Risk Arbitrage in Takeovers*

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Abstract. The paper studies the role of risk arbitrageurs in takeovers and the source of their advantage. We show how the presence of arbitrageurs affects the value of the target shares, since arbitrageurs are more likely to tender. Therefore, an arbitrageur has the informational advantage of knowing he bought shares. In equilibrium, the number of arbitrageurs buying shares and the price they pay are determined endogenously. We also present several empirical implications, including the relationship among trading volume, takeover premium, liquidity of the shares, and the number of risk arbitrageurs investing in one particular deal.

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1. Introduction

It is well known that risk arbitrageurs play an important role in the market for corporate control. After a tender offer, the trading volume increases dramatically in large part because of risk arbitrageurs' activity.[1] They take long positions in the target stock, in the hope that the takeover will go through. They are also usually hedged by taking short positions in the acquirer's stock.

Risk arbitrage used to be a very inconspicuous activity but in the mid-70s, with the emergence of Ivan Boesky and the increasing volume of corporate takeover deals, it became more visible.[2] Attracted by the high rewards, many investment banks started new arbitrage departments and more people became involved in this activity. In more recent years, as the volume of new arbitrage capital increased, spreads have narrowed and after a takeover announcement the share price of the target now rises much more rapidly. The arbitrage community has often come to control, in total, 30 to 40 per cent of the stock and therefore it has become the single most important element in determining the success of a takeover. Moreover, risk arbitrageurs, since they are more likely to tender, are typically perceived as favoring the acquirer.[3]

In this paper we study the role that risk arbitrageurs play in takeover contests and the source of their advantage. We abstract from differences in attitude towards risk and focus on the explanation most commonly given: differences in information. It is often argued that arbitrageurs have better information about the chance of a successful takeover: we argue that it is not necessary to assume that risk arbitrageurs have specific knowledge on the takeover fight. Instead, the information advantage can arise endogenously from the choice of a risk arbitrageur to enter the contest.

[1] Numerous case studies reveal that the increased trading volume is largely due to arbitrage activities. See Harvard Business School case 9-282-065: *Note on Hostile Takeover Bid Defense Strategies.* For specific examples see Harvard Business School (HBS) case 9-285-053: *Gulf Oil Corp—Takeover,* HBS case 9-285-018: *The Diamond Shamrock Tender for Natomas (A)* and D. Commons: *Tender Offer.* On the other hand, it is common knowledge among financial arbitragers that a takeover bid represents one of the best opportunities for them to operate, see Ivan Boesky, *Merger Mania — Arbitrage: Wall Street's Best Kept Money-Making Secret.*

[2] See Welles (1981).

[3] See Grinblatt and Titman (1998).

The intuition is quite simple: if the presence of risk arbitrageurs increases the probability of a takeover, then the fact that one arbitrageur bought shares is per se relevant to their value. Therefore, an arbitrageur who has bought shares has an informational advantage: he knows he bought shares. After all, risk arbitrageurs are often quoted as saying that a crucial part of their activities is trying to predict what other arbitrageurs will do.[4]

We start from a company with diffuse ownership, with no large shareholders who can facilitate the takeover. After a bidder has made a tender offer, arbitrageurs decide whether to buy shares. If they succeed in accumulating non-trivial stakes, they become temporary large shareholders. Unlike small shareholders, they tender a fraction of their shares to the bidder and therefore facilitate the takeover.

The value of the shares depends on the probability that the takeover will take place, and it is higher the larger is the number of risk arbitrageurs buying shares (since they are more likely to tender). Both small shareholders and risk arbitrageurs do not know how many arbitrageurs have entered the contest and update their beliefs by looking at the trading volume. As long as the volume is not too high, the arbitrageurs' have an informational advantage and they may be willing to pay a price which is high enough to persuade the small shareholders to sell their shares.

However, as long as the expected profits are strictly positive, more arbitrageurs will choose to buy shares. If too many arbitrageurs are trying to buy shares, the volume, and consequently the price, will rise too much and they will not buy any shares. We then look at the arbitrageurs' decision to enter the takeover contest and show that there exist symmetric equilibria where each arbitrageur randomizes between entering or not and the takeover has a positive probability to succeed.

In equilibrium, the number of arbitrageurs who choose to take positions, the number of shares they buy, the price they pay and the takeover premium are determined endogenously.

[4]In general, risk arbitrageurs talk to a small subset of other arbitrageurs about whether or not they are involved in a specific deal. In the conclusions, we briefly discuss this possibility.

In a similar spirit, Kyle and Vila (1991) studies a case in which a bidder buys shares before announcing the takeover. Because of noise trading, the bidder succeeds in hiding at least partially his presence. In our paper, since we focus on post-announcement trading, risk arbitrageurs do not have any initial private information: the informational advantage arises endogenously when they start buying. Moreover, since there is more than one arbitrageur with the same informational advantage, we have to check that they do not compete away their rent.

Larcker and Lys (1987) offers a careful empirical study of risk arbitrageurs in takeovers. Their hypothesis is that risk arbitrageurs are better informed than the market about the takeover probability of success. They find that takeovers in which arbitrageurs bought shares have an actual success rate higher than the average probability of success implied by market prices. As a result, they can generate substantial positive returns on their portfolio positions. This is compatible with the results of our model, although we argue that the explanation could be different: the risk arbitrageurs may not know ex ante which takeover attempts are more likely to be successful, but their presence increases the probability of success, since they are more likely to tender.

Focusing on the role of risk arbitrageurs allows us to explain certain empirical patterns during takeover activity and derive testable implications. First of all, we find a positive relationship between trading volume and the probability that the takeover is successful. This also implies a positive relationship between trading volume and stock price. This is consistent with the widely observed phenomenon that, after the takeover announcement, both the stock price and the transaction volume of the target rise tremendously relative to their pre-announcement levels.[5] Moreover, we find that if the opportunity cost of investing in one specific deal increases (for example because the number of alternative deals increases), the number of arbitrageurs expected to invest in the deal decreases but the risk arbitrageurs' profits from investing in that

[5]Based on a sample of mergers before the 1980, Jensen and Ruback (1983) find in a comprehensive survey that the average jump in share price of the target firm ranges from 17% to 35%. For the 1980's merger wave, Bradley, Desai and Kim (1988) find similar results.

deal increase and the takeover premium offered increases. We also find that the more liquid is the target stock, the better risk arbitrageurs can hide their trade. As a consequence, the larger is their presence in the deal and the higher are their returns if they decide to invest in the deal. Since there is less need to convince the arbitrageurs to invest in the deal, the takeover premium offered decreases.

An implication of this paper is that the well known free-riding problem of Grossman and Hart (1980) can be mitigated.[6] The reason is that risk arbitrageurs can play the role of large shareholders, as in Shleifer and Vishny (1986) and Hirshleifer and Titman (1990). The contribution of this paper is to show that, even if at the time of the tender offer there are no large shareholders, there exist equilibria where arbitrageurs enter and buy shares, becoming in this way large shareholders.

This paper is similar in spirit to the literature on shareholders' activism, as in Kahn and Winton (1998), Maug (1998). In these papers, a large shareholder may affect the value of the shares by monitoring the firm's management. If the market price doesn't fully reflect this information, an investor can buy shares and profit from its planned intervention. The active investor is similar to an arbitrageur, who buys shares knowing that he will increase their value by facilitating the takeover. Noe (2001) considers the case of multiple shareholders, as here there are multiple risk arbitrageurs. As in this paper, investors are not endowed with private information, but they produce information through their own actions.

Following the introduction, the model is described in detail. The solution is presented in two steps. Section 3 introduces the case with only one risk arbitrageur, while the rest of the paper studies the case with multiple risk arbitrageurs. Section 4 studies the tendering strategies of risk arbitrageurs, once they have taken position in the target shares. Section 5 studies the choice of risk arbitrageurs to buy shares, Section 6 their decision to enter the contest and Section 7 the bidder's choice of the

[6] Several other papers have shown how this problem can be mitigated in other ways. See, among others, Bagnoli and Lipman (1988), Giammarino and Heinkel (1986), Harrington and Prokop (1993), Holmstrom and Nalebuff (1992), Bebchuk (1989) and Yilmaz (2000). Hirshleifer (1995) gives an overview of the subject.

takeover premium. Section 8 considers several empirical implications. Finally, the conclusions summarize the results and discuss possible extensions.

2. The Model

To focus on the role of arbitrageurs, we assume that at the beginning small shareholders control 100% of the outstanding shares (as in Grossman and Hart, 1980). The model does not consider situations where there are large shareholders, but they could be easily incorporated into the model. The crucial feature is that small shareholders free ride at least partially, so that when the tender offer is made the probability of a successful takeover is less than 1.

At time 0, a bidder announces a cash tender offer of P_T. If more than 50% of the shares are tendered, the bidder purchases all the tendered shares at the price P_T, otherwise all tendered shares are returned.

We assume that P_0 is the initial share price. Both P_0 and P_T are observable to all, as is ΔP, the value improvement per share that the bidder can bring to the firm. Naturally, we assume that[7]

$$P_0 + \Delta P \geq P_T \geq P_0.$$

The bidding price is between the *status quo* share price and the potential improved value of the share. In addition, we assume that if the takeover bid proves to be a failure, the stock price goes back to P_0.[8]

At time 1, arbitrageurs decide whether to enter and speculate. At time 2, stock trading takes place — arbitrageurs take positions, hiding among small investors. At the beginning of time 3 arbitrageurs reveal their position, so that their presence and their holdings are common knowledge. Finally, at time 3 all shareholders decide how many shares to tender and the outcome of the takeover is determined.

Let us now look at the players.

[7] In Section 7 we verify that the bidder never chooses a P_T outside of this range.

[8] The implied assumption is that the occurrence of this takeover bid does not change the probability of new takeover bids and their success. If the stock price falls to a different value than P_0, a similar analysis can still be performed.

The Small Shareholders

A small shareholder controls so few shares that he believes that his decision to tender his shares will have no effect on either the trading price or the outcome of the takeover. We assume that the small shareholders are risk neutral, so that there are no differences in risk preferences between small shareholders and risk arbitrageurs.

If the small shareholders were risk averse and the risk arbitrageurs were risk neutral, this would give an additional advantage to the risk arbitrageurs and reinforce our result.

The Arbitrageurs

There are N potential arbitrageurs, who can choose to take a position. The arbitrageurs are assumed to be risk neutral. After the takeover announcement, each of them has to decide whether to arbitrage or not in the stock of the target firm. If an arbitrageur A_i decides to arbitrage, he must bear a cost c. Such cost can be interpreted as the cost of collecting information or as the opportunity cost of other potential investments, given that risk arbitrageurs, as argued in Shleifer and Vishny (1997), do not have unlimited financial resources. If he decides to enter, the arbitrageur buys a portion δ_i of the total outstanding shares of the firm, where δ_i is endogenously determined.[9] For simplicity, we assume that risk arbitrageurs cannot buy more than $\bar{\delta}$. Later we will mention how this can be obtained as a result if we assume that a risk arbitrageur has to disclose his presence when he buys more than $\bar{\delta}$ shares.[10]

Let us call n the number of arbitrageurs who enter. The entry decision of each arbitrageur is determined endogenously in equilibrium. For the moment, let us call $G(n)$ the distribution of n (with density $g(n)$), which will be endogenously derived in

[9]We are assuming that risk arbitrageurs cannot sell short the shares of the target firm. In reality, risk arbitrageurs usually sell short the bidder's shares in order to hedge. Since risk arbitrageurs are risk neutral in this paper, they have no reason to hedge. In the conclusions, we discuss the possibility that risk arbitrageurs may want to sell short the target's shares for other reasons.

[10]Legally, there is an upper limit $\bar{\delta}$ (which in US is 5%) so that if $\delta_i > \bar{\delta}$, the risk arbitrageur has to declare the amount of shares he owns to comply with Section 13D of the Security Exchange Act.

equilibrium. In order to simplify the notation, n will be treated as a real (continuous) number and, likewise, $g(n)$ as a continuous function.

After the trading session is closed, each arbitrageur learns about the fraction of equity held by other arbitrageurs.[11] He then chooses the portion of shares to tender. For A_i, let us define this portion as $\gamma_i \in [0, 1]$.

Noise Traders and the Total Trade Volume

Shares can be bought also by investors who trade for exogenous reasons—such as liquidity reasons or diversification of their investment portfolio—and independently from the price.[12]

The trading volume from noise traders, ω, is random and independent of both the share price and the demand of arbitrageurs. By definition, the volume of shares traded ω is non-negative and is distributed uniformly on the interval $[0, 1]$. This information is common knowledge.[13]

Let y be the total trading volume of the shares of the firm, then

$$y = \omega + \sum_{i=1}^{n} \delta_i.$$

The number of arbitrageurs n who entered is unknown to both risk arbitrageurs and small shareholders, but everybody knows that n is distributed according to $G(n)$ (which will be determined in equilibrium) and can observe the trading volume y.

[11]It is not necessary for the result of the paper that arbitrageurs reveal their presence. In a previous version of this paper, Cornelli and Li (1998), arbitrageurs do not reveal their presence. They choose the fraction of shares to tender, given their own beliefs on how many other arbitrageurs have bought shares, based on the observed volume. The resulting equilibria are the same as in this version. Moreover, the assumption that the arbitrageurs reveal their presence eliminates cases in which the takeover fails only because of lack of coordination among arbitrageurs, since these failures could be solved by letting the bidder bid again.

[12]They can also be program or package traders whose decision to trade is based on information uncorrelated to the takeover process.

[13]We do not need to impose any constraint on short sales from noise traders. Therefore, in general the upper limit of the support of ω can be $\bar{\omega} > 1$. The notation throughout the paper would only slightly change. In section 8.4, we explicitly introduce $\bar{\omega}$.

In order to solve this model, we use the concept of Perfect Bayesian equilibrium and focus on the symmetric equilibria, where each arbitrageur buys and tenders the same proportions, δ and γ, of shares. To determine the equilibria, we solve the game backwards. We start from the tendering game: after n arbitrageurs entered and bought shares, we determine their optimal tendering strategy, which depends on how many other arbitrageurs are around. Given their tendering strategy, we look at the trading game. We find the rational expectations equilibria and whether there exists an equilibrium where risk arbitrageurs buy shares. Then, we look at the arbitrageurs' choice whether to enter or not. Finally, we look at the choice of the takeover premium by the bidder.

Before solving the general model, we consider a simpler case with only one risk arbitrageur.

3. An Example with One Risk Arbitrageur

This section serves a few different purposes. First of all, it conveys the intuition of the model in a simpler context, since it treats some variables as exogenous and it abstracts from the interaction among risk arbitrageurs. Second, in the general model arbitrageurs buy only a small number of shares and there is perfect competition among them. As a consequence, they are price-takers. In this section we consider instead the case of one single arbitrageur who can buy a large quantity of shares. He takes into account the effect of his demand on the price. This allows us to see how our result goes through even when the risk arbitrageur is not a price taker. Finally, this section considers the case in which there are multiple takeover deals in which the arbitrageurs could invest and the (endogenous) cost of investing in one target's shares is the foregone profit of investing in a different target.

Let us consider one potential arbitrageur, and assume he faces no entry cost ($c = 0$). In this section we assume that the amount of shares the arbitrageur may choose to buy is exogenous and equal to $\delta > \frac{1}{2}$. Therefore, if the risk arbitrageur has bought δ shares, it is optimal for him to tender $\frac{1}{2}$ shares, which is the minimum necessary for the bidder to succeed. As a result he obtains P_T on $\frac{1}{2}$ shares and his remaining $\delta - \frac{1}{2}$ shares are each worth $P_0 + \Delta P$. So if the arbitrageur expects to pay P_1 for the shares, his expected profit from buying δ shares is

$$\frac{1}{2}P_T + (\delta - \frac{1}{2})(P_0 + \Delta P) - \delta P_1.$$

Trading follows a model as in Kyle (1989). The arbitrageur and a liquidity (noise) trader submit orders. The market maker observes the flow (but not the identity of who submitted the orders) and sets a price equal to the expected value of a share conditional on the order flow. With respect to the general model, we restrict the trading choices of the noise trader and assume that he is equally likely to buy 0 or δ shares. Assume the arbitrageur buys δ shares with probability q (to be determined in equilibrium). If the order flow is 0, then the arbitrageur clearly bought no shares. The market maker sets the share price equal to P_0 and the offer fails. If the order flow is δ, the market maker believes that the arbitrageur bought δ shares (in which case the bid succeeds) with probability q and that he bought 0 shares (in which case the bid fails) with probability $1 - q$. The market maker sets the share price equal to $P_0 + q\Delta P$. If the order flow is 2δ, then the arbitrageur clearly bought δ shares. So the market maker sets the share price equal to $P_0 + \Delta P$ and the offer succeeds.[14]

Given this equilibrium, the arbitrageur expects to pay $P_1 = \frac{1}{2}(P_0 + q\Delta P) + \frac{1}{2}(P_0 + \Delta P) = P_0 + \Delta P\frac{1+q}{2}$ per share for δ shares and the arbitrageur's expected profit from buying δ shares equals

$$\pi = \frac{1}{2}(P_T - P_0 - \Delta P) + \delta\Delta P\frac{1-q}{2}.$$

Consider the case in which there is only one takeover bid. The arbitrageur then

[14] In the rest of the paper, we do not model market makers and the trading equilibrium is modeled as a rational expectation equilibrium. In the context of this example, this would imply exactly the same price and quantity of shares traded. The only difference between this example and the general model is that in the second one the risk arbitrageurs are price takers, while in the next step of this example we compute the arbitrageur's choice of q, which takes into account its effect on the price.

chooses q in order to maximize $q\pi$. For a given P_T, the optimal q is

$$q^*(P_T) = \frac{P_T - P_0 - \Delta P}{2\delta \Delta P} + \frac{1}{2}.$$

Now consider the bidder's problem. The bid succeeds with probability $q^*(P_T)$, in which case the bidder buys $\frac{1}{2}$ shares at a price of P_T. The bidder's expected profit equals

$$\pi_b = \frac{1}{2} q^*(P_T)(P_0 + \Delta P - P_T).$$

The bidder's optimal bid is $P_T^* = P_0 + (1 - \frac{\delta}{2})\Delta P$ and the equilibrium probability that the arbitrageur buys shares (and the bid succeeds) is 1/4. The arbitrageur expected profits are $\pi = \frac{\delta \Delta P}{8} > 0$. The larger is the number of shares δ the risk arbitrageur can buy, the lower the bid (since the arbitrageur can buy more shares, the bid doesn't have to be as high to attract him to the deal).

Now consider the case in which there are $t > 1$ takeover deals in which the arbitrageurs could invest. To simplify, let us assume that the arbitrageur can only invest in the shares of one target (for example, because of limited capital). Consider the symmetric case, in which the exogenous variables $P_0, \Delta P$ and δ are the same for all takeover deals. Suppose the arbitrageur invests in target i ($i = 1, \ldots, t$) with probability q_i. We look for a symmetric equilibrium, where all bidders make the same bid P_T and q_i is the same for each target.

If bidder j bids P_T^j, the arbitrageur chooses $\{q_1, \ldots, q_t\}$ in order to maximize the expected profits:

$$\sum_j q_j \pi^j = \sum_j \left[\frac{1}{2}(P_T^j - P_0 - \Delta P) + \delta \Delta P \frac{1 - q_j}{2} \right] q_j$$

subject to the constraint $\sum_j q_j \leq 1$.

We distinguish two cases, depending on whether the constraint is binding or not. If the constraint is not binding, then the equilibrium probability to enter in each of the takeovers is equal to the probability we obtain when there is only one takeover available, i.e. $\frac{1}{4}$. Therefore, for $t < 4$, this is an equilibrium.[15]

If instead $t \geq 4$, the constraint must be binding, and

$$q_j^* = \frac{P_T^j - P_0 - \Delta P}{2\delta \Delta P} + \frac{1}{2} - \frac{\lambda}{\delta \Delta P}$$

where λ is the lagrangian multiplier. If $t-1$ takeovers have the premium P_T and takeover j has a premium P_T^j, then it is easy to show that the probability to enter in takeover j is

$$q_j^*(P_T^j) = \frac{t-1}{t} \frac{P_T^j - P_T}{2\delta \Delta P} + \frac{1}{t}.$$

Therefore, given that all other takeovers have a premium P_T, bidder j chooses P_T^j in order to maximize $\pi_b^j = \frac{1}{2} q_j^*(P_T^j)(P_0 + \Delta P - P_T^j)$. Solving for the symmetric equilibrium in which $P_T^j = P_T$ we find that

$$P_T^* = P_0 + \Delta P - \frac{2\delta \Delta P}{t-1}$$

and $q^* = \frac{1}{t}$.

When takeover activity increases, competition among bidders increases and leads to higher bids. This increases the arbitrageurs' profits but decreases the bidders' profits. As $t \to \infty, P_T \to P_0 + \Delta P$, so that the bidders have zero profits and arbitrageurs appropriate all the surplus.

In this example, we see how one arbitrageur may buy shares and make profits, because he is the only one who knows for sure of his own presence. However, if one arbitrageur, without any specific informational advantage, can make profits in this way, then other arbitrageurs should try to enter, thereby reducing profits. It is therefore necessary to study what happens when more than one arbitrageurs may enter the deal. In what follows we therefore present the general model, with N risk arbitrageurs, who choose simultaneously whether to take a position in a specific deal.

[15] It is easy to check that, for $t < 4$, $q = \frac{1}{4}$ is the only equilibrium. When $t < 4$, at $q = \frac{1}{4}$ the arbitrageur enjoys positive profits. If instead we assume that the constraint is binding for $t = 2$ or $t = 3$, arbitrageurs make negative profits, therefore this cannot be an equilibrium.

While in this example the risk arbitrageur was a monopolist, in the general model we look at the other extreme, where there is perfect competition among risk arbitrageurs who are price-takers.

4. The Tendering Game

We solve the game backward and start from time 3, when arbitrageurs have to choose whether to tender.

The tendering game is played among the arbitrageurs. Small shareholders stay out of the picture, since they take the probability of a takeover as given and therefore, by the Grossman-Hart (1980) argument, they never tender their shares.

At the beginning of period 3, n arbitrageurs have entered. Each arbitrageur A_i reveals that he bought δ_i shares and observes exactly how many other arbitrageurs entered and how many shares they bought. Then, the arbitrageur chooses how many shares to tender, given n and given the strategies of the other risk arbitrageurs.

If the fraction of equity in the hands of the risk arbitrageurs is less than 50%—that is, if $\sum_{i=1}^{n} \delta_i < 0.5$—then there exist multiple equilibria, since the tendering strategy of the risk arbitrageurs is irrelevant: the takeover will fail in any case and the price of the shares will return to P_0.

If instead $\sum_{i=1}^{n} \delta_i \geq 0.5$, in equilibrium the risk arbitrageurs tender exactly 50% of the shares. There are multiple equilibria: in all of them each arbitrageur tenders a fraction γ_i such that $\sum_{i=1}^{n} \gamma_i \delta_i = 0.5$. The unique symmetric equilibrium is given in the following proposition.

Proposition 1: *If $\sum_{i=1}^{n} \delta_i \geq 0.5$, then for each n there exists a unique symmetric equilibrium where each risk arbitrageur tenders a fraction $\gamma > 0$ of its shares such that*

$$\gamma(\delta_1, \delta_2, \ldots, \delta_n) = \frac{0.5}{\sum_{i=1}^{n} \delta_i}.$$

If $\delta_i = \delta$ for all risk arbitrageurs then

$$\gamma(n, \delta) = \frac{0.5}{n\delta}.$$

Proof: The best response of a risk arbitrageur A_i, who owns δ_i shares, given that the other arbitrageurs tender a fraction γ of their shares, is given by

$$\gamma_i = \frac{0.5 - \gamma \sum_{j \neq i} \delta_j}{\delta_i}$$

if $0 \leq \gamma_i \leq 1$ (otherwise $\gamma_i = 0$). From this it follows immediately that the unique symmetric equilibrium is the one given in the proposition.

¶Q.E.D.

Notice that γ is a decreasing function of both n and δ: the more arbitrageurs entered (or the more shares each of them owns), the less each of them has to contribute for the takeover to be successful.

5. The accumulation of shares

Given the symmetric equilibrium of the last subgame, in which risk arbitrageurs tender a positive fraction of their shares, we now look at how risk arbitrageurs buy shares. The players in this stage are the small shareholders—who own all the shares at the beginning of the game and may choose to sell them—the risk arbitrageurs and the noise traders. We want to show that although the price increases when arbitrageurs buy shares, they do succeed in hiding, at least partially, their presence, so that it is profitable for them to buy shares.

We use the concept of rational expectation equilibrium. For each realization of the random variables, n and ω, we characterize an equilibrium where the total trading volume is $y = n\delta + \omega$, and the share price is P_1. The equilibrium is such that (1) given the share price P_1 and the volume y, risk arbitrageurs and small shareholders are maximizing their utility; in particular, this implies that risk arbitrageurs trade if and only if trading implies positive expected (interim) profits and small shareholders sell their shares only if they obtain a price which is at least as high as the profits they expect to obtain by holding the shares; (2) the beliefs of risk arbitrageurs and small shareholders about the probability of success of the takeover (which in turn determine the share price P_1) are consistent with their strategies.

5.1. The Share Price and the Probability of Success of the Takeover. The price is determined as follows. Since we are assuming that the small shareholders own 100% of the shares, if the total demand of shares is less than 100%, then the market price P_1 equals the reservation price of the small shareholders, $\underline{P}$. $\underline{P}$ is such that a small shareholder is indifferent between selling the share and holding it (and waiting to see if the takeover takes place):

$$\underline{P} = \tau(P_0 + \Delta P) + (1 - \tau)P_0 = P_0 + \tau\Delta P \tag{1}$$

where τ is the probability of success of the takeover bid, *as perceived by small shareholders.*

The reservation price V^a of a risk arbitrageur is given by

$$V^a = \tau_i^a[\gamma^e P_T + (1 - \gamma^e)(P_0 + \Delta P)] + (1 - \tau_i^a)P_0 \tag{2}$$

where γ^e is the number of shares the arbitrageur expects to tender and τ_i^a is the probability of success of the takeover *as perceived by the arbitrageur A_i*, who has bought δ_i shares. Note that if $\tau_i^a = \tau$ then $V^a < \underline{P}$ and there is no trade. However, if τ_i^a is sufficiently larger than τ, then there is room for trade between arbitrageurs and small shareholders.

If instead the demand is above 100%, then $P_1 = \underline{P}$ does not clear the market and the competition between risk arbitrageurs in order to obtain the shares will drive the price up to their reservation price, V^a.[16]

Both τ and τ_i^a are endogenous and depend on the transaction volume y, which conveys information about the number of arbitrageurs and their positions. We have therefore to look at the updating process of both arbitrageurs and small shareholders and derive their *posterior* probability that the takeover will be successful.

The following proposition characterizes these probabilities:

[16] This is true only if noise traders are not allowed to sell short. If instead they can sell short, i.e. $\omega \in [0, \bar{\omega}]$ with $\bar{\omega} > 1$, the price remains $P_1 = \underline{P}$ as long as the demand is less than $\bar{\omega}$.

Proposition 2: *If $y < 0.5$, both arbitrageurs and small shareholders think the probability of success of the takeover is zero.*

If $y \geq 0.5$, then the probability τ_i^a that the takeover will succeed, as perceived by an arbitrageur A_i who bought δ_i shares and observed volume y, given that all other arbitrageurs have bought δ shares, is given by

$$\tau^a{}_i = Prob\ [\ (n-1)\delta + \delta_i \geq 0.5\ \ |\ y - \delta_i\ ] = \frac{\int_{\frac{0.5-\delta_i}{\delta}}^{\frac{y-\delta_i}{\delta}} g(s+1)ds}{\int_0^{\frac{y-\delta_i}{\delta}} g(t+1)dt}. \tag{3}$$

In a symmetric equilibrium, $\delta_i = \delta$ and

$$\tau^a = \frac{\int_{\frac{0.5}{\delta}}^{\frac{y}{\delta}} g(s)ds}{\int_1^{\frac{y}{\delta}} g(t)dt}. \tag{4}$$

The probability τ that the takeover will succeed, as perceived by the small shareholders after observing the volume y, is:

$$\tau = \frac{\int_{\frac{0.5}{\delta}}^{\frac{y}{\delta}} g(s)ds}{\int_0^{\frac{y}{\delta}} g(t)dt}. \tag{5}$$

Both τ_i^a and τ are increasing in y and $\tau < \tau^a$.

Proof: See Appendix I.

Since small shareholders do not tender, when $y < 0.5$ it is clear that not enough shares are in the hands of arbitrageurs and $\tau = \tau^a{}_i = 0$. If $y \geq 0.5$ the probabilities are instead positive. The difference in the updating of arbitrageurs and small shareholders is due to the fact that the small shareholders may think that perhaps all volume y is due to noise traders.

The probability of success of takeover as assessed by the risk arbitrageurs can be higher than the probability assessed by the small shareholders. If the difference is sufficiently high, $V^a(y) > \underline{P}(y)$ and the arbitrageurs are willing to buy shares at the price $P_1 = \underline{P}$, which makes the small shareholders indifferent. Even if in general we cannot compare τ and τ_i^a, in the symmetric case we can prove that $\tau^a > \tau$.

5.2. The equilibrium. To derive the equilibrium, we proceed in the following way: for given beliefs τ and τ^a, we derive the optimal choice of δ_i. Given this strategy, we then find the beliefs which are consistent in equilibrium.

In what follows, we show that, given the beliefs, the optimal choice for a risk arbitrageur is always either to buy no shares at all, or to buy up to $\bar{\delta}$ shares. There always exists an equilibrium where arbitrageurs buy no shares at all and the takeover will never succeed (since one arbitrageur alone, by deviating and buying $\bar{\delta}$, cannot make the takeover successful). We want to find out whether there exists an equilibrium where arbitrageurs buy $\bar{\delta}$ shares and the takeover has a positive probability to be successful. In the next proposition we characterize the symmetric equilibria. The intuition is given immediately after.

PROPOSITION 3: *Given that n arbitrageurs entered the contest and the noise trade is ω, then*

a) *If $n\bar{\delta}+\omega < 0.5$, there is a unique equilibrium where risk arbitrageurs buy no shares, the price is P_0 and the trading volume is $y = \omega < 0.5$. The takeover fails.*

b) *If $0.5 \leq n\bar{\delta} + \omega \leq 1$, then in equilibrium risk arbitrageurs buy either $\bar{\delta}$ or 0 shares. If P_T is not too low, then at least for values of y close to 50% there exists an equilibrium where risk arbitrageurs buy $\bar{\delta}$ shares, the share price is $P_0 + \tau\Delta P > P_0$ and the trading volume is $y = n\bar{\delta} + \omega$. The probability that the takeover is successful is strictly positive.*

c) *If $n\bar{\delta}+\omega > 1$ risk arbitrageurs buy no shares, the share price is P_0 and the takeover fails.*

PROOF: See Appendix II.

The intuition of the proposition and the logic of the proof are the following. If the trading volume is less than 50%, everybody knows that the takeover is going to fail and the share price is P_0. In equilibrium risk arbitrageurs are indifferent between buying and not buying shares at the price P_0. Since they are usually not interested in long term positions, we assume that in equilibrium they buy no shares at all. The expectation that the takeover will fail is therefore correct. This gives point (a) of the proposition.

Similarly, when $n\bar{\delta} + \omega > 1$, if each of the risk arbitrageurs demanded $\bar{\delta}$ shares, that level of volume would imply a demand larger than supply and the price would rise up to their reservation price.[17] In equilibrium, risk arbitrageurs buy no shares and the takeover fails. This is case (c).

In all remaining cases, the takeover could be successful if in equilibrium risk arbitrageurs do buy shares. We first show that, given the beliefs τ and τ_i^a, a risk arbitrageur always wants to buy either no shares at all or as many shares as possible.[18]

Once we restrict the choice to 0 or $\bar{\delta}$ shares, we can find out whether for some values of n and ω there exists an equilibrium where arbitrageurs buy $\bar{\delta}$ shares. In order to do this, we compute the expected profits of a risk arbitrageur buying $\bar{\delta}$ shares, given n, ω and the beliefs τ and τ_i^a corresponding to $y = n\bar{\delta} + \omega$. If, when buying $\bar{\delta}$ shares, risk arbitrageurs obtain positive expected profits, this is an equilibrium. If instead the expected profits are negative, then the only equilibrium corresponding to that n and ω is the one in which risk arbitrageurs buy no shares at all.

We show in Appendix II that for low values of volume y, profits are either negative or positive depending on P_T. For P_T sufficiently high (but bounded away from $P_0 +$

[17] If noise traders can sell short, then this happens only if $n\bar{\delta} + \omega > \bar{\omega}$

[18] Throughout the paper we assume that $\delta_i \leq \bar{\delta}$. Note that this can be justified by the following: if a risk arbitrageur has to reveal his transaction as soon as $\delta_i > \bar{\delta}$ (which is what is prescribed, for example, by Section 13D of the Security Exchange Act) then, when $\delta_i > \bar{\delta}$, $\tau = \tau_i^a$ (since everybody knows about the arbitrageur presence) and his expected profits become negative. Holderness and Sheehan (1985) study the price reaction to announcements that some "corporate raiders" had acquired stock in a specific firm. They show that there are positive abnormal returns at the announcement that the investors bought stock in a firm which was target of a reorganization. Unfortunately, they do not distinguish between the case in which the investor was the acquirer and the case in which a third party was the acquirer (although both cases are in the sample).

ΔP) the profits are positive and increasing and it is therefore an equilibrium to buy $\bar{\delta}$ shares. As y increases, the probability of success of the takeover increases, but this has two effects on the profits of the arbitrageurs. On the one hand, more arbitrageurs are likely to be in position and this promises a greater chance of success of the takeover. On the other hand, the price at which arbitrageurs can buy shares increases. For high levels of volume, the informational advantage of the risk arbitrageur becomes thinner, because more arbitrageurs are expected to be present, so the price of the shares increases faster than their benefit. In general, the net effect of an increase in y on the expected (interim) profits $\pi(y)$ can be either positive or negative. In the Appendix we show that the function $\pi(y)$ is in general non-monotonic and could become negative.

Therefore, the (interim) expected profits of risk arbitrageurs, given that they bought $\bar{\delta}$ shares, are the following. If $y < 0.5$ the profits are 0. Starting from $y = 0.5$ profits are positive and increasing (provided that P_T is sufficiently high). As y increases, profits first increase, but then begin to decrease and may eventually become negative. We define $\bar{y}$ as the level of volume at which expected profits are equal to 0. If profits never become negative, $\bar{y}$ is equal to one. All the interval where profits are positive corresponds to values of n and ω for which there exists an equilibrium where risk arbitrageurs buy $\bar{\delta}$ shares. In Appendix II we also show that if P_T is sufficiently high expected profits are always positive for the entire range $0.5 \leq y \leq 1$. This concludes case (b) of Proposition 3.[19]

We have therefore shown that if P_T is not too low there exist trading volumes at which it is an equilibrium for the arbitrageurs to buy shares. In Section 7 we will show that the bidder always chooses a P_T high enough such that this is the case.

[19]Since arbitrageurs can only buy shares without being recognized up to 5%, we have considered this limit small enough for risk arbitrageurs to be price takers. Alternatively, if one thinks arbitrageurs are not price takers, one could modify the trading game as in Kyle (1989), where risk arbitrageurs submit demand functions and therefore take into account the effect that their demand has on the price. As a result, arbitrageurs may buy less than $\bar{\delta}$ shares. It is also possible to consider a model as in Kyle (1985), where risk arbitrageurs demand an amount δ_i independent from the volume and the market makers set the price at the reservation value of the small shareholders based upon total trading volume. This is the case considered in the example with only one risk arbitrageur in Section 3 and we saw there that the same result holds.

6. The entry decision

We now study the arbitrageurs' decision to enter the contest. In the previous stage arbitrageurs always had the option to buy no shares. As a result, the expected profits of an arbitrageur who has chosen to enter are always non-negative. However, when the announcement of the takeover bid is made, some of the arbitrageurs are engaged in other operations and their financial resources—including their bounded debt capacity—are tied up. Shleifer and Vishny (1997) show that arbitrageurs do not have unlimited capital to invest and this is crucial in determining their strategy. Moreover, even before they start buying, arbitrageurs must collect some information, which is costly. We therefore assume that arbitrageurs decide whether to enter or not, where entry has a cost $c > 0$, which can be arbitrarily small.[20]

We focus on the symmetric equilibria. It is clear that the case in which no arbitrageur enters is always an equilibrium. In fact, one arbitrageur alone, who deviates and enters, can at most buy $\bar{\delta}$ shares, and that is not enough for the takeover to succeed. We want to show, however, that there exist other equilibria where the takeover can succeed. First of all, there cannot exist an equilibrium where all arbitrageurs enter with probability one. In this case, the small shareholders would know that there are exactly N arbitrageurs buying with probability one. Therefore the arbitrageurs would have no information advantage at all and there would be no room for trade. The only other possibility for a symmetric equilibrium is a mixed strategy equilibrium where each arbitrageur enters with a probability p which makes him indifferent between entering and not entering (i.e. such that the expected profits from entering are equal to c).

If each arbitrageur randomizes with probability p, the probability that exactly n

[20] Arbitrageurs never earned negative (interim) expected profits in the trading stage, since trading was modelled as a one-shot game. In a dynamic game, arbitrageurs may start buying shares before realizing that too few (or too many) arbitrageurs are likely to be around. As a result, they may have bought shares at a premium and they realize a loss. The cost of entry c may be seen also as an attempt to capture that effect.

out of the N potential arbitrageurs entered is

$$g(n) = \begin{pmatrix} N \\ n \end{pmatrix} p^n (1-p)^{N-n}. \tag{6}$$

Each arbitrageur is able to forecast the equilibrium corresponding to each realization of ω and n. If the equilibrium implies they buy 0 shares, then entry implies a loss equal to c. The ex-ante expected profits can therefore be written as

$$\Pi(p, N, c) \equiv E_{n,\omega} \left[\pi(n, \omega) \right]. \tag{7}$$

where $\pi(n, \omega)$ are the ex post profits for each realization of n and ω.

PROPOSITION 4: *If the cost c is not too high and N not too low, there always exist two symmetric equilibria where each arbitrageur enters with probability p such that $0 < p < 1$ and the ex-ante expected profit in (7) is equal to 0. These are also equilibria of the general game.*

PROOF: See Appendix III.

The condition that c is not too high guarantees that arbitrageurs can break-even after incurring the cost c and the condition on N guarantees that there are potentially enough risk arbitrageurs for the takeover to be successful.

In equilibrium, each of the N arbitrageurs randomizes between entering and not entering with a probability p, where p is endogenously determined so that each risk arbitrageur is indifferent between entering and not entering. Out of the N potential arbitrageurs, n will enter the contest and invest in shares of the target company. Depending on the realization of n and ω, arbitrageurs either buy no shares (and therefore bear a loss c) or buy $\bar{\delta}$ shares. In the first case the takeover will be for sure unsuccessful, while in the second case the takeover has a positive probability to be successful: it is the probability that $0.5 \leq \bar{\delta} n \leq \bar{y} - \omega$ (where $\bar{y}$ is the maximum level of volume for which profits are non-negative).

If S is the ex-ante probability of success of the takeover, in equilibrium the ex-ante expected profits of the small shareholders is $S\Delta P + P_0$. Each of them is indifferent between selling and holding and, if they hold, they get $\Delta P + P_0$ if the takeover is successful and P_0 otherwise.

7. The Takeover Premium

We have seen how the risk arbitrageurs' decision to enter and to buy shares is affected by the takeover premium P_T. Therefore the bidder, when making a takeover offer, will take into account this effect. We now look at the choice of the takeover premium by the bidder. We show that the bidder always offers a P_T strictly lower than $P_0 + \Delta P$. In other words, the bidder does not need to pay out the entire surplus, because he can rely on the presence of the risk arbitrageurs. Moreover, we show that the bidder always offers a takeover premium high enough to encourage the risk arbitrageurs to buy shares. Therefore, the equilibria described in the previous section in which arbitrageurs buy shares and the takeover has a positive probability of success are the ones prevailing.

Given that in a successful bid, $\frac{1}{2}$ shares are tendered, the bidder chooses P_T in order to maximize:

$$\begin{aligned} \text{MAX}_{P_T} \qquad & (P_0 + \Delta P - P_T)\frac{1}{2}\text{Prob}\left\{\delta(y)n > 0.5\right\} = \\ = \quad & (P_0 + \Delta P - P_T)\frac{1}{2}\int_{\frac{0.5}{\bar{\delta}}}^{\frac{\bar{y}}{\bar{\delta}}}\int_0^{\bar{y}-\bar{\delta}n} g(n \mid P_T)d\omega dn = \\ = \quad & (P_0 + \Delta P - P_T)\frac{1}{2}\int_{\frac{0.5}{\bar{\delta}}}^{\frac{\bar{y}}{\bar{\delta}}}(\bar{y} - \bar{\delta}n)g(n \mid P_T)dn. \end{aligned} \tag{8}$$

The first order derivative is:

$$\begin{aligned} - \quad & \int_{\frac{0.5}{\bar{\delta}}}^{\frac{\bar{y}}{\bar{\delta}}}(\bar{y} - \bar{\delta}n)g(n \mid P_T)dn + (P_0 + \Delta P - P_T)\frac{d\bar{y}}{dP_T}\int_{\frac{0.5}{\bar{\delta}}}^{\frac{\bar{y}}{\bar{\delta}}} g(n \mid P_T)dn + \\ + \quad & (P_0 + \Delta P - P_T)\int_{\frac{0.5}{\bar{\delta}}}^{\frac{\bar{y}}{\bar{\delta}}}(\bar{y} - \bar{\delta}n)\frac{\partial g(n)}{\partial p}\frac{dp}{dP_T}dn, \end{aligned} \tag{9}$$

where the first term represents the direct cost of increasing the takeover premium (the bidder has to pay more for the shares) and is negative. The second term is positive and reflects the fact that when P_T increases, profits are positive on a larger range ($\bar{y}$ increases) and this increases the probability of success of a takeover. The last term is the effect that a change in P_T has on the distribution $g(n)$, i.e. on the probability of entry p. This last term can be positive or negative.

The bidder will never choose a $P_T < P^{min}$ (where P^{min} is defined in Appendix II as the minimum takeover premium such that $\pi(y)$ is positive for at least some y) since in that case the probability of success of the takeover is zero. We can now show that it is never optimal for the bidder to set P_T equal to either P^{min} or $P_0 + \Delta P$. In other words, the bidder always offers a strictly positive takeover premium, but still keeps a positive surplus from the takeover.

If P_T is close to $P_0 + \Delta P$ (all the takeover premium is paid out) the last two terms disappear and the first order condition is strictly negative: the bidder has an incentive to reduce P_T and keep some of the surplus created by the takeover. If instead P_T is very close to P^{min}, this implies that $\bar{y}$ is close to 0.5: as $\bar{y} \to 0.5$ all three terms go to zero. However, if we define V as the first order derivative in (9), it can be easily computed that $\frac{\partial V}{\partial P_T}$, at $\bar{y} = 0.5$, is equal to 0 while $\frac{\partial V}{\partial \bar{y}} = (P_0 + \Delta P - P_T)\frac{\bar{y}}{\delta^2}\frac{d\bar{y}}{dP_T}g(\frac{\bar{y}}{\delta})$ which is positive when $\bar{y} = 0.5$. Therefore at $P_T = P^{min}$ (i.e. $\bar{y} = 0.5$) the objective function in (8) increases when P_T increases (through the increase in $\bar{y}$) and it is never optimal to set $P_T = P^{min}$. The bidder has an incentive to increase P_T since this increases the probability that the arbitrageurs actually buy shares and the takeover is successful.

8. Empirical Implications

In this section, we derive several empirical implications of the model. In Section 6 we showed that there are two equilibria where arbitrageurs randomize between entering and not entering with two different probabilities p. Since the two equilibria have similar characteristics, we did not distinguish among them. In this section, however, to derive empirical implications, we focus only on one equilibrium, the one where risk arbitrageurs randomize with the higher probability, p_2^*.

We focus on that equilibrium since the other equilibrium is not stable, due to its not being immune to deviations from coalitions. The intuition of this can be seen by looking at Figure 2 (which was derived in Appendix III). If all arbitrageurs are randomizing with probability p_1^*, one arbitrageur alone obtains zero ex-ante expected profits if he enters. However, if two arbitrageurs agree between themselves to enter with probability 1, they increase the probability that the takeover will succeed and obtain strictly positive ex-ante expected profits. Since arbitrageurs usually talk among small groups, this equilibrium is not stable to deviations by small groups of arbitrageurs. If instead all arbitrageurs are randomizing with probability p_2^*, two arbitrageurs cannot ensure themselves, by deviating, positive ex-ante expected profits. In fact, if they agree to enter with probability 1, ex-ante expected profits are negative. If instead they agree not to enter then their profits are equal to zero. Therefore a coalition of two risk arbitrageurs does not disrupt the equilibrium. For this reason, we focus on the second equilibrium.

8.1. Volume and Price. Since $\frac{d\tau_i^a}{dy} > 0$ and $\frac{d\tau}{dy} > 0$ for $0.5 \leq y \leq \bar{y}$, if $\bar{y} = 1$ the probability of success of the takeover (both the one perceived by the arbitrageurs and the one perceived by other shareholders) increases with y and is equal to zero if $y < 50\%$.[21] If instead $\bar{y} < 1$, then the probability of success of the takeover becomes 0 also for very high volumes ($y > \bar{y}$).

Moreover, if $0.5 \leq y \leq \bar{y}$, the higher is the volume the higher is the price of the shares in the market ($P_0 + \Delta P \tau$).

8.2. Number of Arbitrageurs and their Profits. In Appendix III we derive the function $\bar{\pi}(n) \equiv E_\omega [\pi(n, \omega) \mid n]$, which gives the expected profits per arbitrageur, given the number n of risk arbitrageurs who took positions in the deal. This function is represented in Figure 1. If there are few risk arbitrageurs (less than $\frac{0.5}{\delta}$) the takeover

[21] This is due to the fact that we assumed that initially there are no large shareholders. If we relax this assumption, the takeover may succeed also for lower trading volumes.

cannot succeed and expected profits are negative, since the arbitrageur bears the cost of entry in addition to overpaying for the shares. When n reaches $\frac{0.5}{\bar{\delta}}$, the profits become positive, because the takeover now has a positive chance to be successful. As n increases, however, expected profits decrease, because of two effects. First, as n increases the expected volume increases and the share price increases. Second, for high levels of y the profits $\pi(y)$ may become negative again (and for $y > 1$ they surely are negative): in those cases the arbitrageurs do not buy any shares and the takeover will not be successful. This again reduces expected profits.

Therefore, if $n < \frac{0.5}{\bar{\delta}}$ the takeover never succeeds. If instead $\frac{0.5}{\bar{\delta}} \leq n \leq \frac{\bar{y}}{\bar{\delta}}$ the probability of success of the takeover is equal to $\int_0^{\bar{y}-n\bar{\delta}} f(\omega)d\omega = \bar{y} - n\bar{\delta}$ and is decreasing with n. For higher values of n the probability is again zero. The intuition is quite simple: if not enough arbitrageurs are around the takeover has zero chances to be successful. Once the minimum number is reached, however, the probability of success of the takeover decreases with n since competition among them may drive up the share price so much that they decide not to buy shares at all.

8.3. The Cost of Entry. We have assumed that arbitrageurs face a cost of entry $c > 0$ which is arbitrarily small and justified it as the opportunity cost of investing resources in one specific deal. Jindra and Walkling (1999) argues that there is a cost to arbitrageurs in tying up the funds during the acquisition process and find empirical support to the hypothesis that tender offers with longer time horizon are more costly to arbitrageurs. We have abstracted from the temporal dimension, but tried to depict an analogous situation, where arbitrageurs have to free some resources and this is costly for them. This cost may also be interpreted as a lost opportunity to invest in different deals, because the supply of funds is inelastic.

We now consider how the equilibrium changes when this opportunity cost changes.

Remark 1: The higher is the opportunity cost c of investing in the deal, the higher is the takeover premium P_T offered by the bidder and the lower is the expected number of arbitrageurs who invest in it.

Proof: See Appendix IV.

When c increases, arbitrageurs enter with a lower probability. To compensate for this effect, the bidder increases the takeover premium, which encourages them to enter more often. However, we prove in Appendix IV that the increase in the takeover premium does not fully compensate the effect on p and in the new equilibrium the expected number of arbitrageurs is lower.

Remark 2: The arbitrageurs expected profits, conditional on entering and investing in the deal, are higher the higher is the opportunity cost c.

This derives immediately from the fact that in equilibrium ex-ante expected profits (net of the cost c) are equal to 0. This result is compatible with Mitchell and Pulvino (1999): they find that, for the period between 1963 and 1987, arbitrageurs' returns are higher when takeover activity (measured as the ratio between the sum, across all deals, of the target firms' equity values divided by the total value of all equities traded on the NYSE, AMEX and NASDAQ) is higher. When the takeover activity is higher more deals are available. If we interpret the cost of entry c as the opportunity cost of investing in one deal, then c is higher when there are more alternative deals, which means the returns of the arbitrageurs who entered must be higher. This result is equivalent to the result in Section 3, where we could explicitly introduce different takeover bids and showed that as competition among bids increases the arbitrageurs' profits increase.

Finally, until now we focused on the case in which all arbitrageurs faced the same opportunity cost c. However, some risk arbitrageurs may have a lower opportunity cost, for example because they have easier access to financial resources, and we may want to know what happens if the cost of entry differs across arbitrageurs. Let us still focus on the symmetric equilibrium, i.e. such that the probability to invest in the deal as a function of the cost $p(c_i)$ is the same for all arbitrageurs. Then the arbitrageurs with a lower opportunity cost enter with a higher probability. This can be seen by looking at Figure 2 (and reinterpreting the probability p with which the other risk arbitrageurs enter as the average probability). A higher c lowers the curve and corresponds to a lower p_2^*. Once the arbitrageurs have entered, the rest of the game is the same.

8.4. Shares Liquidity. We assume that noise trade volume is distributed uniformly on $[0,1]$. In reality, some companies' shares trade more frequently than other, and so risk arbitrageurs can hide their trade more easily in some shares than in other. We can then look at the implications of a different level of liquidity.

When shares are less liquid, the trading volume we would normally observe, in absence of arbitrageurs' activity, is lower. We therefore assume that the less liquid the shares are, the lower is the maximum trading volume which can be observed (when no arbitrageurs are around). Therefore, the noise trading volume is distributed uniformly in the interval $[0,\bar{\omega}]$, with $0.5<\bar{\omega}$.[22] A higher $\bar{\omega}$ implies a higher liquidity. We study how the equilibrium changes as $\bar{\omega}$ changes. Using (A1) in Appendix and the fact that $f(y-s\delta)=\frac{1}{\bar{\omega}}$ if $0\leq y-s\delta\leq\bar{\omega}$ and 0 otherwise, we obtain

$$\tau=\frac{\int_{\frac{0.5}{\delta}}^{\frac{y}{\delta}} g(s)ds}{\int_{\frac{y-\bar{\omega}}{\delta}}^{\frac{y}{\delta}} g(t)dt}. \tag{10}$$

Moreover, if $y\geq\bar{\omega}+\delta$, τ^a is equal to τ while if $y<\bar{\omega}+\delta$

$$\tau^a=\frac{\int_{\frac{0.5}{\delta}}^{\frac{y}{\delta}} g(s)ds}{\int_{1}^{\frac{y}{\delta}} g(t)dt}. \tag{11}$$

The intuition is the following. If $y>\bar{\omega}+\delta$, everybody knows that there is at least one risk arbitrageur, therefore the arbitrageur's advantage disappears. If instead $y<\bar{\omega}+\delta$, the risk arbitrageur still has an advantage. As the liquidity decreases, the risk arbitrageur can hide less well: in fact, the difference $\tau^a-\tau$ decreases as $\bar{\omega}$ decreases. As a result, when the liquidity of the target stock decreases, the (interim) expected profits of the risk arbitrageurs decrease. This can be summarized in the following remark:

Remark 1: Given the takeover premium P_T, the more liquid the shares are, the higher the expected profits of the arbitrageurs who bought shares.

[22] $\bar{\omega}>1$ implies that the noise traders can sell short.

However, we know that ex ante profits in equilibrium must be equal to 0, therefore the equilibrium p_2^* must change. In Appendix V we prove the following remark:

Remark 2: The more liquid the shares are, the higher is the expected number of arbitrageurs who invest in the deal and the lower is the takeover premium P_T offered by the bidder.

Proof: See Appendix V.

9. Concluding remarks

We have shown that a risk arbitrageur has an incentive to buy shares in a company which is the target of a takeover offer, even if he has no initial informational advantage. In fact, since he is more likely to tender his shares, the fact that he has bought some of the shares increases the probability of success of the takeover. As long as he manages not to completely reveal his presence in the market, the share price will not fully reflect the increased probability of success and the risk arbitrageur can buy shares and make a profit. We also show that competition among risk arbitrageurs reduces their profits but does not eliminate completely their incentives to invest in the deal.

While the press has often depicted risk arbitrageurs in an unfavorable way, this paper shows that they can actually increase welfare, facilitating takeovers which increase the value of a company. Moreover, small shareholders, who sell their shares to risk arbitrageurs, do not lose money but they actually appropriate the ex ante surplus, which would be lost if the takeover did not succeed.

The model can be extended to take into account other characteristics. We assume that small shareholders take the probability of a takeover for given in order to simplify the analysis. However, the intuition remains the same with more strategic players (as long as the probability of the takeover, at the announcement of the tender offer, is less than one): an individual who owns a larger stake will suffer less of the free-riding problem and tender a larger fraction of his shares, thereby facilitating the takeover. The rest of the analysis would then be the same.

We also focus on the case in which risk arbitrageurs buy shares only after the takeover announcement. In general, risk arbitrageurs do not attempt to forecast acquisition candidates, but rather they try to resolve the uncertainty surrounding an announced proposal. However, the model can be extended to the case in which risk arbitrageurs can take positions also prior to the takeover announcement, speculating on the probability that the announcement will indeed happen. As long as their presence increases the chances of success of a (possible) takeover, the same result holds.

Another extension would be to allow risk arbitrageurs to sell short. Risk arbitrageurs may have an incentive to sell short if their assessment of the probability that the takeover will be successful is lower than the assessment of small shareholders. Therefore, in the cases where we found it was optimal for arbitrageurs not to buy shares, it may become optimal to sell short. An arbitrageur would then have an additional way to make profits since he would be the only one to know for sure whether he is in (or not in).

We could also allow risk arbitrageurs to communicate between themselves. Usually risk arbitrageurs belong to small "clubs" and they talk only to arbitrageurs in the same club. In this case they will be informed not only of their own presence, but also of the presence of everybody in the same club. We could model this behavior by assuming that risk arbitrageurs in the same club commit, before randomizing, to inform each other if they choose to enter the contest. In general, it will be incentive compatible for a risk arbitrageur who entered to tell the truth. If the agreement is among two people only, their ex-ante expected profits increase, so that they will choose to enter more often. However, as the number of members of each club increases, the expected number of arbitrageurs entering increases, until it starts having an adverse effect. As a result, there would be an optimal size of the club.

Finally, this paper allows to study the role of the legal limit to the number of shares an arbitrageur can buy without disclosing his presence (such as the 5% limit according to Section 13D of the Security Exchange Act). This limit is usually interpreted as an obstacle to takeovers, while we show that it can actually favor takeovers. In fact, it reduces competition among risk arbitrageurs. It would therefore be interesting to study how different limits across countries influence the takeover activity.

APPENDIX

Appendix I. Proof of Proposition 2

Since small shareholders do not tender, when $y < 0.5$ it is clear that not enough shares are in the hands of arbitrageurs and ${\tau^a}_i = \tau = 0$. If $y \geq 0.5$

$$
\begin{aligned}
{\tau^a}_i = {\tau^a}_i(y, \delta, \delta_i) &= Prob \ [\ (n-1)\delta + \delta_i \geq 0.5 \ \ | \ y - \delta_i \ ] \\
&= Prob \ [\ n - 1 \geq \frac{0.5 - \delta_i}{\delta} \ \ | \ y - \delta_i \ ]
\end{aligned}
$$

where $y - \delta_i$ is the arbitrageur's observation of the total transaction volume, excluding his own. A_i has to compute the conditional probability distribution of the number of arbitrageurs other than himself. Let d (.) denote the density, then

$$
\begin{aligned}
d \ (n - 1 = s \ \ | \ y - \delta_i \ ) &= \frac{d \ (n - 1 = s, \omega + (n-1)\delta = y - \delta_i \ )}{d \ (\omega + (n-1)\delta = y - \delta_i \ )} \\
&= \frac{d \ (n - 1 = s, \omega = y - s\delta - \delta_i \ )}{d \ (\omega + (n-1)\delta = y - \delta_i \ )}.
\end{aligned}
$$

Under the assumption that, ex ante, n and ω are independent and that the coming of arbitrageurs is mutually independent (which will be shown to be true in equilibrium), it follows

$$
d \ (n - 1 = s, \omega = y - s\delta - \delta_i) = g(s+1) f(y - s\delta - \delta_i)
$$

where $f(\cdot)$ is the density of the noise traders distribution and

$$
Prob \ [\omega + (n-1)\delta = y - \delta_i \ ] = \int_0^{\frac{y-\delta_i}{\delta}} g(s+1) f[y - s\delta - \delta_i] ds.
$$

Therefore, we have

$$
Prob \ [\ n - 1 \geq \frac{0.5 - \delta_i}{\delta}, \ \omega + (n-1)\delta = y - \delta_i \ ] = \int_{\frac{0.5-\delta_i}{\delta}}^{\frac{y-\delta_i}{\delta}} g(s+1) f[y - \delta_i - s\delta] ds \quad \text{(A1)}
$$

and consequently, given that noise traders are uniformly distributed,

$$\tau^a{}_i = \frac{\int_{\frac{0.5-\delta_i}{\delta}}^{\frac{y-\delta_i}{\delta}} g(s+1)ds}{\int_0^{\frac{y-\delta_i}{\delta}} g(t+1)dt}. \tag{A2}$$

In a symmetric equilibrium, $\delta_i = \delta$ and

$$\tau^a = \frac{\int_{\frac{0.5}{\delta}}^{\frac{y}{\delta}} g(s)ds}{\int_1^{\frac{y}{\delta}} g(t)dt}. \tag{A3}$$

The updating of the small shareholders is different, since they may think that perhaps all volume y is due to noise traders. If we repeat the calculation we did for τ_i^a for the case of the small shareholders, the *posterior* probability of success of the takeover (conditional on y) is

$$\tau = \frac{\int_{\frac{0.5}{\delta}}^{\frac{y}{\delta}} g(s)ds}{\int_0^{\frac{y}{\delta}} g(t)dt}. \tag{A4}$$

Even if in general we cannot compare τ and τ_i^a, in the symmetric case it is easy to see that $\tau^a > \tau$. In fact

$$\tau = \text{Prob}\left\{n\delta > \frac{1}{2} \mid y\right\} = \text{Prob}\left\{n\delta > \frac{1}{2} \mid y, n \geq 1\right\} \text{Prob}\left\{n \geq 1 \mid y\right\} \leq \text{Prob}\left\{n\delta > \frac{1}{2} \mid y, n \geq 1\right\} = \tau^a.$$

Moreover, we can see that τ is increasing in y by rewriting:

$$\tau = 1 - \frac{G\left(\frac{0.5}{\delta}\right)}{G\left(\frac{y}{\delta}\right)}$$

where $G\left(\frac{y}{\delta}\right)$ is increasing in y. To see that τ_i^a is increasing in y we can compute the derivative:

$$\frac{\partial \tau_i^a}{\partial y} = \frac{g\left(\frac{y}{\delta}\right)}{\delta} \frac{G\left(\frac{0.5}{\delta}\right) - G(1)}{\left[G\left(\frac{y}{\delta}\right) - G(1)\right]^2} > 0.$$

¶Q.E.D.

Appendix II. Proof of Proposition 3 Let us first look at the choice of δ_i by an arbitrageur, given the beliefs τ and τ_i^a and given that any risk arbitrageur who entered has bought δ shares. The arbitrageur chooses δ_i in order to maximize

$$MAX_{\delta_i}\delta_i\{[\Delta P - \gamma^e(P_0 + \Delta P - P_T)]\ \tau^a{}_i + P_0 - P_1\} \tag{A5}$$

where P_1 is the price paid, which is equal to $\underline{P}$ in equation (1) if $n\bar{\delta} + \omega \leq 1$ and to V^a in equation (2) if $n\bar{\delta} + \omega > 1$ and γ^e is the fraction of shares the arbitrageur expects to tender:

$$\gamma^e = E\left[\frac{0.5}{(n-1)\delta + \delta_i} \mid y - \delta_i\right].$$

Moreover, define

$$P_{\tau_i} \equiv [\Delta P - \gamma^e(P_0 + \Delta P - P_T)]\, \tau_i^a$$

If the total demand $n\bar{\delta} + \omega > 1$, P_1 is equal to the reservation price V^a and the risk arbitrageurs will not buy any shares. If the total demand $n\bar{\delta} + \omega \leq 1$, the objective function becomes

$$MAX_{\delta_i}\delta_i\,[P_{\tau_i} - \Delta P\tau] \tag{A6}$$

the first order derivative is

$$P_{\tau_i} - \Delta P\tau + \delta_i \frac{\partial \tau_i^a}{\partial \delta_i}[\Delta P - \gamma_i(P_0 + \Delta P - P_T)] - \delta_i(P_0 + \Delta P - P_T)\tau_i^a \frac{d\gamma^e}{d\delta_i} \tag{A7}$$

where $\frac{\partial \tau_i^a}{\partial \delta_i} > 0$ and by implicit function theorem $\frac{d\gamma^e}{d\delta_i} < 0$ since, for each possible realization of n, the larger is δ_i the lower is the fraction γ that the arbitrageur has to tender.

If $\delta_i = \delta \leq \bar{\delta}$ (the risk arbitrageur A_i buys as many shares as the others), then $\tau_i^a = \tau^a > \tau$. Moreover, $P_{\tau_i} - \Delta P\tau$ is increasing in δ_i. Therefore, given τ_i^a and τ, define $\hat{\delta}_i$ such that $P_{\tau_i} = \Delta P\tau$ (if such $\hat{\delta}_i > 0$ exists). Then for all $\delta_i < \hat{\delta}_i$ the objective function is negative and the optimum is $\delta_i = 0$. For all $\delta_i \geq \hat{\delta}_i$, the first order derivative is always strictly positive. Therefore, the solution is always a corner solution: the arbitrageur wants to buy either no shares at all or as many shares as possible. The intuition is quite simple: P_{τ_i} is the expected benefit from holding one share, while $\Delta P\tau$ is the actual cost (both in excess of P_0); since risk arbitrageurs are risk neutral, they will buy shares if and only if the expected benefit is higher than the cost.

We know buying 0 shares is an equilibrium, let us now check if it is ever an equilibrium to buy $\bar{\delta}$ shares. For each n and ω, if the arbitrageurs buy $\bar{\delta}$ shares, the volume is $y = n\bar{\delta}+\omega$ and τ and τ^a depend on such y. We have to check whether the profits from buying $\bar{\delta}$ shares are positive or negative. First of all, if $y < 0.5$, then $\tau_i^a = \tau = 0$. The share price is P_0 and the risk arbitrageurs are indifferent between buying and not buying shares. Given that they are not interested in holding long term positions, we assume that in this case they do not buy any shares. This gives us case (a) of Proposition 3. Similarly, in case (c), since $n\bar{\delta} + \omega > 1$, the price should increase up to $P_0 + \tau^a \Delta P$. However, at that price the risk arbitrageurs will not buy shares. Therefore, the unique symmetric equilibrium implies a share price equal to P_0 and the takeover fails.

Let us now consider the case where $0.5 \leq n\bar{\delta} + \omega \leq 1$. If the risk arbitrageur bought $\bar{\delta}$ shares, $\tau^a(y)$ is given by (4). The expected (interim) profits of arbitrageur A_i are

$$\pi(y) \equiv \bar{\delta}\,[P_{\tau_i} - \Delta P\tau] \tag{A8}$$

where

$$\frac{d\pi}{dy} = \frac{\partial \pi}{\partial y} + \frac{\partial \pi}{\partial \gamma^e}\frac{d\gamma^e}{dy}.$$

We know that $\frac{\partial \pi}{\partial \gamma^e} < 0$, moreover by implicit function theorem $\frac{d\gamma^e}{dy} < 0$, since the higher is y the larger is the expected n, therefore the second term is always positive. The intuition is quite clear: the larger is y the less shares the arbitrageur expects to tender and this increases his profits. The first term is given by

$$\frac{\partial \pi}{\partial y} = \bar{\delta}\left[\frac{\partial \tau^a}{\partial y}[\Delta P - \gamma^e(P_0 + \Delta P - P_T)] - \frac{\partial \tau}{\partial y}\Delta P\right] \tag{A9}$$

where $\frac{\partial \tau^a}{\partial y} > 0$ and $\frac{\partial \tau}{\partial y} > 0$. To find out whether $\frac{\partial \pi}{\partial y} > 0$, note that $\frac{\partial \tau^a}{\partial y} > \frac{\partial \tau}{\partial y}$ if and only if $\left[G(\frac{y}{\bar{\delta}})\right]^2 < G(\frac{0.5}{\bar{\delta}})\left[2G(\frac{y}{\bar{\delta}}) - G(1)\right]$, which is always satisfied if y is close to 0.5. Therefore, if P_T is sufficiently high, profits are positive and increasing and it is an equilibrium to buy $\bar{\delta}$ shares at least for values of y sufficiently close to 0.5. If instead P_T is low, profits are negative and decreasing. Let us define as P^{min} the P_T at which (A9) is equal to zero at $y = 0.5$. If $P_T > P^{min}$, interim expected profits are positive at least for y close to 0.5.

For higher level of volumes, however, the profits can be a non-monotonic function of y. The intuition is clear: as y increases it becomes more and more likely that there are arbitrageurs around: the price of the shares goes up more than the value of the shares to the arbitrageurs, since their informational advantage is becoming thinner, so that $\frac{\partial \tau}{\partial y} > \frac{\partial \tau^a}{\partial y}$ and profits start decreasing.

For high levels of y the profits from buying $\bar{\delta}$ shares can become negative (in which case the only equilibrium would be not to buy shares at all). This is possible because, although $\tau^a \geq \tau$ always, the arbitrageur tenders some of its shares and the benefit from one share is therefore less than ΔP. Profits are equal to zero if $P_{\tau_i} = \Delta P \tau$. If we divide both sides by $\Delta P + P_0 - P_T$, define $\phi \equiv \frac{\Delta P}{\Delta P + P_0 - P_T}$ and express τ^a and τ in terms of the distribution $G(s)$, some simple manipulation of the zero profits equation yields $\gamma^e G(\frac{y}{\delta}) = \phi G(1)$. The left hand side is always less than 1 and as $P_T \to P_0 + \Delta P$, $\phi \to \infty$. Therefore, if $G(1)$ is not infinitesimal (which we can check when we endogenize $G(n)$), and P_T is sufficiently high, profits can never become zero, which means the profits are always positive for any $y \geq 0.5$.

To summarize, if P_T is not too low ($P_T \geq P^{min}$) there exists values of n and ω (and therefore y) for which profits are positive and if P_T is sufficiently high profits are positive for any volume larger than 50%. In all this cases it is an equilibrium for the risk arbitrageurs who entered to buy $\bar{\delta}$ shares.

Finally, we have to show that when it is not an equilibrium to buy $\bar{\delta}$ shares, the only symmetric pure strategy equilibrium is the one in which risk arbitrageurs buy 0 shares. Let us assume that each risk arbitrageur buys a quantity $\delta_0 \leq \bar{\delta}$. Then, for given beliefs $\tau(\delta_0)$ a single risk arbitrageur always has an incentive to deviate and buy up to $\bar{\delta}$ shares. Therefore δ_0 cannot be an equilibrium. This gives us case (b) of Proposition 3.

¶Q.E.D.

Appendix III. Proof of Proposition 4

For each (n, ω), we can compute the ex-post profits $\pi(n, \omega)$. We consider the case in which $P_T \geq P^{min}$, otherwise the arbitrageurs never buy shares and the profits are never positive. If (n, ω) is such that $n\bar{\delta} + \omega < 0.5$ or $n\bar{\delta} + \omega > \bar{y}$ (where $\bar{y}$ is either the volume level at which profits become equal to zero or is equal to one) then the arbitrageurs who chose to enter would buy no shares at all and $\pi(n, \omega) = -c$. If instead (n, ω) are such that $0.5 \leq n\bar{\delta} + \omega \leq \bar{y}$ but $n\bar{\delta} < 0.5$ then the arbitrageurs buy $\bar{\delta}$ shares but the takeover fails and their profits are $\pi(n, \omega) = -c - \Delta P \tau \bar{\delta}$. Finally, if $0.5 \leq n\bar{\delta} + \omega \leq \bar{y}$ and

$n\bar{\delta} \geq 0.5$ then the takeover succeeds and the arbitrageurs make positive profits $\pi(n,\omega) = \bar{\delta}\left[\Delta P(1-\tau) - \gamma(n)(\Delta P + P_0 - P_T)\right] - c$.

Let us define $\bar{\pi}(n) \equiv E_\omega\left[\pi(n,\omega) \mid n\right]$ to be the expected profits for a given n of an arbitrageur who has entered. Figure 1 represents $\bar{\pi}(n)$ as a function of n. If $n < \frac{0.5}{\bar{\delta}}$

$$\bar{\pi}(n) = -c - \bar{\delta}\Delta P \int_{0.5-n\bar{\delta}}^{\bar{y}-n\bar{\delta}} \tau(n\bar{\delta}+\omega)d\omega = -c - \bar{\delta}\Delta P \int_{0.5}^{\bar{y}} \tau(\hat{\omega})d\hat{\omega}. \tag{A10}$$

Rewriting the profit as a function of $\hat{\omega}$ shows that the profits are independent of n and therefore constant in that range.

If $\frac{0.5}{\bar{\delta}} \leq n \leq \frac{\bar{y}}{\bar{\delta}}$

$$\bar{\pi}(n) = -c + \bar{\delta} \int_{0}^{\bar{y}-n\bar{\delta}} \pi(n\bar{\delta}+\omega)d\omega = -c + \bar{\delta} \int_{\bar{\delta}n}^{\bar{y}} \pi(\hat{\omega})d\hat{\omega}. \tag{A11}$$

If we take the derivative with respect to n, the lower bound of the integral increases, therefore the profits are a decreasing function of n. It can easily be seen that if $n = \frac{\bar{y}}{\bar{\delta}}$, $\bar{\pi}(n) < 0$, since for any $\omega > 0$ the arbitrageurs earn $-c$. We define $\hat{y}$ as the level of volume such that $\bar{\pi}(\frac{\hat{y}}{\bar{\delta}}) = 0$ (as it is shown in Figure 1).

Finally, if $n > \frac{\bar{y}}{\bar{\delta}}$, $\bar{\pi}(n) = -c$.[23]

The ex-ante expected profits are equal to

$$\Pi(p,N,c) \equiv E_{n,\omega}\left[\pi(n,\omega)\right] = E_n\left[\bar{\pi}(n)\right]. \tag{A12}$$

To find the symmetric equilibria we proceed in the following way. Let us define a distribution of n, $\hat{g}(n)$, such that $\hat{g}(n+1) = g(n)$, where $g(n)$ is the binomial defined in (6). Suppose $N-1$ arbitrageurs randomize their entry decision with probability p and let us consider the entry decision of the N-th arbitrageur. If he decides not to enter, the expected payoff is 0. Suppose he decides to enter with probability 1, then his expected payoff defined as $\tilde{\pi}(p)$ can be expressed in the following way

$$\tilde{\pi}(p) = \sum_{n=1}^{N}\left[\bar{\pi}(n)\hat{g}(n)\right] = A Prob\{n < \frac{0.5}{\bar{\delta}}\} + B Prob\{\frac{0.5}{\bar{\delta}} \leq n \leq \frac{\bar{y}}{\bar{\delta}}\} - c Prob\{n > \frac{\bar{y}}{\bar{\delta}}\},$$

[23] If $N < \frac{\bar{y}}{\bar{\delta}}$ this case never arises.

where $\hat{g}(n)$ is the distribution of the number of arbitrageurs in the game, taking into account the decision of the last arbitrageur, and A and B are the expressions in (A10) and in (A11), respectively. Notice that $A < 0$ always and $B > 0$ if c is sufficiently low.

If all other arbitrageurs randomize with probability $p = 0$, then $E_n[\bar{\pi}(n)] < 0$, since one arbitrageur alone is not enough for the takeover to succeed and the profits are given by (A10). As p increases the ex-ante expected profits increase. We want to prove that, under some conditions, there exists a p, such that $\tilde{\pi}(p) > 0$. Let us take $p = \frac{(0.5+\hat{y})/(2\bar{\delta})}{N}$. The mean of the binomial is $Np = (0.5 + \hat{y})/(2\bar{\delta})$. Therefore if N is large enough, the realized n should be concentrated around Np, falling in the interval corresponding to the expression of (A11). Thus, in the expression of $\tilde{\pi}(p)$, the weight of B is high enough for $\tilde{\pi}(p)$ to be positive.

By continuity, there must exist a value of p—let us call it p_1^*—at which $\tilde{\pi}(p) = 0$. As n increases, however, more and more weight is given to the case in which $n > \frac{\hat{y}}{n}$. Therefore, ex-ante profits become negative again. In particular, take $p = \frac{\bar{y}/\bar{\delta}}{N}$ (or higher) : again, if N is large enough all the weight of the probability is on $\bar{\pi}(n) = -c$ and $\tilde{\pi}(p)$ is negative. By continuity, there must exist another p—let us call it p_2^*, at which ex ante profits are equal to 0.

These two probabilities, p_1^* and p_2^* are the equilibria. Intuitively, when the other arbitrageurs randomize with a probability p such that the expected profits from entering are zero, the last arbitrageur is indifferent between entering and not and then he might as well randomize between the two with probability p. Therefore these two values of p are the two equilibria of the game (in addition to the equilibrium in which nobody enters). Figure 2 represents the ex-ante profits of a risk arbitrageur who enters, given that all other risk arbitrageurs are randomizing with probability p. The level of p for which the ex-ante profits are equal to zero are the equilibria.

Appendix IV. Proof of Remark 1 of Section 8.3

We proceed in the following way. First of all, we prove that, when P_T increases, the expected interim profits of the arbitrageurs increase and the equilibrium probability p_2^* of entry increases. This results will be useful later in the proof. Then we show that, for a given takeover premium P_T , when c increases the expected number of arbitrageurs who invest in the deal decreases. However, the bidder may change the takeover premium in response to such change, so in step 4 we show that p_2^* decreases even after we take into account the bidder's reaction. Finally, we show that P_T increases when c increases.

Step 1: The (interim) expected profits of the risk arbitrageurs, $\pi(y)$, increase with P_T.

This is easy to see, since $\frac{d\pi}{dP_T} = \tau^a \gamma^e \delta(y) \geq 0$ and the inequality is strict when $\delta(y) = \bar{\delta}$. Moreover, the interim expected profits are positive over a larger range of values of y, since by implicit function theorem $\frac{d\bar{y}}{dP_T} > 0$.

Step 2: Given c, the higher is the takeover premium P_T, the higher is p_2^*.

We start by looking at how the expected profits $\bar{\pi}(n)$—derived in Appendix III and represented in Figure 1—change when P_T increases. For $n < \frac{0.5}{\delta}$ there are two cases. If $\bar{y} < 1$, then as P_T increases $\bar{y}$ increases, which implies that $E(\tau)$ increases (i.e. the share price that the risk arbitrageur expects to pay increases) and as a consequence $\bar{\pi}(n)$ decreases. If instead $\bar{y} = 1$, $\bar{\pi}(n)$ remains constant. In the interval $\frac{0.5}{\delta} \leq n \leq \frac{\bar{y}}{\delta}$, $\bar{\pi}(n)$ increases (since $\pi(y)$ increases, from Step 1). Finally, for $n > \frac{\bar{y}}{\delta}$, $\bar{\pi}(n)$ remains equal to $-c$.

Therefore, if $\bar{y} = 1$ the ex-ante expected profits increase when P_T increases and the probability p must increase so that they are again equal to 0. If instead $\bar{y} < 1$, then the sign is not obvious, since $\bar{\pi}(n)$ decreases for $n < \frac{0.5}{\delta}$. However, we can still show that also in that case the ex-ante profits (which in equilibrium should be equal to zero) become strictly positive at p_2^* when P_T increases.

To see this let us assume that $\bar{y} < 1$, i.e. $\frac{d\bar{\pi}}{dP_T}(n) < 0$ for $n < \frac{0.5}{\delta}$. We then proceed in the following way. We know that at the equilibrium p_2^* an individual who unilaterally increases p must receive zero expected profits. We then look at the trade-off facing an individual increasing p: that trade-off guarantees that the ex-ante expected profits are equal to zero. We will show that, in this specific case, when P_T increases the trade-off becomes more favorable. That means that now an individual increasing p would earn strictly positive ex-ante profits. Therefore, the equilibrium p_2^* has to increase.

The trade-off is the following. If one individual increases p from p_2^*, $G(\frac{.5}{\delta})$ (the probability of obtaining a negative payoff, since $\bar{\pi}(n) < 0$ when $n < \frac{0.5}{\delta}$) decreases. As a consequence, his expected payoff increases. The disadvantage, however, is that $1 - G(\frac{\bar{y}}{\delta})$ (the probability of obtaining the negative payoff $-c$, since $\bar{\pi}(n) = -c$ when $n > \frac{\bar{y}}{\delta}$) increases. By definition, these effects must compensate so that ex-ante expected profits are equal to zero. However, when P_T increases, the payoff obtained when $n < \frac{0.5}{\delta}$ becomes even more negative (since we are studying the case where for $n < \frac{0.5}{\bar{\delta}}$ $\frac{d\bar{\pi}}{dP_T} < 0$) and the gain from increasing p is higher. At the same time the cost if $n > \frac{\bar{y}}{\bar{\delta}}$ is still $-c$, but $\bar{y}$ has increased, so it is less likely that the arbitrageur will bear that cost. The remaining mass of probability is in the central part of the support, where profits increased. The trade-off is surely more favorable now, so the probability p must increase.

Step 3: Given the takeover premium P_T, when c increases, p_2^* decreases.

From Figure 2 one sees that an increase in c lowers the entire curve $\bar{\pi}(n)$, which implies that arbitrageurs enter with a lower probability p_2^*.

Step 4: The higher is the opportunity cost c of investing in the deal, the lower is p_2^*.

We proceed in the following way. From Step 2, we know that when P_T increases, the probability of entry p increases. Let us assume that, as an answer to an increase in c (and therefore a decrease in p), P_T increases up to the point in which p would be back to the value it had with the lower c. We then show that the bidder's first order condition computed at that P_T must be negative and therefore the optimal P_T must be lower. Note that we are not showing whether P_T will increase or decrease when c increases. We are only showing that an increase in c cannot cause P_T to increase so much that p_2^* does not decrease as a result.

Let us rewrite the first order conditions in (9) as

$$\Phi(P_T) \equiv -A + (P_0 + \Delta P - P_T)B = 0$$

where $A \equiv \int_{\frac{0.5}{\bar{\delta}}}^{\frac{\bar{y}}{\bar{\delta}}} (\bar{y} - \bar{\delta}n)g(n \mid P_T)dn$ and $B \equiv \frac{d\bar{y}}{dP_T} \int_{\frac{0.5}{\bar{\delta}}}^{\frac{\bar{y}}{\bar{\delta}}} g(n \mid P_T)dn + \int_{\frac{0.5}{\bar{\delta}}}^{\frac{\bar{y}}{\bar{\delta}}} (\bar{y} - \bar{\delta}n)\frac{\partial g(n)}{\partial p}\frac{dp}{dP_T}dn$. Since $A > 0$, this implies that $B > 0$. Moreover $\frac{\partial \Phi}{\partial P_T} = -B < 0$.

Let us assume that P_T, as a response to the increase in c, has increased up to the point in which p is equal to the p_2^* which was optimal before the change. That means that $g(n)$ is also the same as before. Then, let us compare the first order derivative $\Phi(P_T)$ before the change in c (which by definition were equal to 0, since we proved that the optimal P_T was an interior solution) and after both c and P_T have increased. The only change in the two functions is the increase in P_T, while $g(n)$ is the same. We know that $\frac{\partial \Phi}{\partial P_T} < 0$ therefore the first order derivative at the new point must be negative. If the first order derivative is negative, the optimal P_T must be lower. But at that lower P_T, p_2^* is lower than at the initial level.

Step 5: The higher is the opportunity cost c of investing in the deal, the higher is the takeover premium P_T offered by the bidder.

To show that P_T actually increases as a response to an increase in c, let us proceed by contradiction. Let us start from the optimum P_T^* (corresponding to a probability of entry p^*) and let us assume that when c increases P_T decreases to a new level $P_T' < P_T^*$ (corresponding to a probability of entry $p' < p^*$) and let us show this is not possible. Note that at the initial (lower) c the bidder could have achieved p' by choosing a takeover premium $P_T'' < P_T'$. However, a necessary condition for the initial P_T^* to be an optimum was that $\frac{d\Phi}{dP_T} < 0$, therefore the first order derivative must be strictly positive at $P_T' < P_T^*$. Therefore P_T'' cannot be the optimum. ¶Q.E.D.

Appendix V. Proof of Remark 2 of Section 8.4

Let us first show that when $\bar{\omega}$ increases for a given P_T the number of expected arbitrageurs investing in the deal (i.e. p) increases. Then we will endogenize the choice of P_T.

Let us start by seeing how the expected profits $\bar{\pi}(n)$—derived in Appendix III and represented in Figure 1—change when $\bar{\omega}$ increases. As $\bar{\omega}$ increases, $\tau(y)$ decreases (as can be seen from 10) and $\bar{y}$ increase. As a consequence, if $n < \frac{0.5}{\delta}$ from (A10)

$$\frac{d\bar{\pi}}{d\bar{\omega}}(n) = -\bar{\delta}\Delta P \frac{d\bar{y}}{d\bar{\omega}}\tau(\bar{y}) - \bar{\delta}\Delta P \int_{0.5}^{\bar{y}} \frac{\partial \tau}{\partial \bar{\omega}} d\hat{\omega} \tag{A13}$$

where the first term is negative and the second is positive, so the sign is ambiguous. For $\frac{0.5}{\delta} \leq n \leq \frac{\bar{y}}{\delta}$, $\pi(y)$ increases and from (A11)

$$\frac{d\bar{\pi}}{d\bar{\omega}}(n) = \bar{\delta}\frac{d\bar{y}}{d\bar{\omega}}\pi(\bar{y}) + \int_{\delta n}^{\bar{y}} \frac{\partial \pi}{\partial \bar{\omega}} d\hat{\omega} > 0.$$

Finally, for $n > \frac{\bar{y}}{\delta}$, $\bar{\pi}(n)$ remains equal to $-c$.

If $\bar{\pi}(n)$ increases for $n < \frac{0.5}{\delta}$ (i.e. (A13) is positive), the ex-ante expected profits increase for any level of p and therefore p_2^* must increase. If instead $\bar{\pi}(n)$ decreases for $n < \frac{0.5}{\delta}$ (i.e. (A13) is negative), then the effect on the ex-ante profits is ambiguous. However, this last case is exactly the same situation we looked at in Step 2 of Appendix IV. The same logic can be used to show that also in this case the ex-ante profits at p_2^* (which in equilibrium should be equal to zero) become strictly positive when $\bar{\omega}$ increases and therefore the equilibrium p_2^* must increase.

We have then proven that p increases when $\bar{\omega}$ increases for a given P_T. However, if we endogenize the choice of P_T by the bidder, it could be that his response is to decrease P_T by so much that the initial effect on p is completely offset. Once again, exactly the same logic we used in Appendix IV, Steps 4 and 5, applies here: the same arguments can be used to show that P_T must decrease in equilibrium, but that it never decreases up to the point in which the initial effect is completely offset.

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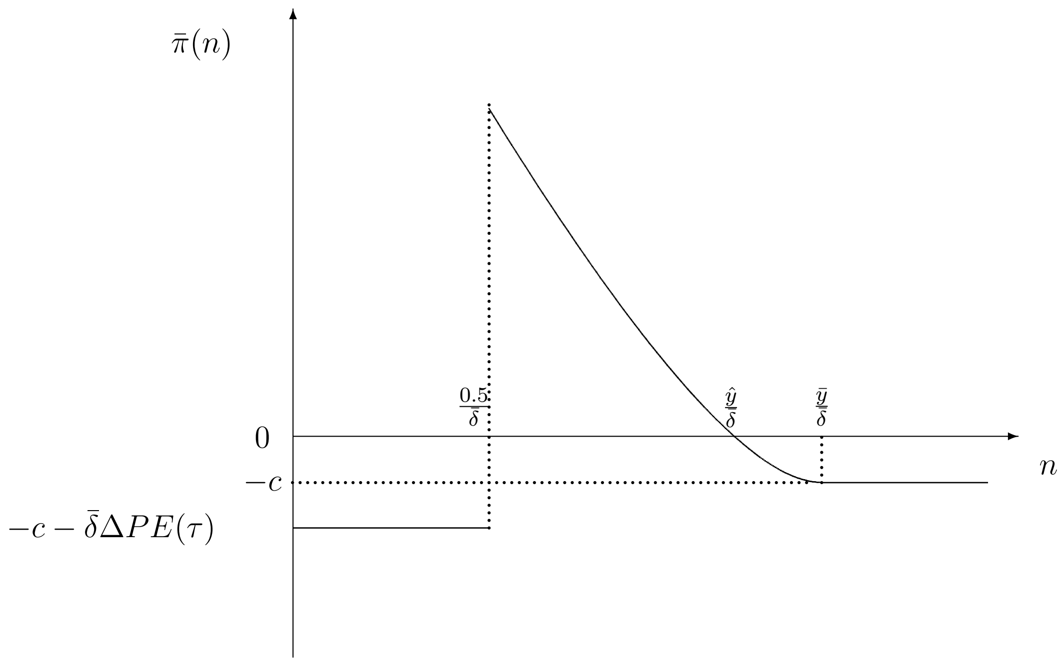

Figure 1: Arbitrageurs profits as a function of their number

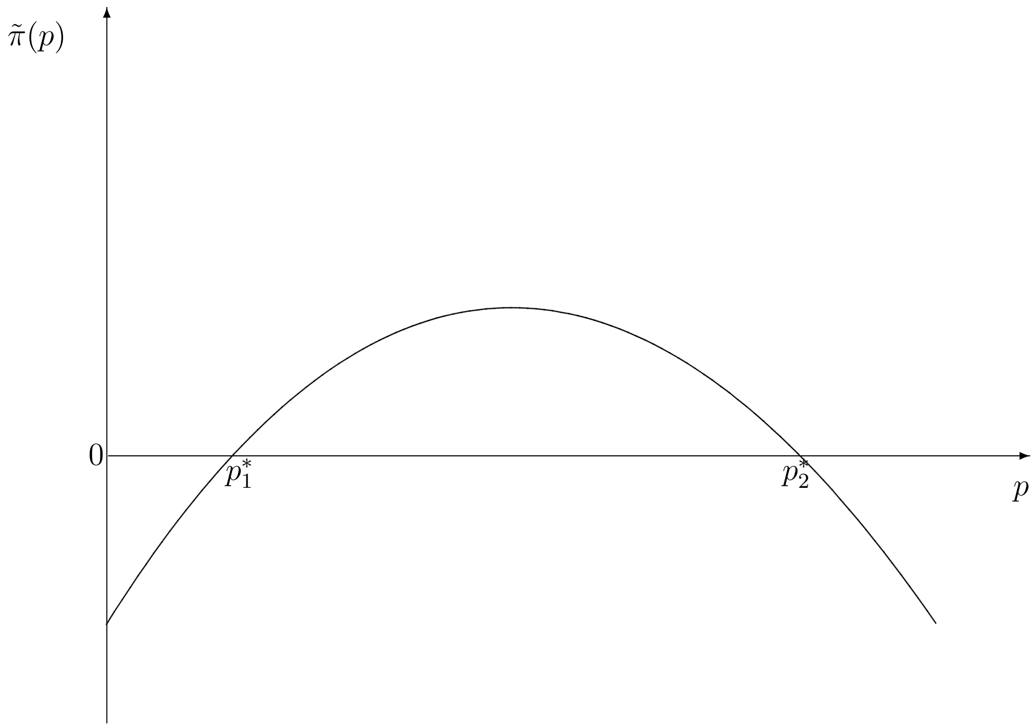

Figure 2: Ex-ante Expected Profits as a function of the randomization p