Optimal capital requirements over the business and financial cycles*

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Abstract

I study economies where banks do not fully internalize the social costs of their lending decisions. In equilibrium, this leads to over-investment by firms. The bank capital requirement that restores investment efficiency depends on the state of the economy. I identify a general equilibrium effect that has two main implications for this optimal capital requirement. First, it is increasing in aggregate bank capital. Second, it is tighter during booms than in recessions. A suboptimal policy that overlooks this effect exacerbates economic fluctuations as it allows for excessive build-up of risk by banks during booms, and contracts credit excessively in recessions.

1 Introduction

It is widely acknowledged that financial institutions have incentives to take on too much risk (Kareken and Wallace (1978), Acharya and Richardson (2009)) and that banking crises are costly for society.[1] Until the 2007-2009 global financial crisis, the main focus of bank prudential regulation was to contain risk taking at the individual bank level. Ever since, the focus

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[1] For instance, the Savings and Loan crisis officially cost the US taxpayer at least $132bn (in 1995 USD). There is no consensus on the 2007-2009 crisis' net fiscal costs, but Laeven and Valencia (2012) estimate the net outlays at 2.1% of GDP in the US, 6.6% in the UK, up to a vertiginous 40% in Ireland. Besides fiscal costs, banking crises appear to severely affect the real economy as they are typically followed by long and painful recessions (Jordà, Schularick and Taylor (2013)) and are associated with large permanent output losses (Cerra and Saxena (2008)).

has evolved towards containing system-wide risk taking. What are the the costs and benefits of such policies? What are the relevant general equilibrium effects to take into account? Is an optimal policy time-varying?

To address such questions, I develop a simple theory of intertwined business and financial cycles, where financial regulation both optimally responds to and influences the business cycle. In the economies I study, regulation is needed because banks do not fully internalize the social costs associated with their lending decisions. They do not internalize these costs for two main reasons. First, in the case of a crisis, some of the costs are ultimately borne by the taxpayer. Second, credit expansion by a bank increases the probability of default of other banks and the losses they make in such an event. As a result, the competitive equilibrium is generally inefficient: banks lend too much, which translates into excessive real investment by firms in a general equilibrium.

The main focus of the paper is on the time-varying bank capital requirement that restores investment efficiency. The first key contribution of the paper is to highlight a general equilibrium effect that is a key driver of this optimal capital requirement, and of its cyclical properties. The second is to provide a detailed analysis of the pecuniary externalities generated by this general equilibrium effect.

The model involves overlapping generations of risk-neutral savers and bankers. Bankers are protected by limited liability. They collect deposits and competitively lend to firms, which operate a risky constant returns-to-scale production function. Some firms succeed, some fail. Those that fail default on their bank loan. Banks are perfectly diversified and their realized return to lending depends on the firm default rate. The return to lending also depends on the equilibrium in the labor and capital markets. Labor supply is fixed, so there are diminishing returns to physical capital, which translates into decreasing marginal returns to lending in a general equilibrium. When the proceeds from lending are insufficient to repay depositors in full, the bank is insolvent and defaults, which generates deadweight losses.

To set up the analysis, I establish the market failure and show that the regulator can restore investment efficiency thanks to a time-varying capital requirement. I then study the cyclical properties of this optimal capital requirement in two steps. First, I present two simple cases that can be solved analytically. This allows me to formally identify the main mechanisms at play. Second, I calibrate the model and solve it numerically to assess the quantitative relevance of these mechanisms.

I find that the optimal capital requirement is *increasing in aggregate bank capital*. To see the intuition, first consider an atomistic bank that doubles its equity. It should simply be allowed to double lending. However, if all banks in the economy double their equity, and if they are allowed to double lending, this could double investment in the economy. Given diminishing returns to capital this cannot be optimal in a general equilibrium. In fact, the optimal policy is to let the banking sector expand, but less than proportionally, which corresponds to an increase in the capital requirement. This is a central result of the paper.

In this economy, aggregate productivity evolves over time. When it is high, the return to lending is high and new bankers also enter the sector with higher wealth (which comes from their wage). Hence, aggregate bank capital is high. Due to the general equilibrium I have just described, this calls for a tightening of the capital requirement. I refer to this effect as the *bank capital channel*.

When productivity is high, future productivity is likely to be high too. Hence, the firm default rate is likely to be low, which means that expected lending profitability is high. Efficiency therefore requires an expansion of aggregate lending. All other things equal, this calls for a loosening of the capital requirement. I refer to this effect as the loan *default rate channel.*

These two channels pull in opposite directions. My analytical results indicate that the bank capital channel is stronger than the loan default rate channel. As a result, the capital requirement is tighter during booms than in downturns. Since the bank capital channel is driven by the general equilibrium effect, these analytical results suggest that this effect could be important.

I calibrate the general model and solve it numerically. Simulation results confirm the analytical insights. First, I consider the economy at steady state and study the response of the optimal capital requirement to a single positive shock (namely, a 1% decrease in the loan default rate). This response is positive: the optimal requirement increase peaks around 20bps after a few periods and then decays over time. Second, I simulate a full series of shocks. I find that the optimal capital requirement is higher in good times (i.e. when the loan default rate is at least one standard deviation below its mean) than in bad times (when the default rate is at least one standard deviation above its mean).

To explore further the quantitative relevance of the general equilibrium effect, I also compare the optimal capital requirement to a suboptimal policy that takes into account the default rate channel but ignores the bank capital channel. Under this suboptimal policy, the cyclical properties of the requirement are reversed: it is tighter in bad times than in good times. As a result, banks take on too much risk in good times (they lend too much and retain less profit) and credit contraction is unnecessarily severe in bad times. According to my calibration, the associated welfare losses amount to 0.04% of steady state consumption, which is in the ballpark of numbers found in the literature on stabilization policy (Lucas, 2003).

The features of the suboptimal policy resonate with the criticisms of the second regime of the international standards for banking regulation (BCBS (2004)), commonly referred to as *Basel II.* I show that my framework can easily be extended to study the cyclical effects of the risk-weights that this regime introduced. Then, taking the cyclical effect of risk-weights as given, I use my calibration results to compute back-of-the-envelope estimates for the optimal time-varying adjustments that would restore investment efficiency. I relate these adjustments to the macroprudential adjustments introduced by Basel III, the current regulatory regime (BCBS (2010)).

It is is well understood that a regulatory regime like Basel II is likely to magnify the business cycle (Kashyap and Stein (2004), Repullo, Saurina and Trucharte (2010)).[2] Adjusting capital requirements to the aggregate state of the economy seems a sensible response. However, how such adjustments should be designed and, more generally, what are their general equilibrium consequences remain open questions.

Kashyap and Stein (2004) argue in favor of adjustments based on the scarcity of aggregate bank capital (relative to lending opportunity). They point out that *a priori* it is not obvious that aggregate bank capital scarcity is greater during a recession, but they interpret the empirical literature on bank capital crunches as generally supporting such a notion. My theoretical model delivers such a result.[3] Repullo and Suarez (2013) find, however, the opposite. They study a theoretical model of optimal bank capital requirements and compare them to Basel I, II, and III. In their setup, even though bank capital is scarcer in bad times, capital requirements should still optimally be tighter than in good times. An important feature of their model is that the production function is linear in investment, which explains why they do not capture the general equilibrium mechanism that drives the result in my model.

Martinez-Miera and Suarez (2014) propose a model where correlated risk-shifting by some banks gives an incentive to other banks to play it safe. The reason is that banks that survive a crisis earn large scarcity rents in the aftermath, an application of the "last-bank-standing effect" (Perotti and Suarez (2002)). They focus on the optimal *level* of a constant capital requirement.[4] Also, they do not consider business cycle dynamics but, in their model, loosening capital requirements after a banking crisis mitigates rents ex-post and induces more systemic risk-taking ex ante. In contrast, in Dewatripont and Tirole (2012), incentives to gamble for resurrection are stronger after a negative macroeconomic shock. In the same vein, Morrison and White (2005) study a model with both moral hazard and adverse selection. They find that the appropriate policy response to a crisis of confidence may be to tighten capital requirements. This happens when the regulator's ability to alleviate adverse selection through banking supervision is relatively low.

In Section 5, I dissect the pecuniary externalities associated with the general equilibrium effect. I show how they interact with bank default costs to generate inefficiencies. I highlight

[2] *Basel I* (BCBS (1988)) imposed a capital requirement of 8% on risk-weighted bank assets. Risk weights where essentially fixed (there were five coarse categories of borrowers, and borrowers would not change categories). To better with risk heterogeneity *in the cross-section*, *Basel II* (BCBS (2004)) introduced risk weights that are directly linked to each loan probability of default. But probabilities of default tend to co-move over the economic cycle, which created effects *in the time-series*. In particular, lower probabilities of default in good times decreased the effective stringency of the requirement (and conversely in a bust). If the purpose of capital requirement is to contain bank risk-taking, effectively tighter requirements in bad times seems desirable. But bank capital (equity in the banking sector) is likely to be scarcer in bad times (because banks have incurred losses), which implies a credit contraction in the economy. The consensus is that such a contraction is excessive from a social point of view.

[3] In a static model, Repullo (2013) finds that capital requirements should be loosened after an exogenous negative shock to bank capital. This is in line with Kashyap and Stein (2004)'s premise and with my results, but the mechanism is different.

[4] Other recent papers focusing on the optimal level include Morrison and White (2005), Van den Heuvel (2008), Admati et al. (2010), Harris, Opp and Opp (2015), and Begenau (2015).

two main classes of externalities. The first, already mentioned above, works through banks' probabilities of default. Credit expansion by a given bank increases aggregate physical capital in equilibrium and, therefore, decreases its marginal productivity. This, in turn, decreases the return to lending of other banks and increases the probability that they default and incur default costs. When making its lending decision, a bank takes its own default costs into account, but not those of others, which leads to inefficiently high levels of lending. The second class of externalities works through the banks' costs given default. Whether these externalities contribute to under- or overlending depends on the specific form of the default cost function. This last result directly speaks to the financial accelerator literature (e.g., Bernanke and Gertler (1989); Bernanke, Gertler and Gilchrist (1999); Carlstrom and Fuerst (1997)). More generally, this analysis contributes to the literature on externalities in macro-finance models (see Davila and Korinek (2017) for a synthesis).[5]

The paper is organized as follows: I present and discuss the environment in Section 2. I establish the market failure and derive the optimal regulatory response in Section 3. I study the cyclical properties of the optimal capital requirement in Section 4; and I analyze and discuss the externalities in Section 5; I discuss the key ingredients of the model in Section 6; and then conclude.

2 The baseline model

There is an infinite number of periods indexed by $t = 0,1,2...$, in which a single consumption good is produced and used as the unit of account, and where generations of agents born at different dates overlap.

Agents All agents are risk neutral, live two periods, and derive utility from their end-of-life consumption. There is a measure 1 of agents born at the beginning of each period. They are endowed with one unit of labor, which they supply inelastically during the first period of their life. After having worked and received their wage, a measure $\eta \ll 1$ of these agents become endowed with banking ability, which enables them to set up a bank under the protection of limited liability. The remaining mass $1-\eta$ of agents become passive savers.

To transfer their wealth across periods, all agents have access to a safe storage technology and bank deposits.[6] The rate of return to storage is normalized to zero. I focus on cases where the storage technology is used in equilibrium, so that the return to storage pins down the expected return on deposits.[7] In addition, bankers can invest their wealth in their bank's equity.

[5] On fire-sales externalities and other amplification mechanisms, see for instance Shleifer and Vishny (1997), Gromb and Vayanos (2002), Krishnamurthy (2003), Lorenzoni (2008), Bianchi (2011), Jeanne and Korinek (2010), Goodhart et al. (2012), Gersbach and Rochet (2012), and Stein (2012), and on aggregate demand externalities see Farhi and Werning (2013), Rognlie, Shleifer and Simsek (2014), and Korinek and Simsek (2016).

[6] The economy can be considered as a small open economy with excess savings, facing the world interest rate.

[7] Alternatively, I could simply allow banks to raise deposits from the rest of the world.

Banks There is a continuum of banks that issue deposits to savers and lend competitively to firms. The rate of return to lending is denoted r. It is a random variable, whose distribution is determined in equilibrium, and taken as given by bankers. When a bank's proceeds from lending are not sufficient to repay its deposits, the bank defaults. In case of bank default, the government compensates depositors for any loss they made. Hence, deposits pay a zero interest rate. The government does not charge an insurance premium ex-ante but, when needed, levies lump-sum taxes on savers in order to break even in each period.

Bank default generates deadweight losses, which I model as a reduction in the value recouped by creditors. These deadweight losses can capture a number of situations (e.g. costly state verification, deadweight losses from taxation generated by an associated bailout, or other forms of spillovers to the real real economy). Throughout the paper, I will consider several loss functions and discuss their interpretations.

Firms In each period, there is a continuum of penniless firms, indexed by i, that operate a constant-return-to-scale production function. Firms competitively hire labor and borrow from banks to invest in capital. For simplicity, firms operate only one period and capital can costlessly be transformed one for one into consumption good, and vice versa. The production function takes the form $a_i k_i^{\alpha} n_i^{1-\alpha}$, where n_i denotes labor, k_i is physical capital ($0 < \alpha < 1$), and $a_i \in \{0,1\}$ is a random variable that captures firm specific productivity.

The realization of a_i is observed by firm i at no cost, but is costly to observe for the banks. As is standard, I assume that such verification costs materialize as a reduction in the liquidation value of the firm. As a result, the optimal contract is a debt contract, whereby the firm repays the principal plus a given interest rate r^l when $a_i = 1$, and enters bankruptcy when $a_i = 0$. To formalize bankruptcy costs, I follow Martinez-Miera and Suarez (2014) and assume that while capital depreciates at a rate δ when $a_i = 1$, it depreciates at a larger rate $\delta + \Delta$ when $a_i = 0$. Finally, I assume that default is independent across firms and denote $A \equiv E[a_i]$ the commonly known probability that a given firm succeeds.

Shocks To trigger fluctuations in this economy, I let A evolve according to an AR(1) process:

$$\ln A_t = (1 - \rho) \ln \bar{A} + \rho \ln A_{t-1} + \epsilon_t, \tag{1}$$

where ρ is a positive parameter capturing the persistence of the shocks, $\bar{A}$ is a positive constant, and ϵ_t represents an *iid* shock with mean zero and standard deviation σ. The realization of A_t is the event that defines the beginning of period t.

3 Market failure and constrained efficiency

3.1 Competitive equilibrium

The problem of the banker Bankers' relevant decisions are how to allocate their wealth between storage and bank equity and how much the bank lends, given its level of equity. This can be formalized as follows.

Consider a representative bank at date t, and denote by e_t its amount of equity. To lend an amount b_t, the bank needs to raise $b_t - e_t$ of deposits. Let v_{t+1} denote the ex-post net worth of the bank, i.e. its value after its return to lending r_{t+1} is realized. That is,

$$v_{t+1} \equiv b_t r_{t+1} + e_t - \Psi_{t+1}$$

where Ψ_{t+1} is the function that captures the deadweight losses from bank default.

Then, consider a representative banker born at date t that has earned a wage w_t. He solves:

$$\max_{e_t, b_t} E_t \left[c_{t+1} \right] \tag{2}$$

subject to the budget constraints and non-negativity conditions:

$$\begin{cases} e_t + s_t = w_t \\ c_{t+1} = v^+_{t+1} + s_t \\ e_t,\, b_t,\, s_t,\, c_{t+1} \geq 0 \end{cases} , \tag{3}$$

where c_{t+1} represent his consumption, s_t is the amount he stores from date t to date $t+1$, and v^+_{t+1} is the realized (private) value of his bank's equity, i.e. the positive part of v_{t+1}:

$$v^+_{t+1} \equiv \left[b_t r_{t+1} + e_t \right]^+ . \tag{4}$$

Note that Ψ_{t+1} does not appear in (4) because it is nil when $b_t r_{t+1} + e_t \geq 0$ (and positive otherwise).

Labor market Labor is hired at the beginning of the period. That is, after A_t is known, but before a_{it}'s are realized. The labor market is competitive. The expected wage is:[8]

$$w_t = (1-\alpha) A_t k_t^\alpha$$

[8] At failed firms, the realized wage is nil. This is unrealistic but, since agents are risk neutral, this is the expected wage that matters. An alternative would be to assume that firms are paid in advance of production (this would require the firm to borrow additional funds).

The return to lending At the end of a given period $t-1$, firms borrow competitively from banks to form capital that they will use for production in period t. Hence, investment takes place before A_t is known.

In a competitive equilibrium, the expected unit repayment to the bank equates the firms' expected marginal return to capital (accounting for bankruptcy cost). Since firms are penniless and protected by limited liability, there cannot be states in which they make strictly positive profits (otherwise they would make profits in expectation). As a result, in an optimal contract, the date-t repayment to the bank by a firm i that had borrowed k_{it} and hired n_{it} workers, must correspond to its realized net share of capital. That is:

$$\begin{cases} \alpha k_{it}^{\alpha} n_{it}^{1-\alpha} + (1-\delta) k_{it} & ;\ a_{it} = 1 \\ (1-\delta-\Delta)\, k_{it} & ;\ a_{it} = 0. \end{cases}$$

Date $t-1$ capital market clearing requires $k_t \equiv \int_i k_{it} = b_{t-1}$, and date t labor market clearing requires $n_t \equiv \int_i n_{it} = 1$. Given constant return to scale, all firms have the same capital to labor ratio in equilibrium. A standard debt contract with an interest rate $r_t^l \equiv \alpha k_t^{\alpha-1} - \delta$ is therefore optimal. Accordingly, the realized rate of return to lending for the bank corresponds to the average net marginal return to capital. That is:

$$r(A_t, k_t) = \alpha A_t k_t^{\alpha-1} - (\delta + (1-A_t)\Delta).$$

Competitive equilibrium definition Given a sequence for the random variables $\{A_t\}_{t=0}^{\infty}$, and initial condition k_0, a competitive equilibrium is a sequence $\{w_t, r_t^l, e_t, b_t, \tau_t\}_{t=0}^{\infty}$, such that: vector $\{w_t, r_t^l\}$ clears the labor and capital markets at date t and $t-1$ respectively; vector $\{e_t, b_t\}$ solves the maximization problem of the representative banker born at date t and τ_t is a lump-sum tax on savers such that the regulator breaks even at all t.

3.2 Efficiency analysis

Investment efficiency Economic surplus in this economy corresponds to output net of depreciation and bank default costs. Formally economic surplus at date $t+1$ is given by:

$$S(A_{t+1}, k_{t+1}, e_t) \equiv A_{t+1} k_{t+1}^{\alpha} - (\delta + (1-A_{t+1})\Delta))\, k_{t+1} - \Psi(A_{t+1}, k_{t+1}, e_t). \tag{5}$$

Investment efficiency requires to maximize expected economic surplus. Define:

$$k_{t+1}^{*} \equiv \arg\max_{k_{t+1}} E_t\left[S(A_{t+1}, k_{t+1}, e_t)\right] \tag{6}$$

Definition 1. Investment at date t is efficient if and only if $k_{t+1} = k_{t+1}^{*}$.

I can now compare the competitive equilibrium investment level, which I denote k_{t+1}^{CE}, with this efficiency benchmark (which I discuss at the end of this section).

Proposition 1. *At all dates, the competitive equilibrium capital stock is inefficiently high. That is:* $k^{CE}_{t+1} > k^*_{t+1}, \forall t$.

Proof. All the proofs are in Appendix A. $\square$

Because of deposit insurance, banks do not fully internalize the losses that occur in bad states. This reflects an implicit subsidy. Banks compete for lending to firms and pass this subsidy onto the firms in a general equilibrium. As a result, firms over invest. In this context, mispriced deposit insurance is a sufficient ingredient to obtain over-investment, but it is not necessary. This is because credit expansion by a bank also increases the probability of default of other banks and the losses they make in such an event. This externality also operates through a general equilibrium effect and also leads to over-investment, even in the absence of deposit insurance. I postpone the detailed study of these externalities to Section 5 to directly turn to the main object of this analysis: the cyclical properties of the optimal regulatory response.

3.3 Regulatory response and constrained equilibrium

The regulator I study the problem of a regulator, whose mission is to restore investment efficiency. The regulatory tool is a time-varying capital requirement $x_t \in [0,1]$ that constrains banks' lending to a multiple of their equity.

$$e_t \geq x_t b_t. \tag{7}$$

Henceforth, I refer to e_t as *bank capital*.

Constrained equilibrium A *constrained equilibrium* is defined as a straightforward extension of the competitive equilibrium. Given the same sequence of random variables and initial condition, it is defined as a sequence of capital requirements $\{x_t\}_{t=0}^{\infty}$ and a vector sequence $\{w_t, r^l_t, e_t, b_t, \tau_t\}_{t=0}^{\infty}$ satisfying the same conditions, with the only difference that $\{e_t, b_t\}$ must solve the problem of the representative banker born at *t subject to the capital requirement* x_t.

The optimal capital requirement I restrict my analysis to the interesting case where bank capital is *scarce* (i.e. $\eta_t w_t < k^*_{t+1}$) at all dates t. If bank capital was plentiful, the optimal capital requirement would be $x_t = 1$, and banks would be fully funded with equity and could not default in equilibrium.[9]

[9]See S. Hanson, A. Kashyap and J. Stein (2011), Stein (2012), and Admati et al. (2010) for discussions on why bank capital is scarce in reality.

Proposition 2. *The regulator can ensure investment efficiency, at all t, with the following capital requirement:*

$$x_t^* = \frac{\eta_t w_t}{k_{t+1}^*}. \tag{8}$$

Corollary 1. *(Equilibrium characterization) In a constrained equilibrium, the capital requirement is binding,* $e_t = \eta_t w_t$*,* $s_t = 0$*,* $b_t = k_{t+1}^*$*, and the equilibrium value of* w_t *and* r_t^l *are pinned down by their respective market clearing conditions.*

Dynamics Since I will focus on lending behavior, it is useful to define $b_t^* \equiv k_{t+1}^*$ as the regulator's desired level of aggregate lending and to express it as a function of A_t and e_t. Such a function is implicitly defined by the regulator's first order condition associated to (6) and the law of motion for A_t (equation 1). Then, the following system of differential equations:

$$\begin{cases} \ln A_t & = (1-\rho)\ln \bar{A} + \rho \ln A_{t-1} + \epsilon_t \\ e_t & = \eta(1-\alpha) A_t \left(b_{t-1}^*\right)^{\alpha} \\ b_t^* & = b_t^*\left(A_t, e_t\right) \end{cases} \tag{9}$$

fully captures the dynamics of the model.[10] The optimal capital requirement can be rewritten as:

$$x_t^* = \frac{e_t}{b_t^*}, \tag{10}$$

which makes clear that its dynamic properties are pined down by the joint dynamics of e_t and b_t^*.

Discussion of the regulatory objective This economy has heterogeneous agents (savers and bankers, of different generations). The choice of a welfare criterion is therefore not trivial.

My approach can be interpreted in terms of constrained efficiency. This is in the following sense. The bank default costs in the model are meant to capture the deadweight losses associated with an unmodeled agency problem between the bank manager-shareholder and its creditors. The scarcity of bankers' wealth implies that the first best allocation cannot be reached. As it is defined, b_t^* can therefore be interpreted as the second-best level of aggregate lending.

In a constrained efficiency (or second-best) problem, the regulator faces the same constraints as the private agents. In this spirit, I want to avoid arbitrary ex-ante transfers to bankers that would alleviate the deadweight losses. It is therefore more appropriate to consider a regulator that maximizes expected net economic surplus at each date rather than one that maximizes an arbitrary intertemporal welfare function.[11] Finally, note that given the welfare criterion,

[10] The second equation obtains by substituting the labor and capital market clearing conditions in the expression for e_t from Corollary 1.

[11] A simple example can illustrate why. Imagine that the regulator would like to transfer wealth from current generation to the next period's banker. He may then be tempted to allow for over-investment today, as a larger

Proposition 2 establishes that focusing on capital requirements is not restrictive since it allows to restore efficiency at all dates.

4 Cyclical properties of the optimal capital requirement

In this Section I study the dynamic properties of x_t^*. First, I identify a key general equilibrium effect that drives the contemporaneous effect of e_t on x_t^*. Then, I use two simple examples to identify and disentangle the main mechanisms by which productivity shocks propagate over time. Then, I calibrate and solve numerically the general model to assess the quantitative relevance of these mechanisms.

Default cost function For this part of the analysis, I consider a simple default cost function where deadweight losses are proportional to the representative bank's financial distress. That is, they are proportional to the extent to which the net worth of the bank is negative. Formally:

$$\Psi_{t+1} \equiv \gamma \left[-\left(b_t r_{t+1} + e_t\right)\right]^+, \tag{11}$$

where γ is a positive parameter, operator $[\,.\,]^+$ selects the positive part of the argument, and $b_t r_{t+1} + e_t$ the realized net worth. Hence, by construction, the costs only occur when net worth is negative, that is, when the bank defaults.

There are several possible interpretations for such a specification. For now, to fix ideas, it is useful to think of deadweight losses as emanating from the bank resolution process, which is arguably longer and more complicated when financial distress is high. Besides being intuitively appealing, default cost function (11) is analytically convenient (Townsend, 1979). However, the main results do not hinge on this specification (see Section 5 for alternative specifications and a full discussion).

4.1 The general equilibrium effect

A key contribution of this paper is to highlight a simple, but nonetheless important, general equilibrium effect. To isolate this effect, it is useful to consider exogenous shocks to aggregate bank capital. To do so, I fix A_t and express x_t^* as a function of e_t:

$$x_t^*(e_t) = \frac{e_t}{b_t^*(e_t)}.$$

It is clear that $x_t^*(e_t)$ is an increasing function unless b_t^* increases more than proportionally with e_t. Denoting κ_t the point elasticity of b_t^* with respect to e_t, I can state the following result.

capital stock will boost tomorrow's wage and, therefore, the wealth of future bankers. This involves negative net-present-value investment, but the regulator could still find it desirable given its objective functions and the constraints it faces.

Proposition 3. $\forall t,\ \kappa_t < 1$. *Hence the optimal capital requirement is increasing in aggregate bank capital.*

More bank capital means that the banking sector can absorb more losses, which decreases expected default costs. Hence κ_t is positive.[12] To see why it is smaller than1, first consider an atomistic bank that doubles its equity. It should simply be allowed to double lending (i.e. a proportional increase). However, if all banks in the economy double their equity, and if they are allowed to double lending, this could double investment in the economy. Given diminishing returns to physical capital this cannot be optimal.

In other words, in a partial equilibrium, an increase in bank capital does not affect the capital requirement but, in general equilibrium, it does because diminishing returns to physical capital translate into diminishing returns to lending.

The source of economic fluctuations in this model is shocks to A_t. One of the novelties of this paper is to study their impact on x_t^* through their effect on e_t. I call this propagation mechanism the aggregate bank capital channel, or simply the *bank capital channel*.

4.2 The main propagation mechanisms

The bank capital channel is only one of the propagation mechanisms in this model. In order to understand how it operates and interacts with other channels, it is useful, for now, to reason in terms of elasticities and/or deviations from a steady state.

Risky steady state Without risk in the economy, there would be no need for a capital requirement (the competitive equilibrium would be efficient). To define a meaningful starting point from where to study fluctuations, I use the simple notion of risky steady state (Coeurdacier, Rey and Winant, 2011). Henceforth, I use the subscript ss to denote variables at the risky steady state, which I will simply refer to as the steady state. Formally, it is defined by:

$$\begin{cases} e_{ss} = & \eta(1-\alpha)\overline{A}\left(b_{ss}^*\right)^{\alpha} \\ b_{ss}^* = & b_{ss}^*\left(\overline{A}, e_{ss}\right). \end{cases}$$

Intuitively, this corresponds to the regulator choosing the optimal policy given the risks facing the economy, but the risks never realizing. Solving numerically for the steady state is straightforward. One can also interpret this state as that the economy would reach under the optimal capital requirement, and after a sufficiently long series of neutral shocks $\epsilon_t = 0$.

[12] A general insight of the macrofinance literature is that an increase in aggregate net worth of financially constrained agents alleviates agency costs and increases the second best level of investment.

Elasticities Defining a_t as the log-deviation of A_t from $\overline{A}$, we have that if the economy is at steady state at date 0:

$$a_t = \sum_{s=1}^{t} \rho^{t-1} \epsilon_t. \tag{12}$$

In Subsection 4.3, I analyze the model globally. But for now, let me focus on system (9) linearized around the steady state. Together with (12), we have:

$$\begin{cases} \boldsymbol{e}_t &= a_t + \alpha \boldsymbol{b}_{t-1} \\ \boldsymbol{b}_t &= \xi a_t + \kappa \boldsymbol{e}_t \\ \boldsymbol{x}_t &= \boldsymbol{e}_t - \boldsymbol{b}_t \end{cases} \tag{13}$$

where bold variables $\boldsymbol{e}_t$, $\boldsymbol{b}_t$, and $\boldsymbol{x}_t$ are the log-deviation from e_{ss}, b^*_{ss}, and x^*_{ss}. Finally, ξ and κ are the point elasticity of b^*_{ss} with respect to A_t and e_t. In the two examples that follow, I consider the effect of a unique initial productivity shock, starting from steady state. That is: $\epsilon_1 = 1$ and $\epsilon_{t\neq 1} = 0$.

4.2.1 The bank capital channel

Let me consider a first example where A_t are *iid* (i.e. $\rho = 0$). In this case, expected productivity is constant at all time. Hence, $\xi = 0$, and

$$\boldsymbol{x}_t = \boldsymbol{e}_t(1-\kappa).$$

The only effect at play is the bank capital channel identified above. In fact, here, ϵ_t can be interpreted as a financial shock, in the sense that it affects the wealth that flows into aggregate bank capital, which impacts banking sector's intermediation capacity, without having other effects on k^*_{t+1}.

From Proposition 3, we know $\kappa < 1$. Hence, the immediate response to the shock is to tighten the capital requirement:

$$\boldsymbol{x}_1 = (1-\kappa).$$

In subsequent periods, there is a feedback effect as $\boldsymbol{e}_t$ increases with $\boldsymbol{b}_{t-1}$ (more lending means more capital, which raises the equilibrium wage and, therefore, future bank capital). Iterating forward, I can solve system (13) for $\boldsymbol{x}_t$:

$$\boldsymbol{x}_t = (\alpha\kappa)^t \, (1-\kappa),$$

which shows that the response is positive at all t and allows me to unambiguously sign the bank capital channel: through it, a positive shock increases the optimal capital requirement.[13]

[13] If $\rho > 0$, the bank capital channel is reinforced because productivity is persistently raised. This boosts wages, which in turn increases future bank equity.

4.2.2 The loan default rate channel

In a second example, I assume away default costs in the banking sector ($\gamma = 0$). In this case economic surplus (5) no longer depends on e_t, and one can solve for b_t^* in closed form:

$$b_t^* = \left(\frac{\alpha E_t[A_{t+1}]}{\delta + (1 - E_t[A_{t+1}])\Delta} \right)^{\frac{1}{1-\alpha}}. \tag{14}$$

This expression makes clear that b_t^* is increasing in A_t (strictly when $\rho > 0$). The intuition is simple: a positive productivity shock raises expected productivity, which calls for more investment, which requires more lending.

In terms of elasticities, this means that $\xi > 0$. Looking at the second equation of system (13), we can see that this effect operates through the term ξa_t. Reinterpreting this channel from a bank lending standpoint, an increase in A_t decreases the loan default rate. This makes lending more profitable in expectation and calls for an expansion in aggregate lending. I refer to this mechanism as the loan default rate channel, or simply the *default rate channel*.

4.2.3 Which channel is likely to dominate?

The bank capital channel and the default rate channel pull in opposite directions. To get a first sense of their relative importance, I assume further that $\Delta = 0$. This is useful because equation (14) then boils down to an isoelastic function, which gives:

$$\xi = \frac{\rho}{1-\alpha}.$$

In this case, the immediate response to the shock is:

$$\boldsymbol{x}_1 = 1 - \frac{\rho}{1-\alpha}.$$

This means that $\boldsymbol{x}_1 < 0$, unless $\rho < 1 - \alpha$. Hence, unless the shock has relatively low persistence, the immediate optimal response to a positive shock is to *loosen* the requirement.

Feedback effect However, any increase in b_t^* raises k_{t+1}, which boosts wages and, therefore, increases e_{t+1}. Looking at the second equation of system (13), we can see that this effect operates through the term $\alpha \boldsymbol{b}_{t-1}$. Since $\gamma = 0$ implies $\kappa = 0$, we get:

$$\boldsymbol{x}_t = \underbrace{a_t + \underbrace{\xi \alpha a_{t-1}}_{\text{feedback effect}}}_{\text{bank capital channel}} - \underbrace{\xi a_t}_{\text{default rate channel}},$$

Hence:

$$\boldsymbol{x}_{t>1} = \rho^{t-1} \left(\frac{1-\rho}{1-\alpha} \right) > 0,$$

which means that the feedback effect is sufficient to ensure that $\boldsymbol{x}_{t>1}$ is always positive. I collect these results in a formal proposition.

Proposition 4. *Assume default is costless ($\gamma = 0$ and $\Delta = 0$). The bank capital channel dominates the default rate channel at any positive lag. In the period where the shock occurs, this is true if the shock is not too persistent. Formally, following a date-1 positive productivity shock, $E_1[\boldsymbol{x}_{t>1}] > 0$, and $\boldsymbol{x}_1 > 0 \Leftrightarrow \rho < 1 - \alpha$.*

In sum, in this example, aggregate bank capital is more procyclical than the efficient level of the capital stock. This suggests that the optimal capital requirement *increases during booms*. Since the positive effect comes from the bank capital channel, this also suggests that the general equilibrium that drives it could be quantitatively important.

4.3 Intertwined business and financial cycles

In the subsection above, I have highlighted the main propagation mechanisms. An increase in productivity raises x_t^* through the bank capital channel, but it decreases it through the default rate channels. In the example above (where $\gamma = \Delta = 0$), the former effect dominate. In this subsection, I assess the quantitative relevance of the bank capital channel for the dynamics of the general model. That is, I now fully allow for feedback effects and non-linearities.[14] I do this in three steps.

First, I solve the model numerically to generate impulse response functions for x_t^*, e_t, and b_t^*. Second, I simulate the economy under x_t^*, and compare the outcome to an economy facing an identical series of shocks but using a different policy function $x_t^{'}$, which takes into account the loan default rate channel but not the bank capital channel. I interpret the difference in economic surplus as a measure of the welfare cost of ignoring the general equilibrium effect. Third, I add to the model a reduced-form version of Basel II's risk-weights and, in the spirit of Basel III, I use my framework to compute back-of-the-envelope estimates for optimal time-varying macroprudential capital buffers.

4.3.1 Methodology

Additional ingredient In real life, retained profits are an important source of capital for banks. The two-period overlapping generation structure of the baseline model rules out profit retention. This is a useful assumption because it allows for transparent analytical results. But since studying the dynamics of x^* in the general case already requires using numerical methods, I enrich the model in order to allow for such a source of bank capital.

[14] Compared to the case in Proposition 3, we get at least three additional effects in the general model. First, $\kappa > 0$ decreases the strength of the bank capital channel. Second, $\Delta > 0$ affects ξ. And third, there are non-linearities (for instance due to the truncation implied by limited liability).

To do so, I use a common modelling trick and impose an exogenous exit rate for banks (see the discussion in Suarez, 2010). That is, I assume that bankers face a constant probability to die λ, which yields the following condition:

$$e_{t+1} \leq \eta w_{t+1} + (1-\lambda) \underbrace{[b_t r_{t+1} + e_t]^+}_{\text{realized net worth}}, \tag{15}$$

where $b_t r_{t+1}$ captures the (potentially negative) profit from lending. The case where $\lambda = 1$ corresponds to the baseline model.

I maintain the assumption that aggregate bank capital is scarce. This is the case in all the simulations I present in this section. As a result, under the optimal capital requirement, surviving bankers always find it optimal to invest all their wealth in bank equity. This means that condition (15) holds with equality and defines the law of motion for aggregate bank capital.

Calibration strategy I interpret one period in the model as 1 year. I choose standard values for the basic technology parameters: $\alpha = 0.35$ and $\delta = 0.05$. The choice of a value for Δ, the extra depreciation of capital due to default, is less obvious as previous studies have used a relatively wide range of values. As a baseline, I follow Repullo and Suarez (2013) and Martinez-Miera and Suarez (2014) and set $\Delta = 0.40$. Their rationale for such a value is that it generates realistic loan losses given default (under Basel II standardized approach, the loss given default for non-rated corporate exposure is 45%).[15]

The exogenous random variable is A_t, which I express here in terms of loan default rate: $D_t \equiv 1 - A_t$. The relevant parameters for its distribution are the constant $\overline{A}$, and the standard deviation and persistence of the shocks, σ and ρ. I calibrate these values to match the average (3.9%), the standard deviation (1.9%), and the (annual) autocorrelation (0.83) of the US bank loan delinquency rate from 1985 to 2016.[16]

This leaves me with three free parameters: λ, η, and γ. The first two are driving aggregate bank capital accumulation. Parameter λ can naturally be linked to the shareholder payout ratio. I choose $\lambda = 0.064$, which is the value induced by a payout ratio of 60% ratio for financial firms (Floyd, Li and Skinner 2015) on a 12% return on equity.[17] Given this value for λ, I pick the value of η to target a bank capital ratio of 4% in the risky steady state, which is in line with

[15] In my model, the loss given default on a loan (in percent) is equal to $\delta + \Delta$, hence the value for Δ. The main alternative approach is to target estimates of bankruptcy costs. For instance, Carlstrom and Fuerst 1997 argue that a reasonable range for estimates that include both the direct and indirect costs is [0.2 to 0.36]. They use a value of 0.25. See Subsection 4.4 for a discussion on how this choice and that of other parameter values affect the results.

[16] Specifically, I use the end of Q4 value of the Delinquency Rate on All Loans, Top 100 Banks Ranked by Assets [DRALT100N], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/DRALT100N, April 3, 2017. I calibrate the parameters to match the moments on a simulated series of 1,000,000 shocks.

[17] Formally, $\lambda = (\text{Payout Ratio}) \times \text{ROE}/(1 + \text{ROE})$. Note that studies adopting a comparable modelling approach typically use higher values for the corresponding parameter, which they link to average survival time for firms or banks (e.g, the exit rate is 0.1 in Gertler and Kiyotaki (2010) and 0.11 in Bernanke, Gertler and Gilchrist, 1999, and 0.2 in Martinez-Miera and Suarez (2014)).

average leverage ratios for banks in the decade before the global financial crisis (Adrian and Shin, 2010). Note that given a typical average risk-weight below 50%, this allows to satisfy the Basel II risked weighted capital ratio of 8%. The last parameter (γ) is specific to my model and captures the intensity of bank default costs. In the absence of empirical estimates, I consider several values, ranging from 0 to 4. I use $\gamma = 2$ as a baseline, which yields a simulated deadweight loss of 5% of steady state GDP, on average, when the representative bank defaults. Table 1 summarizes my calibration strategy.

Table 1: Calibration

		Value	Target
Concavity of the production function	α	0.35	Capital share of output
Baseline depreciation	δ	0.05	Physical capital depreciation
Extra-depreciation in default	Δ	0.40	Loss given default on bank loans
Default rate parameters			**Loan delinquency rate (US)**
Expected success rate	$\overline{A}$	0.9575	Average (1985-2016): 3.9%
Standard deviation of the shock ϵ	σ	0.26	St. dev. (1985-2016): 1.9%
Persistence of the shock ϵ	ρ	0.83	Autocorr. (1985-2016): 0.83
Banker exit rate	λ	0.064	60% payout rate on 12% ROE
Measure of bankers	η	0.0212	4% capital ratio (8% of RWA)
Bank default cost	γ	2	I also consider $\gamma = \{0, \frac{1}{2}, 4\}$

Note: The default rate parameters are such that the moments of a simulated series of 1,000,000 shocks match those of the loan delinquency rates.

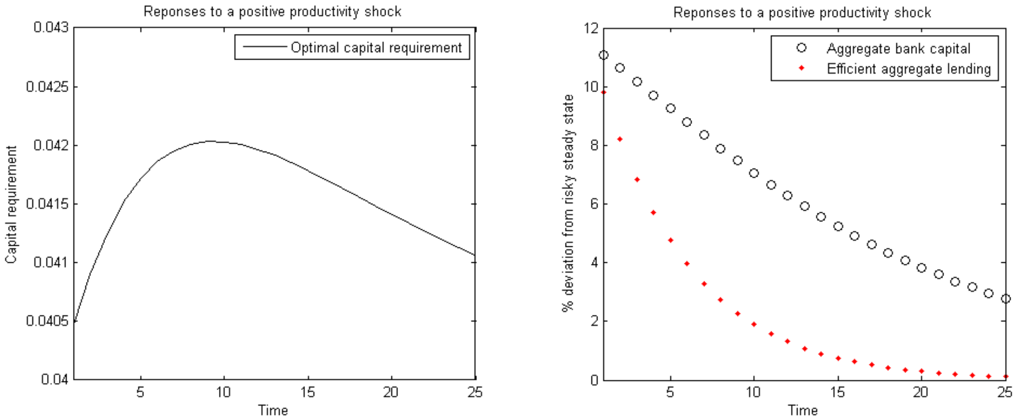
4.3.2 The cyclical properties of the optimal requirement

Figure 1 shows the main impulse response functions for the baseline calibration. Starting from steady state, it depicts the effects of a positive productivity shock that decreases the loan default rate by 1% (from 4.25% to 3.25%). The left panel displays x_t^*, the response of the optimal capital requirement. It is initially positive (by around 5bps) and reaches 20bps after 8 periods before decaying.

The right panel shows the response of aggregate bank capital and of the efficient level of aggregate lending. These responses are expressed in percentage deviation from steady state. They are therefore the non-linear counterparts of $\boldsymbol{e}_t$ and $\boldsymbol{b}_t$. With a slight abuse of notation, I will use the same bold letters to refer to these response functions. To link the two panels, note that $x_t^* - x_{ss} \simeq (\boldsymbol{e}_t - \boldsymbol{b}_t)x_{ss}$, where $x_{ss} = 0.04$. Both responses are positive and decay over time. Note that $\boldsymbol{e}_1$ is only slightly larger than $\boldsymbol{b}_1$, but the latter decays at a faster pace, which explains the hump shape of x_t^*.

The key takeaway from Figure 1 is that it confirms the analytical insights above. In particular: (i) the capital requirement is increasing in aggregate bank capital (as in Proposition 3); and (ii) bank capital is more procyclical than the desired, efficient level of aggregate lending, so that the optimal response to a positive productivity shocks is to tighten the capital requirement

Figure 1: Impulse responses



This figure presents results for the baseline calibration (see Table 1). The left panel shows the path for the optimal capital requirement x_t^*, starting from steady state and following a 1% decrease in the loan default rate. The right panel decomposes the response in two parts by showing the impulse responses for aggregate bank capital (black circles) and the efficient level of aggregate lending (red dots), expressed in percent deviation from their steady state values.

Table 2: Simulation results (baseline calibration)

Mean	x_t^*	$x_t^{'}$
Unconditional	4%	3.9%
In good time	4.4%	3.2%
In bad times	2.9%	5.4%

This table compares simulated moments under the optimal capital requirement (x^*) and the alternative policy ($x^{'}$) defined by equation (16). Both simulations use baseline calibration parameters (Table 1), start at steady state, and follow the exact same sequence of 10,000 random shocks. Good times (bad times) are defined as periods where the loan default rate is at least one standard deviation below (above) the sample mean.

(as in Proposition 4).

These results, and their robustness (see Subsection 4.4), provide support for the quantitative relevance of the bank capital channel and, therefore, of the general equilibrium effect that drives it.

4.3.3 Good times versus bad times

Figure 1 focuses on fluctuations close to steady state. A simple way to confirm that the main insights hold globally is to simulate a full series of shocks and compare the simulated optimal requirement in “good times” versus “bad times”. I define good times (bad times) as periods in which the loan default rate is at least one standard deviation below (above) its mean. Table 2 displays the simulation results. As we can see, x_t^* is indeed tighter in good times (4.4%) than in bad times (2.9%).

An alternative policy To get a sense of the role of the bank capital channel for the results above, it is useful to define an alternative, suboptimal policy for the the capital requirement.

What I want to capture is a policy that takes into account the default rate channel, but ignores the bank capital channel. I model such a policy as follows:

$$x_t' \equiv \frac{e_{ss}}{b_t'}, \tag{16}$$

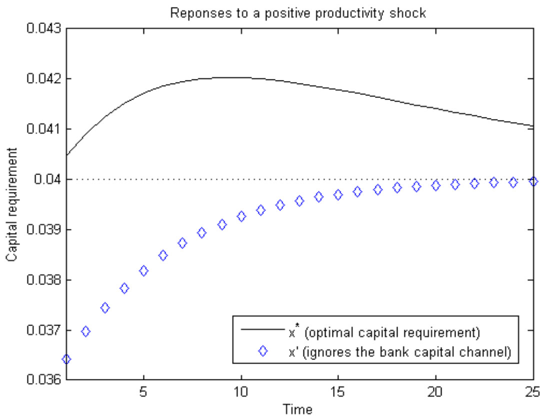
where

$$b_t' \equiv \arg\max_{b_t} E_t\left[S(A_{t+1}, b_t, e_{ss})\right].$$

That is, b_t' corresponds to the efficient level of aggregate lending if $e_t = e_{ss}$. This policy ignores the bank capital channel in the sense that it holds e_t constant to its steady state level.[18] The linearized model of Subsection 4.2 provides a related interpretation: around the steady state $\boldsymbol{b}_t' \simeq \xi a_t$ and, therefore,

$$\boldsymbol{x}_t' \simeq \boldsymbol{x}_t - \boldsymbol{e}_t(1-\kappa).$$

Figure 2: Alternative capital requirement policy



This figure presents results for the baseline calibration (see Table 1). The solid black line corresponds the the left panel of Figure 1. It is the path for the optimal capital requirement x_t^*, starting from steady state and following a 1% decrease in the loan default rate. The sequence of blue diamonds depicts the path of the capital requirement policy that ignores the bank capital channel (see equation 16).

I compare the response to a positive productivity shock under this policy to the optimal response in Figure 2. Ignoring the bank capital channel leads to a loosening of the capital requirement instead of a tightening. The difference is initially around 40bps and then decays over time. Conversely, in case of a negative productivity shock, this policy results in excessively stringent capital requirements.

The last column of Table 2 shows that the same insights hold away from steady state: x_t' is looser in good times and tighter in bad times. As we will see, policy x_t' unnecessarily magnify business cycle fluctuations, which resonates with the critiques of the Basel II regulatory regime. Before exploring this further, let me discuss the quantitative role of bank default costs, and present further details on the simulation results.

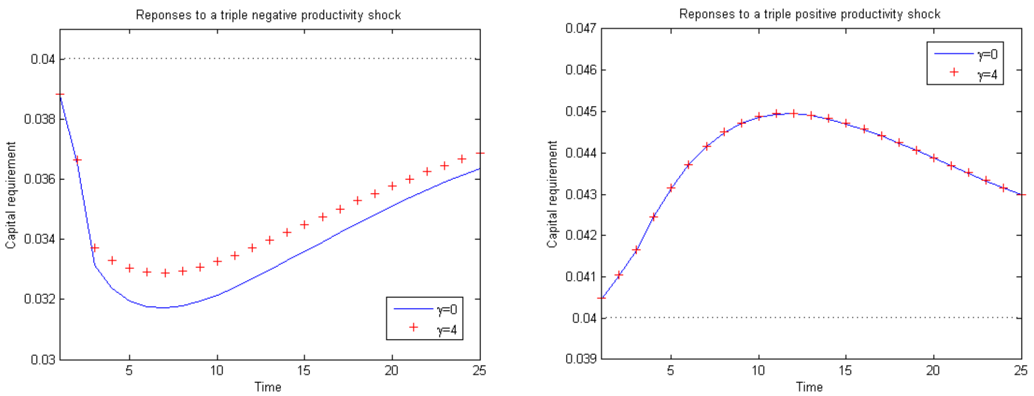
[18] Note that, by construction, policy x_t' also ignores the feedback effect (i.e. the dependence of $\boldsymbol{e}_t$ on $\boldsymbol{b}_{t-1}$).

Table 3: Simulation results (baseline calibration)

	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 2$	$\gamma = 4$
Freq. of large losses	17.5%	17.0%	16.4%	15.9%
in good time	7.5%	7.4%	7.1%	6.9%
in bad times	34.6%	31.8%	29.1%	26.9%
Probability of default	0.64%	0.29%	0.13%	0.08%

This table presents simulated moments for different values of γ. Other parameters are at their baseline values (Table 1). All simulations start at steady state, and follow the exact same sequence of 10,000 random shocks. Good times (bad times) are defined as periods where the loan default rate is at list one standard deviation below (above) the sample mean. Large losses correspond to losses larger than 10% of initial bank capital.

Figure 3: Larger firm default costs



This figure compares the response of the optimal capital requirement to three successive negative productivity shocks (left panel) and three negative ones (right panel), for $\gamma = 0$ (solid blue line) and $\gamma = 4$ (red crosses).

Bank default costs: they mostly matter in bad times Table 3 compares the realized frequencies of large losses and the average probability of default for several values of γ, under the optimal policy. As we can see, going from $\gamma = 0$ to $\gamma = 4$ decreases the probability of default from 0.64% to 0.08%. The frequencies of large losses (i.e. losses larger than 10% of the bank's equity) decrease as well, but not dramatically.

Varying γ in the considered range does not affect much the impulse responses corresponding to Figure to 1. But this is because they are triggered by a single and relatively small positive shock. Considering successive negative shocks gives a clearer role to γ. Figure 3 illustrates such an asymmetry. The left panel displays the response of x_t^* to three successive negative shocks (raising the loan default rate from 4.25% to 7.24%). We can see that the optimal cut in capital requirement is less aggressive when γ is higher. On the contrary, there is no discernible difference in the response to an equivalent series of three positive shocks (see the right panel).

Finally, one could wonder whether the specification for the default cost function strongly affects the impulse response functions. As I show in Appendix B, this is not the case: all the insights derived in this section also apply to the case where bank default costs are proportional to the bank's balance sheet; that is, under the specification $\Psi_{t+1} = \gamma b_t$.

4.3.4 Comparing policies: further simulation results

Table 5 in Appendix C displays a series of additional results for the comparison between x_t^* and $x_t^{'}$. Here are the main findings. First, I find that aggregate lending is more volatile under $x_t^{'}$. Interestingly, the frequency of large bank losses is slightly lower under this suboptimal policy (16% compared to 16.4%). However, distinguishing between good times and bad times gives a nuanced picture. In good times, large losses are more frequent under $x_t^{'}$ (12.1% compared to 7.1% under x_t^*), which is consistent with the popular notion that banks are piling up risks in good times.[19] In bad times, however, banks do not take enough risk under $x_t^{'}$ as large losses are much less frequent than under x_t^* (11.8% compared to 29.12%). This is because x_t^* optimally trades off the costs of risk taking with the benefits of allowing for positive net present value loans to be made. This also explains why the average probability of default is slightly higher under x_t^* .

The excessive risk taking in good times under $x_t^{'}$ materializes in two ways. First, banks do not fully internalize the social costs of of their credit expansion, which means that they lend too much (see Proposition 1 and Section 5 for further intuition). Second, they do not fully retain their equity.[20] This feature resonates with the well-documented dramatic increase in shareholder payouts by banks in the run-up to the global financial crisis (see, e.g., Acharya et al., 2011 and Floyd, Li and Skinner, 2015).

In the model, the relevant measure of welfare is economic surplus (as defined in Equation (5)). At steady state, it corresponds to aggregate consumption. I find that the average welfare loss associated with $x_t^{'}$ amounts to 0.04% of steady state consumption. This number is in the ballpark of those typically associated with the welfare gains from traditional stabilization policy (see e.g. Lucas, 2003). My model is too stylized to take the results of this exercise at face value, but this simple metric is another hint of the bank capital channel quantitative relevance.

4.3.5 Basel II and the countercyclical buffers of Basel III

The model is useful to think about the so-called procyclical effects of Basel II and countercyclical buffers of Basel III.

A Basel II interpretation A key feature introduced by the Basel II regulatory regime is that bank assets are weighted for capital regulation purposes. The weights on individual loans depend on their probability of default; either directly (in the "Advanced" or "Internal Rating Based" approach) or through credit ratings (in the "Standardized" approach).

[19] See Borio and Drehmann (2009) and Boissay, Collard and Smets (2013) for instance.

[20] Under $x_t^{'}$, condition (15) may not be binding in equilibrium. This is in that case that banks pay extra dividends. Here, I allow surviving bankers to re-inject equity in the bank in the future, which somehow alleviates the credit crunch in bad times. In reality, however, banks are typically reluctant to raise capital in bad times, for instance because of stigma or debt overhang considerations. Such ingredients involve considerable technical challenges in a dynamic general equilibrium model (see Bahaj and Malherbe (2016)). I therefore leave this to future research.

These weights were designed to deal with cross-sectional differences in loan riskiness. However, because probabilities of default co-move over the business cycle, the average, or *effective*, risk weight of a given loan portfolio evolves over time. Namely, the effective risk weight goes up during a downturn, and down during a boom. In the context of my model, a simplified version of Basel II's capital requirement can be captured by the following constraint:

$$e_t \geq \overline{x}\omega_t b_t, \tag{17}$$

where $\overline{x}$ is a constant capital requirement (for instance 8% as in the Basel II regime), ω_t is the effective risk weight on the loan portfolio at date t and, as before, e_t and b_t are the representative bank's capital and lending level. In a more general set-up, the effective risk-weight would be a function of both the portfolio composition (b_i's), and the associated probabilities of default (a_i's) and losses given default (which I could capture by indexing Δ_i). However, since only aggregate risk matters in my model, the only relevant variable here is the expected loan default rate. Accordingly, I define

$$\omega_t \equiv \omega\left(E_t[D_{t+1}]\right),$$

where $\omega(\,.\,)$ is an increasing function.

Macroprudential buffers Now, consider a macroprudential regulator that takes as given the rules set by a microprudential regulator. In particular, consider a capital constraint that corresponds to (17) except that the macroprudential regulator can set a time-varying adjustment factor z_t. That is, consider the capital requirement constraint:

$$e_t \geq z_t \overline{x}\omega_t b_t.$$

Assuming it binds in equilibrium (which implies $b_t = b_t^*$), efficiency requires:

$$z_t^* = \frac{e_t}{\overline{x}\omega_t b_t^*}. \tag{18}$$

Log-linearizing around the steady state gives:

$$\boldsymbol{z}_t = \boldsymbol{e}_t - \boldsymbol{b}_t - \boldsymbol{\omega}_t,$$

and the following expression for the optimal macroprudential buffer:

$$z_t^* \simeq \boldsymbol{z}_t \overline{x}.$$

Since ω increases with the loan default rate, the response of $\boldsymbol{\omega}_t$ to a positive productivity shock is negative. Hence, ceteris paribus, this loosens the capital constraint. From a conceptual point of view, this is not necessarily a bad thing because higher productivity calls for more investment, which requires more lending. In the context of the model, such a desired increase in lending due to the increase in credit quality can be captured by $\boldsymbol{b}_t'$, which leads to the following interpretation:

$$z_t \simeq \underbrace{\boldsymbol{e}_t(1-\kappa)}_{\text{bank capital}} - \underbrace{\boldsymbol{b}_t'}_{\text{credit quality}} - \underbrace{\omega_t}_{\text{risk weights}} .$$

As an example, assume that the effect of the change in risk-weights just generates the adjustment required by the change in credit quality. That is, assume $\boldsymbol{\omega}_t = -\boldsymbol{b}_t'$. Then, the only role for the macroprudential buffer is to account for the bank capital channel.

When changes in γ have little effect on $\boldsymbol{x}_t$ this means that $\kappa \simeq 0$. From the discussion above, we now that this is a good approximation in normal and good times. In that case, we have $\boldsymbol{z}_t \simeq \boldsymbol{e}_t$. The baseline impulse response (Figure 1) can then be used in a back-of-the-envelope estimation of the optimal macroprudential buffer. With this approach, the buffer that corresponds to a decrease of 1% in the loan default rate is approximately $11\% \times 8\% = 88$bps at $t = 0$, and then decays over time.

Now, it is widely accepted that the time varying effect of risk weights is excessive. There is no empirical consensus on an estimate for the elasticity of ω_t with respect to productivity, but a series of studies have attempted to measure such an effect. For instance, using a default probability model, Kashyap and Stein (2004) obtain an implied increase of 10% in risk weights from 1998 to 2002 (a period where the US economy contracted and where, according to the data I use for the calibration, there was an increase in a bit less than one percent in delinquency rates).[21]

Assuming $\boldsymbol{\omega}_4 = -10\%$ and, according to the baseline impulse responses, $\boldsymbol{e}_4 = 9.7\%$ and $\boldsymbol{b}_4 = 5.7\%$, yields

$$z_4 \simeq \underbrace{9.7\%}_{\text{bank capital}} - \underbrace{5.7\%}_{\text{credit quality}} + \underbrace{10\%}_{\text{risk weights}} = 14\%,$$

which gives a macroprudential buffer:

$$z_4^* = 14\% \times 8\% = 1.12\%$$

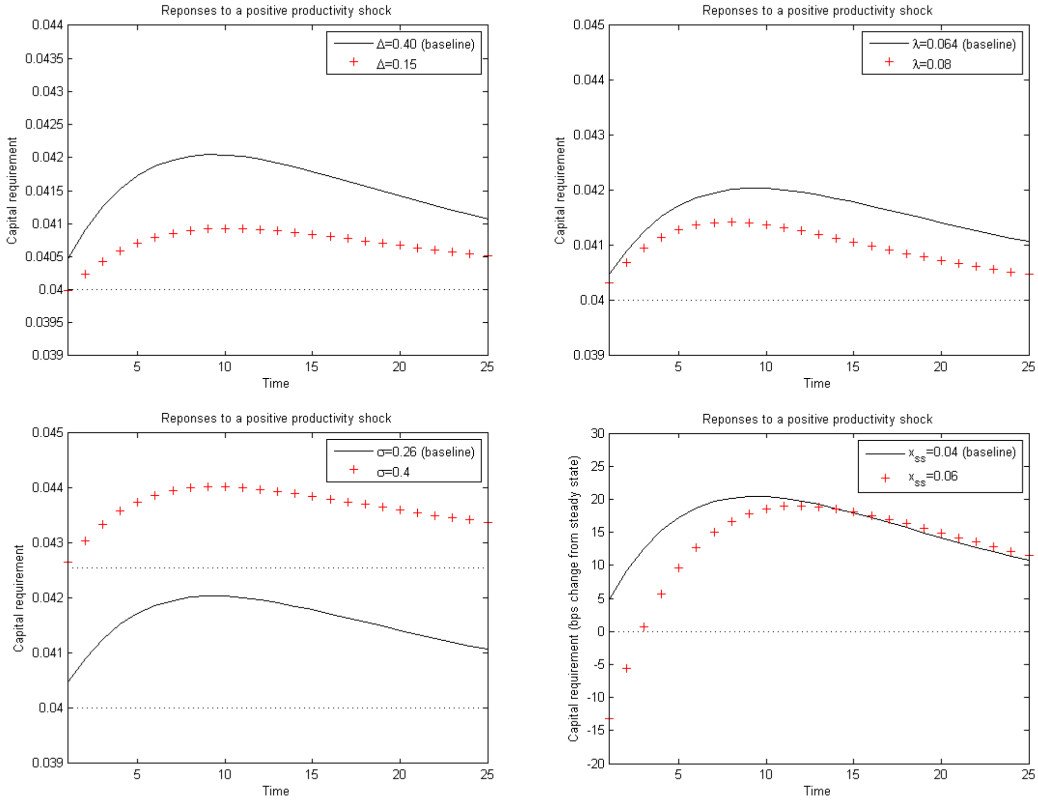
This exercise is very stylized, but it illustrates how the present framework can be used to think formally about the cyclical effects of risk-weights and the so-called countercyclical buffers introduced by Basel III.

4.4 Sensitivity analysis

I have explored the parameter space beyond the set of values of my baseline calibration. The general conclusion from my numerical analysis is that the shape of the impulse response functions and, to some extent, their magnitude are quite robust to alternative parametrizations.

[21] Other approaches typically give higher numbers. Kashyap and Stein (2004) propose several approaches and discuss other estimates found in the literature (see, e.g., Catarineu-Rabell, Jackson and Tsomocos (2005)).

Figure 4: Parameter sensitivity



This figure illustrates how the impulse response for x_t^* is affected by a change of value for parameters Δ, λ, σ, and η. In all but the bottom-right panels, the solid line depicts x_t^* in the baseline calibration (as in Figure 1), and the sequences of red crosses depicts x_t^* for an alternative value of the considered parameter. The bottom right panel compares the impulse response for two different steady-state capital ratio targets: 4% (baseline, solid line) and 6% (alternative, red crosses). The alternative target is achieved by adjusting parameter η. The responses are normalized at their corresponding steady state level.

Figure 4 illustrates how changes in value of non-standard parameters values affect the impulse response for x_t^*. In each cases, we can see that the change tends to dampen the response function, but it remains positive and hump shaped. The top-left panel consider $\Delta = 0.15$, which is much lower than the baseline ($\Delta = 0.4$) and in the lower part of the range used in the literature.

The top-right panel considers a higher value for λ (0.08 instead of 0.064). According to my shareholder payout approach, this could either correspond to a steady state ROE of 15.5% (keeping the shareholder payout ratio constant at 60%) or to a steady-state shareholder payout ratio of 75% (keeping the ROE constant at 12%). Raising the value of λ further continues to dampen the response, but starts to involve steady-state values for the shareholder payout ratio and the ROE that are arguably not reasonable.[22]

The bottom-left panel considers an increase in σ (the standard deviation of the shock). This has two effects. The first one is to raise the steady-state optimal requirement. The second is to slightly dampen the response.

Finally, the bottom-right panel displays the effect of a change in the target for the steady

[22] At values above 0.25 the response is small (a couple of basis points) and starts in negative territory.

state capital ratio (it keeps λ constant and lets η adjust to meet the target). This substantially affects the initial response as it becomes negative. However, the response quickly becomes positive and converges to the baseline case. This pattern is very similar to that described in Proposition 4.

5 Dissecting the externalities

Default costs for banks play several roles in the model. As I have shown above, they asymmetrically affect the optimal response of the capital requirement, with the stronger effect taking place in bad times. But they also interact with the general equilibrium effect to contribute to the need for the capital requirement in the first place: as I show in this section, they typically generate externalities that lead to excessive lending in the competitive equilibrium.

5.1 Setting up the problem

Until now, I have assumed that deposits are guaranteed by the government. Not only because this is a relevant feature of the environment for real life bank, but also because it simplifies the exposition of the main results. In this section, I also consider the case where there are no government guarantees. When there are no government guarantees, if the bank defaults with positive probability in equilibrium, depositors must be compensated by a higher interest rate in the states where the bank stays solvent. I denote $\overline{r} \geq 0$ the interest paid on deposits ($\overline{r} = 0$ in the presence of government guarantees). For this part of the analysis, I focus on a single period and, therefore, omit time subscripts to improve readability.

The default threshold In the general case where $\overline{r} \geq 0$, the representative bank defaults if

$$(b-e)\left(1+\overline{r}\right) > b\left(1+r\right), \tag{19}$$

where the left-hand-side is the total promised repayment to depositors and the right-hand-side is the proceeds from lending.

The equilibrium return to lending is:

$$r(A,k) = \alpha A k^{\alpha-1} - \left(\delta + (1-A)\Delta\right).$$

Substituting in (19), I can solve for the threshold realization for A below which the representative bank defaults:

$$A_0 \equiv \frac{\left(\delta+\Delta+\overline{r}\right)b - e(1+\overline{r})}{\left(\alpha k^{\alpha-1}+\Delta\right)b} \tag{20}$$

The depositors' break even condition The break even condition for the depositors reads:

$$\int_{A_0}^{\infty}(1+\overline{r})f(A)dA+\int_{0}^{A_0}\frac{b(1+r(A))-\Psi(A,\overline{r},b,e,k)}{b-e}f(A)dA=0. \tag{21}$$

The first integral correspond to the states where the bank is solvent and can repay the deposits in full, including the promised interests. The second term, where I now consider a general default cost function $\Psi(A,\overline{r},b,e,k)$, captures what depositors recoup in the default states.

Condition (21), equation (20), and cost function $\Psi(A,\overline{r},b,e,k)$ jointly define an implicit function $\overline{r}(b,k,e)$.[23] Note that if $\Psi(A,\overline{r},b,e,k)=0$, we have:

$$\begin{cases}\frac{\partial\overline{r}}{\partial b}>0\\ \frac{\partial\overline{r}}{\partial e}<0\\ \frac{\partial\overline{r}}{\partial k}>0.\end{cases} \tag{22}$$

The first two inequalities simply reflect that, ceteris paribus, the interest rate required by depositor increases with leverage. The third one, reflects that an increase in aggregate lending reduces the marginal return to lending and, therefore, increases the probability of default (A_0 increases) and decreases the value recovered by depositors in the default states. For what follows, I assume for simplicity that $\Psi(A,\overline{r},b,e,k)$ is such that $\overline{r}(b,e,k)$ is well-behaved. That is, I assume that conditions (22) are satisfied.

The planner's first order condition The planner's problem is

$$\max_{b\geq 0} E[A]b^{\alpha}-(\delta+(1-E[A])\Delta))\,b-\int_{0}^{A_0}\Psi(A,\overline{r},b,e,b)f(A)dA.$$

Assuming the relevant derivative exist, the associated first order condition can be written:

$$\alpha E[A]b^{\alpha-1}-(\delta+(1-E[A])\Delta))-\underbrace{\frac{\partial A_0}{\partial b}f(A_0)}_{\text{ext. margin}}\Psi(A_0)-\int_{0}^{A_0}\underbrace{\frac{\partial\Psi}{\partial b}}_{\text{int. margin}}f(A)dA=0 \tag{23}$$

The first two terms capture the marginal productivity of capital, net of depreciation. The sum of the two other terms captures the marginal effect of b on the expected default cost. The first of the two is the effect on the extensive margin. That is, *the effect on the probability of default*. The second is the effect on the intensive margin. That is, *the effect on the cost given default*.

[23] I assume that such a function exists and is unique. That is, I ignore the cases where economic conditions are so bad that the break-even condition cannot be satisfied. In case there are several values of $\overline{r}$ that satisfy the break-even condition, it is natural to select the the smallest value.

5.2 Externalities

The crux of the matter is that the representative banker, contrarily to the planner, does not take general equilibrium effects into account. In particular, the banker takes k as given, whereas the planner takes into account that $k = b$. As I will now show, this generates inefficiencies through externalities that can operate both via the probability of default (because k affects A_0) and via the costs given default (because k can also affect Ψ).

I first study these two classes of externalities in their general forms and then provide examples on how they play out in equilibrium depending on the specific form of the default cost function.

Probability of default externalities Assume that $\Psi(A_0)f(A_0) \neq 0$, and note the relevant indirect dependence of A_0 on b:

$$\begin{cases} A_0(b,\overline{r},e,k) \equiv \frac{(\delta+\Delta+\overline{r})b-e(1+\overline{r})}{(\alpha K^{\alpha-1}+\Delta)b} \\ \overline{r} = \overline{r}(b,e,k) \\ k = b \end{cases}$$

Accordingly, the total derivative of A_0 with respect to b is:

$$\frac{dA_0(b,\overline{r},e,k)}{db} = \frac{\partial A_0}{\partial b} + \frac{\partial A_0}{\partial \overline{r}}\frac{\partial \overline{r}}{\partial b} + \underbrace{\frac{\partial A_0}{\partial \overline{r}}\frac{\partial \overline{r}}{\partial k}\frac{dk}{db} + \frac{\partial A_0}{\partial k}\frac{dk}{db}}_{\text{wedge}} \tag{24}$$

For what follows, I assume that both the problem of the planner and that of the representative banker are well behaved, in the sense that their optimum is pinned down by their respective first order conditions.[24] Comparing these, it is straightforward to show that: (i) the first two terms in the right-hand-side of (24) appear in both. This is because the banker internalizes the effect of an increase in lending on his own probability of default and, given the distribution for the return to lending, on the interest rate required by the depositors; (ii) The last two terms in (24) only appear in the condition of the planner. This because $\frac{dk}{db} = 0$ from the banker's point of view.

Probability-of-default externalities can be signed: they contribute to over-lending. To see this, note that A_0 is increasing k. Hence, *when a bank increases its lending, this increases the probability of default for other banks*. But there also is a reinforcing indirect effect: because lending becomes less profitable at the margin, the value recovered by depositors in default states decreases, which requires a compensating increase in $\overline{r}$. In turn, a higher $\overline{r}$ also increases the probability of default. In the case where deposits are insured, since $\overline{r} = 0$, this indirect effect does not operate.

[24]The problem of the representative banker is defined as the straightforward extension of problem (2) to the case where $\overline{r} \geq 0$ in combination with the saver's break-even condition (21).

Cost given default externalities Following the same logic as above, consider the general cost function $\Psi\left(A,\bar{r},b,e,k\right)$ and note the relevant indirect dependence:

$$\begin{cases} \bar{r} = \bar{r}(b,e,k) \\ B = b \end{cases}$$

The total derivative is:

$$\frac{\mathrm{d}\Psi\left(A,b,\bar{r},e,k\right)}{\mathrm{d}b} = \frac{\partial\Psi}{\partial b} + \frac{\partial\Psi}{\partial\bar{r}}\frac{\partial\bar{r}}{\partial b} + \underbrace{\frac{\partial\Psi}{\partial\bar{r}}\frac{\partial\bar{r}}{\partial k}\frac{\mathrm{d}k}{\mathrm{d}b} + \frac{\partial\Psi}{\partial k}\frac{\mathrm{d}k}{\mathrm{d}b}}_{\text{wedge}} \tag{25}$$

The last two terms identify another wedge in the banker's first order condition. The last term is the direct effect, and the penultimate the indirect effect through the interest rate required by depositors. Again, only the direct effect would operate if deposits were insured.

Contrarily to above, cost given default externalities can not always be signed, as they depend on the specific form for Ψ. I illustrate this in what follows.

5.3 Examples

Specification 1: Financial distress In the case where $\bar{r} \geq 0$, the cost function used in the main analysis generalizes to:

$$\Psi_1 \equiv \gamma\left[(b-e)(1+\bar{r}(b,e,k)) - b(1+r(A,k))\right]^+.$$

First, by construction, we have $\Psi_1(A_0) = 0$. Hence, even though an increase in lending does increase the probability of default of other banks, this is not a source of inefficiency.[25] Second, from equation (25), lending expansion by a bank increases the costs given default of other banks if

$$\frac{\partial\Psi_1}{\partial k} + \frac{\partial\Psi_1}{\partial\bar{r}}\frac{\partial\bar{r}}{\partial k} \geq 0.$$

And this is the case: an increase in k decreases the marginal return to lending, which, ceteris paribus, increases default costs. Both directly (the first term is positive) and indirectly through an increase in the required interest rate on deposits (the second term is positive too). Hence, we get overlending (and therefore overinvestment) in the competitive equilibrium, even in the absence of government guarantees.[26]

[25] If the support for A is not continuous, then even though $\Psi(A_0) = 0$, probability-of-default externalities can lead to inefficiency. For instance, assuming $A \in \{A_L, A_H\}$, with $A_L < A_H$, it is easy to construct an example where banks default in state A_L in the competitive equilibrium, but not in the socially optimal allocation.

[26] Even though the mechanism at their source is different, this case of the model shares the investment efficiency properties of many fire-sales model (see, e.g. Lorenzoni (2008) and Jeanne and Korinek (2013)). That is, under-investment with respect to first best (i.e. the efficient allocation if bank capital wasn't scarce), but overinvestment with respect to second best (interpreted as k^*_{t+1}).

There are several possible interpretation for Ψ_1. As mentioned in Section 4, one can think of deadweight losses emanating from the bank resolution process.[27] Arguably, this process will be longer and costlier when financial distress is high. Alternatively, in the presence of government guarantees, one can think of the default costs as a proxy for the deadweight losses of taxation associated to bailouts (see, e.g., Acharya et al. (2010) and Bianchi (2013)).

In specification Ψ_1, the costs are linear in financial distress and, therefore, in the amount needed for the bailout (this is the assumption in Acharya et al., 2010). Hence, the externality is not one where financial distress at a bank increases, in itself, the likelihood or the extent of other banks' financial distress. It is therefore quite different from a fire sale externality. That said, making γ an increasing function of aggregate financial distress would be an easy way to add a fire-sale flavor to the model, and to magnify the externality.

Finally, note that specification Ψ_1 is consistent with Townsend's baseline assumptions in his seminal paper that establishes the optimality of standard debt contracts in a costly-state-verification environment (see Proposition 3.1 in Townsend (1979)).[28] Townsend (1979) stresses the analytical convenience of such a specification, but he also considers fixed verification costs. This is the case I now turn to.

Specification 2: Balance sheet size The problem studied by Townsend (1979) is of fixed size. In the present context, the natural interpretation of a fixed verification cost is one that is proportional to the size of the bank. Formally, conditional on default, this gives:

$$\Psi_2 \equiv \gamma b.$$

In this case, there is no cost-given-default externality. Formally, since Ψ_2 does not depend on $\overline{r}$ or k , the wedge in expression (25) is nil. However, probability-of-default externalities do operate (A_0 still depends on $\overline{r}$ and k).

This implies that under specification Ψ_2, as is the case under Ψ_1, the competitive equilibrium exhibits overlending, even in the absence of government guarantees. I have chosen to use specification Ψ_1 for the main analysis because I find it intuitively more appealing, and because of its analytical convenience (having $\Psi(A_0) = 0$ makes the problem particularly well behaved). Arguably, the deadweight losses associated with a bank's insolvency are likely to be increasing both in the extent of financial distress and on the size of the bank (Acharya et al. (2010)). One could therefore argue in favor of a cost function which would mix these two ingredients.

[27] Bankruptcies often involve deadweight losses (Townsend (1979)). In the case of financial institutions, losses can be large (James (1991)), and banking crises are typically followed by long and painful recessions (Reinhart and Rogoff (2009)) involving permanent output losses (Cerra and Saxena (2008)).

[28] Specifically, the equivalence is as follows. In Townsend (1979) and following his own notation, the assumption for Proposition 3.1 is that verification costs are increasing and convex in $\bar{C} - \bar{g}(y_2)$, where $\bar{C}$ is the repayment in absence of default (i.e. when no verification occurs) and $\bar{g}(y_2)$ is the repayment in the case of default (i.e. when verification occurs). Townsend then goes on with the example where verification costs are simply $\lambda(\bar{C} - \bar{g}(y_2))$, where λ is a positive constant, which is thus equivalent to Ψ_1.

However, as I have explained in the previous section, using specification Ψ_2 in the numerical analysis does not substantially affect the results. Using such an hybrid cost function is therefore unlikely to affect them either.

Specification 3: Value of distressed assets Finally, I consider the case where default costs are proportional to the value of distressed assets. That is, conditional on default:

$$\Psi_3 \equiv \gamma b r(A,k)$$

Since Ψ_3 depends on k (through r), the cost-given-default externality operates. The term that banks do not internalize is:

$$\frac{\partial \Psi_3}{\partial k}\frac{\mathrm{d}k}{\mathrm{d}b} = \gamma b \alpha A(\alpha - 1) k^{\alpha-2} < 0.$$

It is negative, which means that bankers do not internalize that their credit expansion *decreases* the default cost for other banks because it increases the losses they make on their loans. Since $\Psi_3(A_0) > 0$, the probability-of-default externality also contributes to the wedge, but with the opposite sign. Hence, *the net effect cannot be signed*. If $f(A_0)$ is small enough, the net effect is negative, and we get underinvestment in the competitive equilibrium (it is easy to generate an example with a discrete probability distribution function for A_t).

For non financial firms, specification Ψ_3 has the natural interpretation that the deadweight losses increase in the value of the firm's capital stock (think of the additional wear and tear of machines that should be reallocated, or as fruit that rots on the shelves during a bankruptcy procedure). In the context of financial assets, the interpretation is less clear, and the sign of the externality is counter-intuitive. This result can, however, be related to the result in Hennessy (2004) that debt overhang problems are worse when the value recovered by creditors in default is high. That financial frictions can lead to counter-intuitive externalities also relates to the work of Davila and Korinek (2017).

A specification equivalent to Ψ_3 was used in Bernanke and Gertler (1989) and in many papers of the *financial accelerator* literature that followed (e.g. Carlstrom and Fuerst (1997); Bernanke, Gertler and Gilchrist (1999)). My analysis sheds new light on the mechanisms by which the financial accelerator operates and on related normative considerations. Under specification Ψ_3, a decrease in an agent's net worth generates, in equilibrium, an increase in the deadweight losses for other banks, which (assuming there are no government guarantees) makes them reduce their lending. To the best of my knowledge, the role of probability-of-default and cost-given-default externalities in financial accelerator models, and in particular that they pull aggregate lending in opposite directions, had not been discussed yet.

6 Discussion of the model's key ingredients

A central result in this paper is that when aggregate bank capital increases, the banking sector should be allowed to expand but less than proportionally. Hence, the capital requirement should be tightened.

This result is robust in the sense that it does not hinge on specific parameter values and is in fact due to very few ingredients. The first of these ingredients is diminishing return to physical capital. This is perhaps the most standard assumption in macroeconomics, but is often abstracted from the literature on banking and financial regulation (notable exceptions include Martinez-Miera and Suarez (2014), and Van den Heuvel (2008)).

The second ingredient, is that bank credit supply matters for physical capital accumulation. In the model, I assume that banks are the only source of funding for firms, but this is not necessary to generate the result. What really matters is that aggregate bank lending affects aggregate real investment *at the margin*. Even though this seems to be a reasonable starting point, this is often abstracted from the macroeconomic literature. It is, however, now at the core of current policy debates and there is a fast growing literature on the subject that builds on contributions such as Holmström and Tirole (1997) for instance.[29]

Third, and perhaps most importantly, capital requirements affect aggregate lending, which need not (always) be the case in reality. There is no consensus on the subject (see Repullo and Suarez (2013)). For instance, there is evidence that banks do hold buffers above the regulatory level (Gropp and Heider, 2010), but there is also evidence of the relevance of bank capital requirements on credit supply in general (Bernanke, Lown and Friedman (1991); Thakor (1996); Ivashina and Scharfstein (2010); Aiyar, Calomiris and Wieladek (2012)). There also is evidence that *changes* in capital requirements affect bank lending (Jimenez et al. (2014); Bahaj and Malherbe (2016)).[30] And indeed, what matters for my analysis is that the requirements are *essentially* constraining lending. That is, what matters is that the capital requirement stance affects their behavior, even if the requirement is not technically binding. The huge resistance of banks (through lobbying for instance) to structural increases in capital adequacy ratios and the strong evidence of "risk-weight optimization" and regulatory arbitrage by the banks operating under the Basel II regime (buying CDS on ABS from AIG was one typical way to explicitly circumvent the regulation for instance, see Yorulmazer (2013)) all indicate that capital requirements do constrain bank decisions (see also Begley, Purnanandam and Zheng (2014); Mariathasan and Merrouche (2014)). In the model, capital requirements are binding because banks do not fully internalize the social cost of lending. This is likely to be the case in reality. First, because deposits are insured in most advanced economies, and there is evidence that it does distort banks' cost of borrowing (Demirguc-Kunt and Detragiache (2002); Ioannidou and Penas (2010)). Second, because large banks benefit from implicit guarantees (Acharya,

[29] See Brunnermeier, Eisenbach and Sannikov (2013) for a survey of recent developments, and Suarez (2010) for an insightful discussion of the various modeling strategies.

[30] See also Kashyap, Tsomocos and Vardoulakis (2014).

Anginer and Warburton (2014); Gandhi and Lustig (2015); Kelly, Lustig and Van Nieuwerburgh (2011); Laeven (2000); Noss and Sowerbutts (2012)). And, last but not least, because financial frictions can generate pecuniary externalities. A number of recent papers have focused on fire sales externalities and alike (see Davila and Korinek (2017) for a synthesis), but the mechanism I highlight in this paper had been overlooked so far.

7 Conclusion

I have studied economies where banks do not fully internalize the social costs of lending. In a competitive equilibrium, this translates into over-investment by firms. The regulator can restore investment efficiency thanks to a time-varying capital requirement. A key of its features is that it is increasing in aggregate bank capital. A regulatory regime that overlooks such a bank capital channel will unnecessarily magnify business and financial cycle fluctuations. A calibration of the model suggests that the associated welfare losses are substantial. I therefore believe that studying the real-world determinants of aggregate bank capital accumulation is an important avenue for future research.[31]

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[31]Our current understanding of bank dynamic capital structure decisions is at best incomplete, especially in general equilibrium (see the discussions in Allen and Carletti (2013), and Repullo and Suarez (2013) for instance, and Rampini and Viswanathan (2014), He and Krishnamurthy (2011), and Coimbra and Rey (2017) for recent advances).

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Appendix A: Proofs

Proposition. *1. At all dates, the competitive equilibrium capital stock is inefficiently high. That is:* $k_{t+1}^{CE} > k_{t+1}^{*}, \forall t$.

Proof. The representative banker maximizes $E_t\left[[b_t r_{t+1} + e_t]^+\right] - e_t$ with respect to $e_t, b_t \geq 0$. The respective first order conditions are:

$$\begin{cases} \int_{\rho_{t+1}}^{\infty} f_t(r_{t+1}) dr_{t+1} \leq 1 \\ \int_{\rho_{t+1}}^{\infty} r_{t+1} f_t(r_{t+1}) dr_{t+1} \leq 0 \end{cases},$$

where f_t is the probability distribution function of r_{t+1} conditional to date-t information, and $\rho_{t+1} \equiv -\frac{e_t}{b_t}$ is the solvency threshold. That is, this is the value for r_{t+1} below which the bank defaults..

Efficiency requires $E_t[r_{t+1}] \geq 0$. Given the banker's first order condition, $E_t[r_{t+1}] > 0$ cannot be true in equilibrium. The only case where the competitive equilibrium could be efficient is when efficiency requires $E_t[r_{t+1}] = 0$. Assume that this is the case and consider the first order conditions.

If $E_t[r_{t+1}] = 0$, the only way for both conditions to be satisfied is for the bank never to default. If this is the case, the bank is locally indifferent between financing the loans with deposits or equity. Now, imagine that the bank substitutes deposits for equity (keeping lending constant) up to the point where it fails if a low A_t realizes. From this point on, decreasing equity further increases expected profits (for each unit of equity withdrawn today, less than a unit is repaid to depositors tomorrow in expectation). Hence, the bank defaults with strictly positive probability in equilibrium and, given the first order conditions, $E_t[r_{t+1}] = 0$ cannot be satisfied in equilibrium.[32] □

Proposition. *2. The regulator can ensure investment efficiency, at all t, with the following capital requirement*

$$x_t^* = \frac{\eta_t w_t}{k_{t+1}^*}. \tag{26}$$

Proof. Given that aggregate bank capital is assumed to be scarce (i.e. $\eta_t w_t < k_{t+1}^*$) and that efficiency requires $E_t[r_{t+1}] \geq 0$, bankers find it profitable to invest all their wealth in equity. Assume the representative bank defaults in equilibrium. Then, following the same logic as in the proof of Proposition 1, the banker always prefer to finance loans with deposits at the margin. Hence the capital requirement must be binding, and $k_{t+1}^* = e_t/x_t^*$ in equilibrium. If the representative bank does not default in the constrained equilibrium, a similar logic applies but the optimal capital requirement is not unique. □

Proposition. *3.* $\forall t$, $\kappa_t < 1$. *Hence the optimal capital requirement is increasing in aggregate bank capital.*

Proof. Since $b_t^* = k_{t+1}^*$, I need to show that $\frac{dk_{t+1}^*}{k_{t+1}^*} / \frac{de_t}{e_t} < 1$. Ignoring subscripts and superscripts, I can rewrite economic surplus:

$$S = E\bar{[}A]k^\alpha - (\delta + (1 - E[A])\Delta))\, k - \gamma \int_0^{A_0} ((\delta + (1 - A)\Delta))\, k - \alpha A k^\alpha - e)\, f(A)\, dA,$$

where

$$A_0 \equiv \frac{(\delta + \Delta)\, k - e}{\alpha k^\alpha + \Delta k}.$$

[32]See the working paper version for a proof of equilibrium existence (anonymized reference).

From the first order condition associated with the maximization of S with respect to k, I can define a function $G(k,e) = 0$

$$G(k,e) \equiv \alpha E[A]k^{\alpha-1} - (\delta + (1 - E[A])\Delta)) - \gamma \int_0^{A_0} \left((\delta + (1 - A)\Delta)) - \alpha^2 A k^{\alpha-1}\right) f(A)\, dA = 0.$$

Applying the implicit function theorem, I need to show that $\left(\frac{\partial G}{\partial e}\right) / \left(-\frac{\partial G}{\partial k}\right) < \frac{k}{e}$. That is,

$$\frac{\gamma\left(-\frac{\partial A_0}{\partial e}\right) f(A_0)\Omega}{\alpha(1-\alpha)E[A]k^{\alpha-2} + \gamma \int_0^{A_0} \left(\alpha^2 (1-\alpha) A k^{\alpha-2}\right) f(A)\, dA + \gamma\left(\frac{\partial A_0}{\partial k}\right) f(A_0)\Omega} < \frac{k}{e}, \tag{27}$$

where

$$\Omega \equiv \left((\delta + (1 - A_0)\Delta)) - \alpha^2 A_0 k^{\alpha-1}\right).$$

We have:

$$\begin{cases} \frac{\partial A_0}{\partial e} = \frac{-1}{\alpha k^\alpha + \Delta k} \\ \frac{\partial A_0}{\partial k} = \frac{(\delta+\Delta)(\alpha k^\alpha + \Delta k) - \left(\alpha^2 k^{\alpha-1} + \Delta\right)((\delta+\Delta)k - e)}{(\alpha k^\alpha + \Delta k)^2}. \end{cases}$$

By the definition of A_0, we have that $(\delta + (1 - A_0)\Delta)) > \alpha A_0 k^{\alpha-1}$. Hence, $\Omega > 0$ and the numerator in (27) is positive. Given that the first two terms of the denominator are also positive, it is sufficient to show that:

$$-\frac{\partial A_0}{\partial e} / \frac{\partial A_0}{\partial k} < \frac{k}{e}. \tag{28}$$

Noting that $\frac{\partial A_0}{\partial k} > 0$, condition (28) becomes:

$$\frac{e}{\alpha k^\alpha + \Delta k} < \frac{(\delta+\Delta)(\alpha k^\alpha + \Delta k) - \left(\alpha^2 k^{\alpha-1} + \Delta\right)((\delta+\Delta)k - e)}{(\alpha k^\alpha + \Delta k)^2} k,$$

which boils down to:

$$k\left(\alpha^2 k^{\alpha-1} + \Delta\right)((\delta+\Delta)k - e) < ((\delta+\Delta)k - e)(\alpha k^\alpha + \Delta k),$$

which holds since $((\delta+\Delta)k - e) > 0$ (otherwise $A_0 = A_L$ and banks do not default in equilibrium), and $\alpha < 1$. □

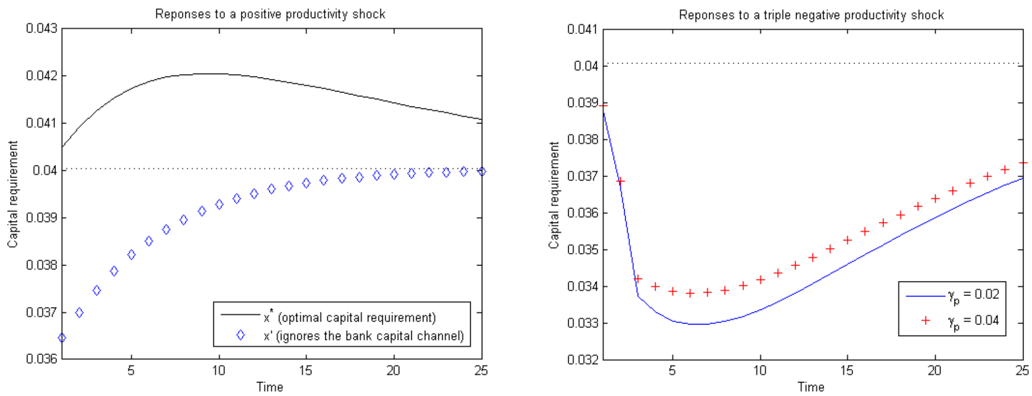
Proposition. *4. Assume default is costless ($\gamma = 0$ and $\Delta = 0$). The bank capital channel dominates the default rate channel at any positive lag. In the period where the shock occurs, this is true if the shock is not too persistent. Formally, following a date-1 positive productivity shock, $E_1[\boldsymbol{x}_{t>1}] > 0$, and $\boldsymbol{x}_1 > 0 \Leftrightarrow \rho < 1 - \alpha$.*

Proof. The proof follows exactly the same logic as the example in the text. The only difference is that one needs to take expectations to account for the realizations of $\epsilon_{t>1}$. □

Appendix B: Alternative default cost function

Figure 5 and Table 4 present results under the alternative bank default cost function (specification $\Psi_{t+1} = b_t \gamma_{\mathrm{p}}$). The value for the cost parameter is chosen so that the expected costs given default are comparable to those under the baseline calibration. That is $\gamma_{\mathrm{p}} = 0.02$, which yields an expected deadweight loss (conditional on default) of 5.88% of steady state GDP. As we can see, both the impulse responses and the loss distributions are very similar to what I get with my baseline specification (see Figures 1 and 2 and Table 3). Finally, note that the welfare loss is essentially unchanged (it remains around 0.04% of steady state consumption).

Figure 5: Bank default costs proportional to balance sheet



This figure presents the results under the assumption that $\Psi_{t+1} = 0.02 b_t$ if the bank defaults, and 0 otherwise. The left panel shows the path for the optimal capital requirement x_t^*, and the alternative policy, starting from steady state and following a 1% decrease in the loan default rate. The right panel shows the path for the optimal capital requirement after a triple negative shock for two different values of γ_{p}.

Table 4: Loss frequencies

	$\gamma = 2$	$\gamma = 4$	$\gamma_{\mathrm{prop}} = 0.02$
Freq. of large losses	16.4%	15.9%	16.0%
in good time	7.1%	6.9%	7%
in bad times	29.1%	26.9%	28%
Probability of default	0.13%	0.08%	0.11%

This table compares simulated moments between the baseline specification for the default cost function (with $\gamma = 2$ and $\gamma = 4$) and the alternative specification $\Psi_{t+1} = 0.02 b_t$. Other parameters are at their baseline value (Table 1). All simulations start at steady state, and follow the exact same arbitrary sequence of 10,000 random shocks. Good times (bad times) are defined as periods where the loan default rate is at list one standard deviation below (above) the sample mean. Large losses correspond to losses larger than 10% of initial bank capital.

Appendix C: Comparing policies

Table 5: Further simulation results (baseline calibration)

		x_t^*	$x_t^{'}$
Lending volatility	$\mathrm{std}\,(b_t)$	1.98	2.19
Probability of default	$\mathrm{mean}\,(\mathrm{Pr}_t\,(v_{t+1} < 0))$	0.13%	$\simeq 0$
Freq. of large losses	ROE $< -10\%$	16.4%	16.0%
	in good time	7.1%	12.1%
	in bad times	29.1%	11.8%
Dividend / recap	$(\hat{e}_t - e_t)\,/\hat{e}_t$	0	1.6%
	in good time	0	7.1%
	in bad times	0	−3.4%
Welfare loss	$\mathrm{mean}\left(S_t^* - S_t^{'}\right)/S_{ss}$	-	0.04%

This table compares simulated moments under the optimal capital requirement (x^*) and the alternative policy ($x^{'}$). Both simulations use baseline calibration parameters (Table 1), start at steady state, and follow the exact same arbitrary sequence of 10,000 random shocks. Good times (bad times) are defined as periods where the loan default rate is at list one standard deviation below (above) the sample mean. ROE is defined as $\left(v_{t+1}^+/e_t\right) - 1$. Variable $\hat{e}_t$ denotes the pre-dividend (and pre-recapitalization) level of aggregate bank equity at the end of date t. Finally, S_t^* and $S_t^{'}$ are the realized surplus under the respective policies.

Appendix D: Summary of intraperiod time-line

1. A_t is realized and publicly observed, firms competitively hire workers.
2. Production takes place and is allocated: wages are paid, solvent firms repay their loan, insolvent firms transfer their residual value to the banks.
3. If solvent, banks repay depositors. If insolvent, banks default and the associated costs are incurred by the depositors. The regulator compensates them for their losses and taxes savers to break even.
4. Young agents learn whether they have banking ability.
5. Young bankers make their investment portfolio decision (storage and/or investment in their bank's equity).
6. Banks borrow from savers and lend to firms, which invest in physical capital. Agents put the remainder of their savings in the storage technology.
7. The old generation consumes and leaves the economy.