

# Portfolio Choice with Internal Habit Formation: A Life-Cycle Model with Uninsurable Labor Income Risk\*

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## Abstract

Motivated by the success of internal habit formation preferences in explaining asset pricing puzzles, we introduce these preferences in a life-cycle model of consumption and portfolio choice with liquidity constraints, undiversifiable labor income risk and stock-market participation costs. In contrast to the initial motivation, we find that the model is not able to simultaneously match two very important stylized facts: a low stock market participation rate, and moderate equity holdings for those households that do invest in stocks. Habit formation increases wealth accumulation because the intertemporal consumption smoothing motive is stronger. As a result, households start participating in the stock market very early in life, and invest their portfolios almost fully in stocks. Therefore, we conclude that, with respect to its ability to match the empirical evidence on asset allocation behavior, the internal habit formation model is dominated by its time-separable utility counterpart.

JEL Classification: E21, G11.

Key Words: Life-Cycle Asset Allocation, Habit Formation, Liquidity Constraints, Stock Market Participation Costs, Uninsurable Labor Income Risk.

# 1 Introduction

Samuelson (1969) and Merton (1969) have shown that, under certain conditions, optimal portfolio allocation between a riskless and a risky asset does not depend on the investment horizon and, as a result, the optimal asset mix between risky and riskless securities should remain constant as the investor ages. The conditions required to obtain this result are homothetic preferences, independently and identically distributed returns, frictionless markets and the absence of labor income. Merton (1971) shows how the same results hold with labor income in a complete markets set-up, in which the investors are allowed to borrow against their human capital and to insure their labor income risk, while Bodie, Merton and Samuelson (1991) extend the model to emphasize how the extent of labor supply flexibility over the life-cycle can change optimal portfolio composition.

In the context of life-cycle asset allocation, the absence of uninsurable labor income can be questioned. Human capital is a large component of a typical investor's wealth, and to the extent that both the level and risk of labor income change over the life-cycle, and markets to insure such idiosyncratic risks are missing, age-varying investment strategies can arise. Indeed, recently, financial economists have begun to study the optimal portfolio allocation behavior in the context of models with stochastic uninsurable labor income and borrowing constraints.<sup>1</sup> Motivated by recent empirical work (Vissing-Jorgensen (2001) and Paiella (1999)) that suggests that small, fixed, entry costs can explain the observed low stock market participation rates, some of these papers have also introduced such costs in the life-cycle model.<sup>2</sup>

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<sup>1</sup>Heaton and Lucas (1996, 1997, 2000), Koo (1998), Viceira (2001) and Haliassos and Michaelides (2003) extend the infinite horizon buffer stock model of Deaton (1991) by including a portfolio choice between a risky and riskless asset. Cocco, Gomes and Maenhout (1999), Campbell, Cocco, Gomes and Maenhout (2001), Dammon, Spatt and Zhang (2001), Cocco (2001) extend the life-cycle model of Carroll (1997) and Gourinchas and Parker (2002), in the same direction. Bertaut and Haliassos (1997) analyze small scale three period models where each period amounts to 20 years, while Constantinides, Donaldson and Mehra (2002) and Storesletten, Telmer and Yaron (2001) analyze the general equilibrium implications of these models.

<sup>2</sup>Considering explicitly the participation decision can also be motivated by recent empirical evidence that for households that do hold equity, the equity premium can potentially be rationalized by the consumption risk of the stock market (for example, Parker, 2001).

However, two important predictions of this literature are still at odds with the observed empirical regularities.<sup>3</sup> First, the calibrated life-cycle portfolio choice model predicts a counterfactually high stock market participation rate. Second, households will invest almost all of their wealth in stocks, a result that is particularly strong for young individuals. Nevertheless, according to the most recent empirical evidence on life-cycle asset allocation, at least 50% of the population (in any country except the U.S.) does not own equities (either directly or indirectly through pension plans) while stock market participation in the U.S. has only passed the fifty percent threshold in 1998-2001 (according to the 2001 Survey of Consumer Finances (SCF)). Moreover, even in a country with a well-developed equity culture like the U.S., the direct ownership of publicly traded stocks was 21.3% in the 2001 SCF. Furthermore, participation rates increase significantly during working life and, even those that do own equities, still invest a large fraction of their financial wealth in alternative assets.

In this paper we extend the finite horizon portfolio choice model with a fixed participation cost by introducing internal habit formation in preferences.<sup>4</sup> The motivation comes from the relative success of habit formation models in solving asset pricing puzzles<sup>5,6</sup> and aggregate consumption dynamics.<sup>7</sup> Habit formation models can be distinguished along two different dimensions. First, while some papers (for example, Campbell and Cochrane (1999) and Chan and Kogan (2002)) use an external habit specification where the habit depends on

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<sup>3</sup>See Guiso, Jappelli and Terlizzese (1996), Bertaut and Haliassos (1997), King and Leape (1998), Poterba and Samwick (1999), Heaton and Lucas (2000), Davis and Willen (2001), Ameriks and Zeldes (2001) and Guiso, Haliassos and Japelli (2002).

<sup>4</sup>Heaton and Lucas (1997) study the impact habit formation in a infinite horizon model with uninsurable labor income risk. Concurrent work by Polkovnichenko (2002) also studies the asset allocation implications of a life-cycle model with internal, additive habit formation preferences.

<sup>5</sup>See Sundaresan (1989), Constantinides (1990), Ferson and Constantinides (1991), Abel (1990, 1999), Detemple and Zapatero (1991), Hindy and Huang (1993), Dybvig (1995), Heaton (1995), Campbell and Cochrane (1999), Lax (2000), Boldrin, Christiano and Fisher (2001), Dai (2001), Brandt and Wang (2002), Chan and Kogan (2002), Wachter (2002), and Menzly, Santos and Veronesi (2002).

<sup>6</sup>Recent work by Chapman (1998), Pijoan-Mas (2001) and Otrok, Ravikumar and Whiteman (2002) questions some of these results.

<sup>7</sup>See Heaton (1993), Carroll, Overland and Weil (2000), Fuhrer (2000), Diaz, Pijoan-Mas and Rios Rull (2000) or Michaelides (2002).

the consumption of a reference group (for instance, aggregate consumption), others assume that the habit depends on the individual's own past consumption (for example Sundaresan (1989) or Constantinides (1990)). Second, some models specify the argument in the utility function as the difference between consumption and habit (additive habit models, such as Constantinides (1990) or Campbell and Cochrane (1999)), while in others utility is a function of the ratio between consumption and habit (multiplicative habit models, such as Abel (1990) or Chan and Kogan (2002)). In this paper, since we are solving an individual agent's decision problem and not an equilibrium model, we specify the habit process as a function of the household's past consumption. With respect to the utility function, we will consider both the ratio and additive specifications. The main conclusions are very similar, and therefore we start by discussing the ratio model for which the intuition is simpler, and present the additive model afterwards.

Contrary to the initial motivation, we find that introducing habit formation preferences in the standard life-cycle asset allocation model actually *decreases* its ability to match the observed empirical regularities. Households increase wealth accumulation early in life because the presence of the habit term leads to a stronger incentive to smooth consumption over time. As a result, relative to the model without habit formation, there is a stronger motive to pay the stock market entry cost and equity investing takes place much earlier in life. In a separate paper (Gomes and Michaelides (2002)) we show that in order to match the participation rates observed in the data, it is important to be able to disentangle risk aversion from the elasticity of intertemporal substitution, so that the latter can be decreased while keeping the former constant (or that both can be decreased simultaneously). With habit formation preferences we obtain exactly the opposite result: for a given coefficient of risk aversion, the EIS is higher than in the corresponding time-separable utility specification. With respect to the share of wealth invested in equities, for those households that do participate in the stock market, the two models deliver essentially the same prediction. After paying the fixed cost, households will invest most of their wealth in stocks, unless we consider very high values of risk aversion. This is the extension of the infinite-horizon result of Heaton and Lucas (1997), and it reflects the equity premium puzzle from an asset allocation perspective.

These conclusions are robust to the functional form of habit formation: both the ratio and the additive specifications produce similar results. In the additive difference habit model, risk aversion is now a function of surplus consumption (consumption relative to the habit level). Therefore, a stronger habit preference, besides making the investor more willing to smooth consumption intertemporally, also increases risk aversion and prudence. As a result, strengthening the habit motive generates even more wealth accumulation early in life than in the ratio habit model. Therefore the investor has a stronger incentive to pay the fixed cost and stock market participation is again close to 100% from very early on in life (within five years of working life).

The rest of the paper is organized as follows. In section two we describe the model and the solution technique. In sections three and four we discuss the results for the multiplicative habit specification for the case without, and with, the fixed entry cost, respectively. In section 5 we discuss the results for the additive habit specification and section six offers some concluding remarks.

## 2 The Model

### 2.1 Preferences

Time is discrete and  $t$  denotes adult age which, following the typical convention in this literature, corresponds to effective age minus 19. Each period corresponds to one year and agents live for a maximum of  $T = 81$  periods (age 100). The probability that a consumer/investor is alive at time  $(t + 1)$  conditional on being alive at time  $t$  is denoted by  $p_t$  ( $p_0 = 1$ ).

As mentioned before, we start by specifying a “ratio habit model” (surplus consumption is given by the *ratio* between consumption and the habit persistence term) and later on we will consider a “difference or additive habit model” (where surplus consumption is given by the *difference* between consumption and the habit persistence term). There is one non-durable consumption good and the period-by-period felicity function is given by

$$U(C_t) = \frac{\{C_t/H_t^\gamma\}^{1-\rho}}{1-\rho} \quad (1)$$

where  $H_t$  is the habit level at the beginning of period  $t$ , and  $\rho$  is the coefficient of relative risk aversion.

The importance of the habit is controlled by the parameter  $\gamma$ , where  $\gamma \in [0, 1]$ . If  $\gamma = 0$ , habits do not affect the felicity function, which then becomes the standard time separable function with CRRA parameter equal to  $\rho$ . The habit level evolves according to the following law of motion:

$$H_t = (1 - \lambda)H_{t-1} + \lambda C_{t-1} \quad (2)$$

where  $\lambda \in [0, 1]$ . We can obtain more intuition by looking at the case of  $\lambda = 1$ , for which the felicity function can be rewritten as

$$U_t = \frac{\{[\frac{C_t}{C_{t-1}}]C_{t-1}^{1-\gamma}\}^{1-\rho}}{1 - \rho} \quad (3)$$

This illustrates the importance of consumption smoothing in habit formation models. The utility in each period is given by a weighted average of both the level and the change in consumption. For  $\gamma = 0$  only current consumption matters, while for  $\gamma = 1$  only consumption growth is important. The investor's subjective discount factor is constant and denoted by  $\beta$ .

## 2.2 Labor Income Process

The labor income process before retirement is the same as the one used by Gourinchas and Parker (2002), or Cocco, Gomes and Maenhout (1999), and it is given by

$$Y_{it} = P_{it}U_{it} \quad (4)$$

$$P_{it} = \exp(f(t, Z_{it}))P_{it-1}N_{it} \quad (5)$$

where  $f(t, Z_{it})$  is a deterministic function of age and household characteristics  $Z_{it}$ ,  $P_{it}$  is a “permanent” component, and  $U_{it}$  a transitory component. We assume that the  $\ln U_{it}$ , and  $\ln N_{it}$  are each independent and identically distributed with mean  $\{-.5 * \sigma_u^2, -.5 * \sigma_n^2\}$ <sup>8</sup>, and variances  $\sigma_u^2$ , and  $\sigma_n^2$ , respectively. The log of  $P_{it}$ , evolves as a random walk with a deterministic drift,  $f(t, Z_{it})$ .

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<sup>8</sup>With this specification the mean of the level of the log random variables equals 1.

Given these assumptions, the growth in individual labor income follows

$$\Delta \ln Y_{it} = f(t, Z_{it}) + \ln N_{it} + \ln U_{it} - \ln U_{it-1}, \quad (6)$$

and its unconditional variance equals  $(\sigma_n^2 + 2\sigma_u^2)$ . This process has a single Wold representation that is equivalent to the MA(1) process for individual earnings growth estimated using household level data (MaCurdy (1982), Abowd and Card (1989), and Pischke (1995)).<sup>9</sup>

For simplicity, retirement is assumed to be exogenous and deterministic, with all households retiring in time period  $K$ , corresponding to age 65 ( $K = 46$ ). Earnings in retirement ( $t > K$ ) follow:  $Y_{it} = \theta P_{iK}$ , where  $\theta$  is the replacement ratio (a scalar between zero and one). This specification considerably facilitates the solution of the model, as it does not require the introduction of an additional state variable (see solution method section).

### 2.3 Assets and wealth accumulation

The investment opportunity set is constant and there are two financial assets, one riskless asset (treasury bills or cash) and one risky asset, stocks. The riskless asset yields a constant gross after tax real return,  $R^f$ , while the risky asset's returns (denoted by  $R_t$ ) are given by

$$R_{t+1} - R^f = \mu + \varepsilon_{t+1} \quad (7)$$

where  $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ .

Before investing in stocks for the first time, the investor must pay a fixed cost. This entry fee represents both the explicit transaction cost from opening a brokerage account and, more importantly, the (opportunity) cost of acquiring information about the stock market. The fixed cost ( $F$ ) is scaled by the level of the permanent component of labor income ( $P_{it}$ ) as this will simplify the solution of the model, but is also motivated by an opportunity cost interpretation for this entry fee.

Following Deaton (1991) we denote cash on hand as the liquid resources available for consumption and saving. We define a dummy variable  $I_P$  which is equal to one when the

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<sup>9</sup>Although these studies generally suggest that individual income changes follow a MA(2), the MA(1) is found to be a close approximation.

cost is incurred for the first time and zero otherwise. When the stock market entry fee has not been paid yet, next period's cash on hand ( $X_{i,t+1}$ ) is given by

$$X_{i,t+1} = S_{it}R_{t+1} + B_{it}R^f + Y_{i,t+1} - FI_P P_{i,t+1} \quad (8)$$

where  $S_{it}$  and  $B_{it}$  denote, respectively, stock holdings and riskless asset holdings (cash) at time  $t$ . If the fixed cost was already paid in the past, we have

$$X_{i,t+1} = S_{it}R_{t+1} + B_{it}R_f + Y_{i,t+1} \quad (9)$$

Moreover, the household must allocate her cash-on-hand ( $X_{it}$ ) between consumption expenditures ( $C_{it}$ ) and savings, so that

$$X_{it} = C_{it} + S_{it} + B_{it} \quad (10)$$

Finally, as in Deaton (1991), we prevent households from borrowing against their future labor income, and also allow no short selling. Specifically, we impose the following restrictions:

$$B_{it} \geq 0 \quad (11)$$

$$S_{it} \geq 0 \quad (12)$$

## 2.4 The optimization problem and solution method

The complete optimization problem can now be written as

$$MAX_{\{S_{it}, B_{it}\}_{t=1}^T} E_1 \sum_{t=1}^T \beta^{t-1} \{\Pi_{j=0}^{t-1} p_j\} U(C_{it}, H_{it}), \quad (13)$$

subject to

$$H_{it} = (1 - \lambda)H_{it-1} + \lambda C_{it-1} \quad (14)$$

$$X_{it+1} = S_{it}R_{t+1} + B_{it}R_f + Y_{it+1} - F * I_P * P_{it+1} \quad (15)$$

$$X_{it} = S_{it} + B_{it} + C_{it} \quad (16)$$

$$B_{it} \geq 0, S_{it} \geq 0 \quad (17)$$

$$R_{t+1} - R_f = \mu + \varepsilon_{t+1}, \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \quad (18)$$

$$Y_{it} = P_{it}U_{it}, \quad P_{it} = \exp(f(t, Z_{it}))P_{it-1}N_{it}, \quad \text{if } t \leq K \quad (19)$$

$$Y_{it} = \theta P_{iK}, \quad \text{if } t > K \quad (20)$$

Analytical solutions to this problem do not exist. We therefore use a numerical solution method based on the maximization of the value function to derive optimal policy functions for total savings and the share of wealth invested in the stock market. The details are given in appendix A, and here we just present the main idea. We first simplify the solution by exploiting the scale-independence of the maximization problem and rewriting all variables as ratios to the permanent component of labor income ( $P_{it}$ ). The equations of motion and the value function can then be rewritten as normalized variables and we use lower case letters to denote them (for instance,  $x_{it} \equiv \frac{X_{it}}{P_{it}}$  and  $h_{it} \equiv \frac{H_{it}}{P_{it}}$ ). This allows us to reduce the number of state variables to four: age ( $t$ ), normalized cash-on-hand ( $x_{it}$ ), normalized habits ( $h_{it}$ ) and participation status (whether the fixed cost has already been paid or not). In the last period the policy functions are trivial (the agent consumes all available wealth) and the value function corresponds to the indirect utility function. We can use this value function to compute the policy rules for the previous period and the corresponding value function. This procedure is then iterated backwards. We optimize over the different choices using grid search.

## 2.5 Parameter Calibration

In our “baseline” set of parameter values we set the coefficient of relative risk aversion equal to 2, the mean equity premium equal to 4 percent and the standard deviation of the risky investment equal to 18 percent. Considering an equity premium of 4% (as opposed to the historical 6%) is a fairly common choice in this literature<sup>10</sup> and this is motivated by two different concerns. First, Campbell et al. (2001) argue that this is actually a better measure of a forward-looking equity premium. Second, even after having paid the fixed entry cost, the average investor still faces non-trivial transaction costs, mostly in the form of mutual fund fees. This adjustment is a short-cut representation for those costs, since the dimensionality

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<sup>10</sup>See, for example, Yao and Zhang (2002), Cocco (2001) or Campbell et al. (2001). Dammon, Spatt and Zhang (2001) use an even lower value.

of the problem prevents us from modelling them explicitly (as in Heaton and Lucas (1997), for example). For the purpose of this paper, if we were to consider a higher equity premium, this would only make our point stronger as it would make it even harder for the model to match the empirical evidence. Nevertheless, we do experiment and present results for equity premia of 2.5% and 5.5%. The annual discount rate,  $\beta$ , is fixed at 0.95 (we also present experiments for lower discount factors) and the constant real interest rate,  $r$ , is set equal to 2%.

Carroll (1992) estimates the standard deviations of the idiosyncratic shocks using data from the *Panel Study of Income Dynamics*, and we use values close to those: around 10 percent per year for  $\sigma_u$  and 8 percent per year for  $\sigma_n$ .<sup>11</sup> The deterministic labor income profile is chosen to reflect the hump shape of earnings over the life cycle and the parameter values are taken from Cocco, Gomes and Maenhout (1999). The retirement transfers are also calibrated using the estimation results from Cocco et al. (1999) and are set to around 68 percent of labor income in the last period of working life (we also present results for replacement rates equal to 45% and 85%). With respect to the fixed cost of participation we consider two cases. One case where we set this cost equal to zero and one case where we set it equal to 0.1, corresponding to 10 percent of the household's expected annual income.

One important correlation is that between labor income shocks and stock returns. Given that there are two earnings shocks, the first issue arises with the correlation between the transitory earnings shock and stock returns. Viceira (2001) and Haliassos and Michaelides (2003) show that varying the correlation between the transitory earnings shock and the stock return does not affect the portfolio choice allocation; similar results hold here and are therefore omitted. Substantial hedging demands do arise, however, when the correlation between permanent earnings shocks and stock returns is positive (especially for higher risk aversion coefficients). The microeconomic evidence on this correlation is scant, however, partly because micro data might not offer a long enough panel to compute the necessary time series correlations with aggregate stock returns. The decomposition of individual earnings growth rates into permanent and transitory shocks, for instance, relies on the cross-sectional

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<sup>11</sup>These values are also consistent with Deaton (1991).

moments (see Abowd and Card (1989), for instance). Computing the correlation of the idiosyncratic earnings innovation with stock returns, however, will need to rely on time series moments, and will probably suffer from a finite sample bias problem (Jermann, (1999)). As a first step, Davis and Willen (2001) have estimated this correlation, and offer estimates between approximately zero and 0.3. On the other hand, Campbell et al. (2001) and Heaton and Lucas (2000) only find such results when considering small subgroups of the population (for instance, self-employed households or households with private businesses). A priori, given that aggregate shocks are a small component of total individual earnings volatility (Pischke (1995)), we might expect the correlation of permanent idiosyncratic earnings innovations and aggregate stock returns to be low. Given the small component of aggregate uncertainty in individual earnings histories and the available empirical evidence to date, we view the zero correlation as a reasonable hypothesis and use it as the benchmark correlation. Results are also offered for the case where the correlation is higher (0.3).

The benchmark values of the habit parameters will be  $\gamma = 0.8$  and  $\lambda = 0.5$ , however we will consider different combinations as well ( $\{\lambda, \gamma\} = \{0.5, 0.8\}, \{0.8, 0.5\}, \{0.8, 0.8\}, \{0.8, 1.0\}$ ). Finally, setting  $\gamma = 0$  generates the portfolio model over the life-cycle without any habits (the CRRA model), and will allow us to identify the effects of the habit on optimal consumption and portfolio choice.

### 3 Results without Fixed Participation Cost

In this section we consider a version of the model without the fixed cost of stock market participation.

#### 3.1 Decision Rules

##### 3.1.1 Consumption Function

In this section we study the behavior of the (normalized) consumption function  $c(x_t, h_t, t)$ . Figure 1.1 plots  $c(h_t)$  at age 25, for our benchmark parameters ( $\gamma = 0.8$ ,  $\lambda = 0.5$  and  $\rho = 2$ )

and different habit levels.<sup>12</sup> The following comments about this policy function are worth making. First, for a given habit level, the shape of the consumption policy rule is the same as in the standard buffer stock saving literature. Consumption equals cash on hand below a certain threshold level, and beyond that level the marginal propensity to consume rapidly falls. Second, this threshold is increasing in the stock of habits since a higher habit level requires additional consumption to maintain the same level of utility.

Figure 1.2 plots the consumption policy functions at age 25 for our benchmark parameters, and for the model with habit formation ( $\gamma = 0$ ). The consumption policy rule collapses in one branch when habits are absent, while for the habit case we only plot the consumption branch that corresponds to the mean habit level associated with the simulations of the model. These two policy functions show that a higher  $\gamma$  generates a higher saving level (when the constraint on saving is not binding). A higher  $\gamma$  implies a stronger motive to smooth consumption over time and thereby generates higher savings.

What explains this result? With multiplicative habit preferences the coefficient of relative risk aversion remains constant and equal to  $\rho$ . Relative to the model without habits, only the elasticity of intertemporal substitution (EIS) changes, as preferences over time and over contingencies are no longer the same. As we increase the strength of the habit in the utility function (starting from  $\gamma = 0$ ), the intertemporal consumption smoothing becomes more important than in the time-separable utility case, thus increasing savings.

### 3.1.2 Portfolio Allocation Rule

We next focus on the asset allocation decision  $\alpha(x_t, h_t, t)$ . Figure 1.3 plots the age-25 portfolio rule as a function of cash-on-hand, for the lowest and highest habit states, using the benchmark preference parameters ( $\gamma = 0.8$ ,  $\lambda = 0.5$  and  $\rho = 2$ ). The qualitative results are also similar to the ones previously obtained in this literature. Even though labor income is risky, wage earnings is a closer substitute for the risk-free asset rather than for equities. As

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<sup>12</sup>We will often use the term consumption function to refer to  $c(x_t)$ , conditional on given values of  $h_t$  and  $t$ . To avoid cluttering, we only plot the policy functions conditional on the habit states that are within 3 standard deviations from the mean habit level in the simulations (while the actual numerical computations were conducted using a total of 50 habit states).

a result, the presence of future labor income increases the demand for stocks and the share invested in equities is a decreasing function of cash-on-hand.

The interesting additional insight involves the behavior of the portfolio allocation, conditional on the habit level. Lower habits generate a higher level of saving than higher habit levels (see figure 1.1). Therefore, there is range of cash-on-hand for which positive savings take place for the lowest habit states, thus generating positive stock holdings, while no savings occur under the highest states. On the other hand, for high levels of liquid wealth a higher habit level is associated with a higher optimal equity allocation since the ratio of savings to future labor income is smaller for a given level of cash on hand.

### 3.1.3 Sensitivity Analysis

The strength of the habit (controlled by the parameter  $\gamma$ ) and the rate with which the habit depreciates ( $1 - \lambda$ ) both have a minor impact on the optimal portfolio rules and as such the results are not shown. This was expected since  $\gamma$  and  $\lambda$  do not affect the investor's attitude toward risk. On the other hand, the consumption functions are quite sensitive to  $\gamma$ , as shown in figure 1.4, which plots  $c(x_t)$  for age 25 and evaluated at the mean habit level. We find that  $\lambda$  is not very important, for example increasing it from 0.5 to 0.8 (for a given  $\gamma$ ) does not affect the results.<sup>13</sup>

## 3.2 Simulation Results

In this section we discuss the implications of the model with respect to consumption, portfolio holdings and wealth accumulation over the life-cycle. It is common practice for researchers to simulate a model over the life-cycle for a large number of individuals (say 1000) to compute the statistics of interest (mean wealth holdings, for instance) for any given age. We use this method as well but complement it using an alternative, equivalent method of computing these statistics that is based on calculating the transition distribution of cash on hand and the habit stock from one age to the next. The main reason for doing this is that we can concisely plot the wealth distributions over the life cycle using this method.

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<sup>13</sup>Again these results are not shown because the policy functions are almost indistinguishable.

The numerical details are delegated to appendix B but the intuitive idea is very simple. Given that normalized cash on hand is a stationary process, it becomes easy to compute the transition probability from one cash on hand state to another from period to period once the optimal policy functions have been computed. Given an initial distribution of cash on hand and habits, the new distribution will be the product of the transition and initial distribution; this will then become the initial distribution for the next period computation.

Figure 2.1 plots the distributions of normalized cash on hand for different habit levels at age 50. Higher levels of cash on hand are associated with higher habit levels and the distributions shift to the right as the habit level increases. Figure 2.2 plots the distributions at mean habit levels during retirement to illustrate how wealth decumulation takes place in the model during this phase of the life-cycle.

In figure 2.3 we plot the average wealth accumulation profiles over the life cycle for both the habit and the no-habit model. We assume that all agents start with zero assets and the lowest possible level of habits. We find this assumption convenient and plausible because it is natural to think that agents who have not inherited large levels of wealth will not have grown in an environment that would generate high levels of habits. Nevertheless, we have experimented with higher initial levels of habits and the conclusions remain unchanged after around five years of working life (we provide further robustness checks to this assumption in a later subsection). Wealth accumulation is higher in the habit model, which is consistent with the consumption policy rules implying higher saving when the strength of the habit is increased. Finally, figure 2.4 illustrates the counterfactual prediction of the model: households invest almost all of their wealth in stocks throughout the life-cycle (and therefore stock market participation takes place as soon as positive saving is undertaken).

## 4 Results with Fixed Participation Cost

We next introduce the positive fixed cost of stock market participation ( $F$ ). We set this cost equal to 10% of the household's expected annual income, which we view as a likely upper bound on the plausible range of fixed stock market participation costs.

## 4.1 Computing the Transition Distributions when the Participation Cost is Positive.

At the beginning of life, the participation rate (proportion of households that have incurred the fixed cost) is zero. Let  $\mathbf{\Pi}_t^O$  denote the mass of agents who have not yet paid the fixed cost. This is a matrix of probabilities for each value of cash on hand, each habit state and each age ( $t$ ). Let  $\mathbf{\Pi}_{th}^O$  denote the vector of probabilities conditional on a given habit state  $h$ . Given their initial savings decisions a new distribution for cash on hand and habits can be evaluated. We can then compute the proportion of households who would be willing to incur the fixed cost, by computing the sum of the probabilities in  $\mathbf{\Pi}_{th}^O$  for which  $x > x_h^*$ , where  $x_h^*$  is the trigger point that causes participation (given by the optimal participation policy rule). The subscript  $h$  emphasizes the dependence of the trigger cash on hand point on the current habit level. The new participation rate is then obtained by adding to the previous period participation rate the percentage of households that choose to pay the fixed cost, times the percentage of households who have never paid the cost in the past. We then compute the distributions for both participants and non-participants, and use the participation rate to compute the unconditional distribution of wealth and habits in the economy. Appendix B explains the computational details.

## 4.2 On the strength of the substitution effect

An interesting comparison, that will prove important for the results on wealth accumulation later on in the paper, is the contrast between total saving when access to the stock market is available, versus the case when wealth can only be invested in the riskless asset. This is done in figure 3.1 that plots the consumption policy rule for age 25 for the mean habit level, under these two alternative scenarios. We conclude that total saving rises in the presence of a higher-earning investment opportunity and therefore the substitution effect outweighs the income effect. Recall that, in the standard textbook two period model with complete markets (and in particular, without non-tradable labor income in the second period), the opposite result is predicted for  $\rho = 2$ . This “reversal” is already present in the life-cycle model without habit formation, but we want to point it out because it will have important

implications for wealth accumulation and the stock market participation rate in the presence of the fixed, stock market entry cost.

### 4.3 Simulation Results

We first plot the evolution for the distributions of cash on hand for the two types of agents: participants and non-participants in the stock market. Households start with zero assets and the initial habit level is set at the lowest point on the habit grid which corresponds to around 25% of mean initial earnings (we later show that the implications for stock market participation and asset allocation are similar if households start from a higher initial habit level). Figure 3.2 illustrates some of the results by plotting the distributions of normalized cash on hand for individuals aged 30 who have incurred the fixed cost and those who have not; the distributions are conditional on the mean habit level to avoid cluttering. These are conditional distributions and the participation rate can be used as a probability weight to generate the unconditional distribution of cash on hand in the cohort. The divergence of the distributions of cash on hand between participants and non-participants is especially significant if one considers that they both start from identical assets, face the same labor income process, have the same preferences and this discrepancy takes place within 10 years from the beginning of working life. These results can be explained by two factors. First, we have just shown that a higher saving rate arises for stock market participants than for non-participants (figure 3.1), thus generating higher wealth accumulation. Second, and perhaps more importantly, given the volatility of the stock market and the high equity premium, wealth invested in the stock market will be subject to higher volatility than wealth invested in the safe asset generating substantial inequality over time between stock market participants and non-participants. Figure 3.3 plots the distributions of cash on hand conditional on the mean habit for age 65 for stockholders (everyone has incurred the participation cost by that time and is a stockholder).

Figure 3.4 plots the unconditional mean consumption and habits over the life-cycle. The distributions for participants and non-participants are used to compute the respective means in the two types and then the proportion of stockholders (the participation rate) is used to

take the unconditional average over the two groups<sup>14</sup>. Figure 3.4 shows that the habit quickly catches up with consumption early in life, and then tracks it closely over the life-cycle.

We next plot the stock market participation rate over the life cycle (figure 3.5). Stock market participation takes place very quickly in this model, as by age 25 all households already find it advantageous to pay the fixed cost. This is explained by the wealth accumulation profiles in figure 2.3: the stronger the habit, the higher the saving rate for any given level of cash on hand and the stronger the incentive to enter the stock market.

Finally, figure 3.6 plots the unconditional portfolio allocation over the life-cycle<sup>15</sup> and the portfolio allocation conditional on participating in the stock market.<sup>16</sup> The asset allocation conditional on participation remains (counterfactually) skewed towards stocks. For the unconditional distribution the share of wealth in stocks does rise over the life-cycle, but does so at a very fast rate, governed entirely by the low participation rate observed during the early working life period.

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<sup>14</sup>Specifically, let  $Part_t$  denote the stock market participation rate in the cohort at time (age)  $t$ , and  $G_x$  and  $G_h$  denote the number of grid points used to discretize the normalized state variables  $x$  (cash on hand) and  $h$  (habits), respectively. The unconditional mean consumption for age  $t$  can then be computed as

$$\bar{c}_t = Part_t \left\{ \sum_{k=1}^{G_x} \sum_{l=1}^{G_h} \pi_{t,kl}^I * c^I(x_k, h_l, t) \right\} + (1 - Part_t) \left\{ \sum_{k=1}^{G_x} \sum_{l=1}^{G_h} \pi_{t,kl}^O * c^O(x_k, h_l, t) \right\}$$

Superscript  $I$  denotes a variable for households participating in the stock market while superscript  $O$  denotes households out of the stock market.

<sup>15</sup>For a given age  $t$ , the unconditional portfolio allocation is computed as:

$$\frac{Part_t * \left\{ \sum_{k=1}^{G_x} \sum_{l=1}^{G_h} \pi_{t,kl}^I * \alpha^I(x_k, h_l, t) * (x_k - c^I(x_k, h_l, t)) \right\}}{Part_t * \sum_{k=1}^{G_x} \sum_{l=1}^{G_h} [\pi_{t,kl}^I * (x_k - c^I(x_k, h_l, t))] + (1 - Part_t) * \sum_{k=1}^{G_x} \sum_{l=1}^{G_h} [\pi_{t,kl}^O * (x_k - c^O(x_k, h_l, t))]}$$

<sup>16</sup>For a given age  $t$ , the portfolio allocation conditional on participation is computed as:

$$\frac{\left\{ \sum_{k=1}^{G_x} \sum_{l=1}^{G_h} \pi_{t,kl}^I * \alpha^I(x_k, h_l, t) * (x_k - c^I(x_k, h_l, t)) \right\}}{\left\{ \sum_{k=1}^{G_x} \sum_{l=1}^{G_h} \pi_{t,kl}^I * (x_k - c^I(x_k, h_l, t)) \right\}}$$

## 4.4 Comparison with the No-Habits Model

Figure 4.1 plots the stock market participation rates over the life-cycle for the two models. In the no-habits case the typical household stays out of the stock market very early in the life cycle until enough wealth has been accumulated to warrant paying the participation fee. As time goes by, a higher percentage of households enters the stock market, and by age 40 everybody has become a stock holder. In the habit model, on the other hand, investors accumulate a much larger buffer stock of wealth at younger ages, and therefore they have a stronger incentive to pay the fixed cost and invest in the stock market. As a result, stock market participation is higher for any given age the stronger the habit (figure 4.1).

Figure 4.2 compares the unconditional mean share of wealth in stocks implied by the different parameter values. The results are just what we would expect after observing figure 4.1, and give little support to the habit formation model as a realistic description of life-cycle portfolio allocation behavior. Introducing habit formation preferences increases the participation rate significantly without improving the model's performance with respect to equity allocation conditional on participation. More precisely, the habit model still predicts that, after paying the fixed cost, the average household will invest virtually all of its wealth in the stock market, at every stage of the life cycle.

## 4.5 Robustness Checks

We now perturb some of the parameters in the model to investigate the robustness of our results.

### 4.5.1 Initial Habit Levels

We first investigate the robustness of our conclusions with respect to a change in initial habit levels. Starting with higher initial habit levels generates lower wealth accumulation early in life than in the benchmark model as the household has to maintain higher consumption to support the higher inherited habit level. Figure 5.1 plots the unconditional mean wealth profiles for the benchmark case, and the cases with initial habits equal to 80 and 100 percent of mean expected labor income, respectively. Higher initial assets generate lower wealth

accumulation as the household needs to maintain a higher consumption level to satisfy the higher initial habit level. Moreover, the difference in the profiles is starkest earlier in life when initial assets are low and the differences get smaller as the average household approaches retirement. Figure 5.2 plots the corresponding stock market participation rates over the life cycle. The lower wealth accumulation associated with the higher initial habits does not affect substantially the decision of whether to incur the fixed cost and enter the stock market since stock market participation is complete within the first five years of working life for all three specifications. The complete portfolio specialization in stocks result is also preserved (not reported for space considerations), leading us to conclude that the asset allocation (conditional on participation) counterfactual implications of the model persist. We conclude that our results are robust to changes in the initial conditions for habit levels.

#### 4.5.2 Equity Premium

We consider equity premia of 2.5 and 5.5 percent (the benchmark is 4 percent).<sup>17</sup> Increasing the equity premium generates higher wealth accumulation because the substitution effect in the model is stronger than the income effect. Stock market participation is naturally increasing in the equity premium but the quantitative effects are very similar across the calibrations. For instance, for the lowest equity premium everybody has incurred the fixed cost (and holds stocks) by age 25, whereas for the highest equity premium this happens by age 24, practically an identical prediction. Asset allocation conditional on participation does not change much in quantitative terms either: the almost complete portfolio specialization in stocks persists for all equity premia throughout the life cycle. Specifically, by age 65 the share of wealth in stocks from the lowest to the highest equity premium is 99.5%, 98% and 96% respectively and the increased diversification as one approaches retirement arises from the higher wealth accumulation associated with the higher equity premium.

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<sup>17</sup>We do not report the results in figures or tables due to space considerations but instead summarize them in the text.

### 4.5.3 Replacement Ratio

We consider two deviations from the benchmark replacement ratio of 68%: a lower level of 45% and an upper level of 85%.<sup>18</sup> A lower replacement ratio during retirement generates higher saving during working life and therefore a stronger incentive to participate in the stock market early on and vice versa for a higher replacement ratio. Quantitatively, however, the implications of the model are again very similar, as in both cases by age 25 everyone in the economy has incurred the fixed cost and has become a stock market participant.

With regards to portfolio allocation conditional on participation, the higher saving associated with the lower replacement ratio induces the household to invest partially in the riskless asset. Nevertheless, the complete portfolio specialization in stocks persists until age 43 and only decreases to 91% by age 65. For the highest replacement ratio model, the complete portfolio specialization in stocks persists throughout the life cycle.

### 4.5.4 CRRA coefficient

We have increased the CRRA coefficient from the benchmark of  $\rho = 2$  to  $\rho = 5$  and  $\rho = 8$ . Higher risk aversion reduces the willingness to hold risky assets but it also implies higher prudence, and thus the household accumulates more wealth over the life cycle. Higher wealth accumulation increases the incentive to incur the fixed cost and actually outweighs the disincentive implied by higher risk aversion. With regards to the decision to incur the fixed cost, the results are similar across parameters because stock market participation takes place very fast (for  $\rho = 2$  everyone has incurred the fixed cost by age 23 while this happens by age 24 for  $\rho = 5$  and  $\rho = 8$ ).

The effects on portfolio choice are more substantial and are shown in figure 5.3. In the beginning of the life cycle, when financial assets as a percentage of total wealth are small, the portfolio is tilted completely in the risky financial asset. For the benchmark model, this result persists almost throughout the life cycle. For the higher risk aversion coefficients, however, the higher wealth accumulation implied by the model and the reduced willingness of

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<sup>18</sup>We do not report the results in figures or tables due to space considerations but instead summarize them in the text.

the household to hold the risky asset, generates a substantial decrease in the share of wealth held in the risky asset market as the household ages. It is instructive to point out the conflict between risk aversion and prudence in matching jointly the stock market participation and the asset allocation decision. Increasing risk aversion generates a higher co-existence between stocks and the riskless asset in the portfolio, but a higher risk aversion also implies higher prudence and therefore higher wealth accumulation increasing the incentive to incur the fixed cost and participate in the stock market.

#### **4.5.5 Discount Rate**

How does the discount rate affect the stock market participation and asset allocation decision? We have increased the discount rate to 7 and 10 percent from the benchmark 5 percent level and the quantitative conclusions from the model remain unchanged. Increased impatience generates lower wealth accumulation and therefore a weaker incentive to incur the fixed cost. Nevertheless, the only change is that everyone incurs the fixed cost by age 25 for  $\delta = 0.07$  and  $\delta = 0.1$  whereas this happens by age 24 for the benchmark  $\delta = 0.05$  (see figure 5.4). Moreover, because of the lower wealth accumulation implied by the higher discount rates, the complete portfolio specialization in stocks result becomes even stronger: for the ten percent discount rate the share of wealth allocated to stocks remains at 100% throughout the life cycle.

#### **4.5.6 Correlation between Permanent Earnings Shocks and Stock Returns**

Consistent with previous results in the literature, the correlation between the transitory and the stock return innovations does not affect the results and is therefore not reported. The correlation between permanent earnings and stock return shocks can, on the other hand, generate substantial hedging demands. We have discussed in the calibration section our choice of the benchmark correlation equal to zero and why an upper bound of around 0.3 might be a reasonable choice based on the empirical evidence. Figures 5.5 and 5.6 compare the unconditional portfolio allocations over the life cycle for  $\rho = 2$  and  $\rho = 5$ , respectively, for the two cases where the correlation is zero and 0.3 (all other parameters are set at their

benchmark values). With regards to the stock market participation decision, the presence of positive correlation does not affect total savings, regardless of the coefficient of relative risk aversion, and the stock market participation decisions are not therefore affected: stock market participation is at 100% within the first 5 years of the working life. The correlation, however, can potentially have important effects on the asset allocation decision. Consistent with previous results in the literature this correlation is not very important for low values of risk aversion. Figure 5.5, for instance, shows that the share of wealth in stocks is not affected much relative to the benchmark zero correlation case. The correlation can have a bigger effect for higher risk aversion coefficients, as is illustrated in figure 5.6. Nevertheless, even for this configuration of parameter values the hedging demand generated by this correlation is very small in magnitude.

## 5 Additive Habit Model

### 5.1 Specification, Risk aversion and Prudence

Until now, we have assumed a ratio habit model: surplus consumption (the argument in the utility function) is given by the ratio between consumption and the habit level. We now consider an “additive habit model”: surplus consumption is given by the difference between consumption and the habit persistence term (following Sundaresan (1989), Constantinides (1990) and Detemple and Zapatero (1991), among others). More precisely, the felicity function is given by:

$$U(C_t) = \frac{\{C_t - \gamma H_t\}^{1-\rho}}{1-\rho} \quad (21)$$

Unlike in the previous specification, both risk aversion and prudence are now dependent on the level of surplus wealth. The coefficient of relative risk aversion ( $ra_t$ ) is given

$$ra_t = \rho \frac{C_t}{C_t - \gamma H_t} \quad (22)$$

while the coefficient of prudence ( $pr_t$ ) is:

$$pr_t = (\rho + 1) \frac{C_t}{C_t - \gamma H_t} \quad (23)$$

As consumption falls towards the habit level, the investor simultaneously becomes more risk averse and more prudent.<sup>19</sup> In the context of a life-cycle model this implies that both prudence and risk aversion will vary with age.

## 5.2 Baseline Results

Relative to the no-habit case ( $\gamma = 0$ ) wealth accumulation now increases for two reasons: in addition to the change in the EIS already present in the ratio habit model, households also become more prudent. From equation (22) we see that risk aversion is a decreasing function of surplus consumption ( $C_t - \gamma H_t$ ) so we expect households to reduce their demand for stocks. In theory, if this effect is strong enough then it could even affect the participation decision early in life.

Figure 6.1 plots the life-cycle patterns of risk-aversion and prudence for the average household, for  $\rho = 2$ ,  $\gamma = 0.5$  and  $\lambda = 0.5$ . The two coefficients move together since, conditional on  $C_t$  and  $H_t$ , one is a linear transformation of the other. Since (for now) we set  $H_1 = 0$ , risk aversion and prudence at age 20 are equal to their CRRA values, 2 and 3 respectively. As surplus consumption ( $C_t - \gamma H_t$ ) decreases both coefficients increase. This effect is quite strong early in life since the habit level is rising very fast, and as a result risk aversion nearly doubles from age 20 to age 30. However, it remains almost unchanged during the rest of the life-cycle, as agents have already accumulated enough wealth to smooth consumption over time. Based on these results, the difference habit model with  $\rho = 2$  should generate results which will be very similar to the ones obtained with a ratio habit model and  $\rho = 4$ , with some small differences during the first few years of the life-cycle.

As expected, the model predicts a counterfactually high wealth accumulation (figure 6.2) and this induces households to pay the entry cost earlier in life (figure 6.3). Finally, the average share of wealth invested in stocks is close to 100% at every age (figure 6.3).

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<sup>19</sup>Naturally, if we set  $\gamma = 0$  we get the standard CRRA results.

## 5.3 Sensitivity Analysis

In this section we restrict the sensitivity analysis to the most important new parameters. Results for the remaining comparative statics are identical to the ones obtained with the ratio habit model and they are available upon request.

### 5.3.1 Initial habit level

The pattern of risk aversion in figure 6.1 is driven by our assumption of a zero initial habit level, at least during the first years of the life-cycle. If we increase the initial habit we reduce surplus wealth of the young households, and consequently increase their risk aversion. Figure 6.4 plots risk-aversion over the life-cycle, for different levels of the initial habit stock. With  $H_1 = 0.8$ <sup>20</sup> the profile is almost flat, and only if we increase  $H_1$  to 1.0 do we obtain a modest increase in risk aversion at age 20. However, even in this case, risk aversion only increases to 4.5, and converges to 4 within 3 years. Figure 6.5 plots the average wealth share invested in stocks for the three different cases and, as expected from the previous results, there is no significant change.

### 5.3.2 Curvature Parameter

Increasing the curvature parameter ( $\rho$ ) yields the same results as in the ratio habit model. Figure 6.6 compares the life-cycle profile of risk aversion for  $\rho = 5$  and  $\rho = 2$ . The two curves are almost identical, except for the scaling factor ( $\rho$ ). With  $\rho = 5$  the investor is more prudent and as a result she accumulates more wealth (figure 6.7). The participation rate (not shown) is virtually unchanged as the wealth effect and the risk aversion effect offset each other almost completely. However, they both lead to reduction of the share of wealth invested in stocks, which is visible in figure 6.8.

### 5.3.3 Habit Strength

A higher value of  $\gamma$  increases both risk aversion and prudence, just like a higher  $\rho$ , but it also makes the investor more willing to smooth consumption intertemporally. Figure 6.9 plots

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<sup>20</sup>This corresponds to 80% of the first year's expected income.

risk aversion for the average household, for  $\rho = 2$  and four different values of  $\gamma$  (0, 0.3, and 0.5). In all cases we set  $H_1 = 0.0$ , but the results are not very sensitive to this choice.<sup>21</sup> As shown before, as we increase the habit motive, the investor accumulates more wealth early in life and therefore she has stronger incentive to pay the fixed cost. This effect is even stronger in the difference habit model, since now a higher  $\gamma$  also implies more prudence with the corresponding increase in precautionary savings. As a result, as shown in figure 6.10, stock market participation rate is in an increasing function of  $\gamma$ .

Since we are considering a difference habit specification, if we increase both  $H_1$  and  $\gamma$  substantially then we can mechanically exclude all households from the stock market, by forcing them to consume all of their income just to keep up with their habit level. However, since labor income is stochastic, this also implies that some households will be unable to meet their minimum consumption requirement, and with this preference specification such an event generates a penalty of minus infinity. Naturally, this creates serious problems for the solution of the model. If the investor cannot rule out states with minus infinity utility then the optimum is not well-defined. We believe that this is also not an economically interesting result as "minus infinity utility" states do not seem to occur often in reality. Moreover, as previously shown, this result does not hold in the ratio habit model, in which this penalty is not equal to minus infinity, or in the difference habit model with  $\gamma$  less than 0.5. Therefore, we choose to limit our analysis to parameter choices for which the probability of those events occurring is either zero or negligible.

## 6 Conclusion

Motivated by the success of models with internal habit formation preferences in solving asset pricing puzzles, we introduce these preferences in a life-cycle portfolio choice model with uninsurable labor income risk and stock market participation costs. If internal habits can explain the equity premium for a given consumption process, then, for a given asset return process, these preferences should also explain the observed low stock market participation rates and portfolio allocations conditional on participation. However, in contrast to this

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<sup>21</sup>More on this on the next paragraph.

intuition, we find that habit formation preferences actually *decrease* the model's ability to match the existing empirical evidence. Intertemporal consumption smoothing becomes more important and households increase their wealth accumulation. As a result, they have a stronger incentive to pay the stock market entry cost and start investing in equities very early in life. Moreover, after paying the fixed cost, they invest virtually all of their wealth in stocks. We conclude that internal habit formation preferences on their own are unlikely to resolve existing portfolio allocation puzzles.

Our results are not inconsistent with the asset pricing literature. First, we have narrowed our analysis to internal habit formation models. We have done so because considering external habits in the context of a partial-equilibrium model would be a substantial extension since the aggregate consumption process cannot be taken as exogenous in this model: aggregate consumption must be the outcome of individual consumption decisions. If aggregate earnings shocks are included and aggregate consumption is time varying and affecting the external habit, then agents need to form expectations about the future evolution of this habit. Unlike the Campbell-Cochrane (1999) external habit model, endogenous aggregate consumption generates an endogenous evolution of the habit and it is not clear what agents' expectations should be to be validated in equilibrium in this heterogeneous agent economy. We think that this is a fruitful area for future research but is beyond the scope of this paper. It should also be pointed out that we have not investigated the implications of stock market mean reversion in this model but we also think that this extension and model uncertainty on behalf of investors might be fruitful areas of future research.

Our results are not inconsistent with the asset pricing literature in a second, arguably more important, dimension. Specifically, habit formation models explain the equity premium puzzle by making investors less willing to hold stocks such that, in equilibrium, equities must offer a higher risk premium for markets to clear. However, in this paper we require the habit formation model to go one step further. Rather than decreasing the willingness to hold stocks by any means, we require the model to generate simultaneously modest stock market participation rates in the population as a whole, and large riskless asset holdings for stock market participants. We find that the model can potentially deliver the latter, which by itself is enough for the risk premium result, but this comes at the expense of not matching

observed median stockholding patterns.

## Appendix A: Numerical Solution

We first simplify the solution by exploiting the scale-independence of the maximization problem and rewriting all variables as ratios to the permanent component of labor income ( $P_{it}$ ). The laws of motion and the value function can then be rewritten in terms of these normalized variables, and we use lower case letters to denote them (for instance,  $x_{it} \equiv \frac{X_{it}}{P_{it}}$ ). This allows us to reduce the number of state variables to four; two continuous state variable (cash on hand and habit) and two discrete state variables (age and participation status). We discretize the state-space along the cash-on-hand and habit level dimensions (the two continuous state variables), so that the relevant policy functions can now be represented on a numerical grid.

We solve the model using backward induction. In the last period ( $t = T$ ) the policy functions are trivial, as the agent consumes all available wealth,  $c_T = x_T$ . As a result the value function corresponds to the indirect utility function,  $V_T(x_T, \cdot) = V(x_T)$ , regardless of whether the fixed cost has been paid before or not, and regardless of the habit level. For every age  $t$  prior to  $T$ , and for each point in the state space, we optimize using grid search. So we need to compute the value associated with each level of consumption, the decision to pay the fixed cost, and the share of liquid wealth invested in stocks. From the Bellman equation these values are given as current utility plus the discounted expected continuation value ( $E_t V_{t+1}(\cdot, \cdot)$ ), which we can compute since we have just obtained  $V_{t+1}$ . We perform all numerical integrations using Gaussian quadrature to approximate the distributions of the innovations to the labor income process and the risky asset returns. We evaluate the value function, for points which do not lie on state space grid, using a cubic spline interpolation.

Once we have computed the value of all the alternatives we just pick the maximum, thus obtaining the policy rules for the current period ( $S_t$ , and  $B_t$ ). At each point of the state space, the participation decision is computed by comparing the value function conditional on having paid the fixed cost (adjusting for the payment of the cost itself) with the value function conditional on non-payment. Substituting these decision rules in the Bellman equation we obtain this period's value function ( $V_t(\cdot, \cdot)$ ), which is then used to solve the previous period's maximization problem. This process is iterated until  $t = 1$ .

## Appendix B: Computing the Transition Distributions

To find the distribution of cash on hand, we first compute the relevant optimal policy rules; bond and stock policy functions for stock market participants and non-participants and the  $\{0, 1\}$  participation rule as a function of normalized cash on hand and normalized habits. Let  $b^I(x, h)$  and  $s^I(x, h)$  denote the bond and stock policy rules respectively for individuals participating in the stock market and let  $b^O(x, h)$  be the savings (bonds) decision for the individual out of the stock market. We assume that households start working life with zero liquid assets from the lowest habit state and first use the bond market as a saving vehicle on account of the fixed, one time, stock market entry cost.

For bond holders in working life, the evolution of normalized cash on hand is given by<sup>22</sup>

$$\begin{aligned} x_{t+1} &= [b^O(x_t, h_t)R_f]\left\{\frac{P_t}{P_{t+1}}\right\} + U_{t+1} \\ &= w(x_t, h_t|\frac{P_t}{P_{t+1}}) + U_{t+1} \end{aligned}$$

where  $w(x, h)$  is defined by the last equality and is conditional on  $\{\frac{P_t}{P_{t+1}}\}$  that includes the deterministically evolving age-specific growth rate of individual labor income  $f(t, Z_t)$ . The normalized habit, moreover, follows

$$h_{t+1} = \{(1 - \lambda)h_t + \lambda c_t(x_t, h_t)\} * \left\{\frac{P_t}{P_{t+1}}\right\}$$

Denote the transition matrix of moving from  $x_j$  to  $x_k$  and  $h_l$  to  $h_m$ ,<sup>23</sup> conditional on being in the bond market as  $T_{kj,ml}^O$ . Let  $\{\Delta x, \Delta h\}$  denote the distance between the equally spaced discrete points of cash on hand and habit, respectively. The random permanent shock  $\frac{P_t}{P_{t+1}}$  is discretized using  $J = 7$  grid points:  $\frac{P_t}{P_{t+1}} = \{N_m\}_{m=1}^{m=J}$ .  $T_{kj,ml}^O = \Pr(x_{t+1}=k, h_{t+1}=m|x_t=j, h_t=l)$

<sup>22</sup>To avoid cumbersome notation, the subscript  $i$  that denotes a particular individual is omitted in what follows.

<sup>23</sup>The normalized cash on hand grid is discretized between (xmin,xmax) where xmin denotes the minimum point on the equally spaced cash on hand grid and xmax the maximum point. Similarly, the normalized habit grid is discretized between (hmin,hmax) where hmin denotes the minimum point on the equally spaced habit grid, and hmax denotes the maximum point.

is found using<sup>24</sup>

$$\sum_{n=1}^{n=J} \Pr(x_{t+1}, h_{t+1} | x_t, h_t, \frac{P_t}{P_{t+1}} = N_n) * \Pr(\frac{P_t}{P_{t+1}} = N_n) \quad (24)$$

Letting the total number of cash on hand grid points equal to  $G_x$  and the total number of habit grid points equal to  $G_h$ , we can construct the transition matrix from one combination of cash and habits to another as a  $G_x * G_h$  by  $G_x * G_h$  matrix. Numerically, probability (24) is calculated using

$$T_{kj,ml,n}^O = \Pr(x_k + \frac{\Delta x}{2} \geq x_{t+1} \geq x_k - \frac{\Delta x}{2}, h_m + \frac{\Delta h}{2} \geq h_{t+1} \geq h_m - \frac{\Delta h}{2} | x_t = x_j, h_t = h_l, \frac{P_t}{P_{t+1}} = N_n)$$

Given that we have conditioned on the permanent innovation,  $h_{t+1}$  is known as of time  $t$ , and we can therefore compute this probability by keeping track of the frequency with which a particular interval in normalized habits is “visited”, while making use of the approximation that for small values of  $\sigma_u^2$ ,  $U \sim N(\exp(\mu_u + .5 * \sigma_u^2), (\exp(2 * \mu_u + (\sigma_u^2)) * (\exp(\sigma_u^2) - 1)))$ . Denoting the mean of  $U$  by  $\bar{U}$  and its standard deviation by  $\sigma$ , the transition probability conditional on  $N_n$  equals

$$T_{kj,ml,n}^O = \Phi\left(\frac{x_k + \frac{\Delta}{2} - w(x_t, h_t | N_n) - \bar{U}}{\sigma}\right) - \Phi\left(\frac{x_k - \frac{\Delta}{2} - w(x_t, h_t | N_n) - \bar{U}}{\sigma}\right)$$

if  $h_m + \frac{\Delta h}{2} \geq h_{t+1} \geq h_m - \frac{\Delta h}{2}$  and is zero otherwise ( $\Phi$  is the cumulative distribution function for the standard normal). The unconditional probability from  $\{x_j, h_l\}$  to  $\{x_k, h_m\}$  is then given by

$$T_{kj,ml}^O = \sum_{n=1}^{n=J} T_{kj,ml,n}^O \Pr(N_n) \quad (25)$$

Given the transition matrix  $\mathbf{T}^O$  (a  $G_x * G_h$  by  $G_x * G_h$  matrix with  $T_{kj}^O$  representing the  $\{k^{th}, j^{th}\}$  element), the next period probabilities of each of the cash on hand and habit states can be found using an appropriating reshaping of

$$\pi_{kmt}^O = \sum_{j,l} T_{kj,ml}^O * \pi_{jlt-1}^O \quad (26)$$

---

<sup>24</sup>The dependence on the deterministically evolving  $\frac{\exp(f(t, Z_t))}{\exp(f(t+1, Z_{t+1}))}$  is implied and is omitted from what follows for expositional clarity.

We next use the vector  $\mathbf{\Pi}_{th}^O$  (this is a  $G_x$  by 1 vector representing the mass of the population out of the stock market at each grid point conditional on a particular habit stock  $h$ ) and the participation policy rule to determine the percentage of households that optimally choose to incur the fixed cost and participate in the stock market. This is found by computing the sum of the probabilities in  $\mathbf{\Pi}_{th}^O$  for which  $x > x_h^*$ ,  $x_h^*$  being the trigger point that causes participation ( $x_h^*$  is determined endogenously through the participation decision rule). These probabilities are then deleted from  $\mathbf{\Pi}_{th}^O$  and are added to  $\mathbf{\Pi}_{th}^I$ , and this is repeated for all habit states. Finally, these conditional distributions  $\{\mathbf{\Pi}_t^O, \mathbf{\Pi}_t^I\}$  are appropriately renormalized to sum to one.

The participation rate can be computed at this stage as

$$Part_t = Part_{t-1} + (1 - Part_{t-1}) * \sum_h \sum_{x_j > x_h^*} \pi_{t,jh}^O$$

The same methodology (but with more algebra and computations) can then be used to derive the transition distribution for cash on hand conditional on being in the stock market  $\mathbf{T}_t^I$ . For stock market participants, the normalized cash on hand evolution equation is

$$\begin{aligned} x_{t+1} &= [b(x_t, h_t)R_f + s(x_t, h_t)\tilde{R}_{t+1}]\left\{\frac{P_t}{P_{t+1}}\right\} + U_{t+1} \\ &= w(x_t, h_t|\tilde{R}_{t+1}, \frac{P_t}{P_{t+1}}) + U_{t+1} \end{aligned} \quad (27)$$

where  $w(x)$  is now conditional on  $\{\tilde{R}_{t+1}, \frac{P_t}{P_{t+1}}\}^{25}$ . The random processes  $\tilde{R}$  and  $\frac{P_t}{P_{t+1}}$  are discretized using Gaussian quadrature with  $J$  grid points respectively:  $R = \{R_z\}_{z=1}^{z=J}$  and  $\frac{P_t}{P_{t+1}} = \{N_n\}_{n=1}^{n=J}$ .  $T_{kj,ml}^I = \Pr(x_{t+1}=k, h_{t+1}=m|x_t=j, h_t=l)$  is found using

$$\sum_{z=1}^{z=J} \sum_{n=1}^{n=J} \Pr(x_{t+1}, h_{t+1}|x_t, h_t, \tilde{R}_{t+1} = R_z, \frac{P_t}{P_{t+1}} = N_n) * \Pr(\tilde{R}_{t+1} = R_z) * \Pr(\frac{P_t}{P_{t+1}} = N_n) \quad (28)$$

where the independence of  $\frac{P_t}{P_{t+1}}$  from  $\tilde{R}_{t+1}$  was used. Numerically, this probability is calculated using

$$\begin{aligned} T_{kj,ml,n}^I &= \Pr(x_k + \frac{\Delta x}{2} \geq x_{t+1} \geq x_k - \frac{\Delta x}{2}, h_m + \frac{\Delta h}{2} \geq h_{t+1} \geq h_m - \frac{\Delta h}{2} \\ |x_t &= x_j, h_t = h_l, \frac{P_t}{P_{t+1}} = N_n, R_{t+1} = R_l) \end{aligned}$$

---

<sup>25</sup>The dependence on the non-random earnings component is omitted to simplify notation.

The transition probability, conditional on  $N_n$  and  $R_z$  equals

$$T_{kj,ml}^I = \Phi \left( \frac{x_k + \frac{\Delta}{2} - w(x_t|N_n, R_z) - \bar{U}}{\sigma} \right) - \Phi \left( \frac{x_k - \frac{\Delta}{2} - w(x_t|N_n, R_z) - \bar{U}}{\sigma} \right)$$

The unconditional probability from  $x_j$  to  $x_k$  is then given by

$$T_{kj,ml}^I = \sum_{z=1}^{z=J} \sum_{n=1}^{n=J} T_{kj,ml}^I \Pr(N_n) \Pr(R_z) \quad (29)$$

Given the matrix  $\mathbf{T}^I$ , the probabilities of each of the states are updated by

$$\pi_{km,t+1}^I = \sum_{j,l} T_{kj,ml}^I * \pi_{jl,t}^I \quad (30)$$

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Figure 1.1: Normalized Consumption (Varying Habit Level, Age 25)

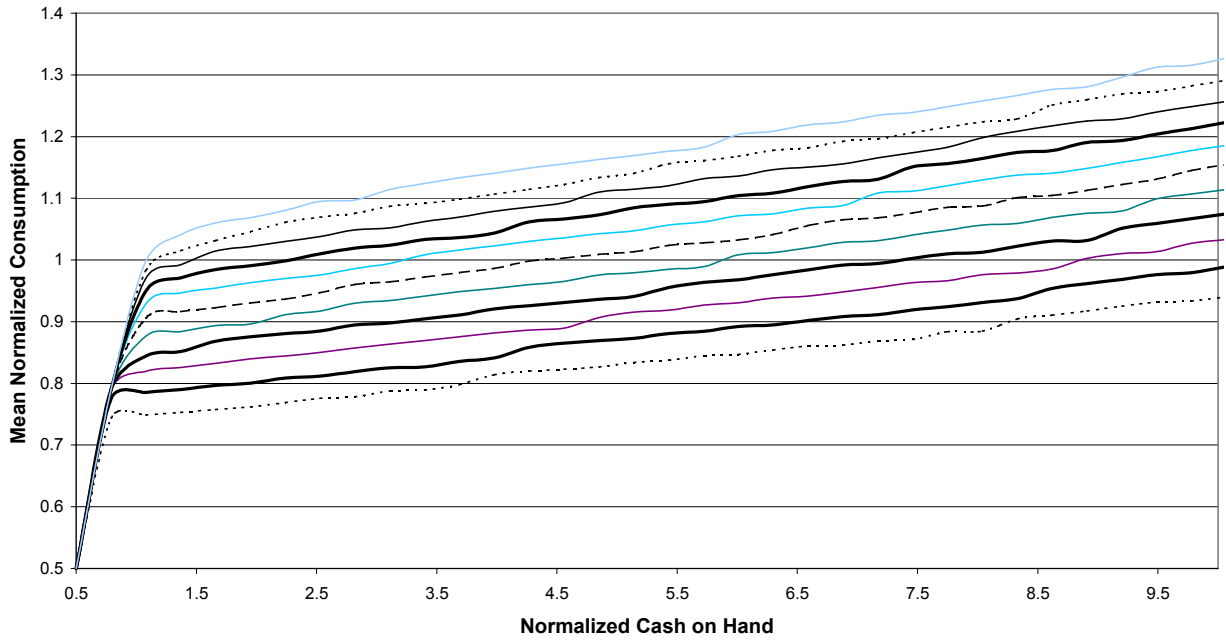


Figure 1.2: Consumption at Mean Habits vs Consumption with No Habits (Age 25)

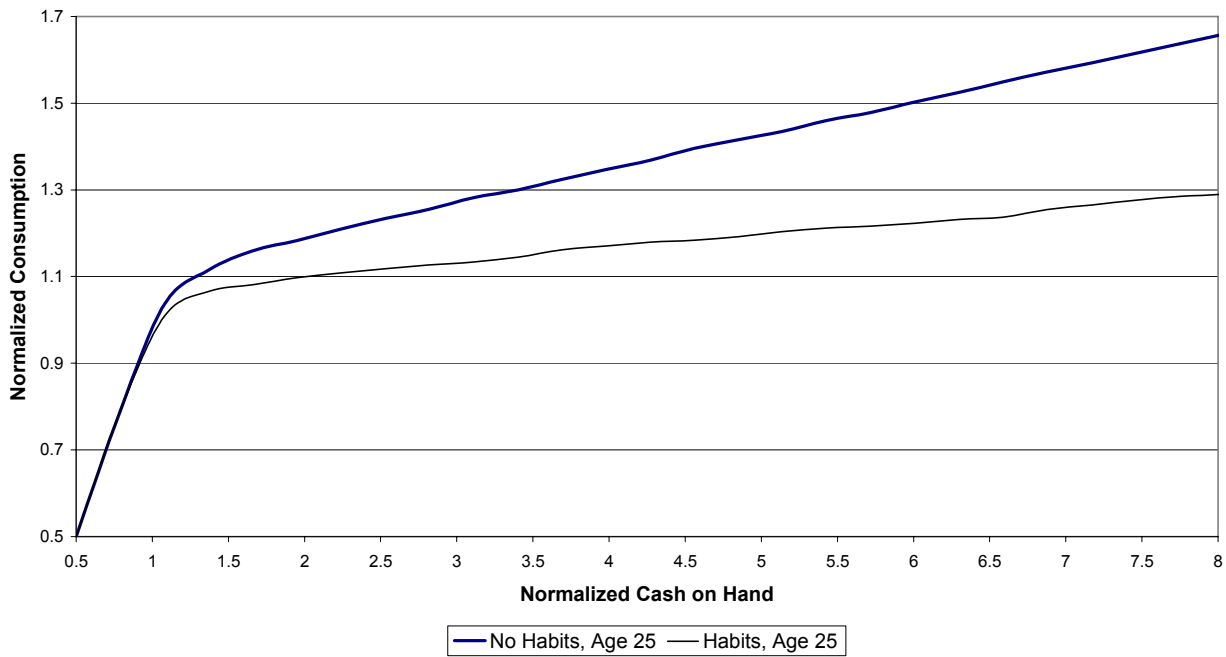


Figure 1.3: Share of Wealth in Stocks (age 25)

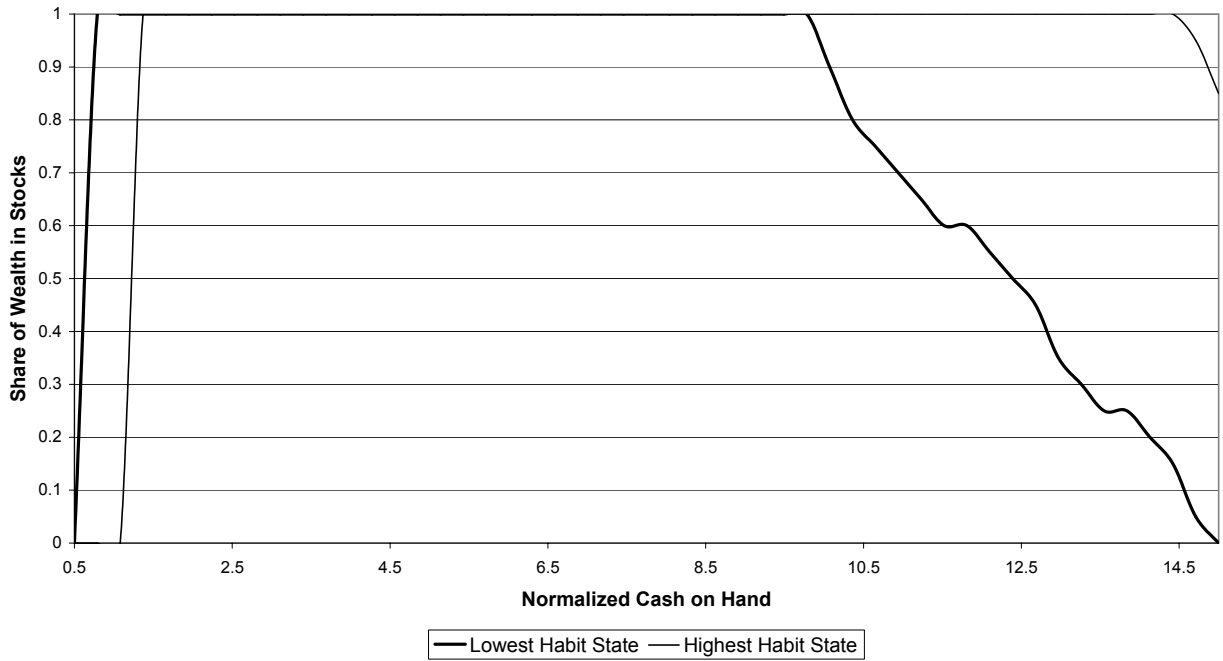
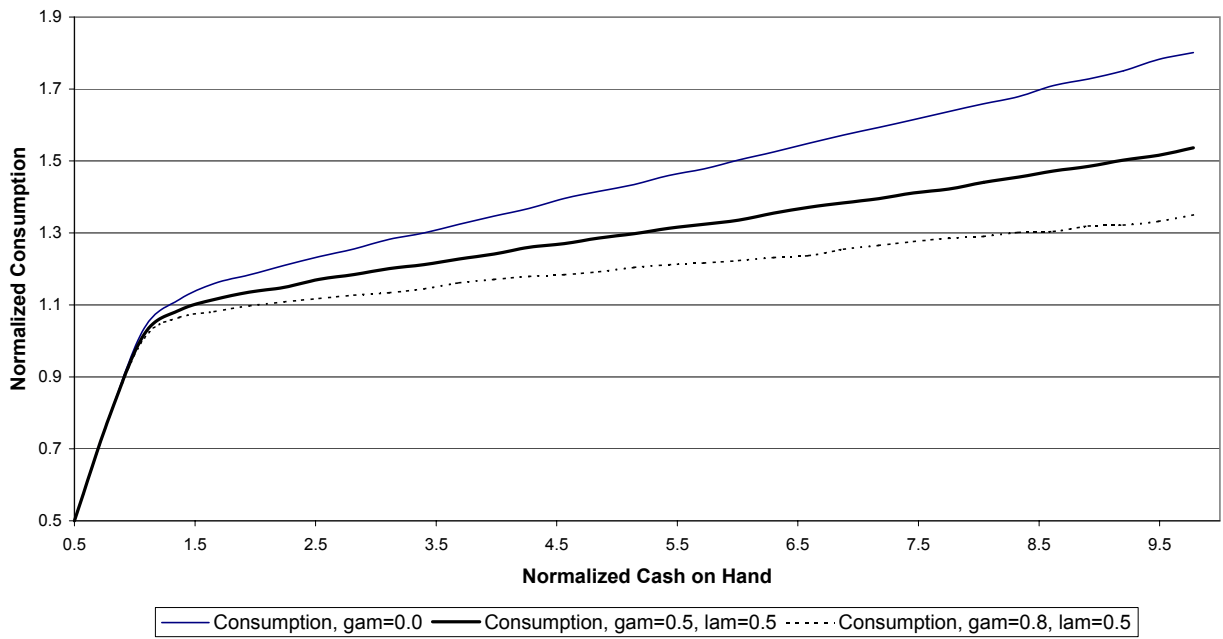
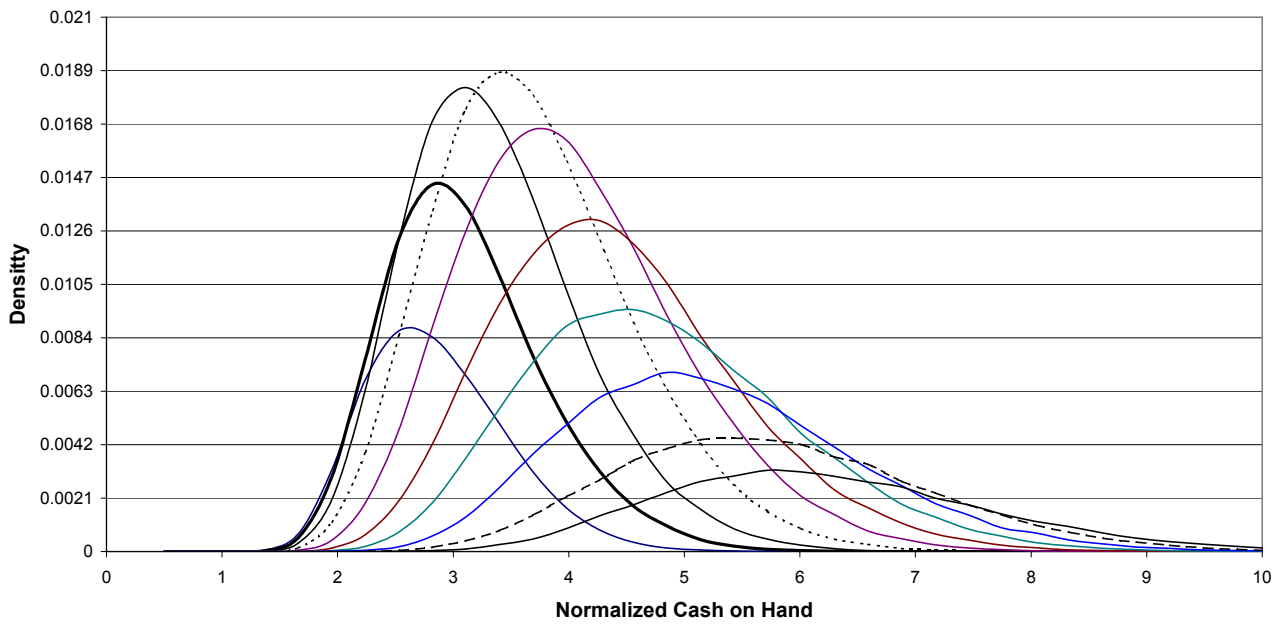


Figure 1.4: Normalized Consumption Varying Habit Strength (Age 25)



**Figure 2.1: Distributions of Normalized Cash on Hand, Conditional on Habit Level at Age 50, No Fixed Cost**



**Figure 2.2: Distributions for Normalized Cash on Hand (Retirement stage) Model without the fixed cost, At Habit = 1.02**

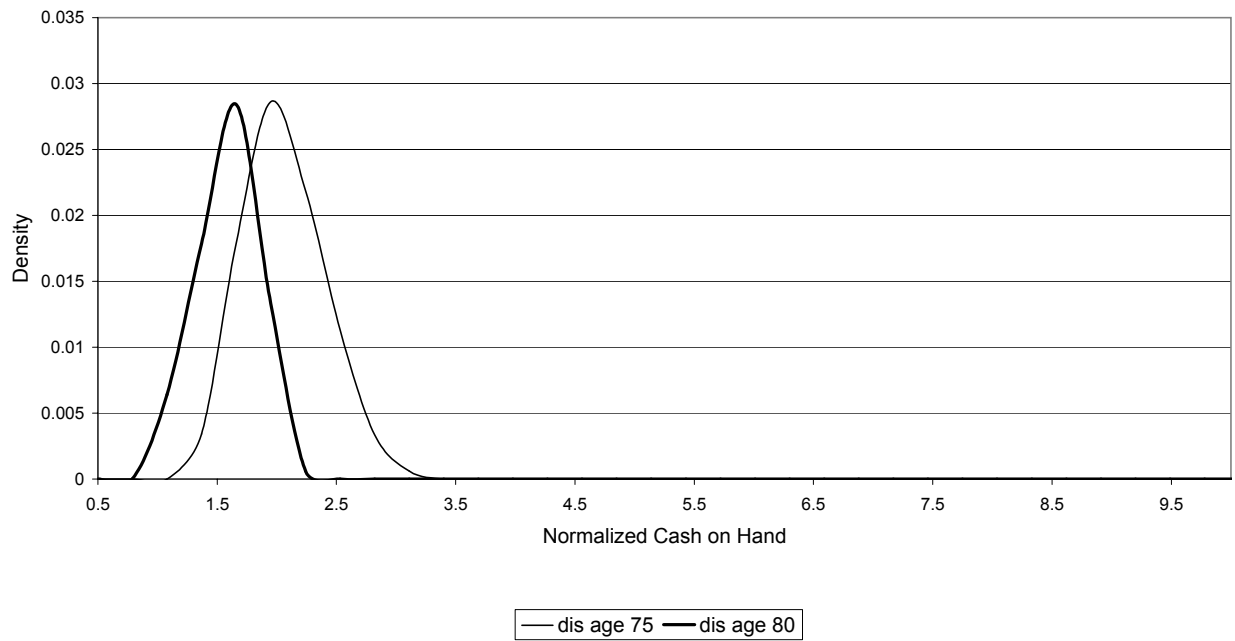


Figure 2.3: Wealth Accumulation without the fixed cost: Habit vs No Habit Model

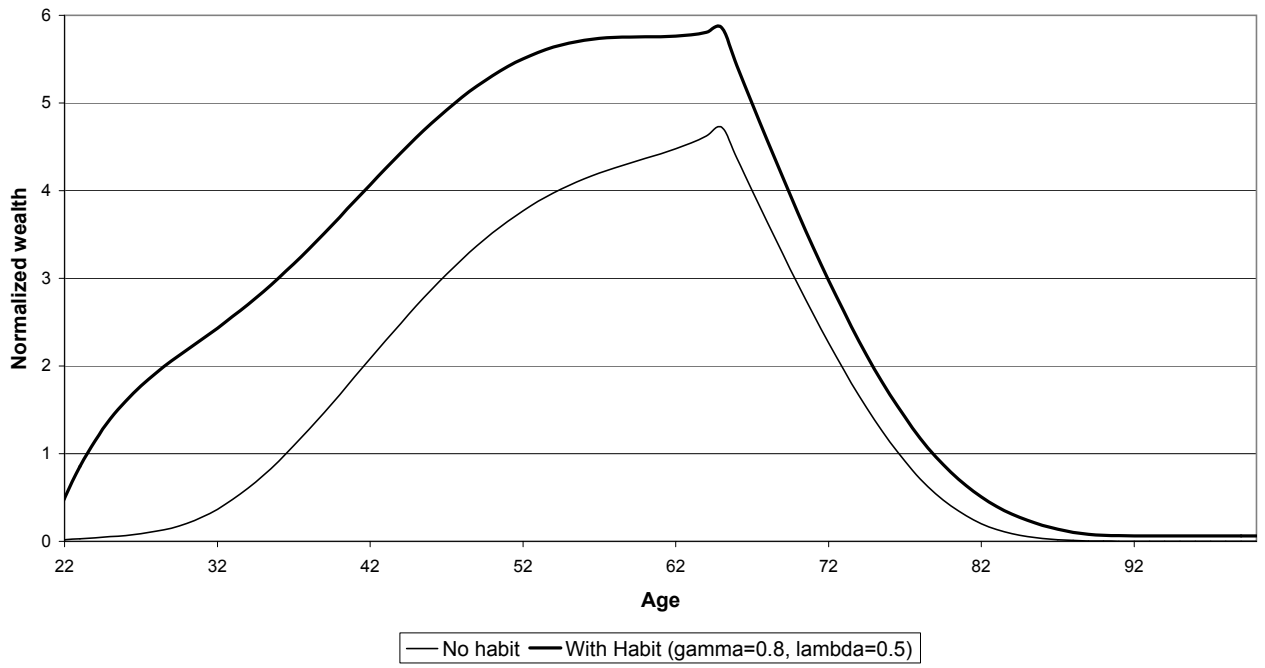


Fig 2.4: Portfolio Allocation over the Life-Cycle (No Fixed Costs)

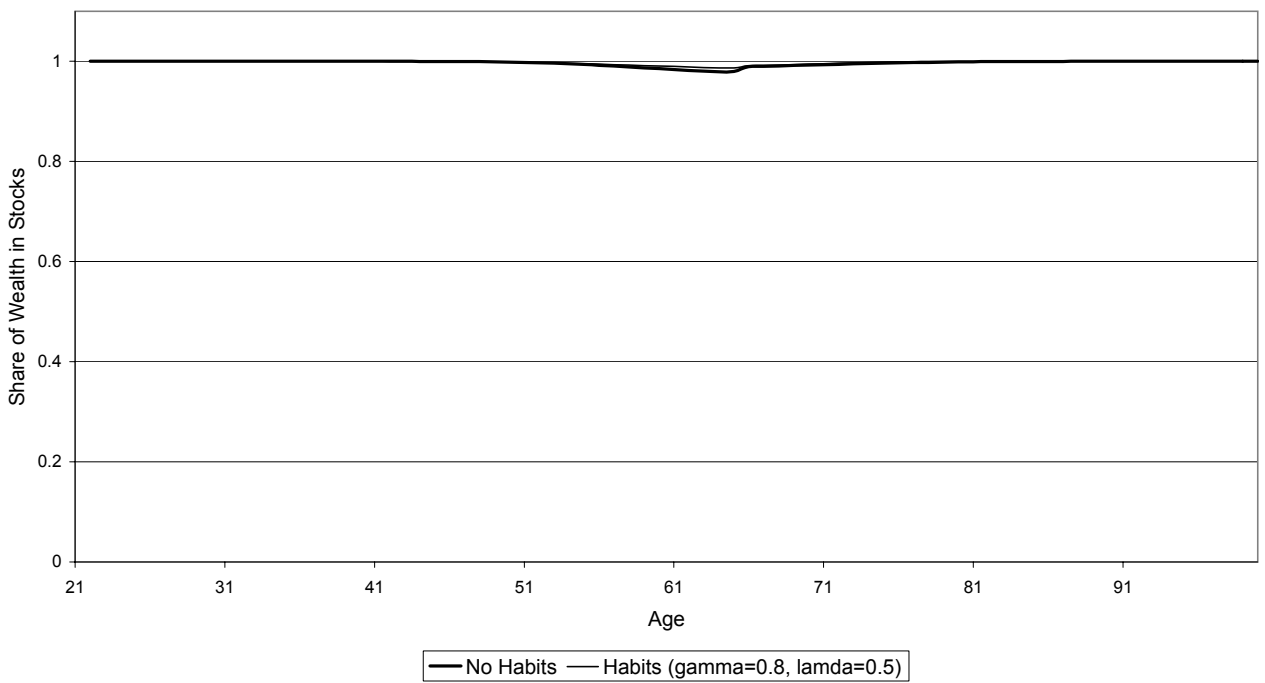


Figure 3.1: Comparing Consumption in and Out of Stock Market at Mean Habit ( $\rho=2$ )

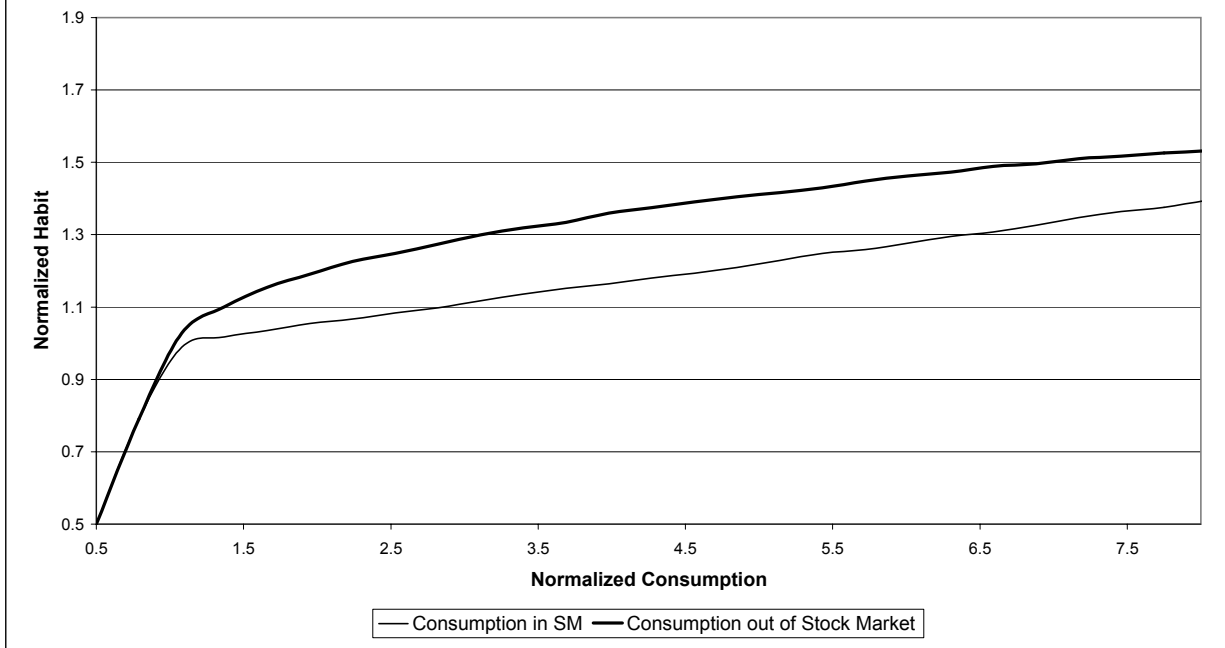


Figure 3.2: Distributions for Normalized Cash on Hand Model with Fixed Cost

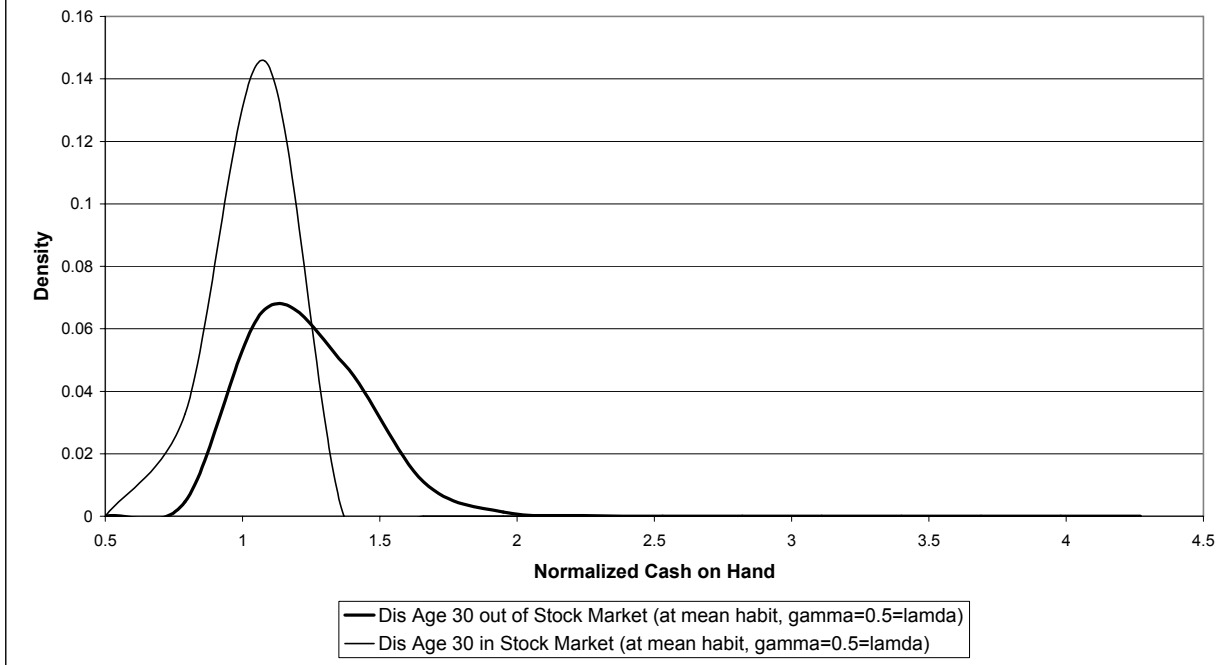


Figure 3.3: Distributions for Normalized Cash on Hand With Fixed Cost at Age 65

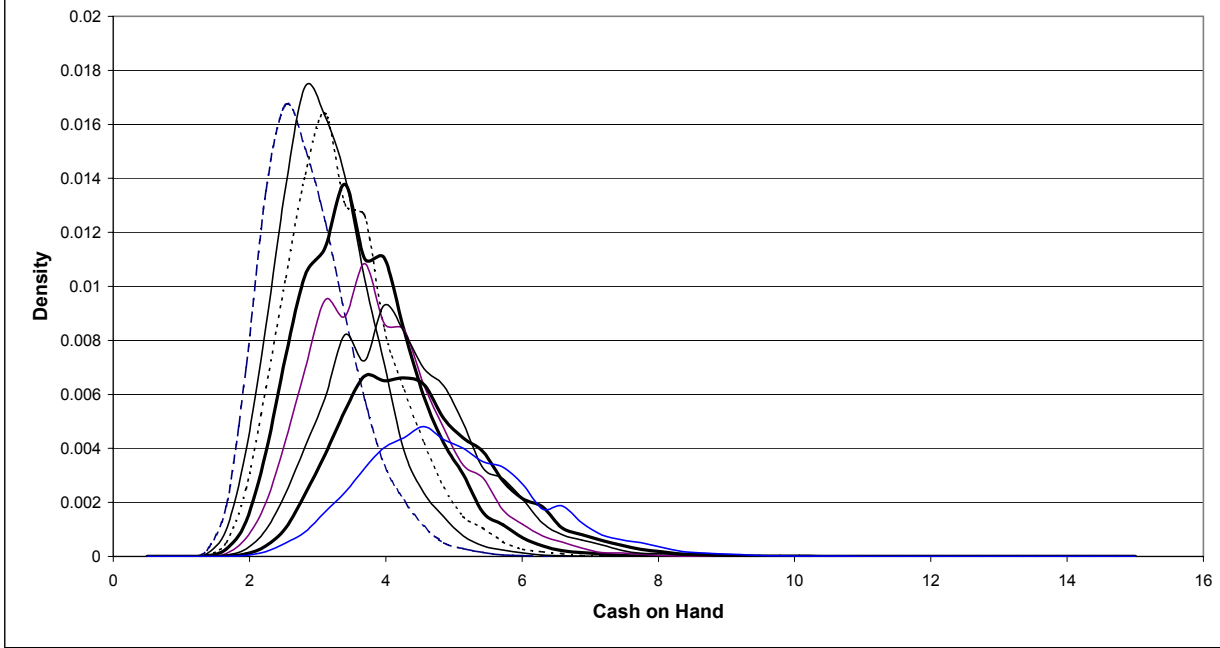


Figure 3.4: Consumption and Habits (with fixed cost)

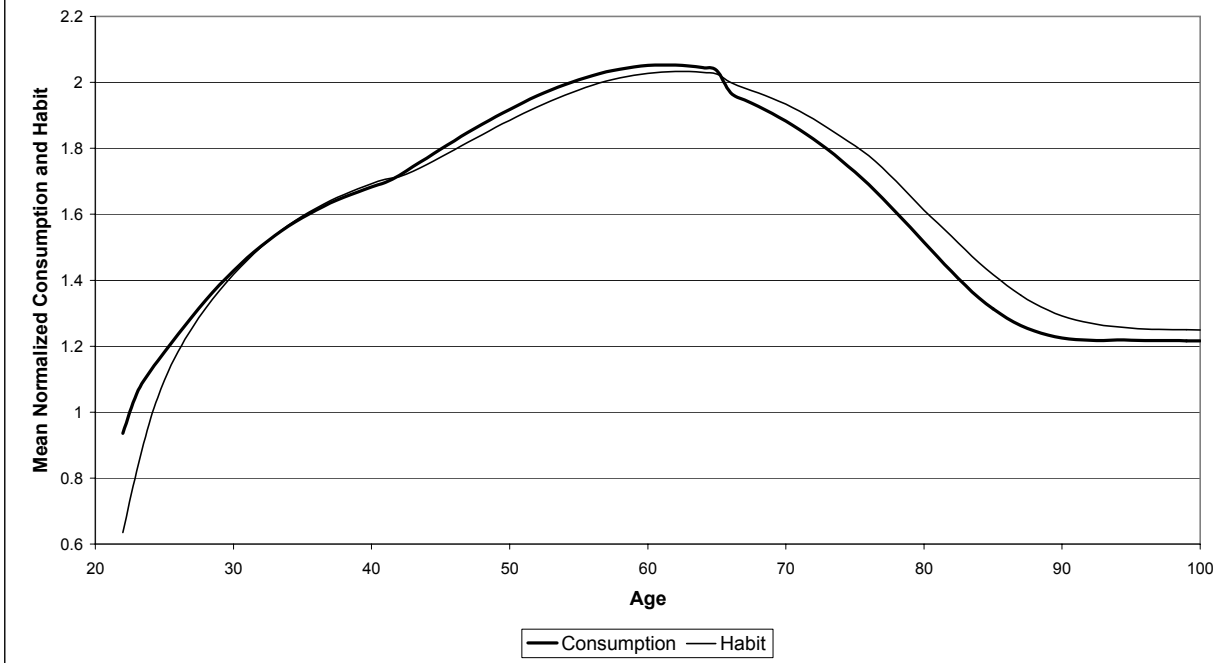


Figure 3.5: Stock Market Participation Rate ( $\gamma=0.8, \lambda=0.5$ )

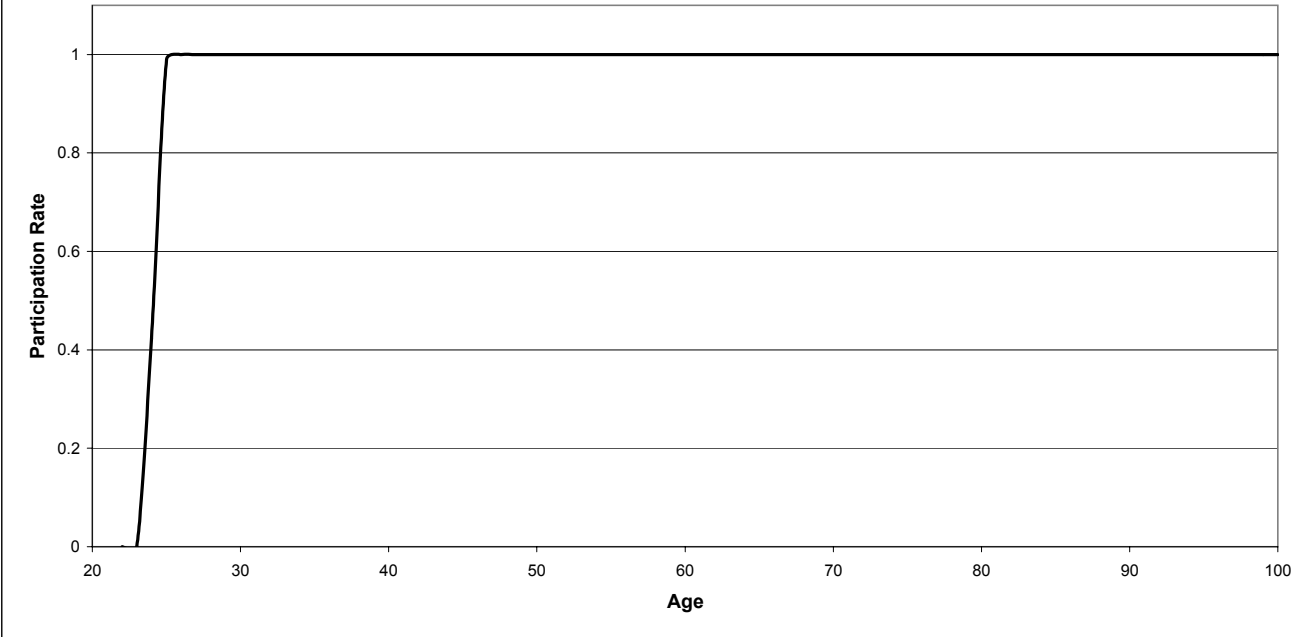
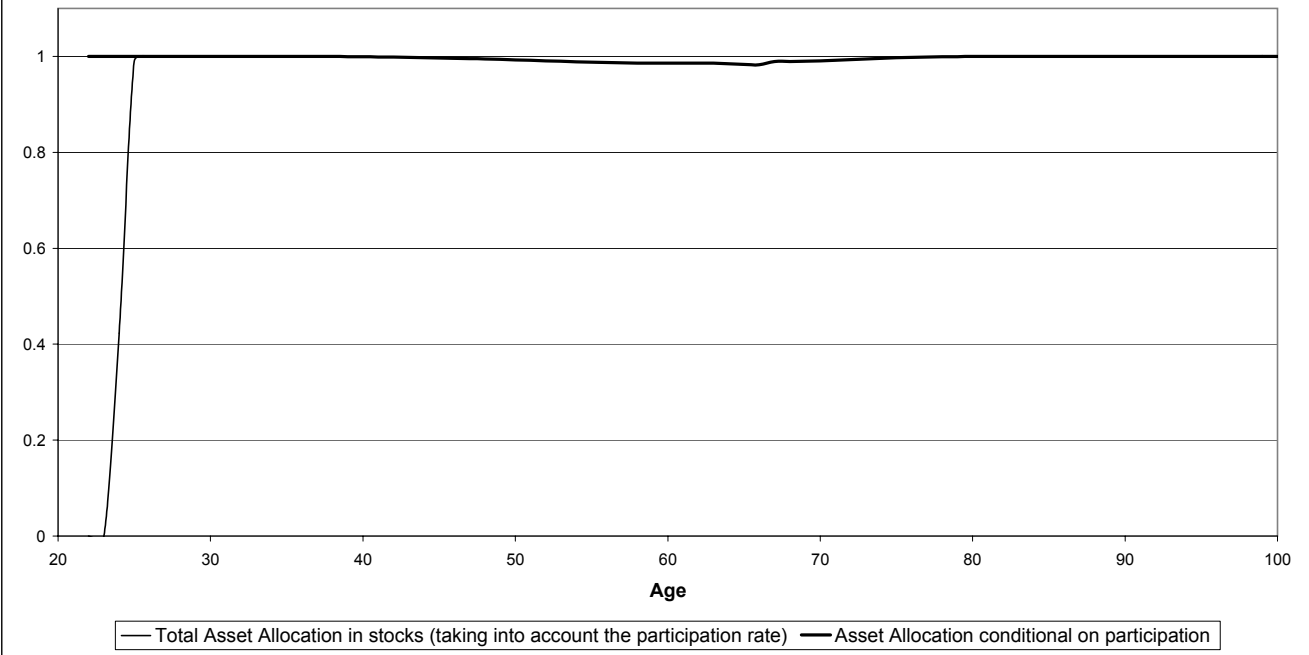
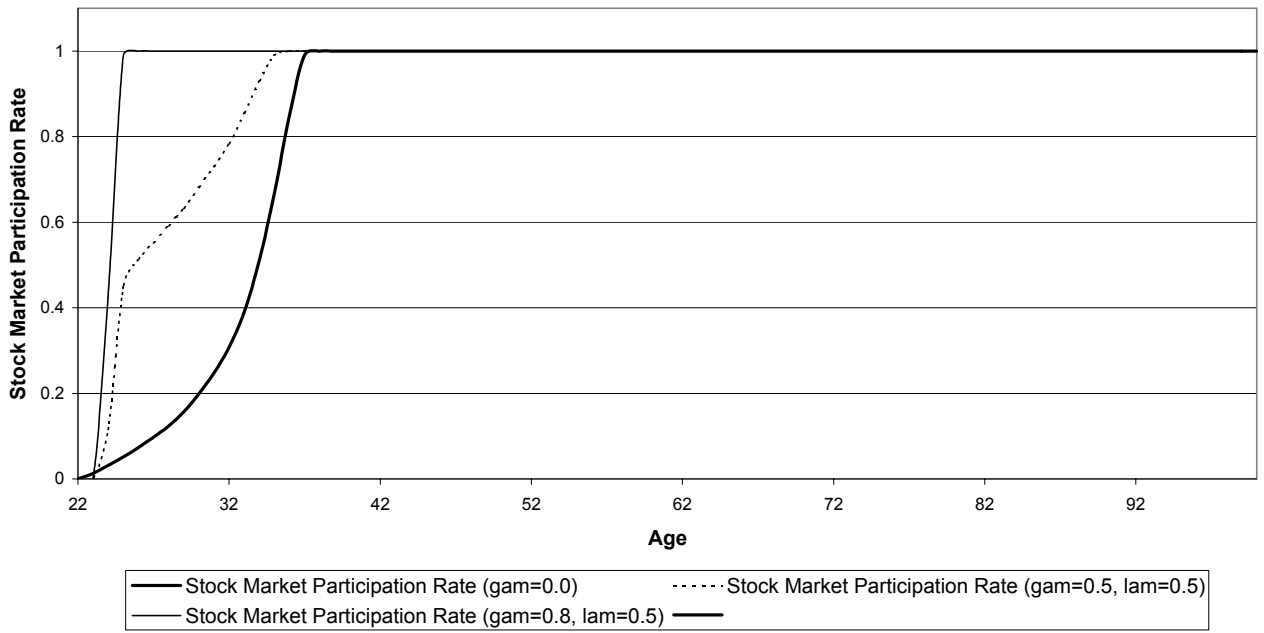


Figure 3.6: Portfolio Allocation over the Life-Cycle with the Fixed Cost



**Figure 4.1: Stock Market Participation varying habit strength**  
( $F=0.1, \rho=2$ )



**Figure 4.2: Unconditional Portfolio Allocations (Habit vs No Habit)**

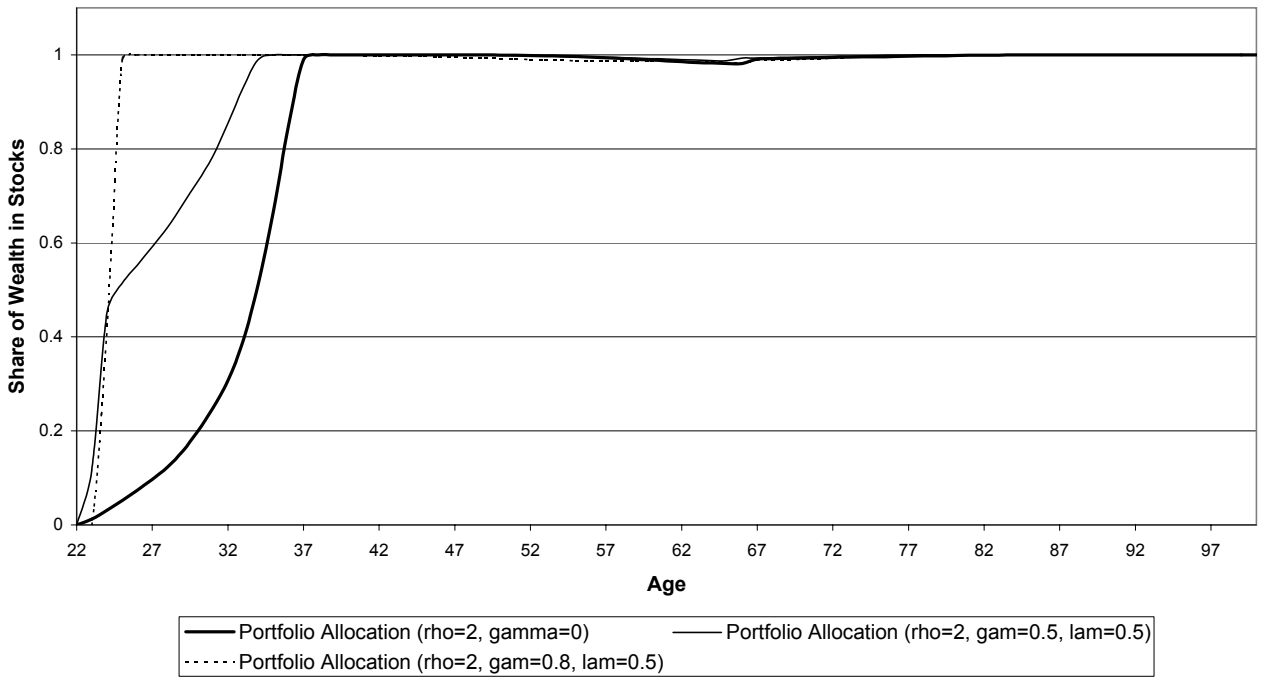


Figure 5.1: Unconditional Mean Wealth, Different Initial Conditions

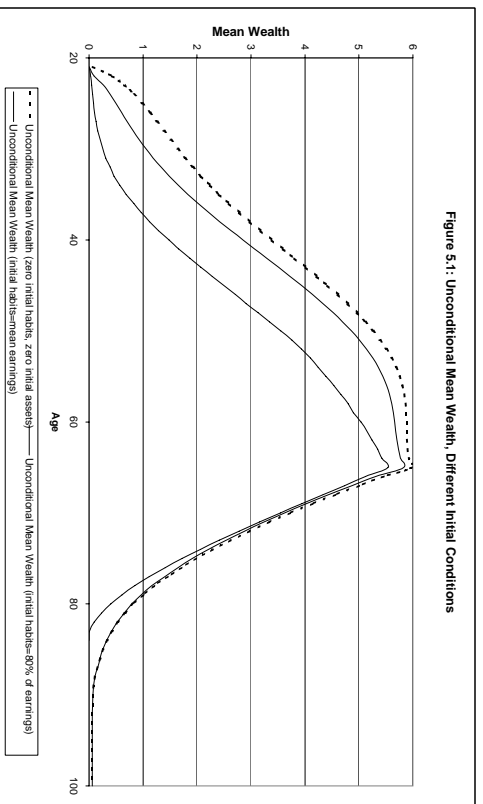


Figure 5.2: Stock Market Participation Varying Initial Habits

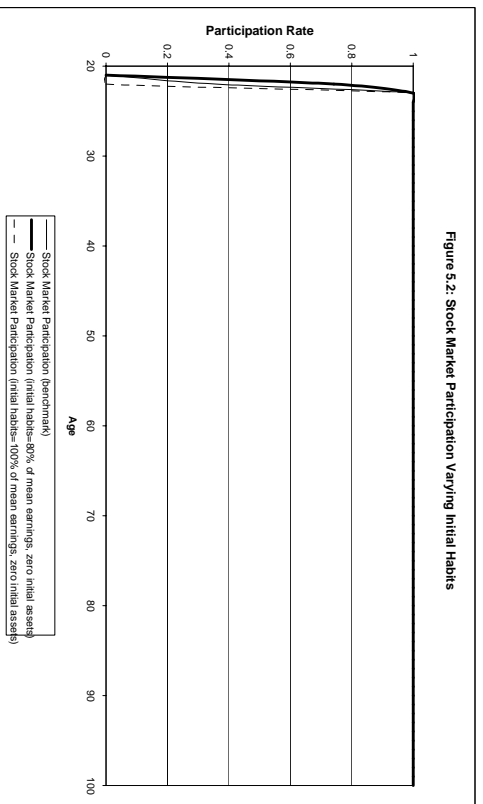


Figure 5.3: Unconditional Portfolio Allocation varying CRRA coefficient, multiplicative habit mod.

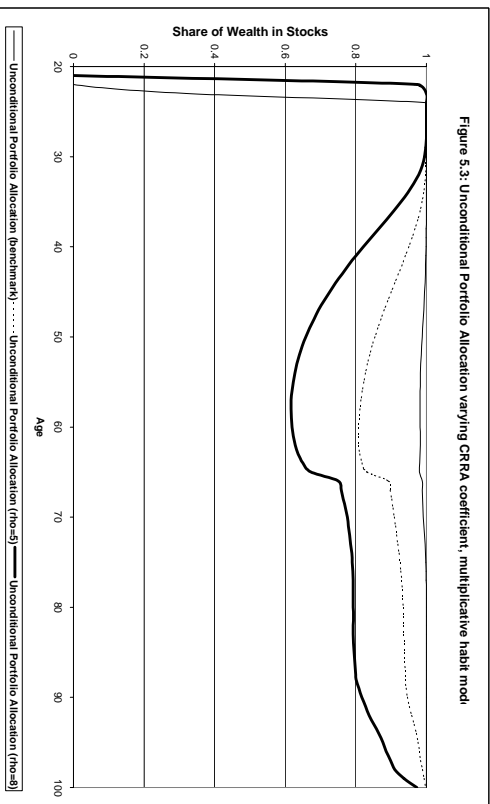


Figure 5.4: Stock Market Participation Varying Discount Rate

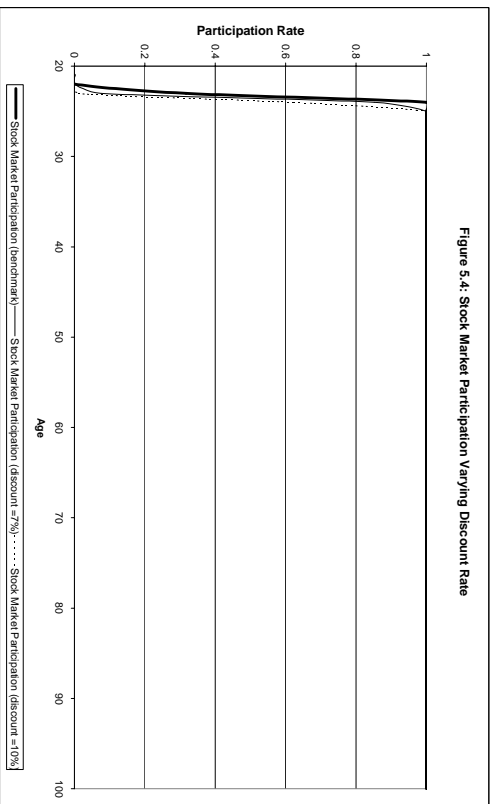


Figure 5.5: Unconditional Portfolio Allocation with Higher Correlation between permanent earnings and stock return innovations

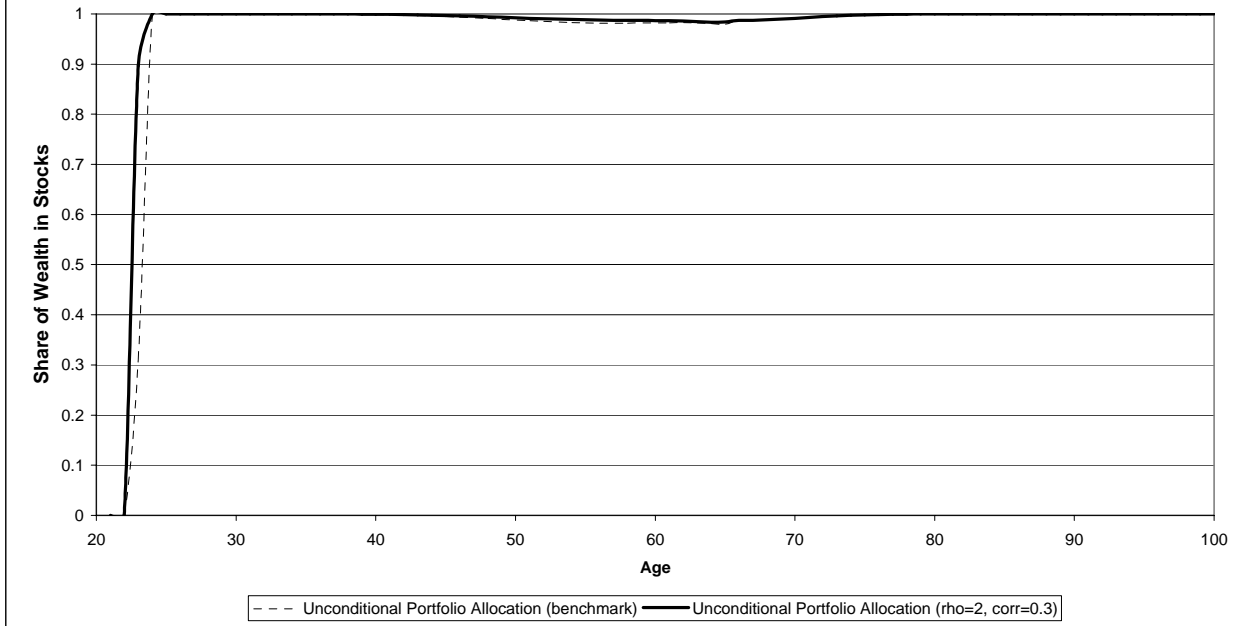


Figure 5.6: Unconditional Portfolio Allocation for rho=5, varying correlation between permanent earnings and stock returns

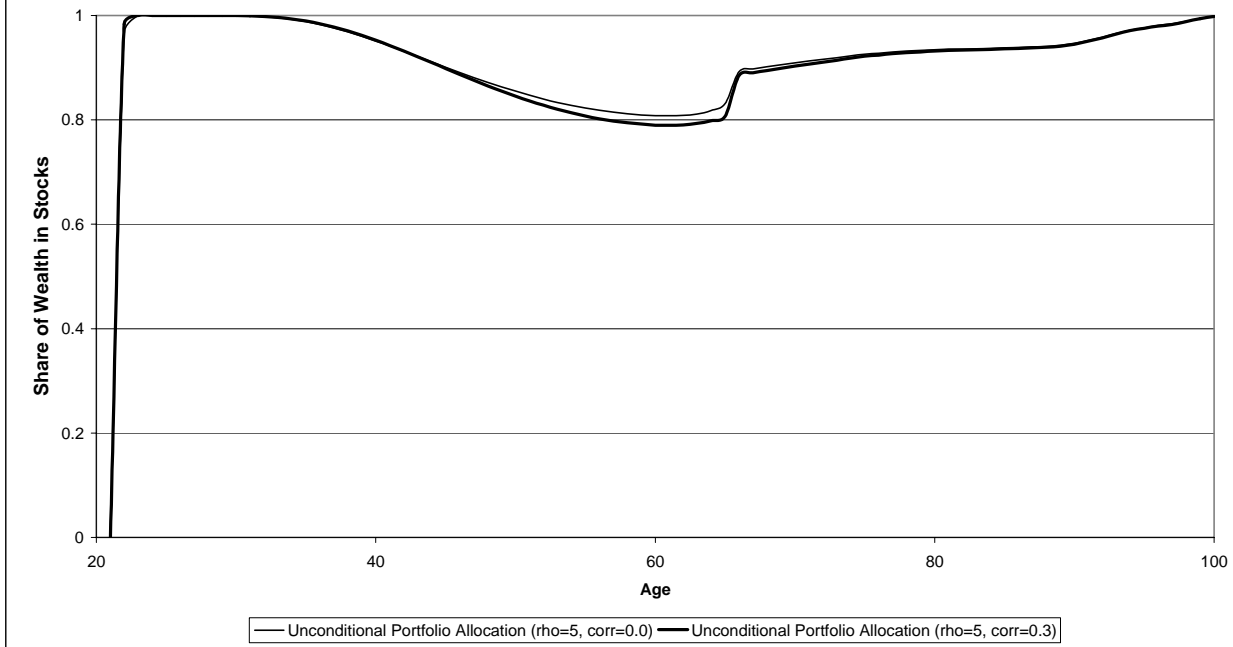


Figure 6.1 - Life-Cycle Patterns of Risk Aversion and Prudence  
(Difference Habit Model with  $\rho=2$ ,  $\gamma=5$ ,  $\lambda=0.5$ , and  $F=0.1$ )

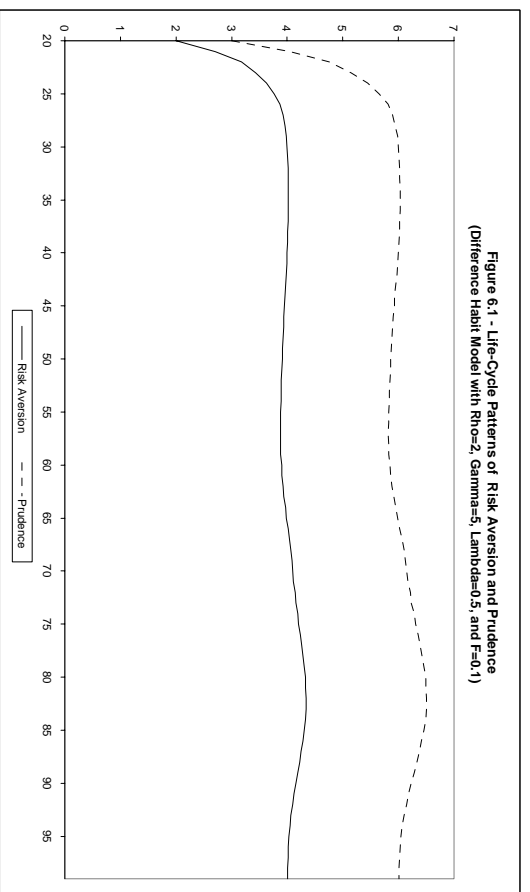


Figure 6.3 - Stock Market Participation Rate and Share of Wealth Invested in Stocks  
(Difference Habit Model with  $\rho=2$ ,  $\gamma=5$ ,  $\lambda=0.5$ , and  $F=0.1$ )

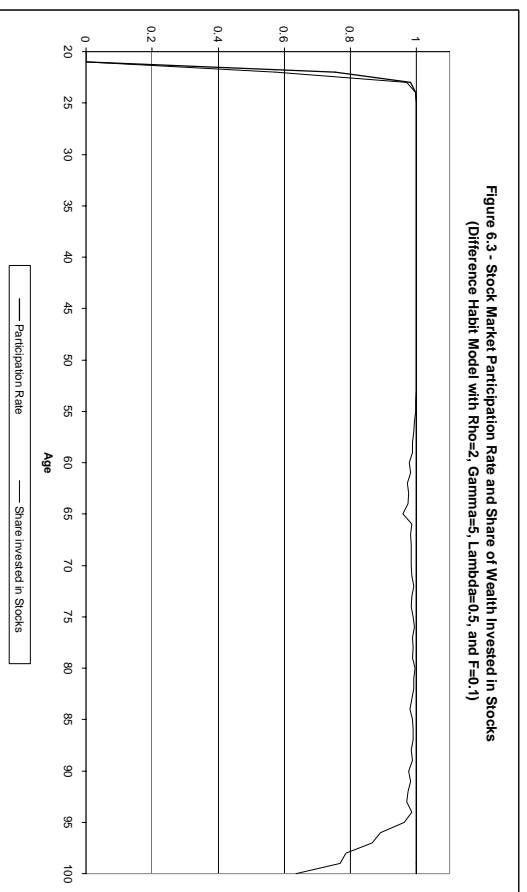


Figure 6.2 - Consumption and Wealth Accumulation  
(Difference Habit Model with  $\rho=2$ ,  $\gamma=5$ ,  $\lambda=0.5$ , and  $F=0.1$ )

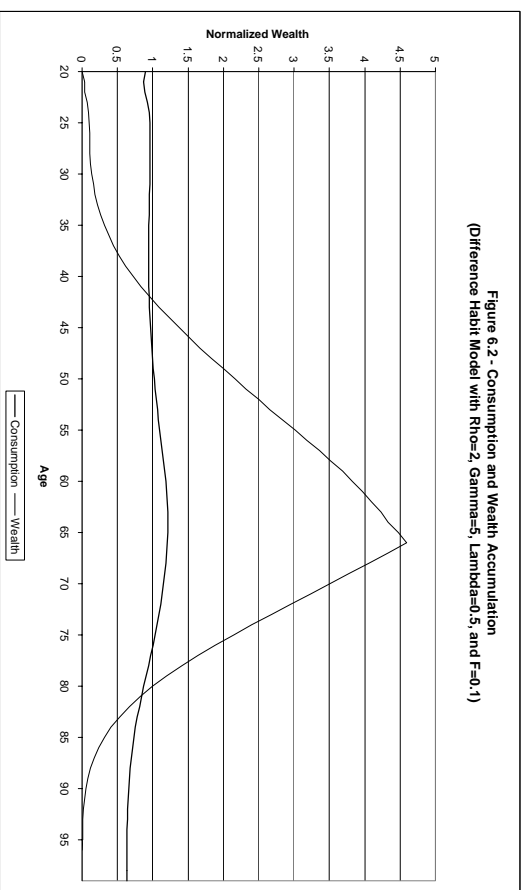


Figure 6.4 - Life-Cycle Patterns of Risk Aversion for different initial habit levels  
(Difference Habit Model with  $\rho=2$ ,  $\gamma=5$ ,  $\lambda=0.5$ , and  $F=0.1$ )

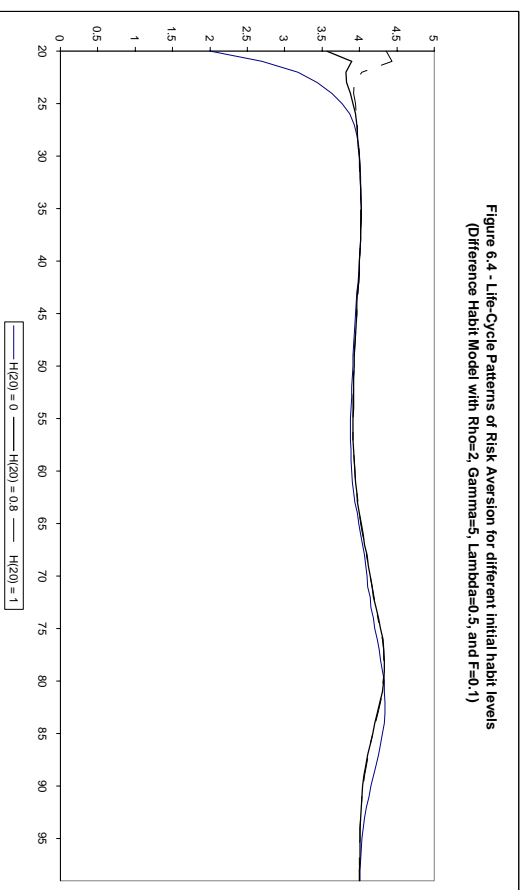


Figure 6.5 - Share Invested in Stocks for different initial habit levels  
(Difference Habit Model with  $\rho=2$ ,  $\gamma=5$ ,  $\lambda=0.5$ , and  $F=0.1$ )

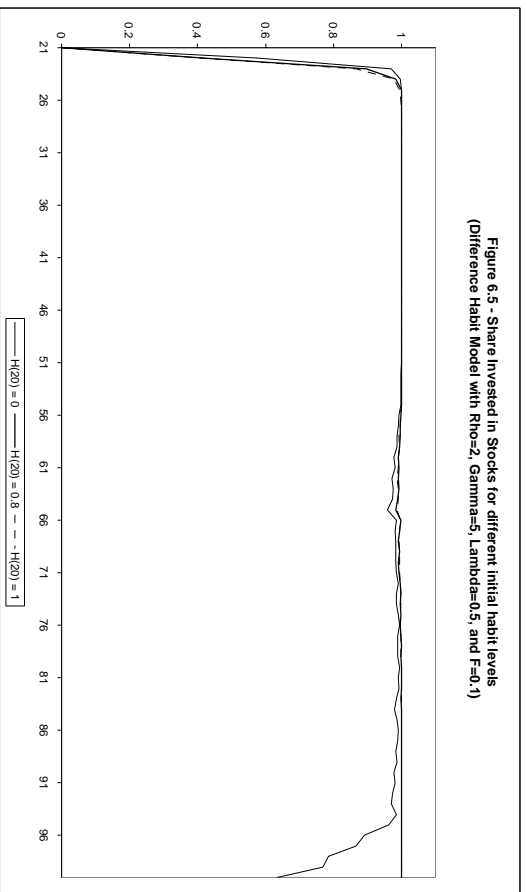


Figure 6.6 - Risk Aversion for different values of  $\rho$   
(Difference Habit Model with  $\gamma=5$ ,  $\lambda=0.5$ , and  $F=0.1$ )

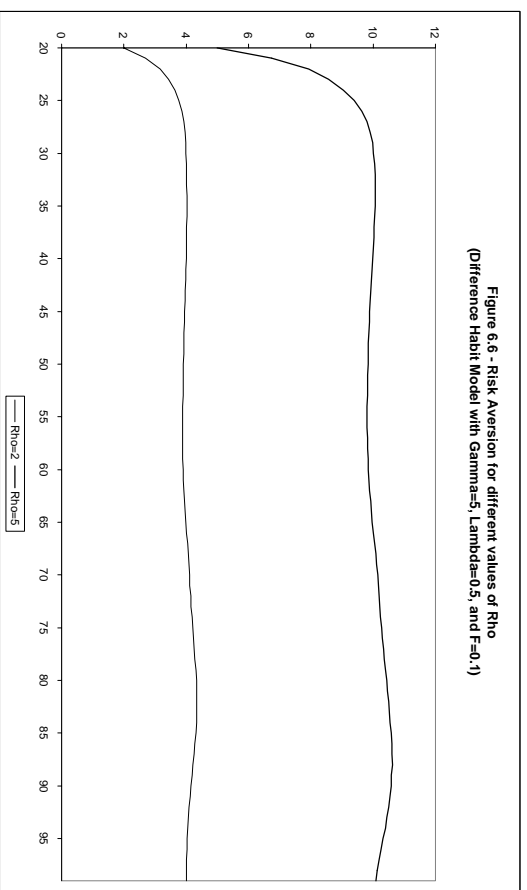


Figure 6.7 - Wealth Accumulation for different values of  $\rho$   
(Difference Habit Model with  $\gamma=5$ ,  $\lambda=0.5$ , and  $F=0.1$ )

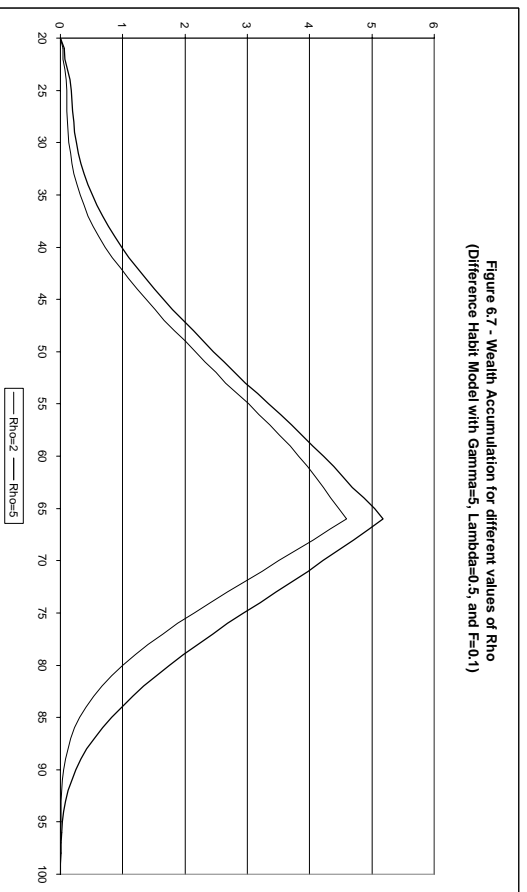


Figure 6.8 - Share Invested in Stocks for different values of  $\rho$   
(Difference Habit Model with  $\gamma=5$ ,  $\lambda=0.5$ , and  $F=0.1$ )

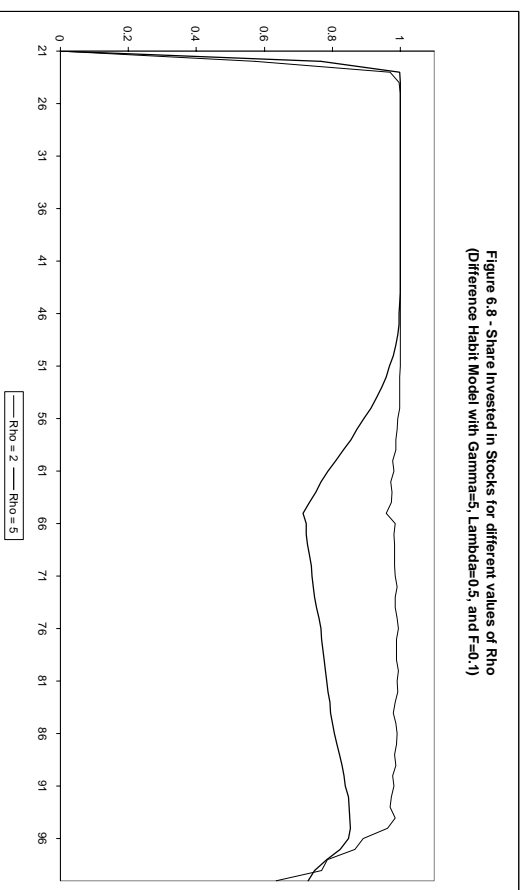


Figure 6.9 - Risk Aversion for different values of Gamma  
(Difference Habit Model with Rho=2, Lambda=0.5, and F=0.1)

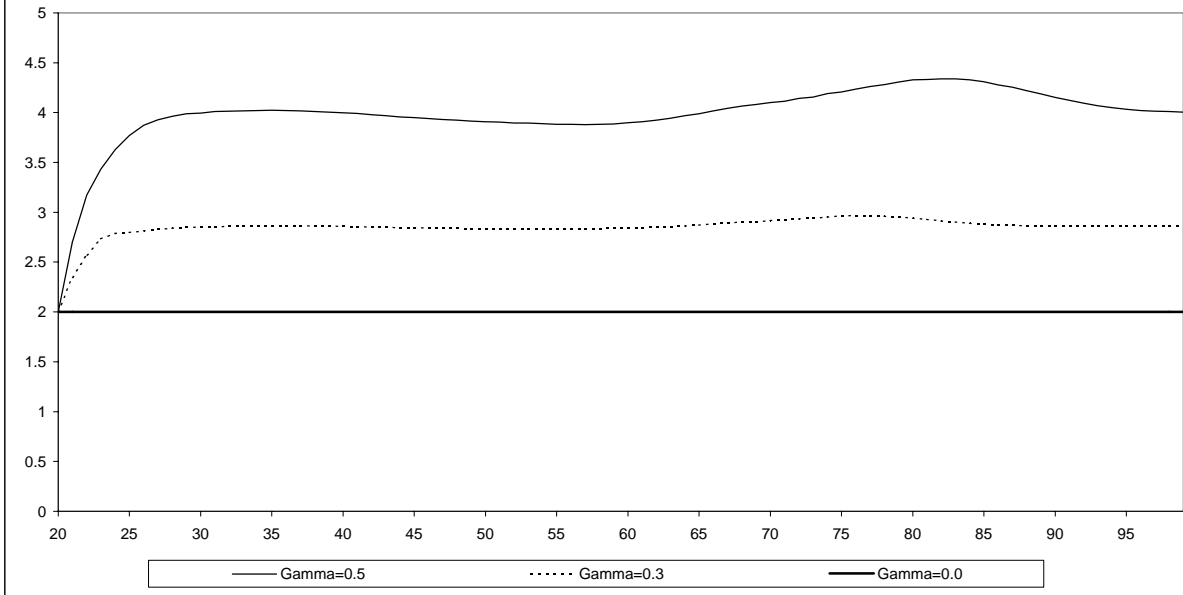


Figure 6.10 - Share Invested in Stocks for different values of Gamma  
(Difference Habit Model with Rho=2, Lambda=0.5, and F=0.1)

