $F(.)$ and R_H such that (12) holds.

Multiplicity under incomplete information

I sketch here a global game version of the model in which agents map a privately observed signal R_{Hi} about the true parameter R_H of the model[25], into an initial investment decision $\lambda(R_{Hi})$, and I show that the best-response to non-decreasing strategies can be non-monotonic and violates the *single-crossing condition* for uniqueness of equilibrium used in the literature.

The intuition for this can be apprehended thanks to an analogy to the binary action game (*to run* or *not to run* the bank) studied by Goldstein and Pauzner (2005). In their game, for a given level of fundamental (θ) the expected utility differential $v(\theta, n)$ of choosing *to run* is increasing in n, the proportion of agents that run, *whenever v is negative* (that is for any n such that *not to run* is preferable). Accordingly, for any level of fundamental (θ), there can be at most one n such that $v(\theta, n) = 0$ and an agent is indifferent between *to run* and *not to run* (there is a single crossing between $v(\theta, n)$ and the *x-axis*). This is the key condition to ensure uniqueness in their model.

Here, if one were to (abusively) summarize all other agents investment decision by their average, call it $\bar{\lambda}$, the example I show below corresponds to a situation in which the secondary market price P is non-monotonic in $\bar{\lambda}$. Therefore, as the marginal utility of λ is strictly increasing in P, it is decreasing in $\bar{\lambda}$ over a certain range, and the best-response is non-monotonic. Crucially, it crosses more than once the level at which externalities become positive, which lets the door to multiple equilibria open (see Bueno de Mesquita, 2011, for binary action games with multiple equilibria when the single-crossing condition is not satisfied, and Mason and Valentinyi, 2010 where single-crossing is one of the sufficient conditions for uniqueness in

[25] This parameter could equally be R_L, p, q, or r.

games with a continuum of actions).

Sketch of the model:

- $R_L = 0$, $q = 0.5$, $r = 0$ (without loss of generality)

- $p = 0.5$ (if $p = 1$, that is if there are no early agents, one can actually prove that multiplicity remains, see Bueno de Mesquita, 2011).

- R_H is drawn from the a uniform distribution with support: $]0+\varepsilon;\infty[$

- Each agent observes a private signal R_{Hi} from a uniformed distribution centered on the true value of R_H: $[R_H - \sigma; R_H + \sigma]$, with $0 < \sigma < \varepsilon$ and maps it into an initial investment decision $\lambda(R_{Hi})$.

First, note that there exist multiple equilibria of the complete information game when $7/3 \leq R \leq 3$. Then, consider the following (non-decreasing) strategies:

$$\lambda_1(R_{Hj}) \equiv \begin{cases} 1/4 & ; R_{Hj} < 2.45 \\ 1/2 & ; 2.45 \leq R_{Hj} \leq 2.5 \\ 1 & ; 2.5 < R_{Hj} \end{cases} \qquad \lambda_2(R_{Hj}) \equiv \begin{cases} 1/4 & ; R_{Hj} < 7/3 \\ 1 & ; 7/3 \leq R_{Hj} \end{cases}$$

Figure 3 displays the price that would result for any realization of R_H if half agents play $\lambda_1(R_{Hj})$ and the other half play $\lambda_2(R_{Hj})$ (call this strategy profile Λ^0).

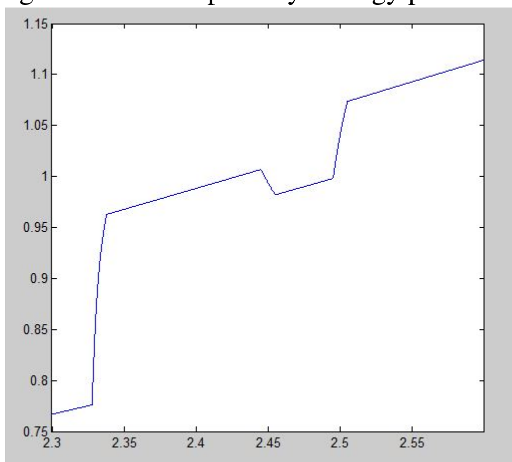
$$P(R_H, \Lambda^0) \equiv \eta(\Lambda^0) R_H,$$

with $\eta(\Lambda^0)$ being the analogous to equation (7).

From this, one can compute the best-response to Λ^0 of an agent that observes a signal R_{Hi}:

$$\lambda^{BR}(\Lambda^0, R_{Hi}) = \arg\max_{\lambda} U_0\left(\lambda, P(R_H, \Lambda^0) \mid R_{Hi}\right)$$

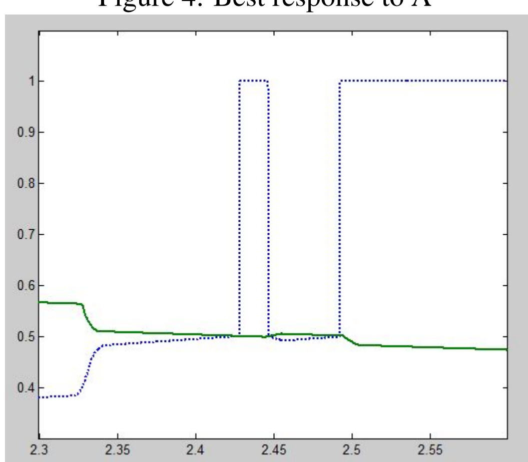
Figure 3: Price implied by strategy profile Λ^0



This graph displays the price (y-axis) implied by Λ^0 for the possible realization of R_H (x-axis).

Figure 4 displays such a best-response, which shows that best-responses to monotonic strategies need not be monotonic, and that, crucially, it crosses several times the locus of the joint values of P and λ from which externalities become positive, that is ($\lambda = 1/2$).

Figure 4: Best response to Λ^0



The dotted line is the best-response ($\lambda^{BR}(\Lambda^0, R_{Hi})$) of an agent (y-axis) to Λ^0, that observes a private signal R_{Hi} (x-axis). Above the solid line is the region corresponding to $L_H(\lambda, P) > 0$, which implies that agents exert positive externalities.

Multiple equilibria in the general model

I prove here that the assumptions about the return to storage, the probability of project success, and the specific form of the utility function were made for simplicity only and the results extent to a general formalization.

Consider the general problem:

$$\max_{\lambda,L_{ij},S_{ij}} U_0 = E_0\left[u(C_1)+\beta_i u(C_2)\right]$$

$$s.t.\begin{cases} C_{1ij}+S_{ij}=1-\lambda+L_{ij}P \\ C_{2ij}=(\lambda-L)R_j+S_{ij}(1+r) \\ 0\le L_{ij}\le\lambda\le 1 \end{cases}$$

Where $i\in\{e,n\}$with $\beta_e=0$, $\beta_n=1$ and $Prob(i=n)=p>0$, and $j\in\{L,H\}$with $Prob(j=H)=q$ and $0<q<1$.

PROPOSITION 2bis (**self insurance generalized**): *Let $\lambda(P)\equiv\arg\max_{\lambda} U_0(\lambda,P)$ be the set of optimal λ to the problem above, given P and the date-1 optimal liquidity and saving policies (L^*_{ij},S^*_{ij}), then:*

$$\lambda(P)\begin{cases} <\lambda^*_{nH}(1+r,R_H) & ;P<1+r \\ =1 & ;P>1+r \\ \in\{[\lambda^*_{nH}(1+r,R_H),1]\} & ;P=1+r \end{cases} \tag{13}$$

With $\lambda^*_{nH}(x,R_H)\equiv\arg\max_\lambda u((1-\lambda)x)+u(\lambda R_H)$ being the optimal endowment share invested in the long-run technology, conditionally on being of type nH, and with $x\equiv\max\{1+r;P\}$ being the best available way to transfer resources to date 1. This proposition states that when the market is expected to be illiquid, the optimal level of investment is strictly smaller than the ex-post optimal level for agent nH.

Proof: Because the rest of the proof of proposition follows exactly the same logic as that of proposition

2, I only show here that agent nH does not participate in the market when $\lambda < \lambda^*_{nH}(1+r,R_H)$.

When $P < 1+r$, $x = 1+r$. By definition of $\lambda^*_{nH}(x,R_H)$, I have : $u^{'}(1-\lambda^*_{nH}(1+r,R_H))(1+r) - u^{'}(\lambda^*_{nH}(1+r,R_H)R_H)R_H = 0$, hence:

$$\frac{\partial U_0}{\partial L_{nH}}\mid_{\lambda<\lambda^*_{nH}(1+r,R_H),L_{nH}=0} < 0$$

□

I can thus define the generalized implied price correspondence $P'(P,R_H) : \left\{\left[\frac{R_L}{1+r},\frac{R_H}{(1+r)}\right],R_H\right\} \to \left[\frac{R_L}{1+r},\frac{R_H}{(1+r)}\right]$:

$$P'(P,R_H) = \begin{cases} P^{'}_{illiq}(P,R_H) = \frac{R_L}{1+r} + \eta_{illiq}(P,R_H)\frac{R_H}{(1+r)} & ;P \leq 1+r \\ P^{'}_{liq}(P,R_H) = \frac{R_L}{1+r} + \eta_{liq}(P,R_H)\frac{R_H}{(1+r)} & ;P \geq 1+r \\ P^{'}(1+r,R_H) \in \left[P^{'}_L(P,R_H),P^{'}_H(P,R_H)\right] & ;P = 1+r \end{cases}$$

With $\eta_{illiq}(P,R_H) = \frac{q-pq}{1-pq} \equiv \eta_{illiq}$, $\eta_{liq}(P,R_H) = \frac{q-pq\lambda^*_{nH}(P,R_H)}{1-pq\lambda^*_{nH}(P,R_H)}$, which combines *proposition 2bis* and the fact that all agents other than nH sell off their assets: $L_{nL}(P,\lambda) = L_{eL}(P,\lambda) = L_{eH}(P,\lambda) = \lambda$.

PROPOSITION 3bis (**multiplicity generalized**): *Let $\bar{\Omega}$ be the range of admissible values*[26] *for parameters* $\{R_L,p,q,r\}$. *Let $\Gamma(\Omega,R_H) = \{(P^*,\lambda^*,\eta^*)\}$ denote the set of stable equilibria defined by (2) for a vector $(\Omega \in \bar{\Omega},R_H)$ of parameters.*

$\forall \Omega \in \bar{\Omega}$, $\exists \underline{R_H} < \overline{R_H}$ such that $\forall R_H \in]\underline{R_H};\overline{R_H}[$, $\Gamma(\Omega,R_H)$ has at least two distinct elements corresponding to equilibria with different level of liquidity.

Before turning to the core of the proof, I establish the continuity of $P^{'}_{liq}(P,R_H)$ and the fact that liquidity is always higher when agent H participates in the market.

Condition 1: $u^{'}(0) > u^{'}(R_H)\frac{R_H}{1+r}$

This (mild) condition will ensure $C^*_{1,nH} > 0$, that is I rule out the case where agent nH optimally chooses not to consume at date 1.

[26] $0 < p \leq 1, 0 < q < 1, 0 \leq R_L < 1+r, r > -1$

LEMMA 1 (**liquidity dominance**): *Under condition 1,* $\eta_{illiq} < \eta_{liq}(P,R_H) < q, \forall P \geq 1+r.$

Proof: First, from $\lambda^*_{nH} \in [0,1]$, I have $\eta_{illiq} \leq \eta_{liq}(P,R_H) \leq q$. Then, *Condition 1* implies that: $u^{'}(0) > u^{'}(R_H)\frac{R_H}{P_{liq}(P,R_H)}$, $\forall P \geq 1+r$. This, in turn, implies that $\lambda^*_{nH}(P,R_H) > 0$ and the first inequality is strict. The second strict inequality comes from: $\frac{\partial U_0}{\partial L_{nH}} |_{L_{nH}=1} < 0$, which is always true since $P \leq R_H$.□

LEMMA 2 (**continuity**): $P^{'}_{liq}(P,R_H)$ *is continuous in P and* R_H*.*

Proof: As $\lambda^*_{nH} \in]0,1[$, $P^{'}_{liq}$ is a continuous function of $\lambda^*_{nH}(P,R_H)$. The implicit functions theorem applied to the first order condition for an interior λ^*_{nH} ensures that $\lambda^*_{nH}(P,R_H)$ is continuous in both its arguments, which implies *Lemma 2*.□

Proof (of PROPOSITION 3bis): As I only consider stable equilibria, I am not interested in the vertical locus for $P'(P,R_H)$ which corresponds to $P = 1+r$. I will consider separately the two functions $P^{'}_{illiq}(P,R_H)$ and $P^{'}_{liq}(P,R_H)$ defined respectively on the sets $\left\{\left[\frac{R_L}{(1+r)}, 1+r\right], R_H\right\}$ and $\left\{\left[1+r, \frac{R_H}{(1+r)}\right], R_H\right\}$ and show that there exists a range of R_H that generates at least an equilibrium for both functions:

The upper bound for a low-liquidity equilibrium: Brouwer's fixed point theorem gives the necessary and sufficient condition on R_H for a unique fixed point $P^{'}_L(P,R_H) = P$:

$$R_H \leq R_L + \frac{(1+r)^2 - R_L}{\eta_{illiq}}$$

There exists thus a unique low-liquidity equilibrium if and only if R_H is low enough, and I can thus set:

$$\overline{R_H} \equiv R_L + \frac{(1+r)^2 - R_L}{\eta_{illiq}}$$

The lower bound for a high-liquidity equilibrium: In order to derive a sufficient condition for the existence of a fixed point $P^{'}_{liq}(P,R_H) = P$, I construct a function $G(P,R_H) \equiv P - P^{'}_{liq}(P,R_H)$, defined on the interval for P: $\left[1+r, \frac{R_H}{(1+r)}\right]$. Clearly, given the continuity of $P^{'}_{liq}(P,R_H)$ (*Lemma 2*), if $G(P,R_H)$ changes sign on its domain, there is a fixed point for $P_H^{'}(P,R_H)$.

Since $P_{liq}^{'}(P,R_H)=\frac{R_L}{1+r}+\eta_{liq}(P,R_H)\frac{R_H}{(1+r)}$ and $\eta_{liq}(P,R_H)$ is bounded above by $q<1$, I have: $P_{liq}^{'}(P,R_H)<\frac{R_H}{(1+r)}$ and thus $P_{liq}^{'}\left(\frac{R_H}{(1+r)},R_H\right)<\frac{R_H}{(1+r)}$. It implies:

$$G\left(\frac{R_H}{(1+r)},R_H\right)>0 \tag{14}$$

For any R_H, given (14), a sufficient condition for the existence of a high-liquidity equilibrium is thus:

$$G((1+r),R_H)\leq 0$$

Which is equivalent to:

$$P_{liq}^{'}(1+r,R_H)\geq(1+r)$$

And thus to:

$$R_H\geq R_L+\frac{(1+r)^2-R_L}{\eta_{liq}(1+r,R_H)} \tag{15}$$

As $\eta_{liq}(P,R_H)$ is bounded, there will always exist a R_H high enough such that condition (15) is satisfied. Yet, I am interested in a lower bound on R_H for this condition to hold. That is, a $\underline{R_H}$ such that:

$$\forall R_H\geq\underline{R_H},\, R_H\geq R_L+\frac{(1+r)^2-R_L}{\eta_{liq}(1+r,R_H)}$$

In order to find $\underline{R_H}$, I construct the function $R_H^{'}(R_H)\equiv R_L+\frac{(1+r)^2-R_L}{\eta_{liq}(1+r,R_H)}$. Given *lemma 1*, it is bounded below by $R_L+\frac{(1+r)^2-R_L}{q}$ and above by $R_L+\frac{(1+r)^2-R_L}{\eta_{illiq}}$. Also, given *Lemma* 2, it is continuous over the range corresponding to these bounds. It admits thus at least a fixed point: $R_H^{'}=R_L+\frac{(1+r)^2-R_L}{\eta_{liq}(1+r,R_H^{'})}$.

For most of the commonly used utility functions (the HARA class for instance), $\eta_{liq}(P,R_H)$ is monotonic in R_H. It implies that there is a unique fixed point; call it $R_H^{'*}$. *Lemma 1* ($\eta_{illiq}<\eta_{liq}(P,R_H)$) implies that $R_H^{'}\left(R_L+\frac{(1+r)^2-R_L}{\eta_{illiq}}\right)<R_L+\frac{(1+r)^2-R_L}{\eta_{illiq}}$ and thus $\forall R_H\geq R_H^{'*}$, $R_H>R_L+\frac{(1+r)^2-R_L}{\eta_{liq}(1+r,R_H)}$. Hence, this unique fixed point is a lower bound on R_H for the existence of a high-liquidity equilibrium. I can thus choose $\underline{R_H} \equiv R_H^{'*}$. If there are multiple fixed points, the correct lower bound is the highest valued fixed point:

$$\underline{R_H} \equiv \max\left\{R_H^{'} : R_H^{'} = R_L + \frac{(1+r)^2 - R_L}{\eta_{liq}(1+r, R_H^{'})}\right\}$$

The range for multiple equilibria: *Lemma 1* implies $\underline{R_H} < \overline{R_H}$, which concludes the proof: $\forall R_H \in]\underline{R_H};\overline{R_H}[$ there exists at least two equilibria.□

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