## Supplemental file: Robustness checks and additional analysis

### Appendix B: The case with convex increasing marginal opportunity cost

We now show that the insight that the intermediary profit $\pi_i$ is unimodal in the number of suppliers $S$ also holds for the following more general marginal opportunity cost function:

$$p_s(q) = s_1 + s_2 q^{\theta}; \text{for } \theta \geq 1.$$

Note that this is a convex increasing monomial function. We address the general setting with $R$ retailers and $I$ intermediaries; the robustness result for the base case can be obtained in the special case when $R=1$ and $I=1$.

The supplier's profit is:

$$\pi_{s,j} = p_s(q_{s,j})q_{s,j} - \int_0^{q_{s,j}} (s_1 + s_2 q^{\theta})dq = \frac{s_2 \theta q_{s,j}^{\theta+1}}{\theta+1}.$$

The intermediary's decision is:

$$\max_{q_{i,l}} \left[ p_i - \left[ s_1 + s_2 \left( \frac{q_{i,l} + Q_{i,-l}}{S} \right)^{\theta} \right] \right] q_{i,l}.$$

The first-order conditions imply:

$$p_i - \left[ s_1 + s_2 \left( \frac{q_{i,l} + Q_{i,-l}}{S} \right)^{\theta} \right] - \frac{s_2 \theta}{S} \left( \frac{q_{i,l} + Q_{i,-l}}{S} \right)^{\theta-1} q_{i,l} = 0;$$

which implies:

$$p_i = s_1 + s_2 \left( \frac{Q}{S} \right)^{\theta} + \frac{s_2 \theta}{I} \left( \frac{Q}{S} \right)^{\theta};$$

and the margin, $m_i$ of the intermediary is:

$$m_i = p_i - p_s = \frac{s_2 \theta}{I} \left( \frac{Q}{S} \right)^{\theta};$$

and the profit, $\pi_i$, of each intermediary is:

$$\pi_i = m_i \frac{Q}{I} = \frac{s_2 \theta S}{I^2} \left( \frac{Q}{S} \right)^{\theta+1}.$$

The retailer's decision problem can be stated as:

$$\max_{q_{r,k}} (p_r - p_i) q_{r,k} = \left[ d_1 - d_2(q_{r,k} + Q_{r,-k}) - \left[ s_1 + s_2 \left( \frac{Q}{S} \right)^{\theta} + \frac{s_2 \theta}{I} \left( \frac{Q}{S} \right)^{\theta} \right] \right] q_{r,k}.$$

The first-order conditions then imply:

$$\Phi(Q,S) = d_2 \left( 1 + \frac{1}{R} \right) Q + s_2 \left( 1 + \frac{\theta}{I} \right) \left( 1 + \frac{\theta}{R} \right) \left( \frac{Q}{S} \right)^{\theta} - (d_1 - s_1) = 0.$$

Now we are ready to determine whether $\pi_i$ is unimodal in $S$. We first calculate $d\pi_i/dS$:

$$\frac{d\pi_i}{dS} = \left( \frac{\partial \pi_i}{\partial Q} \right) \frac{dQ}{dS} + \frac{\partial \pi_i}{\partial S} = \left( \frac{Q}{S} \right)^{\theta} \frac{s_2 \theta}{I^2} \left[ (\theta+1) \frac{dQ}{dS} - \theta \frac{Q}{S} \right];$$

where $dQ/dS$ can be determined using the following relationship:

$$\frac{d\Phi}{dS} = \frac{\partial \Phi}{\partial Q}\frac{dQ}{dS} + \frac{\partial \Phi}{\partial S} = 0;$$

which implies:

$$\frac{dQ}{dS} = \frac{s_2\theta\left(1+\frac{\theta}{I}\right)\left(1+\frac{\theta}{R}\right)\left(\frac{Q^\theta}{S^{\theta+1}}\right)}{d_2\left(1+\frac{1}{R}\right)+s_2\theta\left(1+\frac{\theta}{I}\right)\left(1+\frac{\theta}{R}\right)\left(\frac{Q^{\theta-1}}{S^\theta}\right)} = \frac{\frac{Q}{S}}{1+\left(\frac{d_2(R+1)}{s_2\theta R\left(1+\frac{\theta}{I}\right)\left(1+\frac{\theta}{R}\right)}\right)\left(\frac{S^\theta}{Q^{\theta-1}}\right)} < \frac{Q}{S}. \tag{34}$$

Substituting the above relationship in the expression for $d\pi_i/dS$ and evaluating the terms, we conclude that $d\pi_i/dS < 0$ if and only if $dQ/dS < \theta Q/((\theta+1)S)$ or equivalently, if and only if:

$$\frac{1}{1+\left(\frac{d_2(R+1)}{s_2\theta R\left(1+\frac{\theta}{I}\right)\left(1+\frac{\theta}{R}\right)}\right)\left(\frac{S}{Q}\right)^{\theta-1}S} < \frac{\theta}{\theta+1}.$$

Also note that:

$$\frac{d\left(\frac{Q}{S}\right)}{dS} = \frac{1}{S}\left(\frac{dQ}{dS}-\frac{Q}{S}\right) < 0 \implies \frac{d\left(\frac{S}{Q}\right)}{dS} > 0.$$

Hence, it is easy to verify that:

$$\frac{d}{dS}\left[\frac{1}{1+\left(\frac{d_2(R+1)}{s_2\theta R\left(1+\frac{\theta}{I}\right)\left(1+\frac{\theta}{R}\right)}\right)\left(\frac{S}{Q}\right)^{\theta-1}S}\right] < 0.$$

This implies that, as $S$ is increased, then $d\pi_i/dS$ can change sign from positive to negative at most once. In other words $\pi_i$ is unimodal in $S$.

We also show that for this marginal opportunity cost function the retailer margin does depend on the number of suppliers and intermediaries, as opposed to the case of a linear function. Indeed,

$$m_r = d_1 - d_2 Q - s_1 - s_2\left(1+\frac{\theta}{I}\right)\left(\frac{Q}{S}\right)^\theta$$

and

$$\frac{dm_r}{dS} = \frac{\partial m_r}{\partial Q}\frac{dQ}{dS} + \frac{\partial m_r}{\partial S},$$

where

$$\frac{\partial m_r}{\partial S} = \frac{s_2}{S}\left(1+\frac{\theta}{I}\right)\left(\frac{Q}{S}\right)^\theta,$$

and

$$\frac{\partial m_r}{\partial Q} = -d_2 - \frac{s_2}{S}\left(1+\frac{\theta}{I}\right)\left(\frac{Q}{S}\right)^{\theta-1}.$$

Since both of the two expressions above are independent of $R$ but $dQ/dS$ does depend on $R$ when $\theta > 1$, as is apparent from (34), it is clear that $dm_r/dS \neq 0$ in general.

Similarly,

$$\frac{dm_r}{dI} = \frac{\partial m_r}{\partial Q}\frac{dQ}{dI} + \frac{\partial m_r}{\partial I},$$

where

$$\frac{\partial m_r}{\partial I} = \frac{s_2\theta}{I^2}\left(\frac{Q}{S}\right)^{\theta}.$$

$dQ/dI$ can be deduced from the first-order conditions

$$\Psi(Q,I) = d_2\left(1+\frac{1}{R}\right)Q + s_2\left(1+\frac{\theta}{I}\right)\left(1+\frac{\theta}{R}\right)\left(\frac{Q}{S}\right)^{\theta} - (d_1 - s_1) = 0$$

and the relationship:

$$\frac{d\Psi}{dI} = \frac{\partial\Psi}{\partial Q}\frac{dQ}{dI} + \frac{\partial\Phi}{\partial I} = 0;$$

which implies:

$$\frac{dQ}{dI} = \frac{\frac{s_2\theta}{I^2}\left(1+\frac{\theta}{R}\right)\left(\frac{Q}{S}\right)^{\theta}}{d_2\left(1+\frac{1}{R}\right) + s_2\theta\left(1+\frac{\theta}{I}\right)\left(1+\frac{\theta}{R}\right)\left(\frac{Q^{\theta-1}}{S^{\theta}}\right)} = \frac{\frac{Q}{I^2}}{1+\frac{\theta}{I}+\left(\frac{d_2(R+1)}{s_2\theta(R+\theta)}\right)\left(\frac{S^{\theta}}{Q^{\theta-1}}\right)}. \tag{35}$$

Since both $\partial m_r/\partial I$ and $\partial m_r/\partial Q$ are independent of $R$ but $dQ/dI$ does depend on $R$ when $\theta > 1$, as is apparent from (35), it is clear that $dm_r/dI \neq 0$ in general.

## Appendix C: The case with stochastic demand

We now show that our results are generally robust to the presence of stochasticity in the demand function. We first consider the case with stochastic additive demand, and then with multiplicative stochastic demand. We address the general setting with $R$ retailers and $I$ intermediaries; the robustness results for the base case can be obtained in the special case when $R = 1$ and $I = 1$.

### C.1. The case with stochastic additive demand

Consider the following *stochastic additive linear inverse demand function*:

$$p_r = d_1 - d_2 Q + \epsilon, \tag{36}$$

where $d_1$ is the demand intercept and the random variable $\epsilon$, which has zero mean and standard deviation $\sigma$, represents a random additive perturbation to the demand function. In addition to the conditions we impose on $d_1$ to ensure that the expected retail price is non-negative at equilibrium, the amplitude of perturbation $\epsilon$ is assumed to be small enough so that at equilibrium, the realized retail price remains non-negative.

The $k$th retailer profit is

$$\pi_{r,k} = (p_r - p_i)q_{r,k} = (d_1 - d_2(q_{r,k} + Q_{r,-k}) + \epsilon - p_i)q_{r,k}.$$

Note that the only uncertainty in the retailers' profit function is the random variable $\epsilon$, which determines the realized demand function. The retailer order quantities and market-clearing intermediary price $p_i$ are deterministic because retailers make their orders before demand is realized.

As in Adida and DeMiguel [2011], we assume that retailers have the following mean-standard-deviation utility function:

$$E[\pi_{r,k}] - \gamma\sqrt{\mathrm{Var.}[\pi_{r,k}]} = (d_1 - \gamma\sigma - d_2(q_{r,k} + Q_{r,-k}) - p_i)q_{r,k}.$$

This utility function allows for risk-averse retailers facing a stochastic demand function ($\gamma > 0$ and $\sigma > 0$), but it also covers the case when retailers are risk-neutral ($\gamma = 0$), or when demand is deterministic ($\sigma = 0$).

Then the $k$th retailer chooses its order quantity $q_{r,k}$ to maximize its mean-standard-deviation utility, assuming the rest of the retailers keep their order quantities fixed, and anticipating the intermediary reaction and the intermediary-market-clearing price $p_i$:

$$\max_{q_{r,k},\ p_i} \left[\tilde{d}_1 - d_2(q_{r,k} + Q_{r,-k}) - p_i\right] q_{r,k} \tag{37}$$

$$\text{s.t. } q_{r,k} + Q_{r,-k} = Q_i(p_i), \tag{38}$$

where $Q_{r,-k}$ is the total quantity ordered by the rest of retailers, $Q_i(p_i)$ is the intermediary equilibrium quantity for a price $p_i$, Constraint (38) is the intermediary-market-clearing condition, and we define $\tilde{d}_1 \equiv d_1 - \gamma\sigma$ to be the *risk-adjusted demand intercept*.

From the retailer's objective function (37), it is apparent that the impact of risk on the equilibrium is equivalent to a reduction of the demand intercept to $\tilde{d}_1 \equiv d_1 - \gamma\sigma$. Therefore, the analysis in the main body of our paper applies to the case with stochastic additive demand after replacing the intercept with the risk-adjusted demand intercept.

### C.2. The case with stochastic multiplicative demand

Consider the following stochastic multiplicative linear inverse demand function: $p_r = (d_1 - d_2 Q)\epsilon$ with $\mathrm{E}[\epsilon] = 1$ and $\mathrm{Var}\epsilon = \sigma^2$.

Assuming retailers are risk-averse with a mean-standard-deviation utility function, the $k$th retailer's objective is to maximize

$$\begin{aligned} &[d_1 - d_2(q_{r,k} + Q_{r,-k}) - p_i(q_{r,k} + Q_{r,-k})]\, q_{r,k} - \gamma\sigma(d_1 - d_2(q_{r,k} + Q_{r,-k}))q_{r,k} \\ = &\left[\tilde{d}_1 - \tilde{d}_2(q_{r,k} + Q_{r,-k}) - p_i(q_{r,k} + Q_{r,-k})\right] q_{r,k}, \end{aligned}$$

where $\tilde{d}_1 = d_1(1 - \gamma\sigma)$ and $\tilde{d}_2 = d_2(1 - \gamma\sigma)$.

From the retailer's objective function, it is apparent that the impact of risk on the equilibrium is equivalent to replacing $d_1$ and $d_2$ with $d_1(1-\gamma\sigma)$ and $d_2(1-\gamma\sigma)$, respectively, provided that $\gamma\sigma \leq 1$ and $d_1 - s_1 \geq 0$. Therefore, the analysis in the main body of our paper applies to the case with stochastic multiplicative demand after replacing the intercept and slope with their risk-adjusted counterparts.

## Appendix D: Comparison to other models in the literature

We now give a detailed comparison of the equilibria for our proposed model with the models proposed by C&K and Choi [1991]. To do so, we first briefly state three-tier variants of the models of C&K and Choi that are similar to our model, and then we compare the equilibria of the three models. We address the general setting with $R$ retailers and $I$ intermediaries; the base case can be obtained by focusing on $R = 1$ and $I = 1$.

As discussed in Section 2.2, our model also shares some common elements with that proposed by M&S. Their model, however, does not consider horizontal competition within tiers, and instead focuses on competition between supply *networks*. Moreover, M&S focus on how the equilibrium depends on the position of the leader within the supply network, whereas we fix the retailer in the leader position, which is the situation faced by the supply chain intermediary firms that we are interested in. For these reasons, the equilibrium for M&S's model is not comparable to that for our model. Thus we focus in this section on the equilibria for the models by C&K and Choi, who consider serial multi-tier supply chain models with vertical and horizontal competition similar to our model.

### D.1. A Corbett-and-Karmarkar-type model.

We first consider a three-tier version of C&K's model. The first tier consists of $S$ suppliers who lead the second tier consisting of $I$ intermediaries who lead the third tier consisting of $R$ retailers. There is quantity competition at all three tiers. We focus on the case where only the first tier of suppliers faces production costs, which is the closest to our proposed model of intermediation.

The $k$th retailer chooses its order quantity $q_{r,k}$ to maximize its profit given an inverse demand function $p_r = d_1 - d_2 Q$ and for a given price requested by the intermediary $p_i$:

$$\max_{q_{r,k}} \left[d_1 - d_2(q_{r,k} + Q_{r,-k}) - p_i)\right] q_{r,k}.$$

The $l$th intermediary chooses its order quantity $q_{i,l}$ to maximize its profit for a given price requested by the suppliers $p_s$, and anticipating the price that the retailers are willing to pay for a total quantity $q_{i,l} + Q_{i,-l}$:

$$\max_{q_{i,l}} \left[p_i\left(q_{i,l} + Q_{i,-l}\right) - p_s\right] q_{i,l}.$$

Finally, the $j$th supplier chooses its production quantity $q_{s,j}$ to maximize its profit anticipating the price that the intermediaries are willing to pay for a total quantity $q_{s,j} + Q_{s,-j}$ and given its unit variable cost is $s_1$:

$$\max_{q_{s,j}} \left[p_s\left(q_{s,j} + Q_{s,-j}\right) - s_1\right] q_{s,j}.$$

### D.2. A Choi-type model.

We now consider a three-tier version of Choi's model with the retailers as leaders. Choi assumes that suppliers know the demand function and exploit this knowledge strategically when making production decisions. Moreover, Choi assumes that the suppliers are margin takers with respect to the intermediaries. Thus, the $j$th supplier's decision problem is

$$\max_{q_{s,j}} \ (d_1 - d_2\left(q_{s,j} + Q_{s,-j}\right) - m_r - m_i - s_1)q_{s,j},$$

where $s_1$ is the unit production cost, and $m_r$ and $m_i$ are the retailer and intermediary margins, respectively. Note that because $p_i = m_r + m_i + s_1$ it is apparent that the supplier's decision problem for the Choi-type model is exactly equivalent to the retailer's decision problem for the C&K-type model.

Following the spirit of Choi's model with retailers as leaders, we assume that the intermediary also knows the demand function and exploits this knowledge strategically when making order quantity decisions. The intermediary is a follower (and thus a margin taker) with respect to the retailers, but is a leader with respect to the suppliers. Thus, the intermediary anticipates the price requested by the suppliers to deliver a given quantity. Therefore the $l$th intermediary decision can be written as

$$\max_{q_{i,l}} \ (d_1 - d_2\left(q_{i,l} + Q_{i,-l}\right) - m_r - p_s(q_{i,l} + Q_{i,-l}))q_{i,l}.$$

Finally, the $k$th retailer in Choi's model chooses its order quantity to maximize the profit given the demand function and anticipating the price required by the intermediaries to supply a total quantity $q_{k,r} + Q_{r,-k}$:

$$\max_{q_{k,r}} \ (d_1 - d_2\left(q_{k,r} + Q_{r,-k}\right) - p_i(q_{k,r} + Q_{r,-k}))q_{k,r}.$$

### D.3. Comparing the C&K and Choi models.

C&K give closed-form expressions for the equilibrium quantities for their model. Choi also gives closed-form expressions for the equilibrium quantities for his two-tier model with retailers as leaders, and it is straightforward to extend these closed-form expressions for the three-tier variant of his model that we consider. The resulting closed-form expressions are collected in the second and third columns of Table 1.

The striking realization when comparing the second and third columns in Table 1 is that the equilibrium total order quantities in the C&K and Choi models coincide. Moreover, the aggregate intermediary profits also coincide. Furthermore, a careful look at the expressions for the aggregate profits of the retailers and suppliers reveals that the aggregate retailer profit in C&K's model coincides with the aggregate supplier profit in Choi's model if one replaces the number of retailers by the number of suppliers. Likewise, the aggregate supplier profit in C&K's model coincides with the aggregate retailer profit in Choi's model if one replaces the number of suppliers by the number of retailers. In other words, the equilibria of the C&K and Choi models are equivalent. We believe the reason for this is the assumption in Choi's model that the suppliers have perfect information about the retailer demand function and exploit this strategically when making decisions. This is not a realistic assumption for the intermediation context that we study.

### D.4. Comparing our model to the C&K and Choi models.

The most important difference between the equilibrium for our model and those for the C&K and Choi models is in the intermediary margins and profits. For all three models, the suppliers' market power decreases in the number of suppliers, and their profits become zero in the limit when there is an infinite number of suppliers. The intermediaries' margin and profit, however, behave quite differently for the three models. In C&K's model, the intermediaries' margin increases with the number of suppliers; in Choi's model, it remains constant; and in our model, it decreases. As a result, for the C&K and Choi models, the intermediary profits increase in the number of suppliers, because order quantities also increase. In our model, on the other hand, the increase in the order quantities is not sufficient to offset the decrease in margin and, as a result, the intermediary profits are unimodal in the number of suppliers. The reason for this is that as the number of suppliers grows larger, the retailers know that intermediaries and suppliers will agree to produce at any price above $s_1$. Therefore, the retailers can take advantage of their leading position to extract higher rents, leaving the intermediaries with zero margin and profit for the limiting case where the number of suppliers is infinite.

Comparing the aggregate retailer profits in all three models, we observe that the equilibrium retailer margins for our proposed model and for Choi's model are equal and they are both larger than the equilibrium retailer margin for C&K's model. Moreover, because the equilibrium total quantity in the models by C&K and Choi are identical, this implies that the aggregate retailer profit in the model by Choi is larger than that in the model by C&K. This is not surprising as it is well known that a leading position often results in larger profits for a given player; see Vives [1999].[1]

The question of whether the aggregate retailer profit is larger in our model than in those by C&K and Choi is a bit harder to answer. Note that there is an additional parameter in our model: the marginal cost sensitivity $s_2$. This parameter enters the closed-form expression for the equilibrium quantity in our model and thus it is difficult to compare to the other two models.

[1] Note that the retailer margin and profits in Choi's model coincide with the supplier margin and profits in C&K's model after replacing the number of retailers with the number of suppliers, so as we argue above both models are essentially equivalent.

However, assuming the marginal cost sensitivity $s_2$ equals the demand sensitivity $d_2$, it is easy to show that the equilibrium quantity in our model is larger than in the C&K and Choi models. This implies that the equilibrium aggregate retailer profit utilities in our model are larger not only than those in the C&K model, but also than those in Choi's model (for the case $s_2 = d_2$). Two comments are in order here. First, since in our proposed model the retailers act as leaders, they are able to capture greater profit utilities than in the model by C&K. Second, while Choi also captures the retailers as leaders, he assumes that intermediaries and suppliers know the retailer demand function and exploit this knowledge strategically. This assumption leaves the retailers in a weaker position compared to our model.

We conclude that there are significant differences between the models by C&K and Choi [1991] and our proposed model, particularly the leadership positions and information available within the game, which result in different insights. Our model best fits situations when retailers act as leaders, while C&K's model is more appropriate when suppliers can be considered leaders. Choi's model makes sense when the retail demand function can realistically be known by all players.

**Table 1** Comparison of the equilibrium for the models by C&K, Choi [1991], and the proposed model.

This table gives the equilibrium quantities for variants of the models proposed by C&K and Choi [1991], as well as for our proposed model with multiple retailers and intermediaries. The first column in the table lists the different equilibrium quantities reported: the aggregate supply quantity, the supply price, the intermediary price, the retail price, the supplier margin, the intermediary margin, the retailer margin, the aggregate supplier profit, the aggregate intermediary profit, the aggregate retailer profit, the aggregate supply chain profit, and the efficiency. The second, third, and fourth columns give the expression of the equilibrium quantity for the variants of the models proposed by C&K and Choi [1991], and for our proposed model, respectively. Note that for our proposed model the supplier marginal opportunity cost is not constant, and thus we report the average supplier margin.

| Quantity | Corbett and Karmarkar | Choi | Retailers lead |
|---|---|---|---|
| Aggregate quantity | $\frac{RSI}{d_2(S+1)(I+1)}\frac{d_1-s_1}{R+1}$ | $\frac{RSI}{d_2(S+1)(I+1)}\frac{d_1-s_1}{R+1}$ | $\frac{RSI}{(d_2SI+s_2(I+1))}\frac{d_1-s_1}{R+1}$ |
| Supply price | $d_1-\frac{d_2(R+1)(I+1)}{RI}Q$ | $s_1+\frac{d_2}{S}Q$ | $s_1+\frac{s_2}{S}Q$ |
| Intermediary price | $d_1-\frac{d_2(R+1)}{R}Q$ | $s_1+\frac{d_2}{S}\frac{S+I+1}{I}Q$ | $s_1+\frac{s_2}{S}\frac{I+1}{I}Q$ |
| Retail price | $d_1-d_2Q$ | $d_1-d_2Q$ | $d_1-d_2Q$ |
| Supplier margin | $\frac{d_1-s_1}{S+1}$ | $\frac{d_2Q}{S}$ | $\frac{s_2Q}{2S}$ |
| Intermediary margin | $\frac{d_2Q(R+1)}{RI}$ | $\frac{d_2Q(S+1)}{SI}$ | $\frac{s_2Q}{SI}$ |
| Retailer margin | $\frac{d_1-s_1}{R+1}\frac{IS}{(I+1)(S+1)}$ | $\frac{d_1-s_1}{R+1}$ | $\frac{d_1-s_1}{R+1}$ |
| Agg. supplier profit | $\frac{d_1-s_1}{S+1}Q$ | $\frac{d_2Q^2}{S}$ | $\frac{s_2Q^2}{2S}$ |
| Agg. interm. profit | $\frac{d_2Q^2(R+1)}{RI}$ | $\frac{d_2Q^2(S+1)}{SI}$ | $\frac{s_2Q^2}{SI}$ |
| Agg. retailer profit | $\frac{d_2Q^2}{R}$ | $\frac{d_1-s_1}{R+1}Q$ | $\frac{d_1-s_1}{R+1}Q$ |
| Agg. chain profit | $S\pi_s+I\pi_i+R\pi_R$ | $S\pi_s+I\pi_i+R\pi_R$ | $S\pi_s+I\pi_i+R\pi_R$ |
| Efficiency | $\frac{4SIR(RI+RS+SI+R+S+I+1)}{(S+1)^2(I+1)^2(R+1)^2}$ | $\frac{4SIR(RI+RS+SI+R+S+I+1)}{(S+1)^2(I+1)^2(R+1)^2}$ | $\frac{RI[s_2R(I+2)+2(d_2SI+s_2(I+1))]}{(d_2SI+s_2(I+1))^2}\frac{2d_2S+s_2}{(R+1)^2}$ |

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