

Stock Return Serial Dependence and Out-of-Sample Portfolio Performance*

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Abstract

In this paper, we first show that a vector autoregressive (VAR) model captures daily stock return serial dependence in a statistically significant manner. Second, we characterize (analytically and empirically) the expected return of an arbitrage (zero-cost) portfolio based on the VAR model, and show that it compares favorably to that of other arbitrage portfolios in the literature. Third, we evaluate three investment (positive-cost) portfolios: a *conditional* mean-variance myopic portfolio obtained using the linear VAR model; a conditional mean-variance portfolio using a *nonparametric* autoregressive (NAR) model; and, a portfolio that is *dynamic* rather than myopic in its use of the VAR model. All three investment portfolios have similar performance, and they substantially outperform the traditional (unconditional) mean-variance portfolio, which ignores serial dependence in stock returns. Finally, we find that the portfolios exploiting stock return serial dependence have very high portfolio turnover, and we show how a norm-constraint on portfolio weights can be used to reduce their turnover, while still outperforming the traditional (unconditional) portfolios.

Keywords: Vector autoregression, nonparametric autoregression, portfolio choice, dynamic portfolio choice.

JEL Classification: G11.

1 Introduction

There is extensive empirical evidence that stock returns are serially dependent, and that this dependence can be exploited to produce abnormal positive expected returns. For instance, Jegadeesh and Titman (1993) show that assets with high (low) returns over the last twelve months tend to have high (low) returns for the next six months, and that “strategies which buy stocks that have performed well in the past and sell stocks that have performed poorly in the past generate significant positive returns over 3- to 12-month holding periods.” Lo and MacKinlay (1990) show that returns of large firms lead those of small firms. Specifically, they estimate the cross-autocorrelation matrices for the vector of returns on the five size-sorted quintiles of a sample of stocks from CRSP, and find that “current returns of smaller stocks are correlated with past returns of larger stocks, but not vice versa, a distinct lead-lag relation based on size.” Moreover, they show that a contrarian portfolio that takes advantage of this lead-lag pattern in stock returns by being long past losing stocks and short past winners produces abnormal positive expected returns.¹

Our objective is to study whether investors can exploit the stock return serial dependence that has been documented in the literature to select portfolios of risky assets that perform well *out-of-sample*. We tackle this task in three steps. First, we propose a vector autoregressive (VAR) model to capture stock return serial dependence, and test its statistical significance. Second, we characterize, both analytically and empirically, the expected return of an arbitrage (zero-cost) portfolio based on the VAR model, and compare it to that of other arbitrage portfolios from the literature. Third, we evaluate empirically the out-of-sample gains from using three investment (positive-cost) portfolios that exploit serial dependence in stock returns.

To identify the optimal portfolio weights, our work uses conditional forecasts of *expected returns* for *individual* stocks. This is in contrast to the recent literature on portfolio selection, which finds it optimal to *ignore* estimates of expected returns. Merton (1980) explains the theoretical reason why it is much more difficult to get precise estimates of first moments than second moments. And, Jagannathan and Ma (2003) confirm this in practice: they find that the minimum-variance portfolio (which ignores estimates of expected returns) outperforms portfolios that rely on forecasts of expected returns, even when performance is measured using the Sharpe ratio, which depends on both the portfolio mean and variance. DeMiguel, Garlappi, and Uppal (2009) also find that portfolios that use estimated

¹Other papers that provide evidence of serial- and cross-correlation in stock returns include: Brennan, Jegadeesh, and Swaminathan (1993), Badrinath, Kale, and Noe (1995), Carhart (1997), Moskowitz and Grinblatt (1999), Chordia and Swaminathan (2000), Lewellen (2002), Ramnath (2002), Hou and Moskowitz (2005), Daniel and Titman (2006), Hou (2007), and Menzly and Ozbas (2009).

expected returns perform poorly out of sample, achieving substantially lower Sharpe ratios and higher turnovers compared to portfolios that ignore estimates of expected returns. Consequently, a large part of the literature on portfolio selection has focussed on improving the estimation of the covariance matrix: see, for example, Chan, Karceski, and Lakonishok (1999), Ledoit and Wolf (2003, 2004), DeMiguel and Nogales (2009), and DeMiguel, Garlappi, Nogales, and Uppal (2009). The focus on conditional expected returns for individual stocks distinguishes our work from these papers.

Our paper makes four contributions to the literature on portfolio selection. First, we propose using a vector autoregressive (VAR) model to capture serial dependence in stock returns. Our VAR model allows tomorrow's expected return on every stock to depend linearly on today's realized return on *every* stock, and hence it is general enough to capture any linear relation between stock returns in consecutive periods, irrespective of whether its origin is momentum, lead-lag relations, or some other feature of the data. We verify the validity of the VAR model for stock returns by performing extensive statistical tests on six empirical datasets, and conclude that the VAR model is significant for all datasets. VAR models have been used before for strategic asset allocation—see Campbell and Viceira (1999, 2002); Campbell, Chan, and Viceira (2003); Balduzzi and Lynch (1999); Barberis (2000)—where the objective is to study how an investor should dynamically allocate her wealth across a few asset classes (e.g., a single risky asset (the index), a short-term bond, and a long-term bond), and the VAR model is used to capture the ability of certain variables (such as the dividend yield and the short-term versus long-term yield spread) to predict the returns on the single risky asset.² Our objective, on the other hand, is to study whether an investor can exploit stock return serial dependence to choose a portfolio of multiple risky stocks with better out-of-sample performance, and thus we use the VAR model to capture the ability of today's stock returns to predict tomorrow's expected stock returns.³

Our second contribution is to characterize, both analytically and empirically, the expected return of zero-cost arbitrage portfolios based on the VAR model and to compare them to other arbitrage strategies from the literature. Analytically, we compare the expected return of the VAR arbitrage portfolio to that of the contrarian arbitrage portfolio

²Lynch (2001) considers three risky assets (the three size and three book-to-market portfolios), but he does not consider the ability of each of these risky assets to predict the return on the other risky assets; instead, he considers the predictive ability of the dividend yield and the yield spread. The effectiveness of these predictors in forecasting individual stock returns is examined in Section 6.2.

³VAR models have also been used before to model serial dependence among individual stocks or international indexes. For instance, Tsay (2005, Chapter 8) estimates a vector autoregressive model for a case with only two risky assets, IBM stock and the S&P500 index, Eun and Shim (1989) estimate a VAR model for nine international markets, and Chordia and Swaminathan (2000) estimate a vector autoregressive model for two portfolios, one composed of high-trading-volume stocks and the other of low-trading-volume stocks. However, to the best of our knowledge, our paper is the first to investigate whether a VAR model at the individual stock level can be used to choose portfolios with better out-of-sample performance.

studied by Lo and MacKinlay (1990), who show that the expected return on the contrarian arbitrage portfolio is positive if the stock return autocorrelations are negative and the stock return cross-autocorrelations are positive. We show that the VAR arbitrage portfolio achieves a positive expected return in general, regardless of the sign of the autocorrelations and cross-autocorrelations. Empirically, we show that the VAR arbitrage portfolio substantially outperforms several other arbitrage portfolios such as the contrarian arbitrage portfolio, an arbitrage portfolio based on the unconditional sample mean, an arbitrage portfolio that goes long on small stocks and short on big stocks, and an arbitrage portfolio that goes long on high book-to-market and short on low book-to-market stocks.

Our third contribution is to evaluate the out-of-sample gains associated with investing in three (positive-cost) portfolios that exploit stock return serial dependence. The first portfolio is the *conditional mean-variance portfolio* of a myopic investor who believes stock returns follow the VAR model. This portfolio relies on two assumptions: stock returns in consecutive periods are linearly related and the investor is myopic. The other two portfolios we consider relax each of these two assumptions. Specifically, we consider the conditional mean-variance portfolio of a myopic investor who believes stock returns follow a nonparametric autoregressive (NAR) model, which does *not* require that the relation across stock returns be *linear*. We also consider the *dynamic* portfolio of Campbell, Chan, and Viceira (2003), which is the optimal portfolio of an intertemporally optimizing investor with Epstein-Zin utility. Our empirical results show that all three portfolios that exploit serial dependence in stock returns substantially outperform the traditional (unconditional) portfolios out-of-sample. Moreover, we observe that the gains from exploiting stock return serial dependence come in the form of higher expected return, since the out-of-sample variance of the conditional portfolios is much higher than that of the unconditional (traditional) portfolios; that is, stock return serial dependence can be exploited to forecast stock mean returns much better than using the traditional (unconditional) sample estimator.

We find that the superior out-of-sample performance of the portfolios that exploit time serial dependence in stock returns is accompanied by very high portfolio turnovers. Our fourth contribution is to show how a 1-norm constraint, similar to that studied in DeMiguel, Garlappi, Nogales, and Uppal (2009), can be used to control their turnover while still substantially outperforming the traditional (unconditional) portfolios. We find that the portfolios that exploit time serial dependence in stock returns outperform the traditional portfolios if proportional transactions costs are under 10 basis points.

Finally, we perform several robustness checks. First, we find that the insights from our empirical evaluation are robust to whether we use daily, weekly, or monthly data. Second,

we observe that using the dividend yield and the yield spread, as in Lynch (2001), to predict the returns of the different risky assets can help to improve the out-of-sample performance of mean-variance portfolios, but the improvement is substantially smaller than that from using today’s stock returns as predictors. Third, when regressing the norm-constrained conditional portfolio returns on the Fama-and-French and momentum factors, we find that the slopes are not very different from those obtained by regressing the minimum-variance portfolio return. Fourth, we observe that although all portfolios have very poor performance during the 2008 crisis year, the *relative* performance of the different portfolios is similar to that over the entire period. Fifth, we note that incorporating optimally the effect of transactions costs does *not* improve the out-of-sample performance. The reason for this is that while one can compute the transactions costs accurately, the mean and variance of the portfolio return are estimated with substantial error, and hence an objective function that combines exact transactions costs with the inaccurate estimates of mean and variance results in portfolios that perform poorly out of sample.

The rest of this manuscript is organized as follows. Section 2 describes the datasets and the methodology we use for our empirical analysis. Section 3 states the VAR model of stock returns, and tests its statistical significance. Section 4 characterizes (analytically and empirically) the performance of a VAR zero-cost arbitrage portfolio, and compares it to that of other arbitrage portfolios. Section 5 describes how to compute the different positive-cost portfolios we consider, and discusses their empirical performance. Section 6 describes our robustness checks. Section 7 concludes. Proofs for all propositions are relegated to the appendix, and a second appendix contains tables with additional robustness results.

2 Data and Evaluation Methodology

2.1 Datasets

Table 1 lists the six datasets that we use for our empirical analysis. The table also gives the abbreviations we use to refer to the different datasets when reporting our results, the number of assets in each dataset, the time period covered by the dataset, and the source. Three of the datasets contain the returns on portfolios formed on size and book-to-market. Specifically, we download from Ken French’s website the datasets consisting of 6, 25, and 100 portfolios formed on size and book-to-market (6FF, 25FF, and 100FF). We also consider the datasets containing the returns on the 10 and 48 industry portfolios from Ken French’s website (10Ind, 48Ind). Finally, we consider a dataset containing the CRSP database returns on all stocks that were part of the S&P500 index between 1989 and 2008 (100CRSP). From

this dataset, we select 100 stocks using the following approach. At the beginning of each calendar year, we find the set of stocks for which we have returns for the entire period of our estimation window as well as for the next year. From those stocks, we randomly select 100 and use them for portfolio selection until the beginning of the next calendar year, when we randomly select stocks again.⁴

2.2 Evaluation methodology

We compare the performance of the different portfolios using five criteria, all of which are computed out of sample: (i) portfolio mean return; (ii) portfolio variance; (iii) Sharpe ratio; (iv) portfolio turnover (trading volume); and, (v) certainty equivalent.

We use a “rolling-horizon” procedure to compare the out-of-sample performance of the different portfolios. First, we choose a window over which to perform the estimation. We denote the length of the estimation window by $\tau < T$, where T is the total number of returns in the dataset.⁵ Two, using the return data over the estimation window, we compute the various portfolios. Three, we repeat this procedure for the next period by “rolling” the window forward; that is, by including the return for the next period in the estimation window and dropping the return for the earliest period. We continue doing this until the end of the dataset is reached. At the end of this process, we have generated $T - \tau$ portfolio-weight vectors for each strategy; that is, w_t^k for $t = \tau, \dots, T - 1$ and for each strategy k . Holding the portfolio w_t^k for one day gives the *out-of-sample* return at time $t + 1$: $r_{t+1}^k = (w_t^k)^\top r_{t+1}$, where r_{t+1} denotes the asset returns.

We use the time series of returns and weights for each portfolio to compute their out-of-sample mean, variance, Sharpe ratio, and turnover:

$$\hat{\mu}^k = \frac{1}{T - \tau} \sum_{t=\tau}^{T-1} r_{t+1}^k, \quad (1)$$

$$(\hat{\sigma}^k)^2 = \frac{1}{T - \tau - 1} \sum_{t=\tau}^{T-1} (r_{t+1}^k - \hat{\mu}^k)^2, \quad (2)$$

$$\widehat{\text{SR}}^k = \frac{\hat{\mu}_k}{\hat{\sigma}_k}, \quad (3)$$

⁴Note that the way we select the data is not critical for our results, because we are only comparing results for different portfolio strategies and the data selection should affect all strategies in a similar way.

⁵For our experiments, we use an estimation window lengths of $\tau = 1000$ for daily data, $\tau = 260$ for weekly data, and $\tau = 105$ for monthly data. We have chosen these estimation window lengths because they result in good out-of-sample performance for the benchmark traditional minimum-variance portfolio, while leaving a reasonable amount of data available for out-of-sample evaluation. Note also that τ needs to be strictly greater than 100, which is the number of assets in our two largest datasets.

$$\text{Turnover} = \frac{1}{T - \tau - 1} \sum_{t=\tau}^{T-1} \sum_{j=1}^N \left(\left| w_{j,t+1}^k - w_{j,t}^k \right| \right), \quad (4)$$

where in the definition of turnover, $w_{j,t}^k$ denotes the portfolio weight under strategy k *before* rebalancing but at $t + 1$, and $w_{j,t+1}^k$ the desired portfolio weight at time $t + 1$ (after rebalancing).

To compute the certainty equivalent, we assume the investor has an initial wealth of \$1 and wishes to maximize the Epstein-Zin utility of lifetime consumption. As in Campbell, Chan, and Viceira (2003), we consider the case where the investor's annual subjective discount rate $\delta = 0.92$, and elasticity of intertemporal substitution $\psi = 1$, which implies that optimal consumption is equal to the fixed proportion $(1 - \delta)$ of the investor's wealth at each date. We then compute the certainty equivalent as the level of initial wealth that would generate a level of Epstein-Zin utility equal to the discounted Epstein-Zin utility of consumption.⁶

To measure the statistical significance of the difference between the Sharpe ratios (or certainty equivalents) of two given portfolios, we use the (non-studentized) stationary bootstrap of Politis and Romano (1994) to construct a two-sided confidence interval for the difference between the Sharpe ratios (or certainty equivalents). We use 1,000 bootstrap resamples and an expected block size equal to 5. Then we use the methodology suggested in Ledoit and Wolf (2008, Remark 3.2) to generate the resulting bootstrap p-values.

Finally, to measure the impact of proportional transactions costs on the performance of the different portfolios, we also compute the portfolio returns net of transactions costs as

$$r_{t+1}^k = \left(1 - \kappa \sum_{j=1}^N \left| w_{j,t}^k - w_{j,(t-1)+}^k \right| \right) (w_t^k)^\top r_{t+1}, \quad (5)$$

where κ is the proportional cost to be paid. We then compute the Sharpe ratio and certainty equivalent as described above, but using the out-of-sample returns net of transactions costs.⁷

3 A Vector Autoregressive (VAR) Model of Stock Returns

We now introduce the VAR model and test its statistical significance. In Section 3.1, we state the VAR model of stock returns. In Section 3.2, we test the statistical significance of

⁶Note that the out-of-sample time series of consumption from our empirical evaluation allows us to measure variation in consumption over time, but does not allow us to measure the variation in consumption across states, because it is a single out-of-sample realized time series of consumption. As a consequence, the *out-of-sample* certainty equivalent of the discounted Epstein-Zin utility depends only on the elasticity of intertemporal substitution ψ , but not on the risk aversion parameter γ .

⁷In Section 6.5 we also consider the case where the investor's optimization problem incorporates transaction costs explicitly.

the VAR model for the six datasets listed in Table 1. Finally, in Section 3.3, we use statistical tests to understand the nature of the relation between stock returns in consecutive periods.

3.1 The VAR model

We use the following vector autoregressive (VAR) model to capture serial dependence in stock returns:⁸

$$r_{t+1} = a + Br_t + \epsilon_{t+1}, \quad (6)$$

where $r_t \in \mathbb{R}^N$ is the stock return vector for period t , $a \in \mathbb{R}^N$ is the vector of intercepts, $B \in \mathbb{R}^{N \times N}$ is the matrix of slopes, and ϵ_{t+1} is the error vector, which is independently and identically distributed as a multivariate normal with zero mean and covariance matrix $\Sigma_\epsilon \in \mathbb{R}^{N \times N}$, assumed to be positive definite.

Our VAR model considers multiple stocks and assumes that tomorrow's expected return on each stock (conditional on today's return vector) may depend linearly on today's return on any of the multiple stocks. This linear dependence is characterized by the slope matrix B (for instance, B_{ij} represents the marginal effect of $r_{j,t}$ on $r_{i,t+1}$ conditional on r_t). Thus, our model is sufficiently general to capture any linear relation between stock returns in consecutive periods, independent of whether its source is momentum, mean-reversion, or some other time-series feature of the data.

In this section, we assume that r_t is a jointly covariance-stationary process with mean $\mu = E[r_t]$ and autocovariance matrices $\Gamma_k = E[(r_{t-k} - \mu)(r_t - \mu)^\top]$ for $k = 0, 1$. We also assume that the covariance matrix Γ_0 is positive definite.

3.2 Significance of the VAR model

To test whether the VAR model is statistically significant, we use the VAR-significance test in Tiao and Box (1981). The null hypothesis is

$$H_0 : B = 0, \quad (7)$$

where B is the matrix of slopes in equation (6). The test statistic for hypothesis (7) is

$$M = -(\tau - N - 2.5) \ln \left(\frac{|\hat{\Omega}_1|}{|\hat{\Gamma}_0|} \right), \quad (8)$$

⁸To conserve space, we report only the results for the first-order vector autoregressive model, VAR(1), which is given in equation (6), but we have also estimated a general p th-order vector autoregressive model, VAR(p) using Schwarz's Bayesian criterion (Schwarz (1978)) to choose the order, and we have found the order $p = 1$ to be optimal.

where $\hat{\epsilon}$ is the matrix of residuals obtained after fitting the VAR equation (6) to the data, and $\hat{\Omega}_1 = \frac{1}{\tau-3} \hat{\epsilon}^\top \hat{\epsilon}$ is the residual covariance matrix. Under general conditions (see Tiao and Box (1981)) M follows asymptotically a chi-squared distribution with N^2 degrees of freedom.

We perform this test every year (roughly 250 days) of the time period spanned by each of the six datasets listed in Table 1, using an estimation window of 1,000 days each time. For more than 97% of the estimation windows, the test rejects hypothesis (7) at a 1% significance level. Hence, we infer that the VAR model is statistically significant for the six datasets we consider.

3.3 Interpretation of the VAR model

To understand the specific character of the serial dependence in stock returns, we test the significance of each of the elements of the estimated slope matrix B . For exposition purposes, we first study two small datasets with only two assets each, and we then provide summary information for the full datasets.

3.3.1 Results for two portfolios formed on size

We consider a dataset with one small-stock portfolio and one large-stock portfolio. The return on the first asset is the average equally-weighted return on the three small-stock portfolios in the 6FF dataset with six assets formed on size and book-to-market, and the return on the second asset is the average equally-weighted return on the three large-stock portfolios.

We first estimate the VAR model for a particular 1,000-day estimation window and test the significance of each element (i, j) of the matrix of slopes B with the null hypothesis: $H_0 : B_{ij} = 0$. Using ordinary least squares or maximum likelihood estimation, we have under general conditions that the estimated elements of the matrix of slopes are asymptotically normal. The estimated VAR model (including only the intercepts and slopes that are significant at the 5% level) is:

$$\begin{aligned} r_{t+1,\text{small}} &= 0.0005 + 0.138r_{t,\text{small}} + 0.198r_{t,\text{big}}, \\ r_{t+1,\text{big}} &= 0.0006 + 0.196r_{t,\text{big}}. \end{aligned}$$

The significance of the $(1, 2)$ element of matrix B indicates that small-stock returns depend on the returns of big stocks in the previous period. On the other hand, because the $(2, 1)$ element of matrix B is not significant, we have no evidence that big-stock returns depend

on small-stock returns on the previous period. Also, both small and large-stock portfolio returns have significant first-order autocorrelations. This indicates that, besides significant first-order autocorrelations, there is also a significant lead-lag relation between the return on large stocks and the return on small stocks.

To test whether this result holds for the entire period spanned by the dataset, we estimate the VAR model for every year covered by the dataset using an estimation window of 1,000 days each time. The conclusion is that for 47% of the years, the (1, 2) element of matrix B is significant at a 1% level, consistent with the findings of Lo and MacKinlay (1990); and for 86% of the years, the (2, 1) element of matrix B is *not* significant at the 1% level. Moreover, the diagonal terms of B are significant at the 1% significance level for more than 50% of the years.

3.3.2 Results for two portfolios formed on book-to-market ratio

We now study a second dataset with one low book-to-market stock portfolio (growth portfolio) and one high book-to-market stock portfolio (value portfolio). The return on the first asset is the average equally-weighted return on the two portfolios corresponding to low book-to-market stocks in the 6FF dataset, and the return on the second portfolio is the average equally-weighted return on the two portfolios corresponding to high book-to-market stocks.

The estimated VAR model for a particular 1,000-day estimation window (reporting only the intercepts and slopes that are significant at the 5% level) is:

$$\begin{aligned} r_{t+1,\text{growth}} &= 0.0007 + 0.441r_{t,\text{growth}} - 0.319r_{t,\text{value}}, \\ r_{t+1,\text{value}} &= 0.0006 + 0.193r_{t,\text{value}}. \end{aligned}$$

Note that the (1, 2) element of matrix B is significant and negative: growth-stock returns are negatively correlated with value-stock returns for the previous period. Moreover, the (2, 1) element of matrix B is not significant, showing no evidence that value-stock returns depend on previous growth-stock returns. Finally, growth-stock returns have large significant first-order autocorrelations.

To test whether this result holds for the entire period spanned by the dataset, we estimate the VAR model for every year using an estimation window of 1,000 days each time. The conclusion is that for 39% of the years, the (1, 2) element of matrix B is significant and negative (at a 1% level), but for 86% of the years the (2, 1) element of matrix B is not significant (at the same level). Moreover, for 61% of the years there exist significant autocorrelations in growth-portfolio returns, and for about 36% of the years the autocorrelations

for value-portfolio returns are significant. The results confirm that besides significant first-order autocorrelations, there is a negative lead-lag relation between the return on growth stocks and the return on value stocks.

3.3.3 Results for full datasets

We now summarize our findings for the six datasets listed in Table 1. Table 3 gives a *lead-lag relation table* for the 6FF dataset, which gives the percentage of years for which each of the elements of the slope matrix B estimated using a 1,000-day window is significantly different from zero at the 1% level. The symbols “+” and “−” indicate whether the element is significantly positive or negative. The table shows that there exist strong first-order autocorrelations in small-growth and big-growth portfolio returns. Moreover, there is strong evidence that big-growth portfolios lead small-growth and small-neutral portfolios, and evidence (although weaker) that small-growth portfolios lead small-neutral ones. We have obtained similar insights from the tests on the 25FF and 100FF datasets, but to conserve space we do not report the results in the manuscript.

We now turn to the industry datasets, and to make the interpretation easier, we start with the dataset with five industry portfolios downloaded from Ken French’s website. Table 4 gives the lead-lag relations for this dataset, where the following labels are used for the five industries: Cnsmr (Consumer Durables, NonDurables, Wholesale, Retail, and Some Services), Manuf (Manufacturing, Energy, and Utilities), HiTec (Business Equipment, Telephone and Television Transmission), Hlth (Healthcare, Medical Equipment, and Drugs), Other (Mines, Constr, BldMt, Trans, Hotels, Bus Serv, Entertainment, Finance). We observe from Table 4 that there exist strong first-order autocorrelations in HiTec, Hlth and Other-portfolio returns. Moreover, there is strong evidence that HiTec-industry companies lead all the other companies, and evidence (although weaker) that Manuf-industry companies lead (in a negative manner) Cnsmr and HiTec-companies. The conclusions are similar for the 10Ind and 48Ind datasets, but we do not report the results in the manuscript to conserve space.

Finally, for the 100CRSP dataset, we find that around 20% of the stocks lead other stocks (more than 50% of the time periods), and around 35% of the stocks follow other stocks (more than 50% of the time periods).

4 Analysis of VAR Arbitrage Portfolios

To gauge the potential of the VAR model for portfolio selection, we study the performance of an arbitrage (zero-cost) portfolio based on the VAR model, and compare it analytically and empirically to that of other arbitrage portfolios considered in the literature.

4.1 Analytical comparison

In this section, we compare *analytically* the expected return of the VAR arbitrage portfolio to that of the contrarian arbitrage portfolio studied by Lo and MacKinlay (1990).⁹

4.1.1 The contrarian arbitrage portfolio

To study whether contrarian profits are due exclusively to market overreaction, Lo and MacKinlay (1990) consider the following contrarian (“c”) arbitrage portfolio:

$$w_{c,t+1} = -\frac{1}{N}(r_t - r_{et}e), \quad (9)$$

where $e \in \mathbb{R}^N$ is the vector of ones and $r_{et} = e^\top r_t / N$ is the return of the equally-weighted portfolio at time t . Note that the weights of this portfolio add up to zero, and thus it is an *arbitrage* portfolio. Also, the portfolio weight for every stock is equal to the negative of the stock return in excess of the return of the equally-weighted portfolio. That is, if a stock obtains a high return at time t , then the contrarian portfolio assigns a negative weight to it for period $t + 1$, and hence this is a *contrarian* portfolio. Lo and MacKinlay (1990) show that the expected return of the contrarian arbitrage portfolio is:

$$E[w_{ct}^\top r_t] = C + O - \sigma^2(\mu), \quad (10)$$

where

$$C = \frac{1}{N^2}(e^\top \Gamma_1 e - \text{tr}(\Gamma_1)), \quad (11)$$

$$O = -\frac{N-1}{N^2}\text{tr}(\Gamma_1), \quad (12)$$

$$\sigma^2(\mu) = \frac{1}{N} \sum_{i=1}^N (\mu_i - \mu_m)^2, \quad (13)$$

and where μ_i is the mean return on the i th stock, μ_m is the mean return on the equally-weighted portfolio, and “tr” denotes the trace of matrix. Note that C is a positive multiple

⁹Note that Lo and MacKinlay (1990) did not propose the contrarian strategy as a practical investment strategy for choosing portfolios of stocks, but rather to show that contrarian profits are not necessarily due to stock market overreaction. We, however, find the comparison between the VAR and contrarian arbitrage portfolios helpful in the context of testing the potential of the VAR model for portfolio selection.

of the sum of the cross covariances of stock returns, O is a negative multiple of the sum of the autocovariances, and $\sigma^2(\mu)$ is the cross-sectional variance of expected stock returns. Therefore, equation (10) shows that the contrarian arbitrage portfolio has a positive expected return if the cross covariances are positive, the autocovariances are negative, and their combined effect on the expected return, measured through the sum $C + O$, is larger than the cross-sectional variance of expected stock returns; that is, if $C + O > \sigma^2(\mu)$.

4.1.2 The VAR arbitrage portfolio

We consider the following VAR (“ v ”) arbitrage portfolio:

$$w_{v,t+1} = \frac{1}{N}(a + Br_t - r_{vt}e), \quad (14)$$

where $a + Br_t$ is the VAR model forecast of the stock return at time $t + 1$ conditional on the return at time t , and $r_{vt} = (a + Br_t)^\top e/N$ is the VAR model prediction of the equally-weighted portfolio return at time $t + 1$ conditional on the return at time t . Note that the weights of $w_{v,t+1}$ add up to zero, and thus it is also an arbitrage portfolio. Also, the portfolio $w_{v,t+1}$ assigns a positive weight to those stocks whose VAR-based conditional expected return is above that of the equally-weighted portfolio, and a negative weight to the rest of the stocks.

The following proposition gives the expected return of the VAR arbitrage portfolio, and shows that it is positive in general. For tractability, in the proposition we assume we can estimate the VAR model exactly, and hence we set $B = \Gamma_1^\top \Gamma_0^{-1}$ and $a = (I - B)\mu$, which are the VAR parameters that result in a stock return process with an expected return equal to μ , a covariance matrix equal to Γ_0 , and a lag-1 autocovariance matrix equal to Γ_1 . Note that we do *not* make this assumption in our empirical analysis in Section 4.2, and instead estimate the VAR model from empirical data.

Proposition 1 *Assume that r_t is a jointly covariance-stationary process with mean μ and autocovariance matrices $\Gamma_k = E[(r_{t-k} - \mu)(r_t - \mu)^\top]$ for $k = 0, 1$. Assume also that the covariance matrix Γ_0 is positive definite. Finally, assume we can estimate the VAR model exactly; that is, let $B = \Gamma_1^\top \Gamma_0^{-1}$ and $a = (I - B)\mu$. Then the expected return of the VAR arbitrage portfolio is*

$$E[w_{vt}^\top r_t] = G + \sigma^2(\mu) \geq 0, \quad (15)$$

where

$$G = \frac{\text{tr}(\Gamma_1^\top \Gamma_0^{-1} \Gamma_1)}{N} - \frac{e^\top \Gamma_1^\top \Gamma_0^{-1} \Gamma_1 e}{N^2} \geq 0, \quad (16)$$

and

$$\sigma^2(\mu) = \frac{1}{N} \sum_{i=1}^N (\mu_i - \mu_m)^2 \geq 0, \quad (17)$$

where μ_i is the i th stock mean return and μ_m is the equally-weighted portfolio mean return.

Proposition 1 shows that the expected return of the VAR arbitrage portfolio is the sum of two terms, $G + \sigma^2(\mu)$. From (16) we see that G depends only on the covariance matrix Γ_0 and the lag-one cross-covariance matrix Γ_1 , while from equation (17) we see that $\sigma^2(\mu)$ depends exclusively on the stock mean returns. Moreover, the proposition shows that each of these two terms makes a *nonnegative* contribution to the expected return of the VAR arbitrage portfolio. Furthermore, Proposition 1 also shows that the expected return of the VAR arbitrage portfolio is *strictly* positive in general because $\sigma^2(\mu) > 0$ except for the degenerate case where all assets have the same expected return.

4.1.3 Analytical comparison

Proposition 1 shows that the VAR arbitrage portfolio can always exploit the structure of the covariance and cross-covariance matrix, as well as that of the mean stock returns, to obtain a strictly positive expected return. This result contrasts with that obtained for the contrarian arbitrage portfolio. Essentially, the VAR arbitrage portfolio can exploit the autocorrelations and cross-autocorrelations in stock returns regardless of their sign, whereas, as explained above, the expected return of the contrarian portfolio is positive if the autocorrelations are positive and the cross correlations negative.

Note also that the cross-sectional variance of mean stock returns enters the expression for the contrarian portfolio expected return as a negative term, but it enters the expression for the VAR portfolio expected return as a positive term. The reason for this is that the contrarian portfolio assigns a negative weight to those assets whose realized return at time t is above that of the equally-weighted portfolio and, as a result, the contrarian portfolio tends to assign a negative weight to assets with a mean return that is above average. This results in the negative contribution of the cross-variance of mean stock returns to the expected return of the contrarian arbitrage portfolio.

In the remainder of this section we compare the contrarian and VAR arbitrage portfolios for four different return generating processes: independently and identically distributed returns, the return generating process used by Lo and MacKinlay (1990) to model stock market overreaction, the case where stock returns follow the VAR model defined in Equation

(6), and the return generating process used by Khandani and Lo (2010) to model market autocorrelation and liquidity shocks.¹⁰

Example 1. Independently and identically distributed (iid) stock returns

The following proposition shows that for the case with iid returns, the contrarian arbitrage portfolio has a negative expected return $E[w_{ct}^\top r_t] = -\sigma^2(\mu)$, whereas the VAR arbitrage portfolio has a positive expected return $E[w_{vt}^\top r_t] = \sigma^2(\mu)$. The intuition for this result is that when stock returns are iid, the contrarian arbitrage portfolio tends to assign a negative weight to stocks with high mean return, because these stocks tend to have returns above that of the equally-weighted portfolio. This results in a negative contribution to the expected return. Moreover, this negative contribution *cannot* be compensated by the stock return autocovariances and the lag-one cross-covariances because they are all zero for the case with iid returns. The VAR arbitrage portfolio, on the other hand, is able to exploit the difference in mean returns for the different stocks and, as a result, has a positive expected return. This is summarized in the following proposition.

Proposition 2 *Let the conditions of Proposition 1 hold. Moreover, let stock returns be independently and identically distributed. Then, the contrarian arbitrage portfolio has a negative expected return $E[w_{ct}^\top r_t] = -\sigma^2(\mu)$, whereas the VAR arbitrage portfolio has a positive expected return $E[w_{vt}^\top r_t] = \sigma^2(\mu)$.*

Example 2. Stock market overreaction

Lo and MacKinlay (1990) model stock market overreaction through a situation where all stock return autocovariances are negative and all cross-covariances are zero; that is, they argue that overreaction is characterized by negative stock return autocovariances, and choose zero cross-covariances for simplicity. We now compare the expected return of the contrarian and VAR arbitrage portfolios for this particular case; that is, for the case where the cross-covariance matrix Γ_1 is diagonal with elements $\gamma_i < 0$. The following proposition states that if elements γ_i are not too negative (that is, if the stock return autocovariances are not too negative), then the VAR policy attains a better expected return than the contrarian policy. Note that the assumption that stock return autocovariances are not too negative is not a strong one; for instance, Campbell, Lo, and MacKinlay (1997, page 74) show that “individual security returns are weakly negatively autocorrelated.”

¹⁰It is easy to show that only under rather extreme conditions on the return generating process will the contrarian and VAR arbitrage portfolio coincide. In particular, they coincide if the mean returns of all stocks are identical, and the autocovariance matrices satisfy $\Gamma_1 = -\Gamma_0$. This latter condition is particularly restrictive, as we would typically expect the covariances of stock returns to be of a much larger magnitude than the autocovariances.

Proposition 3 *Let the conditions of Proposition 1 hold. Moreover, let the cross-covariance matrix Γ_1 be diagonal with the vector $\gamma < 0$ containing the diagonal terms, and let $\tilde{\gamma}$ be the vector containing the elements in the diagonal of Γ_0^{-1} . If*

$$\frac{N-1}{N^2} \sum_i \gamma_i \geq -2\sigma^2(\mu) - G, \quad (18)$$

where $G = \frac{1}{N}(\sum_i \gamma_i^2 \tilde{\gamma}_i) - \frac{1}{N^2} \gamma^\top \Gamma_0^{-1} \gamma$, with $\sigma^2(\mu)$ and G positive numbers, then

$$E[w_{vt}^\top r_t] \geq E[w_{ct}^\top r_t]. \quad (19)$$

Example 3. Vector autoregressive stock returns

We now compare the contrarian and VAR arbitrage portfolios for the case where stock returns follow the VAR model defined in Equation (6). The following proposition states that, if the autocovariances (diagonal terms in Γ_1) are not too negative and the strict cross-covariances (off-diagonal terms) are not too positive, then the VAR policy attains a better expected return than the contrarian policy.

Proposition 4 *Let the conditions of Proposition 1 hold. Moreover, let the returns follow the VAR model defined in (6), where Σ_ϵ is the covariance matrix of the error vector. If*

$$\frac{1}{N} \text{tr}(\Gamma_1) - \frac{1}{N^2} e^\top \Gamma_1 e \geq -2\sigma^2(\mu) - G, \quad (20)$$

where $G = \frac{1}{N} \text{tr}(\Gamma_0 - \Sigma_\epsilon) - \frac{1}{N^2} e^\top (\Gamma_0 - \Sigma_\epsilon) e$, with $\sigma^2(\mu)$ and G positive numbers, then

$$E[w_{vt}^\top r_t] \geq E[w_{ct}^\top r_t]. \quad (21)$$

Example 4. A model with market autocorrelation and liquidity shocks

To capture the effect of market return autocorrelation and liquidity shocks on stock returns, Khandani and Lo (2010) consider a linear multivariate two-factor model of stock returns, where the first factor is the market return, which follows a first-order autoregressive process, and the second factor represents the cumulative effect of liquidity shocks on stock returns, and follows an infinite-order moving-average process. Specifically, they consider the following model:

$$r_t = a + \beta \nu_t + \lambda_t + \eta_t, \quad (22)$$

$$\nu_t = \rho \nu_{t-1} + \zeta_t, \quad (23)$$

$$\lambda_t = \Theta \lambda_{t-1} - \epsilon_t + \epsilon_{t-1}, \quad (24)$$

where $r_t \in \mathbb{R}^N$ is the stock return vector, $a \in \mathbb{R}^N$ is the vector of intercepts, $\nu_t \in \mathbb{R}$ is the market return, $\beta \in \mathbb{R}^N$ is the vector of stock betas, $\lambda_t \in \mathbb{R}^N$ represents the cumulative

effect of the liquidity shocks on the stock return vector, ζ_t is a univariate white-noise random variable, η_t and ϵ_t are N -dimensional white-noise random variables with diagonal covariance matrix; that is, the components of η_t and ϵ_t are serially and cross-sectionally uncorrelated, $\rho \in (-1, 1)$ is the market return serial autocorrelation, and Θ is the diagonal matrix $\Theta = \text{diag}(\theta_1, \theta_2, \dots, \theta_N)$ with $\theta_i \in (0, 1)$.

The following proposition characterizes the relation between the expected return of the VAR and contrarian arbitrage portfolios for this linear factor model. For expositional purposes, we assume that for all i , $\theta_i = \theta$ and $\sigma_{\lambda_i} = \sigma_\lambda$, where σ_{λ_i} is the unconditional standard deviation of the i th component of λ_t ; it is, however, straightforward to extend the analysis to the general case and the insights are similar.

Proposition 5 *Let the conditions of Proposition 1 hold. Moreover, let the returns follow the linear factor model defined in (22)–(24), and let $\theta_i = \theta$ for all i , and $\sigma_{\lambda_i} = \sigma_\lambda$ for all i . Then, if*

$$\rho\sigma_v^2\sigma(\beta)^2 - \left(\frac{N-1}{N}\right) \left(\frac{1-\theta}{2}\right) \sigma_\lambda^2 \geq -2\sigma^2(\mu) - G, \quad (25)$$

where σ_v^2 is the unconditional market return variance, $\sigma(\beta)^2$ is the cross-sectional variance of the beta coefficients, both $\sigma^2(\mu)$ and G are positive numbers, and G is given by equation (16) with

$$\Gamma_0 = \beta\beta^T\sigma_v^2 - \frac{1-\theta}{2\theta}\sigma_\lambda^2 I + \Sigma_\eta, \quad (26)$$

$$\Gamma_1 = \beta\beta^T\rho\sigma_v^2 - \frac{1-\theta}{2\theta}\sigma_\lambda^2 I, \quad (27)$$

then $E[w_{vt}^\top r_t] \geq E[w_{ct}^\top r_t]$.

Proposition 5 indicates that provided the market autocorrelation ρ is not too negative, and the variance of the cumulative effect of the liquidity σ_λ^2 is not too large, then the expected return of the VAR arbitrage portfolio will be higher than that of the contrarian arbitrage portfolio.

To gain some understanding about the magnitude of the market return autocorrelation necessary for the VAR arbitrage portfolio to outperform the contrarian portfolio, we compare the expected return of these two arbitrage portfolios for the specific calibration of the parameters used by Khandani and Lo (2010) in Figure 5 of their paper. Specifically, we assume that there are 100 stocks, $\Theta = I/2$, the cross-sectional variance of stock mean returns $\sigma^2(\mu) = 0$, the market return annual standard deviation $\sigma_v = 0.2$, $\sigma_\lambda^2 = \sigma_v^2/50$, and we generate the stock betas randomly using a Normal distribution with mean one and

standard deviation 1/16. For this specific calibration, we find that the VAR arbitrage portfolio attains a higher expected return provided the serial autocorrelation coefficient of the market return ρ is larger than 5%. Moreover, note that in this calibration it is assumed that the cross-sectional variance of stock mean returns is zero $\sigma^2(\mu) = 0$, which benefits the contrarian arbitrage portfolio relative to the VAR arbitrage portfolio. Thus, our result from comparing the two portfolios for the case of a model with market autocorrelation and liquidity shocks suggests that the VAR arbitrage portfolio will attain higher expected returns provided the market return autocorrelation is not very negative.

4.2 Empirical comparison

In this section, we compare *empirically* the performance of the VAR arbitrage portfolio to: (i) the contrarian arbitrage portfolio, (ii) an arbitrage portfolio based on sample mean returns, (iii) an arbitrage portfolio based on size, and (iv) an arbitrage portfolio based on book-to-market ratio. We first compare the *in-sample* expected return of the contrarian and VAR arbitrage portfolios by using the analytical expressions in Equation (10) and Proposition 1. We then compare the *out-of-sample* expected return and Sharpe ratio of the different arbitrage portfolios, using the rolling horizon methodology described in Section 2.2.

4.2.1 In-sample comparison of performance

The first panel in Table 5 gives the in-sample values of C , O , $\sigma^2(\mu)$, G , as well as the in-sample expected returns of the contrarian and VAR arbitrage portfolios, which are calculated using equations (10)–(13) and (15)–(16), for the six datasets listed in Table 1. The results show that the contrarian portfolio achieves a positive in-sample expected return only for the 100CRSP dataset. This is not surprising because the contrarian strategy makes sense in the context of individual stocks, as is the case for the 100CRSP dataset. The rest of the datasets we consider consist of assets that are portfolios of stocks, and it is well known—see Campbell, Lo, and MacKinlay (1997)—that portfolio returns have positive autocorrelation, which implies that O is negative, and hence the contrarian strategy has a negative expected return. Note also that the in-sample expected return of the VAR arbitrage portfolio is positive for all datasets and larger than that of the contrarian portfolio for the 100CRSP.

4.2.2 Out-of-sample comparison of performance

We now compare the out-of-sample expected return and Sharpe ratio of the VAR arbitrage portfolio to that of four other arbitrage portfolios: (i) the contrarian arbitrage portfolio

(CON) given in (9); (ii) an arbitrage portfolio based on the *unconditional* sample mean return (UNC), which we compute as:

$$w_{s,t+1} = \frac{1}{N} \left(\hat{\mu} - \frac{\hat{\mu}^\top e}{N} e \right), \quad (28)$$

where $\hat{\mu}$ is the sample mean return vector, and $\hat{\mu}^\top e/N$ is the equally-weighted portfolio sample mean return; that is, this portfolio assigns a positive weight to stocks that have a larger sample mean return than the equally-weighted portfolio, and a negative weight to the rest; (iii) an arbitrage portfolio formed by going long small stocks and short big stocks; and, (iv) an arbitrage portfolio formed by going long high book-to-market stocks and going short low book-to-market stocks. To make a fair comparison between the expected return of the different arbitrage portfolios, we normalize the arbitrage portfolios so that the sum of all positive weights equals one for both portfolios.¹¹

The second and third panels in Table 5 give the out-of-sample expected returns and Sharpe ratios, respectively, of the contrarian, VAR, and unconditional arbitrage portfolios computed using the rolling-horizon methodology described in Section 2.2. We first compare the VAR and contrarian arbitrage portfolios. For the 100CRSP dataset, the average out-of-sample expected return of the VAR arbitrage portfolio is around 20% higher than that of the contrarian arbitrage portfolio. Moreover, the contrarian portfolio gets negative expected returns for all other datasets, while the VAR arbitrage portfolio obtains positive expected returns for all datasets, which are also substantially larger than the expected returns of the contrarian arbitrage portfolio in absolute value.¹² Similar results are obtained for the out-of-sample Sharpe ratios. The VAR arbitrage portfolio attains positive Sharpe ratios for all datasets, while the contrarian arbitrage portfolio attains a negative Sharpe ratio for all datasets except the 100CRSP dataset, where its Sharpe ratio is still substantially lower than that of the VAR arbitrage portfolio. As with the in-sample results in the previous subsection, the reason for the negative value of the out-of-sample expected return and Sharpe ratio of the arbitrage contrarian portfolio is that the assets in the Fama-and-French and industry datasets are portfolios of stocks, which tend to be positively autocorrelated, and it is intuitively clear that contrarian portfolios will, in general, have negative returns when applied to datasets with positively autocorrelated assets. We also observe that the out-of-sample expected return and Sharpe ratio of the VAR arbitrage portfolio are much larger than those of the arbitrage portfolio based on the unconditional sample mean.

¹¹We have tested also the raw (non-normalized) arbitrage portfolios, and the insights are similar.

¹²Therefore the VAR arbitrage portfolio outperforms also the arbitrage portfolio obtained by reversing the sign of the contrarian portfolio weights.

We also compare the VAR arbitrage portfolio to the small-minus-big (SMB) and the high-minus-low (HML) arbitrage portfolios on the datasets formed on size and book-to-market ratio.¹³ The results, not reported in the tables to conserve space, show that both the expected return and the Sharpe ratio of the VAR arbitrage portfolio are substantially larger than those of the SMB and HML arbitrage portfolios.¹⁴

Last but not least, the VAR arbitrage portfolio attains surprisingly high out-of-sample Sharpe ratios (ranging from 1.98 for the 100CRSP dataset to 5.35 for the 25FF dataset). We must note, however, that these high Sharpe ratios are associated with very high trading volumes, and hence it is not clear whether the VAR arbitrage portfolios can be implemented in the presence of transaction costs. We study this issue in the next section, where we evaluate the performance of the conditional mean-variance portfolios from VAR both in the absence and in the presence of transaction costs.

5 Analysis of VAR Mean Variance Portfolios

In this section, we describe the various investment (positive-cost) portfolios whose out-of-sample empirical performance we analyze. We discuss in Section 5.1 portfolios that ignore stock return serial dependence, in Section 5.2 portfolios that exploit stock return serial dependence, and in Section 5.3 shortsale- and norm-constrained portfolios as well as the impact of proportional transactions costs. In Section 5.4, we analyze what proportion of the gains from exploiting serial dependence in stock returns comes from exploiting auto-covariances in stock returns and what proportion comes from exploiting cross-covariances.

5.1 Portfolios that *ignore* stock return serial dependence

We describe below three portfolios that do not take into account serial dependence in stock returns: the equally-weighted ($1/N$) portfolio, the minimum-variance portfolio, and the mean-variance portfolio.

¹³For the 6FF dataset, the small minus big arbitrage portfolio is computed by assigning an equal positive weight to the 3 small stock portfolios and an equal negative weight to the 3 large stock portfolios; for the 25FF dataset by assigning an equal positive weight to the 5 small stock portfolios and an equal negative weight to the 5 large stock portfolios; and, for the 100FF dataset by assigning an equal positive weight to the 10 small stock portfolios and an equal negative weight to the 10 large stock portfolios. For the 6FF dataset, the high minus low arbitrage portfolio is computed by assigning an equal positive weight to the 2 high book-to-market stock portfolios and an equal negative weight to the 2 low book-to-market portfolios; for the 25FF dataset by assigning an equal positive weight to the 5 high book-to-market stock portfolios and an equal negative weight to the 5 low book-to-market portfolios; and, for the 100FF dataset by assigning an equal positive weight to the 10 high book-to-market stock portfolios and an equal negative weight to the 10 low book-to-market portfolios.

¹⁴For instance, for the 6FF dataset, the VAR arbitrage portfolio obtains a Sharpe ratio of 4.32, the SMB portfolio of 0.21, and the HML portfolio of 0.65, and for the 25FF dataset, the VAR arbitrage portfolio obtains a Sharpe ratio of 5.35 and the SMB portfolio of 0.15, and the HML portfolio of 0.74.

5.1.1 The $1/N$ portfolio

The $1/N$ portfolio studied by DeMiguel, Garlappi, and Uppal (2009) is simply the portfolio that assigns an equal weight to all N stocks. In our evaluation, we consider the $1/N$ portfolio with rebalancing; that is, we rebalance the portfolio every day so that the weights for every asset are equal.

5.1.2 The minimum-variance portfolio

The minimum-variance portfolio is the solution to the problem

$$\min_{\mathbf{w}} \quad \mathbf{w}^\top \Sigma \mathbf{w}, \quad (29)$$

$$\text{s.t.} \quad \mathbf{w}^\top \mathbf{e} = 1, \quad (30)$$

where $\Sigma \in \mathbb{R}^{N \times N}$ is the covariance matrix of stock returns, $\mathbf{w}^\top \Sigma \mathbf{w}$ is the portfolio return variance, and the constraint $\mathbf{w}^\top \mathbf{e} = 1$ ensures that the portfolio weights sum up to one. For our empirical evaluation, we consider the *traditional (unconditional) minimum-variance portfolio*, which can be computed by solving problem (29)–(30), after replacing the covariance matrix by its estimator. We use the sample covariance matrix after applying the shrinkage approach proposed by Ledoit and Wolf (2003).

5.1.3 The mean-variance portfolio

The mean-variance portfolio is the solution to:

$$\min_{\mathbf{w}} \quad \mathbf{w}^\top \Sigma \mathbf{w} - \frac{1}{\gamma} \mathbf{w}^\top \boldsymbol{\mu}, \quad (31)$$

$$\text{s.t.} \quad \mathbf{w}^\top \mathbf{e} = 1, \quad (32)$$

where $\boldsymbol{\mu}$ is the mean stock return vector and γ is the risk-aversion parameter. For our empirical evaluation, we compute the *traditional (unconditional) mean-variance portfolio* by solving problem (31)–(32) after replacing the mean stock return vector by its sample estimate, and the covariance matrix by its sample estimator after applying the shrinkage approach proposed by Ledoit and Wolf (2003). We consider values of the risk aversion parameter $\gamma = \{1, 2, 10\}$.

5.1.4 Empirical performance

Table 6 gives the out-of-sample Sharpe ratio of the different portfolios together with the p-value that the Sharpe ratio is different from that of the minimum-variance portfolio.

We observe from the top panel of Table 6 that the minimum-variance portfolio (MIN) attains a substantially higher out-of-sample Sharpe ratio than the equally-weighted portfolio for all datasets. The reason is that the estimator of the covariance matrix we use (the shrinkage estimator of Ledoit and Wolf (2003) based on an estimation window of 1,000 days) is a very accurate estimator and, as a result, the performance of the minimum-variance portfolio is very good.

We also observe that the unconditional mean-variance portfolios based on the sample mean have substantially lower Sharpe ratio than the minimum-variance portfolio for every dataset and risk-aversion parameter (UNC1, UNC2, UNC10). This confirms the insight from the literature that the estimation error in the sample mean is very large, and as a result the unconditional mean-variance portfolios perform poorly out of sample.

Finally, we observe from our turnover calculations (not reported in the tables to conserve space) that the $1/N$ portfolio has a relatively low turnover ranging from 0.3% to 1.1% per day for the different datasets, the minimum-variance portfolio has a moderate turnover, ranging from 2% to 21%, but the mean-variance portfolio has a substantially higher turnover—even for the largest value of the risk aversion parameter of $\gamma = 10$, turnover ranges from 28% to 1,000% for the six different datasets.

5.2 Portfolios that exploit stock return serial dependence

We consider three portfolios that exploit stock return serial dependence. The first portfolio is the *conditional mean-variance portfolio* of a myopic investor who believes stock returns follow the VAR model. This portfolio relies on two assumptions: stock returns in consecutive periods are linearly related and the investor is myopic. The other two portfolios we consider relax each of these two assumptions. Specifically, we consider the conditional mean-variance portfolio of a myopic investor who believes stock returns follow a nonparametric autoregressive (NAR) model, which does not require that stock returns be linearly related, and we consider the dynamic portfolio of Campbell, Chan, and Viceira (2003), which is the optimal dynamic portfolio of an investor with Epstein-Zin utility function.

5.2.1 The conditional mean-variance portfolio from the VAR model

One way to exploit serial dependence in stock returns is to use the *conditional mean-variance portfolios based on the VAR model*. These portfolios are computed by solving problem (31)–(32) after replacing the mean and covariance matrix of asset returns with their conditional estimators obtained from the VAR model. Specifically, these portfolios are computed from

the mean of tomorrow’s stock return conditional on today’s stock return:

$$\mu_V = a + Br_t, \tag{33}$$

where a and B are the least squares estimators of the coefficients of the VAR model obtained from historical data, and the conditional covariance matrix of tomorrow’s stock returns:

$$\Sigma_V = \frac{1}{\tau} \sum_{i=t-\tau+1}^t (r_i - a - Br_{i-1})(r_i - a - Br_{i-1})^\top. \tag{34}$$

In addition, we apply the shrinkage approach of Ledoit and Wolf (2003) to obtain a more stable estimator of the conditional covariance matrix. We consider values of the risk aversion parameter $\gamma = \{1, 2, 10\}$. These portfolios are optimal for a myopic investor (who cares only about the returns tomorrow) who believes stock returns follow a linear VAR model.

5.2.2 The conditional mean-variance portfolio from the NAR model

One assumption underlying the VAR model is that the relation between stock returns in consecutive periods is linear. To gauge the effect of this assumption, we consider a nonparametric autoregressive (NAR) model.¹⁵ We focus on the nonparametric technique known as *nearest-neighbor regression*. Essentially, we find the set of, say, 50 historical dates when asset returns were closest to today’s asset returns, and we term these 50 historical dates the “*nearest neighbors*”. We then use the empirical distribution of the 50 days following the 50 nearest-neighbor dates as our conditional empirical distribution of stock returns for tomorrow, conditional on today’s stock returns. The main advantage of this nonparametric approach is that it does not assume that the time serial dependence in stock returns is of a linear type, and in fact, it does not make any assumptions about the type of relation between them.

The *conditional mean-variance portfolios based on the NAR model* are obtained by solving the problem (31)–(32) after replacing the mean and covariance matrix of asset returns with their conditional estimators obtained from the NAR model. That is, we use the mean of tomorrow’s stock return conditional on today’s return:

$$\mu_N = \frac{1}{k} \sum_{i=1}^k r_{t_i+1}, \tag{35}$$

where t_i for $i = 1, 2, \dots, k$ are the time indexes for the k nearest neighbors in the historical time series of stock returns, and the covariance matrix of tomorrow’s stock return conditional

¹⁵See Györfi, Kohler, Krzyzak, and Walk (1987) for an in-depth discussion of nonparametric regression and Györfi, Udina, and Walk (2008, 2007) for an application to portfolio selection, and Mizraeh (1992) for an application to exchange rate forecasting.

on today's return:

$$\Sigma_N = \frac{1}{k-1} \sum_{i=1}^k (r_{t_{i+1}} - \mu_N)(r_{t_{i+1}} - \mu_N)^\top. \quad (36)$$

In addition, we apply the shrinkage approach of Ledoit and Wolf (2003) to obtain a more stable estimator of the conditional covariance matrix. We consider values of the risk aversion parameter $\gamma = \{1, 2, 10\}$. The conditional mean-variance portfolios from NAR are the optimal portfolios of a myopic investor who believes stock returns follow a nonparametric autoregressive (NAR) model.

5.2.3 Dynamic portfolios from the VAR model

An important assumption underlying the conditional mean-variance portfolios from VAR and NAR is that the investor is myopic; that is, the investor cares only about the utility of returns tomorrow, and does not take into account returns beyond tomorrow when making investment decisions. To gauge the effect of this assumption, we consider the dynamic portfolios of Campbell, Chan, and Viceira (2003), which are an approximation to the solution of the optimal consumption and portfolio choice problem of an investor with infinite horizon and Epstein-Zin utility of consumption who believes stock returns follow the VAR model and who is not subject to shortsale constraints. As in some of the experiments in Campbell, Chan, and Viceira (2003), we set the elasticity of intertemporal substitution $\psi = 1$, and the annual time-discount factor $\delta = 0.92$. We consider risk aversion parameters $\gamma = \{1, 2, 10\}$. Campbell, Chan, and Viceira (2003) indicate that for the case with $\psi = \gamma = 1$, the approximate dynamic portfolio coincides with the log-utility portfolio and thus it gives the exact solution to the problem. For $\psi = 1$ but $\gamma > 1$, the approximate dynamic portfolios are accurate approximations of the exact solution when the time period used is small, which is the case for the daily datasets we consider.

5.2.4 Empirical performance

Table 6 gives the out-of-sample Sharpe ratios of the different portfolios.¹⁶ Our main observation is that all three portfolios that exploit stock return serial dependence substantially outperform the three traditional (unconditional) portfolios in terms of out-of-sample Sharpe ratio. Nevertheless, we also observe from our turnover calculations (not reported to conserve space) that the high Sharpe ratios of the conditional mean-variance and dynamic portfolios

¹⁶Note that we have not been able to compute the approximate dynamic portfolios for the datasets with large number of assets (48Ind, 100FF, 100CRSP) because of computational constraints. The computation of the approximate dynamic portfolios requires solving a quadratic system of equations numerically, and this task is computationally intensive.

have extremely high turnovers, much higher than those of the traditional unconditional portfolios. These turnovers decrease with the value of the risk aversion parameter,¹⁷ but are still very large even for large risk aversion parameters—the conditional mean-variance and dynamic portfolios require trading in excess of 1,000% of wealth daily for all datasets and for all values of the risk aversion parameter. Obviously, such large trading volumes render the *unconstrained* conditional mean-variance and dynamic portfolios impractical. In Section 5.3, we study how one can impose constraints on the conditional mean-variance portfolio weights to control the turnover of these portfolios and make them practical.

Another interesting insight from Table 6 is that the performance of the conditional mean-variance portfolios from NAR is slightly better than that of the conditional portfolios from VAR. The reason is that the linearity assumption underlying the VAR model results in portfolio weights that are a bit more extreme than those obtained from the NAR model. Finally, we observe that the out-of-sample Sharpe ratio of the dynamic portfolios is similar to that of the conditional mean-variance portfolios from VAR.¹⁸

5.3 Constrained portfolios

To control portfolio turnover, we consider imposing two different types of constraints on the mean-variance portfolios. First, we consider imposing shortsale constraints, which have been shown in the literature to reduce turnover and improve performance; see Jagannathan and Ma (2003). Second, we consider imposing a 1-norm-constraint on the difference between the mean-variance portfolio and the minimum-variance portfolio; see DeMiguel, Garlappi, Nogales, and Uppal (2009) for an analysis of norm constraints in the context of portfolio selection. We impose these constraints on both the unconditional and conditional portfolios. Also, to conserve space, we report the results only for a risk aversion parameter $\gamma = 2$, but we have also evaluated the portfolios corresponding to $\gamma = 1$ and 10 and the insights from these are similar.

Sections 5.3.1 and 5.3.2 describe the shortsale and norm constraints, respectively, and Sections 5.3.3 and 5.3.4 discuss the results in the absence and presence of proportional transaction costs, respectively.

¹⁷We observe from our calculations that increasing the risk-aversion parameter decreases both the out-of-sample mean and variance, but does not change the out-of-sample Sharpe ratio substantially.

¹⁸We note from our empirical results that, due to the extreme weights of the *unconstrained* conditional mean-variance and dynamic portfolios, these portfolios lead the investor (eventually) to reach negative wealth—for most datasets and most values of the risk-aversion parameter γ . Consequently, we do not report the certainty equivalent of consumption for these unconstrained portfolios.

5.3.1 Shortsale-constrained portfolios

We compute the shortsale-constrained minimum-variance and mean-variance portfolios by solving problems (29)–(30) and (31)–(32), respectively, after imposing the additional constraint that all portfolio weights must be nonnegative; that is, after imposing that $w \geq 0$.

To the best of our knowledge, there is no computationally feasible approach to compute the dynamic portfolios of Campbell, Chan, and Viceira (2003) in the presence of constraints for the case with multiple risky assets.¹⁹ As an approximation to the shortsale-constrained dynamic portfolios, we compute the *unconstrained* dynamic portfolios using the approach of Campbell, Chan, and Viceira (2003), we then set all negative portfolio weights equal to zero, and finally we scale all positive portfolio weights so that they sum to one. This is the same approach used in Brandt, Santa-Clara, and Valkanov (2009).

5.3.2 Norm-constrained portfolios

An alternative approach to control portfolio turnover is to impose a 1-norm constraint on the difference between the conditional mean-variance portfolio weights and the minimum-variance portfolio weights.

We compute the norm-constrained mean-variance portfolios by solving problem (31)–(32) after imposing the additional constraint that the norm of the difference between the mean-variance portfolio and the traditional minimum-variance portfolio is smaller than a certain threshold δ ; that is, after imposing that $\|w - w_0\|_1 = \sum_{i=1}^N |w_i - (w_0)_i| \leq \delta$, where w_0 is the unconditional minimum-variance portfolio. We use the traditional minimum-variance portfolio as the target because of the stability of its portfolio weights. We consider values of the threshold parameter $\delta = \{0.05, 0.25, 0.50\}$.

To approximate the norm-constrained dynamic portfolios, we first compute the *unconstrained* dynamic portfolios w_d using the approach of Campbell, Chan, and Viceira (2003). If the 1-norm of the difference between the unconstrained dynamic portfolio and the minimum-variance portfolio w_0 is smaller than δ , then we set the norm-constrained dynamic portfolio $w_{d,c}$ equal to the unconstrained dynamic portfolio w_d . If the difference is greater than δ , however, then we set the norm-constrained dynamic portfolio

$$w_{d,c} = w_0 + \delta(w_d - w_0) / \|w_d - w_0\|_1.$$

¹⁹Note that it is possible to compute the constrained dynamic portfolios for the case with a single state variable using numerical dynamic programming; see, for instance, Campbell, Cocco, Gomes, and Viceira (2001) and Lynch (2001). This approach, however, is difficult to apply to our framework because it would require a number of state variables equal to the number of risky assets, which would result in a discretized state space whose size grows exponentially with the number of risky assets.

5.3.3 Empirical performance

The out-of-sample Sharpe ratio, turnover, and certainty equivalent of the constrained portfolios are given in Tables 7, 8, and 9, respectively.

Shortsale constraints

The tables show that although the shortsale constraints reduce substantially the turnover of all portfolios considered (minimum-variance, unconditional and conditional mean-variance, and dynamic portfolios), they do not provide a satisfactory solution for any of the portfolios considered: for the minimum-variance portfolio the shortsale constraints worsen the performance; for the unconditional mean-variance portfolios they improve performance, but the improvement is not significant; for the conditional mean-variance portfolios they reduce turnover, but the reduction is not substantial; and, for the dynamic portfolios they reduce turnover a bit more than for the conditional portfolios, but they also worsen the performance a bit more.

Norm constraints

The out-of-sample Sharpe ratio, turnover, and certainty equivalent of the norm-constrained portfolios are given in Tables 7, 8, and 9, respectively.

We observe that imposing norm-constraints is an effective approach for reducing the turnover of the conditional mean-variance portfolios from VAR while preserving their good out-of-sample performance. Specifically, although the Sharpe ratio of the conditional mean-variance portfolios from VAR decreases, in general, when we make the norm constraint tighter (decrease δ), it stays substantially larger than the Sharpe ratio of the unconstrained minimum-variance portfolio for all datasets. Moreover, the turnover of the norm-constrained conditional mean-variance portfolios from VAR decreases drastically as we make the norm constraint tighter. For the case with $\delta = 5\%$, the turnover of the conditional mean-variance portfolio from VAR stays below 16% for all datasets, for the case with $\delta = 25\%$, it stays below 58%, and for the case with $\delta = 50\%$, it stays below 107%.

The norm constraints have an effect on the conditional mean-variance portfolios from NAR similar to that on the conditional portfolios from VAR. That is, they reduce turnover, and they also reduce Sharpe ratio, which are still substantially larger than those of the minimum-variance portfolios for all datasets. We also note that the out-of-sample Sharpe ratios of the conditional portfolios from VAR and NAR are quite similar; that is, the norm constraints tend to homogenize the performance of the VAR and NAR portfolios and reduce

the advantage of the NAR portfolios over the VAR portfolios observed in the absence of constraints.²⁰

The norm-constraints also have a similar effect on the dynamic portfolios; that is, they reduce the turnover of the dynamic portfolios, and the reduction is a bit larger than that of the turnover of the conditional mean-variance portfolios, but the norm-constrained dynamic portfolios tend to have an out-of-sample Sharpe ratio a bit smaller than that of the conditional portfolios, although it is still larger than that of the unconditional minimum-variance portfolios for the three datasets for which we compute the dynamic portfolios (6FF, 25FF, 10Ind).

We observe from our empirical results on out-of-sample mean and variance (not reported in the tables to conserve space) that the gains from using the norm-constrained portfolios come in the form of higher expected return, since the out-of-sample variance of these portfolios is much higher than that of the unconditional (traditional) portfolios; that is, stock return serial dependence can be used to obtain stock mean return forecasts that are much better than those from the traditional sample estimator.

Finally, from the last panel of Table 9, we note that the relative performance of the norm-constrained and unconstrained portfolios when evaluated in terms of out-of-sample certainty equivalent is the same as when evaluated in terms of out-of-sample Sharpe ratio.

5.3.4 Empirical performance in the presence of transaction cost

We now evaluate the relative performance of the norm-constrained portfolios in the presence of proportional transactions costs. Tables 10 and 11 give the out-of-sample Sharpe ratio and certainty equivalent, respectively, of the different portfolios after subtracting a proportional transaction cost of 5 basis points.²¹

From Table 10 we observe that, in the presence of a proportional transactions cost of 5 basis points, the norm-constrained conditional portfolios with $\delta = 5\%$ outperform the minimum-variance portfolio for all datasets except the 100CRSP dataset. The same pattern, but a bit more pronounced, is observed when we use a 1-norm constraint with a larger threshold of $\delta = 25\%$. In the presence of a transactions cost of 5 basis points, the norm-constrained conditional portfolios from VAR and NAR outperform the minimum-

²⁰We also observe that imposing a 1-norm-constraint on the difference between the unconditional mean-variance portfolio weights and the minimum-variance portfolio weights reduces the turnover of the unconditional mean-variance portfolio, and improves its out-of-sample Sharpe ratio. Comparing the norm-constrained unconditional and conditional mean-variance portfolios, we see that the norm-constrained conditional portfolios have larger turnover, but also a substantially larger Sharpe ratio than the norm-constrained unconditional portfolios.

²¹The transaction costs are subtracted as indicated in Section 2.2.

variance portfolio for the 6FF, 25FF, 10Ind, and 48Ind datasets, but their performance is similar to that of the minimum-variance portfolio for the 100FF dataset, and they are outperformed by the minimum-variance portfolio for the 100CRSP dataset. The behavior of the norm-constrained dynamic portfolios in the presence of transaction costs of 5 basis points is similar to that of the conditional mean-variance portfolios: they tend to have slightly smaller turnover than the conditional portfolios, and slightly smaller out-of-sample Sharpe ratio, but they also outperform the minimum-variance portfolio for the three small datasets for which we compute the dynamic portfolios (6FF, 25FF, 10Ind).

Table 11 shows that the same conclusion holds when performance is measured in terms of certainty equivalent. The reason why the norm-constrained conditional portfolios do not substantially outperform the minimum-variance portfolio for the 100CRSP dataset in the presence of transactions costs is that even the norm-constrained conditional portfolios have a much larger turnover (for the 100CRSP dataset) than the minimum-variance portfolio, and as a result the transactions costs erode the gains from the superior estimates of expected returns used by the conditional portfolios.²² We conclude that the norm-constrained portfolios are practical only if the transactions costs are below 10 basis points.

5.4 Exploiting auto-covariances versus cross-covariances

In this section, we investigate what proportion of the gains from exploiting time serial dependence in stock returns is obtained by exploiting auto-covariances in stock returns, and what proportion is obtained by exploiting cross-covariances. To do this, we compare the performance of the conditional mean-variance portfolios from VAR defined in Section 5.2.1, with that of a conditional mean-variance portfolio obtained from a *diagonal* VAR model, which is a VAR model estimated under the additional constraint that only the diagonal elements of the slope matrix B can be different from zero.

The results in Table 12 demonstrate that for the datasets on portfolios sorted by size and book-to-market a very substantial part of the gains comes from exploiting cross-covariances in stock returns. This is not a surprising result as we found in Section 3.3 that there exist statistically significant lead-lag relations between large and small stocks, and growth and value stocks, and these lead-lag relations imply that there are substantial cross-covariances. For the industry portfolio datasets, on the other hand, most of the gains come from exploiting the auto-covariances in stock returns. Finally, for the 100CRSP dataset a substantial part of the gains comes also from exploiting cross-covariances in stock returns.

²²Tables A1 and A2 demonstrate that in the presence of a transactions cost of 10 basis points, the conditional and dynamic portfolios no longer outperform the minimum-variance portfolio in terms of their out-of-sample Sharpe ratios or certainty equivalent.

6 Robustness Checks

In this section, we report the results of several additional analysis that we have run to test the robustness of our findings.

6.1 Performance with weekly and monthly data

We evaluate the performance of the different portfolios for the six datasets listed in Table 1, but using weekly data (with weekly rebalancing), and monthly data (with monthly rebalancing).

We find that even with weekly and monthly data the norm-constrained conditional mean-variance and dynamic portfolios with $\delta = 5\%$ generally outperform the minimum-variance portfolios in terms of Sharpe ratio on all datasets except 100CRSP, where the performance is slightly worse.²³ We also note that the performance of the norm-constrained conditional and dynamic portfolios (relative to that of the minimum-variance portfolio) improves as the frequency of the data increases. The reason for this may be that the degree of predictability in stock returns is larger as we increase the data frequency from monthly to daily.

We also find that the norm-constrained conditional and dynamic portfolios with $\delta = 5\%$ tend to outperform the minimum-variance portfolio for most weekly and monthly datasets even in the presence of proportional transactions costs of 5 basis points, but the differences are not substantial.²⁴ That is, the result that the norm-constrained conditional and dynamic portfolios outperform the unconditional portfolio for transactions costs of around to 5 basis points still holds as we decrease the frequency of the data and trading. This is a bit surprising because as one decreases the amount of trading, one would expect that the transactions costs associated with the conditional mean-variance portfolios would be smaller, and hence these portfolios would perform better than their daily-rebalanced counterparts. But as we discussed previously, the degree of predictability decreases with data frequency, and hence the advantage of trading less frequently (and thus incurring lower transactions costs) is offset by the lower degree of predictability in the lower frequency data.

The analysis in this section throws some light over the issue of whether the daily stock return serial dependence documented in this manuscript may be due solely to asynchronous or infrequent trading at low frequencies. The results for weekly and monthly return data

²³Tables A3 and A4 give the out-of-sample Sharpe ratios attained by the constrained portfolios for weekly and monthly data, respectively. For the weekly data, we use an estimation window of 260 weeks, whereas for the monthly data, we use 105 months. We need the estimation window to be larger than 100 because there are two datasets with 100 assets (100FF and 100CRSP).

²⁴Tables A5 and A6 give the out-of-sample Sharpe ratios attained by the constrained portfolios with proportional transactions costs of 5 basis points for weekly and monthly data, respectively.

show that only part of the serial dependence in daily data may be due to these features. Specifically, the performance of the norm-constrained conditional and dynamic portfolios shows that there is stock return serial dependence in the weekly and monthly data, and these weekly and monthly data are much less likely to be affected by asynchronous or infrequent trading than the daily data. This result is in agreement with the observation by Lo and MacKinlay (1990, p. 197) that the lead-lag relations in stock returns they document cannot be completely attributed to asynchronous or infrequent trading.

6.2 Using the dividend yield and the yield spread as predictors

We now study whether the out-of-sample performance of the conditional mean-variance portfolios can be improved by using as predicting variables the dividend yield and yield spread.²⁵ Note that these variables are obvious choices when expected stock returns are related to the business cycle; see Lynch (2001); Campbell, Chan, and Viceira (2003). We find that the use of predictive variables (such as the dividend yield and term spread) helps to improve slightly the out-of-sample performance of mean-variance portfolios, but the improvement is much smaller than that obtained from exploiting stock return serial dependence.²⁶

6.3 Sensitivity of portfolio returns to Fama-French & momentum factors

We compare the risk characteristics of the norm-constrained conditional mean-variance portfolios from VAR to those of the minimum-variance portfolio. In particular, we regress the portfolio returns in excess of the riskfree rate on the market return in excess of the riskfree rate (MKT), the return on the “small-minus-big” portfolio (SMB), the return on

²⁵We compute the dividend yield as the logarithm of the difference between the continuously compounded 12-month dividend yield on the value-weighted S&P500 index with and without dividends downloaded from CRSP. We compute the yield spread as the difference between the zero-coupon yield on the five-year bond obtained from the Federal Reserve website (see Gurkaynak, Sack, and Wright (2007)) and the one-month Treasury bill rate obtained from Ken French’s website.

²⁶Table A7 contains three panels that give the out-of-sample Sharpe ratios of three types of portfolios. The first panel gives the results for the unconditional (traditional) portfolios that ignore stock return serial dependence and ignore also dividend yield and yield spread predictability. The second panel gives the conditional portfolios that exploit stock return serial dependence *and* also exploit dividend yield and yield spread predictability; these portfolios include the conditional mean-variance portfolios based on VAR and NAR and the dynamic portfolios of Campbell, Chan, and Viceira (2003). And, the third panel gives the portfolios that exploit *only* dividend yield and yield spread predictability, but ignore stock return serial dependence. Comparing the second panel in Table A7 with the second panel in Table 6, we observe that practically all of the out-of-sample gains are generated by exploiting the time serial dependence in stock returns, and that there is little or no additional out-of-sample gains to be made by exploiting the dividend yield and the yield spread in addition to the serial dependence in stock returns. Moreover, the third panel in Table A7, which gives the Sharpe ratios of the portfolios that exploit only the dividend yield and yield spread, seems to confirm this finding. In particular, we note that although the portfolios that exploit only these two predicting variables outperform the unconditional (traditional) mean-variance portfolios (at least for some of the datasets), they are clearly outperformed by the portfolios that exploit only the serial dependence in stock returns. We find that the same relative results are obtained when we consider the out-of-sample certainty equivalent as the performance measure, and when we use weekly and monthly data.

the “high-minus-low” portfolio (HML), and the return on the “up-minus-down” momentum portfolio (UMD), downloaded from Ken French’s website. We run these regressions for data at the daily frequency, weekly frequency, and also the monthly frequency, for all six datasets that we are using in our analysis. In order to conserve space, the tables with these results for each of the six datasets at the daily, weekly, and monthly frequencies are not included in the manuscript, but they are available from the authors and we discuss the main insights below.

We find that the betas of the MIN, VAR_{2,5}, VAR_{2,25} and VAR_{2,50} portfolio returns on the three Fama-French factors and the momentum factor are very similar.²⁷ The similarity of the betas across these four portfolio strategies holds for daily, weekly, and monthly data across all six datasets. We also find that the R² values drop as we go from the MIN portfolio to the VAR_{2,50} portfolio,²⁸ and the values for alpha increase.²⁹

The fact that the betas of the conditional mean-variance portfolios are similar to those of the minimum-variance portfolio seems to indicate that the norm-constrained conditional mean-variance portfolios do not involve much more factor-related risk than the minimum-variance portfolio. The fact that the values for alpha are larger for the conditional portfolios, on the other hand, seems to imply that they may perform better than the minimum-variance portfolio.

6.4 Performance during periods of market crisis

In order to analyze the behavior of the different portfolios during periods of financial-market crisis, we compare their out-of-sample performance during the 2008 calendar year. We observe that the 1/*N* and minimum-variance portfolios attain negative out-of-sample Sharpe ratios for all datasets during this period. This is not surprising as this was a period of large losses in the financial markets, and the 1/*N* and minimum-variance portfolios are not very different from the market indexes that attained large negative returns during this period. Surprisingly, however, in the absence of transactions costs, the unconditional mean-variance portfolios achieve positive Sharpe ratios for all datasets except 100CRSP. And, the conditional mean-variance and dynamic portfolios also attain positive Sharpe ratios

²⁷For example, for the the 25FF data at a daily frequency, the beta on the excess market return is 0.4794 for the MIN portfolio, 0.4781 for the VAR_{2,5} portfolio, 0.4732 for the VAR_{2,25} portfolio, and 0.4632 for the VAR_{2,50} portfolio; the beta on the SMB factor is 0.2926 for the MIN portfolio, 0.2895 for the VAR_{2,5} portfolio, 0.2756 for the VAR_{2,25} portfolio, and 0.2585 for the VAR_{2,50} portfolio; the beta on the HML factor is 0.4801 for the MIN portfolio, 0.4821 for the VAR_{2,5} portfolio, 0.4915 for the VAR_{2,25} portfolio, and 0.45028 for the VAR_{2,50} portfolio; and, the beta on the UMD factor is 0.0505 for the MIN portfolio, 0.0499 for the VAR_{2,5} portfolio, 0.0483 for the VAR_{2,25} portfolio, and 0.0459 for the VAR_{2,50} portfolio.

²⁸For the 25FF dataset at a daily frequency, the R² is 0.5449 for the MIN portfolio, 0.5433 for the VAR_{2,5} portfolio, 0.5211 for the VAR_{2,25} portfolio, and 0.4667 for the VAR_{2,50} portfolio.

²⁹For the 25FF dataset at a daily frequency, the alpha is 0.0003 for the MIN portfolio, 0.0004 for the VAR_{2,5} portfolio, 0.0006 for the VAR_{2,25} portfolio, and 0.0010 for the VAR_{2,50} portfolio.

for all datasets. Moreover, we observe that, just like in regular periods, during the crisis periods the good performance of the conditional mean-variance and dynamic portfolios is accompanied by very large trading volumes. When testing the impact of transactions costs, we observe that our results from Section 5.3.4 still hold; that is, the norm-constrained portfolios outperform the minimum-variance portfolios for transactions costs of under 10 basis points.

6.5 Optimal portfolios with proportional transactions costs

We have used norm constraints to control the high turnover of the conditional mean-variance portfolios and reduce the impact of transactions costs. An alternative approach is to impose the transactions costs explicitly in the mean-variance portfolio optimization problem, and thus, obtain a portfolio that is optimal (at least in-sample) in the presence of proportional transactions costs. In particular, one could solve the following mean-variance problem with proportional transactions costs:

$$\min_{\mathbf{w}} \quad \mathbf{w}^\top \Sigma \mathbf{w} - \frac{1}{\gamma} \mathbf{w}^\top \boldsymbol{\mu} + \kappa \|\mathbf{w} - \mathbf{w}_0\|_1, \quad (37)$$

$$\text{s.t.} \quad \mathbf{w}^\top \mathbf{e} = 1, \quad (38)$$

where κ is the rate of proportional transactions cost, \mathbf{w}_0 is the portfolio before trading, $\|\mathbf{w} - \mathbf{w}_0\|_1$ is the one norm of the difference between the portfolio weights before and after trading, and hence, $\kappa \|\mathbf{w} - \mathbf{w}_0\|_1$ is the transactions cost.

We have computed the conditional mean-variance portfolios from VAR and NAR with transactions costs, but we find that their out-of-sample performance in the presence of transactions costs is poor; specifically, much worse than that of the norm-constrained conditional mean-variance portfolios studied in Section 5.3.³⁰ One explanation for this is that the objective function in (37) combines two terms that are subject to estimation error (variance and expected return) with one term that is estimation-error free (the transaction cost). As a result, what is optimal in sample (according to the objective function in (37)) performs very poorly out of sample because of the disparity in the estimation error in the first two terms and the third term.

Finally, note that there are two main differences between the norm constraint considered in Section 5.3 ($\|\mathbf{w} - \mathbf{w}_0\|_1 \leq \delta$) and the transaction cost term in equation (37). First, while the target portfolio (\mathbf{w}_0) for the transaction cost term is the portfolio before trading, the target portfolio for the norm constraint is the minimum-variance portfolio. Second, while the transaction cost is added directly to the objective function, and thus

³⁰To conserve space we have not included the tables with these results, but they are available upon request.

weighted against the expected return and the return variance, the norm constraint is imposed as a separate constraint. This second difference is the key to the better performance of the norm-constrained portfolios compared to the portfolios optimal with transaction costs because while the latter try to balance in the same objective function terms that are subject to estimation error with others that are not, the former impose the norm constraint independent of the objective function.

7 Conclusion

In this paper, we have shown that the vector-autoregressive (VAR) model is statistically significant for all six datasets considered. Moreover, we have shown (analytically and empirically) that the performance of a simple VAR zero-cost *arbitrage* portfolio compares favorably to that of other arbitrage portfolios from the literature (contrarian, unconditional sample mean, SMB, and HML). We also find that the positive-cost portfolios that exploit time-serial dependence in stock returns outperform substantially the traditional unconditional portfolios.

We find that the major limitation of the portfolios that exploit time-serial dependence is that they entail very high turnover. Our analysis shows that shortsale constraints are not very effective at reducing turnover. We demonstrate how norm-constraints on the portfolio-weight vector can be used to reduce turnover of these portfolios, while still outperforming traditional portfolios. We also consider the effect of transactions costs, and we find that the norm-constrained conditional mean-variance portfolios outperform the traditional portfolios for low transactions costs that are under 10 basis points at which only the biggest investors can trade, but not for the higher transactions costs that individual investors are likely to face.

Proofs for all the Propositions

Proof for Proposition 1

We first show that $E[\mathbf{w}_{vt}^\top r_t] = G + \sigma^2(\mu)$. To see this, note that

$$E[\mathbf{w}_{vt}^\top r_t] = c_1 - c_2, \quad (1)$$

where

$$c_1 = \frac{1}{N} E[(a + Br_{t-1})^\top r_t], \quad (2)$$

$$c_2 = \frac{1}{N} E[(r_{v,t-1}e)^\top r_t]. \quad (3)$$

We start by computing c_1 :

$$\begin{aligned} Nc_1 &= E((a + Br_{t-1})^\top r_t) \\ &= E(a^\top r_t) + E(r_{t-1}^\top B^\top r_t) \\ &= a^\top \mu + E(\text{tr}(r_{t-1}^\top B^\top r_t)) \\ &= \mu^\top (1 - B^\top) \mu + E(\text{tr}(B^\top r_t r_{t-1}^\top)) \\ &= \mu^\top \mu - \mu^\top B^\top \mu + \text{tr}(B^\top E(r_t r_{t-1}^\top)) \\ &= \mu^\top \mu - \mu^\top B^\top \mu + \text{tr}(B^\top (\Gamma_1^\top + \mu \mu^\top)) \\ &= \mu^\top \mu - \mu^\top B^\top \mu + \text{tr}(B^\top \Gamma_1^\top) + \mu^\top B^\top \mu \\ &= \mu^\top \mu + \text{tr}(\Gamma_1^\top \Gamma_0^{-1} \Gamma_1). \end{aligned} \quad (4)$$

We now compute c_2 :

$$\begin{aligned} c_2 &= \frac{1}{N} E(r_{v,t-1} e^\top r_t) \\ &= \frac{1}{N^2} E(e^\top a^\top e r_t + e^\top (r_{t-1}^\top B^\top e) r_t) \\ &= \frac{1}{N^2} e^\top (a^\top) e \mu + \frac{1}{N^2} (\text{tr}(B^\top e e^\top E(r_t r_{t-1}^\top))) \\ &= \frac{1}{N^2} ((\mu^\top e)^2 - e^\top (\mu^\top B^\top e) \mu) + \frac{1}{N^2} (e^\top \Gamma_1^\top \Gamma_0^{-1} \Gamma_1 e + e^\top (\mu^\top B^\top e) \mu). \end{aligned} \quad (5)$$

Hence

$$\begin{aligned}
c_1 - c_2 &= \frac{1}{N}(\text{tr}(\Gamma_1^\top \Gamma_0^{-1} \Gamma_1) - \frac{1}{N} e^\top \Gamma_1^\top \Gamma_0^{-1} \Gamma_1 e) + \frac{1}{N}(\mu^\top \mu - \frac{1}{N}(\mu^\top e)^2) \\
&= \frac{1}{N}(\text{tr}(\Gamma_1^\top \Gamma_0^{-1} \Gamma_1) - \frac{1}{N} e^\top \Gamma_1^\top \Gamma_0^{-1} \Gamma_1 e) + \frac{1}{N} \sum_{i=1}^N (\mu_i - \mu_m)^2, \\
&= G + \frac{1}{N} \sum_{i=1}^N (\mu_i - \mu_m)^2 = G + \sigma^2(\mu),
\end{aligned} \tag{7}$$

which proves that equation (15) holds.

We now show that the two terms on the right-hand side of (15) are nonnegative. The term $\sigma^2(\mu)$ is obviously greater than or equal to zero, so it only remains to show that G is nonnegative. To see this, note that

$$\begin{aligned}
G &= \frac{1}{N}(\text{tr}(\Gamma_1^\top \Gamma_0^{-1} \Gamma_1) - \frac{1}{N} e^\top \Gamma_1^\top \Gamma_0^{-1} \Gamma_1 e) \\
&= \frac{1}{N}(\|\Gamma_0^{-1/2} \Gamma_1\|_F^2 - \frac{1}{N} \|\Gamma_0^{-1/2} \Gamma_1 e\|_2^2),
\end{aligned} \tag{8}$$

where $\Gamma_0^{-1/2}$ is the Cholesky factor of the positive definite matrix Γ_0 , and $\|A\|_F = \sum_{i=1}^N \sum_{j=1}^N a_{ij}^2$ is the Frobenius norm of matrix A . Also, note that

$$\|\Gamma_0^{-1/2} \Gamma_1 e\|_2 \leq \|\Gamma_0^{-1/2} \Gamma_1\|_F \|e\|_2 = \sqrt{N} \|\Gamma_0^{-1/2} \Gamma_1\|_F, \tag{9}$$

where the inequality follows from basic matrix norm properties; see Golub and Loan (1996, Ch. 2). Equation (8) together with inequality (9) imply that $G \geq 0$.

Proof for Proposition 2

When stock returns are independently and identically distributed (iid), we have that $\Gamma_1 = 0$, and for this case Lo and MacKinlay (1990) show that the contrarian arbitrage portfolio has a negative expected return $E[w_{ct}^\top r_t] = -\sigma^2(\mu)$. Also, from Proposition 1 it is easy to see that in this case the expected return of the VAR arbitrage portfolio is $E[w_{vt}^\top r_t] = \sigma^2(\mu)$.

Proof for Proposition 3

Remember the contrarian and VAR portfolios are $w_{c,t+1} = -\frac{1}{N}(r_t - r_{et}e)$ and $w_{v,t+1} = \frac{1}{N}(a + Br_t - r_{vt}e)$, respectively. Then, the expected return for the VAR portfolio is

$$\begin{aligned}
E[w_{vt}^\top r_t] &= \frac{1}{N} E[(a + Br_t - r_{vt}e)^\top r_t] \\
&= \frac{1}{N} \text{tr}(\Gamma_1^\top \Gamma_0^{-1} \Gamma_1) - \frac{1}{N^2} e^\top (\Gamma_1^\top \Gamma_0^{-1} \Gamma_1) e + \sigma^2(\mu) \\
&= G + \sigma^2(\mu)
\end{aligned} \tag{10}$$

where $G = \frac{1}{N}(\sum_i \gamma_i^2 \tilde{\gamma}_i) - \frac{1}{N^2} \gamma^\top \Gamma_0^{-1} \gamma$ and $G \geq 0$ always.

On the other hand, the expected return for the contrarian policy is

$$\begin{aligned} E[\mathbf{w}_{ct}^\top r_t] &= \frac{1}{N} E[-(r_t - r_{et}e)^\top r_t] \\ &= -\frac{1}{N} \text{tr}(\Gamma_1) + \frac{1}{N^2} e^\top \Gamma_1 e - \sigma^2(\mu) \\ &= -\frac{N-1}{N^2} \sum_i \gamma_i - \sigma^2(\mu). \end{aligned} \quad (11)$$

Note $-\frac{N-1}{N^2} \sum_i \gamma_i \geq 0$ under the stock market overreaction assumption ($\gamma_i \leq 0$).

Hence, we will have $E[\mathbf{w}_{vt}^\top r_t] \geq E[\mathbf{w}_{ct}^\top r_t]$ if

$$\frac{N-1}{N^2} \sum_i \gamma_i \geq -2\sigma^2(\mu) - G, \quad (12)$$

where $\sigma^2(\mu)$ and G are both positive.

Proof for Proposition 4

Remember the contrarian and VAR portfolios are $\mathbf{w}_{c,t+1} = -\frac{1}{N}(r_t - r_{et}e)$ and $\mathbf{w}_{v,t+1} = \frac{1}{N}(a + Br_t - r_{vt}e)$, respectively. Then we have that the expected return for the VAR portfolio is

$$\begin{aligned} E[\mathbf{w}_{vt}^\top r_t] &= \frac{1}{N} E[(a + Br_t - r_{vt}e)^\top r_t] \\ &= \frac{1}{N} \text{tr}(\Gamma_1^\top \Gamma_0^{-1} \Gamma_1) - \frac{1}{N^2} e^\top (\Gamma_1^\top \Gamma_0^{-1} \Gamma_1) e + \sigma^2(\mu). \end{aligned} \quad (13)$$

Under the VAR(1) model defined in (6), the following relations hold:

$$\Gamma_0 = B\Gamma_0 B^\top + \Sigma_\epsilon, \quad (14)$$

and

$$B = \Gamma_1^\top \Gamma_0^{-1}. \quad (15)$$

Hence,

$$\Gamma_0 = \Gamma_1^\top \Gamma_0^{-1} \Gamma_1 + \Sigma_\epsilon, \quad (16)$$

from which we can obtain the following expression:

$$\Gamma_1^\top \Gamma_0^{-1} \Gamma_1 = \Gamma_0 - \Sigma_\epsilon. \quad (17)$$

Finally,

$$\begin{aligned}
E[\mathbf{w}_{vt}^\top r_t] &= \frac{1}{N} E[(a + Br_t - r_{vt}e)^\top] \\
&= \frac{1}{N} \text{tr}(\Gamma_0 - \Sigma_\epsilon) - \frac{1}{N^2} e^\top (\Gamma_0 - \Sigma_\epsilon) e + \sigma^2(\mu).
\end{aligned} \tag{18}$$

Note $G = \frac{1}{N} \text{tr}(\Gamma_0 - \Sigma_\epsilon) - \frac{1}{N^2} e^\top (\Gamma_0 - \Sigma_\epsilon) e \geq 0$ always.

On the other hand, the expected return for the contrarian policy is

$$\begin{aligned}
E[\mathbf{w}_{ct}^\top r_t] &= \frac{1}{N} E[-(r_t - r_{ct}e)^\top r_t] \\
&= -\frac{1}{N} \text{tr}(\Gamma_1) + \frac{1}{N^2} e^\top \Gamma_1 e - \sigma^2(\mu).
\end{aligned} \tag{19}$$

Hence, we will have $E[\mathbf{w}_{vt}^\top r_t] \geq E[\mathbf{w}_{ct}^\top r_t]$ if

$$\frac{1}{N} \text{tr}(\Gamma_1) - \frac{1}{N^2} e^\top \Gamma_1 e \geq -2\sigma^2(\mu) - G, \tag{20}$$

where $\sigma^2(\mu)$ and G are both positive.

Proof for Proposition 5

Khandani and Lo (2010) show that the expected return of the contrarian arbitrage portfolio for this linear factor model is $(N - 1)(1 - \theta)/(2N)\sigma_\lambda^2 - \rho\sigma_v^2\sigma(\beta)^2 - \sigma^2(\mu)$. This result and Proposition 1 yield equation (25). The expressions for Γ_0 and Γ_1 follow from the definition of the linear factor model.

Table 1: List of datasets considered

This table lists the various datasets analyzed, the abbreviation used to refer to each dataset in the tables reporting the performance of the various portfolio strategies, the time period spanned by the dataset, and the source of the data.

#	Dataset	Abrv.	Time Period	Source
1	6 Fama and French portfolios of firms sorted by size and book to market	6FF	02/01/1970–31/12/2008	K. French
2	25 Fama and French portfolios of firms sorted by size and book to market	25FF	02/01/1970–31/12/2008	K. French
3	100 Fama and French portfolios of firms sorted by size and book to market	100FF	02/01/1970–31/12/2008	K. French
4	10 industry portfolios representing the U.S. stock market	10Ind	02/01/1970–31/12/2008	K. French
5	48 industry portfolios representing the U.S. stock market	48Ind	02/01/1970–31/12/2008	K. French
6	100 stocks from S&P500 index	100CRSP	03/01/1989–31/12/2008	CRSP

Table 2: List of portfolios considered

This table lists the various portfolio strategies we consider. The last column of the table gives the abbreviation that we use to refer to the strategy in the tables in which we compare the performance of the different portfolios.

#	Model	Abbreviation
<i>Panel A: Arbitrage portfolios</i>		
Arbitrage portfolios based on returns		
1	Arbitrage contrarian portfolio based on last period return – eq. (9)	CON
2	Arbitrage portfolio based on VAR conditional mean returns – eq. (14)	VAR
3	Arbitrage portfolio based on unconditional sample mean returns – eq. (28)	UNC
Arbitrage portfolios based on firm-specific characteristics		
4	Arbitrage portfolio based on size	SMB
5	Arbitrage portfolio based on book-to-market ratio	HML
<i>Panel B: Mean-variance portfolios</i>		
Portfolios that ignore stock return serial dependence		
6	Equally-weighted ($1/N$) portfolio	$1/N$
7	Minimum-variance portfolio	MIN
8	Unconditional mean-var portfolio with risk aversion γ	$UNC\gamma$
Portfolios that exploit stock return serial dependence		
9	Conditional mean-var portfolio from VAR with risk aversion γ	$VAR\gamma$
10	Conditional mean-var portfolio from NAR with risk aversion γ	$NAR\gamma$
11	Approximate dynamic portfolio from VAR with risk aversion γ	$DYN\gamma$
12	Conditional mean-var portfolio from diagonal VAR with risk aversion γ	$DIA\gamma$
Shortsale-constrained portfolios		
13	Shortsale-constrained minimum-variance portfolio	MIN
14	Unconditional shortsale-constrained mean-var portfolio	$UNC\gamma$
15	Conditional shortsale-constrained mean-var portfolio from VAR	$VAR\gamma$
16	Conditional shortsale-constrained mean-var portfolio from NAR	$NAR\gamma$
17	Shortsale-constrained approximate dynamic portfolio from VAR	$DYN\gamma$
Norm-constrained portfolios		
18	Unconditional 1-norm-constrained mean-var portfolio with threshold δ	$UNC\gamma,\delta$
19	Conditional 1-norm-constrained mean-var portfolio from VAR with threshold δ	$VAR\gamma,\delta$
20	Conditional 1-norm-constrained mean-var portfolio from NAR with threshold δ	$NAR\gamma,\delta$
21	1-norm-constrained approximate dynamic portfolio from VAR with threshold δ	$DYN\gamma,\delta$

Table 3: Lead-lag relation for the 6FF dataset

This table shows the percentage of periods for which each of the estimated elements of the slope matrix B is significantly different from zero at the 1% level for the dataset with six portfolios formed on size and book-to-market. The symbols “+” and “-” indicate whether the element is significantly positive and negative, respectively.

	Small Growth	Sm. Neutral	Sm. Value	Big Growth	Bg Neutral	Bg Value
Small Growth	0.42(+)	0.11(+)	0.17(-)	0.50(+)	0.12(-)	0.00(+)
Small Neutral	0.25(+)	0.11(+)	0.05(-)	0.28(+)	0.05(-)	0.00(+)
Small Value	0.17(+)	0.11(+)	0.11(+)	0.19(+)	0.05(+)	0.14(+)
Big Growth	0.11(-)	0.05(+)	0.05(-)	0.64(+)	0.10(-)	0.05(-)
Big Neutral	0.17(-)	0.14(+)	0.05(-)	0.14(+)	0.14(+)	0.03(+)
Big Value	0.14(-)	0.08(+)	0.05(-)	0.14(+)	0.03(+)	0.08(+)

Table 4: Lead-lag relation for the 5Ind dataset

This table shows the percentage of periods for which each of the estimated elements of the slope matrix B is significantly different from zero at the 1% level for the dataset with five industry portfolios. The symbols “+” and “-” indicate whether the element is significantly positive and negative, respectively. The table uses the following industry labels: Cnsmr (Consumer Durables, NonDurables, Wholesale, Retail, and Some Services), Manuf (Manufacturing, Energy, and Utilities), HiTec (Business Equipment, Telephone and Television Transmission), Hlth (Healthcare, Medical Equipment, and Drugs), Other (Mines, Constr, BldMt, Trans, Hotels, Bus Serv, Entertainment, Finance).

	Cnsmr	Manuf	HiTec	Hlth	Other
Cnsmr	0.14(+)	0.21(-)	0.33(+)	0.07(+)	0.17(+)
Manuf	0.17(-)	0.17(+)	0.21(+)	0.05(+)	0.14(+)
HiTec	0.09(-)	0.21(-)	0.52(+)	0.02(-)	0.09(-)
Hlth	0.09(+)	0.05(-)	0.38(+)	0.64(+)	0.09(-)
Other	0.11(-)	0.05(-)	0.35(+)	0.00(+)	0.57(+)

Table 5: In-sample expected returns for arbitrage (zero-cost) portfolios

This table reports the in- and out-of-sample characteristics of three arbitrage portfolios—contrarian (CON), VAR, and unconditional (UNC)—for the six datasets listed in Table 1. The first panel gives the in-sample values of C , O , $\sigma^2(\mu)$, G , as well as the in-sample expected returns of the contrarian and VAR arbitrage portfolios, which are calculated using equations (10)–(13) and (15)–(16). The second and third panels give the out-of-sample expected return and Sharpe ratios, respectively, of the contrarian, VAR, and unconditional arbitrage portfolios computed using the rolling-horizon methodology described in Section 2.2.

Quantity/strategy	6FF	25FF	100FF	10Ind	48Ind	100CRSP
In-sample values of C, O, $\sigma^2(\mu)$, G						
C	0.002110	0.002938	0.003253	0.001298	0.002894	-0.000549
O	-0.002447	-0.003354	-0.003625	-0.002293	-0.003858	0.001164
$\sigma^2(\mu)$	0.000003	0.000004	0.000004	0.000001	0.000003	0.000007
G	0.000105	0.000301	0.000508	0.000207	0.000582	0.002961
In-sample expected returns						
CON	-0.000340	-0.000420	-0.000376	-0.000995	-0.000967	0.000608
VAR	0.000107	0.000304	0.000513	0.000208	0.000585	0.002968
Out-of-sample expected returns						
CON	-0.2101	-0.2065	-0.1326	-0.4354	-0.2969	0.2186
VAR	0.3728	0.4110	0.2941	0.5184	0.4065	0.2507
UNC	0.0459	0.0507	0.0415	-0.0013	0.0263	-0.0349
Out-of-sample Sharpe ratios						
CON	-2.3196	-2.5587	-1.7908	-3.3914	-2.2557	1.2551
VAR	4.3221	5.3457	4.6197	4.3738	3.7587	1.9826
UNC	0.5754	0.7310	0.7263	-0.0104	0.2205	-0.2041

Table 6: Sharpe ratios for investment (positive-cost) portfolios

This table reports the annualized out-of-sample Sharpe ratios for the different portfolios and datasets, together with the P-value that the Sharpe ratio for a strategy is different from that for the unconstrained minimum-variance portfolio. Due to computational constraints, we have not been able to compute the dynamic portfolios (DYN) for the three largest datasets (48Ind, 100FF, and 100CRSP).

Strategy	6FF	25FF	100FF	10Ind	48Ind	100CRSP
Portfolios that ignore stock return serial dependence						
1/N	0.8518 (0.00)	0.8947 (0.00)	0.9112 (0.00)	0.7654 (0.01)	0.7740 (0.16)	0.6389 (0.51)
MIN	1.7259 (1.00)	2.1961 (1.00)	2.4300 (1.00)	1.1211 (1.00)	1.0054 (1.00)	0.7710 (1.00)
UNC1	0.7957 (0.00)	1.0470 (0.00)	0.8323 (0.00)	0.4450 (0.03)	0.5389 (0.12)	-0.4722 (0.00)
UNC2	0.9006 (0.00)	1.1063 (0.01)	0.8617 (0.00)	0.5004 (0.02)	0.5595 (0.15)	-0.4554 (0.00)
UNC10	1.5028 (0.29)	1.5444 (0.04)	1.0921 (0.00)	0.8470 (0.23)	0.7119 (0.34)	-0.3168 (0.00)
Portfolios that exploit stock return serial dependence						
VAR1	3.5744 (0.00)	4.6524 (0.00)	3.6167 (0.00)	4.4365 (0.00)	4.2010 (0.00)	1.7828 (0.01)
VAR2	3.5848 (0.00)	4.6575 (0.00)	3.6182 (0.00)	4.4406 (0.00)	4.2025 (0.00)	1.7836 (0.01)
VAR10	3.6643 (0.00)	4.6976 (0.00)	3.6299 (0.00)	4.4715 (0.00)	4.2149 (0.00)	1.7895 (0.01)
NAR1	4.6409 (0.00)	5.9758 (0.00)	3.9136 (0.00)	4.8219 (0.00)	4.6265 (0.00)	1.9177 (0.00)
NAR2	4.6636 (0.00)	5.9904 (0.00)	3.9189 (0.00)	4.8330 (0.00)	4.6324 (0.00)	1.9198 (0.00)
NAR10	4.8092 (0.00)	6.0947 (0.00)	3.9608 (0.00)	4.8937 (0.00)	4.6740 (0.00)	1.9361 (0.00)
DYN1	3.3732 (0.00)	4.4641 (0.00)	-	3.4273 (0.00)	-	-
DYN2	3.3189 (0.00)	4.5272 (0.00)	-	3.3784 (0.00)	-	-
DYN10	3.3464 (0.00)	4.5915 (0.00)	-	3.3665 (0.00)	-	-

Table 7: Sharpe ratios for constrained investment (positive-cost) portfolios

This table reports the annualized out-of-sample Sharpe ratios for the different constrained portfolios and datasets, together with the P-value that the Sharpe ratio for a strategy is different from that for the unconstrained minimum-variance portfolio.

Strategy	6FF	25FF	100FF	10Ind	48Ind	100CRSP
Unconstrained portfolios						
1/N	0.8518 (0.00)	0.8947 (0.00)	0.9112 (0.00)	0.7654 (0.01)	0.7740 (0.17)	0.6389 (0.52)
MIN	1.7259 (1.00)	2.1961 (1.00)	2.4300 (1.00)	1.1211 (1.00)	1.0054 (1.00)	0.7710 (1.00)
UNC2	0.9006 (0.00)	1.1063 (0.01)	0.8617 (0.00)	0.5004 (0.02)	0.5595 (0.14)	-0.4554 (0.00)
VAR2	3.5848 (0.00)	4.6575 (0.00)	3.6182 (0.00)	4.4406 (0.00)	4.2025 (0.00)	1.7836 (0.00)
NAR2	4.6636 (0.00)	5.9904 (0.00)	3.9189 (0.00)	4.8330 (0.00)	4.6324 (0.00)	1.9198 (0.00)
DYN2	3.3189 (0.00)	4.5272 (0.00)	- -	3.3784 (0.00)	- -	- -
Shortsale-constrained portfolios						
MIN	1.1087 (0.00)	1.0809 (0.00)	0.9801 (0.00)	0.9498 (0.01)	1.0153 (0.93)	0.7456 (0.82)
UNC2	0.9067 (0.00)	1.0105 (0.00)	0.9098 (0.00)	0.5259 (0.00)	0.6212 (0.07)	0.2986 (0.13)
VAR2	2.1118 (0.01)	2.4825 (0.12)	1.7011 (0.00)	2.2831 (0.00)	1.6055 (0.00)	0.8245 (0.89)
NAR2	2.0964 (0.01)	2.5605 (0.04)	2.0390 (0.09)	1.8838 (0.00)	1.7636 (0.00)	1.1138 (0.24)
DYN2	1.7360 (0.94)	1.5738 (0.00)	- -	2.0014 (0.00)	- -	- -
Norm-constrained portfolios						
UNC2,50	1.8049 (0.10)	2.4328 (0.00)	2.4454 (0.84)	1.0404 (0.31)	1.0141 (0.96)	0.5783 (0.40)
UNC2,25	1.7757 (0.04)	2.3458 (0.00)	2.4466 (0.52)	1.1002 (0.61)	1.0765 (0.31)	0.7072 (0.69)
UNC2,5	1.7338 (0.01)	2.1965 (0.00)	2.4300 (0.00)	1.1205 (0.89)	1.0055 (0.34)	0.7658 (0.72)
VAR2,50	2.6472 (0.00)	3.7717 (0.00)	3.5154 (0.00)	2.4396 (0.00)	1.9613 (0.00)	1.0593 (0.24)
VAR2,25	2.2075 (0.00)	3.0540 (0.00)	3.1430 (0.00)	1.8390 (0.00)	1.6683 (0.00)	1.0537 (0.05)
VAR2,5	1.8208 (0.00)	2.3558 (0.00)	2.5079 (0.00)	1.2665 (0.00)	1.1624 (0.00)	0.8596 (0.00)
NAR2,50	2.6643 (0.00)	3.8362 (0.00)	4.0323 (0.00)	2.1071 (0.00)	1.8625 (0.00)	1.1720 (0.07)
NAR2,25	2.2159 (0.00)	3.0872 (0.00)	3.3778 (0.00)	1.6641 (0.00)	1.5777 (0.00)	1.1273 (0.01)
NAR2,5	1.8155 (0.00)	2.3106 (0.00)	2.4424 (0.00)	1.2235 (0.00)	1.0820 (0.00)	0.8510 (0.00)
DYN2,50	2.3140 (0.00)	2.7408 (0.00)	- -	1.9809 (0.00)	- -	- -
DYN2,25	2.0236 (0.00)	2.4708 (0.00)	- -	1.5583 (0.00)	- -	- -
DYN2,5	1.7859 (0.00)	2.2513 (0.00)	- -	1.2093 (0.00)	- -	- -

Table 8: Turnovers for constrained investment (positive-cost) portfolios

This table reports the daily turnovers for the different constrained portfolios and datasets.

Strategy	6FF	25FF	100FF	10Ind	48Ind	100CRSP
Unconstrained portfolios						
1/N	0.0026	0.0031	0.0044	0.0044	0.0064	0.0127
MIN	0.0219	0.0518	0.1002	0.0171	0.0489	0.0590
UNC2	2.4357	13.6721	57.1962	2.7263	15.6544	16.1168
VAR2	923.4151	1339.6338	11412.2774	349.0354	2707.5650	5594.6237
NAR2	138.4652	589.0565	3992.1590	189.2437	983.9278	2320.8044
DYN2	296.5781	1986.7716	-	333.1255	-	-
Shortsale-constrained portfolios						
MIN	0.0048	0.0113	0.0061	0.0050	0.0215	0.0243
UNC2	0.0289	0.0730	0.1007	0.0660	0.0759	0.0872
VAR2	1.3989	1.7604	1.9325	1.5600	1.8095	1.9259
NAR2	1.4428	1.7314	1.8318	1.5771	1.6818	1.7905
DYN2	1.1563	1.3155	-	1.2146	-	-
Norm-constrained portfolios						
UNC2,50	0.0376	0.0742	0.1469	0.0522	0.0846	0.1180
UNC2,25	0.0286	0.0631	0.1076	0.0340	0.0651	0.0914
UNC2,5	0.0225	0.0518	0.1002	0.0187	0.0489	0.0597
VAR2,50	0.6933	0.9127	1.0610	0.7820	0.9521	1.0222
VAR2,25	0.3532	0.4775	0.5776	0.3962	0.4976	0.5386
VAR2,5	0.0746	0.1084	0.1220	0.0859	0.1325	0.1521
NAR2,50	0.6895	0.8584	0.9818	0.8082	0.8877	0.9593
NAR2,25	0.3513	0.4476	0.4916	0.4090	0.4648	0.5060
NAR2,5	0.0693	0.0772	0.1007	0.0794	0.0838	0.0974
DYN2,50	0.5854	0.6639	-	0.6081	-	-
DYN2,25	0.2953	0.3383	-	0.3049	-	-
DYN2,5	0.0669	0.0902	-	0.0648	-	-

Table 9: Certainty equivalents for constrained investment (positive-cost) portfolios

This table reports the annualized out-of-sample certainty equivalent for the different constrained portfolios and datasets, together with the P-value that the certainty equivalent for a strategy is different from that for the unconstrained minimum-variance portfolio.

Strategy	6FF	25FF	100FF	10Ind	48Ind	100CRSP
Unconstrained portfolios						
1/N	0.00064 (0.17)	0.00069 (0.25)	0.00071 (0.44)	0.00047 (0.75)	0.00052 (0.85)	0.00047 (0.92)
MIN	0.00122 (1.00)	0.00118 (1.00)	0.00095 (1.00)	0.00060 (1.00)	0.00047 (1.00)	0.00040 (1.00)
Shortsale-constrained portfolios						
MIN	0.00082 (0.16)	0.00078 (0.29)	0.00072 (0.45)	0.00051 (0.66)	0.00057 (0.60)	0.00048 (0.82)
UNC2	0.00090 (0.39)	0.00076 (0.54)	0.00078 (0.66)	0.00034 (0.70)	0.00069 (0.85)	0.00035 (0.78)
VAR2	0.00899 (0.04)	0.01674 (0.01)	0.00478 (0.20)	0.04202 (0.00)	0.04493 (0.05)	0.00260 (0.55)
NAR2	0.00772 (0.05)	0.01613 (0.01)	0.00827 (0.08)	0.01509 (0.01)	0.02864 (0.02)	0.00487 (0.24)
DYN2	0.00327 (0.31)	0.00210 (0.68)	- -	0.00688 (0.00)	- -	- -
Norm-constrained portfolios						
UNC2,50	0.00147 (0.33)	0.00147 (0.13)	0.00117 (0.30)	0.00055 (0.98)	0.00057 (0.58)	0.00039 (0.97)
UNC2,25	0.00134 (0.31)	0.00132 (0.14)	0.00096 (0.66)	0.00058 (0.97)	0.00053 (0.54)	0.00039 (0.96)
UNC2,5	0.00123 (0.34)	0.00118 (0.22)	0.00095 (0.26)	0.00060 (0.97)	0.00047 (0.79)	0.00039 (0.83)
VAR2,50	0.00472 (0.00)	0.00627 (0.00)	0.00323 (0.00)	0.00660 (0.00)	0.00406 (0.01)	0.00095 (0.26)
VAR2,25	0.00240 (0.00)	0.00272 (0.00)	0.00176 (0.00)	0.00200 (0.00)	0.00138 (0.00)	0.00063 (0.21)
VAR2,5	0.00139 (0.00)	0.00137 (0.00)	0.00100 (0.00)	0.00076 (0.00)	0.00058 (0.00)	0.00043 (0.21)
NAR2,50	0.00449 (0.00)	0.00599 (0.00)	0.00385 (0.00)	0.00342 (0.00)	0.00299 (0.00)	0.00095 (0.19)
NAR2,25	0.00235 (0.00)	0.00266 (0.00)	0.00189 (0.00)	0.00143 (0.00)	0.00119 (0.01)	0.00062 (0.15)
NAR2,5	0.00137 (0.00)	0.00129 (0.00)	0.00096 (0.00)	0.00070 (0.00)	0.00054 (0.03)	0.00042 (0.12)
DYN2,50	0.00285 (0.00)	0.00205 (0.00)	- -	0.00232 (0.00)	- -	- -
DYN2,25	0.00186 (0.00)	0.00155 (0.00)	- -	0.00118 (0.00)	- -	- -
DYN2,5	0.00133 (0.00)	0.00124 (0.00)	- -	0.00069 (0.00)	- -	- -

Table 10: Sharpe ratios for constrained investment (positive-cost) portfolios and transactions costs of 5 basis points

This table reports the annualized out-of-sample Sharpe ratios for the different constrained portfolios and datasets in the presence of a proportional transactions cost of 5 basis points, together with the P-value that the Sharpe ratio for a strategy is different from that for the unconstrained minimum-variance portfolio.

Strategy	6FF	25FF	100FF	10Ind	48Ind	100CRSP
Unconstrained portfolios						
1/N	0.8496 (0.00)	0.8921 (0.00)	0.9075 (0.00)	0.7618 (0.01)	0.7688 (0.25)	0.6295 (0.68)
MIN	1.7019 (1.00)	2.1229 (1.00)	2.2608 (1.00)	1.1026 (1.00)	0.9425 (1.00)	0.7094 (1.00)
UNC2	0.5609 (0.00)	0.0812 (0.00)	-1.1456 (0.00)	0.1940 (0.00)	-0.2122 (0.00)	-1.0785 (0.00)
VAR2	-0.1789 (0.00)	-1.8043 (0.00)	-1.5664 (0.00)	-0.1004 (0.01)	-1.0153 (0.00)	-0.3887 (0.00)
NAR2	0.1778 (0.00)	-0.6349 (0.00)	-0.9190 (0.00)	-0.2256 (0.00)	-0.4943 (0.00)	-0.7531 (0.00)
DYN2	-0.5158 (0.00)	-0.6197 (0.00)	- -	0.1098 (0.00)	- -	- -
Shortsale-constrained portfolios						
MIN	1.1042 (0.00)	1.0705 (0.00)	0.9746 (0.00)	0.9449 (0.03)	0.9931 (0.69)	0.7250 (0.85)
UNC2	0.8849 (0.00)	0.9522 (0.00)	0.8366 (0.00)	0.4860 (0.00)	0.5841 (0.11)	0.2678 (0.14)
VAR2	1.0529 (0.00)	1.2193 (0.00)	0.5154 (0.00)	1.2935 (0.26)	0.8240 (0.56)	0.3053 (0.22)
NAR2	0.9863 (0.00)	1.2799 (0.00)	0.7987 (0.00)	0.8991 (0.24)	0.9382 (0.97)	0.5906 (0.70)
DYN2	0.7734 (0.00)	0.4480 (0.00)	- -	1.0293 (0.57)	- -	- -
Norm-constrained portfolios						
UNC2,50	1.7652 (0.17)	2.3313 (0.01)	2.2272 (0.74)	0.9890 (0.17)	0.9296 (0.87)	0.5005 (0.36)
UNC2,25	1.7448 (0.08)	2.2569 (0.00)	2.2667 (0.85)	1.0644 (0.36)	0.9993 (0.45)	0.6260 (0.60)
UNC2,5	1.7092 (0.01)	2.1233 (0.00)	2.2609 (0.00)	1.1003 (0.69)	0.9425 (0.28)	0.7035 (0.60)
VAR2,50	1.9058 (0.00)	2.5589 (0.00)	2.0165 (0.03)	1.6518 (0.00)	1.0619 (0.39)	0.4301 (0.25)
VAR2,25	1.8234 (0.00)	2.3914 (0.00)	2.2219 (0.48)	1.4173 (0.00)	1.0862 (0.08)	0.5938 (0.41)
VAR2,5	1.7393 (0.00)	2.2032 (0.00)	2.3025 (0.00)	1.1734 (0.00)	0.9923 (0.01)	0.7027 (0.85)
NAR2,50	1.9273 (0.00)	2.6873 (0.00)	2.5697 (0.01)	1.3004 (0.00)	0.9675 (0.84)	0.5476 (0.43)
NAR2,25	1.8343 (0.00)	2.4637 (0.00)	2.5739 (0.00)	1.2310 (0.00)	1.0193 (0.25)	0.6805 (0.87)
NAR2,5	1.7399 (0.00)	2.2015 (0.00)	2.2722 (0.00)	1.1377 (0.00)	0.9741 (0.00)	0.7496 (0.06)
DYN2,50	1.6779 (0.33)	1.8126 (0.00)	- -	1.3289 (0.00)	- -	- -
DYN2,25	1.7019 (0.95)	1.9969 (0.00)	- -	1.2287 (0.00)	- -	- -
DYN2,5	1.7129 (0.00)	2.1244 (0.47)	- -	1.1391 (0.00)	- -	- -

Table 11: Certainty equivalents for constrained investment (positive-cost) portfolios and transactions costs of 5 basis points

This table reports the annualized out-of-sample certainty equivalents for the different constrained portfolios and datasets in the presence of a proportional transactions cost of 5 basis points, together with the P-value that the certainty equivalent for a strategy is different from that for the unconstrained minimum-variance portfolio.

Strategy	6FF	25FF	100FF	10Ind	48Ind	100CRSP
Unconstrained portfolios						
1/N	0.00064 (0.17)	0.00068 (0.29)	0.00071 (0.52)	0.00047 (0.79)	0.00052 (0.79)	0.00047 (0.84)
MIN	0.00119 (1.00)	0.00111 (1.00)	0.00084 (1.00)	0.00059 (1.00)	0.00044 (1.00)	0.00038 (1.00)
Shortsale-constrained portfolios						
MIN	0.00082 (0.20)	0.00077 (0.35)	0.00072 (0.54)	0.00051 (0.70)	0.00055 (0.57)	0.00048 (0.73)
UNC2	0.00088 (0.39)	0.00069 (0.49)	0.00068 (0.67)	0.00032 (0.66)	0.00063 (0.87)	0.00032 (0.78)
VAR2	0.00173 (0.69)	0.00199 (0.91)	0.00043 (0.43)	0.00669 (0.22)	0.00504 (0.52)	0.00057 (0.84)
NAR2	0.00129 (0.54)	0.00192 (0.87)	0.00086 (0.68)	0.00223 (0.69)	0.00382 (0.41)	0.00114 (0.82)
DYN2	0.00081 (0.14)	0.00042 (0.04)	-	0.00163 (0.65)	-	-
Norm-constrained portfolios						
UNC2,50	0.00141 (0.39)	0.00135 (0.18)	0.00097 (0.42)	0.00052 (0.92)	0.00051 (0.65)	0.00036 (0.98)
UNC2,25	0.00130 (0.39)	0.00122 (0.19)	0.00084 (0.78)	0.00055 (0.93)	0.00049 (0.61)	0.00036 (1.00)
UNC2,5	0.00120 (0.36)	0.00111 (0.22)	0.00084 (0.25)	0.00058 (0.93)	0.00044 (0.78)	0.00038 (0.80)
VAR2,50	0.00208 (0.10)	0.00208 (0.01)	0.00086 (0.80)	0.00260 (0.01)	0.00126 (0.41)	0.00043 (0.92)
VAR2,25	0.00158 (0.09)	0.00152 (0.01)	0.00086 (0.77)	0.00124 (0.01)	0.00075 (0.39)	0.00041 (0.95)
VAR2,5	0.00127 (0.01)	0.00120 (0.00)	0.00086 (0.12)	0.00068 (0.00)	0.00049 (0.34)	0.00039 (1.00)
NAR2,50	0.00193 (0.09)	0.00206 (0.00)	0.00113 (0.10)	0.00127 (0.22)	0.00102 (0.61)	0.00044 (0.92)
NAR2,25	0.00152 (0.06)	0.00153 (0.00)	0.00099 (0.04)	0.00087 (0.22)	0.00068 (0.55)	0.00042 (0.88)
NAR2,5	0.00126 (0.01)	0.00118 (0.00)	0.00084 (0.00)	0.00063 (0.10)	0.00047 (0.37)	0.00040 (0.40)
DYN2,50	0.00141 (0.76)	0.00091 (0.00)	-	0.00113 (0.01)	-	-
DYN2,25	0.00131 (0.99)	0.00103 (0.00)	-	0.00082 (0.01)	-	-
DYN2,5	0.00122 (0.13)	0.00112 (0.87)	-	0.00063 (0.00)	-	-

Table 12: Sharpe ratios for conditional portfolios based on VAR and diagonal VAR model

This table reports the annualized out-of-sample Sharpe ratios for the conditional mean-variance portfolios from the VAR and diagonal VAR models, together with the P-value that the Sharpe ratio for a strategy is different from that for the unconstrained minimum-variance portfolio.

Strategy	6FF	25FF	100FF	10Ind	48Ind	100CRSP
Portfolios that ignore stock return serial dependence						
1/N	0.8518 (0.00)	0.8947 (0.00)	0.9112 (0.00)	0.7654 (0.01)	0.7740 (0.17)	0.6389 (0.49)
MIN	1.7259 (1.00)	2.1961 (1.00)	2.4300 (1.00)	1.1211 (1.00)	1.0054 (1.00)	0.7710 (1.00)
UNC2	0.9006 (0.00)	1.1063 (0.00)	0.8617 (0.00)	0.5004 (0.03)	0.5595 (0.16)	-0.4554 (0.01)
Portfolios that exploit stock return serial dependence						
VAR2	3.5848 (0.00)	4.6575 (0.00)	3.6182 (0.00)	4.4406 (0.00)	4.2025 (0.00)	1.7836 (0.00)
DIAG2	3.0512 (0.00)	3.4297 (0.00)	2.1681 (0.43)	4.0981 (0.00)	4.7707 (0.00)	1.0626 (0.43)

Additional tables

Table A1: Sharpe ratios for constrained investment (positive-cost) portfolios and transactions costs of 10 basis

This table reports the annualized out-of-sample Sharpe ratios for the different constrained portfolios and datasets in the presence of a proportional transactions cost of 10 basis points, together with the P-value that the Sharpe ratio for a strategy is different from that for the unconstrained minimum-variance portfolio.

Strategy	6FF	25FF	100FF	10Ind	48Ind	100CRSP
Unconstrained portfolios						
1/N	0.8474 (0.00)	0.8894 (0.00)	0.9037 (0.00)	0.7583 (0.02)	0.7637 (0.41)	0.6201 (0.92)
MIN	1.6780 (1.00)	2.0497 (1.00)	2.0919 (1.00)	1.0841 (1.00)	0.8795 (1.00)	0.6479 (1.00)
UNC2	0.2202 (0.00)	-0.6983 (0.00)	-2.1040 (0.00)	-0.1131 (0.00)	-0.6882 (0.00)	-1.5439 (0.00)
VAR2	-0.2882 (0.00)	-2.3073 (0.00)	-1.6387 (0.00)	-0.6896 (0.00)	-1.2224 (0.00)	-0.4204 (0.00)
NAR2	-1.9561 (0.00)	-0.9021 (0.00)	-0.9685 (0.00)	-1.5031 (0.00)	-0.5840 (0.00)	-0.8086 (0.00)
DYN2	-1.5691 (0.00)	-0.7165 (0.00)	- -	-1.0078 (0.00)	- -	- -
Shortsale-constrained portfolios						
MIN	1.0998 (0.00)	1.0601 (0.00)	0.9691 (0.00)	0.9400 (0.04)	0.9709 (0.42)	0.7044 (0.68)
UNC2	0.8631 (0.00)	0.8940 (0.00)	0.7633 (0.00)	0.4461 (0.00)	0.5469 (0.12)	0.2371 (0.17)
VAR2	-0.0040 (0.00)	-0.0431 (0.00)	-0.6726 (0.00)	0.3025 (0.00)	0.0411 (0.00)	-0.2149 (0.00)
NAR2	-0.1217 (0.00)	-0.0010 (0.00)	-0.4437 (0.00)	-0.0858 (0.00)	0.1122 (0.00)	0.0667 (0.05)
DYN2	-0.1905 (0.00)	-0.6796 (0.00)	- -	0.0569 (0.00)	- -	- -
Norm-constrained portfolios						
UNC2,50	1.7255 (0.29)	2.2298 (0.01)	2.0093 (0.40)	0.9376 (0.06)	0.8451 (0.85)	0.4227 (0.33)
UNC2,25	1.7139 (0.08)	2.1680 (0.00)	2.0870 (0.76)	1.0287 (0.21)	0.9221 (0.50)	0.5449 (0.52)
UNC2,5	1.6846 (0.02)	2.0501 (0.00)	2.0919 (0.00)	1.0801 (0.58)	0.8796 (0.36)	0.6412 (0.59)
VAR2,50	1.1643 (0.00)	1.3470 (0.00)	0.5183 (0.00)	0.8633 (0.01)	0.1618 (0.00)	-0.1998 (0.00)
VAR2,25	1.4392 (0.00)	1.7291 (0.00)	1.3021 (0.00)	0.9952 (0.02)	0.5039 (0.00)	0.1336 (0.00)
VAR2,5	1.6578 (0.00)	2.0507 (0.88)	2.0974 (0.57)	1.0803 (0.62)	0.8223 (0.00)	0.5459 (0.00)
NAR2,50	1.1888 (0.00)	1.5361 (0.00)	1.1040 (0.00)	0.4930 (0.00)	0.0717 (0.00)	-0.0773 (0.00)
NAR2,25	1.4522 (0.00)	1.8394 (0.00)	1.7691 (0.00)	0.7977 (0.00)	0.4607 (0.00)	0.2335 (0.00)
NAR2,5	1.6643 (0.02)	2.0924 (0.00)	2.1023 (0.00)	1.0518 (0.00)	0.8662 (0.16)	0.6483 (0.93)
DYN2,50	1.0414 (0.00)	0.8841 (0.00)	- -	0.6765 (0.00)	- -	- -
DYN2,25	1.3800 (0.00)	1.5231 (0.00)	- -	0.8990 (0.00)	- -	- -
DYN2,5	1.6400 (0.00)	1.9975 (0.00)	- -	1.0689 (0.00)	- -	- -

Table A2: Certainty equivalents for constrained investment (positive-cost) portfolios and transactions costs of 10 basis points

This table reports the annualized out-of-sample certainty equivalents for the different constrained portfolios and datasets in the presence of a proportional transactions cost of 5 basis points, together with the P-value that the certainty equivalent for a strategy is different from that for the unconstrained minimum-variance portfolio.

Strategy	6FF	25FF	100FF	10Ind	48Ind	100CRSP
Unconstrained portfolios						
1/N	0.00064 (0.18)	0.00068 (0.34)	0.00071 (0.66)	0.00047 (0.80)	0.00051 (0.74)	0.00047 (0.78)
MIN	0.00115 (1.00)	0.00104 (1.00)	0.00074 (1.00)	0.00057 (1.00)	0.00041 (1.00)	0.00037 (1.00)
Shortsale-constrained portfolios						
MIN	0.00081 (0.20)	0.00076 (0.38)	0.00071 (0.65)	0.00050 (0.74)	0.00054 (0.50)	0.00047 (0.68)
UNC2	0.00086 (0.38)	0.00062 (0.50)	0.00059 (0.64)	0.00029 (0.61)	0.00058 (0.89)	0.00030 (0.74)
VAR2	0.00033 (0.00)	0.00023 (0.02)	0.00004 (0.00)	0.00104 (0.37)	0.00054 (0.46)	0.00013 (0.26)
NAR2	0.00021 (0.00)	0.00022 (0.02)	0.00009 (0.01)	0.00032 (0.09)	0.00049 (0.52)	0.00026 (0.51)
DYN2	0.00020 (0.00)	0.00008 (0.00)	- -	0.00038 (0.04)	- -	- -
Norm-constrained portfolios						
UNC2,50	0.00136 (0.47)	0.00124 (0.22)	0.00081 (0.59)	0.00048 (0.81)	0.00046 (0.71)	0.00033 (0.92)
UNC2,25	0.00126 (0.44)	0.00113 (0.23)	0.00073 (0.90)	0.00053 (0.84)	0.00045 (0.64)	0.00034 (0.94)
UNC2,5	0.00117 (0.40)	0.00104 (0.24)	0.00074 (0.27)	0.00057 (0.89)	0.00041 (0.77)	0.00036 (0.77)
VAR2,50	0.00091 (0.00)	0.00068 (0.01)	0.00023 (0.00)	0.00102 (0.51)	0.00039 (0.19)	0.00019 (0.14)
VAR2,25	0.00104 (0.00)	0.00085 (0.01)	0.00041 (0.00)	0.00077 (0.54)	0.00040 (0.22)	0.00027 (0.18)
VAR2,5	0.00116 (0.14)	0.00106 (0.96)	0.00074 (0.82)	0.00061 (0.88)	0.00042 (0.31)	0.00035 (0.23)
NAR2,50	0.00082 (0.00)	0.00070 (0.05)	0.00033 (0.01)	0.00047 (0.02)	0.00034 (0.09)	0.00020 (0.23)
NAR2,25	0.00099 (0.00)	0.00087 (0.09)	0.00051 (0.08)	0.00052 (0.02)	0.00038 (0.11)	0.00028 (0.28)
NAR2,5	0.00116 (0.27)	0.00107 (0.02)	0.00074 (0.00)	0.00057 (0.16)	0.00042 (0.67)	0.00037 (0.95)
DYN2,50	0.00070 (0.00)	0.00041 (0.00)	- -	0.00055 (0.00)	- -	- -
DYN2,25	0.00091 (0.00)	0.00068 (0.00)	- -	0.00057 (0.00)	- -	- -
DYN2,5	0.00113 (0.00)	0.00100 (0.00)	- -	0.00059 (0.12)	- -	- -

Table A3: Sharpe ratios for constrained investment (positive-cost) portfolios with weekly returns

This table reports the annualized out-of-sample Sharpe ratios for the different constrained portfolios and datasets using weekly returns, together with the P-value that the Sharpe ratio for a strategy is different from that for the unconstrained minimum-variance portfolio.

Strategy	6FF	25FF	100FF	10Ind	48Ind	100CRSP
Unconstrained portfolios						
1/N	0.8738 (0.00)	0.8927 (0.00)	0.8984 (0.00)	0.8496 (0.04)	0.8026 (0.72)	0.6315 (0.14)
MIN	1.4678 (1.00)	1.7737 (1.00)	1.7736 (1.00)	1.1404 (1.00)	0.8617 (1.00)	0.9527 (1.00)
UNC2	0.9050 (0.05)	1.2328 (0.11)	1.0356 (0.02)	0.3512 (0.00)	0.4393 (0.11)	-0.4053 (0.00)
VAR2	1.5254 (0.70)	1.6316 (0.67)	0.9255 (0.00)	1.1890 (0.86)	0.9937 (0.60)	0.8420 (0.83)
NAR2	2.2741 (0.00)	2.6867 (0.00)	2.1539 (0.23)	0.8975 (0.25)	0.9579 (0.70)	0.5303 (0.22)
DYN2	1.4893 (0.70)	1.5712 (0.50)	- -	1.1523 (0.91)	- -	- -
Shortsale-constrained portfolios						
MIN	1.0298 (0.00)	1.0682 (0.00)	1.1196 (0.00)	1.0320 (0.04)	1.0334 (0.09)	0.7646 (0.16)
UNC2	0.9373 (0.00)	1.0297 (0.00)	0.9358 (0.00)	0.7053 (0.01)	0.6932 (0.49)	0.4042 (0.12)
VAR2	1.5759 (0.48)	1.5051 (0.14)	1.0448 (0.00)	0.8988 (0.15)	0.7725 (0.72)	-0.0356 (0.00)
NAR2	1.4534 (0.92)	1.6947 (0.71)	1.7200 (0.80)	0.8524 (0.10)	0.6136 (0.17)	0.5035 (0.16)
DYN2	1.3663 (0.40)	1.0462 (0.00)	- -	1.0456 (0.54)	- -	- -
Norm-constrained portfolios						
UNC2,50	1.5440 (0.10)	1.9468 (0.03)	1.9133 (0.16)	1.0653 (0.44)	0.9138 (0.65)	0.5201 (0.09)
UNC2,25	1.5157 (0.04)	1.8839 (0.01)	1.8784 (0.04)	1.1257 (0.74)	0.9105 (0.39)	0.7448 (0.21)
UNC2,5	1.4794 (0.01)	1.7990 (0.00)	1.7735 (0.98)	1.1423 (0.81)	0.8798 (0.19)	0.9339 (0.58)
VAR2,50	1.9107 (0.00)	2.3529 (0.00)	1.9338 (0.13)	1.2647 (0.08)	0.8848 (0.89)	0.5898 (0.10)
VAR2,25	1.7032 (0.00)	2.1012 (0.00)	1.9063 (0.01)	1.2297 (0.00)	0.9260 (0.29)	0.8120 (0.28)
VAR2,5	1.5168 (0.00)	1.8442 (0.00)	1.8092 (0.00)	1.1623 (0.01)	0.8862 (0.06)	0.9439 (0.75)
NAR2,50	1.8894 (0.00)	2.5136 (0.00)	2.5553 (0.00)	1.1911 (0.48)	0.8573 (0.99)	0.9204 (0.88)
NAR2,25	1.6913 (0.00)	2.1847 (0.00)	2.2362 (0.00)	1.1858 (0.25)	0.9077 (0.48)	1.0231 (0.56)
NAR2,5	1.5140 (0.00)	1.8608 (0.00)	1.8357 (0.00)	1.1534 (0.12)	0.8821 (0.12)	0.9963 (0.14)
DYN2,50	1.7277 (0.00)	1.8921 (0.00)	- -	1.3183 (0.00)	- -	- -
DYN2,25	1.6000 (0.00)	1.8338 (0.00)	- -	1.2332 (0.00)	- -	- -
DYN2,5	1.4945 (0.00)	1.7858 (0.00)	- -	1.1595 (0.00)	- -	- -

Table A4: Sharpe ratios for constrained investment (positive-cost) portfolios with monthly returns

This table reports the annualized out-of-sample Sharpe ratios for the different constrained portfolios and datasets using monthly returns, together with the P-value that the Sharpe ratio for a strategy is different from that for the unconstrained minimum-variance portfolio.

Strategy	6FF	25FF	100FF	10Ind	48Ind	100CRSP
Unconstrained portfolios						
1/N	0.7981 (0.00)	0.7978 (0.00)	0.7975 (0.00)	0.8770 (0.10)	0.7720 (0.66)	0.3748 (0.55)
MIN	1.1557 (1.00)	1.3450 (1.00)	1.3302 (1.00)	1.0826 (1.00)	0.8520 (1.00)	0.2754 (1.00)
UNC2	0.8265 (0.23)	0.9743 (0.26)	0.6911 (0.04)	0.2547 (0.00)	0.2474 (0.05)	-0.3840 (0.14)
VAR2	0.9240 (0.40)	0.7545 (0.11)	0.0616 (0.00)	0.2418 (0.00)	-0.0639 (0.00)	-0.4996 (0.01)
NAR2	0.8717 (0.31)	0.8857 (0.13)	0.8604 (0.08)	0.3442 (0.02)	0.3045 (0.07)	-0.1024 (0.36)
DYN2	0.9969 (0.60)	0.6837 (0.06)	- -	0.2442 (0.00)	- -	- -
Shortsale-constrained portfolios						
MIN	0.8889 (0.00)	0.9099 (0.00)	0.9391 (0.01)	1.0223 (0.23)	1.0106 (0.17)	0.3672 (0.44)
UNC2	0.9680 (0.09)	0.9461 (0.00)	0.9690 (0.01)	0.6848 (0.09)	0.5063 (0.16)	0.1479 (0.72)
VAR2	1.0571 (0.48)	1.0172 (0.05)	0.5959 (0.00)	0.8907 (0.29)	0.2424 (0.03)	0.3418 (0.82)
NAR2	0.9486 (0.16)	0.8933 (0.00)	0.9629 (0.03)	0.8308 (0.14)	0.7187 (0.60)	-0.0701 (0.26)
DYN2	0.9553 (0.09)	0.8563 (0.00)	- -	0.9554 (0.28)	- -	- -
Norm-constrained portfolios						
UNC2,50	1.2384 (0.18)	1.4053 (0.50)	1.3512 (0.80)	1.0189 (0.60)	0.7733 (0.52)	0.2968 (0.99)
UNC2,25	1.2048 (0.09)	1.3937 (0.34)	1.3554 (0.69)	1.0798 (0.97)	0.8336 (0.82)	0.3383 (0.79)
UNC2,5	1.1662 (0.10)	1.3587 (0.19)	1.3396 (0.39)	1.0887 (0.57)	0.8571 (0.74)	0.2950 (0.67)
VAR2,50	1.3431 (0.00)	1.6281 (0.00)	1.2039 (0.10)	1.1080 (0.73)	0.8057 (0.74)	0.2188 (0.84)
VAR2,25	1.2582 (0.00)	1.5090 (0.00)	1.2837 (0.25)	1.1239 (0.31)	0.8796 (0.66)	0.2615 (0.92)
VAR2,5	1.1774 (0.00)	1.3811 (0.00)	1.3244 (0.53)	1.0954 (0.13)	0.8669 (0.24)	0.2777 (0.96)
NAR2,50	1.3088 (0.02)	1.4741 (0.27)	1.5831 (0.03)	1.0406 (0.67)	0.8378 (0.84)	0.0176 (0.25)
NAR2,25	1.2429 (0.00)	1.4358 (0.16)	1.4931 (0.01)	1.0902 (0.90)	0.8848 (0.63)	0.1490 (0.46)
NAR2,5	1.1749 (0.00)	1.3674 (0.05)	1.3690 (0.00)	1.0897 (0.51)	0.8668 (0.32)	0.2575 (0.63)
DYN2,50	1.2539 (0.00)	1.3928 (0.00)	- -	1.1169 (0.26)	- -	- -
DYN2,25	1.2060 (0.00)	1.3692 (0.00)	- -	1.1035 (0.15)	- -	- -
DYN2,5	1.1660 (0.00)	1.3499 (0.00)	- -	1.0874 (0.11)	- -	- -

Table A5: Sharpe ratios for constrained investment (positive-cost) portfolios and transactions costs of 5 basis points with weekly returns

This table reports the annualized out-of-sample Sharpe ratios for the different constrained portfolios and datasets with weekly returns, in the presence of a proportional transaction cost of 5 basis points, together with the P-value that the Sharpe ratio for a strategy is different from that for the unconstrained minimum-variance portfolio.

Strategy	6FF	25FF	100FF	10Ind	48Ind	100CRSP
Unconstrained portfolios						
1/N	0.8727 (0.00)	0.8915 (0.00)	0.8968 (0.00)	0.8477 (0.06)	0.8002 (0.93)	0.6272 (0.18)
MIN	1.4554 (1.00)	1.7283 (1.00)	1.6507 (1.00)	1.1287 (1.00)	0.8179 (1.00)	0.9136 (1.00)
UNC2	0.8013 (0.02)	0.8911 (0.01)	-0.4811 (0.00)	0.2541 (0.00)	-0.1668 (0.00)	-0.6668 (0.00)
VAR2	1.0963 (0.27)	-0.2867 (0.00)	-0.9687 (0.00)	0.1197 (0.00)	-0.4266 (0.00)	-0.8858 (0.00)
NAR2	1.3766 (0.76)	0.4648 (0.00)	-0.8249 (0.00)	-0.0996 (0.00)	-0.5766 (0.00)	-0.4810 (0.00)
DYN2	0.1413 (0.01)	-1.2929 (0.00)	- -	-0.0703 (0.00)	- -	- -
Shortsale-constrained portfolios						
MIN	1.0260 (0.00)	1.0608 (0.00)	1.1048 (0.00)	1.0274 (0.03)	1.0237 (0.05)	0.7499 (0.24)
UNC2	0.9274 (0.00)	1.0114 (0.00)	0.9112 (0.00)	0.6904 (0.03)	0.6800 (0.57)	0.3926 (0.14)
VAR2	1.3535 (0.47)	1.2508 (0.01)	0.7991 (0.00)	0.6792 (0.01)	0.6046 (0.36)	-0.1576 (0.00)
NAR2	1.2164 (0.08)	1.4239 (0.10)	1.4565 (0.37)	0.6165 (0.00)	0.4308 (0.06)	0.3893 (0.07)
DYN2	1.1741 (0.03)	0.8289 (0.00)	- -	0.8382 (0.05)	- -	- -
Norm-constrained portfolios						
UNC2,50	1.5258 (0.14)	1.8932 (0.02)	1.7849 (0.19)	1.0424 (0.32)	0.8671 (0.69)	0.4790 (0.08)
UNC2,25	1.5008 (0.07)	1.8341 (0.01)	1.7507 (0.05)	1.1081 (0.64)	0.8632 (0.49)	0.7003 (0.20)
UNC2,5	1.4667 (0.01)	1.7529 (0.00)	1.6505 (0.96)	1.1296 (0.95)	0.8350 (0.23)	0.8926 (0.57)
VAR2,50	1.7585 (0.00)	2.1032 (0.00)	1.6022 (0.62)	1.0870 (0.52)	0.6786 (0.20)	0.4206 (0.03)
VAR2,25	1.6220 (0.00)	1.9520 (0.00)	1.6693 (0.77)	1.1331 (0.93)	0.7860 (0.59)	0.6875 (0.08)
VAR2,5	1.4935 (0.00)	1.7820 (0.00)	1.6656 (0.15)	1.1359 (0.30)	0.8235 (0.70)	0.8862 (0.30)
NAR2,50	1.7368 (0.00)	2.2848 (0.00)	2.2353 (0.00)	1.0143 (0.08)	0.6594 (0.17)	0.7764 (0.51)
NAR2,25	1.6102 (0.00)	2.0461 (0.00)	2.0077 (0.00)	1.0885 (0.29)	0.7722 (0.44)	0.9122 (0.99)
NAR2,5	1.4909 (0.00)	1.8002 (0.00)	1.7065 (0.00)	1.1268 (0.83)	0.8205 (0.86)	0.9404 (0.37)
DYN2,50	1.6039 (0.00)	1.7139 (0.26)	- -	1.1790 (0.08)	- -	- -
DYN2,25	1.5361 (0.00)	1.7336 (0.36)	- -	1.1620 (0.03)	- -	- -
DYN2,5	1.4757 (0.00)	1.7359 (0.00)	- -	1.1402 (0.00)	- -	- -

Table A6: Sharpe ratios for constrained investment (positive-cost) portfolios and transactions costs of 5 basis points with monthly returns

This table reports the annualized out-of-sample Sharpe ratios for the different constrained portfolios and datasets with monthly returns, in the presence of a proportional transaction cost of 5 basis points, together with the P-value that the Sharpe ratio for a strategy is different from that for the unconstrained minimum-variance portfolio.

Strategy	6FF	25FF	100FF	10Ind	48Ind	100CRSP
Unconstrained portfolios						
1/N	0.7975 (0.00)	0.7972 (0.00)	0.7967 (0.00)	0.8760 (0.09)	0.7708 (0.75)	0.3730 (0.49)
MIN	1.1515 (1.00)	1.3282 (1.00)	1.2837 (1.00)	1.0773 (1.00)	0.8325 (1.00)	0.2615 (1.00)
UNC2	0.7951 (0.17)	0.8663 (0.14)	-0.2293 (0.00)	0.2255 (0.00)	-0.0198 (0.01)	-0.5552 (0.07)
VAR2	0.7487 (0.17)	0.1282 (0.00)	-0.0188 (0.00)	-0.1570 (0.00)	-0.1108 (0.00)	0.4767 (0.53)
NAR2	0.7231 (0.14)	0.3050 (0.00)	-0.1860 (0.00)	0.1064 (0.00)	0.1587 (0.03)	-0.2984 (0.10)
DYN2	0.6230 (0.07)	-0.2478 (0.00)	- -	-0.2117 (0.00)	- -	- -
Shortsale-constrained portfolios						
MIN	0.8872 (0.00)	0.9067 (0.00)	0.9339 (0.01)	1.0199 (0.30)	1.0070 (0.12)	0.3620 (0.39)
UNC2	0.9646 (0.08)	0.9389 (0.00)	0.9599 (0.04)	0.6804 (0.08)	0.5017 (0.25)	0.1436 (0.72)
VAR2	1.0085 (0.32)	0.9619 (0.03)	0.5447 (0.00)	0.8370 (0.22)	0.2027 (0.01)	0.3188 (0.87)
NAR2	0.8987 (0.08)	0.8389 (0.00)	0.9067 (0.04)	0.7786 (0.08)	0.6768 (0.48)	-0.0923 (0.26)
DYN2	0.9174 (0.05)	0.8110 (0.00)	- -	0.9034 (0.19)	- -	- -
Norm-constrained portfolios						
UNC2,50	1.2320 (0.19)	1.3858 (0.54)	1.3037 (0.79)	1.0100 (0.59)	0.7536 (0.56)	0.2834 (0.94)
UNC2,25	1.1996 (0.11)	1.3755 (0.35)	1.3077 (0.68)	1.0725 (0.94)	0.8136 (0.84)	0.3238 (0.76)
UNC2,5	1.1619 (0.08)	1.3417 (0.21)	1.2928 (0.40)	1.0831 (0.66)	0.8376 (0.78)	0.2807 (0.68)
VAR2,50	1.3116 (0.00)	1.5745 (0.00)	1.1228 (0.02)	1.0644 (0.89)	0.7497 (0.49)	0.1827 (0.73)
VAR2,25	1.2408 (0.00)	1.4740 (0.00)	1.2188 (0.11)	1.0990 (0.60)	0.8389 (0.97)	0.2326 (0.80)
VAR2,5	1.1715 (0.00)	1.3618 (0.00)	1.2752 (0.31)	1.0871 (0.29)	0.8438 (0.40)	0.2606 (0.97)
NAR2,50	1.2749 (0.02)	1.4258 (0.41)	1.5043 (0.05)	0.9971 (0.41)	0.7858 (0.63)	-0.0169 (0.27)
NAR2,25	1.2247 (0.01)	1.4031 (0.20)	1.4294 (0.01)	1.0652 (0.80)	0.8463 (0.86)	0.1207 (0.33)
NAR2,5	1.1688 (0.00)	1.3483 (0.07)	1.3200 (0.00)	1.0816 (0.69)	0.8440 (0.44)	0.2406 (0.53)
DYN2,50	1.2293 (0.00)	1.3555 (0.01)	- -	1.0823 (0.90)	- -	- -
DYN2,25	1.1928 (0.00)	1.3448 (0.00)	- -	1.0852 (0.62)	- -	- -
DYN2,5	1.1607 (0.00)	1.3326 (0.00)	- -	1.0807 (0.30)	- -	- -

Table A7: Sharpe ratios for investment (positive-cost) portfolios using dividend yield and yield spread as predictors

This table reports the annualized out-of-sample Sharpe ratios for the different portfolios that use the dividend yield and the yield spread as predictors, together with the P-value that the Sharpe ratio for a strategy is different from that for the unconstrained minimum-variance portfolio.

Strategy	6FF	25FF	100FF	10Ind	48Ind	100CRSP
Unconditional portfolios						
1/N	0.8518 (0.00)	0.8947 (0.00)	0.9112 (0.00)	0.7654 (0.01)	0.7740 (0.17)	0.6389 (0.48)
MIN	1.7259 (1.00)	2.1961 (1.00)	2.4300 (1.00)	1.1211 (1.00)	1.0054 (1.00)	0.7710 (1.00)
UNC1	0.7957 (0.00)	1.0470 (0.00)	0.8323 (0.00)	0.4450 (0.02)	0.5389 (0.16)	-0.4722 (0.00)
UNC2	0.9006 (0.01)	1.1063 (0.00)	0.8617 (0.00)	0.5004 (0.03)	0.5595 (0.13)	-0.4554 (0.00)
UNC10	1.5028 (0.24)	1.5444 (0.04)	1.0921 (0.00)	0.8470 (0.19)	0.7119 (0.26)	-0.3168 (0.00)
Portfolios that exploit stock return serial dependence dividend yield and yield spread						
VAR1	3.5708 (0.00)	4.6866 (0.00)	3.5579 (0.00)	4.3693 (0.00)	4.1949 (0.00)	1.6271 (0.02)
VAR2	3.5810 (0.00)	4.6917 (0.00)	3.5594 (0.00)	4.3733 (0.00)	4.1965 (0.00)	1.6278 (0.01)
VAR10	3.6593 (0.00)	4.7321 (0.00)	3.5711 (0.00)	4.4026 (0.00)	4.2087 (0.00)	1.6337 (0.01)
NAR1	4.6436 (0.00)	5.9743 (0.00)	3.9131 (0.00)	4.8213 (0.00)	4.6266 (0.00)	1.9177 (0.00)
NAR2	4.6664 (0.00)	5.9889 (0.00)	3.9185 (0.00)	4.8324 (0.00)	4.6325 (0.00)	1.9198 (0.00)
NAR10	4.8125 (0.00)	6.0932 (0.00)	3.9604 (0.00)	4.8930 (0.00)	4.6741 (0.00)	1.9361 (0.00)
DYN1	3.3779 (0.00)	4.4849 (0.00)	- -	3.4041 (0.00)	- -	- -
DYN2	3.3300 (0.00)	4.5403 (0.00)	- -	3.3500 (0.00)	- -	- -
DYN10	3.3553 (0.00)	4.5931 (0.00)	- -	3.3341 (0.00)	- -	- -
Portfolios that exploit dividend yield and yield spread						
VAR1	0.8532 (0.00)	0.8154 (0.00)	0.0849 (0.00)	0.5754 (0.05)	0.7577 (0.44)	-0.9746 (0.00)
VAR2	0.8971 (0.01)	0.8374 (0.00)	0.0951 (0.00)	0.5972 (0.05)	0.7650 (0.38)	-0.9690 (0.00)
VAR10	1.2130 (0.06)	1.0105 (0.00)	0.1769 (0.00)	0.7566 (0.10)	0.8214 (0.50)	-0.9221 (0.00)
NAR1	1.3361 (0.16)	1.3980 (0.02)	0.3796 (0.00)	1.1082 (0.95)	1.2097 (0.46)	-1.0597 (0.00)
NAR2	1.3896 (0.23)	1.4257 (0.02)	0.3877 (0.00)	1.1350 (0.98)	1.2167 (0.46)	-1.0570 (0.00)
NAR10	1.7367 (0.99)	1.6364 (0.06)	0.4528 (0.00)	1.3141 (0.40)	1.2688 (0.37)	-1.0344 (0.00)

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