Stage Financing and the Role of Convertible Securities*

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Abstract

Venture capital financing is characterized by extensive use of convertible securities and stage financing. In a model where a venture capitalist provides staged financing for a project, we illustrate an advantage of convertible debt (or warrants) over a mixture of debt and equity. Essentially, when the venture capitalist retains the option to abandon the project, the entrepreneur has an incentive to engage in window dressing and bias positively the short-term performance of the project, reducing the probability that it will be liquidated. An appropriately designed convertible security prevents such behavior because window dressing also increases the probability that the venture capitalist will exercise the conversion option becoming the owner of a substantial fraction of the project's equity.

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1 Introduction

Projects backed by venture capital exhibit great uncertainty and high failure risk, and are typically not financed by banks. Lerner and Gompers (1999) provide an overview for the reasons why traditional sources of financing are not suitable for such projects, and stress that tools employed by venture investors are a response to their special nature. In this paper, we focus on two such financing tools—stage financing and convertible securities.

A widely used financing technique in venture capital is infusion of capital over time. The venture capitalist retains the option to abandon the venture whenever the forward looking net present value of the project is negative. Financing rounds are usually related to significant stages in the development process such as completion of design, pilot production, first profitability results, or the introduction of a second product. At every stage, new information about the venture is released (Sahlman 1990).[1] Also widely used in venture capital backed projects are convertible securities issued by entrepreneurs in exchange for funds.[2] We provide a rationale for the combined use of stage financing and convertible securities in venture capital financing.

Stage financing is appealing to venture capitalists for two reasons. First, the option to abandon is essential because an entrepreneur will almost never quit a failing project as long as others are providing capital (Admati and Pfleiderer 1994). Second, the threat to abandon creates incentives for the entrepreneur to maximize value and meet goals. But this threat has the drawback that it might induce the entrepreneur to focus on meeting the immediate hurdle of the next stage.[3] To illustrate, the entrepreneur can make the conditions under which a project will be evaluated more favorable, whether it is the test of a prototype or a market test, increasing the likelihood of good interim performance. This phenomenon is commonly described as "window dressing." The venture capitalist then has to decide whether to continue investing on the basis of interim performance that has been artificially improved. Stage financing thus creates a conflict of interest between the venture capitalist, who provides the funds, and the entrepreneur, who wants to continue with the project.

[1]See also Lerner (1995). Gompers (1995) provides evidence that the number of financing rounds increases with the fraction of intangible assets. Egli, Ongena, and Smith (2000) study a model where the choice of stage financing arises endogenously. Neher (1999) studies a model where the number of financing rounds and the duration of the project are determined endogenously. Black and Gilson (1998), Table 4, report the percentage of capital infused on average at various venture capital financing rounds in the US and Germany.

[2]Kaplan and Strömberg (2002) provide a comprehensive characterization of the contracts used in venture capital financing and document the extensive use of convertible securities by venture capitalists in the US.

[3]Sahlman (1988, p.32) observes that having a "fume date" may serve as a motivating factor, but "may create incentives to aim for short-term success rather than long-term value creation."

We model window dressing as follows. Venture capitalist and entrepreneur commonly observe the interim performance of the project. For brevity, we call interim performance the "signal." Before the signal is realized, the entrepreneur can manipulate its distribution so that a good signal realization is more likely to appear, without affecting the actual quality of the project. We show that this change in the distribution of the signal should *not* be thought of as increasing or reducing the signal's noise, because this latter type of signal manipulation does not constitute a conflict of interest: both the venture capitalist and the entrepreneur want to increase the signal's precision. To capture the conflict of interest that arises from stage financing, window dressing must be modeled as a shift of probability mass from low to high realizations of the signal so that low realizations are less likely.

It is often hard to specify unequivocally what "good interim performance" means. For instance, to decide if a "working prototype" indeed works, or to evaluate consumer responses to a market test, much subjective judgement must be exercised. To capture this feature, we assume that although the interim performance signal is observed by the venture capitalist and the entrepreneur, it is nonverifiable.[4]

Window dressing reduces the venture capitalist's payoff because his refinancing/liquidation decision is based on lower quality information. This can render stage financing nonviable so that a profitable project will not be financed altogether. Yet, with debt-equity financing (debt, equity, or a combination of both) the entrepreneur will always window dress. We show that an appropriately designed convertible security makes window dressing no longer advantageous: the gain from reducing the likelihood of liquidation is more than offset by the increased likelihood of debt conversion (conditional on refinancing). This is because although window dressing renders low quality projects harder to identify (reducing the probability of liquidation), it renders high quality projects easier to identify. This, in turn, increases the probability that in the event of refinancing the venture capitalist will exercise the conversion option becoming the owner of a substantial fraction of the venture and appropriating much of the project's value. If the terms of conversion are set in advance to be sufficiently favorable to the venture capitalist, this effect dominates in terms of payoffs, and the entrepreneur will not engage in short-term window dressing. A central element of our analysis is the determination of an appropriate conversion ratio and a suitable amount of convertible security that will ensure that the project is financed, but at the same time will deter the entrepreneur from window dressing.

[4]We also devote a section to the version of the model where the signal is verifiable because in the real world there are situations where interim performance can be measured objectively (e.g., sales revenue).

In the main part of the paper we focus on convertible debt, but warrants and convertible preferred equity are also commonly used in venture capital financing. Convertible preferred equity has similar features to convertible debt, but is special in several respects. Until conversion, a convertible preferred stock promises a fixed dividend (and hence is similar to convertible debt). Unlike debt, failure to pay the dividend does not trigger liquidation; rather, the unpaid dividends accrue and must be paid before any dividends are paid out to common stock holders. Since we do not consider payoffs at interim stages, our model applies to convertible preferred equity as well.[5] We will show that our analysis is virtually unchanged if the entrepreneur issues a combination of straight debt and warrants. This extension is important as empirical evidence suggests that the deadline feature of warrants fits our model well.

We first analyze the model abstracting from renegotiation, and then consider it as follows. First, we show that convertible debt prevents window dressing equally well even with renegotiation. Then, we ask whether convertible debt is *necessary* for preventing window dressing if debt-equity contracts can be renegotiated. We distinguish between spontaneous and designed renegotiation in the sense of Aghion, Dewatripont, and Rey (1994), and explain why debt-equity financing cum spontaneous renegotiation cannot prevent window dressing.[6] Then, we turn to designed renegotiation adapting Aghion, Dewatripont, and Rey's (1994) definition of "simple renegotiation" to the context of our model. We show that debt-equity with simple renegotiation indeed prevents window dressing. The point is that debt-equity cum simple renegotiation is "isomorphic" to convertible debt. In this sense, convertible debt is interpreted as a simple way to design renegotiation and introduce it in a contract.

Explanations for the use of convertible securities are often centered on ex-ante asymmetry of information between managers and investors. Stein (1992) focuses on ex-ante asymmetry of information regarding the quality of the issuing firm. Brennan and Schwartz (1988) focus on ex-ante asymmetry of information regarding the risk of the issuing firm. Since the value of the option embedded in the convertible security increases with the volatility of the underlying security, offering a conversion option is less "costly" for low risk firms who signal to investors that they are of low risk by issuing convertible debt.[7]

At the time of initial venture capital financing, the entrepreneur and the financier are often

[5] Other major advantages of preferred shares are the favorable tax treatment and the fact that preferred shares carry votes. Since our model abstracts from taxes and control rights, it would make little difference if we used convertible preferred equity rather than convertible debt or warrants. For an analysis of transfers of control in venture capital financing situations see, for example, Chan, Siegel, and Thakor (1990) and Cestone (2001).

[6] Aghion, Dewatripont, and Rey use the term "voluntary" rather than "spontaneous."

[7] See also Harris and Raviv (1985) and Nyborg (1995).

equally informed regarding the project's chances of success, and the true quality is gradually revealed to both. The main conflict of interest is the asymmetry of information about future *actions* of the entrepreneur. Our model stresses this aspect of venture capital financing rather than ex-ante information asymmetry. Green (1984) also focuses on asymmetric information regarding the entrepreneur's actions. In his model, a mix of convertible securities and debt is superior to straight debt because the conversion option reduces the inclination of the entrepreneur to engage in risky projects. In terms of economic intuition, the difference between the models is that in Green (1984) the entrepreneur reduces the value of the project by taking excessive risk ("asset substitution"), whereas in our model the entrepreneur reduces the value of the project through window dressing. Thus, in our model negative incentives arise due to stage financing, inducing the entrepreneur to focus on short-term objectives. It is an implication of our model that stage financing and convertible securities should be used together. Moreover, with a sufficiently high proportion of equity financing, the danger of excessive risk taking would not arise in Green's model, whereas in our model the conflict of interest between the entrepreneur and the venture capitalist arises whether the project is financed with debt, equity, or any combination of both.

In our model, interim performance is observed by the venture capitalist and the entrepreneur. Therefore, the entrepreneur cannot engage in mis-reporting of information ("cooking the books"), but can manipulate the "technology" that generates the signal. In models such as Milgrom and Roberts (1986) and Shin (1994), this technology is fixed but the signal is observed by only one party that has an incentive to mis-report its private information ex-post. In our model, if the entrepreneur had obtained information about the project that the venture capitalist does not have, the danger of mis-reporting would arise.

Some recent papers provide other explanations for the use of convertible debt in venture capital situations. Repullo and Suarez (1998) focus on the advisory role of the venture capitalist and the optimal contractual arrangement when there is double sided moral hazard. Schmidt (2000) also focuses on double sided moral hazard and shows that convertible debt can induce efficient investment. Hellmann (2000) focuses on convertibility at the IPO stage, while Biais, Bisière, and Décamps (1998) argue that the choice of securities (e.g., convertible debt) is affected by the trade-off between anticipated costs of financial distress and the need to provide incentives to managers. Berglöf (1994) looks at the role of convertible securities in mitigating distributional conflicts associated with a future sale of the firm. None of these papers, however, look at the combined use of convertible securities and staged infusion of capital which are so common in venture capital financing.

Several other papers are concerned with venture capital financing but not with convertible securities. Hellmann (1998) provides an explanation for why an entrepreneur might agree to be replaced by the venture capitalist. Marx (1998) argues that a mix of debt and equity dominates only equity and only debt by generating incentives for the venture capitalist to intervene as a response to poor performance.[8] Bergemann and Hege (1999, 2000) study a dynamic model with venture capital financing, focusing on the optimal compensation of the entrepreneur in order to provide him with the right incentives for the allocation of funds, and Ueda (2000) asks why start-up firms typically prefer venture capital rather than bank financing.[9] Finally, von Thadden (1995) shows that a contract that resembles a long-term credit line can reduce short-term behavior in stage financing situations.

In the next section we present the model under the assumption that contracts cannot be renegotiated. In Sections 3 and 4 we allow, respectively for contract renegotiation and straight debt cum warrants financing. Section 5 analyzes the model under the assumption that the signal is verifiable, Section 6 studies other forms of signal manipulation, and Section 7 concludes.

2 The model

2.1 The basic set-up without signal manipulation

At time 0 an entrepreneur considers undertaking a project with uncertain returns. There are two possible states of nature: with probability 1/2 the state is "good," which we denote h, and with probability 1/2 the state is "bad," l. The project generates a (stochastic) output, π_i, distributed exponentially on the interval $[0, \infty)$, with density $\lambda_i e^{-\lambda_i \pi_i}$ where $1/\lambda_i$ is the mean of π_i for $i = h, l$. Let

$$2\,\frac{1}{\lambda_l} > \frac{1}{\lambda_h} > \frac{1}{\lambda_l}\,, \tag{1}$$

i.e., the difference between the expectation of output in the two states is not "too large."[10]

The entrepreneur has no capital and will ask a venture capitalist for the required funds.

[8] See also Marx (1999).

[9] As mentioned, Gompers and Lerner (1999) offer reasons for why banks typically do not finance high risk and "high-tech" projects. First, banks insist on collateral which these projects do not have since they are not intensive in physical assets. Second, banks are perceived as having comparative advantage in monitoring through long-term relationships with borrowers, whereas in the "high-tech" industry change is very rapid, and effective monitoring requires skills that banks do not have. Third, in the United States, the Glass-Steagall Act prohibited banks from owning the equity of nonfinancial firms, preventing banks from providing financing via convertible securities. It is early to tell whether the repeal of Glass-Steagall in 1999 will induce entry of banks into the "high-tech" market niche.

[10] This assumption is not a necessary condition for the main result to hold, but it greatly facilitates the analysis.

The total amount of money necessary for the project is $I_1 + I_2$, of which I_1 must be invested at time 0, and I_2 can be delayed until time 1, provided that the project is not liquidated.

Stage financing

Stage financing means that I_1 is invested at time 0, and I_2 is delayed. At time 1, a signal, x, of the project's quality (the state of nature) is realized and is observed by the entrepreneur and the venture capitalist. The signal can be interpreted as short-term performance, and both parties update their expectations about the project's quality. We assume that x is nonverifiable. After observing it, the venture capitalist decides whether to refinance the project and supply I_2 to the entrepreneur, or liquidate. We assume that the payoff to both parties after liquidation is zero.[11] If the project is refinanced, the output is realized at time 2 and is shared by the entrepreneur and the venture capitalist according to the contract that has been agreed upon.

In principle, the venture capitalist can commit at time 0 to supply $I_1 + I_2$, irrespectively of the signal, but to underline the problem created by window dressing we assume that

$$\frac{1}{2}\frac{1}{\lambda_l} + \frac{1}{2}\frac{1}{\lambda_h} - (I_1 + I_2) < 0 \ , \tag{2}$$

which implies that committing $I_1 + I_2$ at time 0 is not viable. This assumption allows us to illustrate our point in a sharp manner: if the venture capitalist does not prevent window dressing, the project will not be financed, underscoring the loss of surplus.[12]

Ex-post payoffs with debt-equity financing

Suppose the venture capitalist can provide financing to the entrepreneur in the form of debt, equity, or a combination of the two. We denote a debt-equity contract (d, s), where d is the amount of debt (cum interest) the entrepreneur owes the venture capitalist, and s is the fraction of the enterprise's equity owned by the venture capitalist. In this section, we abstract from the possible renegotiation of debt-equity contracts at time 1.

Consider the debt-equity contract (d, s). The magnitudes d and s are determined at time 0

[11] For our purposes, this is an innocuous assumption. In general, liquidation value can be of great importance. In Neher (1999), the central economic force is the gradual embodiment of the entrepreneur's human capital in the project's physical capital, and as a consequence, the project's liquidation value increases over time.

[12] Assumption (2) is not necessary; even if profits without stage financing were positive, the venture capitalist would prefer to finance the project in stages because he thus keeps the valuable option to abandon after the realization of the signal, and the entrepreneur would want to window dress the signal in order to reduce the probability of liquidation.

and do not change throughout the life-time of the venture. At time 0, I_1 is supplied. At time 1, if the project is not liquidated, I_2 is supplied. At time 2, the output is divided according to the sharing rule induced by the debt-equity contract (d, s).[13] The promised amount d may be larger or smaller than $I_1 + I_2$, the total amount of financing.

We assume $s \in [0, 1)$, i.e. the entrepreneur must own some shares, justified on the grounds that otherwise the entrepreneur will not make effort for the project to succeed in the long-run.[14] Since the signal about the project's quality (observed by both parties at time 1) is nonverifiable, it is not possible to write in the contract at time 0 that the provision of I_2 (refinancing) will depend on the realized value of the signal.

Assuming limited liability and zero bankruptcy costs, the ex-post payoffs (after the state of nature is revealed but before profits are realized) to the venture capitalist and the entrepreneur from a debt-equity contract, (d, s), for $i = l, h$, are:

$$\begin{aligned} \mathcal{VC}_i^{DE} &= \int_0^d \pi_i \lambda_i e^{-\lambda_i \pi_i} d\pi_i + \int_d^\infty [d + s(\pi_i - d)] \lambda_i e^{-\lambda_i \pi_i} d\pi_i \\ &= \frac{1}{\lambda_i}[1 - e^{-\lambda_i d}(1 - s)], \\ \mathcal{E}_i^{DE} &= \int_d^\infty (1 - s)(\pi_i - d) \lambda_i e^{-\lambda_i \pi_i} d\pi_i \\ &= \frac{1}{\lambda_i} e^{-\lambda_i d}(1 - s). \end{aligned} \tag{3}$$

Ex-post payoffs with convertible debt financing

Alternatively, the venture capitalist and the entrepreneur can sign a convertible debt contract which we characterize in the following way. At time 0, the initial debt and equity holdings by the venture capitalist are d_0 and s_0. Suppose $d_0 > 0$, and let part of the debt be convertible into equity at the conversion ratio γ so that $d = d_0 - \gamma(s - s_0)$, where $d \geq 0$ and $s \in [s_0, 1)$

[13] Both parties are likely to take into account the entire amount $I_1 + I_2$ when specifying d and s at time 0. If not, one of them may be held up at time 1. The intuition is as follows. If the entrepreneur has enough bargaining power, then at date 1 the venture capitalist may be "locked in." For example, if the entrepreneur has all the bargaining power, at time 1 he will offer a new debt-equity contract which guarantees the venture capitalist just enough (expected) revenue to cover I_2. The venture capitalist will accept the offer since I_1 is sunk. Foreseeing this, the venture capitalist will prefer to have d and s specified at time 0. A similar logic applies to the entrepreneur if his participation in the project requires an arbitrarily small amount of effort at time 0. Then, at time 1 the entrepreneur would be "locked in" since his investment in effort is sunk. As a result, the entrepreneur will prefer to specify d and s at time 0. Endogenizing the time at which d and s are set requires more structure (how negotiation about d and s takes place, what happens in case of disagreement, etc.) and is beyond the scope of this paper. We believe that, in the context of our model, it is reasonable to specify the venture capitalist's share of profits, s, and the (cum interest) debt, d, relative to the entire amount of financing, $I_1 + I_2$.

[14] In a previous version of the paper (London Business School working paper no. 253, 2001) we allowed for entrepreneurial effort that affects long-run (time 2) profits. We showed that, indeed, $s < 1$ is a necessary condition for inducing the entrepreneur to exert such effort.

are the post-conversion debt and equity positions after *all* the convertible debt is converted, and $s - s_0$ is the fraction of the equity purchased. A special case is 100 percent initial debt financing ($s_0 = 0$). Another special case is that all the debt is convertible ($d = 0$). A contract with convertible debt is, therefore, characterized by (d_0, s_0, γ, s).

The ex-post payoffs to the venture capitalist and the entrepreneur from a convertible debt contract, (d_0, s_0, γ, s), for $i = l, h$, are:

$$\begin{aligned} \mathcal{VC}_i^{CD} &= \tfrac{1}{\lambda_i}\,[1 - e^{-\lambda_i\,[d_0 - \gamma(\hat{s} - s_0)]}\,(1 - \hat{s})], \\ \mathcal{E}_i^{CD} &= \tfrac{1}{\lambda_i} e^{-\lambda_i\,[d_0 - \gamma(\hat{s} - s_0)]}\,(1 - \hat{s})\,, \end{aligned} \tag{4}$$

where $\hat{s} \in [s_0, s]$ is the venture capitalist's post-conversion equity stake. Denoting by $\hat{d} \in [d, d_0]$ the post-conversion debt, we have $\hat{d} = d_0 - \gamma(\hat{s} - s_0)$. If the venture capitalist converts the entire amount of convertible debt, then $\hat{s} = s$ and $\hat{d} = d$.

The joint distribution of signal and output

At time 1, a signal x about the state of nature is realized and observed by the entrepreneur and the venture capitalist. The signal has a continuous distribution on the interval $[0, 1]$. Let $\alpha_h(x)$ denote the joint density function of x and h, and $\alpha_l(x)$ the joint density function of x and l. Let $q_h(x) = \frac{\alpha_h(x)}{\alpha_h(x)+\alpha_l(x)}$ denote the probability of h conditional on the signal realization x, where the denominator denotes the marginal density of x. For clarity of exposition, we concentrate on a special case where the joint density of signal and output is linear:

$$\alpha_h(x) = x \;, \quad \alpha_l(x) = 1 - x\,. \tag{5}$$

The intuition for $\alpha_h(x)$ being an increasing function of x is that in state h high realizations of x are more likely than low realizations of x (and analogously for $\alpha_l(x)$ being a decreasing function). The conditional probability of state h is

$$q_h(x) = \frac{x}{x + 1 - x} = x\,. \tag{6}$$

The signal x is strictly informative in the sense that the interim probability, $q_h(x)$, is strictly increasing in x; see Figure 1.

Interim payoffs with debt-equity and convertible debt financing

Consider the debt-equity contract, (d, s). Interim payoffs (after the realization of the signal x) are averages of the ex-post payoffs in equation (3), weighted by the interim conditional probabilities $q_h(x)$ and $1 - q_h(x)$, minus I_2 for the venture capitalist:[15]

$$\begin{aligned} E[\mathcal{VC}]_x^{DE} &= [1 - q_h(x)]\,\frac{1}{\lambda_l}\left[1 - e^{-\lambda_l d}(1 - s)\right] \\ &\quad + q_h(x)\,\frac{1}{\lambda_h}\left[1 - e^{-\lambda_h d}(1 - s)\right] - I_2, \end{aligned} \tag{7}$$

$$\begin{aligned} E[\mathcal{E}]_x^{DE} &= [1 - q_h(x)]\,\frac{1}{\lambda_l}\,e^{-\lambda_l d}(1 - s) \\ &\quad + q_h(x)\,\frac{1}{\lambda_h}\,e^{-\lambda_h d}(1 - s). \end{aligned} \tag{8}$$

For the convertible debt contract, (d_0, s_0, γ, s), interim payoffs are similarly obtained as averages of the ex-post payoffs in equation (4):

$$\begin{aligned} E[\mathcal{VC}]_x^{CD} &= [1 - q_h(x)]\,\frac{1}{\lambda_l}\,[1 - e^{-\lambda_l[d_0 - \gamma(\hat{s} - s_0)]}(1 - \hat{s})] \\ &\quad + q_h(x)\,\frac{1}{\lambda_h}\,[1 - e^{-\lambda_h[d_0 - \gamma(\hat{s} - s_0)]}(1 - \hat{s})] - I_2, \end{aligned} \tag{9}$$

$$\begin{aligned} E[\mathcal{E}]_x^{CD} &= [1 - q_h(x)]\,\frac{1}{\lambda_l}\,e^{-\lambda_l[d_0 - \gamma(\hat{s} - s_0)]}(1 - \hat{s}) \\ &\quad + q_h(x)\,\frac{1}{\lambda_h}\,e^{-\lambda_h[d_0 - \gamma(\hat{s} - s_0)]}(1 - \hat{s})\,, \end{aligned} \tag{10}$$

where $\hat{s}$ is the post-conversion equity held by the venture capitalist. If he converts the entire amount of convertible debt, then $\hat{s} = s$, while if he converts nothing, $\hat{s} = s_0$.

The refinancing decision and ex-ante payoffs

The entrepreneur never wants to liquidate the project once it has started since he provides no financing and (because of limited liability) always obtains a positive payoff as long as $s < 1$.[16] The driving force of our model is the conflict of interest between entrepreneur and venture capitalist. The former always wants to proceed with the project, while the latter wants to refinance it only if the interim news regarding the probability of success are sufficiently favorable. To compute ex-ante payoffs, this refinancing/liquidation decision must be taken into account.

[15] Notice that, at time 1, I_1 is sunk.

[16] Moreover, we can imagine that any investment by the entrepreneur in human capital is firm-specific and, hence, sunk.

We calculate the maximal ex-ante surplus of the project, defined as the ex-ante surplus given efficient liquidation, as a weighted average of the interim surplus over all signal realizations, x, for which refinancing is efficient, minus the time 0 investment, I_1. The interim surplus is the sum of the interim payoffs of the entrepreneur and the venture capitalist and is equal to $[1-q_h(x)]\,\frac{1}{\lambda_l} + q_h(x)\,\frac{1}{\lambda_h} - I_2$. Denote by x' the realization of the signal that makes the interim surplus zero (x' is the socially efficient refinancing/liquidation cut-off), determined by

$$[1-q_h(x')]\,\frac{1}{\lambda_l} + q_h(x')\,\frac{1}{\lambda_h} - I_2 = 0\,. \tag{11}$$

Recalling that $q_h(x) = x$, we can solve for x' in terms of the model's parameters:

$$x' = \frac{I_2 - 1/\lambda_l}{1/\lambda_h - 1/\lambda_l}\,. \tag{12}$$

The project's maximal ex-ante surplus is then calculated, and is assumed to be strictly positive:

$$\begin{aligned} & \int_{x'}^{1} \left\{ [1-q_h(x)]\,\tfrac{1}{\lambda_l} + q_h(x)\,\tfrac{1}{\lambda_h} - I_2 \right\} [\alpha_l(x)+\alpha_h(x)]\,dx - I_1 \\ = & \int_{x'}^{1} \left\{ \alpha_l(x)\,\tfrac{1}{\lambda_l} + \alpha_h(x)\,\tfrac{1}{\lambda_h} - I_2\,[\alpha_l(x)+\alpha_h(x)] \right\} dx - I_1 \\ = & \tfrac{(1-x')^2}{2}\,\tfrac{1}{\lambda_l} + \tfrac{1-(x')^2}{2}\,\tfrac{1}{\lambda_h} - (1-x')\,I_2 - I_1 > 0\,. \end{aligned} \tag{13}$$

Since x' is expressed in terms of the model's parameters (equation (12)), this assumption constitutes a condition on the parameters of the model.[17]

The assumption that the maximal ex-ante surplus is strictly positive implies that, without signal manipulation, the project is worth financing in stages, thanks to the information revealed by the signal. Therefore, debt-equity and convertible debt contracts exist so that, with stage financing but without signal manipulation, both the entrepreneur and the venture capitalist participate in the venture. Given such contracts, the refinancing/liquidation cut-off value of x is determined by the interim payoff of the venture capitalist and is denoted x_0, defined as the smallest realization of the signal that makes the interim payoff of the venture capitalist positive. This cut-off value and the resulting ex-ante payoffs of the venture capitalist and the entrepreneur will be introduced as the need arises.

[17] Tedious calculations, using (12), yield $\left\{ \frac{1}{2}\left[\left(\frac{1}{\lambda_h}\right)^2 - I_2^2\right] - \left(\frac{1}{\lambda_l} - I_2\right)\right\} \Big/ \left(\frac{1}{\lambda_h} - \frac{1}{\lambda_l}\right) - I_1 > 0$.

2.2 Signal manipulation

To reduce the chances that the project will be liquidated, the entrepreneur can manipulate the distribution of the signal in period 0. We require that after signal manipulation, the unconditional distribution of profits is unchanged. This means that the ex-ante probabilities of the good and bad states, h and l, continue to be 1/2. Namely, signal manipulation affects the distribution of the signal x without affecting the true performance of the project. The venture capitalist cannot observe whether the entrepreneur manipulates the signal.[18]

We model signal manipulation (window dressing) as a rightward shift of probability mass which reduces the information content of low values of the signal. Since the entrepreneur does not know, at time 0, whether the project is good or bad (whether the state of nature is l or h), it is unlikely that he can shift mass to the right only in one state of nature. Rather, we assume that he can engage in activities that give an appearance of better short-term performance in general by shifting probability mass from left to right for various values of x and in both states.

We start by explaining how signal manipulation must be modelled to capture the conflict of interest that arises as a consequence of stage financing. We establish that when continuation depends on the realization of a signal, the entrepreneur would always choose to increase the precision of the signal and the venture capitalist benefits from this choice. As a consequence, the venture capitalist does not worry about manipulation in the form of signal jamming, and does not need to design securities to prevent it. This fact is summarized in

Result 1 *Signal manipulation that increases (or reduces) the precision of the signal by removing (or adding) noise in a mean preserving manner does not capture the conflict of interest between entrepreneur and venture capitalist regarding the liquidation of the project.*

The proof is in two steps. First, we establish that the entrepreneur will not jam the signal in the commonly used sense of adding noise to it (in other words, it is not in the interest of the entrepreneur to reduce the precision of the signal). We follow the literature and model signal jamming as a transfer of mass of size ϵ from the center to each of the tails of the marginal distributions $\alpha_h(x)$ and $\alpha_l(x)$ in a mean preserving manner, as illustrated in panel (a) of Figure 2. Suppose that in period 0 the entrepreneur can jam the signal in this way. Given any refinancing/liquidation cut-off value of x, denoted $\bar{x}$, the entrepreneur does not want to shift mass from

[18] For simplicity, we assume that the entrepreneur can manipulate the signal costlessly. In a previous version (London Business School working paper no. 253, 2001) we introduced a direct cost (born by the entrepreneur), and an indirect cost in the form of effort that is shifted away from improvement of long-run performance. The results do not change in a meaningful way.

the right of $\bar{x}$ to its left since this would increase the probability of liquidation. Moreover, the mass shifted from the center to the *right* tail of the distribution does not affect the probability of liquidation or the expected return of the project. Therefore, jamming the signal involves only drawbacks and no benefits for the entrepreneur. It follows that: (a) the venture capitalist believes that the signal is not jammed; (b) this belief pins down a refinancing/liquidation cut-off value of x; and (c) the entrepreneur does not jam the signal.

Next, suppose the entrepreneur increases the precision of the signal by transfering mass of size ϵ from the tails to the center of the marginal distributions $\alpha_h(x)$ and $\alpha_l(x)$ in a mean preserving manner, as illustrated in panel (b) of Figure 2.[19] Then both entrepreneur and venture capitalist benefit from such activity. In fact, given any beliefs of the venture capitalist (that pin down a refinancing/liquidation cut-off value, $\bar{x}'$) the entrepreneur benefits from shifting mass of size ϵ from the tails to the center because this reduces the probability of liquidation. The venture capitalist also benefits from such manipulation, and would not discourage it. The intuition is as follows. On the one hand, the probability of liquidating a bad project decreases, since mass is shifted in state l from the left of $\bar{x}'$ to its right. On the other hand, the probability of liquidating a good project also decreases since mass is shifted in state h from the left of $\bar{x}'$ to its right. The latter effect dominates because, for small values of x, $\alpha_h(x) < \alpha_l(x)$, so the impact of a shift of mass of given size is higher in state h.[20] The information content of the signal increases on the interval $[x_1, x_2]$ because a signal realization in this interval indicates with greater certainty that the state of nature is l (namely, q_h declines on this interval). Thus, the decision rule used by the venture capitalist—liquidate if and only if $x < \bar{x}'$—is based on better information, increasing the value of the option to abandon. This completes the proof of Result 1.

It follows that if the entrepreneur can choose the precision of the signal, he chooses the most

[19] Reducing the noise in the signal in this manner can be carried out as long as there is probability mass to the left of the cut-off value of x both under $\alpha_l(x)$ and $\alpha_h(x)$.

[20] The proof is as follows. We assume that $\bar{x}' < 1/3$, otherwise shifting mass from the left of $\bar{x}'$ to the center is meaningless in state l. (The conditional mean of $\alpha_l(x)$ is 1/3.) The venture capitalist's ex-ante payoff, prior to signal manipulation, is

$$\int_{\bar{x}'}^{1} \left\{ [1 - q_h(x)] \frac{1}{\lambda_l} \left[1 - e^{-\lambda_l d}(1-s)\right] + q_h(x) \frac{1}{\lambda_h} \left[1 - e^{-\lambda_h d}(1-s)\right] - I_2 \right\} \{\alpha_l(x) + \alpha_h(x)\}\, dx - I_1$$

$$= \left\{ \left[1 - e^{-\lambda_l d}(1-s)\right] - I_2 \right\} \int_{\bar{x}'}^{1} \alpha_l(x) dx + \left\{ \left[1 - e^{-\lambda_h d}(1-s)\right] - I_2 \right\} \int_{\bar{x}'}^{1} \alpha_h(x) dx - I_1 \,.$$

Signal manipulation, as displayed in Figure 2b, increases both $\int_{\bar{x}'}^{1} \alpha_l(x)dx$ and $\int_{\bar{x}'}^{1} \alpha_h(x)dx$ by the same amount, ϵ. The effect on the venture capitalist's ex-ante payoff thus depends on the sign and magnitude of $A \equiv [1 - e^{-\lambda_l d}(1 - s)] - I_2$ and $B \equiv [1 - e^{-\lambda_h d}(1-s)] - I_2$. Since $\bar{x}'$ is determined by the equation $\frac{\alpha_l(\bar{x}')}{\alpha_l(\bar{x}') + \alpha_h(\bar{x}')} A + \frac{\alpha_h(\bar{x}')}{\alpha_l(\bar{x}') + \alpha_h(\bar{x}')} B = 0$, we have $A < 0$ and $B > 0$. By the definition of $\alpha_i(x)$ in equation (5), and since $\bar{x}' < 1/3$, we have $\alpha_l(\bar{x}') > \alpha_h(\bar{x}')$ implying $B > |A|$. Hence, the venture capitalist's ex-ante payoff increases as a result of signal manipulation.

precise signal available, and the venture capitalist benefits from this choice. Since we want to model the tension described in Sahlman (1988), we focus instead on signal manipulation that constitutes a conflict of interest between entrepreneur and venture capitalist.[21] To capture this conflict, it is essential that mass is shifted from the left of the refinancing/liquidation cut-off to its right (which is in the interest of the entrepreneur), and that more mass is shifted in state l than in state h (this damages the venture capitalist). (We elaborate on this point in Section 6 where other forms of signal manipulation are discussed.) We will now present one type of signal manipulation that fits this description.

Window dressing

We have shown that if the main concern of the entrepreneur is to avoid liquidation, he has an incentive to engage in signal manipulation that shifts probability mass from the left of the refinancing/liquidation cut-off value of x to its right. Suppose that at time 0, the entrepreneur can affect the distribution of the signal by reducing the probability of observing low realizations of x and increasing the probability of observing high realizations of x without affecting the probability of states h and l, which remains 1/2. Let us call such activity window dressing and denote it $a = 1$, while $a = 0$ denotes no window dressing. We now turn to a specific model that captures the essential economic forces at work.

Denote $\alpha_h(x, 0)$ the joint density function of x and h when $a = 0$, and similarly for $\alpha_l(x, 0)$. Denote $q_h(x, 0) = \frac{\alpha_h(x,0)}{\alpha_h(x,0)+\alpha_l(x,0)}$ the probability of state h conditional on the signal realization x when $a = 0$. This is just different notation for the expressions in equations (5) and (6). Thus, for x in the interval $[0, 1]$:

$$\alpha_h(x, 0) = x \ , \quad \alpha_l(x, 0) = 1 - x \ , \quad q_h(x, 0) = \frac{x}{x + 1 - x} = x \, , \tag{14}$$

i.e., the signal is strictly informative, and $\int_0^1 \alpha_h(x, 0)dx = \int_0^1 \alpha_h(x, 1)dx = 1/2$.

We express the rightward shift of mass in terms of a parameter k. Window dressing involves: (a) transferring mass in state l from the interval $[0, k]$ to the interval $(k, (1 + k)/2)$; and (b) transferring mass in state h from the interval $(k, (1 + k)/2]$ to the interval $[(1 + k)/2, 1)$, as illustrated in Figure 3. The logic is that as a result of window dressing, poor projects (state l) are less likely to exhibit low interim performance, but good projects (state h) are more likely to

[21] We implicitly assume that any signal manipulation that is feasible and desirable to both parties is already incorporated in the primitive joint distribution of signal and output.

exhibit high interim performance. Thus,

$$\alpha_h(x,1) = \begin{cases} x & x \le k \\ k & k < x < \frac{1+k}{2} \\ 1 & x \ge \frac{1+k}{2}, \end{cases}$$

$$\alpha_l(x,1) = \begin{cases} 1-k & x \le k \\ 1-x+\frac{k^2}{1-k} & k < x < \frac{1+k}{2} \\ 1-x & x \ge \frac{1+k}{2}, \end{cases} \tag{15}$$

implying

$$q_h(x,1) = \begin{cases} \frac{x}{x+1-k} & x \le k \\ \frac{k}{k+1-x+k^2/(1-k)} & k < x \le \frac{1+k}{2} \\ \frac{1}{2-x} & x > \frac{1+k}{2}. \end{cases} \tag{16}$$

Window dressing increases the conditional (interim) probability of state h on the intervals $(0,k)$ and $((1+k)/2,1)$, where $q_h(x,1) > q_h(x,0)$. Notice that $q_h(x,1)$ is strictly informative (strictly increasing in x) everywhere except at $x = k$. This is due to a specific feature of the joint distribution of signal and output that we are using, in particular to the discontinuity of $\alpha_l(x,1)$ at $x = k$. It plays no role in the analysis nor does it drive any of the results, and is assumed only in order to simplify calculations.[22]

Result 2 below establishes that such window dressing reduces the maximal ex-ante surplus of the project. An immediate consequence is that the venture capitalist wants to prevent window dressing because, for any sharing rule, window dressing reduces *his* payoff. Define $x'_{a=1}$ as the socially efficient refinancing/liquidation cut-off value when the signal is manipulated. For simplicity of exposition, we restrict attention to the case where $x'_{a=1}$ is unique, as illustrated in panel (b) of Figure 3. A sufficient condition for this is $x' < k'$, where x' is given in (12) and k' can be derived as shown in Figure 3. From the figure it is also evident that $x'_{a=1}$ is smaller than x'. (Uniqueness of $x'_{a=1}$ is convenient but by no means necessary. If it were not unique, the socially efficient liquidation/refinancing decision when there is window dressing would be nonmonotone in x. The analysis would go through but would be cumbersome.[23])

Result 2 *Window dressing reduces the maximal ex-ante surplus.*

[22]In Section 6 we study another specification of the joint distribution of signal and output where both $q_h(x,0)$ and $q_h(x,1)$ are strictly informative.

[23]Moreover, the possibility of nonuniqueness of the refinancing/liquidation cut-off for $a = 1$ arises only because $q_h(x,1)$ is not strictly informative at $x = k$, a feature that can be eliminated without difficulty (see Section 6).

The intuition is that for low realizations of x the information content of the signal decreases as a result of window dressing, because a low signal realization indicates with less certainty that the state of nature is l. The refinancing/liquidation decision is thus based on lower quality information, reducing the value of the option to abandon. By a similar logic, for any sharing rule (and its corresponding refinancing/liquidation cut-off value of x), window dressing reduces the payoff of the venture capitalist because he is the one investing the additional money on the basis of less accurate information.

Proof of Result 2. The maximal ex-ante surplus when $a = 0$ is

$$\int_{x'}^{1} \left\{ [1 - q_h(x,0)]\,\frac{1}{\lambda_l} \;+\; q_h(x,0)\,\frac{1}{\lambda_h} \;-\; I_2 \right\} [\alpha_l(x,0) + \alpha_h(x,0)]\, dx \;-\; I_1 \,. \tag{17}$$

We will now show that it is larger than the maximal ex-ante surplus when $a = 1$. The expression in (17) is larger than

$$\int_{x'_{a=1}}^{1} \left\{ [1 - q_h(x,0)]\,\frac{1}{\lambda_l} \;+\; q_h(x,0)\,\frac{1}{\lambda_h} \;-\; I_2 \right\} [\alpha_l(x,0) + \alpha_h(x,0)]\, dx \;-\; I_1 \tag{18}$$

because the payoff in (18) is derived using a socially sub-optimal liquidation cut-off.[24] In turn, the expression in (18) is larger than the ex-ante surplus when $a = 1$,

$$\int_{x'_{a=1}}^{1} \left\{ [1 - q_h(x,1)]\,\frac{1}{\lambda_l} \;+\; q_h(x,1)\,\frac{1}{\lambda_h} \;-\; I_2 \right\} [\alpha_l(x,1) + \alpha_h(x,1)]\, dx \;-\; I_1 \,, \tag{19}$$

which can be seen as follows. If the integrals in (18) and (19) were taken on the entire interval $[0, 1]$, they would be equal (because window dressing does not affect the project's true performance). Therefore, showing that (18) is larger than (19) is equivalent to showing that the integrand in (18) integrated on $[0, x'_{a=1}]$ is smaller than the integrand in (19) integrated on the same interval. Using the definitions of $\alpha_h(x,0)$, $\alpha_h(x,1)$, $q_h(x,0)$, and $q_h(x,1)$ (equations (14), (15), and (16)), this is equivalent to showing that $\left(\frac{1}{\lambda_l} - I_2\right) \int_0^{x'_{a=1}} [(1-x) - (1-k)]\, dx < 0$.[25] Since on the interval $[0, x'_{a=1}]$, $x < k$, we have $\int_0^{x'_{a=1}} [(1-x) - (1-k)]\, dx > 0$, and from equation (11) we have $\frac{1}{\lambda_l} - I_2 < 0$, establishing the result. □

To underline the damage caused by window dressing, we assume that the decline in the maximal ex-ante surplus as a result of window dressing is sufficiently pronounced so that the

[24]More specifically, on the interval $[x'_{a=1}, x']$ the integrand is negative, by definition of x'.

[25]Notice that on the interval $[0, x'_{a=1}]$ window dressing does not affect α_h.

expression in (19) is strictly negative. This condition can be written as

$$\int_{x'_{a=1}}^{1} \left\{ \alpha_l(x,1)\,\frac{1}{\lambda_l} \;+\; \alpha_h(x,1)\,\frac{1}{\lambda_h} \;-\; I_2\left[\alpha_l(x,1)+\alpha_h(x,1)\right] \right\} \,dx \;-\; I_1 \;<\; 0\,, \tag{20}$$

namely, the project is *not* worth financing. Thus, when it is known that $a = 1$, there is no contract such that the venture capitalist's ex-ante payoff is positive (as he bears all the costs and what he can obtain is bounded above by the maximal ex-ante surplus, which is negative).

2.3 Window dressing with a debt-equity contract

We showed that window dressing reduces the probability of liquidation but, at the same time, reduces the benefit of stage financing (by reducing the value of the option to abandon), rendering the project nonviable. We now show that, nevertheless, once debt-equity financing has been obtained, the entrepreneur will always want to window dress the distribution of the signal. Anticipating this, the venture capitalist will not provide financing at time 0.[26]

Proposition 1 *If the project is financed with a debt-equity contract,* (d, s)*, the entrepreneur will always manipulate (window dress) the distribution of the signal.*

The proof and the intuition are one and the same. Suppose that (for some reason) the project is financed with the debt-equity contract, (d, s). Suppose further that the venture capitalist believes that $a = 0$. The contract and this belief pin down a refinancing/liquidation cut-off x_0.[27] Given x_0, the entrepreneur wants to shift mass from the left of x_0 to its right because this reduces the probability of liquidation (with no other effect on the entrepreneur's payoff). The effect of window dressing on the probability of refinancing varies according to whether x_0 is smaller than k, larger than $(1 + k)/2$, or between the two. Consider Figure 3. If $x_0 \leq k$, the rightward shift of mass does not affect the probability of refinancing in state h, but increases the probability of refinancing in state l. If $k < x_0 \leq (1+k)/2$, the probability of refinancing in both states increases, and if $x_0 > (1 + k)/2$, only the probability of refinancing in state h increases. In all three cases, the payoff of the entrepreneur is higher when $a = 1$. Therefore, $a = 0$ cannot be part of an equilibrium. If the venture capitalist believes that $a = 1$, there will be another

[26]This feature of the model resembles Stein (1989) where managers inflate the price of the company shares. Although the market perfectly anticipates this behavior, the managers nevertheless engage in signal manipulation because they take market expectations as given. In Stein's model, the damage caused by signal manipulation is not large enough to prevent trade in the company shares.

[27]Since the venture capitalist obtains only part of the maximal ex-ante surplus, x_0 is greater than x', the socially efficient refinancing/liquidation cut-off.

cut-off value of x and by an analogous argument, given this cut-off, the entrepreneur wants to shift mass from its left to its right, namely, $a = 1$ in line with the venture capitalist's belief. Thus, *if* the project is financed with a straight debt-equity contract, $a = 1$; anticipating this, financing will not be provided as the contract fails to provide incentives for no window dressing.

2.4 The role of convertible debt in preventing window dressing

We will now show that with an appropriately designed financing scheme that combines equity, straight debt, and convertible debt the project will be financed and the entrepreneur will not window dress at the ex-ante stage.[28] The result is summarized in:

Proposition 2 *There is a convertible debt contract, (d_0, s_0, γ, s), such that the entrepreneur does not window dress and the project is financed.*

A detailed proof is provided in Appendix 1. We present here the main arguments. The proof consists of constructing a convertible debt contract, (d_0, s_0, γ, s) and cut-off values, x_0 and x_1, satisfying $0 < x_0 < k < x_1 < 1$, such that in equilibrium the venture capitalist (a) provides I_1 at time 0; (b) liquidates the project after observing $x < x_0$, but not otherwise, where x_0 is the signal realization that renders his interim payoff zero; (c) does not convert any debt after observing $x \in [x_0, x_1)$; (d) converts the maximal amount of debt allowed, $d_0 - d$, after observing $x \geq x_1$, where $d = d_0 - \gamma(s - s_0)$; and the entrepreneur chooses $a = 0$.

The conversion ratio, γ, and the venture capitalist's post-conversion share, s, play a key role in the construction of the contract. If γ is low, converting debt into equity is cheap and the venture capitalist will convert if he decides to refinance the project, whereas if γ is high, he will not. In either case, the option to convert does not constitute a threat that prevents window dressing. A conversion ratio, γ, that is not too high or too low must be selected to provide incentives for the entrepreneur. The attributes "low" and "high" depend on s, the venture capitalist's post-conversion share of the enterprise. The larger is s, the smaller is the entrepreneur's post-conversion share of the enterprise (i.e., the bigger is the "punishment" for window dressing), and the larger γ can be and still be effective. The γ and s that jointly prevent window dressing are such that when the venture capitalist chooses to convert debt, conversion takes place at a convenient rate for the venture capitalist, hurting the entrepreneur (in other

[28] Typically, there will be many convertible debt contracts that induce $a = 0$. The choice among them will depend, among other things, on the relative bargaining power of entrepreneur and venture capitalist.

words, the venture capitalist buys underpriced equity). The loss to the entrepreneur more than offsets the advantage from the fact that liquidation is less likely to take place.

We begin by fixing an arbitrary value for s_0. This illustrates that the result in Proposition 2 does not rely on "razor's edge" arguments, and there is considerable leeway in constructing convertible debt contracts that prevent window dressing. We want to choose the other parameters of the contract and the two cut-off values of x, so that the venture capitalist will not convert any debt after observing $x \in [x_0, x_1)$, but will convert all the convertible debt after observing $x \geq x_1$. To exploit the model's "natural structure," we design the contract so that $x_1 = (1+k)/2$. We choose d_0, γ, and s such that the following condition is satisfied:

$$\begin{aligned} & \tfrac{1-k}{2}\tfrac{1}{\lambda_l}\left[1 - e^{-\lambda_l d_0}(1-s_0)\right] + \tfrac{1+k}{2}\tfrac{1}{\lambda_h}\left[1 - e^{-\lambda_h d_0}(1-s_0)\right] \\ = \; & \tfrac{1-k}{2}\tfrac{1}{\lambda_l}\left[1 - e^{-\lambda_l[d_0-\gamma(s-s_0)]}(1-s)\right] + \tfrac{1+k}{2}\tfrac{1}{\lambda_h}\left[1 - e^{-\lambda_h[d_0-\gamma(s-s_0)]}(1-s)\right]. \end{aligned} \tag{21}$$

The left-hand side is the venture capitalist's interim payoff if he converts no debt, and the right-hand side is his interim payoff if he converts all the convertible debt. (See equations (7) and (8), and recall that $q(x,0) = x$, i.e., $q(x_1, 0) = (1+k)/2$.) Thus, condition (21) states that the signal realization $x = (1+k)/2$ renders the venture capitalist indifferent between not converting debt and converting all the convertible debt. Later in the proof, we show that, for any signal realization, the venture capitalist's interim payoff is maximized either by converting no debt or by converting all the convertible debt. Therefore, if $x \geq (1+k)/2$, the venture capitalist converts the entire amount of convertible debt, and if $x < (1+k)/2$ he converts none. Condition (21) defines an implicit relation between γ, s, and d_0. Throughout the proof, we require that this equality is satisfied. In particular, any argument involving a change in γ and d_0 entails a corresponding change in s so that (21) is satisfied.

Next, we impose a condition that ensures that the entrepreneur prefers not to manipulate the signal. That is, given the convertible debt contract and the cut-off values, x_0 and $x_1 = (1+k)/2$, his ex-ante payoff with $a = 0$ is larger than his ex-ante payoff with $a = 1$ (given that the venture capitalist believes that $a = 0$). Spelling out this condition explicitly is tedious (see equation (30) in Appendix 1), but after rearranging it boils down to the following inequality:

$$e^{\lambda_h \gamma(s-s_0)}\,\frac{1-s}{1-s_0} \leq 1 + \frac{\frac{1}{2}x_0(x_0-2k)}{\frac{1}{8}(1-k)^2}\,\frac{\lambda_h}{\lambda_l}\,e^{-(\lambda_l-\lambda_h)d_0}. \tag{22}$$

We rearrange equation (21) so that its left-hand side equals the left-hand side of (22)—see Appendix 1—to obtain

$$e^{\lambda_l \gamma(s-s_0)} \frac{1-s}{1-s_0} = \frac{\frac{1-k}{2}\frac{1}{\lambda_h} e^{-\lambda_h[d_0-\gamma(s-s_0)]} e^{-(\lambda_l-\lambda_h)d_0} + \frac{1+k}{2}\frac{1}{\lambda_h} e^{-\lambda_h[d_0-\gamma(s-s_0)]}}{\frac{1-k}{2}\frac{1}{\lambda_l} e^{-\lambda_l[d_0-\gamma(s-s_0)]} + \frac{1+k}{2}\frac{1}{\lambda_h} e^{-\lambda_h[d_0-\gamma(s-s_0)]}}. \quad (23)$$

This facilitates the construction of a convertible debt contract such that both (22) and (23)—i.e. (21)—hold. To construct this contract, we proceed as follows. We increase γ and d_0 so that (a) for each value of γ and d_0, we choose s that satisfies (23); and (b) the post-conversion debt, $d_0 - \gamma(s - s_0)$, remains positive and finite.[29] (In Appendix 1, we show formally how this is accomplished.) The driving economic force in this construction is the change in γ and the corresponding change in s, as discussed above.

We then show that as γ and d_0 increase, the right-hand side of (22) approaches 1. We further show that, as γ and d_0 increase, the right-hand side of (23) approaches a number strictly between 0 and 1, and so must the left-hand side in order to preserve the equality. Since the left-hand side of (23) is equal to the left-hand side of (22), the inequality in (22) is eventually satisfied. Therefore, there are values of γ, d_0, and s that ensure that both (23) and (22) hold.

Next, we verify that the venture capitalist's interim payoff is maximized either at s_0—"no debt conversion," or at s—"full debt conversion" which implies that the formulation of condition (23) is consistent with optimization by the venture capitalist. Finally, we turn to the venture capitalist's ex-ante participation constraint, and show that it is satisfied.

Economic intuition for Proposition 2

Figure 4 displays *interim* payoffs of venture capitalist and entrepreneur as a function of the signal realization x. Panel (a) displays these payoffs when there is no window dressing ($a = 0$). The dashed lines represent the payoffs for straight debt-equity financing, (d_0, s_0). The venture capitalist's interim payoff can be written as $L' - I_2 + (H' - L')x$, where $L' = \frac{1}{\lambda_l}\left[1 - e^{-\lambda_l d_0}(1 - s_0)\right]$ and $H' = \frac{1}{\lambda_h}\left[1 - e^{-\lambda_h d_0}(1 - s_0)\right]$. Similarly, the entrepreneur's interim payoff can be written as $L'' + (H'' - L'')x$, where $L'' = \frac{1}{\lambda_l} e^{-\lambda_l d_0}(1 - s_0)$ and $H'' = \frac{1}{\lambda_h} e^{-\lambda_h d_0}(1 - s_0)$.[30] The venture capitalist's interim payoff at x_0 is zero because x_0 is defined as the value of x that makes him indifferent between liquidating and investing I_2. The entrepreneur, who does not invest money,

[29]This condition guarantees that we are considering "pure" convertible debt. In Section 4, we introduce the use of warrants (instead of convertible debt) and this type of condition is no longer required.

[30]The sum of these payoffs is the total interim surplus, $\frac{1}{\lambda_l} + \left(\frac{1}{\lambda_h} - \frac{1}{\lambda_l}\right) x - I_2$.

has a strictly positive interim payoff at x_0.

The solid lines represent the payoffs for convertible debt financing, (d_0, s_0, γ, s), in equilibrium, i.e. when the venture capitalist converts all the convertible debt following $x \geq (1+k)/2$. The upward kink in the venture capitalist's payoff and the corresponding downward kink in the entrepreneur's payoff at $x = (1+k)/2$ illustrate the benefit to the venture capitalist from converting debt at a favorable ratio and the corresponding damage to the entrepreneur.[31]

The ex-ante payoff for each party is computed as a weighted average—over the signal realizations, x—of the interim payoff (minus $I_1 + \int_{x_0}^{1} I_2 \left[\alpha_l(x,0) + \alpha_h(x,0)\right] dx$ for the venture capitalist). The shaded area illustrates the damage to the entrepreneur from debt conversion by the venture capitalist. (Had the venture capitalist not converted, the interim payoffs would be those represented by the dashed lines.) However, the shaded area serves only as an illustration because the actual ex-ante payoff is computed as a weighted average of the interim payoffs, where the weight given to signal realization x is $\alpha_l(x,0) + \alpha_h(x,0)$.

Panel (b) similarly displays the interim payoffs when there is window dressing ($a = 1$), assuming that the venture capitalist believes that there is no window dressing (thus, the refinancing/liquidation cut off, x_0, is the same as in panel (a)). To decide whether to manipulate the signal, the entrepreneur compares his ex-ante payoff when $a = 0$ (top-right graph) and when $a = 1$ (bottom-right graph). For this comparison, the most relevant intervals are $[0, x_0]$ and $[(1+k)/2, 1]$. On $[0, x_0]$, his interim payoff is zero (whether $a = 0$ or 1) but for $a = 1$ it occurs with lower probability, which constitutes the incentive to window dress. The main force against window dressing is the damage to him from debt conversion by the venture capitalist, as illustrated by the shaded areas on the interval $[(1+k)/2, 1]$. Ex-ante, the entrepreneur does not know for sure in which interval the signal will be realized. But he does know that window dressing increases the likelihood that the signal will be in the interval where he is damaged (in other words, window dressing increases the weights given to the payoffs on the interval $[(1+k)/2, 1]$). An appropriately designed convertible debt contract ensures that the latter force dominates in terms of the entrepreneur's ex-ante payoff.[32]

The parameters d_0 and s_0 shift the payoff schedule up or down, and are chosen to satisfy individual rationality constraints. For the entrepreneur the constraint is trivially satisfied: due to limited liability, and since he does not invest his own money, his payoff cannot be negative.

[31] The sum of the two slopes is $\frac{1}{\lambda_h} - \frac{1}{\lambda_l}$, as is the case for straight debt-equity financing; see footnote 30.

[32] For an exhaustive analysis, we should also study the interval $[x_0, (1+k)/2]$. This discussion was only meant to provide the intuition, but the proof of Proposition 2 takes into account all the effects.

Thus, d_0 and s_0 are chosen to guarantee a sufficiently high (ex-ante) payoff to the venture capitalist. These parameters only affect the division of the surplus, and there are degrees of freedom in their choice.[33]

A numerical example

We simulated the model and generated a host of examples, of which we report one in order to illustrate that the model generates plausible results. The project's output, π_i, is distributed exponentially on the interval $[0, \infty)$, with density $\lambda_i e^{-\lambda_i \pi_i}$, $i = h, l$, and mean $\frac{1}{\lambda_l} = 56,680$ and $\frac{1}{\lambda_h} = 110,667$. The required investment at each stage is $I_1 = 10,871$ and $I_2 = 74,830$. Thus, $\frac{1}{2}\frac{1}{\lambda_l} + \frac{1}{2}\frac{1}{\lambda_h} - (I_1 + I_2) = -2,028$, namely, stage financing is necessary.[34]

The signal is distributed on $[0, 1]$ as illustrated in Figure 1, with $k = 0.3794$ and $(1+k)/2 = 0.6897$. The socially efficient liquidation cut-off value of the signal (when there is no window dressing) is $x' = 0.3362$, and the maximal ex-ante surplus is 1024. If, however, there is window dressing, the socially efficient liquidation cut-off value of the signal is $x_{a=1} = 0.3143$, and the maximal ex-ante surplus is negative (-257). Thus, without window dressing the project is worth financing, but with window dressing it is not.

Consider the debt-equity contract $s_0 = 0.3$ and $d_0 = 420,819$.[35] The liquidation cut-off value of the signal (when there is no window dressing), that makes the venture capitalist's interim payoff zero, is $x_0 = 0.348$. But given this contract (and the venture capitalist's belief that there is no window dressing), the entrepreneur will window dress (Proposition 1): his ex-ante payoff with window dressing is 767 while without window dressing it is 765.

Consider the convertible debt contract $s_0 = 0.3$, $d_0 = 420,819$, $\gamma = 317,538$, and $s = 0.8645$, implying $d = 420,819 - 317,538\ (0.8645 - 0.3) = 241,569$. The amount of convertible debt is $d_0 - d = 420,819 - 241,569 = 179,250$. The liquidation cut-off value of the signal (when there is no window dressing) is $x_0 = 0.348$, and the debt conversion cut-off is $x_1 = (1+k)/2 = 0.6897$. Without window dressing, the venture capitalist's ex-ante payoff is 261, and the entrepreneur's ex-ante payoff is 759. Given this contract (and the venture capitalist's correct belief that there is no window dressing), the entrepreneur will not window dress (Proposition 2): we verified

[33] In the proof of Proposition 2 (see Step 9 in Appendix 1), we fix s_0 and vary d_0 to satisfy the venture capitalist's participation constraint. Varying only d_0 is sufficient for this purpose because the project's output (π_i, $i = l, h$) is unbounded. Then, for fixed s_0, the share of the surplus that goes to the venture capitalist increases monotonically with d_0 (and approaches unity as d_0 approaches infinity).

[34] The displayed numbers are rounded.

[35] Kaplan and Strömberg (2002) report that in more than 98 percent of the cases in their sample the liquidation rights of the venture capitalist (which correspond to d) are much larger than his cumulative investment.

that his ex-ante payoff without window dressing is larger than his payoff with window dressing (while the venture capitalist believes that there is no window dressing) by calculating both sides of inequality (22). We also plotted the venture capitalist's interim payoff as a function of $\hat{s}$ for several values of $x \in [x_0, x_1)$ and $x \in [x_1, 1]$, and verified that it achieves a corner maximum, at $\hat{s} = s_0 = 0.3$ and $\hat{s} = s = 0.8645$, respectively. Thus, following a signal $x \geq x_1 = 0.6897$, the venture capitalist exchanges about 40 percent of the debt (which is all the convertible debt) for 55 percent of the project's equity, becoming the main equity holder. (Notice that the initial equity holding by the venture capitalist does not give him control over the project, whereas after full debt conversion he holds 86.45 percent of the equity. In our study, we overlooked the issue of control rights and benefits of control. These issues no doubt play a role in the design of these securities, and are left for future work.)

To get a sense of the "riskiness" of the project, we calculated the following probabilities: (1) the probability of refinancing ($x \geq x_0$) in states l and h is, respectively, 0.21 and 0.44 (the ex-ante probability of refinancing is the average of these numbers, 0.325; this is slightly high compared to real world "common wisdom," although in the sample studied by Gompers and Lerner, 1999, only about 15 percent of the companies are liquidated); (2) the (unconditional) probability of debt conversion ($x \geq x_1$) in states l and h is, respectively, 0.05 and 0.48; (3) the (unconditional) probability that profits will exceed post-conversion debt ($\pi_i \geq d = 241,669$) in states l and h are 0.01 and 0.11, respectively; thus, for example, the ex-ante probability that there will be refinancing cum debt conversion *and* the entrepreneur will walk away with *any* money, is 0.5*0.05*0.01+0.5*0.48*0.11=0.02665, i.e., about 2.7 percent. (For the entrepreneur to walk away with money if there is no debt conversion, ex-post profits must exceed $d_0 = 420,819$ which happens with very small probability.) Therefore, although the ex-ante expected surplus from the project is not high at all relative to the required investment, with small probability both the entrepreneur and the venture capitalist can walk away with a huge amount of money, which is exactly what is observed in the real world.

Allowing for renegotiation with convertible debt financing

If the venture capitalist refinances (i.e., if $x \geq x_0$), there will be no renegotiation of the terms of the convertible debt contract even if renegotiation is allowed. The reason is that there is no mutual gain at time 1 from changing the terms of the contract because the entrepreneur and the venture capitalist are risk neutral, and the terms of the contract cannot be altered so that one party benefits without hurting the other.

Renegotiation can arise only if there is an inefficiency that induces the parties to change the contract in a Pareto improving manner. This arises if the project is liquidated when instead there is a positive surplus from continuation. In other words, there is renegotiation after signal realizations such that $x' \leq x < x_0$, where x' is the socially efficient cut-off value of x. Only when $x' = x_0$ there is no renegotiation. But this can happen only if $s_0 = 1$, which is inconsistent with the basic assumption of our model that $s < 1$ (and hence, $s_0 < 1$). Therefore, although an appropriately designed convertible debt contract ensures that financing is provided at time 0, inefficient liquidation at time 1 after some realizations of x is inevitable.[36] In that sense, convertible debt financing is constrained optimal (where the constraint is determined by the need to provide incentives to the entrepreneur to undertake long-term effort).

In the proof of Proposition 2 (Appendix 1, step 9) we showed that the convertible debt contract can be constructed so that x_0 is arbitrarily close to x'. In other words, inefficient liquidation and the ensuing renegotiation can be made to occur with an arbitrarily small probability, implying that renegotiation can be rendered negligible in terms of payoffs. This implies that the possibility of renegotiation at time 1 does not affect the feasibility of the project because none of the arguments we made depend on strict indifference.

The deadline for conversion

In the model, the deadline for conversion must precede time 2 and it is, indeed, optimal for the parties to set this deadline before time 2. If conversion were *at* time 2, the venture capitalist would wait until then, when the uncertainty regarding the state of nature is resolved, to decide whether and how much to convert. This decision would be independent of the realization of the signal (at time 1). But then the logic of Proposition 1 applies: because the threat of debt conversion at favorable terms in response to a good signal is no longer present, the entrepreneur wants to window dress in order to reduce the probability of liquidation, and the project will not be financed. The parties thus have a common interest to set the deadline for debt conversion strictly before time 2.

This feature of the securities in the model does not match the convertible debt (or, more precisely, preferred shares) observed in the reality, which typically have infinite maturity, so that

[36] Note that this inefficiency would happen with *any* contract that does not render the venture capitalist the sole owner of the project's equity. This well understood type of inefficiency has nothing to do with the phenomenon we are studying—preventing window dressing with a convertible security—and is due to the nonverifiability of the signal. It does not arise when the signal is verifiable, as we will show in Section 5, because in that case the liquidation decision and the allocation of the surplus among the parties are not linked.

venture capitalists can decide whether to convert only when they exit the investment (upon sale or IPO). However, in the reality warrants are also widely used in venture capital financing and they do have a maturity date (see Kaplan and Strömberg 2002). Thus, it can be argued that the convertible security in our model best represents a combination of straight debt (or preferred shares) and warrants. In Section 4 we consider this case explicitly.

3 Allowing for renegotiation of the debt-equity contract

In the previous sections we assumed that the debt-equity contract could not be renegotiated at the interim stage, after the signal is observed by the parties. In this section, we explicitly consider this possibility, and argue that there is a role for convertible securities even when debt-equity contracts can be renegotiated.

Spontaneous renegotiation of the debt-equity contract

Parties renegotiate a contract whenever there is space for a Pareto improvement. In our model, this can happen only if there is inefficient liquidation by the venture capitalist. When the contract does not specify how renegotiation will be carried out (e.g., sequence of moves, permissible offers), we define it as "spontaneous." Can such renegotiation prevent window dressing? Clearly, the answer depends on the specific renegotiation that will arise "spontaneously." To illustrate the logic, we start by assuming that, following inefficient liquidation, the parties renegotiate so that the project is refinanced and the surplus is split in proportion μ to the venture capitalist and $1-\mu$ to the entrepreneur.[37]

Suppose that the project is financed with a debt-equity contract, (d, s), and let x_0 be the corresponding refinancing/liquidation cut-off that makes the venture capitalist's interim payoff zero. Suppose, first, that $x_0 = 1$ (in other words, (d, s) gives the venture capitalist a payoff so low that he never wants to refinance). Two situations can arise: either $x < x'$, the project is liquidated, and his payoff is zero, or $x \geq x'$, and there is renegotiation that determines his payoff. Renegotiation means that the parties agree on a division of the ex-post surplus such that, in expectation, they are entitled to fractions $\mu, 1-\mu$ of the interim surplus, $\left[(1-q(x,0))\frac{1}{\lambda_l} + q(x,0)\frac{1}{\lambda_h} - I_2\right]$.[38] Ex-ante, at time 0, when the entrepreneur chooses whether

[37] We thank a referee for suggesting this approach. We will show that even costless renegotiation does not prevent window dressing. *A fortiori* it will not do so when it entails a cost $c > 0$. In that case, the socially efficient liquidation cut-off is some $x'_c > x'$, there is renegotiation following $x \geq x'_c$, and the project's surplus net of c is split between the parties according to the shares μ and $1-\mu$.

to window dress, his payoff is equal to the expectation over x of the post-renegotiation interim payoff (which is a constant fraction μ of the interim surplus).

This payoff can be achieved through debt-equity contracts, as follows. For each realization of $x \geq x'$, and consequent renegotiation, the parties can agree on a debt-equity contract that gives the entrepreneur (in expectation) a constant share, $1-\mu$, of the interim surplus. Such a debt-equity contract is easily built: choose $s = \mu$ and $d(x)$ such that the expected revenue to the venture capitalist from $d(x)$ equals $(1-\mu)I_2$. (That is, the venture capitalist receives a share μ of the revenue and bears a share μ of the interim investment, I_2, namely, he obtains a share μ of the surplus.) To compute $d(x)$, remember that if the state of nature is i the expected revenue from debt with face value d is

$$\int_0^d \pi_i \lambda_i e^{-\lambda_i \pi_i} d\pi_i + \int_d^{+\infty} d\lambda_i e^{-\lambda_i \pi_i} d\pi_i = \frac{1}{\lambda_i}\left[1 - e^{-\lambda_i d}\right],$$

and, therefore, $d(x)$ is the unique solution of

$$q(x,0)\frac{1}{\lambda_h}\left[1 - e^{-\lambda_h d}\right] + (1 - q(x,0))\frac{1}{\lambda_l}\left[1 - e^{-\lambda_l d}\right] = (1-\mu)I_2.$$

Notice that $d'(x) < 0$. (This follows from the fact that $q(x,0)$ is continuously differentiable and strictly increasing in x.) If d were a constant, the proof of Proposition 1 would go through unchanged. The fact that d decreases with x constitutes an *additional* incentive for the entrepreneur to window dress. Thus, *a fortiori,* the payoff profile arising from renegotiation does not prevent window dressing, and $a = 0$ is not part of the equilibrium. Anticipating this, the venture capitalist will not finance the project.

Suppose now $x_0 < 1$. Following $x \geq x_0$ the project is refinanced, there is no renegotiation, and the payoffs to the venture capitalist and the entrepreneur are those determined by the debt-equity contract, (d,s). Following $x' \leq x < x_0$, inefficient liquidation would occur and there is renegotiation which results in the above surplus sharing rule in proportions μ, $1-\mu$. Notice that renegotiation on the interval $[x', x_0)$ occurs because with the existing contract, (d,s), the venture capitalist's payoff is not high enough to cover I_2. Renegotiation must give him a higher payoff than with (d,s) (because following renegotiation he is willing to refinance the project but without renegotiation he is not). This implies that renegotiation gives the entrepreneur a lower payoff than he would have gotten with (d,s) (had the project been refinanced). Thus, the

[38] In the analysis, we maintain the equilibrium belief $a = 0$, which is necessary for the project to be financed.

entrepreneur's share of the interim surplus is higher on the interval $[x_0, 1]$ than on the interval $[x', x_0)$. (To see this, notice that at x_0, the venture capitalist's interim payoff is zero and the entrepreneur's interim payoff is positive; see Figure 4. To the left of x_0, the venture capitalist's interim payoff rises from negative to positive due to renegotiation, and as a consequence, the entrepreneur's interim payoff falls below what it would have been without renegotiation. Thus, to the left of x_0 the entrepreneur's payoff after renegotiation is lower than to the right of x_0.)

This payoff profile cannot prevent window dressing. To prevent window dressing, the entrepreneur's payoff profile must be such that his share of the surplus after "medium" realizations of x is large relative to his share after "high" realizations of x so as to provide an incentive *not* to shift mass from left to right.

More generally, to prevent window dressing, the entrepreneur's bargaining power must decrease with x, so that his share of the surplus is smaller the larger is x. Otherwise, he will have an incentive to window dress. In most situations, spontaneous renegotiation will not be such that the bargaining power changes endogenously as a function of x in this particular manner. In such cases, the entrepreneur will window dress even when he anticipates renegotiation.

Designed renegotiation of the debt-equity contract

Whenever "spontaneous" renegotiation does not prevent window dressing, parties can include in the contract the rules of renegotiation. Aghion, Dewatripont, and Rey (1994) demonstrate that, in situations with "under-investment," efficiency can be achieved by explicitly specifying these rules in the contract. Because renegotiation can take many forms, they focus on "simple" renegotiation. Renegotiation is said to be simple if it is characterized by two rules: (i) a default allocation in the event that renegotiation breaks down or fails to occur; and (ii) all the bargaining power is given to one party.

In our model, window dressing cannot be prevented if all the bargaining power is given to one party. If the venture capitalist has all the bargaining power, the entrepreneur's payoff will remain constant (at the default). If the entrepreneur has all the bargaining power, his share of the surplus will rise with x because the (expected) size of the total "pie" increases with x. In neither case the resulting profile of interim payoffs to the entrepreneur prevents window dressing. Therefore, "simple" renegotiation as defined by Aghion, Dewatripont, and Rey will not prevent window dressing. We can, however, extend their definition, adapting it to the context of our model, in a way that prevents window dressing. The important point is that this "simple" renegotiation is isomorphic to a convertible debt contract.

We call renegotiation "simple" if (i) it specifies a default allocation in the event that renegotiation breaks down or fails to occur; (ii) it specifies a menu of debt-equity contracts from which the parties must choose at time 1 if renegotiation takes place; and (iii) the right to choose from this menu is allocated to one party ex-ante (at time 0). Requirements (i) and (iii) correspond to requirements (i) and (ii) in Aghion, Dewatripont, and Rey's "simple" renegotiation. In addition, we introduce (ii) which has the effect of limiting the bargaining power allocated in (iii), since the party with the right to choose the sharing rule cannot choose any such rule, but only one that is specified in the menu.

Since in our model renegotiation can occur only following inefficient liquidation, the default allocation in (i) is (trivially) an interim payoff of zero to each party, because failure to renegotiate entails liquidation of the project. Let the menu of contracts in (ii) be $[(d_0, s_0), (d, s)]$, and give the venture capitalist the right to choose a contract from this menu after x is realized.[39] We set these values equal to those of the convertible debt contract we constructed in Proposition 2, with $d = d_0 - \gamma(s - s_0)$. We know from Proposition 2 that in such case the venture capitalist will choose (d_0, s_0) after observing $x < (1 + k)/2$ and (d, s) after observing $x \geq (1 + k)/2$, and the entrepreneur will choose not to window dress. Therefore, this designed renegotiation is equivalent to a convertible debt contract. The venture capitalist's right to choose a contract from the menu in (ii) is equivalent to his choice whether to convert no debt or convert all the convertible debt in case of refinancing. Finally, (iii) does not give *all* the bargaining power to the venture capitalist because he is constrained by the menu of contracts in (ii), namely, by the conversion ratio, γ, and the maximal equity stake he can achieve, s. Thus, convertible debt financing that incorporates an option to convert debt at a pre-specified conversion ratio can be interpreted as "simple" renegotiation in the spirit of Aghion, Dewatripont, and Rey.[40]

4 Straight debt cum warrants

With a convertible debt contract, debt is automatically reduced upon conversion. Now we consider the alternative of straight debt cum warrants, such that when the venture capitalist converts he has to pay money but the debt is not reduced. This case is important because in

[39] This menu is the simplest possible, since there are only two choices.

[40] As in the case of renegotiation of a convertible debt contract, here too there is an inevitable inefficiency (which can be made arbitrarily small) due to the need to provide incentives to the entrepreneur to undertake long-term effort. This need implies $s < 1$ (and hence $s_0 < 1$) which in turn implies inefficient liquidation (see the discussion in footnote 36).

the reality the warrants used by the venture capitalists have finite maturity—not necessarily equal to the debt maturity—see Kaplan and Strömberg (2002). The contract we constructed has this feature. We will show that, with minor modifications, our model applies to the case of straight debt cum warrants.

The initial debt and equity holdings by the venture capitalist are d_0 and s_0, where d_0 is straight debt. In addition, the entrepreneur issues to the venture capitalist warrants that can be converted to equity at the rate γ. If (some) warrants are exercised, the venture capitalist receives $\hat{s} - s_0$ shares and in exchange pays $\hat{m} = \gamma(\hat{s} - s_0)$ dollars. The ex-post payoff to the venture capitalist from this contract, for $i = l, h$, is:

$$\begin{aligned} & \int_0^{d_0} \pi_i \lambda_i e^{-\lambda_i \pi_i} d\pi_i + \int_{d_0}^{\infty} [d_0 + \hat{s}(\pi_i - d_0)] \lambda_i e^{-\lambda_i \pi_i} d\pi_i \; - \; \hat{m} \; + \; \hat{s}\, \hat{m} \\ = \; & \tfrac{1}{\lambda_i} [1 - e^{-\lambda_i d_0} (1 - \hat{s})] \; - \; (1 - \hat{s})\, \hat{m} \, . \end{aligned} \tag{24}$$

The first two terms in the first line of (24) are the venture capitalist's share of the profits, the third term is what he pays when exercising the warrants, and the fourth term is the fraction of this payment that is rebated to him (as the owner of a fraction $\hat{s}$ of the project). Thus, the entrepreneur's ex-post payoff is

$$\frac{1}{\lambda_i} e^{-\lambda_i d_0} (1 - \hat{s}) \; + \; (1 - \hat{s})\, \hat{m} \, . \tag{25}$$

From here on, the analysis is basically the same. The proof of Proposition 1—with debt-equity financing the entrepreneur will window dress—is identical, and the following proposition is analogous to Proposition 2:

Proposition 3 *There is a straight debt cum warrants contract such that the entrepreneur does not window dress and the project is financed.*

The proof is similar but not identical, due to the somewhat different form of the ex-post payoffs, and is provided in Appendix 2.

5 Verifiable signal

In many circumstances, interim performance can be measured objectively and in a verifiable manner. In such cases, the initial contract does not need to allocate to one party the option to convert debt. Instead, an automatic conversion clause can be specified, contingent on a pre-specified threshold of a verifiable signal. A security with such a clause is equivalent to a contract which specifies different debt-equity combinations contingent on the realization of a verifiable signal. In this section, we analyze such contracts.

An efficient contingent contract must ensure that the project is liquidated only when it is socially optimal to do so (i.e., when the expected surplus from continuation is negative). This requirement is easily satisfied: the contract at time 0 should specify that the project will be liquidated if and only if $x < x'$, where x' is the socially efficient refinancing/liquidation cut-off.

An efficient contingent contract must also ensure that the entrepreneur does not manipulate the signal. From Proposition 1 we know that this cannot be achieved by specifying a debt-equity sharing rule that does not vary as a function of the interim performance signal. The sharing rule must change so that the entrepreneur "loses" if the signal is too favorable, rendering $a = 0$ optimal from his perspective. When the signal is verifiable, it is straightforward to render the sharing rule contingent on the realization of the signal since this can be specified explicitly in the contract. The "simplest" manner to do so is to formulate a *contingent debt-equity contract* with two debt-equity sharing rules, (d_0, s_0) and (d, s), and a cut-off value of the signal that triggers the switching from one sharing rule to the other.

To show that such a contract exists, we can mimic the convertible debt contract constructed in Proposition 2: (a) set the initial debt-equity allocation equal to the pre-conversion debt-equity allocation, (d_0, s_0); (b) set the refinancing/liquidation cut-off equal to x'; (c) set the trigger for switching to the alternative debt-equity allocation equal to the debt conversion trigger, $x_1 = (1 + k)/2$; and (d) set the debt-equity allocation for $x \geq x_1$ equal to the post-conversion allocation (when all the convertible debt is converted), (d, s).

Contingent debt-equity financing exhibits an improvement with respect to convertible debt financing: socially efficient liquidation. This is because efficient liquidation can be specified in the contract regardless of the sharing rule.

The analogy between contingent debt-equity and convertible debt financing highlights a fundamental role of the debt conversion option which is to adjust the debt-equity structure to new information that is revealed during the life-time of the project. This feature bears resemblance to provisions allowing the provider of financing to take control of the project at an interim stage in response to nonverifiable information, a feature that is very common in incomplete contracts environments. In our model, convertible debt financing allows the provider of financing to alter the financial structure of the project (changing the output allocation rule) at an interim stage in response to nonverifiable information. If only part of the information that is expected to arise is verifiable, convertible debt financing may be superior to contingent debt-equity financing because convertible debt prevents window dressing whether the signal is verifiable or not (at the cost of inefficient liquidation) whereas contingent debt-equity financing prevents window dressing only if information is verifiable.[41] That could explain why convertible securities used in venture capital financing typically leave the venture capitalist with the option to convert even when there is an automatic conversion clause.

The study by Kaplan and Strömberg (2002) is consistent with this reasoning. In their sample of venture capital financing contracts, automatic conversion is contingent only on an IPO taking place and satisfying certain conditions (e.g., regarding the issue price, proceeds, or market capitalization), but never on other verifiable interim information such as sales revenue, profitability, or market share. This suggests that while the "final" outcome (the occurrence and terms of an IPO) is a well-defined verifiable signal, pre-IPO performance signals are not. Therefore, it may be better to leave the venture capitalist with the option to convert debt into equity at pre-specified terms.

6 The joint distribution of signal and output

Until now we focused on one manner in which window dressing can affect the joint distribution of signal and output (Figure 3). In this section, we discuss the robustness of that particular form of signal manipulation and show that, although specific features were introduced in order to simplify the exposition, signal manipulation as presented in Figure 3 captures well the phenomenon of window dressing.

In Result 1, we considered the possibility of modifying the precision of the signal by removing (or adding) noise in a mean preserving manner. We showed that with a debt-equity contract, in situations where the entrepreneur wants to pass a minimum performance threshold, he will always choose to increase the precision of the signal and the venture capitalist will benefit from it. Therefore, there is no conflict of interest and the venture capitalist does not need to use instruments like convertible debt to prevent window dressing. The problem is not whether our suggested convertible contract alleviates the conflict of interest between entrepreneur and venture capitalist, but rather that there is no such conflict to begin with.[42]

In Result 1 we showed that the venture capitalist benefits from higher precision of the signal because, for low realizations of x, the mass that is shifted from left to right, ϵ, is larger in state h

[41] Moreover, the cost of inefficient liquidation can be made arbitrarily small; see step 9 in Appendix 1.

[42] According to Sahlman (1990), conflict of interest between entrepreneurs and venture capitalists regarding refinancing decisions is a central reason for window dressing by entrepreneurs.

than in state l *relative to the density of the signal.* As a result, the advantage of not liquidating good projects more than compensates the fact that fewer bad projects are liquidated. This can be generalized: *any* type of signal manipulation that has this property does not create conflict of interest between entrepreneur and venture capitalist because it improves the quality of information on which the decision to liquidate is based.

To create such a conflict of interest, the amount of mass shifted from left to right must be *smaller* (relative to the density of the signal) in state h than in state l. The case displayed in Figure 3 is an extreme case where *no* mass is shifted from left to right in state h. This feature is not crucial for our main result—it is simply easier to treat. Our results go through with an intermediate situation, displayed in Figure 5, where mass on the interval $[0, k]$ is shifted to the right in both states. (It can be shown that q_h changes as presented in the figure.) In this case, signal manipulation hurts the venture capitalist and is a good representation of window dressing. We prefer to focus on the case displayed in Figure 3 (a limit case) because it saves the need to verify algebraically the conditions on parameters ensuring that signal manipulation is actually harmful to the venture capitalist.

The discussion so far established that to capture the essence of window dressing mass should be moved from the left of the refinancing/liquidation cut-off value of x to its right so that the information content of low values of x decreases. It still remains to be determined *where* to the right this mass should be moved. In Figure 3, no mass is moved to the interval $[(1 + k)/2, 1]$ in state l. Again, this is an extreme case that simplifies the computations, but may be attenuated so that some mass is moved to the interval $[(1 + k)/2, 1]$ in both states. As long as q_h does not decrease for high values of x, our result goes through.

If, instead, q_h decreases for high values of x, convertible debt may still be effective in preventing window dressing as long as the decrease in q_h is not substantial (which depends on parameter values). Notice that for q_h to decrease substantially for high values of x, it is necessary that more mass (relative to the density of the signal) is moved to high values of x in state l than in state h. The economic interpretation is quite intuitive: if the entrepreneur succeeds in increasing substantially the performance of "bad" projects without increasing by too much the performance of "good" projects, the venture capitalist is hurt when he converts securities into equity following a good signal realization, and there is no threat that prevents window dressing. While we cannot exclude this possibility *a priori*, we believe that it is not very realistic since it should be easier to improve the interim performance of a project that is fundamentally good than of a project that is fundamentally poor.

In Figure 6, we present a case that is similar in all respects to the case presented in Figure 3 with the only difference that when $a = 1$, the signal is informative everywhere. It illustrates that the discontinuity in $q_h(x, 1)$ at $x = k$ is not instrumental for our results. This type of window dressing represents a situation where, when the project is "bad," the probability of observing different values of the signal is constant on a large interval (from 0 to 2/3). In other words, the entrepreneur succeeds in manipulating the short-term performance so that, when the project is "bad," poor interim performance is as likely as medium performance. The analysis is very similar to that of previous sections and is omitted. We can show that there is a convertible debt contract such that in equilibrium the venture capitalist liquidates if $x < 1/3$, refinances but does not convert any debt after observing $1/3 \leq x < 2/3$, refinances and converts all the convertible debt after observing $x \geq 2/3$, and the entrepreneur does not window dress.[43]

7 Conclusion

We showed that convertible securities dominate a mix of debt and equity in stage financing situations. When the venture capitalist retains the option to abandon the project if in the medium term he receives bad news, the entrepreneur has an incentive to engage in "window dressing," or short-termism, and bias the process that generates these interim news in order to reduce the probability of liquidation. Convertibles reduce this incentive for short-termism through the threat of conversion.

The fundamental role of the conversion option is to adjust the ownership (debt-equity) structure to new information. This feature resembles provisions allowing the provider of financing to alter the financial structure of an existing contract at an interim stage in response to nonverifiable information, a feature that is common in incomplete contracts environments. The need to commit in advance to the terms of debt conversion is in line with the logic of Aghion, Dewatripont, and Rey (1994) who show that by specifying in the contract the bargaining procedure to be used in future renegotiation, an efficient outcome can be achieved.

Venture capital convertible debt contracts sometimes incorporate, in addition to the voluntary debt conversion, automatic conversion agreements.[44] In such situations, where the signal

[43] The example can be generalized so that the cut-off values of x are parameters (rather than the numbers 1/3 and 2/3).

[44] Gompers (1996, Table 4) documents the use of such agreements, but Kaplan and Strömberg (2002) report that, in their sample of venture capital financing contracts, automatic conversion is contingent only on an IPO taking place at certain conditions, but never on other verifiable interim information such as sales revenue, profitability, or market share.

that triggers conversion is verifiable, there is no gain from endowing the financier with the power to *unilaterally* alter the debt-equity structure by converting debt. In situations where signals are not verifiable, authority to unilaterally change the terms of the contract is crucial. Our model captures well the contingent financial structure aspect that is relevant in these situations, and is especially suitable for settings where interim signals are not verifiable.

We conclude by pointing out an important generalization of our 3-period set-up. We can think of a set-up with more periods where window dressing can occur at various stages where additional financing is needed. The venture capitalist can decide not to convert debt at a given stage yet still keeps the option to do so in a future financing round. The possibility of issuing new securities at interim stages then arises. This is an important extension that should be studied in future work.

Appendix 1: Proof of Proposition 2

Step 1. Fix $s_0 < 1$. The value of s_0 will not change throughout the proof.[45]

Step 2. Given s_0, we impose the following condition on γ, s, and d_0:

$$\begin{array}{c} \frac{1-k}{2}\frac{1}{\lambda_l}\left[1-e^{-\lambda_l d_0}(1-s_0)\right] \quad + \quad \frac{1+k}{2}\frac{1}{\lambda_h}\left[1-e^{-\lambda_h d_0}(1-s_0)\right] \\ = \\ \frac{1-k}{2}\frac{1}{\lambda_l}\left[1-e^{-\lambda_l[d_0-\gamma(s-s_0)]}(1-s)\right] + \frac{1+k}{2}\frac{1}{\lambda_h}\left[1-e^{-\lambda_h[d_0-\gamma(s-s_0)]}(1-s)\right]. \end{array} \tag{26}$$

This is precisely condition (21) in the main text and states that the signal realization $x = (1+k)/2$ renders the venture capitalist indifferent between not converting debt and converting the entire amount of convertible debt. In step 8, we will show that, for any signal realization, the venture capitalist's interim payoff is maximized either by converting no debt or by converting the full amount of convertible debt available. Therefore, if $x \geq (1+k)/2$, the venture capitalist converts the entire amount of debt, and if $x < (1+k)/2$ he converts no debt.

Dividing both sides of (26) by $(1-s_0)$, solving for $(1-s)/(1-s_0)$, and dividing the numerator and denominator of the right hand side by $e^{-\lambda_h d_0}$ yields

$$\frac{1-s}{1-s_0} = \frac{\frac{1-k}{2}\frac{1}{\lambda_l}e^{-(\lambda_l-\lambda_h)d_0} + \frac{1+k}{2}\frac{1}{\lambda_h}}{\frac{1-k}{2}\frac{1}{\lambda_l}e^{-(\lambda_l-\lambda_h)d_0}e^{\lambda_l\gamma(s-s_0)} + \frac{1+k}{2}\frac{1}{\lambda_h}e^{\lambda_h\gamma(s-s_0)}}. \tag{27}$$

This equation defines an implicit relation between γ, s, and d_0. Throughout the proof, we will require that this equality is satisfied. In particular, any argument involving a change in γ and d_0 entails a corresponding change in s so that (27) is satisfied and, consequently, the debt conversion cut-off value remains $x_1 = (1+k)/2$.

Step 3. We establish that along any sequence such that $\gamma \to \infty$, $d_0 \to \infty$, and (27) holds, $s \to 1$. This is true because as $d_0 \to \infty$, the first term in the numerator of the right-hand side of (27) approaches zero (since $\lambda_l > \lambda_h$) and, therefore, the numerator approaches the constant $\frac{1+k}{2}\frac{1}{\lambda_h}$, while the denominator approaches infinity.[46]

Step 4. To construct a convertible debt contract that prevents window dressing, we increase γ and d_0 so that: (a) $\gamma \to \infty$; (b) $d_0 \to \infty$; (c) condition (27) is satisfied along the sequence; and (d) the expression $d_0 - \gamma(s - s_0)$ approaches a finite nonnegative number.[47]

[45] This illustrates that the result in Proposition 2 does not rely on "razor's edge" arguments, and that there is considerable leeway in constructing convertible debt contracts that prevent window dressing.

[46] The second term in the denominator approaches infinity as $\gamma \to \infty$, and the first term is bounded below by zero.

[47] This is easily accomplished. Consider a sequence such that $\gamma \to \infty$, $d_0 \to \infty$, and (27) holds. Since along the sequence, $s \to 1$, the limit of $d_0 - \gamma(s - s_0)$ is the same as the limit of $d_0 - \gamma(1 - s_0)$. There are many sequences of d_0 and γ such that this expression approaches a finite limit (e.g., if $d_0(n) = n$ and $\gamma(n) = (n-2)/(1-s_0)$, then $d_0(n) - \gamma(n)(1-s_0) = n - \frac{n-2}{1-s_0}(1-s_0) = 2$).

Step 5. We establish that if γ and d_0 increase as in step 4, then $\gamma(1-s) \to 0$, a fact that will be used later in the proof. To see this, multiply both sides of (27) by $e^{\lambda_h \gamma (s-s_0)}$ and then multiply the numerator and denominator of the right-hand side by $e^{-\lambda_h d_0}$, to obtain

$$e^{\lambda_h \gamma(s-s_0)} \, \frac{1-s}{1-s_0} \; = \; \frac{\frac{1-k}{2}\,\frac{1}{\lambda_l}\,e^{-\lambda_h [d_0-\gamma(s-s_0)]}\,e^{-(\lambda_l-\lambda_h)d_0} \; + \; \frac{1+k}{2}\,\frac{1}{\lambda_h}\,e^{-\lambda_h [d_0-\gamma(s-s_0)]}}{\frac{1-k}{2}\,\frac{1}{\lambda_l}\,e^{-\lambda_l [d_0-\gamma(s-s_0)]} \; + \; \frac{1+k}{2}\,\frac{1}{\lambda_h}\,e^{-\lambda_h [d_0-\gamma(s-s_0)]}}\,, \tag{28}$$

which is condition (23) in the main text. First, we establish that the right-hand side approaches a finite limit as $\gamma \to \infty$ and $d_0 \to \infty$. By step 4, $d_0 - \gamma(s - s_0)$ approaches a finite limit and, therefore, both terms in the denominator of the right-hand side as well as the second term in the numerator on the right-hand side approach a finite limit. By contrast, the first term in the numerator on the right-hand side approaches zero. Thus, denoting z the (strictly positive finite) limit of $d_0 - \gamma(s - s_0)$, the right-hand side of (28) approaches the finite limit

$$0 \; < \; \frac{\frac{1+k}{2}\,\frac{1}{\lambda_h}\,e^{-\lambda_h z}}{\frac{1-k}{2}\,\frac{1}{\lambda_l}\,e^{-\lambda_l z} \; + \; \frac{1+k}{2}\,\frac{1}{\lambda_h}\,e^{-\lambda_h z}} \; < 1\,. \tag{29}$$

To preserve the equality, so must the left-hand side of (28) implying that $s \to 1$ at the same rate at which $e^{\lambda_h \gamma(s-s_0)} \to \infty$ and, therefore, $s \to 1$ at a faster rate than that at which $\gamma \to \infty$. Thus, $\gamma(1-s) \to 0$ as $\gamma \to \infty$.

Step 6. Denote x_0 the liquidation/refinancing cut-off value. This cut-off value changes when d_0 changes. The entrepreneur's ex-ante payoff with $a = 0$ is greater than with $a = 1$ (when the venture capitalist believes that $a = 0$) if the following condition is satisfied:

$$\begin{aligned}
& \int_{x_0}^{\frac{1+k}{2}} (1-x)\,\frac{1}{\lambda_l}\,e^{-\lambda_l d_0}\,(1-s_0)\,dx \quad + \quad \int_{x_0}^{\frac{1+k}{2}} x\,\frac{1}{\lambda_h}\,e^{-\lambda_h d_0}\,(1-s_0)\,dx \\
& \int_{\frac{1+k}{2}}^{1} (1-x)\,\frac{1}{\lambda_l}\,e^{-\lambda_l [d_0-\gamma(s-s_0)]}\,(1-s)\,dx \quad + \quad \int_{\frac{1+k}{2}}^{1} x\,\frac{1}{\lambda_h}\,e^{-\lambda_h [d_0-\gamma(s-s_0)]}\,(1-s)\,dx \\
\geq \\
& \int_{x_0}^{k} (1-k)\,\frac{1}{\lambda_l}\,e^{-\lambda_l d_0}\,(1-s_0)\,dx \quad + \quad \int_{x_0}^{k} x\,\frac{1}{\lambda_h}\,e^{-\lambda_h d_0}\,(1-s_0)\,dx \\
& \int_{k}^{\frac{1+k}{2}} \left(1-x+\frac{k^2}{1-k}\right)\frac{1}{\lambda_l}\,e^{-\lambda_l d_0}\,(1-s_0)\,dx \quad + \quad \int_{k}^{\frac{1+k}{2}} k\,\frac{1}{\lambda_h}\,e^{-\lambda_h d_0}\,(1-s_0)\,dx \\
& \int_{\frac{1+k}{2}}^{1} (1-x)\,\frac{1}{\lambda_l}\,e^{-\lambda_l [d_0-\gamma(s-s_0)]}\,(1-s)\,dx \quad + \quad \int_{\frac{1+k}{2}}^{1} 1\,\frac{1}{\lambda_h}\,e^{-\lambda_h [d_0-\gamma(s-s_0)]}\,(1-s)\,dx\,,
\end{aligned} \tag{30}$$

which, after manipulation, yields

$$e^{\lambda_h \gamma(s-s_0)}\,\frac{1-s}{1-s_0} \; \leq 1 + \frac{\frac{1}{2}x_0(x_0-2k)}{\frac{1}{8}(1-k)^2}\,\frac{\lambda_h}{\lambda_l}\,e^{-(\lambda_l-\lambda_h)d_0}\,, \tag{31}$$

which is equation (22) in the main text.

As $\gamma \to \infty$ and $d_0 \to \infty$, the second term on the right-hand side of (31) approaches 0 (notice that

x_0 changes along the sequence but is bounded so it does not affect the limit of the right-hand side). Therefore, the right-hand side approaches 1. From step 5, the left-hand side approaches a limit that is strictly smaller than 1 (the left-hand side of (31) is also the left-hand side of (28)), implying that far enough along the sequence the inequality (31) is (strictly) satisfied, namely, the entrepreneur does not gain by deviating from $a = 0$ to $a = 1$

Step 7. We now establish two simple auxiliary results that will be used in the next step. In step 4, we showed that as $\gamma \to \infty$, $\gamma(1-s) \to 0$. An implication of this fact is that for γ large enough, $\frac{1}{\lambda_h} - \gamma(1-s) > \frac{1}{\lambda_l} - \gamma(1-s) > 0$ (since $\frac{1}{\lambda_h} > \frac{1}{\lambda_l}$). In addition, for γ large enough, $\frac{1}{\lambda_l} - \gamma(1-s_0) < \frac{1}{\lambda_h} - \gamma(1-s_0) < 0$ (since s_0 is fixed and $\gamma \to \infty$).

Step 8. The venture capitalist decides how much debt to convert, thus determining his post-ownership equity stake, $\hat{s} \in [s_0, s]$. We will now show that after any signal realization, x, the venture capitalist will convert no debt ($\hat{s} = s_0$) or will convert all the convertible debt ($\hat{s} = s$).

The venture capitalist's interim payoff as a function of $\hat{s}$ and x is:[48]

$$[1 - q_h(x,a)]\,\frac{1}{\lambda_l}\,[1 - e^{-\lambda_l[d_0 - \gamma(\hat{s}-s_0)]}(1-\hat{s})] + q_h(x,a)\,\frac{1}{\lambda_h}\,[1 - e^{-\lambda_h[d_0 - \gamma(\hat{s}-s_0)]}(1-\hat{s})] - I_2\,.$$

The first derivative of the venture capitalist's interim payoff with respect to $\hat{s}$ is

$$[1 - q_h(x,a)]\,\frac{1}{\lambda_l}\,e^{-\lambda_l[d_0 - \gamma(\hat{s}-s_0)]}[\frac{1}{\lambda_l} - \gamma(1-\hat{s})] + q_h(x,a)\,\frac{1}{\lambda_h}\,e^{-\lambda_h[d_0 - \gamma(\hat{s}-s_0)]}[\frac{1}{\lambda_h} - \gamma(1-\hat{s})]\,, \qquad (32)$$

and the second derivative of this payoff with respect to $\hat{s}$ is

$$[1 - q_h(x,a)]\,\frac{1}{\lambda_l}\,\lambda\gamma e^{-\lambda_l[d_0 - \gamma(\hat{s}-s_0)]}[\frac{2}{\lambda_l} - \gamma(1-\hat{s})] + q_h(x,a)\,\frac{1}{\lambda_h}\,\lambda\gamma e^{-\lambda_h[d_0 - \gamma(\hat{s}-s_0)]}[\frac{2}{\lambda_h} - \gamma(1-\hat{s})]\,. \quad (33)$$

By step 7, we know that for γ large enough, $\frac{1}{\lambda_l} - \gamma(1-s_0) < 0$ and $\frac{1}{\lambda_l} - \gamma(1-s) > 0$, implying that there exists $\bar{s} \in (s_0, s)$ for which $\frac{1}{\lambda_l} - \gamma(1-\bar{s}) = 0$. Analogously, there exists $\underline{s} \in (s_0, s)$ for which $\frac{1}{\lambda_h} - \gamma(1-\underline{s}) = 0$. Moreover, $\underline{s} < \bar{s}$ (since $\lambda_l > \lambda_h$). Consequently, $\frac{1}{\lambda_l} - \gamma(1-\hat{s})$ evaluated at $\underline{s}$ is strictly negative and $\frac{1}{\lambda_h} - \gamma(1-\hat{s})$ evaluated at $\bar{s}$ is strictly positive. It follows that the first derivative of the venture capitalist's interim payoff is strictly negative to the left of $\underline{s}$, strictly positive to the right of $\bar{s}$, and zero at a value $s^* \in (\underline{s}, \bar{s})$.

An analogous set of arguments applies to the second derivative of the venture capitalist's interim payoff, which is strictly negative to the left of $\underline{s}'$, strictly positive to the right of $\bar{s}'$, zero at a value $s^{**} \in (\underline{s}', \bar{s}')$, where $\underline{s}'$ and $\bar{s}'$ satisfy $\frac{2}{\lambda_h} - \gamma(1-\underline{s}') = 0$ and $\frac{2}{\lambda_l} - \gamma(1-\bar{s}') = 0$.

From $\frac{2}{\lambda_l} - \gamma(1-\bar{s}') = 0$ and $\frac{1}{\lambda_h} - \gamma(1-\underline{s}) = 0$ (see the previous two paragraphs) and $\frac{2}{\lambda_l} > \frac{1}{\lambda_h}$ (see assumption (1)), we obtain $\bar{s}' < \underline{s}$. Therefore, the interval $(\underline{s}', \bar{s}')$ lies to the left of the interval $(\underline{s}, \bar{s})$, with no overlap. Thus, when the venture capitalist's interim payoff function becomes flat, at $s^* \in (\underline{s}, \bar{s})$,

[48] This is the same expression as in equation (9).

the second derivative of this function is strictly positive.

It follows that the interim payoff of the venture capitalist, as a function of his post-conversion equity stake, achieves a unique interior minimum on the interval (s_0, s). Therefore, the function is maximized either at s_0—"no debt conversion," or at s—"full debt conversion," and the formulation of conditions (26) and (31) is consistent with optimization by the venture capitalist.

Step 9. We now turn to the venture capitalist's ex-ante participation constraint. To show that this constraint is satisfied, it is sufficient to show that his payoff from the debt-equity contract (d_0, s_0) is positive, since the option to convert can only increase this payoff. We will show that as $\gamma \to \infty$ and $d_0 \to \infty$, this payoff becomes strictly positive, ensuring that the venture capitalist will provide financing at time 0.

Consider the ex-ante payoff of the venture capitalist from a debt-equity contract (d_0, s_0), when $a = 0$ and with the refinancing/liquidation cut-off x_0. This payoff is computed in an analogous way to the maximal ex-ante surplus in (13), and is given by

$$\int_{x_0}^{1} [\,(1-x)\,\frac{1}{\lambda_l}\,[1-e^{-\lambda_l d_0}(1-s_0)] \,+\, x\,\frac{1}{\lambda_h}\,[1-e^{-\lambda_h d_0}(1-s_0)] - \,I_2\,]\,dx\, -I_1\,, \tag{34}$$

where x_0 is determined by

$$[1-q(x_0,0)]\,\frac{1}{\lambda_l}\,[1-e^{-\lambda_l d_0}(1-s_0)] \,+\, q(x_0,0)\,\frac{1}{\lambda_h}\,[1-e^{-\lambda_h d_0}(1-s_0)] \,-\, I_2 \,=\, 0\,. \tag{35}$$

As $d_0 \to \infty$, $1 - e^{-\lambda d_0}(1-s_0) \to 1$. Therefore, by equations (35) and (11), x_0 approaches x' (the socially efficient cut-off value of x). By the strict inequality in (13)—namely, by the assumption that the maximal ex-ante surplus with $a = 0$ is strictly positive)—far enough along the sequence, the expression in (34) is strictly positive. □

Appendix 2: Proof of Proposition 3

Step 1. Fix $s_0 < 1$. The value of s_0 will not change throughout the proof.

Step 2. We can always choose an initial debt d_0 and a corresponding refinancing/liquidation cut-off, x_0 so that: (1) $x_0 < k$; and (2) without window dressing the ex-ante payoff of the venture capitalist is strictly positive. To show that this can be done, consider the venture capitalist's ex-ante payoff from a debt-equity contract (d, s), when $a = 0$, and with the refinancing/liquidation cut-off, x_0,[49]

$$\int_{x_0}^{1} [\,(1-x)\,\frac{1}{\lambda_l}\,[1-e^{-\lambda_l d}(1-s)] \,+\, x\,\frac{1}{\lambda_h}\,[1-e^{-\lambda_h d}(1-s)] - \,I_2\,]\,dx\, -I_1 \tag{36}$$

[49]These equations are analogous to (34) and (35) which were written for a specific contract (d_0, s_0), whereas (36) and (37) are written for a generic contract (d, s).

where x_0 is determined by

$$[1-x_0]\,\frac{1}{\lambda_l}\,[1-e^{-\lambda_l d}(1-s)]\,+\,x\,\frac{1}{\lambda_h}\,[1-e^{-\lambda_h d}(1-s)]\,-\,I_2\,=\,0\,. \tag{37}$$

Let $s=s_0$ (step 1). As d increases, $1-e^{-\lambda_i d}(1-s)$ approaches unity. From equations (36), (37), (13), and (11) it follows that x_0 approaches x' which, by assumption, is smaller than k. Thus, there is a value d_0' that is large enough so that, for all $d_0 \geq d_0'$, the corresponding cut-off x_0 is smaller than k. Moreover, by the assumption that the total ex-ante surplus with $a=0$ and efficient liquidation is strictly positive—see (13)—the expression in (36) is strictly positive.

Step 3. Given s_0 and d_0, we impose the following condition on γ and s:

$$\begin{aligned}&\tfrac{1-k}{2}\,\tfrac{1}{\lambda_l}\,\left[1-e^{-\lambda_l d_0}(1-s_0)\right]+\,\tfrac{1+k}{2}\,\tfrac{1}{\lambda_h}\,\left[1-e^{-\lambda_h d_0}(1-s_0)\right]=\\ &=\tfrac{1-k}{2}\,\tfrac{1}{\lambda_l}[1-e^{-\lambda_l d_0}(1-s)]+\,\tfrac{1+k}{2}\,\tfrac{1}{\lambda_h}[1-e^{-\lambda_h d_0}(1-s)]\,-\,(1-s)\gamma(s-s_0)\,.\end{aligned} \tag{38}$$

This condition states that the signal realization $x=(1+k)/2$ renders the venture capitalist indifferent between not exercising the warrants, and exercising all the warrants.[50] Manipulation yields

$$\frac{1-s}{1-s_0}\;=\;\frac{\tfrac{1-k}{2}\,\tfrac{1}{\lambda_l}\,e^{-\lambda_l d_0}\,+\,\tfrac{1+k}{2}\,\tfrac{1}{\lambda_h}\,e^{-\lambda_h d_0}}{\tfrac{1-k}{2}\,\tfrac{1}{\lambda_l}e^{-\lambda_l d_0}\,+\,\tfrac{1+k}{2}\,\tfrac{1}{\lambda_h}e^{-\lambda_h d_0}\,+\gamma(s-s_0)}\,, \tag{39}$$

which defines an implicit relation between γ, s, s_0, and d_0. Throughout the proof, we will require that this equality be satisfied. In particular, any argument involving a change in γ (given s_0 and d_0) entails a change in s so that (27) is satisfied.

Step 4. Along any sequence such that $\gamma\to\infty$ and (39) holds, $s\to1$. This is because all the terms on the right-hand side of (39) are constant except γ so the right-hand side approaches zero as $\gamma\to\infty$.

Step 5. Next, we establish that $\gamma(1-s)$ approaches a strictly positive finite limit. This can be seen by multiplying both sides of (39) by $\gamma(s-s_0)$, obtaining

$$\gamma(s-s_0)\,\frac{1-s}{1-s_0}\;=\;\frac{\gamma(s-s_0)\,\left(\tfrac{1-k}{2}\,\tfrac{1}{\lambda_l}\,e^{-\lambda_l d_0}\,+\,\tfrac{1+k}{2}\,\tfrac{1}{\lambda_h}\,e^{-\lambda_h d_0}\right)}{\tfrac{1-k}{2}\,\tfrac{1}{\lambda_l}e^{-\lambda_l d_0}\,+\,\tfrac{1+k}{2}\,\tfrac{1}{\lambda_h}e^{-\lambda_h d_0}\,+\gamma(s-s_0)}\,. \tag{40}$$

As $\gamma\to\infty$, the right-hand side approaches the strictly positive finite limit $\frac{1-k}{2}\,\frac{1}{\lambda_l}\,e^{-\lambda_l d_0}+\frac{1+k}{2}\,\frac{1}{\lambda_h}\,e^{-\lambda_h d_0}\equiv X$, and so must the left-hand side. Taking the limit of the left-hand side, we have $\lim_{\gamma\to\infty}\gamma(s-s_0)\,\frac{1-s}{1-s_0}=\lim_{\gamma\to\infty}\gamma(1-s)\,\lim_{\gamma\to\infty}\frac{s-s_0}{1-s_0}=X$. As $\gamma\to\infty$, $s\to1$, implying $\lim_{\gamma\to\infty}\frac{s-s_0}{1-s_0}=1$ and, therefore, $\lim_{\gamma\to\infty}\gamma(1-s)=X$ which is strictly positive and finite. (This means that $s\to1$ at the same rate as $\gamma\to\infty$.)

[50] The interim payoff on the right-hand side of (38) uses the ex-post payoffs in (24) evaluated at $\hat{s}=s$ and $\hat{m}=m=\gamma(s-s_0)$.

Step 6. The entrepreneur's ex-ante payoff with $a = 0$ is greater than with $a = 1$ (when the venture capitalist believes that $a = 0$) if the following condition is satisfied:

$$\begin{aligned} &\int_{x_0}^{\frac{1+k}{2}} (1-x)\,\tfrac{1}{\lambda_l}\,e^{-\lambda_l d_0}\,(1-s_0)\,dx \quad + \quad \int_{x_0}^{\frac{1+k}{2}} x\,\tfrac{1}{\lambda_h}\,e^{-\lambda_h d_0}\,(1-s_0)\,dx+ \\ &+\int_{\frac{1+k}{2}}^{1} (1-x)\,[\,(1-s)\,\tfrac{1}{\lambda_l}\,e^{-\lambda_l d_0} + (1-s)m\,]\,dx \quad + \quad \int_{\frac{1+k}{2}}^{1} x\,[\,(1-s)\,\tfrac{1}{\lambda_h}\,e^{-\lambda_h d_0} + (1-s)m\,]\,dx \\ \geq \\ &\int_{x_0}^{k} (1-k)\,\tfrac{1}{\lambda_l}\,e^{-\lambda_l d_0}\,(1-s_0)\,dx \quad + \quad \int_{x_0}^{k} x\,\tfrac{1}{\lambda_h}\,e^{-\lambda_h d_0}\,(1-s_0)\,dx+ \\ &+\int_{k}^{\frac{1+k}{2}} \left(1-x+\tfrac{k^2}{1-k}\right)\,\tfrac{1}{\lambda_l}\,e^{-\lambda_l d_0}\,(1-s_0)\,dx \quad + \quad \int_{k}^{\frac{1+k}{2}} k\,\tfrac{1}{\lambda_h}\,e^{-\lambda_h d_0}\,(1-s_0)\,dx+ \\ &+\int_{\frac{1+k}{2}}^{1} (1-x)\,[\,(1-s)\,\tfrac{1}{\lambda_l}\,e^{-\lambda_l d_0} + (1-s)m\,]\,dx \quad + \quad \int_{\frac{1+k}{2}}^{1} [\,(1-s)\,\tfrac{1}{\lambda_h}\,e^{-\lambda_h d_0} + (1-s)m\,]\,dx\,, \end{aligned} \tag{41}$$

which, after manipulation and multiplication of both sides by $\gamma(s - s_0)$, yields

$$\gamma(s-s_0)\,\frac{1-s}{1-s_0} \leq \frac{\gamma(s-s_0)\,\left[\frac{1}{2}\,x_0(x_0-2k)\frac{1}{\lambda_l}\,e^{-\lambda_l d_0} \;+\; \frac{1}{8}\,(1-k)^2\,\frac{1}{\lambda_h}\,e^{-\lambda_h d_0}\right]}{\frac{1}{8}\,(1-k)^2\frac{1}{\lambda_h}\,e^{-\lambda_h d_0} \;+\; \frac{1}{8}\,(1-k)^2\,\gamma(s-s_0)}. \tag{42}$$

From step 5, it follows that, as $\gamma \to \infty$, the left-hand side of (42) approaches $\frac{1-k}{2}\,\frac{1}{\lambda_l}\,e^{-\lambda_l d_0} + \frac{1+k}{2}\,\frac{1}{\lambda_h}\,e^{-\lambda_h d_0}$ (in (40) and (42) the left-hand side is the same). The right-hand side of (42) approaches $\frac{\frac{1}{2}\,x_0(x_0-2k)}{\frac{1}{8}\,(1-k)^2}\,\frac{1}{\lambda_l}\,e^{-\lambda_l d_0} + \frac{1}{\lambda_h}\,e^{-\lambda_h d_0}$. It is easily verified that, in the limit, the right-hand side exceeds the left-hand side if $\frac{\lambda_l}{\lambda_h}\,e^{(\lambda_l-\lambda_h)d_0} \geq \frac{1-k}{2} + \frac{4x_0(2k-x_0)}{(1-k)^2}$. As $d_0 \to \infty$, the left-hand side of this expression also approaches infinity, while the right-hand side remains constant and finite. Therefore, there exists a debt value $d_0^{''}$ such that, for all $d_0 \geq d_0^{''}$, the entrepreneur will not window dress.

Step 7. The venture capitalist decides how many warrants to exercise, thus determining his post-ownership equity stake, $\hat{s} \in [s_0, s]$. We will now show that after any signal realization, x, the venture capitalist will exercise no warrants ($\hat{s} = s_0$) or will exercise all the warrants ($\hat{s} = s$).

The venture capitalist's interim payoff as a function of $\hat{s}$ and x is:

$$[1-x]\,\frac{1}{\lambda_l}\,[1-e^{-\lambda_l d_0}(1-\hat{s})] + x\,\frac{1}{\lambda_h}\,[1-e^{-\lambda_h d_0}(1-\hat{s})] - (1-\hat{s})\gamma(\hat{s}-s_0) - I_2\,.$$

The first derivative of the venture capitalist's interim payoff with respect to $\hat{s}$ is

$$[1-x]\,\frac{1}{\lambda_l}\,e^{-\lambda_l d_0} + x\,\frac{1}{\lambda_h}\,e^{-\lambda_h d_0} - \gamma(2\hat{s}-s_0-1)\,, \tag{43}$$

and the second derivative of this payoff with respect to $\hat{s}$ is $2\gamma > 0$.

In other words, the venture capitalist interim payoff is a parabola that achieves its minimum at $\tilde{s} = \left\{\gamma\,(1+s_0) \,+\, d_0 \,-\, \left[(1-x)\,\frac{1}{\lambda_l} \,+\, x\,\frac{1}{\lambda_h}\right]\right\}\,/\,2\gamma$, implying that, as $\gamma \to \infty$, $\tilde{s} \to (1+s_0)\,/\,2$, where

$s_0 < (1 + s_0)\,/\,2 < 1$. Therefore, for γ large enough, the venture capitalist's interim payoff is strictly decreasing in $\hat{s}$ as he begins exercising warrants and begins to rise if enough warrants are exercised, at $\hat{s} = (1 + s_0)/2$. From that point on, the payoff keeps increasing as more warrants are exercised until $\hat{s}$ reaches its maximal value, s. Thus, the venture capitalist's interim payoff is maximized either at $\hat{s} = s_0$ or at $\hat{s} = s$.

Step 8. To be show that the venture capitalist's ex-ante participation constraint is satisfied, it is sufficient to show that his payoff from the debt-equity contract (d_0, s_0) is positive, since the option to exercise warrants can only increase this payoff. The payoff is the same as in equation (34).[51] As $d_0 \to \infty$, this payoff becomes strictly positive. Thus, there is a value $d_0^{'''}$ that is large enough so that, for all $d_0 \geq d'''_0$, the venture capitalist's ex-ante participation constraint is satisfied.

Step 9. We design the contract so that $d_0 \geq \max(d_0', d_0^{''}, d_0^{'''})$ (see steps 2, 6, and 8).[52] $\square$

[51]The determination of x_0 is the same as in equations (35) and (37).

[52]In step 2, d_0 was chosen so that $x_0 < k$; in step 6, d_0 was chosen so that the entrepreneur will not window dress; and in step 8, d_0 was chosen so that the venture capitalist's ex-ante participation constraint is satisfied. Notice that in the proof of Proposition 2, as γ increased, d_0 increased along with it to ensure that the post-conversion debt, $d_0 - \gamma(s - s_0)$, remains positive. As d_0 increased, x_0 had to be adjusted to render the venture capitalist indifferent between refinancing and liquidating. Here, there is no requirement that $d_0 - \gamma(s - s_0)$ remain positive. Therefore we can fix d_0, and a corresponding x_0, at the end and not worry about it when building the sequence as γ increases. This highlights that straight debt cum warrants financing provides more degrees of freedom in the design of the contract, making it "easier" to prevent window dressing.

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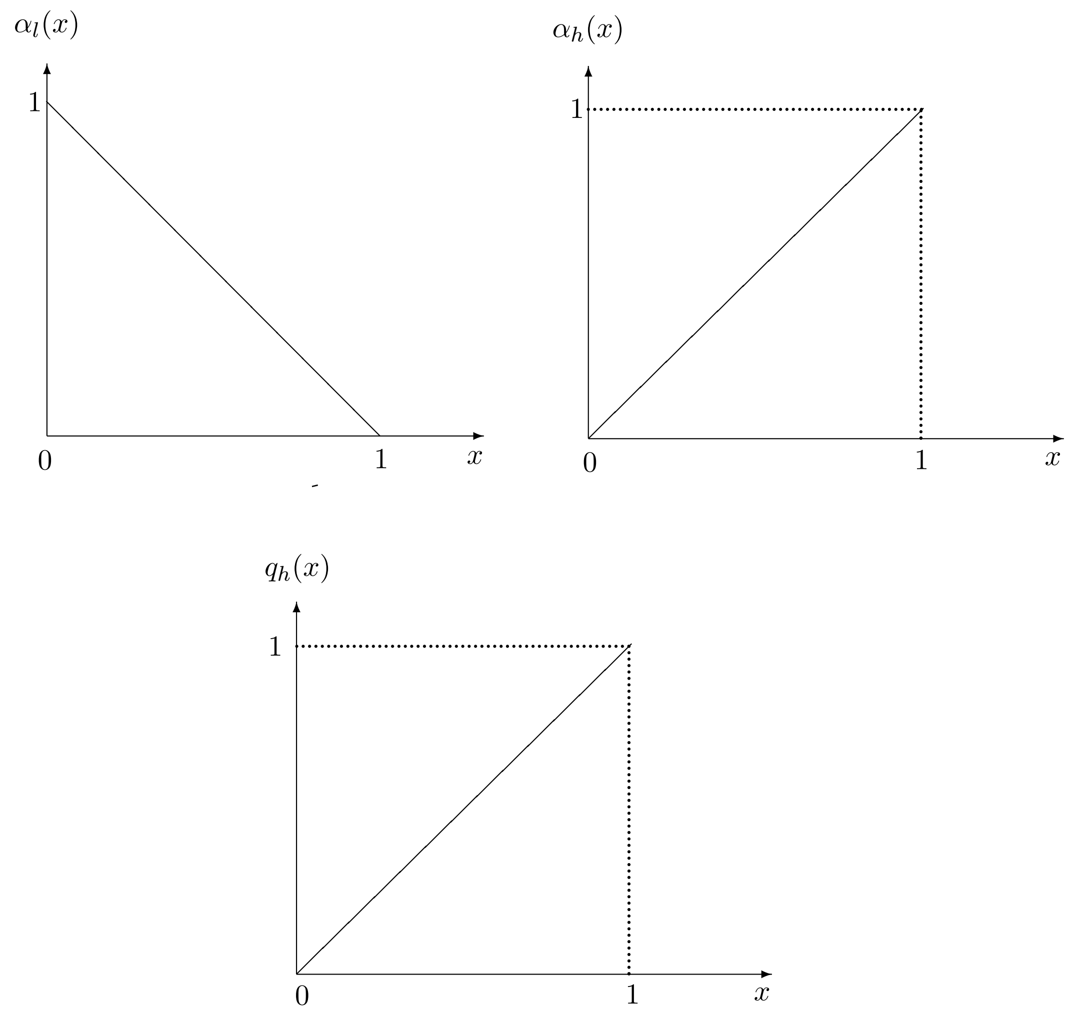


Figure 1: The joint distribution of signal and output

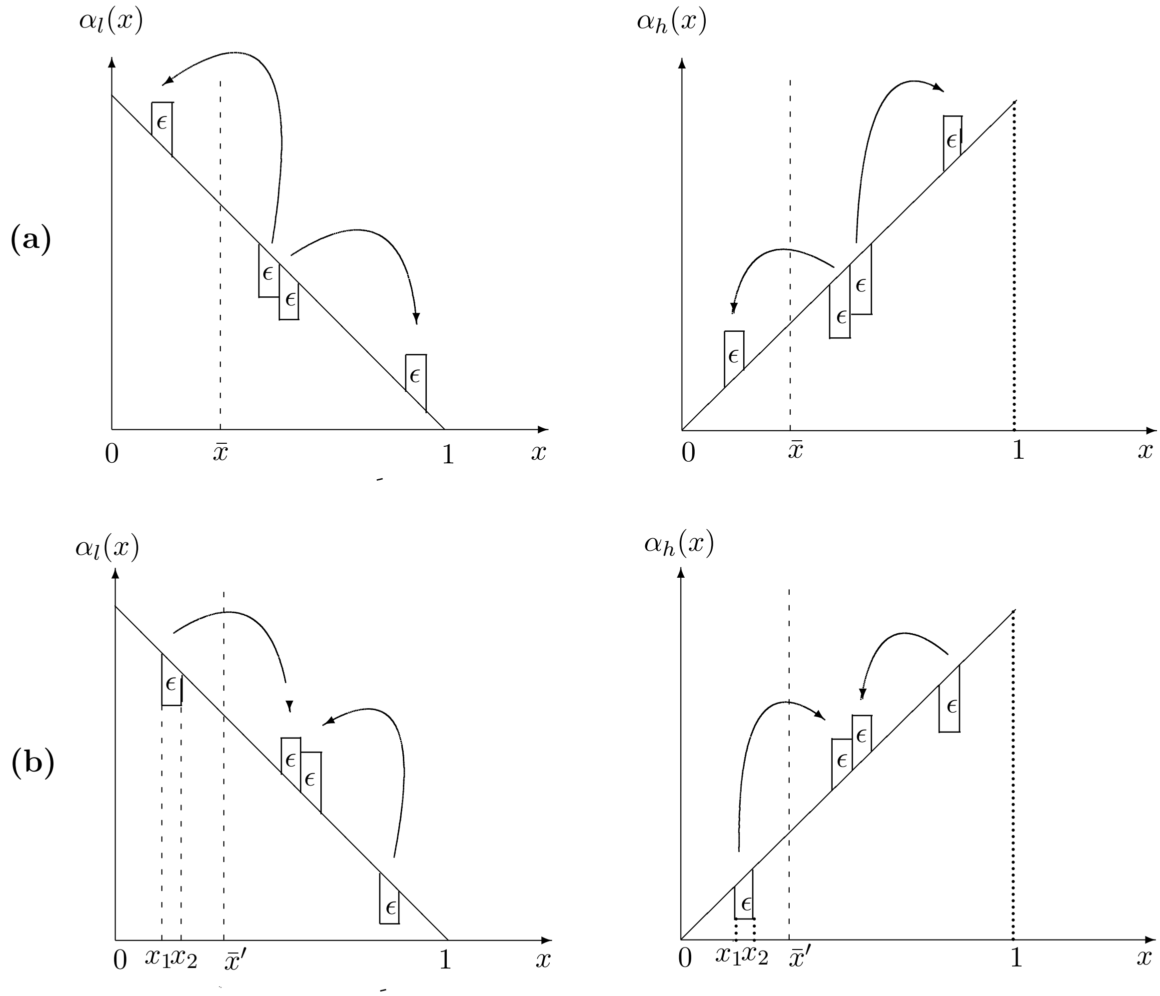


Figure 2: (a) Signal jamming; (b) The opposite of signal jamming

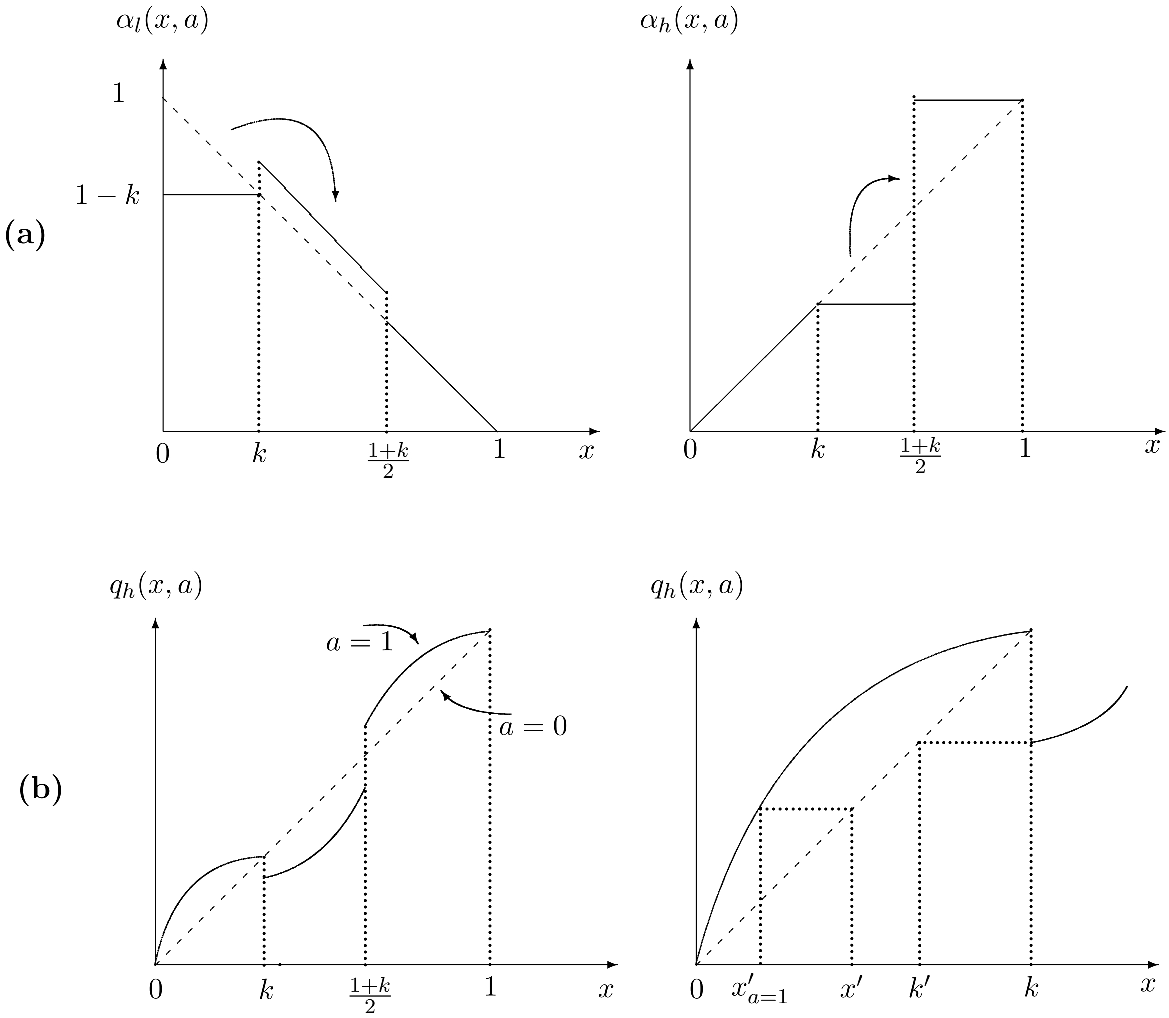
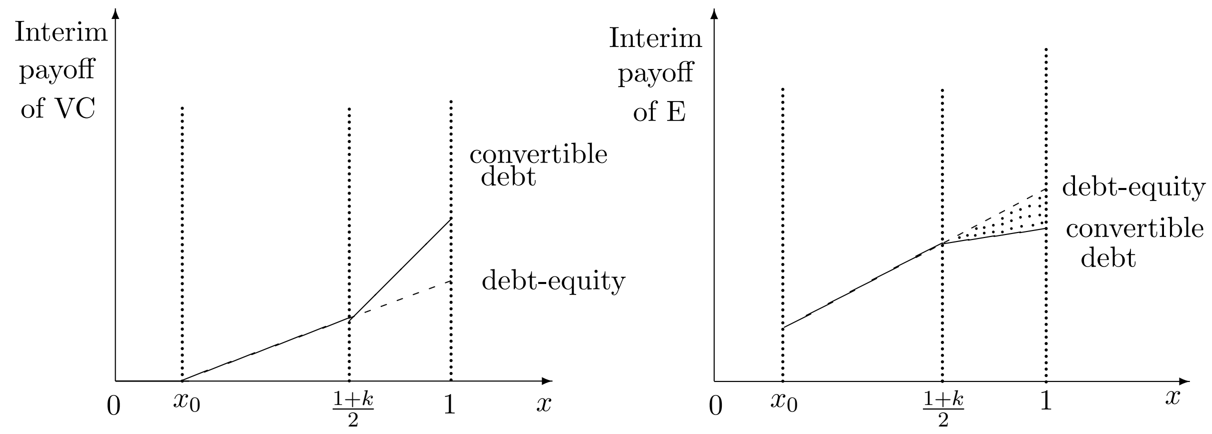


Figure 3: Window dressing: (a) the shift of mass in states l and h; (b) the effect on the conditional probability q_h; the figure on the right is an enlargement of the figure on the left, on the interval $x \in [0, k]$.

(a): $a = 0$



(b): $a = 1$

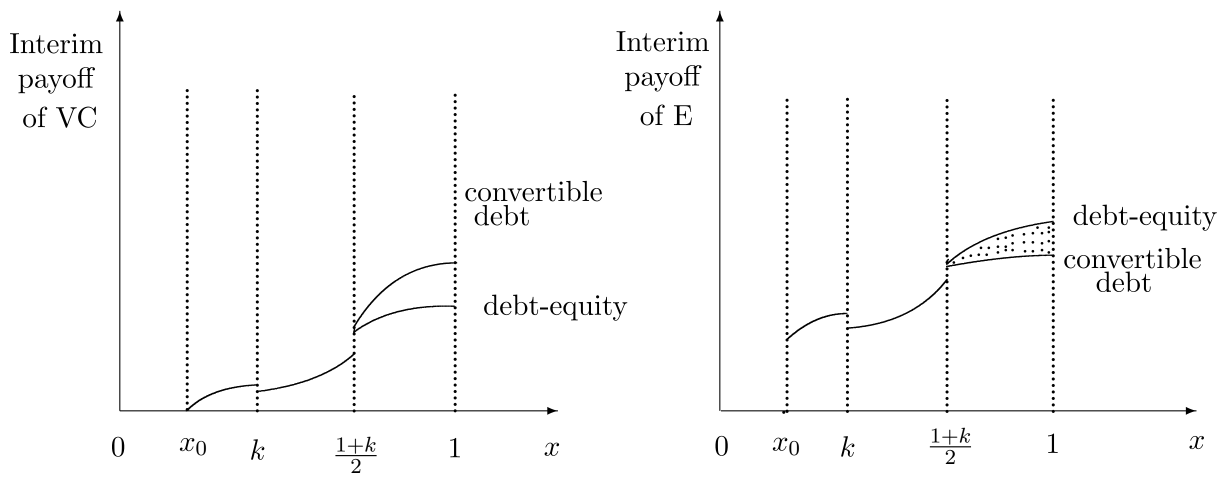


Figure 4: Interim payoffs as a function of the signal realization

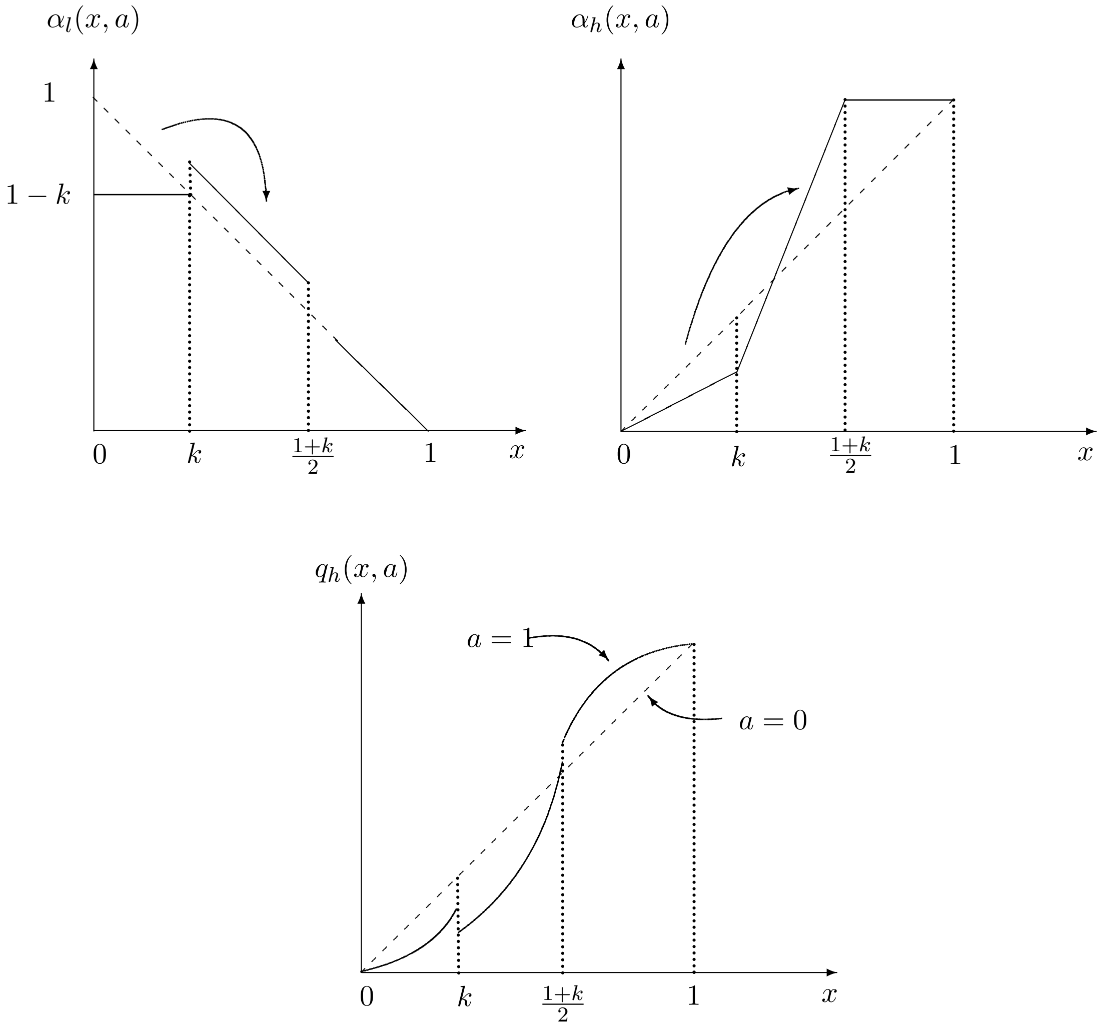


Figure 5: An alternative form of signal manipulation that is desirable to both venture capitalist and entrepreneur

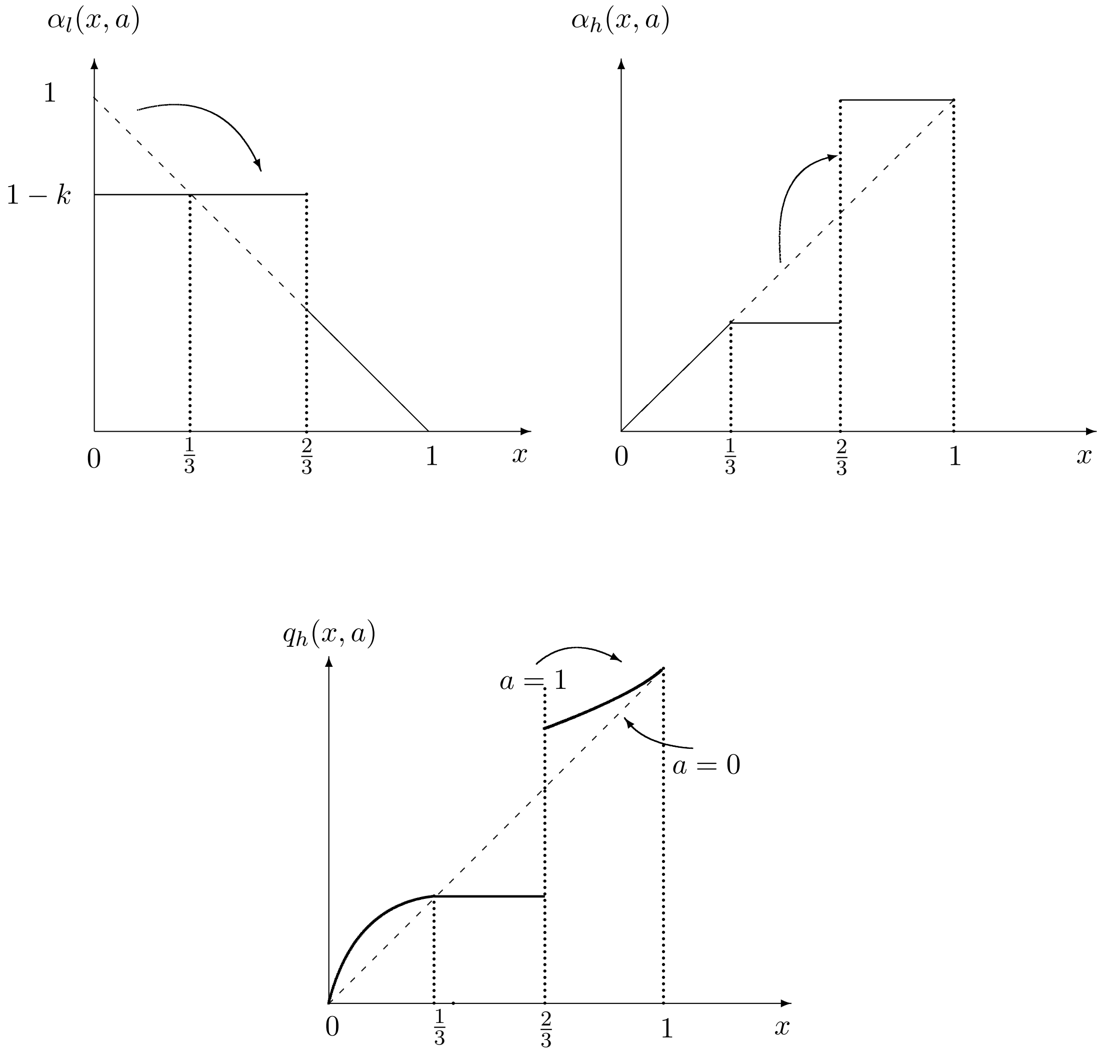


Figure 6: An alternative form of signal manipulation with a strictly informative signal