The cross-sectional dynamics of the US business cycle: 1950–1999☆

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Abstract

Modern interest in business cycles has focussed on the co-movements and correlations in the major macroeconomic aggregates. In this paper we offer another dimension to business cycle analysis which looks at the time series of cross-sectional distributions of the growth rates of sales by US quoted companies from 1950 to 1999. We detect correlations between aggregate business cycle fluctuations and the higher moments of the cross-sectional distribution. We find a significant negative correlation between the rate of growth of gdp and the cross-sectional variance and skewness of growth rates of sales. On the other hand there is positive correlation, at business cycle frequencies with kurtosis. In order to explore this further we turn to the dynamic evolution of firms and analyse the sensitivity of growth rates to aggregate shocks conditioning on firm size. The results suggest that despite considerable heterogeneity macroeconomic shocks have pervasive effects that are, however, more pronounced for firms in the middle range of growth. This has implications for both macro and industrial economics. © 2002 Published by Elsevier Science B.V.

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1. Introduction

Modern approaches to the analysis of business cycles focus on co-movements and correlations among the major macroeconomic aggregates. Until recently the cross-sectional behaviour of individual firms or households was not considered important for understanding the aggregate, or else there was insufficient data. This has changed over the last decade. Not only is there an increasing availability of highly disaggregated, longitudinal microeconomic and sectorial data for sufficiently long periods of time, but it has been argued that particular forms of microeconomic adjustment—in particular when

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they are lumpy—can mean that the higher moments of the cross-sectional distribution of microeconomic actions can affect the dynamics of macroeconomic aggregates. This has been put most forcefully by Haltiwanger (1997).\(^1\)

It is becoming increasingly apparent that changes in the key macroaggregates at cyclical and secular frequencies are best understood by tracking the evolution of the cross-sectional distribution of activity and changes at the micro level.

The proper understanding of business cycles requires knowledge of the cross-sectional distributions as well as the behaviour of aggregate economic variables over time. Caballero (1992), Caballero and Engel (1992, 1993) and Haltiwanger (1993) have suggested that statistical agencies should report the higher moments of economic activity; for example, the distribution of aggregate output.\(^2\)

In this paper we provide some new empirical evidence on the US business cycle since 1950 that uses information on firm level sales obtained from the COMPUSTAT database of quoted company accounts. We use a mixture of parametric and non-parametric methods in order to explore the relationship between aggregate business cycle fluctuations and the cross-sectional distribution of firm sales growth. In the standard models used in macroeconomics, the mean behaviour of firms subject to idiosyncratic and aggregate shocks should be correlated with the aggregate economy but the higher moments ought to be independent. However, we find a distinct cyclical pattern to the higher moments of the cross-sectional distribution of firm growth rates. We find that cross-section of firm growth rates exhibit significant degrees of skewness and kurtosis that varies with the business cycle. Moreover, the dispersion of growth rates also varies over time in a secular fashion. These preliminary empirical findings are reported in Section 3.

The co-movements and persistence in aggregate measures that are the usual concern of macroeconomists, may need to be augmented with these new stylised facts from the cross-section. There is a need for theories that can explain not just the persistence of the business cycle and the comovements in output, consumption, hours, employment and investment, but also why the distribution of firm behaviour changes markedly over the cycle, and how that can matter in determining the amplitude of the cycle and the real economic processes of job generation and destruction.

The rest of the paper is organised as follows. A framework that organises our empirical analysis is presented in Section 2. The stylised facts about the dynamics of the cross-sectional distribution of growth rates are reported in Section 3. In Sections 4 and 5 we become more specific and apply the framework of Section 2 to discriminate between competing hypotheses about what drives the cross-sectional distribution of firm growth over the business cycle. Section 6 concludes.

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\(^1\) McGuckin (1995) has made similar points.

\(^2\) Hildenbrand (1998) has argued that we may not need an explicit model of individual behaviour if we are interested in the aggregate activity of a large and heterogeneous group of economic agents. He examines the cross-sectional distribution of household incomes in the UK and while they are not time invariant, the distributions change only slowly over time.
2. Cross-sectional heterogeneity in firm growth

Here we present a simple framework to analyse the cross-sectional heterogeneity in firm growth rates. Consider a population of firms with the $i$th firm producing output according to some standard production function:

$$y_{it} = f(A_{it}, K_{it}, L_{it}),$$

with $y$ the output/sales, $A$ the level of technical efficiency, and $K$ and $L$ inputs of capital and labour. We assume that the firm is subjected to a variety of shocks both real and nominal: some, idiosyncratic, some, industry specific and some, economy wide. So the total shock experienced in period $t$ by the $i$th firm is

$$\varepsilon_{it} = \varepsilon_{it} + \zeta_{jt} + \eta_{it},$$

with $\varepsilon_{it}$ the firm specific shock, $\zeta_{jt}$ the $j$th industry shock and $\eta_{it}$ the economy wide disturbance. While idiosyncratic and industry specific shocks are not likely to have obvious cyclical patterns, aggregate shocks, whether real or nominal, can be expected to be at the business cycle frequencies.

The growth rate of the individual firm about its mean can be thought of as made up of firm specific responses to shocks:

$$g_{it} = \bar{g}_{it} + \varepsilon_{it} + \kappa_{it}\zeta_{jt} + \lambda_{it}\eta_{it},$$

For the $i$th firm, in period $t$, $\lambda_{it}$ is its response to the aggregate shock, and $\kappa_{it}$, its response to industry shocks. In any period, the set of shocks and firm level responses generate the probability distribution of firm growth rates. Denote this by $h_{t}(g)$. Our primary interest is in characterising and exploring patterns in $h_{t}(g)$ over time and how it might vary with the business cycle.

We consider two ways in which firm specific responses to shocks could be characterised. One possibility is that aggregate shocks modify the systematic relationship between the growth of the firm and its characteristics. Such changes (in the systematic component of firm growth) may drive the cross-sectional distribution of growth rates over the business cycle. For example, following up the attention paid to the growth-size relationship in the industrial economics literature on firm growth, we could usefully ask whether large firms grow faster than small firms in recoveries, and if the converse was true in recessions. Are cycle related changes in the growth-size relationship (combined with changes in the size distribution itself) the main driving forces behind the evolution of cross-sections over time?

The other possibility is that growth responses of firms at business cycle frequencies do not have as much to do with features such as firm size, but are better explained by the differential impact of aggregate shocks across the population of firms. Suppose that all firms were affected in the same way by an aggregate shock (so $\lambda_{it} = \lambda$, for all $i$).

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3 We are being deliberately agnostic about what the aggregate shocks might be.

4 See also Adadir and Talmain (2000). $\lambda_{it}$ can be thought akin to the $\beta$ of the corporate finance literature. As a first approximation we can assume that each firm responds in the same way to the industry level shock. Although in this paper we do not condition on industry, results for the UK reported in Higson et al. (2001) show that including an industry dummy does not alter the results.
In this case the cross-sectional distribution would experience a spread preserving shift in the mean of the distribution. To the extent to which firms are differentially affected by an aggregate shock, we would expect that the higher moments of the distribution to alter at business cycle frequencies. It is a commonplace of the finance literature that some types of firm are cyclical and others not.

In formal terms, denote variables relevant to the growth rate of the firm by $Z_t$ (a key element of which is firm size, $s_t$), and the probability distribution of growth rates conditional on $Z_t$ by $h_t(g|Z_t)$. If the way firm growth rates, $g_{it}$, depend on firm specific features is represented by $f_i(Z_t)$, observed growth rates of firms are given by $E(f_i(Z_t) + v_t)$ where $v_t$ is that portion of the growth rate that cannot be ascribed to any systematic firm specific influence. Denote the distribution of $v_t$ by $l_i(v_t)$.

The dynamics of the cross-section of growth rates, $h_t(g)$, depends on changes in $h_t(g|Z_t)$, and changes in the distribution of $Z_t$ ($m_t(Z_t)$) across the population of firms. The conditional distribution $h_t(g|Z_t)$, is comprised of the conditional mean growth, $f_i(Z_t)$, and the residual, $l_i(v_t|Z_t)$. The relative importance of these two components as they evolve over the cycle will indicate the balance of evidence between our two hypotheses.

In this paper, we specialise $Z_t$ to firm size, $s_t$, limiting consideration to a single key determinant of firm growth found in the literature. In those terms, the moments of cross-sectional distribution of growth rates are driven by, on the one hand, changes in the growth-size relationship, $f_i(s_t)$ and on the other by the distribution of the disturbance, $l_i(v_t)$. If the cross-sectional dynamics of the growth rate distribution is driven by aggregate shocks changing the growth-size relationship, we would expect to see the growth-size relationship change over the business cycle. By examining changes in $f_i(s_t)$ we can draw inferences on whether, for example, small firms grow faster relative to large in recoveries, and large firms contract less relative to small in recessions. If these kinds of changes are the major driving force, $f_i(s_t)$ will dominate $h_t(g)$. If on the other hand, the influence of aggregate shocks on the growth of firms is independent of size we would expect $l_i(v_t)$ to dominate $h_t(g)$.

3. Empirical evidence for the US

In this section we turn to an examination of some cross-sectional evidence on the US postwar economy using the COMPUSTAT database of quoted firm accounts for
1950–1999. We have adopted the following procedures:

1. For all firms that are on the database we have extracted gross nominal sales. No filtering according to industry was used.
2. All sales were deflated by the gdp chained deflator and logarithmic changes computed and multiplied by 100.
3. A cut-off of ±25 percent growth rate was then used to truncate the data and exclude outliers. 9
4. Continuous growth rates were computed for the largest third of firms, for the smallest third of firms and the third in the middle.
5. The moments of the cross-sectional distributions of growth rates in each year were computed.

Fig. 1 provides a visual representation of the cross-sections of all firms for 1951–1999. It is a contour map of the kernel densities fitted to yearly cross-sections of continuous growth rates. The density estimates of the cross-sectional distribution of real growth rates were generated with a Gaussian kernel and an automatic, Silverman bandwidth. These non-parametric estimates of the cross-sectional distributions should be regarded as largely impressionistic. Nevertheless, they do suggest some interesting features of the evolution of cross-sectional growth rates in the postwar

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9 Alternative cut-offs ranging to ±100% did not change results. We have not explicitly addressed the issue of internal growth as against growth via mergers.
period. Because the average growth rate of the US economy was positive in the postwar period, the central mass of the distribution lies to the right of zero. There is considerable dispersion in performance reflecting a considerable amount of churning at the sectorial and individual firm level. The central mass also moves with the aggregate growth rate of the macroeconomy. What is also striking is that these fluctuations in the mean are associated with contortions in the whole cross-sectional distribution. This shows up clearly in the contour map of Fig. 1. This picture suggests that there are significant deviations from normality and that these deviations are associated with the aggregate business cycle.\(^\text{10}\)

In Fig. 2 we have plotted the moments of the growth rates cross-sections against the annual rate of growth of gdp (shown as a bar in the plot). There is a close visual correspondence between the mean and median of the cross-sections and the aggregate economy. The standard deviation appears to have an upward trend, at least until the 1970s and skewness seems to be counter-cyclical. Excess kurtosis appears to be positively correlated with the business cycle but also there is an indication of a break in the shape of the distribution in the mid-1970s. Whereas in the first half of the sample excess kurtosis is positive, it is negative in the second half of the sample.

In Table 1, we provide simple regressions of the cross-sectional moments on current and lagged gdp growth rates. These regressions indicate considerable co-movement and persistence in the cross-sectional moments and the aggregate growth rate of the US economy. Both the standard deviation and skewness are negatively correlated with the aggregate growth rate while kurtosis is positively correlated. The spread (standard deviation) of the distribution narrows during upswings in the economy, but a negative skewness is observed and there is an increase in kurtosis. In an economic downturn the spread of performance widens, and a positive skewness is observed.

The use of unweighted growth rates means that we are treating all firms the same, but there is enormous variation in the size of firms measured by sales. If aggregate output is dominated by a few very large firms then the behaviour of smaller firms—though of interest to industrial economists—may tell us little about the aggregate economy. In Tables 2–4 we show the relationship between the aggregate economy and the cross-sectional moments when we divide the growth rates into three classes given by the size of the firm in the previous year. We note that there is still a significant relationship between the aggregate economy and the central moments calculated for large, medium and small firms.

We can summarise our findings so far as follows:

- There is considerable cross-sectional heterogeneity in the growth rates of US firms which varies with the business cycle.

\(^{10}\) To establish whether these cross-sectional distributions represent significant deviations from normality, we used the Bowman and Shenton (1975) statistic to test whether the cross-sectional series is normally distributed. The omnibus test statistic measures the difference of the skewness and kurtosis of the series with those from the normal distribution. This is simply computed as \(N/6(sk^2 + \frac{1}{4}(k - 3)^2)\); \(sk\) is the skewness, \(k\) is the kurtosis, and \(N\) the number of firms in each year. Where the statistic is distributed as a \(\chi^2(2)\). The omnibus tests suggested that there is significant departure from normality for almost the whole of the sample. Further details of these tests can be obtained from the authors.
Fig. 2. Moments of truncated \((-25, +25)\) cross sections: 1951–99 against GDP growth (BAR).

- The spread (standard deviation) of growth rates has an upward trend; it also appears to have a counter-cyclical tendency; the distribution narrows in an upswing and widens in a downturn.
- The skewness of growth rates is counter-cyclical. In recessions there is a positive skew to the distribution; in upturns, a negative skew.
Table 1
All firms. Sample: 1952–1999, OLS estimates. t-statistics below coefficient estimates. LM(2) is a Lagrange Multiplier test for serial correlation distributed as $\chi^2$

<table>
<thead>
<tr>
<th>Mean</th>
<th>Median</th>
<th>SD</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.5207</td>
<td>-0.1834</td>
<td>18.1379</td>
<td>-0.0344</td>
</tr>
<tr>
<td>-1.12</td>
<td>-0.38</td>
<td>2.46</td>
<td>-0.82</td>
<td>1.61</td>
</tr>
<tr>
<td>0.4301</td>
<td>0.3740</td>
<td>0.9100</td>
<td>0.2772</td>
<td>0.8056</td>
</tr>
<tr>
<td>Variable</td>
<td>3.29</td>
<td>2.85</td>
<td>15.33</td>
<td>2.04</td>
</tr>
<tr>
<td>$\Delta Ln(gdp)$</td>
<td>0.9443</td>
<td>0.9302</td>
<td>-3.6070</td>
<td>-0.0596</td>
</tr>
<tr>
<td>11.33</td>
<td>11.03</td>
<td>-6.83</td>
<td>-7.82</td>
<td>8.23</td>
</tr>
<tr>
<td>$\Delta Ln(gdp_{t-1})$</td>
<td>-0.1450</td>
<td>-0.0991</td>
<td>1.1965</td>
<td>0.0022</td>
</tr>
<tr>
<td>-1.06</td>
<td>-0.71</td>
<td>2.18</td>
<td>0.21</td>
<td>-3.98</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.743</td>
<td>0.733</td>
<td>0.860</td>
<td>0.596</td>
</tr>
<tr>
<td>LM(2)</td>
<td>0.026</td>
<td>0.042</td>
<td>1.433</td>
<td>1.255</td>
</tr>
</tbody>
</table>

Table 2
Top third of firms by size in previous year. Sample: 1952–1999, OLS estimates. t-statistics below coefficient estimates. LM(2) is a Lagrange Multiplier test for serial correlation distributed as $\chi^2$

<table>
<thead>
<tr>
<th>Mean</th>
<th>Median</th>
<th>SD</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.142</td>
<td>-0.0485</td>
<td>8.2164</td>
<td>-0.0239</td>
</tr>
<tr>
<td>-0.83</td>
<td>-0.07</td>
<td>2.81</td>
<td>-0.99</td>
<td>2.69</td>
</tr>
<tr>
<td>0.3402</td>
<td>0.2919</td>
<td>0.8757</td>
<td>0.3027</td>
<td>0.6354</td>
</tr>
<tr>
<td>Variable</td>
<td>2.39</td>
<td>2.04</td>
<td>12.5</td>
<td>2.13</td>
</tr>
<tr>
<td>$\Delta Ln(gdp)$</td>
<td>0.3196</td>
<td>1.1559</td>
<td>-1.2848</td>
<td>-0.0196</td>
</tr>
<tr>
<td>9.91</td>
<td>8.8</td>
<td>-5.99</td>
<td>-4.22</td>
<td>4.23</td>
</tr>
<tr>
<td>$\Delta Ln(gdp_{t-1})$</td>
<td>-0.0311</td>
<td>-0.0595</td>
<td>0.2104</td>
<td>0.0007</td>
</tr>
<tr>
<td>-0.6</td>
<td>-0.3</td>
<td>0.94</td>
<td>0.13</td>
<td>-2.23</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.687</td>
<td>0.631</td>
<td>0.808</td>
<td>0.334</td>
</tr>
<tr>
<td>LM(2)</td>
<td>0.24</td>
<td>0.212</td>
<td>1.212</td>
<td>0.84</td>
</tr>
</tbody>
</table>

- Kurtosis is positively related to the cycle. In an upswing, the tendency for the distribution of growth rates to narrow is also associated with more peakedness so that there is more mass about the mean as well as the tails.

4. Systematic and stochastic growth of firms over the business cycle

In the previous section we have identified some stylised facts about the cross section of growth rates. Now we turn to an examination of the data to decide between the two mechanisms set out in Section 2 above, nesting them in a simple model of firm growth. The standard approach to the cross-sectional growth-size relationship has been
Table 3
Middle third of M@rms by size in previous year. Sample: 1952–1999, OLS estimates. \(t\)-statistics below coefficient estimates. LM(2) is a Lagrange Multiplier test for serial correlation distributed as \(\chi^2\).

<table>
<thead>
<tr>
<th>Regression of cross-sectional moments on gdp growth</th>
<th>Mean</th>
<th>Median</th>
<th>SD</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.1921</td>
<td>-0.2808</td>
<td>6.9355</td>
<td>-0.0145</td>
<td>0.3217</td>
</tr>
<tr>
<td>Lagged dependent</td>
<td>-1.28</td>
<td>-0.59</td>
<td>3.33</td>
<td>-0.71</td>
<td>2.46</td>
</tr>
<tr>
<td>(\Delta Ln(gdp))</td>
<td>0.3932</td>
<td>0.3191</td>
<td>0.8664</td>
<td>0.3435</td>
<td>0.544</td>
</tr>
<tr>
<td>Variable</td>
<td>3.06</td>
<td>2.54</td>
<td>12.15</td>
<td>2.47</td>
<td>4.18</td>
</tr>
<tr>
<td>(\Delta Ln(gdp_{t-1}))</td>
<td>11.8</td>
<td>11.41</td>
<td>-4.67</td>
<td>-4.59</td>
<td>4.75</td>
</tr>
<tr>
<td>(\Delta Ln(gdp_{t-1}))</td>
<td>-0.0321</td>
<td>-0.0258</td>
<td>0.3612</td>
<td>0.0024</td>
<td>-0.0103</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.363</td>
<td>0.209</td>
<td>2.164</td>
<td>1.018</td>
<td>4.267</td>
</tr>
</tbody>
</table>

LM(2)                                            | 0.761 | 0.749 | 0.793 | 0.353 | 0.399 |

Table 4
Smallest third of M@rms by size in previous year. Sample: 1952–1999, OLS estimates. \(t\)-statistics below coefficient estimates. LM(2) is a Lagrange Multiplier test for serial correlation distributed as \(\chi^2\).

<table>
<thead>
<tr>
<th>Regression of cross-sectional moments on gdp growth</th>
<th>Mean</th>
<th>Median</th>
<th>SD</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.1687</td>
<td>-0.1679</td>
<td>6.6012</td>
<td>0.006</td>
<td>0.6032</td>
</tr>
<tr>
<td>Lagged dependent</td>
<td>-1.06</td>
<td>-0.35</td>
<td>2.35</td>
<td>0.3</td>
<td>4.75</td>
</tr>
<tr>
<td>(\Delta Ln(gdp))</td>
<td>0.5145</td>
<td>0.4472</td>
<td>0.894</td>
<td>0.1566</td>
<td>0.2553</td>
</tr>
<tr>
<td>Variable</td>
<td>4.17</td>
<td>3.54</td>
<td>13.09</td>
<td>1.14</td>
<td>1.81</td>
</tr>
<tr>
<td>(\Delta Ln(gdp_{t-1}))</td>
<td>0.3173</td>
<td>0.9474</td>
<td>-1.3974</td>
<td>-0.0236</td>
<td>0.0348</td>
</tr>
<tr>
<td>(\Delta Ln(gdp_{t-1}))</td>
<td>10.92</td>
<td>11.15</td>
<td>-6.34</td>
<td>-6.04</td>
<td>3.54</td>
</tr>
<tr>
<td>(\Delta Ln(gdp_{t-1}))</td>
<td>-0.0737</td>
<td>-0.1792</td>
<td>0.5553</td>
<td>-0.0018</td>
<td>-0.0018</td>
</tr>
<tr>
<td>(\Delta Ln(gdp_{t-1}))</td>
<td>-1.63</td>
<td>-1.29</td>
<td>2.43</td>
<td>-0.37</td>
<td>-0.17</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.733</td>
<td>0.74</td>
<td>0.813</td>
<td>0.441</td>
<td>0.237</td>
</tr>
<tr>
<td>LM(2)</td>
<td>0.245</td>
<td>0.295</td>
<td>2.295</td>
<td>0.101</td>
<td>1.512</td>
</tr>
</tbody>
</table>

To estimate a first order Galton–Markov model which generalises the Gibrat model to allow past size to influence current size:

\[
z_{it} = \beta z_{it-1} + \varepsilon_{it}, \tag{4}\]

where \(z_{it}\) is the deviation from the log of size of firm \(i\) at time \(t\) from the mean of the logs of sizes of firms at time \(t\), \(\beta\) is the size growth coefficient and \(\varepsilon_{it}\) is the disturbance. Gibrat’s law holds if \(\beta\) is close enough to unity. A value of \(\beta < 1\) would suggest regression towards the mean with small firms, on average, growing faster than large; a value of \(\beta > 1\) would suggest that large firms, on average, grow faster than small. Eq. (4) is then equivalent to

\[
z_{it} - z_{it-1} = (\beta - 1)z_{it-1} + \varepsilon_{it}. \tag{5}\]
Empirical work on direct tests of Gibrat’s law started in the 1950s and has generally found that it serves as an approximation (Scherer, 1980; Hart and Oulton, 1995). But earlier studies (Samuels, 1965; Singh and Whittington, 1968; Prais, 1976) found a tendency for large firms to grow faster than small, while later studies (Kumar, 1985; Acs and Audretsch, 1990; Hall, 1987; Evans, 1987a, b; Dunne et al., 1989; Dunne and Hughes, 1994) found a tendency for small firms to grow faster.\(^{11}\) In a time series study, Geroski et al. (1997) established that growth patterns of UK firms are captured by a random walk with a drift; growth rates are not persistent.

If there is no significant serial correlation and if \(\hat{\varepsilon}_{it}\) is independent of \(z_{it-1}\), then the variance of growth rates evolves according to

\[
V(\Delta z_{it}) = (\beta_t - 1)^2 V(z_{it-1}) + V(\varepsilon_{it}).
\]

The third central moment, which measures skewness evolves as

\[
E(\Delta z_{it} - \bar{z}_{it})^3 = (\beta_t - 1)^3 E(z_{it-1} - \bar{z}_{it-1})^3 + E(\varepsilon_{it} - \bar{\varepsilon}_{it})^3.
\]

It is easy to see that the coefficient of skewness, (7) normalised by the standard deviation to be dimensionless, evolves as

\[
sk(\Delta z_{it}) = [(\beta_t - 1)^3 sk(z_{it-1})\sigma(z_{it-1})^3 + sk(\varepsilon_{it})\sigma(\varepsilon_{it})^3] \frac{1}{\sigma(\Delta z_{it})^3}.
\]

We estimate the Galton process in (5) by OLS, for successive pairs of years using data on firms that survive from one year to the next. The decompositions of the higher moments of the growth rate distributions given by (6) and (8) are plotted in Figs. 3 and 4. The constant captures the linear effect of aggregate shocks.

\(^{11}\) In the UK studies have found standardised estimates of \(\beta\) in the range 0.77–1.12; Prais (1976), Dunne and Hughes (1994), Hart and Oulton (2001).
Fig. 5 shows the distinct ways in which Gibrat’s law is violated over the up and down phases of the business cycle. A regression of the estimated $\beta$ on GDP growth gave the following result.

$$
\beta_t = 0.1565 + 0.3897\beta_{t-1} + 0.4509\beta_{t-2} + 0.09151\Delta \text{Ln}(\text{gdp}_t),
$$

with

$$
(1.52) \quad (4.03) \quad (4.89) \quad (4.59)
$$

$$
R^2 = 0.663, \quad \text{LM}(2) = 3.47.
$$

This suggests a pro-cyclical pattern to the $\beta$ coefficient, suggesting that in the expansion phase, smaller firms on average grow faster than large, while in a contraction, this tendency is tempered in favour of large firms.

What is the force driving the upward drift of the cross-sectional dispersion of growth rates? In the decomposition (6), the first term on the r.h.s. represents a systematic mechanism with two components that work together upon the dispersion of growth rates: the relationship between growth and size works with the distribution of firm sizes. Fig. 3 clearly shows that it is the variance of the purely idiosyncratic disturbance that has driven the increasing in variance of growth rates. Empirical studies of firm growth have established that growth rates of firms cannot be predicted well by size, or indeed, other explanatory factors. What is suggested here is that the degree of unpredictability, the volatility in the growth rates of firms, has increased over time.

What is the force driving the counter-cyclical skewness in the growth rate distribution? The pattern noted in Section 3 implies that there are relatively more firms with above average growth rates in recoveries than in recessions. This could be due to changes in the growth-size behaviour depending on the business cycle. In recoveries,

12 Except for the extreme business cycle years we do not find that $\beta$ departs significantly from unity.
the median growth rate may be higher than the mean because a larger number of small firms grow faster than the few larger firms, while in a recession, the median may be lower than the mean growth rate, due to many small firms having lower growth rates than those of larger firms. If this is the main driving force, we would expect to see $\beta_i$ to be counter-cyclical and for the first term to dominate the total in (8). Figs. 3 and 4 clearly shows the dominance of the second term, and this implies that it is the aggregate shocks that hold the key to the observed pattern.

One important point about short run growth is that the transitory components may dominate permanent components in the short run. Transitory components bias the OLS estimate of $\beta_i$ downwards; firms that are of transitorily low size will show higher growth rates than firms that are of transitorily high size. It is possible to treat this as an error in variables problem, as Hart and Oulton (1995) have, and control for the transitory influences by estimating a reverse regression to get a compromise estimate of $\beta_i$ (the geometric mean of the standard coefficient and the inverse of the reverse regression coefficient).\(^{13}\) In Fig. 5 we have also presented the estimates of the compromise coefficient. We find that this coefficient is quite close to the standard Galtonian coefficient. It is clear that the transitory components are not responsible for the increasing dispersion or the counter-cyclical skew of the growth rate distribution.

\(^{13}\)See Prais (1958) and Maddala (1992). This also assumes there is zero correlation between errors in dependent and independent variables. It may be that transitory components are larger among the small firms than the large.
5. The role of aggregate shocks

We now turn to a closer examination of the effect of aggregate shocks on firms differentiated by growth rates. To do this we examine the time series properties of our data set. However, we cannot rely upon a sufficient run of growth rates for individual firms. The population of quoted companies is subject to continuous attrition through takeover and merger as well as new firms entering or firms exiting through bankruptcy. Instead we select percentiles of the order statistics. This provides us with 99 time series made up of the growth rates of firms at each percentile in each year. Since the growth rate of the firm is very weakly correlated with size, each percentile is going to be largely a random draw from the size distribution of firms.

We have plotted the deciles of these growth rates in Fig. 6. Even for the top and bottom deciles there appears to be a cyclical pattern. Clearly aggregate shocks have a pervasive effect on firms growing at different rates. However, if we calculate the correlation matrix for the deciles and include the rate of growth of gdp, as shown in Table 5, we note that the correlation with gdp growth diminishes as we move away from the 5 and 6th deciles with the weakest correlation between the 1st decile of (contracting) firms and gdp. To the extent to which the aggregate growth rate captures aggregate shocks, the smaller effect on the high and low deciles could imply the sort of distortions in the cross-sectional distributions over business cycles that we have observed. The central mass of the growth rate distribution responds more strongly to the aggregate shock than the tails. So a negative shock moves the central mass closer to the left of the distribution leaving the right tail behind and generates positive skewness. A positive shock shifts the central mass to the right, closer to the group of rapidly growing firms and away from the group of declining firms. So negative skewness results.

We can refine the analysis further if we look more closely at the affect of aggregate shocks at the percentile level. In Table 6 we report GMM estimates of the 5th, 30th,
Table 5
Correlation matrix of deciles and GDP growth, 1951–1999

<table>
<thead>
<tr>
<th>Decile 1st</th>
<th>Decile 2nd</th>
<th>Decile 3rd</th>
<th>Decile 4th</th>
<th>Decile 5th</th>
<th>Decile 6th</th>
<th>Decile 7th</th>
<th>Decile 8th</th>
<th>Decile 9th</th>
<th>%gdp</th>
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<tbody>
<tr>
<td>1.000</td>
<td>0.980</td>
<td>0.954</td>
<td>0.921</td>
<td>0.870</td>
<td>0.791</td>
<td>0.704</td>
<td>0.590</td>
<td>0.473</td>
<td>0.655</td>
</tr>
<tr>
<td>0.980</td>
<td>1.000</td>
<td>0.992</td>
<td>0.974</td>
<td>0.936</td>
<td>0.869</td>
<td>0.791</td>
<td>0.688</td>
<td>0.572</td>
<td>0.721</td>
</tr>
<tr>
<td>0.954</td>
<td>0.992</td>
<td>1.000</td>
<td>0.994</td>
<td>0.970</td>
<td>0.918</td>
<td>0.852</td>
<td>0.759</td>
<td>0.647</td>
<td>0.761</td>
</tr>
<tr>
<td>0.921</td>
<td>0.974</td>
<td>0.994</td>
<td>1.000</td>
<td>0.989</td>
<td>0.951</td>
<td>0.895</td>
<td>0.812</td>
<td>0.705</td>
<td>0.787</td>
</tr>
<tr>
<td>0.870</td>
<td>0.936</td>
<td>0.970</td>
<td>0.989</td>
<td>1.000</td>
<td>0.984</td>
<td>0.947</td>
<td>0.881</td>
<td>0.787</td>
<td>0.805</td>
</tr>
<tr>
<td>0.791</td>
<td>0.869</td>
<td>0.918</td>
<td>0.951</td>
<td>0.984</td>
<td>1.000</td>
<td>0.986</td>
<td>0.943</td>
<td>0.866</td>
<td>0.812</td>
</tr>
<tr>
<td>0.704</td>
<td>0.791</td>
<td>0.852</td>
<td>0.895</td>
<td>0.947</td>
<td>0.986</td>
<td>1.000</td>
<td>0.981</td>
<td>0.929</td>
<td>0.803</td>
</tr>
<tr>
<td>0.590</td>
<td>0.688</td>
<td>0.759</td>
<td>0.812</td>
<td>0.881</td>
<td>0.943</td>
<td>0.981</td>
<td>1.000</td>
<td>0.970</td>
<td>0.772</td>
</tr>
<tr>
<td>0.473</td>
<td>0.572</td>
<td>0.647</td>
<td>0.705</td>
<td>0.787</td>
<td>0.866</td>
<td>0.929</td>
<td>0.970</td>
<td>1.000</td>
<td>0.727</td>
</tr>
<tr>
<td>0.655</td>
<td>0.721</td>
<td>0.761</td>
<td>0.787</td>
<td>0.805</td>
<td>0.812</td>
<td>0.803</td>
<td>0.772</td>
<td>0.727</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table 6
The effects of aggregate GDP on percentiles of cross-sectional growth rates: GMM estimates of the model:

\[(1 - \lambda_1 L - \lambda_2 L^2) p_i = \beta_0 + \beta_1 \ln(gdp_t)\]

<table>
<thead>
<tr>
<th>Percentile</th>
<th>(\beta_0)</th>
<th>(\lambda_1)</th>
<th>(\lambda_2)</th>
<th>(\beta_1)</th>
<th>(R^2)</th>
<th>DW</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-9.68803</td>
<td>0.549615</td>
<td>0.049033</td>
<td>0.988155</td>
<td>0.687</td>
<td>1.696</td>
<td>0.416</td>
</tr>
<tr>
<td></td>
<td>-17.48</td>
<td>7.92</td>
<td>0.79</td>
<td></td>
<td>7.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>-5.84198</td>
<td>0.361554</td>
<td>0.02755</td>
<td>1.500577</td>
<td>0.701</td>
<td>1.335</td>
<td>0.402</td>
</tr>
<tr>
<td></td>
<td>-14.42</td>
<td>5.74</td>
<td>0.42</td>
<td></td>
<td>14.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>-1.85065</td>
<td>0.352367</td>
<td>0.010346</td>
<td>1.291015</td>
<td>0.663</td>
<td>1.666</td>
<td>0.521</td>
</tr>
<tr>
<td></td>
<td>-3.21</td>
<td>4.97</td>
<td>0.16</td>
<td></td>
<td>12.44</td>
<td></td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>2.111926</td>
<td>0.379314</td>
<td>-0.02684</td>
<td>1.124021</td>
<td>0.663</td>
<td>1.735</td>
<td>0.671</td>
</tr>
<tr>
<td></td>
<td>2.29</td>
<td>4.73</td>
<td>-0.42</td>
<td></td>
<td>10.94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>95</td>
<td>8.090188</td>
<td>0.334958</td>
<td>0.176554</td>
<td>0.56316</td>
<td>0.662</td>
<td>1.635</td>
<td>0.284</td>
</tr>
<tr>
<td></td>
<td>5.10</td>
<td>5.10</td>
<td>4.25</td>
<td></td>
<td>8.70</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

50th, 70th and 95th percentiles regressed on GDP growth. We estimate the relationship:

\[(1 - \lambda_1 L - \lambda_2 L^2) p_i = \beta_0 + \beta_1 \ln(gdp_t)\]

where \(p_i\) is the \(i\)th percentile and \(L\) is the lag operator. \(t\)-statistics are shown below parameter estimates. \(J\) is the \(p\)-value for the Newey–West \(J\)-statistic with 5 over-identifying restrictions. The instrument set includes all lagged variables plus current and lagged values of short- and long-term interest rates as well as Tobin’s \(Q\) (obtained from Smithers and Wright, 2000). The effect of the aggregate is lower for the highest and lowest percentiles. This is shown visually in Fig. 7 where we have plotted the coefficient, \(\beta_1\) for the 1st–99th percentile; ±2 standard errors are also plotted. The shape of this profile confirms what we have found but with some refinements. The aggregate shock seems to be at its strongest at the 28th decile and declines rapidly away towards the lowest deciles (firms that are declining). The effect declines more or less monotonically as we move up the percentiles until the 70th percentile after which it falls away rapidly for the fastest growing firms.
This cross-sectional pattern to the correlation between the aggregate economy and growth rates provides a heuristic explanation for some of the cross-sectional features we have observed empirically. Note that we have found that the spread and skewness are negatively correlated with the business cycle, while kurtosis is positively correlated. Since aggregate shocks impact on firms growing at different rates according to the pattern shown in Fig. 7, we can explain the effects on the higher moments. Suppose that there is a positive aggregate shock and we divided the growth rate percentiles into 4 groups: percentiles 2–8, 9–30, 31–85 and 85–99. The middle two groups are shunted closer to the fourth group and away from the first group, which tends to generate negative skewness. Equally the second group also moves closer to the third, so this causes the standard deviation to decline during the economic upswing. At the same time the concentration of mass in the centre of the distribution and in the tails increases kurtosis. Nevertheless, this still leaves unexplained why aggregate shocks affect firms growing at different rates in different ways.

6. Conclusions

Our empirical results provide some lessons for both macro and industrial economists. We offer another dimension to the analysis of business cycles by using a database of firm sales from 1950 to 1999 for the USA. This allows us to observe how the cross-section of sales and its growth rate evolves over long periods of time and varies with the business cycle. We find that there is a cyclical pattern to the higher moments of the cross-sectional distribution.

We have investigated two explanations for these stylised facts drawing in part on the literature in industrial economics concerning the relationship between the size of the firm and its growth. We find that the size-growth relationship contributes little to an explanation of the stylised facts. However, we find that aggregate shocks have different correlations with firms depending upon how rapidly they are declining or expanding. Firms in the middle of the growth distribution are more correlated with aggregate
shocks so an adverse shock shifts the mass of the distribution towards the left, leaving rapidly growing M@rms behind so that positive skewness emerges. With a positive shock, the mass of the distribution shifts to the right, in this case leaving declining M@rms behind and generating negative skewness. Corporate policy has generally focussed on the growth performance of small firms relative to large, over the business cycle and over the longer horizon. If firms most susceptible to fluctuations in the economic climate are those in the middle range in terms of growth, then the objective of modulating the amplitude of the business cycle is best served by shifting the focus of policy to firms in the middle range of growth.

There are a number of aspects of this analysis that requires more attention. We have identified both size and growth as important aspects but we have ignored the industrial dimension in the sense that we do not allow for products or sectors that may be more or less cyclical because of the structure of demand and its sensitivity to shocks to permanent income. We have also been vague about the precise form of the aggregate shocks. It would be useful to explore what sort of aggregate supply or demand shocks contribute to the cross-sectional dynamics we observe. There is also an extensive literature on the credit channel (Greenwald and Stiglitz, 1993) and how the size of the firm may matter for how monetary policy impacts on firm behaviour. The extent to which the distortions to the cross-sectional distributions we find are due to supply and/or demand shocks remains an unresolved issue.

Acknowledgements

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