Structural Estimation of Systemic Risk: Measuring Contagion in the Sub-Prime Crisis

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August 2014

Preliminary and Incomplete

Abstract

This paper develops a measure of international systemic risk using a semi-structural approach. In particular, we work with a multi-country dynamic equilibrium setting, placing a constraint on the portfolio volatility of one of the countries. The tightening of this constraint is a channel through which shocks are propagated internationally in our model. The propagation of this financial constraint is what we define as systemic risk. In this paper we offer two estimation alternatives. First, we present a semi-structural approach in which we measure the tightness of the constraint using cross-equation restrictions from the model. The second approach is to implement a full structural estimation. We compare these two estimates with simple measures of systemic risk from principal components. We apply the two methodologies to the measurement of the systemic risk during the 2007-08 sub-prime crisis.

Keywords: International finance, asset pricing, terms of trade, dynamic VAR constraints, contagion.

1. Introduction

Since the first quarter of 2007 the US equity markets have been hit with a sequence of shocks related to the sub-prime crisis. Despite the fact that prior to this crisis, developed and developing countries seemed to have decoupled from one another, meaning that correlations of output, consumption, inflation, interest rates, and stock market returns had dropped significantly from the previous decade, the events after the summer of 2007 made it very clear that systemic risk, in the form of international contagion, was back and with a vengeance.

Measuring the degree of systemic risk shares some of the same challenges the contagion literature has had. The main reason for this is that systemic risk and contagion are usually estimated using reduced-form representations, which complicate the interpretation of coefficients as well as the separation between different theories behind the propagation of shocks. For example, from the computation of simple correlations to linear models, from copulas to GARCH models, from principal components to probit regressions, all these methodologies are agnostic about the different theories of behind the propagation mechanism. [1] Furthermore, in most cases, different theories of why shocks are transmitted internationally are tested in a regression based framework, by introducing interaction terms as some of regressor on the right hand side. Although this approach has taught us a great deal about the crises that prevailed during the 1990s, it has had its limitations with regards to determine the strength of international linkages which generate systemic consequences.

The purpose of this paper is twofold. Firstly, we develop an estimation method for international contagion that is semi-structural. As opposed to the standard methods in the literature, our estimation of the strength of global transmission mechanisms for shocks is based on cross-equation restrictions that arise from a formal model of financial constraints. We do not fit the model entirely, which would constitute a fully fledged structural estimation, but rather use the restrictions derived from the model. We view this as the next (and necessary) step in the empirical contagion literature, disentangling the different chan-

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[1] For principal component studies, see e.g., Calvo and Reinhart (1996) and Kaminsky and Reinhart (2007). For evidence of contagion based on reduced-form linear latent factor models, see e.g., Corsetti, Pericoli, and Sbracia (2005), Dungey, Fry, González-Hermosillo, and Martin (2005), and Rigobon (2003), and for a copula approach see e.g., Hartmann, Straetmans, and de Vries (2004) and Rodriguez (2007).

nels of propagation before fitting a full structural model. Secondly, we develop a full fledge structural estimation using the model presented in this paper. The advantage of the first methodology is that it is extremely easy to implement, and it has a clear structural interpretation. This methodology shares several of the advantages of the simple reduced form techniques commonly used in the literature. It has the additional nice characteristic that it has a structural interpretation and therefore it measures the contagion due to tightening of financial constraints. The advantage of the second methodology is that the assumptions required for its estimation are less stringent than the semi-structural approach, it not only measures the contagion due to financial constraints but also the propagation of shocks that occur through standard channels. Henbce, it is a better measure of the degree of systemic risk – or in other words, it measures the systemic consequences of all the shocks in the estimation.

We use both methodologies and study the sub-prime crisis which originated in the US. Clearly there has been dramatic systemic risk (and contagion) across the world caused by the eruption of this crisis, and this has been exacerbated by the tightening of global financial constraints. In order to relate our measure of financial contagion to the actual contagion generated by the sub-prime meltdown, we evaluate our contagion measure with regards to its predictability on news and asset price co-movement in almost 50 stock markets in the world, around the time of the crisis.

For the first purpose of our study, the estimation of systemic risk, Sections 2 and 3 of the paper develop a dynamic equilibrium model of asset prices, in which contagion is transmitted via the (daily) tightening of portfolio volatility constraints[2]. We assume that volatility constraints are placed on the portfolio of a representative financial institution in the base or 'Center' country, as in the model of Pavlova and Rigobon (2007). The reasoning behind this choice of constraint is that restrictions on volatility are very similar in spirit to

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[2]Related literature on two-good two-country asset pricing models includes Helpman and Razin (1978), Cole and Obstfeld (1991) and Zapatero (1995), which do not feature portfolio constraints. Related work on portfolio constraints in the asset pricing literature includes, for example, Basak and Croitoru (2000), Basak and Cuoco (1998), Detemple and Murthy (1994), Detemple and Serrat (2003), Gallmeyer and Hollifield (2004) and Shapiro (2002). Also see Cass, Siconolfi, and Villanacci (2001) for analysis of a general equilibrium model with portfolio constraints. For more applied analyses, see Boyer, Kumagai, and Yuan (2006), Geanakoplos (2003), Gromb and Vayanos (2002), Mendoza and Smith (2002), and Yuan (2005).

limits on Value-at-Risk (VAR), which form an extremely important part of risk regulation for banks across the globe. In particular, the definition of VAR is the worst expected loss of a portfolio over a given horizon at a particular confidence level. Therefore, a constraint on VAR is equivalent to a limit on the downside risk of a portfolio, whereas a constraint on volatility simply restricts overall risk. The similarity between these two types of constraint is thus very clear; however, it is far more tractable to model restrictions on portfolio volatility rather than VAR.

The presence of these realistic financial constraints can then be directly tied to the contagious effects of crises such as the sub-prime collapse[3]. Indeed, the contagion measure which we derive from our model of volatility restrictions is demonstrated to be a common factor that affects the covariance of all countries across the world at the same time; this is explained in greater detail in Section 4, which evaluates some of the implications of our model in the data. Therefore, our paper provides evidence which links actual international contagion to an increase in the tightening of financial constraints facing banks across the globe.

For the second purpose of our paper, namely our empirical analysis in Section 4, we collect information on the stock market indices of 49 countries, and their respective exchange rates relative to the US, (which we take as our 'Center' or constrained country; this makes sense given that the US was the country of origin of the sub-prime crisis). It is important to highlight that with the information we have,(at daily frequencies), we cannot fit the whole model. The reason is that we do not have enough moments in the data to pin down all the parameters. This is why we adopt a semi-structural approach.

Our model has two very important implications that distinguish our approach from the standard one used in the literature. Firstly, when there is a shock that tightens the constraint imposed on the Center country, the co-movement of all stock markets in the world is affected. In other words, a tightening of the constraint not only affects the co-movement between the Center country and any other country, but it also affects the co-movement between any two other countries in the world. The second implication from our model is that if all stock

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[3]There are several papers in the empirical literature which argue that financial risk-regulation has indeed been largely responsible for contagion in developing and emerging markets over the last twenty years or so. For example, see Calvo (2002), Kaminsky and Reinhart (2000), and Van Rijckeghem and Weder (2003).

markets are measured in the same currency, then this change in the co-movement of any two stock markets has to be identical in magnitude across all possible pairs of countries in the world. This means that contagion cannot be measured in our model simply by looking at the increase in the correlation across countries after a shock to the constraint. Rather, it has to be the case that, as a result of a tightening of the constraint in the Center country, covariances across all countries increase by the exact same amount, and only this proportion of their co-movement can be associated with global financial contagion in our set-up.

Another main difference between our approach and the standard methodology in the literature is that we have to include as many countries as possible in our analysis. Contagion papers rarely do this; indeed, most of the time they analyze a subset of countries across the world. In our case, however, because of the implications of the model, contagion can only be measured from the co-movement implied by the joint shift in covariances of all countries.

Our approach is not exempt from criticism, given that our model has to make several simplifying assumptions that are unlikely to be true in reality. Firstly, it assumes that consumers' utility has a log representation. Secondly, it assumes that markets are complete. Thirdly, we have an endowment economy without labor and capital, and our supply shocks are assumed to be independent across countries. Fourthly, our model is a real model where the demand shocks are represented by consumer expenditure shifts; therefore, it does not include nominal, monetary, fiscal, and other sources of shocks. As a result, it is clear that we are deriving our cross-equation restrictions from a very stylized setting.

Moreover, for the semi-structural estimation, our empirical implementation requires us to extend the assumptions in our model even further. For example, we not only have to assume that supply shocks are independent across countries, but we also have to assume that their heteroscedasticity is independent across countries as well, (see Section 4). In other words, we are not only assuming independence of first moments, but for the estimation procedure we also need to assume independence of second moments. Furthermore, we end up implementing our contagion measure using a rolling window to estimate the variance-covariance matrix of all countries in the world. If daily options data on bundles of stock market indices and individual stock markets existed, we could at least estimate the implied variances of

countries from financial markets at a daily frequency. Unfortunately, this information does not exist and we are left with having to use a rolling window to estimate variances and covariances, (and, in turn, our contagion measure), instead. For the structural estimation this assumptions are not needed.

Despite the presence of several of these simplifying assumptions, the major result from our empirical analysis is that our measures of contagion has strong predictability effects on the daily number of news articles in world newspapers documenting the sub-prime crisis. There is also a high daily positive correlation between changes in our contagion estimator and changes in the co-movement of stock prices across the globe, following the sub-prime collapse. Therefore, our new contagion measure appears to be a strong measures of the crisis in this regard.

2. The Model

Our first objective in this paper is to model how a realistic example of a constraint on global financial institutions, (a restriction on volatility), affects the international co-movement of asset prices. Extending the model of Pavlova and Rigobon (2007), we initially work with a three country model, where all three countries are assumed to be identical in size. (The justification for this will be provided later on; in particular, we find that in both constrained and unconstrained environments, all three countries always hold an identical number of shares in the stocks of each country.) In this section and the next, we then specialize the model of Pavlova and Rigobon (2007) to consider a specific constraint on the volatility of the portfolio of one of the countries. We examine the implications of this constraint for the dynamics of stock prices and terms of trade at a partial and general equilibrium level.

2.1. The Economic Setting

The setting is almost exactly the same as in Pavlova and Rigobon (2007). We briefly recall their main assumptions and results here. We consider a continuous-time pure-exchange world economy with a finite horizon, $[0, T]$. We start with three countries in the world economy,

indexed by $j \in \{0, 1, 2\}$. Countries 0, 1 and 2 are assumed to be identical in size. (For ease of reference, however, we refer to Country 0 as the Center country and Countries 1 and 2 as the Periphery Countries). Each Country j has a strictly positive output process modeled as a Lucas (1978) tree:

$$dY^j(t) = \mu_{Y^j}(t)\, Y^j(t)\, dt + \sigma_{Y^j}(t)\, Y^j(t)\, dw^j(t), \qquad j \in \{0, 1, 2\}, \qquad (1)$$

where μ_{Y^j} and $\sigma_{Y^j} > 0$ are the respective processes for the mean growth rate and volatility of output for Country j, and w^j is the independent Brownian motion representing an output or supply shock to Country j. The price of the good produced by Country j is denoted by p^j. We define a numeraire basket containing $\beta \in (0, 1)$ units of the good produced in Country 0 and $(1-\beta)/2$ units of each of the remaining two goods and normalize the price of this basket to be equal to unity. We think of β as the size of the Center country relative to the world economy. Given our assumption that all countries are identical in size, we therefore set β equal to $\frac{1}{3}$ in our model.

In order to complete markets, investment opportunities are represented by four securities. Each Country j issues a stock S^j, a claim to its output. All stocks are in unit supply. There is also the "world" bond B, which is riskless in units of the numeraire, and is in zero net supply. We define the terms of trade from the viewpoint of the Center country (Country 0): $q^1 \equiv p^1/p^0$ and $q^2 \equiv p^2/p^0$ are the terms of trade of the Periphery countries 1 and 2, respectively, with the Center country.

A representative consumer-investor of each country is endowed at time 0 with a supply of the stock market of his country; the initial wealth of agent i is denoted by $W_i(0)$. Each consumer i chooses nonnegative consumption of each good $(C_i^0(t), C_i^1(t), C_i^2(t))$, $i \in \{0, 1, 2\}$, and a portfolio of the available risky securities $x_i(t) \equiv (x_i^{s^0}(t), x_i^{s^1}(t), x_i^{s^2}(t))^\top$, where x_i^j denotes a fraction of wealth W_i invested in security j. For the sake of tractability, preferences of consumer i are represented by a time-additive log utility function defined over consumption of all three goods:

$$E\left[\int_0^T u_i\big(C_i^0(t), C_i^1(t), C_i^2(t)\big)\, dt\right],$$

where

$$u_0\big(C_0^0,\, C_0^1,\, C_0^2\big) \;=\; \alpha_0 \log C_0^0(t) + \frac{1-\alpha_0}{2} \log C_0^1(t) + \frac{1-\alpha_0}{2} \log C_0^2(t),$$

$$u_1\big(C_1^0,\, C_1^1,\, C_1^2\big) \;=\; \frac{1-\alpha_1(t)}{2} \log C_1^0(t) + \alpha_1(t) \log C_1^1(t) + \frac{1-\alpha_1(t)}{2} \log C_1^2(t),$$

$$u_2\big(C_2^0,\, C_2^1,\, C_2^2\big) \;=\; \frac{1-\alpha_2(t)}{2} \log C_2^0(t) + \frac{1-\alpha_2(t)}{2} \log C_2^1(t) + \alpha_2(t) \log C_2^2(t).$$

We set the preference weight on the domestically-produced good, α_i, to be greater than $1/3$ (and less than 1), for each Country i. This is in order to generate a home bias in preferences. We also include demand shifts as a key source of uncertainty in our model[4]. In particular, an increase in α_i in our model represents a demand shift towards domestically produced goods in Country i. We take each α_i to be a martingale (i.e., $E[\alpha_i(s)|\mathcal{F}_t] = \alpha_i(t)$, $s > t$), and hence can be represented as

$$d\alpha_1(t) = \sigma_{\alpha_1}(t)^\top dw(t), \quad d\alpha_2(t) = \sigma_{\alpha_2}(t)^\top dw(t).$$

We keep the preference parameter of the Center country, α_0, fixed, in order to keep the focus on the Periphery Countries.

2.2. Countries' Optimization and Benchmark Unconstrained Equilibrium

In an environment with no portfolio constraints, we have the following results reported from Pavlova and Rigobon (2007).

Proposition 1. *(i) We can define a representative agent who is endowed with the aggregate supply of securities and consumes the aggregate output. His utility is given by*

$$U(C^0,\, C^1,\, C^2;\, \lambda_0,\, \lambda_1,\, \lambda_2) \;=\; E\left[\int_0^T u(C^0(t),\, C^1(t),\, C^2(t);\, \lambda_0,\, \lambda_1,\, \lambda_2)dt\right],$$

with

$$u(C^0,\, C^1,\, C^2;\, \lambda_0,\, \lambda_1,\, \lambda_2) \;=\; \max_{\sum_{i=0}^2 C_i^j = C^j,\, j \in \{0,1,2\}} \sum_{i=0}^2 \lambda_i u_i(C_i^0,\, C_i^1,\, C_i^2),$$

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[4]Demand shifts are modeled along the lines of Dornbusch, Fischer, and Samuelson (1977). In particular, in the absence of demand shocks, free trade in goods would imply extremely high correlation of stock markets and therefore irrelevancy of financial structure, as established by Helpman and Razin (1978), Cole and Obstfeld (1991) and Zapatero (1995).

where $\lambda_i > 0$, $i = 0, 1, 2$, are the weights on consumers 0, 1, and 2, respectively. These weights are constant in the unconstrained economy, but will be stochastic in the economy with portfolio constraints. In the unconstrained case, these weights are simply the inverses of the Lagrange multipliers on the consumers' intertemporal budget constraints. Since in equilibrium these multipliers, and hence the weights, cannot be individually determined, we adopt a normalization $\lambda_0 = 1$.

(ii) Terms of trade are given by the relevant marginal rates of substitution processes

$$q^1(t) = \frac{u_{C^1}(Y^0(t), Y^1(t), Y^2(t); \lambda_1, \lambda_2)}{u_{C^0}(Y^0(t), Y^1(t), Y^2(t); \lambda_1, \lambda_2)} = \frac{\frac{1-\alpha_0}{2} + \lambda_1\alpha_1(t) + \lambda_2\frac{1-\alpha_2(t)}{2}}{\alpha_0 + \lambda_1\frac{1-\alpha_1(t)}{2} + \lambda_2\frac{1-\alpha_2(t)}{2}} \frac{Y^0(t)}{Y^1(t)}, \qquad (2)$$

$$q^2(t) = \frac{u_{C^2}(Y^0(t), Y^1(t), Y^2(t); \lambda_1, \lambda_2)}{u_{C^0}(Y^0(t), Y^1(t), Y^2(t); \lambda_1, \lambda_2)} = \frac{\frac{1-\alpha_0}{2} + \lambda_1\frac{1-\alpha_1(t)}{2} + \lambda_2\alpha_2(t)}{\alpha_0 + \lambda_1\frac{1-\alpha_1(t)}{2} + \lambda_2\frac{1-\alpha_2(t)}{2}} \frac{Y^0(t)}{Y^2(t)}. \qquad (3)$$

(iii) The prices of the stocks of the Center and the Periphery countries are given by

$$S^0(t) = \frac{1}{\beta + \frac{1-\beta}{2}q^1(t) + \frac{1-\beta}{2}q^2(t)} Y^0(t)(T - t), \qquad (4)$$

$$S^1(t) = \frac{q^1(t)}{\beta + \frac{1-\beta}{2}q^1(t) + \frac{1-\beta}{2}q^2(t)} Y^1(t)(T - t), \qquad (5)$$

$$S^2(t) = \frac{q^2(t)}{\beta + \frac{1-\beta}{2}q^1(t) + \frac{1-\beta}{2}q^2(t)} Y^2(t)(T - t). \qquad (6)$$

(iv) Wealth distribution is constant, determined only by the initial shareholdings:

$$\frac{W_1(t)}{W_0(t)} = \lambda_1 \quad and \quad \frac{W_2(t)}{W_0(t)} = \lambda_2. \qquad (7)$$

(v) In the unconstrained equilibrium, all agents hold an equal number of shares of stocks S^0, S^1, and S^2. This number is given by $\frac{\lambda_i}{\lambda_0+\lambda_1+\lambda_2}$, where $\lambda_0 = 1$. No shares of the bond are held in equilibrium.

(vi) The joint dynamics of the terms of trade and the three stock markets in the benchmark unconstrained economy, (suppressing the drift term), are given by

$$\begin{bmatrix} \frac{dq^1(t)}{q^1(t)} \\ \frac{dq^2(t)}{q^2(t)} \\ \frac{dS^0(t)}{S^0(t)} \\ \frac{dS^1(t)}{S^1(t)} \\ \frac{dS^2(t)}{S^2(t)} \end{bmatrix} = I(t)dt + \underbrace{\begin{bmatrix} a(t) & b(t) & 1 & -1 & 0 \\ \widetilde{a}(t) & \widetilde{b}(t) & 1 & 0 & -1 \\ -X_{\alpha_1}(t) & -X_{\alpha_2}(t) & \beta M(t) & \frac{1-\beta}{2}\frac{M(t)}{q^1(t)} & \frac{1-\beta}{2}\frac{M(t)}{q^2(t)} \\ a(t) - X_{\alpha_1}(t) & b(t) - X_{\alpha_2}(t) & \beta M(t) & \frac{1-\beta}{2}\frac{M(t)}{q^1(t)} & \frac{1-\beta}{2}\frac{M(t)}{q^2(t)} \\ \widetilde{a}(t) - X_{\alpha_1}(t) & \widetilde{b}(t) - X_{\alpha_2}(t) & \beta M(t) & \frac{1-\beta}{2}\frac{M(t)}{q^1(t)} & \frac{1-\beta}{2}\frac{M(t)}{q^2(t)} \end{bmatrix}}_{\Theta_u(t)} \begin{bmatrix} d\alpha_1(t) \\ d\alpha_2(t) \\ \sigma_{Y^0}(t)dw^0(t) \\ \sigma_{Y^1}(t)dw^1(t) \\ \sigma_{Y^2}(t)dw^2(t) \end{bmatrix}$$

The drift term I and quantities X_{α_1}, X_{α_2}, M, a, $\widetilde{a}$, b, and $\widetilde{b}$ are defined in Pavlova and Rigobon (2007).

2.3. Volatility Constraints

Here, we specify that the Center country now faces a constraint on the volatility of its stock portfolio. We make the plausible assumption that the constraint is placed on the portfolio of a representative financial institution, such as a commercial bank, in this country. As described in Section 1, the reasoning behind this choice of constraint is that a restriction on volatility is very similar to a limit on portfolio Value-at-Risk (VAR), but is far more tractable to model. In particular, VAR is a risk measure used by banks across the world, which calculates the worst expected loss of a portfolio over a given horizon at a particular confidence level. Therefore, a constraint on VAR is a limit on the downside risk of a portfolio, whereas a volatility constraint simply restricts overall risk[5].

Suppose we then have the following, exogenously set dynamic[6]constraint on the volatility of Country 0's portfolio x_0, (where σ represents the volatility matrix of all stock returns in the economy).

$$x_0(t)^\top \sigma(t)\sigma(t)^\top x_0(t) \le \phi(t). \tag{8}$$

Equation (8) thus provides us with the exact form of the constraint we are going to use when analyzing the properties of general equilibrium in the economy with volatility limits. We assume a daily[7]horizon here for changes in the degree of the volatility constraint, $\phi(t)$, over

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[5]We recognize here that we do not model separate sets of investors or economic agents in Country 0 who are constrained, and do not get into exact details as to why a VAR constraint, or its proxy, a volatility constraint, may be imposed. It is important to stress, however, that VAR limits are a very realistic example of financial restrictions facing institutions across the globe, given that they function as a tool for controlling risk. For example, according to the Basel Committee on Banking Supervision (2001) report, the Basel Committee stipulates that international commercial banks should use VAR as an internal risk measure, subject to the fulfillment of certain sets of conditions and the approval of their supervisory committees. Under these circumstances, banks have no choice but to comply with this regulation, allowing us to justify the imposition of a VAR or volatility restriction on a representative financial institution, or commercial bank, in Country 0.

[6]We can rely on a wealth of evidence from current practices to support the assumption of a stochastic limit on portfolio VAR or volatility. For example, dynamic VAR constraints are often used by banks and firms to help set position limits for traders, as well as to trim the risk of different business units of a corporation. These VAR limits are stochastic because they are usually tied to movements of the market. For instance, if market volatility increases, more stringent controls on risk may have to be imposed via tighter VAR constraints, so that financial positions are scaled down appropriately. (On the other hand, if market volatility jumps up suddenly, a risk manager may alternatively want to relax his VAR limits to avoid a liquidation under difficult market conditions; thus, the implementation and flexibility of dynamic restrictions always depends on the judgement of the risk manager).

[7]Relating this to actual practice, we note that dynamic VAR limits, while used widely to control risk across all financial institutions, firms and banks, are best suited to fast-paced trading environments where turnover

time.

3. Equilibrium in the Economy with Volatility Constraints

We now describe general equilibrium under the specific volatility constraint considered in Section 2.3. As described in Pavlova and Rigobon (2007), in a constrained economy, the only change to be made to Proposition 1 is that the representative agent must now be defined using stochastic weighting processes, (introduced by Cuoco and He (2001)), with these stochastic weights representing the effects of the new constraint[8]. The following results are reported from Pavlova and Rigobon (2007).

Proposition 2. *(i) In an equilibrium with the portfolio constraint, the weighting processes* λ_1 *and* λ_2 *are the same up to a multiplicative constant, which we denote as* λ. *The dynamics of* λ *are given below:*

$$d\lambda(t) = \lambda(t)[r(t) - r_0(t) + m(t)^\top (m_0(t) - m(t))]dt - \lambda(t)(m_0(t) - m(t))^\top dw(t). \quad (9)$$

where m_0 *and* r_0 *are, respectively, the effective market price of risk and interest rate faced by Country 0 due to the portfolio constraint[9], and* m *and* r *are, respectively, the vector market price of risk and interest rate of the unconstrained Periphery Countries.*
(ii) When such an equilibrium exists, the joint dynamics of the terms of trade and three stock markets in the economy with the portfolio constraint are given by

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is rapid, leverage is high, and portfolio positions change several times a day, (see Jorion (2000)). These environments are most prevalent in commercial banks, which usually have high turnover and liquidity in their portfolios, and thus need to adjust risk exposure frequently. Furthermore, a daily horizon is most commonly used by commercial banks across the world in order to calculate trading VARs, (with the added advantage that a daily VAR is consistent and easily comparable with daily profit and loss measures). Therefore, we can justify our assumption here that a daily volatility constraint, (as a proxy for a daily VAR limit), is placed on the portfolio of a representative commercial bank in Country 0

[8]See, for example, Basak and Croitoru (2000), Basak and Cuoco (1998), and Shapiro (2002). For the original solution method see Negishi (1960).

[9]The optimization problem of the Center under portfolio constraints is formally equivalent to a dual problem with no constraints but with the Center facing a fictitious 'tilted' investment opportunity set (Cvitanić and Karatzas (1992)). Cvitanić and Karatzas show that the tilt in the fictitious investment opportunity set is characterized by the multipliers on the portfolio constraints. Consequently, Country 0's market price of risk under the constraint, m_0, and interest rate, r_0 (derived in the Appendix), differ from those faced by the unconstrained investors in the Periphery; these are denoted respectively as m and r. Given that the difference in investment opportunity sets faced by the Center and Periphery countries is due to the presence of the portfolio constraint on Country 0, it is clear that all investors will only hold the same portfolio in the unconstrained case, where the tilted market price of risk $m_0(t)$ coincides with $m(t)$, and $r_0(t)$ coincides with $r(t)$.

$$
\begin{bmatrix}
\frac{dq^1(t)}{q^1(t)} \\
\frac{dq^2(t)}{q^2(t)} \\
\frac{dS^0(t)}{S^0(t)} \\
\frac{dS^1(t)}{S^1(t)} \\
\frac{dS^2(t)}{S^2(t)}
\end{bmatrix}
= I_c(t)dt +
\left[
\begin{array}{c|c}
\begin{matrix}
A(t) \\
\widetilde{A}(t) \\
-X_\lambda(t) \\
A(t) - X_\lambda(t) \\
\widetilde{A}(t) - X_\lambda(t)
\end{matrix}
& \Theta_u(t)
\end{array}
\right]
\begin{bmatrix}
\frac{d\lambda(t)}{\lambda(t)} \\
d\alpha_1(t) \\
d\alpha_2(t) \\
\sigma_{Y^0}(t)dw^0(t) \\
\sigma_{Y^1}(t)dw^1(t) \\
\sigma_{Y^2}(t)dw^2(t)
\end{bmatrix}
$$

where $\lambda(t) \equiv \lambda_1(t)$, I_c, A, $\widetilde{A}$ and X_λ are reported in Pavlova and Rigobon (2007), and the unconstrained dynamics matrix $\Theta_u(t)$ is as defined in Proposition 1.

The only departure from the corresponding expression in the benchmark economy is in the addition of the first, $d\lambda/\lambda$, term. Therefore, a movement in λ represents a tightening or a loosening of the portfolio constraint; this will be explored in greater detail later in this section. Furthermore, given the definition of λ in part (iv) of Proposition 1 in terms of the Periphery countries' relative wealth, an increase (decrease) in lambda also reflects a wealth transfer in the world economy to (away from) the Periphery countries. This mechanism of endogenous wealth transfers is the channel through which changes in λ affect the tightness of the portfolio constraint in our model[10].

We can now define all equilibrium quantities in our constrained setting. The set of equations required to do this is presented in the following proposition.

Proposition 3. *When equilibrium exists, the equilibrium market price of risk processes faced by the Center and the Periphery are related as follows:*

When $\quad x_0(t)^\top \sigma(t)\sigma(t)^\top x_0(t) = m(t)^\top m(t) \le \phi(t),$

$$
m_0(t) = m(t), \quad \psi(t) = 0, \quad \text{(Constraint not binding)}, \tag{10}
$$

otherwise,

$$
m_0(t) = \frac{m(t)}{1 + 2\psi(t)}, \tag{11}
$$

$$
(1 + 2\psi(t))^2 = \frac{m(t)^\top m(t)}{\phi(t)}, \quad \text{(Constraint binding)},
$$

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[10]For other papers which explore contagion as a wealth effect, see Kyle and Xiong (2001) and Cochrane, Longstaff, and Santa-Clara (2008).

where ψ is the multiplier on the volatility constraint, and σ is the volatility matrix of the three stock returns in the economy; see Pavlova and Rigobon (2007). Furthermore,

$$\sigma_{Y^0}(t)i_0 \ - \ \frac{\left(\lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}\right)(m(t) - m_0(t)) - \frac{\lambda_1(t)}{2}\sigma_{\alpha_1}(t) - \frac{\lambda_2(t)}{2}\sigma_{\alpha_2}(t)}{\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}}$$

$$= X_\lambda(t)(m(t) - m_0(t)) + X_{\alpha_1}(t)\sigma_{\alpha_1}(t) + X_{\alpha_2}(t)\sigma_{\alpha_2}(t)$$
$$+ \frac{1-\beta}{2}M(t)(q^1(t) + q^2(t))\sigma_{Y^0}(t)\,i_0 - \frac{1-\beta}{2}M(t)q^1(t)\sigma_{Y^1}(t)\,i_1$$
$$- \frac{1-\beta}{2}M(t)q^2(t)\sigma_{Y^2}(t)\,i_2 + m_0(t). \tag{12}$$

where $i_0 \equiv (1,\,0,\,0)^\top$, $i_1 \equiv (0,\,1,\,0)^\top$, and $i_2 \equiv (0,\,0,\,1)^\top$.

Equations (10)–(11) are the complementary slackness conditions from the constrained portfolio optimization of the Center country. When the portfolio volatility constraint is not binding, the market price of risk faced by the Center coincides with that faced by the Periphery countries. Therefore, the portfolio of the Center is equivalent to the Periphery countries' unconstrained portfolios. When the constraint is binding, however, there is now a difference between the market prices of risk faced by the Center and the Periphery, expressed in (11). Equation (12) is derived from market clearing in the consumption goods.

Finally, given (10)–(11), we can derive an analogous result to (v) of Proposition 1 in the constrained environment. This provides us with the justification for our assumption that all three countries are identical in size in our model:

Corollary 4. *In the constrained equilibrium, all agents still hold an equal number of shares of stocks S^0, S^1, and S^2. Their holdings of the bond, however, are different, although by assumption, net demand = net supply = 0 in equilibrium.*

The relationship between λ and the multiplier on the volatility constraint ψ, (or in other words, the link between λ and the tightness of the portfolio volatility constraint), can be expressed in a simple plot, Figure 1. Here, we fix all time-dependent/state variables in our model, except for the wealth shares of the Periphery countries, $\lambda_1(t)$ and $\lambda_2(t)$. The horizontal axes measure λ_1 and λ_2 and the vertical axis measures ψ. The parameters used in the analysis are summarized in Table 1.

β	$\frac{1}{3}$	ϕ	0.003		
α_0	0.75	$Y^0(t)$	1.0	$\sigma_{Y^0}(t)$	0.1
α_1	0.75	$Y^1(t)$	1.0	$\sigma_{Y^1}(t)$	0.1
α_2	0.75	$Y^2(t)$	1.0	$\sigma_{Y^2}(t)$	0.1
$\lambda_1(t) \in [0.75, 1.25]$					
$\lambda_2(t) \in [0.75, 1.25]$					
$\sigma_{\alpha_1}(t)$	(0, 0.2, 0)				
$\sigma_{\alpha_2}(t)$	(0, 0, 0.2)				

Table 1: Parameter choices

From Pavlova and Rigobon (2007), the reasoning behind the choice of these parameters is described as follows. In our model, as previously mentioned, all three countries in the world are identical in size, so in the numeraire consumption basket they each represent $\frac{1}{3}$ of the world. We set 75 percent as the share of expenditures on the domestic good, which is a conservative estimate given the share of the service sector in GDP. It is also found that in the data, demand shocks are positively correlated with domestic supply innovations. Therefore, we assume that a demand shift in Country j has a positive loading on w^j and zero loadings on the remaining Brownian motions.

We can observe from Figure 1 that roughly along the diagonal of the horizontal plane, where λ_1 is equal or close to λ_2 in magnitude, the volatility constraint does not bind, and the multiplier ψ is zero. This is our unconstrained region. As λ_1 and λ_2 diverge from each other, however, the constraint does become more binding and the multiplier increases. This means that the constraint on the volatility of the Center country's portfolio becomes tighter as λ_1 and λ_2 become increasingly different, or as the relative wealth shares of the two other countries in the world diverge. These results make sense; indeed, given that we are imposing a volatility constraint on the portfolio, or, equivalently, on the overall wealth of Country 0, it is natural to expect that this constraint would become more (less) binding as the wealth flows of Country 0 to and from Countries 1 and 2 diverge from (converge to) each other, or as Countries 1 and 2 diverge (converge) in relative wealth. Given that in equilibrium λ_1 and λ_2 are the same up to a multiplicative constant, which we denote as λ, Figure 1 thus displays how movements in λ_1, λ_2, and therefore λ, tighten or loosen our portfolio constraint. In particular, as λ_1 and λ_2 diverge (converge), the absolute magnitude of λ will diverge from

(converge to) 1, and this will consequently make the volatility constraint tighter (looser) in our model.
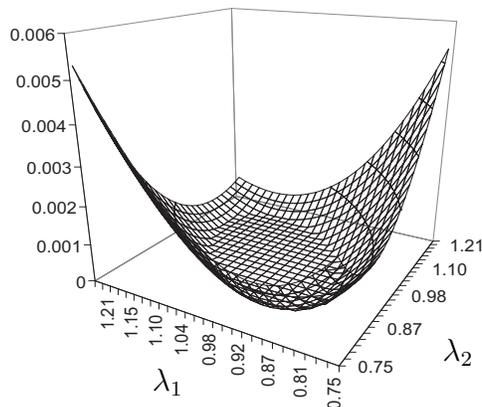

Figure 1: Value of the multiplier on the portfolio volatility constraint ψ

4. Empirical Estimation

In this section we estimate the contagious effect derived from the US sub-prime crisis, taking the US to be the Center or constrained country in our model, and the rest of the countries as the periphery. The sub-prime crisis has had small output effects from the beginning of 2007 until February 2008. However, it has had a large effect on financial markets. One of the implications of the sub-prime meltdown is the tightening of cash constraints in the US financial system, which can be interpreted as a tightening of the portfolio risk constraint in our model, or a change in λ. Of course, this is an oversimplification. Nevertheless, by taking our model literally and estimating it in the data during the sub-prime collapse, we are in effect measuring the contagion generated by such a crisis due to the tightening of portfolio constraints.

Our model is extremely difficult to estimate in the data given that we would need to observe both output and consumption, as well as asset prices, at high frequencies. In particular, as mentioned before, the effects of the crisis on stock markets have been relatively short lived. Therefore, these effects would be undetectable at the lower frequencies at which

output is typically observed. Therefore, only some parts of the model can be taken to the data. In this section, we explore two of those approaches. In the first one we make several simplifying assumptions that lead to a very simple implementation. In the second one we make fewer assumption but the estimation becomes more intensive.

4.1. Description of the Data

The data we have collected on global stock markets and exchange rates[11] in order to estimate our covariance matrix originates from DataStream 4.0. Daily stock market data was obtained from the "DataStream Total Market" index for each country listed in the database, (there were 51 countries in total reporting this index), both in US dollars and local currency, from February 9th 2007 to February 11th 2008. The US dollar and local currency values of the stock market index for each country were then used to calculate individual exchange rates. For countries which did not report the "DataStream Total Market" index in both US dollars and their local currency, daily spot exchange rates were downloaded instead.

The full list of countries reporting daily stock market index data from DataStream comprises, (in alphabetic order): Argentina, Australia, Austria, Belgium, Brazil, Bulgaria, Canada, Chile, China, Columbia, Cyprus, Denmark, Finland, France, Germany, Greece, Hong Kong, Hungary, India, Indonesia, Ireland, Israel, Italy, Japan, South Korea, Luxembourg, Malaysia, Mexico, Netherland, New Zealand, Norway, Pakistan, Peru, Philippines, Poland, Portugal, Romania, Russia, Singapore, Slovenia, South Africa, Spain, Sri Lanka, Sweden, Switzerland, Taiwan, Thailand, Turkey, United Kingdom, United States of America and Venezuela.

Two of these countries, Columbia and Venezuela, were dropped from our sample due to inadequate exchange rate data, leaving us with 49 countries in total.

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[11]Strictly speaking, given that we are taking our common currency to be the US dollar, in order to compute the global covariance of relative asset prices we need daily data on the terms of trade, or the relative price of a representative consumption basket, for all other countries in the world relative to the US. It is not possible, however, to obtain data on global terms of trade at a daily frequency, meaning that we have to use daily exchange rate data instead

4.2. Multiple countries

We extend our model to include N+1 countries. Manipulating part (ii) of Proposition 2, we can express the joint dynamics of all relative asset prices in the economy as follows:

$$
\begin{bmatrix}
\frac{dS^0(t)}{S^0(t)} \\
\frac{dS^1(t)}{S^1(t)} - \frac{dq^1(t)}{q^1(t)} \\
\frac{dS^2(t)}{S^2(t)} - \frac{dq^2(t)}{q^2(t)} \\
\vdots \\
\frac{dS^n(t)}{S^n(t)} - \frac{dq^n(t)}{q^n(t)}
\end{bmatrix}
= \widetilde{I}(t)dt +
\left[
\begin{array}{c|c}
\begin{matrix} -X_\lambda(t) \\ -X_\lambda(t) \\ -X_\lambda(t) \\ \vdots \\ -X_\lambda(t) \end{matrix} & \Theta_v(t)
\end{array}
\right]
\begin{bmatrix}
\frac{d\lambda(t)}{\lambda(t)} \\
\{d\alpha_1(t), \ldots, d\alpha_n(t)\}' \\
\{\sigma_{Y^0}(t)dw^0(t), \ldots, \sigma_{Y^n}(t)dw^n(t)\}'
\end{bmatrix}
$$

where matrix $\Theta_v(t)$ is defined below as:

$$
\begin{bmatrix}
\underbrace{\begin{matrix} -X_{\alpha_1}(t) & \cdots & -X_{\alpha_n}(t) \\ -X_{\alpha_1}(t) & \cdots & -X_{\alpha_n}(t) \\ -X_{\alpha_1}(t) & \cdots & -X_{\alpha_n}(t) \\ \vdots & & \vdots \\ -X_{\alpha_1}(t) & \cdots & -X_{\alpha_n}(t) \end{matrix}}_{demand}
\underbrace{\begin{matrix} \beta M(t) & \frac{1-\beta}{2}M(t)q^1(t) & \frac{1-\beta}{2}M(t)q^2(t) & \cdots & \frac{1-\beta}{2}M(t)q^n(t) \\ \beta M(t)-1 & \frac{1-\beta}{2}M(t)q^1(t)+1 & \frac{1-\beta}{2}M(t)q^2(t) & \cdots & \frac{1-\beta}{2}M(t)q^n(t) \\ \beta M(t)-1 & \frac{1-\beta}{2}M(t)q^1(t) & \frac{1-\beta}{2}M(t)q^2(t)+1 & \cdots & \frac{1-\beta}{2}M(t)q^n(t) \\ \vdots & \vdots & \vdots & & \vdots \\ \beta M(t)-1 & \frac{1-\beta}{2}M(t)q^1(t) & \frac{1-\beta}{2}M(t)q^2(t) & \cdots & \frac{1-\beta}{2}M(t)q^n(t)+1 \end{matrix}}_{supply}
\end{bmatrix}
$$

and where the vectors $\{d\alpha_1(t), \ldots, d\alpha_n(t)\}'$ and $\{\sigma_{Y^0}(t)dw^0(t), \ldots, \sigma_{Y^n}(t)dw^n(t)\}'$ represent all the demand and supply shocks in the economies.

We make several assumptions here regarding the demand and supply shocks in this economy. Firstly, we have already assumed that the supply shocks or Brownian motions are independent across countries. Secondly, we have assumed that the demand shocks are linear functions of the supply shocks that have different loadings on each shock. Furthermore, we note that $\Theta_v(t)$ has exactly the same loading for all countries on the demand shocks, while the loadings on the supply shocks are different across all countries. This means that the product of $\Theta_v(t)$ and the demand and supply shocks is a linear function that will have different loadings on every supply shock for each of the N+1 countries in the economy. Finally, any change in (λ), (equivalent to a tightening or loosening of the portfolio volatility constraint), depends on both the supply shocks and also on changes in the degree of the constraint.

Tracking the impact of any particular supply shock becomes, therefore, a very difficult empirical exercise, given that the total effect is a convolution of supply, demand, and portfolio constraint dynamics. In order to tackle this problem, let us consider a change in the tightness

of the portfolio constraint, or a change in (λ), when all the country supply shocks are zero. In this case, an increase in the tightening of the constraint shifts λ, and this change is reflected equally in all relative asset prices, (given that the loading on changes in λ, $-X_\lambda(t)$, is the same for all countries in the model). Thus, an alternative to estimating the impact of a tightening of the constraint is to find periods in time in which just the constraint shifts. The impact of the constraint will then be equal to the average change in all relative asset prices at that time. This method involves making a very strong assumption, however, given that supply shocks are unlikely to be zero at any point in time.

Nevertheless, the same intuition can be used if we think about shifts in second moments instead. As can be inferred from part (i) of Proposition 2, shocks to the degree of tightening of the constraint change the conditional heteroscedasticity of the λ process. Let us now assume that shocks to the output supply for all countries are homoscedastic. If this is the case, then any heteroscedasticity observed across relative asset prices has to be explained by changes in the portfolio constraint factor. Therefore, just estimating this heteroscedasticity would be a measure of the contagious effect of the portfolio constraint. Once again, the assumption that all supply shocks are homoscedastic is a strong one. However, given we have assumed that the supply shocks are independent across countries, if we are instead willing to make the assumption that these shocks are heteroscedastic, but their conditional heteroscedasticity is independent across countries as well, then we can estimate the impact of the portfolio constraint as the average change in the heteroscedasticity of all moments of relative stock prices.

More precisely, if a change in the tightening of the portfolio constraint shifts $\frac{d\lambda(t)}{\lambda(t)}$, then all variances and covariances across all countries will shift exactly by a factor of $X_\lambda^2(t) \cdot var\left(\frac{d\lambda(t)}{\lambda(t)}\right)$. If a supply shock is heteroscedastic, on the other hand, then some variances and covariances between relative asset prices will increase in response, while others will decrease. Furthermore, the magnitude of the increases and decreases will vary across countries as a result of any supply shock. However, if the heteroscedasticity of supply shocks is independent across countries, (meaning that the average change in the variance of supply shocks across all countries is equal to zero), then the average change in covariance of relative asset prices

coming from supply shocks will be equal to zero as well.

In summary then, the change in the tightening of the portfolio constraint implies a shift in second moments that is common to all pairs in the data, assuming that all stock markets are measured in the same currency. Supply shocks do not share this property. We use this difference to implement our estimator of contagion. Denote the covariance matrix of all relative stock prices as

$$\Omega_t = covar \begin{bmatrix} \frac{dS^0(t)}{S^0(t)} \\ \frac{dS^1(t)}{S^1(t)} - \frac{dq^1(t)}{q^1(t)} \\ \frac{dS^2(t)}{S^2(t)} - \frac{dq^2(t)}{q^2(t)} \\ \vdots \\ \frac{dS^n(t)}{S^n(t)} - \frac{dq^n(t)}{q^n(t)} \end{bmatrix}$$

then, the average change in all the elements of Ω_t is proportional to the change in variance of $\frac{d\lambda(t)}{\lambda(t)}$, induced by a change in the tightening of the portfolio constraint. The average covariance change of relative asset prices is thus our new contagion measure at any point in time.

Although the assumptions required to rationalize this measure of contagion are strong, (namely, output shocks are independent and their conditional volatility is also independent across countries), its implementation is extremely simple. The first step is to compute all stock markets in one common currency, (we use the US dollar as our base). Secondly, we estimate the covariance matrix displayed above at a daily frequency using a rolling window; the justification for this methodology is provided shortly. (Given that we have assumed a dynamic daily portfolio constraint, we need to use a time horizon of one day in our contagion estimation procedure). Thirdly, we estimate the daily change in the average of all the elements in this covariance matrix in order to identify changes in the constraint, and thus contagion, over time.

4.3. Semi-structural approach

We implement our contagion estimator from February to December 2009. This is a period of time during which news about the sub-prime crisis dominated financial markets in the

US. We estimate the daily covariance matrix for relative asset prices across all countries, (measuring stock prices in US dollars), using a rolling window of 5 days. (We also estimate the covariance matrix using 10 and 20 day rolling windows and the results are qualitatively the same. Our preferred specification uses the highest frequency possible.)[12] Using the rolling window we then compute the change in the average of all the elements of the covariance matrix from day to day. We depict the results in Figure 2. We also present the conditional daily standard deviation of the US stock market, computed using the same 5 day rolling window, for discussion purposes only. All daily stock market and exchange rate data is measured in logs.

It is interesting to compare our measure of contagion to the stock market standard deviation in the US. Clearly, the sub-prime crisis was a shock which originated in the US, and therefore it may initially make sense to use the US stock market volatility as a measure of the size of that shock. However, that would be incorrect, given that the US stock markets have been hit by a multitude of shocks which may not have been propagated with the same intensity to other countries. For instance, the implicit expected monetary policy response to the sub-prime crisis has been accounted for in the US markets, but might not have been reflected in other countries. Furthermore, fiscal packages in the US will have affected US asset prices, but might not have been accounted for to the same degree elsewhere in the world. Our estimator of contagion, however, captures exclusively the increase in co-movement across asset prices that is compounded from all sources of shocks in the US; these include the automatic responses, and the surprise responses as well, of fiscal and monetary policy.

Interestingly, even though in the figure our measure of contagion and the US stock market standard deviation appear to be correlated, their correlation in levels is only 52.9 percent, and -2.8 percent in changes. In other words, these variables do indeed appear to be measuring

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[12]One important question is if we can compute the daily implied variance-covariance matrix for relative asset prices from the data. Unfortunately we cannot, and thus have to use a rolling window to estimate covariances instead. In particular, we may have daily information about implied volatility in some countries from options on the stock market indices, but we certainly do not have information that we could use to estimate daily covariances. It is also worth highlighting that we cannot estimate the conditional variance-covariance matrix in our case either. The reason is that we are allowing the covariance terms across countries to be as unrestricted as possible, and are just using their average shift to estimate contagion.
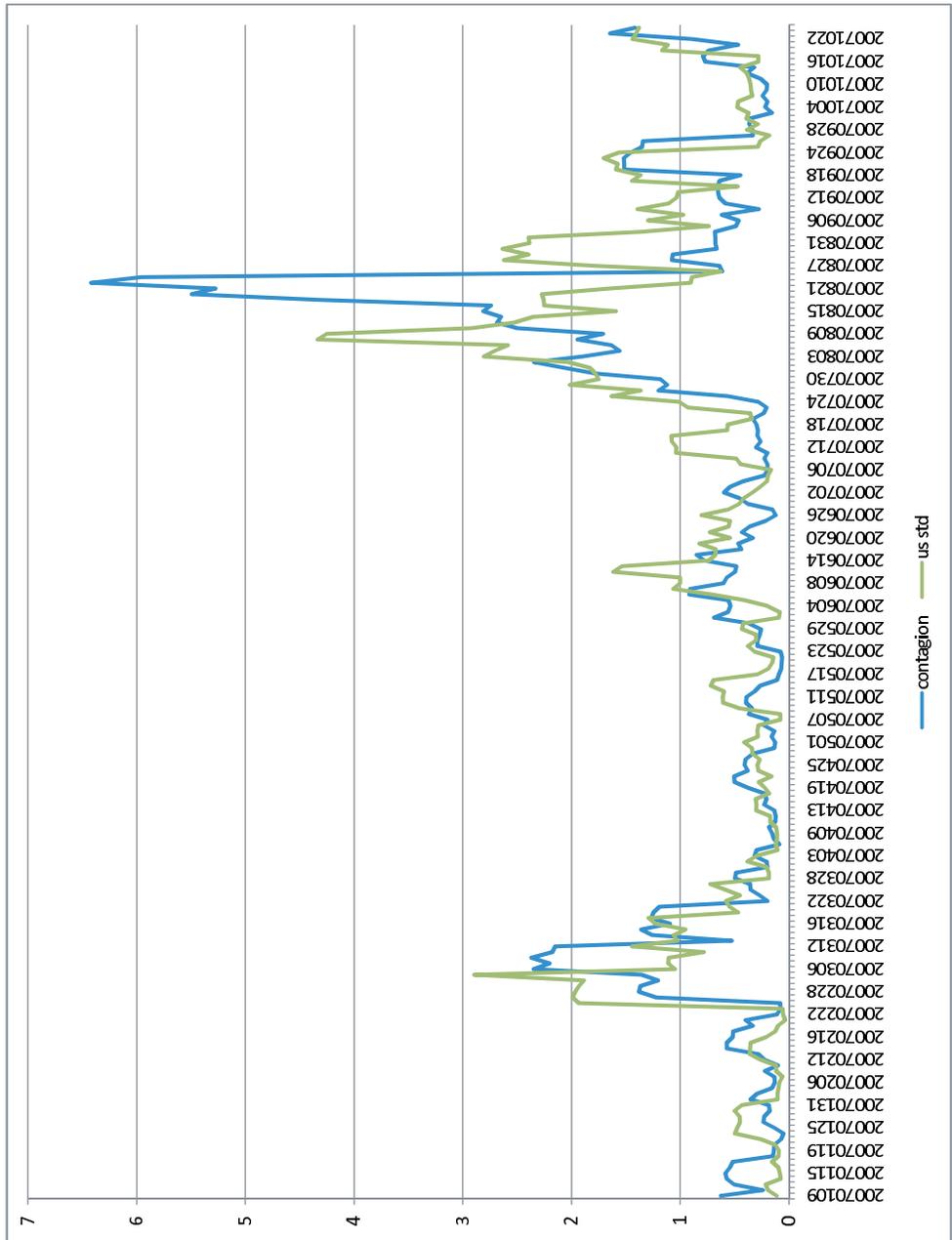
Figure 2: Contagion Measure and Rolling Standard Deviation of US Stock Market.

very different aspects of the US sub-prime crisis.

In Figure 2 we can clearly see two major instances in which the contagion measure increased significantly. One event starts in February 26th, 2007, and the other starts after July 19th, 2007. In Figure 3 we present all 49 stock market prices measured in dollars and normalized to have a price of one on February 26th of 2007; the scale for all stock market prices is displayed on the left vertical axis of the graph. The corresponding scale for the contagion measure, represented by the separate thick line in the chart, is displayed on the right vertical axis. The idea here is to observe the co-movement of stock prices globally, and how it increases, when our contagion estimator increases. We notice that except for a few countries, all stock markets in the world collapse together immediately following the February 26th event, and our contagion measure increases sharply at the same time. The opposite trend can be observed around the beginning of the second week of March, when stock markets start to pick up again and diverge from each other, and our contagion measure drops at the same time. Therefore, there appears to be a strong positive correlation here between changes in the contagion estimator and co-movement of stock prices across the globe, following the outbreak of the sub-prime crisis.

Figure 4 presents the same exercise for the July 19th event. In the first case in February, our measure of contagion increases from an average of 0.29 to a maximum of 2.37. During the July 19th event, the average is 0.39 and increases to 6.42. Because the same methodology is used throughout, we can therefore say that the second event is more contagious, in the sense that the sub-prime shock should increase the degree of co-movement across countries much more. Indeed, not surprisingly, asset prices do appear to co-move with higher intensity after the shock in July 19th, than after the corresponding shock in February. Furthermore, we observe that the same positive relationship between changes in the contagion estimator and co-movement of stock prices holds for the July contagious event as well, now magnified to an even greater extent.
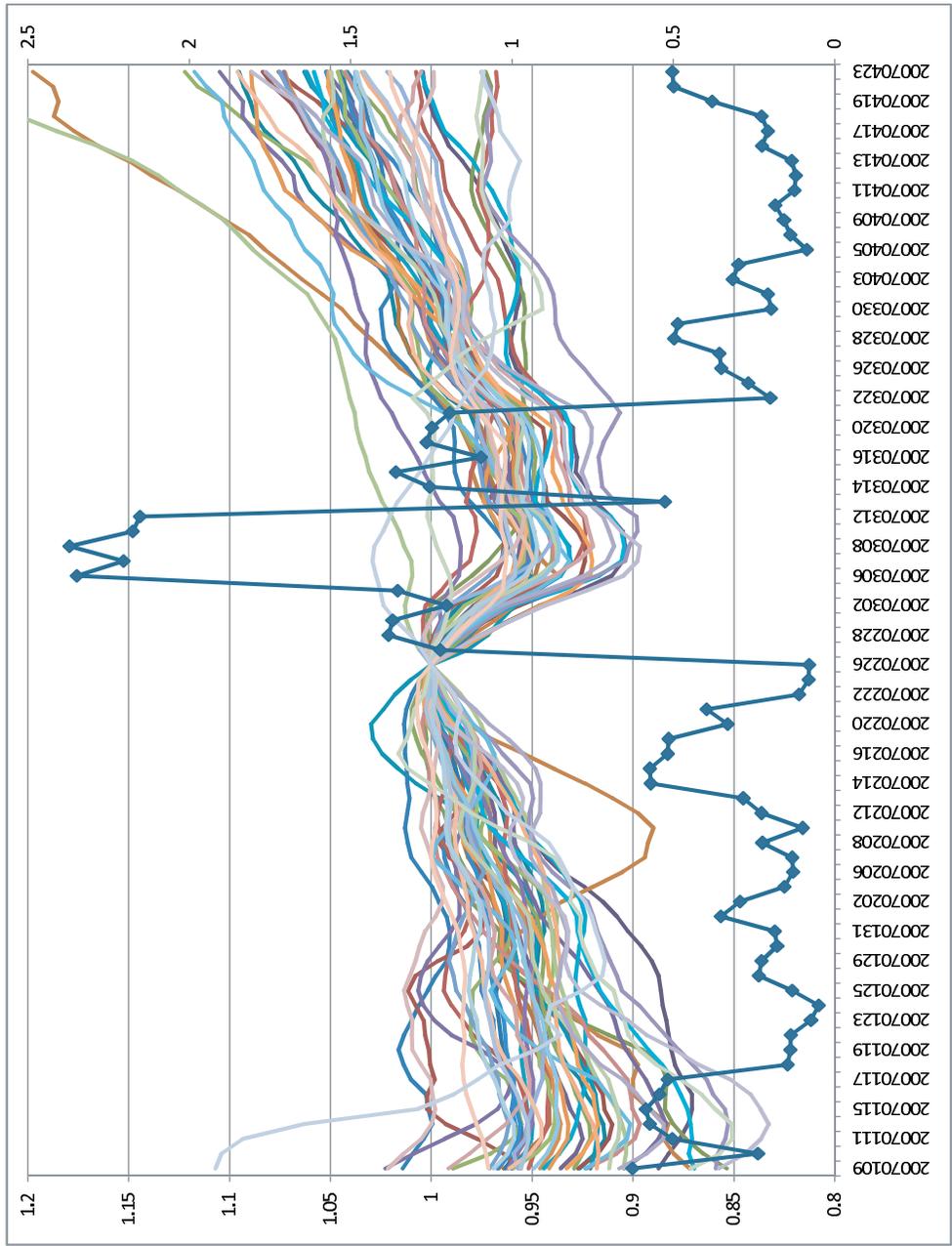
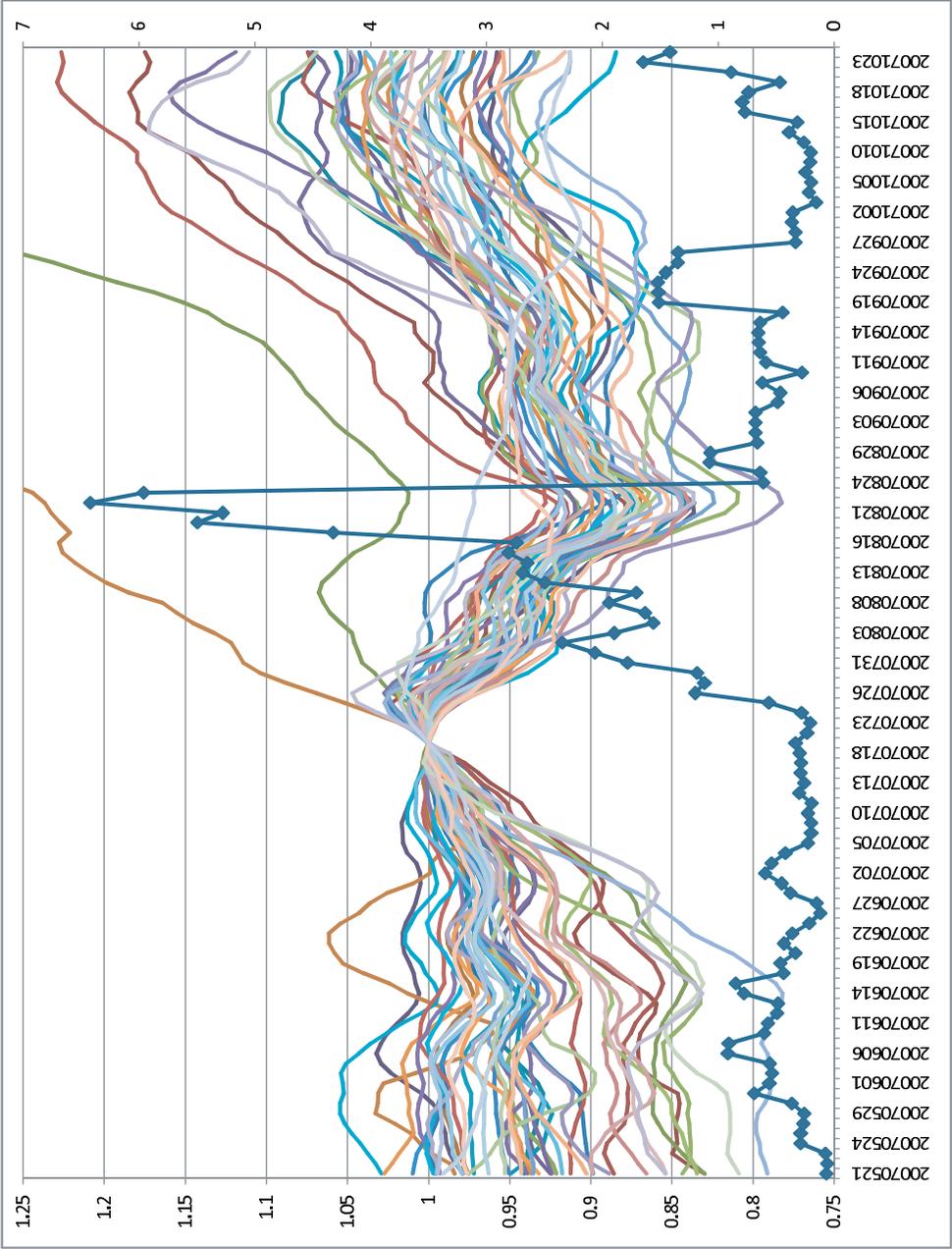Figure 3: Sub-prime Crisis: February Contagious Event.

Figure 4: Sub-prime Crisis: July Contagious Event.

4.4. Structural approach

The results from the previous subsection highlight the significant synchronization that appeared during the sub-prime crisis in all countries – and more interestingly, the fact that this co-movement is detected across any pair of countries even when the US is not one of them. Even though this simple estimation proves to be quite robust and informative, in this subsection we take the further steps and estimate using the full model.

The procedure of estimation is performed using simulated GMM. The idea is to simulate our model given some parameters: variances, drifts, demand shocks, etc. and compute moments from the simulation and compare them to the moments estimated in the data. Because of the complexity of the model we have decided to estimate using only three countries or regions, and only concentrating on the variance-covariance matrix.

[to be completed: we are in the process of programming the estimation]

5. Conclusion

In conclusion, our paper develops a measure of international financial contagion which is derived from the cross-equation restrictions of a model with portfolio constraints. This semi-structural estimation procedure generates a contagion measure which, under certain assumptions about the first and second moments of output shocks, reflects the average change in the joint covariance of all countries in the world with each other. Therefore, our estimator of contagion here differs from standard contagion measures which just focus on calculating the covariance between a 'constrained' country, or the country which suffers an initial shock, (in the case of the sub-prime crisis, this would be the US), and other countries in the world.

In our empirical tests, we demonstrate that changes in our new global contagion estimator are positively correlated with changes in the number of daily news articles documenting the sub-prime crisis. This correlation is far higher than the corresponding correlation between changes in our contagion measure and changes in the standard deviation of the US stock market, and also between changes in the number of news articles and changes in the standard

deviation of the US stock market. These results indicate that the US stock market variance does not measure the degree of the sub-prime crisis to the same extent as either our contagion measure or the volume of news. Furthermore, there is evidence here that our estimator of contagion is itself a strong measure of the sub-prime crisis. These findings are further reinforced by the fact that there appears to be a positive relationship between changes in our contagion measure and co-movement of stock prices across the globe following the sub-prime collapse. Finally, we obtain the result that our estimator of contagion has a highly significant total cumulative effect on changes in the number of news articles reporting the crisis. For example, when we include up to ten leads and lags of the contagion measure, in a regression of changes in news articles on changes in the contagion measure, the null hypothesis that all coefficients are equal to zero is rejected very strongly at the 5 percent level. Moreover, the contemporaneous coefficient on contagion is significantly positive at the 5 percent level.

In sum, these findings reveal that our estimator of contagion is indeed a robust measure, and even predictor, of the sub-prime crisis. The implication from our empirical analysis is then that we can explain the spread of international financial contagion due to the sub-prime collapse, at least to a certain extent, via the tightening of financial constraints facing banks or institutions in the US. This result, as well as the new estimation procedure for contagion introduced in the paper, represents our most important contribution to the literature.

Appendix: Proofs

Proof of Propositions 1 and 2   See Pavlova and Rigobon (2007)

Proof of Proposition 3   Proposition 3 can be proved as follows. Let us consider the partial optimization problem of Country 0 , where the representative agent is maximizing his expected utility over trading strategies $x_0(t)$, subject to the constraint in (8). In particular, from Pavlova and Rigobon (2007), we can represent the objective function of country 0 in the form

$$E \int_0^T [\log W_0(t) - K(t)] \, dt.$$

where $K(t) = \alpha_0 \log(p^0(t)(T - t)) + \frac{1-\alpha_0}{2} \log(p^1(t)(T - t)) + \frac{1-\alpha_0}{2} \log(p^2(t)(T - t))$. The investor of country 0 takes prices in the good markets p^j, $j = 0$, 1, 2 as given, and hence from his viewpoint $K(t)$ is exogenous at any time t. Therefore, the constrained maximization problem for the Center Country reduces to the following:

$$\max_{W_0(t)} E \left[\int_0^T \log W_0(t) dt \right] \tag{A.1}$$

$$\text{subject to} \quad x_0(t)^\top \sigma(t) \sigma(t)^\top x_0(t) \le \phi(t). \tag{A.2}$$

Since we know that, for Country 0:

$$dW_0(t) = W_0(t)[(r(t)1 + x_0(t)^\top (\mu(t) - r(t)1)dt + (x_0(t)^\top \sigma(t))dw(t)], \tag{A.3}$$

(where $\mu(t)$ and $\sigma(t)$ are, respectively, the vector of expected stock returns and the volatility matrix of the investment opportunity set), then it follows from (A.3) that:

$$d \log W_0(t) = [(r(t)1 + x_0(t)^\top (\mu(t) - r(t)1) - \frac{1}{2}|x_0(t)^\top \sigma(t)|^2)dt + x_0(t)^\top \sigma(t)dw(t)] \tag{A.4}$$

$$W_0(t) = W_0(0)e^{\int_0^t (r(t)1 + x_0(t)^\top (\mu(t) - r(t)1) - \frac{1}{2}|x_0(t)^\top \sigma(t)|^2)dt + \int_0^t x_0(t)^\top \sigma(t)dw(t)} \tag{A.5}$$

Therefore, if we assume in addition that $x_0(t)^\top \sigma(t) \in (H^2)^3$,(the set of square-integrable processes),so that $E \left[\int_0^T x_0(t)^\top \sigma(t)dw(t) \right] = 0$, then substituting (A.4) into (A.1) implies that (A.1) reduces to the following problem:

$$\max_{W_0(t)} \left[\int_0^T x_0(t)^\top (\mu(t) - r(t)1) - \frac{1}{2}|x_0(t)^\top \sigma(t)|^2 dt \right] \tag{A.6}$$

$$\text{subject to} \quad x_0(t)^\top \sigma(t) \sigma(t)^\top x_0(t) \le \phi(t) \tag{A.7}$$

where ψ is the multiplier on the volatility constraint above (so that $\psi = 0$ when the constraint is not binding).

The first order conditions from (A.6) at time t imply that:

$$x_0(t) = \frac{\sigma^{-1}(t)^\top m(t)}{1 + 2\psi} \tag{A.8}$$

where $m(t) = \sigma(t)^{-1}(\mu(t) - r(t))$.

Now using the fact, from Pavlova and Rigobon (2007), that the countries' optimal portfolios of risky assets are given by:

$$x_0(t) = (\sigma(t)^\top)^{-1} m_0(t), \quad x_i(t) = (\sigma(t)^\top)^{-1} m(t), \quad i \in \{1, 2\}.$$

we can solve for $m_0(t)$ by reexpressing (A.8) as:

$$m_0(t) = \sigma(t)^\top x_0(t) = \frac{m(t)}{1 + 2\psi} \tag{A.9}$$

Substituting (A.8) into (8),(with the inequality constraint binding now as an equality), we can solve for $\psi(t)$:

$$(1 + 2\psi(t))^2 = \frac{m(t)^\top m(t)}{\phi(t)} \qquad \text{(Constraint binding)} \tag{A.10}$$

$$\psi(t) = 0 \qquad \text{(Constraint not binding)} \tag{A.11}$$

Therefore, to summarize, in the case where the constraint is not binding , $\psi(t) = 0$, and therefore $m_0(t) = m(t)$. This is reported in (10). In the case where the constraint is binding,(A.9) and (A.10) give us the complementary slackness conditions presented in (11). Equation (12) follows from market clearing, coupled with the investors' first-order conditions.

Q.E.D.

Proof of Corollary 4. We first present the proof of Corollary 4, and show that the analogous result in the unconstrained environment, i.e. result (v) of Proposition 1, is just a special case of this. Consider Country 0's portfolio. From (iii) of Proposition 1, and expressions for optimal consumption allocations and sharing rules for aggregate endowment from Pavlova and Rigobon (2007), we have:

$$S^0(t) = p^0(t)Y^0(t)(T - t)$$

$$S^1(t) = p^1(t)Y^1(t)(T - t)$$

$$C_0^0(t) = \frac{1}{p^0(t)(T - t)}\alpha_0 W_0(t) = \frac{Y^0(t)}{\alpha_0 + \lambda_1 \frac{1 - \alpha_1(t)}{2} + \lambda_2 \frac{1 - \alpha_2(t)}{2}}\alpha_0$$

$$C_0^1(t) = \frac{1}{p^1(t)(T - t)}\frac{1 - \alpha_0}{2}W_0(t) = \frac{Y^1(t)}{\frac{1 - \alpha_0}{2} + \lambda_1\alpha_1(t) + \lambda_2 \frac{1 - \alpha_2(t)}{2}}\frac{1 - \alpha_0}{2}$$

Therefore, after some manipulation:

$$\frac{W_0(t)}{S^0(t)} = \frac{1}{\alpha_0 + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \frac{1-\alpha_2(t)}{2}}$$

$$\frac{W_0(t)}{S^1(t)} = \frac{1}{\frac{1-\alpha_0}{2} + \lambda_1 \alpha_1(t) + \lambda_2 \frac{1-\alpha_2(t)}{2}}$$

Following the exact same procedure for the portfolios of Countries 1 and 2, we can report the following results for the Countries' wealth as a proportion of stock prices. (Here, we just present results for S^0 and S^1 for simplicity - the expressions for the Countries' wealth as a proportion of the stock price S^2 are derived analogously):

$$\frac{W_1(t)}{S^0(t)} = \frac{\lambda_1}{\alpha_0 + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \frac{1-\alpha_2(t)}{2}}$$

$$\frac{W_1(t)}{S^1(t)} = \frac{\lambda_1}{\frac{1-\alpha_0}{2} + \lambda_1 \alpha_1(t) + \lambda_2 \frac{1-\alpha_2(t)}{2}}$$

$$\frac{W_2(t)}{S^0(t)} = \frac{\lambda_2}{\frac{1-\alpha_0}{2} + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \alpha_2(t)}$$

$$\frac{W_2(t)}{S^1(t)} = \frac{\lambda_2}{\frac{1-\alpha_0}{2} + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \alpha_2(t)}$$

Therefore, letting $N_j^i(t)$ be the number of shares Country i holds in stock j, then we have:

$$N_j^i(t) = \frac{x_i(t)^\top i_j W_i(t)}{S^j(t)} \qquad i = 0,\ 1,\ 2, \qquad j = 0,\ 1,\ 2, \qquad \text{(A.12)}$$

Using the securities market clearing equation for all stocks j, we also have:

$$N_j^0(t) + N_j^1(t) + N_j^2(t) = 1 \qquad j = 0,\ 1,\ 2 \qquad \text{(A.13)}$$

Therefore, applying (A.12) and (A.13) for stocks 0 and 1,(i.e.for $j = 0,1$), we have:

$$1 = \frac{x_0(t)^\top i_0 + x_1(t)^\top i_0 \lambda_1 + x_2(t)^\top i_0 \lambda_2}{\alpha_0 + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \frac{1-\alpha_2(t)}{2}} \qquad \text{(A.14)}$$

$$1 = \frac{x_0(t)^\top i_1 + x_1(t)^\top i_1 \lambda_1 + x_2(t)^\top i_1 \lambda_2}{\frac{1-\alpha_0}{2} + \lambda_1 \alpha_1(t) + \lambda_2 \frac{1-\alpha_2(t)}{2}} \qquad \text{(A.15)}$$

Now, from the expression above, (in the proof of Proposition 3), for countries' optimal portfolios of risky assets, and equation (11), we can easily derive the following expression for Country 0's portfolio in terms of the portfolio of the periphery countries,(letting $x(t) = x_1(t) = x_2(t)$ denote the portfolio held by each of the Periphery countries, given that these will always be identical in both the constrained and unconstrained cases):

$$x_0(t) = \frac{x(t)}{1 + 2\psi} \tag{A.16}$$

Therefore, using (A.16), (A.14) and (A.15) can be reexpressed as:

$$1 = \frac{x_0(t)^\top i_0(1 + (\lambda_1 + \lambda_2)(1 + 2\psi))}{\alpha_0 + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \frac{1-\alpha_2(t)}{2}} \tag{A.17}$$

$$1 = \frac{x_0(t)^\top i_1(1 + (\lambda_1 + \lambda_2)(1 + 2\psi))}{\frac{1-\alpha_0}{2} + \lambda_1 \alpha_1(t) + \lambda_2 \frac{1-\alpha_2(t)}{2}} \tag{A.18}$$

Using the above two equations, it follows directly that:

$$\frac{x_0(t)^\top i_0}{\alpha_0 + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \frac{1-\alpha_2(t)}{2}} = \frac{x_0(t)^\top i_1}{\frac{1-\alpha_0}{2} + \lambda_1 \alpha_1(t) + \lambda_2 \frac{1-\alpha_2(t)}{2}} \tag{A.19}$$

The left hand side of (A.19), is equal to $N_0^0(t)$, and the right hand side is equal to $N_1^0(t)$, i.e. we have shown that Country 0 holds the same number of shares in stocks 0 and 1. This argument can be easily extended to show that Country 0 holds the same number of shares in all stocks, (using the securities market clearing equation for S^2 and manipulating in the same way). The same logic applies to Countries 1 and 2. The proof of the analogous result for the unconstrained setting is then just a special case of this argument with $\psi = 0$. Therefore, in both the unconstrained and constrained settings, all countries hold an equal number of shares in stocks 0, 1 and 2.

Furthermore, in the unconstrained case, since $\psi = 0$, it can also be shown very easily from manipulating equations (A.12) and (A.13), (for all i and j), that for all Countries i and Stocks j:

$$N_j^i(t) = \frac{\lambda_i}{\lambda_0 + \lambda_1 + \lambda_2}, \qquad \lambda_0 = 1 \tag{A.20}$$

In other words, when there is no constraint, the number of shares of each stock purchased by any given Country is equal to its relative wealth share in the world economy. It follows that in the benchmark equilibrium, agents in each Country invest all their wealth in their respective stock portfolios. Therefore, no shares of the bond are traded in equilibrium. We do not arrive at the same result in the constrained case, however, since we have the extra term ψ appearing in equation (A.13) for all i and j, (implying that equation (A.20) no longer holds). In this scenario, therefore, agents have different holdings of the bond in equilibrium.

Q.E.D.

Derivation of r_0 (for Country 0) Equations (10) and (11) are derived at a partial equilibrium level. The partial-equilibrium constrained optimization problem of country 0 is

an example of the class of problems considered by Cvitanić and Karatzas, (Cvitanić and Karatzas (1992)). In particular, as a result of the volatility constraint (8), portfolio values x_0 are restricted to lie in a closed, convex, non-empty subset $K \in R^3$. We can define the constraint set K as follows:

$$K = \{x_0(t); \ x_0(t)^\top \sigma(t)\sigma(t)^\top x_0(t) \leq \phi(t)\}.$$

As previously mentioned, the problem of Country 0 facing a portfolio constraint is equivalent to a fictitious problem, based on a different financial market with no constraints. In particular, this fictitious market is tilted, or rather, the (constrained) Center country's effective interest rate and the market price of risk, r_0 and m_0, are tilted to reflect the extent to which the country's investments are constrained. Let this tilt be captured by a three-dimensional vector of multipliers, $\nu(t)$, so that for each process $\nu(t)$, (in the set of processes adapted to $\{\mathcal{F}_t; t \in [0, T]\}$), we define a fictitious market M^ν, in which the three stocks and the global bond are traded. Define the support function, $\delta(\nu(t))$ as follows:

$$\delta(\nu(t)) = \sup_{x_0(t) \in K} (-x_0(t)^\top \nu(t))$$

We can also define the effective domain of the support function, $\widetilde{K}$, as :

$$\widetilde{K} = \{\nu(t); \ \delta(\nu(t)) \leq infinity\}.$$

Now we can define the following tilted processes in the fictitious, unconstrained market :

$$r_0(t) \ = \ r(t) + \delta(\nu(t)) \tag{A.21}$$

$$m_0(t) \ = \ m(t) + \sigma^{-1}(t)\nu(t) \tag{A.22}$$

$$d\xi^\nu(t) \ = \ -\xi^\nu(t)[(r(t) + \delta(\nu(t)))dt + (\sigma^{-1}(t)\nu(t) + m(t)^\top)dw(t)] \tag{A.23}$$

Given the quadratic form of our constraint, and the definition of $\delta(\nu(t))$ and K above, we can easily solve for $\delta(\nu(t))$ from a simple constrained minimization problem. We obtain:

$$\delta(\nu(t)) = (\phi(t)\nu(t)^\top(\sigma(t)\sigma(t)^\top)^{-1}\nu(t))^{1/2} \qquad \nu(t) \in \widetilde{K}. \tag{A.24}$$

In the fictitious market characterized by $\nu(t)$, Country 0 will maximize his expected utility of consumption subject to his budget constraint,(with state price density process $\xi^\nu(t)$ replacing $\xi_0(t)$). The dynamic portfolio choice problem in the fictitious market without constraints, M^ν, can be equivalently expressed in a static form:

$$Q^\nu = \sup_{C_0^0, C_0^1, C_0^2} E\left[\int_0^T u_0(C_0^0(t), C_0^1(t), C_0^2(t)) \, dt\right] \tag{A.25}$$

subject to $\qquad E[\xi^\nu(T)W_0(T)] \leq W_0(0).$ \tag{A.26}

where Q^ν is the value function defined in market M^ν, (and we define $\widetilde{\psi}$ to be the multiplier on the wealth constraint above). In addition, from Cvitanić and Karatzas, we also know that for any admissible choice of ν, the value function Q^ν gives an upper bound for the optimal value function of the original, constrained problem. Therefore, if we minimize the value function of the fictitious problem, Q^ν, with respect to ν, (and the minimum corresponds to a feasible trading policy), then we have found an optimal portfolio policy under the constraint on Country 0. (Cvitanić and Karatzas outline sufficient conditions under which the solution of the minimization problem exists and leads to the optimal portfolio policy, i.e. conditions under which there is no duality gap).

Referring to the proof of Proposition 1, we can substitute Country 0's log utility function into (A.25) to obtain the following equivalence relation between the minimization problems:

$$\min_{\nu \in \widetilde{K}} Q^\nu = \min_{\nu \in \widetilde{K}} E\left[\sup_{W_0(t)} \int_0^T \log W_0(t)dt - W_0(T)\widetilde{\psi}\xi^\nu(T)\right] = \min_{\nu \in \widetilde{K}} \widetilde{U}(\widetilde{\psi}\xi^\nu(T))$$

We can now form the Bellman equation for this problem, where the value function, $Q(t,y)$ is a function of t and state variable y, $Q(T,y) = \widetilde{U}(y)$, and $y = \widetilde{\psi}\xi^\nu(t)$. (Here, we have suppressed the dependence of y on t). Therefore, using (A.23) to obtain the drift of $\xi^\nu(t)$, (which in turn determines the drift of y over time), the standard Bellman equation for this problem is as follows:

$$\min_{\nu \in \widetilde{K}}(Q_t - yQ_y(\delta(\nu(t)) + r(t)) + \frac{1}{2}Q_{yy}y^2(m(t) + \sigma^{-1}(t)\nu(t))^2) = 0 \tag{A.27}$$

Since we have a log utility function for Country 0, (which is a special case of a power function with curvature/elasticity of intertemporal substitution set equal to 1), the Value function $Q(t,y)$ solving (A.27) will take the following standard form:

$$Q(t,y) = \log y f(t)$$

for some function of time, $f(t)$.

In particular, for all power functions, yQ_y and $y^2 Q_{yy}$ will be constant, so the optimal $\nu(t)$ from (A.27) will not depend on the state variable y. Therefore, equation (A.27) reduces to the minimization problem below:

$$\min_{\nu \in \widetilde{K}}((m(t) + \sigma^{-1}(t)\nu(t))^2 + \delta(\nu(t)) = 0 \tag{A.28}$$

Therefore, let $\nu^*(t)$ be the solution to (A.28) so that:

$$\nu^*(t) = argmin((m(t) + \sigma^{-1}(t)\nu(t))^2 + \delta(\nu(t))$$

Substituting (A.24) into the above, we have, equivalently:

$$\nu^*(t) = argmin((m(t) + \sigma^{-1}(t)\nu(t))^2 + (\phi(t)\nu(t)^\top(\sigma(t)\sigma(t)^\top)^{-1}\nu(t))^{1/2})$$

A nonzero solution to this minimization problem will exist if and only if the original volatility constraint,(8), is binding, (so that, obviously, $\nu^* = 0$ when Country 0 is unconstrained and does not face a fictitious set of investment opportunities).

Let us consider the case where (8) is binding. From (A.22), we have :

$$\nu^*(t) = \sigma(t)(m_0(t) - m(t)) \tag{A.29}$$

Substituting (A.9) into (A.29), we obtain the following expression for ν^* in terms of the multiplier $\psi(t)$:

$$\frac{-2\psi(t)}{1 + 2\psi(t)} \sigma(t)\, m(t) = \nu^*(t) \tag{A.30}$$

From (A.22) and (A.24), we then have:

$$r_0(t) = r(t) + (\phi(t)\nu^*(t)^\top(\sigma(t)\sigma(t)^\top)^{-1}\nu^*(t))^{1/2} \tag{A.31}$$

When (8) is not binding:

$$\nu^*(t) = 0 \tag{A.32}$$
$$m_0(t) = m(t) \tag{A.33}$$
$$r_0(t) = r(t) \tag{A.34}$$

Q.E.D.

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