Firms can enhance the reliability of their supply through process improvement and overproduction. In decentralized supply chains, however, these mitigating actions may be the supplier’s responsibility yet are often not contractible. We show that wholesale price contracts, despite their simplicity, can perform well in inducing reliable supply, and we identify when and why they perform well. This could explain the widespread use of wholesale price contracts in business settings with unreliable supply. In particular, we investigate how the performance of wholesale price contracts depends on the interplay between the nature of supply risk and the type of procurement process. Supply risk is classified as random capacity when events such as labor strike disrupt the firm’s ability to produce, or as random yield when manufacturing defects result in yield losses. The procurement process is classified as control when the buyer determines the production quantity, or as delegation when instead the supplier does. Analyzing the four possible combinations, we find that for random capacity, irrespective of the procurement process type, contract performance monotonically increases with the supplier’s bargaining power; thus, wholesale price contracts perform well when the supplier is powerful. However, this monotonic trend is reversed for random yield with control: in that case, wholesale price contracts perform well when instead the buyer is powerful. For random yield with delegation, wholesale price contracts perform well when either party is powerful.

Key words: supplier reliability; random capacity; random yield; wholesale price contracts

1. Introduction

In an era of outsourcing and globalization, reliability of supply is an increasingly important aspect of supply chain management. Supply disruptions are often classified as either random capacity or random yield (Wang et al. 2010). Random capacity disruptions affect the supplier’s production capacity, as when there is a labor strike or machine breakdown. By contrast, random yield disruptions affect the supplier’s production yield—for example, when firms in the semiconductor sector or in high-tech electronics suffer from manufacturing defects. A critical aspect of such disruptions is that suppliers can exert \textit{ex ante} effort to improve their processes and thus their reliability. For instance, suppliers can proactively invest in labor relations to avoid strikes (Lexology 2010), or undertake projects to improve yield (Zhang and Armer 2013). However, such effort is often costly and not contractible. Since suppliers capture only a fraction of the overall margin, they do not fully internalize the consequences of suboptimal effort, and this leads to a \textit{moral hazard} problem.

There is a vast literature on coordinating contracts that deals with aligning incentives in a supply chain. Indeed, we verify the conventional wisdom that the moral hazard problem just mentioned can be resolved by using contracts that incorporate a penalty for the non-delivery of goods and make the supplier the residual claimant. However, such coordinating penalty contracts can be quite difficult to implement. For instance, the supplier’s limited liability may induce an upper limit on the imposed penalty, and thus impede coordination (Sappington 1983). Another problem is that suppliers may view penalties as unfair and hence resist agreeing to such contracts (LinkedIn Discussion 2011). Therefore, firms often prefer to use wholesale price contracts that are easier to implement (e.g., Kalkanci et al. 2011).

For instance, wholesale price contracts are often employed in semiconductor manufacturing, where yield issues are a serious concern. In work done jointly with managers at KLA-Tencor Corporation
and based on interviews with ten “fabless” companies, Chatterjee et al. (1999) report the widespread use of wholesale price contracts for outsourcing production of integrated circuits—which remains the preferred mode of contracting nowadays in the semiconductor industry. In conversations with the authors, a leading semiconductor foundry based in East Asia acknowledged that it typically uses wholesale price contracts regardless of the buyer and duration of the interaction, despite concerns about improving yield. Wholesale price contracts are also frequently used in the agriculture industry. In floriculture, for instance, wholesalers often outsource the cultivation of flowers and plants to contract growers (MacDonald et al. 2004). Due to the nature of floriculture, contract growers experience yield issues. Nevertheless, MacDonald et al. (2004) document that wholesale price contracts are typically used.

Motivated by this evidence that wholesale price contracts are often preferred, we investigate when and why such contracts perform well—that is, result in high supply chain efficiency. We examine in particular the four cases typically observed in practice, which are characterized by the combination of two types of supply risk and two types of procurement process. As described previously, the two types of supply risk correspond to random capacity and random yield; the two types of procurement process correspond to control and delegation.

The procurement process is classified as control if the buyer determines the production quantity or as delegation if the supplier does. As with the type of supply risk, the type of procurement process is often determined by the business context. For instance, the control scenario is observed in the floriculture industry. Although contract growers experience yield issues, the wholesalers provide them with the inputs of seedlings, fertilizers, and chemicals—thus effectively controlling the production quantity (MacDonald et al. 2004). By contrast, we observe the delegation scenario in the semiconductor industry. Semiconductor manufacturers, such as Samsung and TSMC, often experience severe yield issues but can identify most defects through internal testing. These manufacturers therefore inflate their production quantities beyond the buyer’s order, so that they can meet customer demand without shipping any defects.

With regard to the four cases considered, we summarize our main findings in the $2 \times 2$ matrix in Figure 1. The two columns correspond to the two types of supply risk, and the two rows to the two types of procurement process. For each of the four cases, the figure indicates whether the efficiency of the wholesale price contract is increasing, decreasing, or V-shaped with respect to the supplier’s bargaining power.

<table>
<thead>
<tr>
<th>Type of Supply Risk</th>
<th>Procurement Process Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Capacity</td>
<td>Control: Increasing</td>
</tr>
<tr>
<td></td>
<td>Delegation: Increasing</td>
</tr>
<tr>
<td>Random Yield</td>
<td>Control: Decreasing</td>
</tr>
<tr>
<td></td>
<td>Delegation: V-Shaped</td>
</tr>
</tbody>
</table>

Specifically, in the random capacity case—irrespective of the procurement process type—the efficiency of the wholesale price contract increases monotonically with the supplier’s bargaining power (and with the wholesale price). The reason is that a more powerful supplier has a bigger margin and thus a greater incentive to invest effort and thereby improve efficiency. This suggests that, if the supplier is powerful, a wholesale price contract may be preferred to more complex contracts that perform better in theory but are difficult to implement in practice.

In the random yield case, however, if the buyer controls the production quantity decision then the monotonic trend in efficiency associated with the wholesale price contract is reversed. As before, a more powerful supplier, with his bigger margin, has a greater incentive to invest in improving reliability. However, in addition to the supplier’s effort, overproduction (inflating the order quantity above demand) can also mitigate yield risk—and a more powerful supplier gives the buyer less incentive to inflate the order quantity. Importantly, the buyer’s order quantity has a dominating effect on the supply chain efficiency compared with the supplier’s effort, because the order quantity plays a dual role: it directly influences proportional yield and indirectly generates incentives for the supplier to invest effort via a larger order size. As a result, the efficiency of the wholesale price contract decreases with the supplier’s bargaining power, and thus the wholesale price contract may be preferred when the buyer is powerful.

If, in the random yield case, the buyer delegates the production quantity decision to the supplier, then efficiency exhibits a V-shaped pattern: efficiency is high when either the buyer or the supplier is powerful. Specifically, efficiency is monotonically decreasing in the supplier’s bargaining power (as in the control scenario) up to a threshold value, whereafter it increases monotonically. It is intuitive that, since the supplier determines the production quantity, the buyer’s order quantity no longer plays a dual role, but can provide the supplier with only an indirect incentive to exert effort. Therefore, even though the efficiency trend parallels the one observed under the
control scenario up to a threshold value of the supplier’s bargaining power, beyond that point it is no longer profitable for the buyer to inflate production. Then the supplier unilaterally determines both the effort and the production quantity, and thus efficiency is increasing in the supplier's bargaining power as in the random capacity case.

Overall, in the face of unreliable supply, our analysis provides guidance as to when and why the wholesale price contract is preferable to a more complex coordinating contract.

2. Related Literature

Supply reliability has received considerable attention in recent years. Early studies have focused on situations in which supply reliability is exogenously given, and explored buyer-led risk management strategies such as multisourcing (Babich et al. 2007, Dada et al. 2007, Federgruen and Yang 2008, 2009b, Tang and Kouvelis 2011, Tomlin 2006, 2009, Tomlin and Wang 2005); carrying inventory (Tomlin 2006); or using a backup production option (Yang et al. 2009). Building upon this research, later studies expanded the scope of attention to endogenous reliability. Some studies investigate how buyers can intervene directly to improve suppliers’ reliability (Liu et al. 2010, Wang et al. 2010). Others examine how buyers can indirectly induce suppliers to improve either product quality (Baiman et al. 2000, Kaya and Özer 2009) or reliability—for example, by way of horizontal competition (Federgruen and Yang 2009a) or investment subsidies and/or order inflation (Tang et al. 2014).

Of greater relevance to our work are papers exploring, in the case of endogenous reliability, contractual incentives as a means to induce suppliers’ investment in reliability. However, the coordinating contract invariably takes on a complex form. In the context of vaccine manufacturing, for instance, Dai et al. (2016) identify the coordinating contract that can induce suppliers to deliver products on time. This contract combines buy-back features with a rebate for timely delivery (in effect, a penalty for late delivery). Similarly, Ang et al. (2017) find that when facing the risk of supply disruption, firms have to resort to coordinating contracts that impose penalties—thereby inducing their suppliers to choose appropriate upstream network partners. Yet as discussed in the introduction, such penalty features are difficult to implement in practice.

At the same time, some theoretical and behavioral studies have argued that, in practice, simple contracts are often preferred to complex coordinating contracts. In this strand of literature, the popularity of simple contracts is attributed to two transaction-cost arguments: simple contracts are (i) easier to design and negotiate (Holmstrom and Milgrom 1987, Kalkanci et al. 2011, 2014, Lariviere and Porteus 2001) and (ii) easier to enforce legally (Schwartz and Watson 2004).

A few papers in economics and supply chain management have also explored the efficiency of simple contracts. The economics papers that assess the performance of simple contracts (e.g., Chu and Sappington 2007, Rogerson 2003) focus on their mathematical form, such as different combinations of a fixed price and a cost reimbursement scheme. As such, they do not consider the enabling role of operational factors. In supply chain management, Cachon (2004), Lariviere and Porteus (2001), and Perakis and Roels (2007) investigate the performance of wholesale price contracts and the role of operational factors in settings with demand uncertainty.

In this study, we take our cue from and build upon the above literature. Indeed, as we mentioned in the introduction, we observe widespread use of wholesale price contracts even in the context of endogenous supply reliability. However, the insights obtained from the literature dealing with demand uncertainty we mentioned above do not carry over to the case of supply uncertainty. The latter setting, and hence our own model, poses two distinct challenges. First, besides investing in process improvement, firms can influence yield distribution by inflating the production quantity; inflating the order quantity does not influence the demand distribution when demand is uncertain. Second, the contract performance critically depends on the interplay between the nature of supply risk and the two types of procurement process (control versus delegation). We therefore focus on the performance of wholesale price contracts when supply is unreliable.

Our objective in this research is to focus on the widely used and easy-to-implement wholesale price contract and understand its efficiency properties. While we do characterize the coordinating contract for the various scenarios that we study, we do so for illustrating the benchmark performance and offering an alternative to the wholesale price contract when its efficiency is low. However, to keep the focus of our manuscript, we do not consider other contracts with intermediate efficiency. A case-in-point is the two-part tariff (Feldstein 1972). This particular contract form can coordinate the supply chain in our setting. However, the coordinating contract is such that the buyer pays the supplier for delivered units, and the supplier makes a lump-sum payment to the buyer to “buy out” the business and become the residual claimant. Such bidirectional payment exchanges may be difficult to implement in some situations, and they are observed mainly as slotting allowances in the retail sector (Lariviere and Padmanabhan 1997). Moreover, capital-constrained suppliers cannot always
We now describe our basic model. In sections 4 and 5, we show how the basic model can be applied to the cases with random capacity and random yield, respectively. We consider a supply chain with one supplier (he) and one buyer (she) who faces deterministic demand for a single selling season. Both the supplier and buyer are risk neutral. Demand \( D \) is modeled as being deterministic so that we can focus on the effect of supply uncertainty; this approach is in line with a large share of the existing literature (Deo and Corbett 2009, Dong and Tomlin 2012, Gümos et al. 2012, Yang et al. 2009, 2012). We will model supply uncertainty with uniform distributions in sections 4 and 5, and we check the robustness of our results with general distributions in Appendix A.

As illustrated in Figure 2, the sequence of events is as follows. After observing the demand \( D \), the buyer and supplier bargain over the wholesale price \( w \). Next, the buyer orders quantity \( q \) from the supplier, and the supplier then exerts unverifiable effort \( e \) to improve his reliability and chooses production quantity \( x \). Given the effort \( e \), a corresponding random loss is associated with production; as a result, the supplier delivers \( \tilde{q} \leq q \) units. Finally, the buyer fulfills demand at unit price \( p \).

In principle, we would like to analyze the four cases illustrated in Figure 1: \{random capacity, random yield\} × \{control, delegation\}. Note, however, that the supplier’s production quantity decision is relevant only for the case of random yield with delegation. This is because it is not optimal for the supplier to inflate his production quantity under random capacity and because the supplier is not allowed to inflate his production quantity under random yield with control. We therefore assume \( x = q \) until section 5.2, where we study the case of random yield with delegation.

We introduce our basic model in terms of the linear wholesale price contract, where the buyer pays a wholesale price \( w \) for each delivered unit. The expected profits for buyer and supplier can thus be written as

\[
\begin{align*}
\pi_b(q, e, w) &= pS(q, e) - wy(q, e), \\
\pi_s(q, e, w) &= wy(q, e) - c(q, e),
\end{align*}
\]

where \( y(q, e), S(q, e), \) and \( c(q, e) \) are the expected values for the delivered quantity, sales, and cost, respectively. In the centralized supply chain, there is no bargaining over the wholesale price \( w \), and the order quantity is the same as the production quantity, denoted by \( q \). All decisions are made by a central planner who seeks to maximize the expected supply chain profit, \( \Pi(q, e) = pS(q, e) - c(q, e) \).

In the decentralized supply chain, the buyer and supplier first bargain over the wholesale price \( w \). To model this process, we use the Nash bargaining model with asymmetric bargaining power (Binmore et al. 1986, Roth 1979). Let \( \pi^*_b(w) \) and \( \pi^*_s(w) \) be the equilibrium expected profits of the buyer and supplier, respectively, once the wholesale price \( w \) has been determined. Each firm bargains while anticipating those expected profits, and the resulting wholesale price solves the optimization problem

\[
\max_w \pi^*_b(w)^x \pi^*_s(w)^{1-x},
\]

where \( x \in (0, 1) \) is the supplier’s relative bargaining power. We assume that the firms will reach an agreement provided the resulting expected profit is non-negative (i.e., both firms’ threat points are zero).
Once the wholesale price is negotiated, the buyer acts as a Stackelberg leader by deciding the order quantity. However, the buyer faces a moral hazard problem because the supplier’s effort is not contractible. The buyer’s decision problem, which determines the firms’ equilibrium profits, $\pi_b(w)$ and $\pi_s(w)$, can therefore be written as follows:

$$\max_{q, e} \quad \pi_b(q, e, w),$$

subject to

$$e = \arg\max_{e \geq 0} \pi_s(q, e, w), \tag{3}$$

$$\pi_s(q, e, w) \geq 0.$$ 

The first constraint ensures incentive compatibility for the supplier—in other words, the supplier chooses the effort $e$ that maximizes his expected profit. The second constraint ensures the supplier’s participation by providing the supplier with at least his reservation profit, which we normalize to zero.

A brief discussion about this model is in order. It is prohibitively difficult to solve problem (2) analytically because this problem takes as an input the solution to problem (3). However, we can greatly simplify the analysis by showing that we can represent the supplier’s relative bargaining power not only by $\alpha$ but also—and equivalently—by the wholesale price $w$. Interpreting the wholesale price $w$ as the supplier’s relative bargaining power allows us to just focus on problem (3) without solving problem (2). This approach suffices for our analysis, because we are interested only in how the expected profits of buyer and supplier, $\pi_b(w)$ and $\pi_s(w)$, depend on their relative bargaining power. For that purpose, we establish the following result.

**Proposition 1.** The optimal solution $w^*$ to problem (2) is monotonically increasing in $x$ if both $\pi_s(w)$ and $\pi_b(w)$ are continuous in $w$ and either $\pi_s(w)$ is strictly increasing or $\pi_b(w)$ is strictly decreasing in $w$.

Appendix B includes additional results showing that, for all cases we consider, the requirements of Proposition 1 hold under some mild conditions. Proposition 1 suggests that a higher wholesale price $w$ implies that the supplier has greater bargaining power. Therefore, in the rest of the study, we will solve only problem (3) and interpret the wholesale price $w$ as the supplier’s relative bargaining power.

To avoid trivial results and simplify the exposition, we make the following assumption.

**Assumption 1.** The following conditions hold:

(i) In the centralized supply chain, it is profitable to produce a strictly positive amount even when the supplier does not exert any effort; that is, $\partial \Pi(q, e)/\partial q|_{q=0, e=0} > 0$.

(ii) If the buyer is indifferent among order quantities $Q \subset [0, D]$, then she chooses the largest quantity $q = \sup Q$.

Assumption 1(i) ensures that the optimal production quantity in the centralized supply chain will be strictly positive. Assumption 1(ii) implies that the buyer will satisfy demand provided her profit is not hurt, thus precluding outcomes that are Pareto suboptimal. We commence our analysis with the random capacity scenario.

### 4. Random Capacity

Here we model disruptions that destroy part or all of the supplier’s capacity (Ciarallo et al. 1994, Wang et al. 2010), where the capacity loss is independent of the production quantity. Examples include labor strikes, machine breakdowns, fire, etc. In section 4.1, we show how the basic model of section 3 can be applied to the case of random capacity, state our assumptions, and characterize the optimal decisions in the centralized supply chain. We analyze the decentralized setup in section 4.2.

#### 4.1. Model and Centralized Supply Chain

In the random capacity case, production quantity does not affect the capacity loss. Hence the supplier has no incentive to inflate his production quantity beyond the buyer’s order quantity. We therefore assume, without loss of generality, that the production quantity $x$ is equal to the order quantity $q$ (i.e., the scenario of random capacity with delegation converges to the one of random capacity with control). Consequently, the supplier delivers a random quantity $\tilde{q} = \min\{q, K - \tilde{\zeta}\}$, where $q$ is the order quantity, $K$ is the supplier’s nominal capacity, and $\tilde{\zeta}$ is the random capacity loss. We assume the random loss to be $\zeta = \psi/(e + 1)$, where $\psi$ is uniformly distributed in $[0, K]$ and hence the support of $\tilde{\zeta}$ is $[0, K/(e + 1)]$. The density and cumulative distribution function (CDF) of the random loss $\tilde{\zeta}$—conditional on the effort $e$—are $g(\tilde{\zeta}|e) = (e + 1)/K$ and $G(\tilde{\zeta}|e) = (e + 1)\tilde{\zeta}/K$, respectively.

Finally, the expected delivered quantity is $\bar{y}(q, e) = E_\tilde{\zeta}[\tilde{q}]$, and the expected sales are $S(q, e) = E_\tilde{\zeta}[\min\{\tilde{q}, D\}]$. In Appendix A, we extend this model to the case where the random loss $\tilde{\zeta}$ follows a general distribution.

We assume that the supplier initiates production after the random loss is realized. Hence the expected cost is $c(q, e) = cy(q, e) + v(e)$, where $c$ is the unit production cost and $v(e)$ is the cost of effort to improve production.
reliability. We shall also need the following technical assumption.

**Assumption 2.** The following conditions hold:

(i) The cost of effort is thrice continuously differentiable, and satisfies \( v(0) = 0, \) and \( v'(e) > 0 \) and \( v''(e) \geq 0 \) for \( e > 0. \)

(ii) In equilibrium, the effort level is strictly positive.

Part (i) implies that the cost of the supplier’s effort is convex and increasing. Part (ii) precludes the trivial case of zero effort level, thus simplifying the exposition.

We find that if the order quantity is smaller than demand \( (q < D) \) then so is the delivered quantity \( \bar{q} \), in which case the buyer is able to sell everything. As a result, the expected sales and delivered quantities coincide \( (S(q, e) = y(q, e)) \) and increase with the order quantity \( q \). Yet producing a quantity that exceeds demand does not affect the likelihood of satisfying that demand, and so does not affect expected sales, because the capacity loss is independent of the order quantity. Therefore, the expected sales function \( S(q, e) \) has a kink at \( q = D \). The technical properties of \( S(q, e) \) and \( y(q, e) \) are summarized in Lemma 1 (see Appendix B.1).

We now characterize the optimal order quantity and effort level in the centralized supply chain.

**Proposition 2.** Let Assumptions 1 and 2 hold. Then, in the centralized supply chain, there exist unique optimal decisions \( (q^*, e^*) \), under which the optimal order quantity \( q^* \) is equal to demand \( D \).

The optimal order quantity \( q^* \) is equal to the demand \( D \) because producing more than \( D \) does not increase expected sales. Hence the only way to mitigate risk is for the supplier to exert effort. Given that effort is unverifiable, we next address the efficiency of the wholesale price contract in a decentralized supply chain.

**4.2. Performance of the Wholesale Price Contract**

We study the wholesale price contract and show that supply chain efficiency is generally increasing in the wholesale price, which is a proxy for the supplier’s bargaining power (per Proposition 1). Therefore, a wholesale price contract may be the preferred mode of contracting for supply chains with sufficiently powerful suppliers—even if a theoretically superior (yet complex) contract exists.

**Proposition 3.** Let Assumptions 1 and 2 hold. Then, in the random capacity case, the efficiency of the wholesale price contract is monotonically increasing in \( w \) if \( p < 3c \).

The intuition behind this observed efficiency trend is as follows. The buyer will typically order \( D \) units for any “reasonable” size of the gross margin, \( p - c \). Then, a more powerful supplier (higher wholesale price) has a bigger margin and thus a greater incentive to invest effort, and thereby improve efficiency. We illustrate the efficiency trend visually in panel (a) of Figure 3. Note that the supplier with all the bargaining power will choose the maximum wholesale price \( w = p \), and the buyer with all the bargaining power will choose the minimum wholesale price \( w = c \), as follows from Proposition 1. Therefore, it suffices to use the wholesale price contract in achieving high supply chain efficiency if the supplier is powerful.

Then which contract should firms use when the buyer has greater bargaining power? It is a well-known result that under a moral hazard setting with risk-neutral players, the first-best outcome is achieved by transferring the risk to the agent and making the agent the residual claimant. We find that unit-penalty contracts coordinate the supply chain through this mechanism, while allowing for flexibility in the allocation of profits between buyer and supplier. Under such contracts, the buyer imposes a penalty \( z \) for each unit of shortage, while paying \( w \) for each unit delivered. The next result formalizes our findings.

**Proposition 4.** Let Assumptions 1 and 2 hold. Then, there exists \( \bar{z} > 0 \) such that the following unit penalty contracts coordinate the supply chain: \( w^* = p - \chi \) and \( z^* = \chi \) for \( \chi \in [0, \bar{z}] \); and the buyer’s expected profit is \( \pi_b = \chi D \).

By setting the unit-penalty \( z \) equal to her unit margin \( p - w \), the buyer transfers the entire risk to the supplier and thereby induces the supplier’s first-best effort. Flexible profit allocation is achieved by varying the unit penalty \( z^* = \chi \), where the upper bound on the penalty \( \bar{z} \) ensures that the supplier earns non-negative profit and that the buyer does not order more than \( D \) to exploit the high penalty fee.

Although the unit penalty contract coordinates in principle, there are complexities associated with its implementation that may make wholesale price contacts preferable. More specifically, unit penalty contracts have the following two sources of transaction cost. First, suppliers may view penalties as unfair and may resist such contracts; this would likely increase both the length and cost of the bargaining process (LinkedIn Discussion 2011). Second, the bounded rationality and limited cognitive ability of individuals prevents them from fully optimizing the contractual parameters, even with contract forms that seem to be relatively simple.

Let \( T \) represent this transaction cost, which we express as a percentage of the centralized supply
chain profit. For wholesale prices \( w \) such that the efficiency of the wholesale price contract exceeds \( 100 - T\% \), it is optimal—from the aggregate supply chain perspective—to use the wholesale price contract. This is illustrated in panel (b) of Figure 3, which shows the transaction cost \( T \) of the unit penalty contract as well as the wholesale price \( w_t \) for which the efficiency of the wholesale price contract is \( 100 - T\% \). It follows that, for any wholesale price \( w \geq w_t \), it is optimal to use the wholesale price contract from an aggregate supply chain perspective.

Moreover, one can argue that the wholesale price contract is optimal not only from the aggregate supply chain’s perspective but also from the individual firm’s perspective. In other words, the wholesale price contract offers a Pareto improvement (with respect to the coordinating contract), for buyer and supplier both, when transaction costs are taken into account. To see this, we should compare the two contracts under the same level of supplier bargaining power \( \alpha \). We can then show the following: if the transaction cost \( T \) is above some threshold, then there always exists a set of wholesale prices such that the wholesale price contract achieves a Pareto improvement. The intuition is that in the theoretically extreme case when \( T \) approaches 100%, both firms’ expected profits approach zero. Since we can always find a wholesale price contract under which both firms earn strictly positive profits, it follows that wholesale price contracts can offer Pareto improvement when the transaction cost \( T \) is sufficiently high. For more details, see Proposition 15 in Appendix B.4.

Next we study how our insights change in the context of random yield.

5. Random Yield

In this section, we model disruptions in which the random loss is stochastically proportional to the production quantity; that is, a larger production quantity increases the likelihood of obtaining a larger amount of usable output (Federgruen and Yang 2008, 2009a,b, Tang and Kouvelis 2011). This model applies, for example, when manufacturers of semiconductor or biotech products face uncertain yields in their manufacturing processes. The key distinguishing feature of random yield versus random capacity is that supply risk can be mitigated not only by increasing effort but also by producing more than demand.

We study two different cases that depend on the supplier’s decision regarding production quantity. In section 5.1, we examine the control scenario, in which the buyer’s order fully determines the supplier’s production quantity decision. In section 5.2, we investigate the delegation scenario, in which the supplier independently determines his production quantity after receiving the buyer’s order. For each case, we
show how the basic model of section 3 can be applied and discuss the performance of the wholesale price contract.

5.1. Control Scenario

In the control scenario, the supplier’s production quantity equals the buyer’s order quantity. Recall from the introduction that, in the floriculture industry, where yield issues are an important concern, wholesalers often outsource production to contract growers and provide them with seedlings, fertilizers, and chemicals; thus the wholesalers effectively control contract growers’ production quantities (MacDonald et al. 2004). More generally, Federgruen and Yang (2009a) argue that the control scenario is commonly observed when the supplier cannot test the quality of the manufactured products at his site. Full inspection at the supplier’s site is often impossible or impractical (e.g., Baiman et al. 2000, Balachandran and Radhakrishnan 2005). This is the case (i) when the testing technology is proprietary, in which case the buyer deliberately limits—because of intellectual property concerns—the supplier’s ability to detect failures (Doucakis 2007, p. 23); or (ii) when the end product consists of multiple interacting components, each procured independently, which rules out testing of any component in isolation by the supplier (Li et al. 2011). In such cases, the supplier cannot find out which product will pass the buyer’s quality test. Furthermore, the buyer will seldom accept a delivery in excess of her order quantity because she incurs appraisal cost. Therefore, the supplier has no choice but to deliver only the ordered quantity, and thus the supplier’s production quantity must equal the buyer’s order quantity.

5.1.1. Model and Centralized Supply Chain. The supplier delivers a random quantity \( \tilde{q} = (1 - \xi)q \), where \( q \) is the order quantity and \( \xi \) is the random proportional loss. To focus on the effect of the random proportional loss, we assume here that the supplier has no capacity constraints. We further assume that the random loss is \( \xi = \psi/(e + 1) \), where \( \psi \) is uniformly distributed in \([0, 1]\) and the support of \( \xi \) is \([0, 1/(e + 1)]\). Let the density and CDF of the random proportional loss be denoted \( h(\xi|e) = e + 1 \) and \( H(\xi|e) = (e + 1)\psi \), respectively, and let \( E[\xi] = \mu_\xi \) be the expected random loss. Then the expected delivered quantity is \( y(q, e) = E[\tilde{q}] = q + v(e) \). Assumption 2 applies also in this random yield model.

An interesting property of the random yield model with control is that, in contrast with the random capacity model, expected sales are increasing in the order quantity even when that quantity exceeds demand \( (q > D) \). The reason is that the random loss is stochastically proportional to the order quantity \( q \), which means that ordering more could increase the supplier’s likelihood of delivering \( D \) units and thus could increase expected sales. The technical properties of expected sales, \( S(q, e) \), and of the expected delivered quantity, \( y(q, e) \), are summarized in Lemma 2 (see Appendix B.2). We are now in a position to characterize optimal decisions in the centralized supply chain.

\[ \text{PROPOSITION 5. Let Assumptions 1 and 2 hold. Then, in the centralised supply chain with random yield, there exist both an optimal order quantity } q^* \text{ and an optimal effort level } e^* \; \text{; moreover, the optimal order quantity } q^* \; \text{is strictly greater than the demand } D. \]

The optimal decisions under random yield differ qualitatively from those, described in Proposition 2, under random capacity; it is now optimal to order (or, equivalently, to produce) more than the demand \( (q^* > D) \). Therefore, the decision maker can increase expected profit not only by exerting additional effort but also by ordering more. The reason is that, as we mentioned, expected sales increase with order quantity even when the order quantity exceeds demand. We now investigate how, in a decentralized setting, the need to coordinate both the buyer’s order inflation and the supplier’s effort results in dynamics that differ from those under random capacity.

5.1.2. Performance of the Wholesale Price Contract. Our main finding is that the efficiency of the wholesale price contract generally decreases with the supplier’s bargaining power. Therefore, the wholesale price contract is more likely to be preferred when the buyer is powerful. This result contrasts sharply with the result for random capacity, where the efficiency of the wholesale price contract increases with \( w \). The reason is as follows. Just as under random capacity, a higher wholesale price increases the supplier’s incentive to invest in reliability, which in turn increases efficiency. At the same time, however, increasing \( w \) also reduces efficiency by diminishing the buyer’s incentive to inflate her order quantity. Moreover, the buyer’s order quantity has a dominating effect on the supply chain efficiency compared with the supplier’s effort, because the order quantity plays a dual role: it directly influences proportional yield and indirectly generates incentives (via a larger
order size) for the supplier to invest effort. Note that
the supplier bears no overage cost, which makes a larger
order quantity more effective at inducing effort. There-
fore, the supply chain efficiency—like the order quan-
tity—decreases with the wholesale price.

We now discuss the results that lead us to conclude
that efficiency is generally decreasing in the wholesale
price under random yield with control.

**Proposition 6.** Let Assumptions 1 and 2 hold. Then, for
random yield with control, the efficiency of the wholesale
price contract is monotonically decreasing in \( w \) for
\( w \in [w_c, p] \) if either of these sufficient conditions hold:

1. \( v(e) = \theta e \), and

\[
wc = \frac{(8pk^2 - 32c - 2p) + \sqrt{(8pk^2 - 32c - 2p)^2 + 4(p(16k + 9)(7p + 32c))}}{2(16k + 9)},
\]

where \( k = 2 - \sqrt{\frac{2c}{m}} \) or

2. \( \omega_c = c, \ v(e) = \theta e^m \) where \( m > 1 \), and either \( \theta \) or

\( m \) is sufficiently large.

Part (i) gives sufficient conditions for efficiency to be
monotonically decreasing when the cost of effort is
linear. In particular, it provides an explicit expression
for the threshold \( w_c \). Part (ii) shows that efficiency is
decreasing in the entire range of wholesale prices—
provided the cost of effort is sufficiently large.

Figure 4 shows how the range of wholesale prices
for which the sufficient condition in Proposition 6(i)
holds depends on the cost of effort \( \theta \). The solid hori-
zontal line represents the retail price \( (p = 12) \). The
dashed line shows the minimum feasible wholesale price,
which is the minimum wholesale price for which the
supplier’s participation constraint can be satisfied;
hence equilibria exist only above this line. The line
drawn through the square markers is the threshold \( w_c \)
given in Proposition 6(i), and it shows that the suffi-
cient conditions hold for a fairly wide range of whole-

s
ale prices. Moreover, Proposition 6(i) gives only
sufficient conditions and thus the monotonic trend
can hold also for wholesale prices even lower than \( w_c \).

Indeed, the line drawn through triangular markers
plots the actual threshold wholesale price—above
which efficiency is monotonically decreasing—and
shows that the actual threshold is less than \( w_c \). We
remark that we can only compute the actual threshold
numerically. Finally, for costs of effort \( \theta \geq 40 \), the
monotonic trend holds over the entire range of feasible
wholesale prices; this result is consistent with
Proposition 6(ii).

To verify the robustness of our analytical findings,
we conduct a comprehensive numerical investigation.
Toward that end, we use the following cost function:
\( c(q, e) = cq + \theta e^m \), where \( c, \theta > 0 \) and \( m \geq 1 \).

Figure 5 plots supply chain efficiency as a function
of wholesale price for the same set of parameters used
to generate Figure 3. First of all, we observe a very
robust decreasing trend in efficiency. Recall that the
supplier with all the bargaining power will choose
the maximum wholesale price and that the buyer
with all the bargaining power will choose the mini-
mum wholesale price. Therefore, the efficiency is
fairly high (about 98%) when the buyer is powerful
but is relatively low (about 80%) when the supplier is
powerful. We then repeat our experiment with differ-
ent parameter combinations, choosing seven values
for each for \( c, \theta \), and \( m \) in the following ranges:
\( c \in [0.1, 5] \), \( \theta \in [1, 100] \), and \( m \in [1, 5] \). We therefore
perform our analysis with \( 7^3 = 343 \) different combi-

dinations of parameters. We find that supply chain effi-
ciency is monotonically decreasing with wholesale
price in 84.3% of all cases. The remaining 15.7% of
cases exhibit a slight increase in efficiency (typically
less than 0.1%) for small \( w \) only, yet otherwise the
general decreasing trend prevails.

Having assessed the efficiency of the wholesale
price contract, we now turn to the question of which
contract to use when the wholesale price contract
results in low efficiency. The unit penalty contract no
longer coordinates because—even though it is optimal
for the buyer to inflate her order quantity above
demand—the supplier shares none of the overage
risk. One obvious way to share the overage cost is
through a buy-back agreement. Indeed, we find that
a unit-penalty with buy-back contract coordinates
the supply chain while allowing for arbitrary profit allo-
cation between the buyer and supplier.
There are a number of contexts in which the buyer constraint cannot be satisfied. Then there is no feasible solution. If the wholesale price $c = \min \{ q, (1 - \xi) x \}$, where $q$ is the order quantity, $x$ is the production quantity, and $\xi$ is the random proportional loss defined in section 5.1. The expected delivered quantity is $y(q, x, e) = E(c[\xi])$ and the expected sales are $S(q, x, e) = E[c[\min \{ q, D \}]$. The cost is $c(x, e) = cx + v(e)$.

The delegation scenario differs from the control scenario in that, if the order quantity is larger than the demand ($q > D$) then, as in the random capacity model, expected sales become constant. This is because, as long as $q \geq D$, the probability of receiving $D$ units depends only on the production quantity $x$ and the effort $e$. Therefore, ordering more than $D$ does not directly increase expected sales, and thus $S(q, x, e)$ has a kink at $q = D$. Note, however, that a higher $q$ does give the supplier an incentive to choose higher $x$ and $e$, which would indirectly increase expected sales. The properties of $S(q, x, e)$ and $y(q, x, e)$ are summarized in Lemma 3 (see Appendix B.3).

In the centralized supply chain, the order quantity is redundant, and the decision maker chooses only the production quantity $x$ and the effort $e$. Therefore, the optimal decisions in the centralized supply chain are the same as for the control scenario discussed in section 5.1 except that (a) we replace the order quantity $q$ with the production quantity $x$ in Proposition 5 and (b) we refer to the optimal production quantity as $x^\ast$. We examine the centralized setup next.

### 5.2.1. Model and Centralized Supply Chain

The model is similar to that of the control scenario in section 5.1 except that the supplier determines his own production quantity $x$. Therefore, we present only those parts of the model that differ from the control scenario. The supplier delivers a random quantity $\tilde{q} = \min \{ q, (1 - \xi) x \}$, where $q$ is the order quantity, $x$ is the production quantity, and $\xi$ is the random proportional loss defined in section 5.1. The expected delivered quantity is $y(q, x, e) = E(c[\xi])$ and the expected sales are $S(q, x, e) = E[c[\min \{ \tilde{q}, D \}]$. The cost is $c(x, e) = cx + v(e)$.

The delegation scenario differs from the control scenario in that, if the order quantity is larger than the demand ($q > D$) then, as in the random capacity model, expected sales become constant. This is because, as long as $q \geq D$, the probability of receiving $D$ units depends only on the production quantity $x$ and the effort $e$. Therefore, ordering more than $D$ does not directly increase expected sales, and thus $S(q, x, e)$ has a kink at $q = D$. Note, however, that a higher $q$ does give the supplier an incentive to choose higher $x$ and $e$, which would indirectly increase expected sales. The properties of $S(q, x, e)$ and $y(q, x, e)$ are summarized in Lemma 3 (see Appendix B.3).

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5.2. Delegation Scenario

There are a number of contexts in which the buyer may delegate the production quantity decision to the supplier (see e.g., Chick et al. 2008, Tang et al. 2014). In these contexts, it is possible that the buyer inflates the order quantity above demand, and the supplier further inflates the production quantity. This aspect of the scenario introduces an additional source of inefficiency into the supply chain: The supplier might inflate his production quantity, even if the buyer has already padded her order quantity to buffer against yield losses. These buffers may accumulate and thus exacerbate inefficiency. We now examine whether this is indeed the case.

### 5.2.2. Performance of the Wholesale Price Contract

We find that the efficiency associated with the wholesale price contract exhibits a V-shaped pattern as the wholesale price (supplier’s bargaining power) increases. We therefore argue that if either party has most of the bargaining power then the wholesale price contract is likely to be preferred over more complex contracts that could theoretically offer better performance. This result contrasts with the case of random capacity and also with the case of random yield with control.
to exert effort. Therefore, the buyer whose profit margin is $p - w$ finds it profitable to inflate the order quantity only up to a threshold wholesale price. The efficiency trend for delegation parallels the one in the control scenario up to this threshold. Above the threshold wholesale price, the buyer does not inflate at all and the supplier unilaterally determines his effort as well as the production quantity. Hence supply chain efficiency increases in the supplier’s bargaining power, or equivalently, in the wholesale price (as in the case of random capacity), thus giving rise to the V-shape.

We now discuss the results that lead us to conclude that efficiency exhibits a V-shaped pattern. First, we analytically show that if the wholesale price $w$ is sufficiently large, then the efficiency of the wholesale price contract monotonically increases with $w$. This trend corresponds to the right-hand side of the V-shape.

**Proposition 8.** Let Assumptions 1 and 2 hold. Then, there exists a $w < p$ such that the efficiency associated with the wholesale price contract is monotonically increasing in $w \in [w, p]$.

Second, we numerically verify the results by examining the efficiency pattern of the wholesale price contract across the entire bargaining power spectrum. For this purpose, we use the same cost function as for the numerical analysis in section 5.1: $c(q, e) = cq + \theta e^m$, where $c$, $\theta > 0$ and $m \geq 1$. Figure 6 illustrates that efficiency follows a clear V-shaped pattern as a function of the wholesale price. The lowest efficiency is 95.9%, at the bottom of the V-shape. When $w$ is low, the efficiency rises to 98.3%, and when $w$ is high, it rises to 100%. We analyze 343 different cases—using the same parameter combinations as in section 5.1—and observe an unambiguous V-shape in 88.9% of them. In 11.1% of the cases, we again observe a prominent V-shape, but with a slight increase (typically less than 0.1%) in efficiency when $w$ is very low, at the left extreme of the bargaining power spectrum, followed by the expected V-shaped pattern.

Our results suggest that when using incentives to improve supply reliability in a decentralized supply chain, one must consider whether the buyer controls or delegates the production quantity decision, in addition to bargaining power and the nature of supply risk. An interesting result in the delegation scenario is that as the supplier—the agent undertaking unverifiable action—earns greater margin (and therefore payoff), the efficiency trend is neither increasing (as with random capacity) nor decreasing (as with random yield with control); instead, the efficiency follows a V-shaped pattern.

To consolidate this insight, there is still one remaining loose end—namely, which contract coordinates in the delegation scenario? We find that the unit-penalty contract coordinates the supply chain by making the supplier the residual claimant, as it does under random capacity.

**Proposition 9.** Let Assumptions 1 and 2 hold. Then, there exists a $\bar{\xi} > 0$ such that the following unit penalty contracts coordinate the supply chain: $w^* = p - \bar{\xi}$, $z^* = \bar{\xi}$, where $0 \leq \xi \leq \bar{\xi}$. The buyer’s expected profit is then $\pi_b = \xi D$.

The unit penalty contract coordinates the supply chain with flexible profit allocation despite the additional dimension of moral hazard (production quantity) as compared with the control scenario, in which coordination requires the more complex unit-penalty with buy-back contract. The intuition is that if the penalty fee is set equal to the margin (and is not too large), then the buyer does not inflate the order, because she can earn her margin through either a sale or the penalty imposed on the supplier. Therefore, the supplier faces exactly the same trade-offs as the centralized decision maker and thus chooses the first-best effort and production quantity.

6. Conclusions

Our research is motivated by the widespread prevalence of wholesale price contracts in settings with unreliable supply that could be improved by the supplier. Specifically, we investigate when and why
wholesale price contracts can be used to generate efficient outcomes in a decentralized supply chain. We characterize how the performance of the wholesale price contract depends on the interplay between the type of supply risk (random capacity versus random yield) and type of procurement process (control versus delegation). In this way, we demonstrate that careful appreciation of the operational features of the setting is instrumental in identifying the circumstances under which wholesale price contracts perform well.

When a supplier invests unverifiable effort (e.g., reliability investment), coarse intuition might suggest that the supplier will invest more effort if he can negotiate a higher share of the supply chain profit, thereby increasing efficiency. We find this intuition to be correct in our setting with random capacity. However, the scenario of random yield with control exhibits the exact opposite trend, and for random yield with delegation, we find that supply chain efficiency is no longer monotonic in bargaining power but instead is V-shaped.

Even though they are suboptimal in theory, wholesale price contracts may be preferred to complex contracts that come with a transaction cost—a phenomenon we refer to as a preference for “appropriate” contracts. Our findings translate into simple insights that have the potential to inform managerial decisions. To illustrate, compare the following two scenarios: (i) Nike sources garments from Sabrina Garment Manufacturing, and (ii) Apple sources semiconductor chips from Samsung (Bloomberg Business 2015, CNN 2013). Sabrina is based in Cambodia, and has encountered labor disruptions (random capacity), whereas Samsung has to contend with random yield. Both Nike and Apple form the bulk of their respective suppliers’ business, and thus have greater bargaining power in the respective relationships (The Wall Street Journal 2013). Although both firms face supply risk, our results indicate that the wholesale price contract would generate high efficiency for Apple’s supply chain, but result in low efficiency for Nike’s.

We believe our findings offer guidance regarding when and why to use the wholesale price contract—or instead a more complex coordinating contract—and identify the role played by the types of supply risk and procurement process in this outcome.

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Appendix A. General Model

Unless otherwise specified, all the results in Appendix B (technical results), Appendix C (tables), and online Appendix S1 (proofs of all results) are based on the general model we define in this Appendix A.

A.1. Random Capacity

We generalize the assumption that the random loss $\zeta$ is uniformly distributed. The random loss is $\zeta = f_\psi(e, \epsilon)$, where $\psi$ is a random variable that captures the underlying supply risk and $f_\psi$ is a function that models the dependence of the random loss on the supplier’s effort $\epsilon$. We make the following technical assumption, which our basic model with the uniform distribution satisfies.

ASSUMPTION 3. The following conditions hold:

(i) The random loss $\zeta$ has support $[0, a_\epsilon(e)]$, where both $G(\zeta|\epsilon)$ (the CDF) and $a_\epsilon(e)$ are twice continuously differentiable with finite derivatives in $e \geq 0$ and $\zeta \in [0, a_\epsilon(e)]$.

(ii) Either of the following holds:

(a) $a_\epsilon(e) = K$, $\partial G(\zeta|\epsilon)/\partial e > 0$, and $\partial^2 G(\zeta|\epsilon)/\partial e^2 < 0$ for $e \geq 0$ and $\zeta \in (0, K)$.

(b) $a_\epsilon(0) = K$, $a_\epsilon'(e) < 0$, $\partial G(\zeta|\epsilon)/\partial e > 0$, and $\partial^2 G(\zeta|\epsilon)/\partial e^2 \leq 0$ for $e \geq 0$ and $\zeta \in (0, a_\epsilon(e))$.

Part (ii) implies that the effort $\epsilon$ mitigates the random loss $\zeta$ in the sense of first-order stochastic dominance (FOSD) with decreasing returns to scale. Moreover, the supplier’s effort achieves the stochastic dominance either by shifting the CDF upward while preserving the support of the random loss (as in part (a)) or by reducing the support of the random loss (as in part (b)).

With this generalized model and Assumption 3, both Proposition 2 and Proposition 4 continue to hold. However, Proposition 3 no longer holds; we have instead the following weaker results for monotonicity.

PROPOSITION 10. Let Assumptions 1, 2, and 3 hold. Then, for random capacity, the efficiency of the wholesale price contract is monotonically increasing in $w$ for $w \in [c, p]$ if either of these sufficient conditions holds:

(i) $w$ is sufficiently close to the unit price $p$; or
(ii) $w = c$ and the buyer’s optimal order quantity is equal to the demand $D$.

In part (i) we show that the supply chain efficiency is monotonically increasing in the wholesale price (or in the supplier’s bargaining power) when the wholesale price is sufficiently high. In (ii), we show that this monotonicity result holds over the entire interval, $w \in [c, p]$, provided the buyer always orders $D$ units.
We have also numerically verified that this efficiency trend continues to hold when the loss distribution is triangular or of beta type; for the sake of brevity, we omit the details.

A.2. Random Yield: Control Scenario

We generalize the assumption that the random loss $\xi$ is uniformly distributed. The random loss is $\xi = f_y(\psi, e)$, where $\psi$ is a random variable that captures the underlying supply risk and $f_y$ is a function that models the dependence of the random loss on the supplier’s effort $e$. We make the following technical assumption, which our basic model with the uniform distribution satisfies.

**Assumption 4.** The following conditions hold:

(i) The random loss $\xi$ has support $[0, a_y(e)]$, where both $H(\xi|e)$ (the CDF) and $a_y(e)$ are thrice continuously differentiable with finite derivatives in $e \geq 0$ and $\xi \in [0, a_y(e)]$.

(ii) Either of the following holds:

(a) $a_y(e) = 1$, $\partial H(\xi|e)/\partial e > 0$, and $\partial^2 H(\xi|e)/\partial e^2 < 0$ for $e \geq 0$ and $\xi \in (0, 1)$.

(b) $a_y(0) = 1$, $a_y'(0) < 0$, $\partial H(\xi|e)/\partial e > 0$, and $\partial^2 H(\xi|e)/\partial e^2 \leq 0$ for $e \geq 0$ and $\xi \in (0, a_y(e)]$.

Part (ii) implies that the effort $e$ mitigates the random loss $\xi$ in the sense of FOSD with decreasing returns to scale. Moreover, the supplier’s effort achieves the stochastic dominance either by shifting the CDF upward while preserving the support of the random loss (as in part (a)) or by reducing the support of the random loss (as in part (b)).

With this generalized model and Assumption 4, the statements in Proposition 5, Proposition 6(ii), and Proposition 7 continue to hold. Now, however, Proposition 6(i) no longer holds; instead we obtain the following weaker result.

**Proposition 11.** Let Assumptions 1, 2, and 4 hold. Then, under random yield with control, the efficiency of the wholesale price contract is monotonically decreasing in $w$ for $w \in [w_c, p]$, provided that $w_c$ is sufficiently close to $p$.

Proposition 11 shows that, if the wholesale price is sufficiently high, then supply chain efficiency decreases with the supplier’s bargaining power (or the wholesale price).

Moreover, we numerically verify the monotonicity result. Recall that our model assumes the loss distribution to have a bounded support and to exhibit first-order stochastic dominance as effort increases. The uniform distribution we use in the basic model exhibits FOSD as the support shrinks with greater effort. We consider two other loss distributions that have bounded and fixed support: a triangular distribution and a beta-type distribution. The triangular distribution exhibits FOSD as the mode approaches zero with greater effort, while the support remains fixed. For the beta-type distribution we consider, the CDF has a closed-form expression, and this function exhibits FOSD as the mode approaches zero while the support remains fixed (see Jones 2009). This beta-type distribution is quite flexible and encompasses a variety of bell shapes.

We conduct a comprehensive numerical analysis with the triangular and beta-type distributions, and find that the decreasing efficiency trend is robust. Specifically, we explore the triangular distribution with the same parameter combinations as used for examining the uniform distribution and find that efficiency is completely monotonic in every case. For the beta-type distribution, we conduct the numerical analysis with a narrower range of parameters to ensure feasibility, and again find that efficiency is monotonic in 97.8% of the cases. The remaining 2.2% of the cases exhibit a slight increase for small $w$, but again the general decreasing trend persists.

A.3. Random Yield: Delegation Scenario

The basic model for the uniform distribution in section 5.2 introduces only those modeling elements that differ from the control scenario. That basic model can be immediately extended to the general model without any changes. We need to only make the following assumption for tractability, which our basic model with the uniform distribution satisfies.

**Assumption 5.** The expected delivered quantity $y(q, x, e)$ is jointly concave in $q$ and $e$, and also in $x$ and $e$, in the feasible region of problem (3).

Under Assumption 5, both Proposition 8 and Proposition 9 continue to hold. We conduct a comprehensive numerical analysis with the triangular and beta-type distributions and with the same combinations of parameters as used in Appendix A.2, finding that the V-shaped pattern is robust. With the triangular distribution, we observe an unambiguous V-shape in 85.1% of the cases; in 1.4% of the cases, we find a prominent V-shape but with a slight increase in efficiency at the left extreme of the V. In 13.5% of the cases, efficiency is just increasing. However, these are exceptional cases in which the unit production cost $c$ is so high that feasible solutions exist only when the wholesale price $w$ is greater than 90% of the retail price $p$. For the beta-type distribution, we observe an unambiguous V-shape in 97.1% of the cases. In 1.9% of the cases, we find a prominent V-shape but with a slight increase in efficiency at the left extreme of the V, and in 1% of the cases, the efficiency was just increasing.
Appendix B. Technical Results

B.1. Random Capacity
The following lemma holds under the general model defined in Appendix A, and it establishes both the properties of and the relationship between expected sales and expected delivered quantity.

**Lemma 1.** Let Assumptions 2 and 3 hold. Then the following holds.

(i) \( S(q, e) = y(q, e) \) if \( q \leq D \), and \( S(q, e) = y(D, e) \) if \( q > D \).
(ii) \( y(q, e) \) is increasing and concave in both \( q \) and \( e \).
Also, \( y(q, e) \) is twice continuously differentiable in \( q \) and \( e \), except when \( q = K - a_y(e) \), in which case \( y(q, e) \) is once continuously differentiable.

The next proposition holds under our basic model with the uniform distribution, and—together with Proposition 1—allows us to interpret the wholesale price \( w \) as a proxy for the supplier's bargaining power \( \pi \).

**Proposition 12.** The buyer's and the supplier's expected profits at equilibrium, \( \pi^*_b(w) \) and \( \pi^*_s(w) \), satisfy the following: (i) \( \pi^*_b(w) \) and \( \pi^*_s(w) \) are both continuous in \( w \) and (ii) \( \pi^*_s(w) \) is strictly increasing in \( w \) if \( p < 3c \).

B.2. Random Yield: Control Scenario
The following lemma holds under the general model defined in Appendix A, and it establishes both the properties of and the relationship between expected sales and expected delivered quantity.

**Lemma 2.** Let Assumptions 2 and 4 hold. Then the following holds.

(i) If \( q \leq D \), then \( S(q, e) = y(q, e) \). If \( q > D \), then \( S(q, e) < y(q, e) \).
(ii) \( y(q, e) \) and \( S(q, e) \) are increasing and concave in both \( q \) and \( e \).
Also, \( y(q, e) \) and \( S(q, e) \) are thrice continuously differentiable in \( q \) and \( e \), except that \( S(q, e) \) is once continuously differentiable (a) in \( q \) when \( q = D \), and (b) in \( q \) and \( e \) when \( q = D/ (1 - a_y(e)) \).

The next proposition holds under the general model defined in Appendix A, and—together with Proposition 1—allows us to interpret the wholesale price \( w \) as a proxy for the supplier's bargaining power \( \pi \).

**Proposition 13.** The buyer's and the supplier's expected profits at equilibrium, \( \pi^*_b(w) \) and \( \pi^*_s(w) \), satisfy the following: (i) \( \pi^*_b(w) \) and \( \pi^*_s(w) \) are both continuous in \( w \) and (ii) \( \pi^*_s(w) \) is strictly decreasing in \( w \).

B.3. Random Yield: Delegation Scenario
The following lemma holds under the general model defined in Appendix A, and it establishes both the properties of and the relationship between expected sales and expected delivered quantity. The difference from the control scenario is that the expected sales do not depend on the order quantity \( q \) when the buyer orders at least \( D \) units. Hence, the expected sales \( S(q, x, e) \) have a kink at \( q = D \).

**Lemma 3.** Let Assumptions 2 and 4 hold. If the supplier determines his own production quantity, then the following holds.

(i) If \( q \leq D \), then \( S(q, x, e) = y(q, x, e) \). If \( q > D \), then \( S(q, x, e) = y(D, x, e) \).
(ii) \( y(q, x, e) \) is increasing and concave in both \( x \) and \( e \).
Also, \( y(q, x, e) \) is thrice continuously differentiable in \( x \) and \( e \), except that it is once continuously differentiable (a) in \( x \) when \( x = q \), and (b) in \( x \) and \( e \) when \( x = q/(1 - a_y(e)) \).

The next proposition holds under our basic model with the uniform distribution, and—together with Proposition 1—allows us to interpret the wholesale price \( w \) as a proxy for the supplier's bargaining power \( \pi \).

**Proposition 14.** The buyer's and the supplier's expected profits at equilibrium, \( \pi^*_b(w) \) and \( \pi^*_s(w) \), satisfy the following: (i) \( \pi^*_b(w) \) and \( \pi^*_s(w) \) are both continuous in \( w \) and (ii) \( \pi^*_s(w) \) is strictly decreasing in \( w \) if \( p \leq 2c \) and \( \nu(e) = \theta e \), where \( \theta \geq (5/7) \times cD \).

B.4. Other Results
The next lemma gives the conditions equivalent to Assumption 1(i) for each type of supply risk.

**Lemma 4.** Assumption 1(i) is equivalent to (i) \( p > c \) for random capacity, and (ii) \( p(1 - \mu_y^0) > c \) for random yield.

The following proposition states that if the transaction cost \( T \) of a coordinating contract is sufficiently high, then the wholesale price contract is actually Pareto efficient compared to the coordinating contract because of the high transaction cost.

**Proposition 15.** There exists a threshold \( T' > 0 \) such that if the transaction cost of a coordinating contract \( T \) exceeds this threshold \( T' \), then there exists a set of
wholesale prices for which the wholesale price contract is Pareto efficient compared to the coordinating contract.

Appendix C. Tables
The results in these tables apply to the general model defined in Appendix A. We use “≥” and “≤” to signify “strictly positive” and “strictly negative,” respectively.

<table>
<thead>
<tr>
<th>Table C1 Derivatives of y(q, e) under Random Capacity</th>
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<tr>
<td>Derivatives</td>
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<table>
<thead>
<tr>
<th>Table C2 Derivatives of y(q, e) and S(q, e) under Random Yield with Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Derivatives</td>
</tr>
<tr>
<td>0 ≤ q ≤ D</td>
</tr>
<tr>
<td>D &lt; q ≤ K – a(d)</td>
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<tr>
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</tbody>
</table>

Notes

1 Supply chain efficiency is the ratio of the expected profit in the decentralized supply chain to the optimal expected profit in the centralized supply chain.

2 We use the Nash bargaining model with asymmetric bargaining power to model the bargaining process, and Proposition 1 establishes that the equilibrium wholesale price increases with the supplier’s bargaining power. Hence, throughout the study, we use wholesale price as a proxy for the supplier’s bargaining power.

3 We do not directly compare the efficiencies of wholesale price contracts under random capacity and random yield on a point-by-point basis, because they are two different models. Instead, we compare only whether the efficiency under each model is increasing or decreasing with the supplier’s bargaining power.

4 There is also an emergent stream of literature that investigates the role of network configuration in determining supply reliability (see e.g., Ang et al. 2017, Bimpikis et al. 2013, Jain et al. 2014, Lim et al. 2013).

5 Over the past two decades, coordinating contracts have received much attention in the supply chain management literature. Numerous studies have identified coordinating contracts in different business contexts (for a review, see Cachon 2003).

6 In the context of random yield, such contracts are also referred to as quality-based incentive pricing (Q-pricing) contracts (Baiman et al. 2004).

7 Zhou (1997) shows that the Nash bargaining model with asymmetric bargaining power can be extended to the case when the payoff set (i.e., \( \{ \pi_1(w), \pi_2(w) \} \)) in our case is non-convex, as in our model, for the bargaining power parameter \( x \in (0, 1) \). The supplier has all the bargaining power in the limit as \( x \to 1 \), whereas the buyer has all the bargaining power in the limit as \( x \to 0 \).

8 Monotonicity and convexity/concavity results are all used in the weak sense throughout the study, unless stated otherwise.

9 We remark that analyses that calibrate the performance of simple contracts are typically hard problems that require specific assumptions about the functional form of the uncertainty distributions involved (Chu and Sappington 2007, Rogerson 2003).

10 We use a similar argument to that in the last paragraph of section 4, one can argue that the wholesale price contract is Pareto efficient compared to the coordinating contract when the buyer is powerful once transaction costs are considered.

11 The reason why the decreasing monotonic trend may not hold when \( w \) is sufficiently close to \( c \) is that both effort and quantity exhibit diminishing marginal impact (returns to scale) on expected sales and therefore on efficiency; the relevant derivatives can be found in Table C2 (see Appendix C). This means that, as \( w \) increases, the marginal increase in efficiency due to additional effort is greater when \( w \) is close to \( c \), and the baseline effort is small; the corresponding marginal loss in efficiency due to a reduced order quantity is less significant. It follows that the monotonic trend in efficiency could be contravened only if \( w \) were close to \( c \), a conclusion in line with Proposition 6.

12 Although Proposition 7 checks only the necessary KKT conditions, we numerically confirm that a unit-penalty with buy-back contract does coordinate the supply chain under the parameters used in our numerical analysis.

13 Specifically, the CDF is \( F_2(x) = 1 – (1 – e^{-x})^a \), where \( a \) is a parameter and \( e \) is the effort level.

14 We choose seven values each for \( c, \theta, \) and \( m \) in the following ranges: \( c \in [0.1, 5], \theta \in [1, 100], \) and \( m \in [1, 5] \).
therefore perform our analysis with $7^3 = 343$ different combinations of parameters.

The beta-type distribution generally results in higher yield losses than other distributions do, and these losses are increasing in the parameter $a$ in the CDF. Therefore, we use a narrower range of parameters to ensure feasible solutions. Specifically, we choose five values each for $a$, $c$, $\theta$, and $m$ in the following ranges: $a \in [1, 3]$, $c \in [0.1, 1]$, $\theta \in [1, 100]$, and $m \in [1, 5]$, where (i) $\tilde{c} = 3$, $\tilde{\theta} = 100$ for $a = 1$ and 1.5; (ii) $\tilde{c} = 2$, $\tilde{\theta} = 50$ for $a = 2$ and 2.5; and (iii) $\tilde{c} = 1$, $\tilde{\theta} = 50$ for $a = 3$. These combinations result in $5^4 = 625$ cases.

The detailed derivatives are summarized in Table C1 (see Appendix C).

References


**Supporting Information**

Additional supporting information may be found online in the supporting information tab for this article:

**Appendix S1:** Proofs of All Results.