Offsetting the Implicit Incentives:
Benefits of Benchmarking in Money Management[*]

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This draft: November 2007

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Abstract

Money managers are rewarded for increasing the value of assets under management. This gives a manager an implicit incentive to exploit the well-documented positive fund-flows to relative-performance relationship by manipulating her risk exposure. The misaligned incentives create potentially significant deviations of the manager's policy from that desired by fund investors. In the context of a familiar continuous-time portfolio choice model, we demonstrate how a simple risk management practice that accounts for benchmarking can ameliorate the adverse effects of managerial incentives. Our results contrast with the conventional view that benchmarking a fund manager is not in the best interest of investors.

1. Introduction

The money management industry has been growing at a mind-boggling pace. More and more individual investors' money is put into the care of professional managers. The question of these managers' incentives is thus of utmost importance to households and policymakers. Incentives of individual investors and professional managers are not always perfectly aligned. In particular, contrary to investors' objectives, managers take into account of their performance relative to some index in their portfolio choice. This is because capital inflows into funds follow good relative performance. Therefore, a significant factor in a rational fund manager's decision problem is an *implicit incentive* that arises from the relationship between fund flows and her performance relative to an index. Previous research has shown that if the manager is unrestricted in her portfolio choice, she has an incentive to boost the riskiness of her portfolio when underperforming her index and lock in her gains upon catching up.[1] In this paper, we consider mechanisms aimed at limiting such behavior.

Towards this, we consider risk management practices that account for benchmarking. Establishing an economic role for such widely observed practices is also of independent theoretical interest given the arguments made against them in the academic literature (Roll, 1992; Admati and Pfleiderer, 1997). We focus on a simple constraint, referred to as a "minimum performance constraint" or a "benchmarking restriction," which prohibits a shortfall in the manager's return relative to a reference portfolio to exceed a pre-specified level over a certain horizon. If a manager violates the constraint, she incurs a large penalty; in our model, we assume that she simply loses her job.[2] This simple, yet versatile, constraint is also closely related to some popular risk management practices such as stop-loss limits, portfolio insurance, value-at-risk (VaR) and tracking error limits. Such a constraint can be either explicitly or implicitly imposed on the manager by her superiors. The parameter governing the stringency of the benchmarking restriction in our model is the manager's allowed shortfall relative to the benchmark. We demonstrate that as the (appropriate) benchmarking restriction becomes more stringent, the impact of the fund-flows induced incentives on the manager's policy weakens, and beyond a certain allowed shortfall the convexity in the flow-performance relationship ceases to have an effect on the manager's optimal policy.

Absent the benchmarking restriction, the asset allocation choice of the manager is not necessarily in the best interest of fund investors, who care about the risk and return of their investment and not about attracting capital into the fund. Moreover, the risk tolerance of fund investors need not coincide with that of the fund manager. We compare the manager's policy when acting in the best interest of fund investors with when following an asset allocation policy optimal from

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[1]See Chevalier and Ellison (1997) and Basak, Pavlova, and Shapiro (2007) who argue that convexities in the fund flow-performance relationship give rise to such behavior.

[2]See, for example, Del Guercio and Tkac (2002) and their references to surveys by Greenwich Associates for evidence on underperformance-related manager termination decisions.

her viewpoint. A simple calibration reveals that the costs of misaligned incentives could be quite significant. Most of our cost estimates, measured in units of an investor's initial wealth, are within the 1.4–8% range. We show that a benchmark that is less risky than the index can temper deviations from the investors' desired risk exposure in states where the manager is tempted to deviate the most, and hence is beneficial. Across most of our calibrations, the percentage loss recouped (in terms of the investor's initial wealth) by benchmarking the manager ranges from 60% to 80%. Our results thus provide a rationale for benchmarking-type restrictions observed in this industry. Our analysis, however, also demonstrates how complicated fine-tuning the manager's behavior may be in the presence of implicit flow-driven incentives. Indeed, as we illustrate, when the benchmark is (sufficiently) riskier than the index, benchmarking the manager may actually exacerbate her risk exposure.

Our work is related to the literature on (adverse) consequences of benchmarking. In a mean-variance setting, Roll (1992) argues that benchmarking a money manager to an index results in her choosing a portfolio that is not mean-variance efficient. Admati and Pfleiderer (1997), in a similar context but with an asymmetrically informed investor and portfolio manager, also advocate against benchmarking the manager, and particularly linking compensation to the types of benchmarks observed in practice. The spirit of these results is that, in an economy without fund-flows induced considerations, benchmarking induces a manager to deviate from choosing a mean-variance efficient portfolio that is desired by investors (with mean-variance preferences). Our viewpoint is that money managers are concerned with attracting fund inflows, which we accept as a fact of life. The role of our benchmarking restriction is to (partially) alleviate the adverse effects of the ensuing managerial incentives, thus benefitting investors.

There is a strand of literature, growing out of Bhattacharya and Pfleiderer (1985), investigating optimal contracting in the context of delegated portfolio management, where the manager typically has superior information or ability, or expends costly effort. In this vein is also Starks (1987). Dybvig, Farnsworth, and Carpenter (2001) consider restrictions on the investment opportunity set (trading strategies) as part of an optimal contract, while Gómez and Sharma (2006) analyze the effect of short-selling constraints on a manager's optimal contract. Also within a static principal-agent framework, Agarwal, Gómez, and Priestley (2007) study the incentive effects of relative (to a benchmark) performance evaluation in the presence of portfolio constraints. In a dynamic portfolio choice model, Cadenillas, Cvitanić, and Zapatero (2004) consider a principal-agent problem in which a risk-averse manager compensated with options chooses the riskiness of the projects she invests in. Our focus in this paper is different. Instead of solving for an optimal contract, we look for alternative mechanisms aimed at counteracting the manager's adverse incentives.[3] In that spirit, Jorion (2003) further analyzes Roll's static setup and considers how imposing additional

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[3]Effectively, instead of altering the manager's compensation structure, we alter her investment opportunity set so as to temper undesirable swings in her risk exposure in the targeted states of the world.

constraints can move optimal portfolios closer to mean-variance efficiency, while Alexander and Baptista (2007) extends Jorion's work by incorporating a VaR constraint. Brennan (1993) and Gómez and Zapatero (2003) study the equilibrium implications of Roll-type setting and derive a two-beta CAPM.

Closer to our message is van Binsbergen, Brandt, and Koijen (2007), who also advocate the use of benchmarking in money management. Their way of addressing benchmarking, however, differs from ours in that they model managers as deriving utility from the ratio of their terminal portfolio value over a benchmark. Hence, unlike in our analysis, the managers care only about relative performance, and not absolute. The ensuing effects of the benchmark on the managers' behavior are also different. Since managers are risk averse in van Binsbergen et. al, they try to reduce the variability of the portfolio-benchmark ratio even when outperforming the benchmark. In our setting, the manager is affected disproportionately more when her performance relative to the benchmark is poor. Basak, Pavlova, and Shapiro (2007) examine fund-flows induced incentives of fund managers. The special case of our manager's choice being unrestricted coincides with theirs. However, they do not examine the effects of restricting the manager's investment policy, which is the goal of this paper. There is also a recent literature examining benchmarking absent adverse incentives and fund-flows considerations. In a dynamic setting like ours, Teplá (2001), and Basak, Shapiro, and Teplá (2006) study the optimal policies of an agent subject to a benchmarking restriction. Our formulation that incorporates simultaneously both the fund-flows and benchmarking considerations permits a study of the economic role of investment restrictions, the main focus of this paper. In particular, the insights of our analysis here cannot be gained from examining the benchmarking restriction or the fund-flows induced incentives alone.

The rest of the paper is organized as follows. Section 2 describes the model primitives and discusses the fund-flows induced implicit incentives and the benchmarking restriction. Section 3 derives the manager's optimal policy under benchmarking, and Section 4 evaluates cost/benefits of benchmarking to investors. Section 5 concludes, and the Appendix provides the proofs and other material omitted in the body of the paper.

2. The Economic Setting

2.1 Economic Primitives

We adopt the familiar Black and Scholes (1973) economy for the financial investment opportunities. We consider a continuous-time, finite horizon, $[0, T]$ economy, in which uncertainty is driven by a Brownian motion w. Available for investing are a riskless money market account and a risky stock.[4] The money market provides a constant interest rate r. The stock price, S, follows a

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[4]The extension to multiple stocks and multiple sources of uncertainty is possible and does not change the main message delivered by our basic economic setting.

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geometric Brownian motion

$$dS_t = \mu S_t dt + \sigma S_t dw_t,$$

where the stock mean return, μ, and volatility, σ, are constant.

We consider a fund manager who dynamically allocates the fund's assets, initially valued at W_0, between the risky stock and the money market. The manager's compensation, due at the horizon T, is proportional to the terminal value of assets under management. The manager is guided by constant relative risk aversion (CRRA) preferences, defined over the value of assets under management at time T:

$$u(A_T) = \frac{A_T^{1-\gamma}}{1-\gamma}, \ \gamma > 0, \tag{1}$$

where the risk aversion parameter γ is assumed to be observable. This formulation of the manager's objective is consistent with the linear fee structure prevalent in the mutual fund industry.[5] Her portfolio value process, W, follows

$$dW_t = [(1 - \theta_t)r + \theta_t\mu] W_t dt + \theta_t \sigma W_t dw_t, \tag{2}$$

where θ denotes the fraction of the portfolio invested in the risky stock, or the *risk exposure*. Absent flows into the fund, the time-T value of assets under management A_T coincides with the market value of the manager's portfolio W_T. In this case, maximizing the expected objective function (1) subject to (2) yields the manager's optimal risk exposure, θ^N, henceforth the *normal risk exposure*, given by (Merton (1971)):

$$\theta_t^N = \frac{1}{\gamma}\frac{\mu - r}{\sigma^2}.$$

The manager's performance triggering fund flows is evaluated relative to an *index*, Y, a value-weighted portfolio with a fraction β invested in the stock market and $(1 - \beta)$ in the money market, following

$$dY_t = (1 - \beta)rY_t dt + \beta(Y_t/S_t)dS_t = [(1 - \beta)r + \beta\mu]Y_t dt + \beta\sigma Y_t dw_t.$$

We define the risk exposure of the index portfolio, θ^Y, as the fraction of the index invested in the risky asset:

$$\theta_t^Y = \beta.$$

We denote the (continuously compounded) returns on the manager's portfolio and on the index over the period $[0, t]$ by $R_t^W = \ln\frac{W_t}{W_0}$ and $R_t^Y = \ln\frac{Y_t}{Y_0}$, respectively, where we normalize $Y_0 = W_0$, without loss of generality. It turns out that in our analysis we need to distinguish between two (mirror) subcases depending on whether the manager normally desires higher risk exposure than that of her index ($\theta^N > \theta^Y$) or not ($\theta^N < \theta^Y$). We refer to the former as economies (a) and the

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[5]In particular, the fund may have a linear fee structure, αA_T, $\alpha > 0$. Such a linear structure would be optimal in our model absent any incentive problems, that is, if there were no flows-induced incentives ($f_T = 1$ in Section 2.2) and a hypothetical fund investor had CRRA preferences with the relative risk aversion γ.

latter as economies (b). For expositional simplicity, in the body of the paper we focus mainly on economies (a), maintaining the assumption that $\theta^N > \theta^Y$. Note that this condition is imposed on exogenous quantities, and is likely to be satisfied when the market risk premium is high, volatility is low, or the risk aversion of the manager is low. In economies (b), the solution to the manager's problem has a similar structure, and the main message of the paper on the benefits of benchmarking remains valid (Table 2b). For brevity, we relegate its analysis to the Appendix.

2.2 Fund Flows and Implicit Incentives

Tying the compensation to performance provides the manager with an incentive to increase the final value of the portfolio W_T. Perhaps just as significant to the manager's choices are *implicit incentives* due to inflows or outflows of funds in response to her performance relative to the index. Well-performing managers are rewarded further by attracting new capital into a fund and thereby increasing their fees, while poor-performing managers are penalized. We model this by augmenting the fund time-T asset value by the quantity f_T, representing fund flows, so that

$$A_T = W_T f_T.$$

The flow rate f_T is understood in the proportion-of-portfolio terms; for example if $f_T > 1$, the manager gets an inflow, otherwise if $f_T < 1$, gets an outflow.

The empirical literature has suggested several specifications of the functional form for the flow-performance relationship f_T. Chevalier and Ellison (1997) argue that for mutual funds it resembles a bull spread or a collar of option pricing: a flat segment when managers are underperforming, followed by an upward-sloping approximately linear segment straddling the break-even relative performance, and then again a flat one. Additionally, for extreme values of outperformance, the relationship becomes increasing and convex. Sirri and Tufano (1998) find that as a function of the fund's rank, the relationship is relatively flat for underperformers but increasing and convex for outperformers. For pension funds, Del Guercio and Tkac (2002) document that inflows are triggered when managers beat an index; the magnitude of excess returns is not significantly related to flow. While different studies emphasize different features of the flow-performance relationship, they all agree on two distinguishing characteristics: the flow-performance function is (i) increasing and (ii) exhibits convexities.

The function we consider here is the simplest possible one that captures above characteristics in reduced-form. There are two fund flow rates: high, f_H, and low, f_L; $f_H \geq f_L > 0$. At the terminal date, the manager receives fund flows at rate

$$f_T = \begin{cases} f_L & \text{if} & R_T^W - R_T^Y < \eta, \\ f_H & \text{if} & R_T^W - R_T^Y \geq \eta. \end{cases} \tag{3}$$

The pivotal difference in returns η, which we will call the *flow threshold*, can be either positive, zero, or negative. While being consistent with the findings of Del Guercio and Tkac, this form of

the flows for performance relationship may appear somewhat simplistic. Nonetheless, it is capable of capturing most of the implications of more complex functions for the manager's behavior. Basak, Pavlova, and Shapiro (2007) consider a variety of such flow-performance relationships (collar-type, linear-convex, linear-linear, etc.), some of which they interpret as part of the manager's compensation package, and show that the manager's behavior is quite similar across all specifications, as long as the functions are increasing and exhibit convexities. They argue that the presence of a convexity is the first-order effect; the exact form of the flow function around the convexity ends up either playing a minor role or being completely immaterial for the manager's optimization. By adopting our simple two-tier flow function we thus avoid their computational difficulties and distill their main insights.

In the presence of implicit incentives, there are two considerations affecting the manager's behavior. First is her attitude towards risk, driving the normal policy, second is the risk-shifting incentive induced by nonconcavities due to fund flows. To understand the latter, note that the nonconcave payoff to the manager can be expressed as

$$A_T \;=\; \underbrace{f_L W_T}_{\text{linear component}} \;+\; \underbrace{(f_H - f_L) W_T 1_{\{R_T^W - R_T^Y \geq \eta\}}}_{\text{option-like component}},$$

where the first term is a linear function of the terminal portfolio value, and the second is a position in $f_H - f_L$ "asset-or-nothing" binary options with a stochastic strike. When the manager is following her normal policy, her optimal portfolio value process is a geometric Brownian motion, and hence an exact pricing formula for the binary option is readily available. In particular, the volatility of the underlying, W/Y, depends on the difference between the volatility of the manager's portfolio and that of the index, and is given by $\sigma|\theta_t^N - \theta_t^Y|$.[6] As emphasized in the vast risk-shifting literature (Jensen and Meckling (1976)), to increase the value of her compensation, the manager has an incentive to deviate from her normal policy by boosting the volatility of the underlying. Note that an increase in the volatility (or risk exposure) of the manager's portfolio W, $\sigma\theta_t$, does not always result in an increase of the volatility of the underlying W/Y. This motivates us to treat economies (a) and (b) separately; in the latter, the manager chooses to decrease her portfolio volatility to boost the value of her payoff.

2.3 A Benchmarking Restriction

Our final ingredient describing a realistic investment environment of a fund manager involves risk management restrictions either explicitly or implicitly imposed on her by her superiors. Such restrictions limit the allowed shortfall in the manager's portfolio, measured in absolute terms or relative

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[6]The binary option with the payoff $W_T 1_{\{R_T^W \geq R_T^Y + \eta\}} = W_T 1_{\{W_T \geq e^\eta Y_T\}}$ is essentially an option on the ratio W/Y. The properties of such an option closely resemble those of an exchange option. For discussion of binary and exchange options, see Hull (2005).

to a pre-specified benchmark. While a slight underperformance relative to the benchmark can be ascribed to "bad luck," a significant underperformance is likely to have more serious consequences.

To capture this, we consider a *benchmarking restriction* of the following form

$$R_T^W - R_T^X \geq \varepsilon, \qquad (4)$$

where X is a value-weighted portfolio, with a fraction δ invested in the stock market and $(1 - \delta)$ in the money market, and ε is the manager's *allowed shortfall*.[7] Here, the benchmark X is not the same as the index Y since the index triggering fund flows from retail investors need not coincide with the benchmark used by the manager's superiors. We view this benchmark as a potential policy instrument aimed at mitigating adverse incentives. The allowed shortfall can also act as a device towards aligning incentives. For example, $\varepsilon = -\infty$ means that the manager is completely unrestricted, while $\varepsilon = -5\%$ implies that the maximal shortfall of the manager's return over that of the benchmark may not exceed 5%. When the benchmark is simply the money market, this restriction is a familiar stop-loss limit, routinely imposed on professional traders. By considering a stochastic benchmark X, we make the restriction more general – capable of also capturing, for example, a termination rule for managers whose performance is unsatisfactory. Termination of money managers is typically linked to underperformance relative to a benchmark, rather than underperformance in absolute terms.

Our hard constraint can easily be generalized to a softer tracking error-type constraint, which may permit exceeding the allowed shortfall with some probability or allow violating the benchmarking restriction at a cost. Alternatively, our restriction can be interpreted as a limiting case of a VaR constraint. A VaR constraint specifies a floor (in our case, possibly stochastic) which the manager's portfolio has to exceed with a pre-specified probability. Our constraint establishes a floor which has to be maintained at all times. In that sense, it is closer to portfolio insurance. For simplicity, we do not consider soft constraints in this paper. Finally, it is of independent interest to explore the effects of benchmarking in light of the well-known theoretical work cautioning against the use of benchmarking in money management (Roll, 1992; Admati and Pfleiderer, 1997).

Anticipating our results, to alleviate the effects of implicit incentives, we require that the risk exposure of the benchmark, $\theta^X = \delta$, is less than that of the index, θ^Y. To simplify our presentation below, we further restrict the risk exposure of the benchmark to be below the manager's normal exposure. To summarize, we assume that $\theta^X \leq \min\{\theta^N, \theta^Y\}$. In the sequel, we comment on the manager's optimal behavior when these two conditions are violated.

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[7]We normalize $X_0 = W_0$, without loss of generality.

3. Unwinding the Manager's Incentives with Benchmarking

The optimization problem of the manager is given by:

$$\max_{\theta} E[u(A_T)] \tag{5}$$

subject to the budget constraint (2), the flow-performance relationship (3), and the benchmarking restriction (4).

As is well known (e.g., Karatzas and Shreve, 1998), the driving economic state variable in an agent's dynamic investment problem is the so-called state price density. In the complete-markets Black and Scholes (1973) economy, this state price density process, ξ, is given by $d\xi_t = -r\xi_t dt - \kappa \xi_t dw_t$, where $\kappa \equiv (\mu - r)/\sigma$ is the constant market price of risk in the economy. Proposition 1 characterizes the solution to the manager's problem (5) in terms of the primitive economic state variable ξ.[8]

Proposition 1. *The optimal risk exposure and terminal portfolio value of a fund manager facing implicit incentives and a benchmarking restriction are given by*

$$
\begin{aligned}
\theta_t^* \;=\; & \theta^N + \Big[\mathcal{N}(d(\hat{\kappa}, \check{\xi})) - \mathcal{N}(d(\hat{\kappa}, \xi_{YH})) \Big] (\gamma/\hat{\kappa} - 1)\, B\, \theta^N Z(\hat{\kappa}) \xi_t^{-1/\hat{\kappa}} / W_t^*\, 1_{\{a2,\,a3,\,a4\}} \\
& + \mathcal{N}(-d(\check{\kappa}, \check{\xi}))(\gamma/\check{\kappa} - 1)\, C\, \theta^N Z(\check{\kappa}) \xi_t^{-1/\check{\kappa}} / W_t^* \\
& + \Big\{ \Big[\phi(d(\hat{\kappa}, \check{\xi})) - \phi(d(\hat{\kappa}, \xi_{YH})) \Big] B\, Z(\hat{\kappa}) \xi_t^{-1/\hat{\kappa}} 1_{\{a2,\,a3,\,a4\}} \\
& + \Big[\phi(d(\gamma, \tilde{\xi})) f_H^{(1/\gamma - 1)} + \Big(\phi(d(\gamma, \check{\xi})) - \phi(d(\gamma, \xi_a)) \Big) f_L^{(1/\gamma - 1)} 1_{\{a4\}} \Big] Z(\gamma)(y\xi_t)^{-1/\gamma} \\
& - \phi(d(\check{\kappa}, \check{\xi})) C\, Z(\check{\kappa}) \xi_t^{-1/\check{\kappa}} \Big\} \gamma \theta^N / (\kappa \sqrt{T-t}\, W_t^*)\,, \\[4pt]
W_T^* \;=\; & \frac{1}{f_H} J\left(\frac{y}{f_H} \xi_T \right) 1_{\{\xi_T < \tilde{\xi}\}} + e^{\eta} Y_T 1_{\{\tilde{\xi} \le \xi_T < \check{\xi};\, a2,\, a3,\, a4\}} + \frac{1}{f_L} J\left(\frac{y}{f_L} \xi_T \right) 1_{\{\check{\xi} \le \xi_T < \check{\xi};\, a4\}} \\
& + e^{\varepsilon} X_T 1_{\{\check{\xi} \le \xi_T\}}\,,
\end{aligned}
$$

where the arguments of the indicator function $1_{\{\cdot\}}$ refer to the subeconomies identified below, y solves $E[\xi_T W_T^*] = W_0$, $J(\cdot)$ is the inverse function of $u'(\cdot)$, $\mathcal{N}(\cdot)$ and $\phi(\cdot)$ the standard-normal cumulative distribution and density functions respectively, $\hat{\kappa} = \kappa/(\beta\sigma)$, $\check{\kappa} = \kappa/(\delta\sigma)$, $d(v, x) = (\ln x/\xi_t + (r + (2 - v)/(2v)\kappa^2)(T - t))/(\kappa\sqrt{T-t})$, $B = W_0 e^{[\eta/T + (1-\beta)r + \beta(\mu - \beta\sigma^2/2 - (r + \kappa^2/2)\sigma/\kappa)]T}$, $C = W_0 e^{[\varepsilon/T + (1-\delta)r + \delta(\mu - \delta\sigma^2/2 - (r + \kappa^2/2)\sigma/\kappa)]T}$, $Z(v) = e^{\frac{1-v}{v}\left(r + \frac{\kappa^2}{2v}\right)(T-t)}$, $g(\xi) = (\gamma(y\xi/f_L)^{1-1/\gamma} - (\xi^{1/\hat{\kappa}}/(Bf_H))^{\gamma-1})/(1 - \gamma) + yB\xi^{1-1/\hat{\kappa}}$, $\xi_{YH} = (yB^{\gamma}/f_H^{1-\gamma})^{1/(\gamma/\hat{\kappa}-1)}$, $\xi_{XL} = (yC^{\gamma}/f_L^{1-\gamma})^{1/(\gamma/\check{\kappa}-1)}$,

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[8]The analysis presented here is equally valid for a fund-flow to relative-performance specification along the lines of that estimated by Chevalier and Ellison (1997) who argue that it resembles a collar of option pricing. We may consider a collar-type function f_T given by

$$
f_T = \begin{cases}
f_L & \text{if} \quad R_T^W - R_T^Y < \eta_L, \\
f_L + \psi(R_T^W - R_T^Y - \eta_L) & \text{if} \quad \eta_L \le R_T^W - R_T^Y < \eta_H, \\
f_H \equiv f_L + \psi(\eta_H - \eta_L) & \text{if} \quad R_T^W - R_T^Y \ge \eta_H,
\end{cases} \tag{6}
$$

with $\psi > 0$, $\eta_L \le \eta_H$, assuming additionally that $\frac{\gamma}{1-\gamma}\left(\frac{f_H + \psi}{f_L}\right)^{1-1/\gamma} + \frac{f_H + \psi}{f_H} - \frac{1}{1-\gamma} \ge 0$ (satisfied for empirically plausible parameter values). Indeed, Proposition 1 is valid for this flow function, but with η now replaced by η_H.

$\xi_{XH} = (yC^\gamma / f_H^{1-\gamma})^{1/(\gamma/\check{\kappa}-1)}$, $\quad \xi_{XY} = (B/C)^{\kappa/(\sigma(\beta-\delta))}$, $\quad \xi_a > \xi_{YH}$ *satisfies* $g(\xi_a) = 0$, *and* W_t^* *is as given in the proof.*

There are four possible subeconomies a1–a4, as summarized in Table 1, with $\tilde{\xi} = \xi_{XH}$ *in a1 and* $\tilde{\xi} = \xi_{YH}$ *otherwise;* $\check{\xi} = \xi_{XY}$ *in a2 and* $\check{\xi} = \xi_a$ *in a3, a4;* $\check{\xi} = \xi_{XH}, \xi_{XY}, \xi_a, \xi_{XL}$, *in a1, a2, a3, a4, respectively.*

Proposition 1 reveals that the manager's optimal behavior has a different pattern (economies a1–a4) depending on the stringency of the benchmarking restriction, as governed by the allowed shortfall ε. While the expressions in the proposition appear complicated, the intuition behind the manager's optimal policy is simple, and can be articulated in two easy steps. To best highlight this, Figure 1 plots the manager's optimal risk exposure as a function of her performance relative to the index for varying levels of the allowed shortfall. First, consider the dotted plot that corresponds to the case of the benchmarking restriction being infinitely loose, and hence the manager being effectively unrestricted. Note that when underperforming the index ($R_t^W - R_t^Y < \eta$), the manager attempts to avoid getting a low flow f_L at the terminal date by increasing her risk exposure, and when outperforming ($R_t^W - R_t^Y > \eta$), she "locks in" her gains. Because of her risk aversion, however, the increase in the poor-performing manager's risk exposure is bounded. The risk exposure reaches its maximum when the manager is sufficiently behind the index, where a further scaling up the size of her gamble required to catch up with the index becomes prohibitively risky. Nonetheless, when unrestricted, the manager significantly increases her tracking error in the underperformance region, which is not in the best interest of fund investors whose utility is unaffected by fund flows.
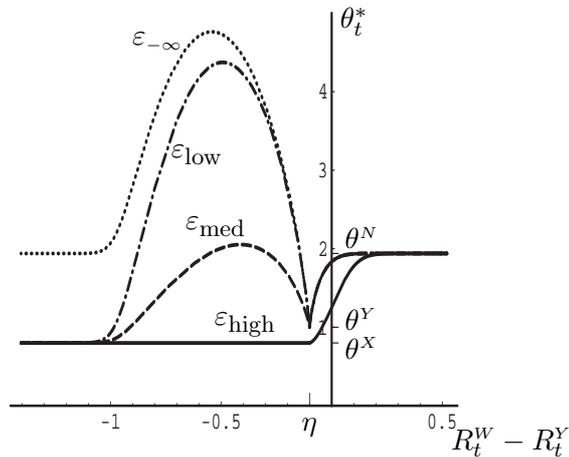
Figure 1. The effects of the benchmarking restriction. The dotted plot corresponds to the case where the manager is unrestricted ($\varepsilon = -\infty$). The remaining plots are for the varying degrees of the stringency of the benchmarking restriction ($\varepsilon > -\infty$).[9]

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[9]The figure is typical. Parameter values are chosen for demonstrative purposes. Their values are $\gamma = 1.0$, $f_L = 0.85$, $f_H = 1.15$, $\beta = 1.0$, $\eta = -0.1$, $\mu = 0.08$, $r = 0.01$, $\sigma = 0.19$, $W_0 = 1$, $t = 0.8$, $T = 1$, $\varepsilon_{\text{low}} = -1.0$, $\varepsilon_{\text{med}} = -0.5$, $\varepsilon_{\text{high}} = -0.05$, and $\delta = 0.8$.

Second, consider the effects of a potentially binding benchmarking restriction ($\varepsilon > -\infty$). The figure underscores the importance of imposing a state-dependent restriction on the manager for the purposes of reducing her implicit incentives-induced tendencies to gamble. In the states in which the manager is outperforming the index, the benchmarking restriction does not drastically affect her behavior. In contrast, in the underperformance states, the benchmark has a significant effect by forcing the manager to tilt her risk exposure closer towards the risk exposure of the benchmark X. Since by construction, the benchmark we propose is safer than both the manager's normal policy and the index, it acts in the direction of reducing the manager's risk exposure. The lever controlling how much power the benchmarking restriction has in reducing the risk exposure is the allowed shortfall ε. As ε increases, the manager's risk exposure is forced to approach that of the benchmark, converging to the latter when ε reaches its upper bound ($\varepsilon = 0$). It is this lever ε that gives rise to a range of subcases, as reported in Table 1 of Proposition 1. For the subcases corresponding to a very low ε (economies $a4$), the manager is allowed to underperform the benchmark by a large amount, and so the benchmarking restriction has practically no effect in the range where the manager gambles (dash-dotted plot in Figure 1). By increasing ε (economy $a3$), we move to the subeconomy in which the benchmarking restriction is strong enough to target the risk-exposure humps induced by implicit incentives (dashed plot). Finally, for high enough ε (economies $a1$, $a2$), we reach the subcases where the gambling behavior is no longer present (solid plot).
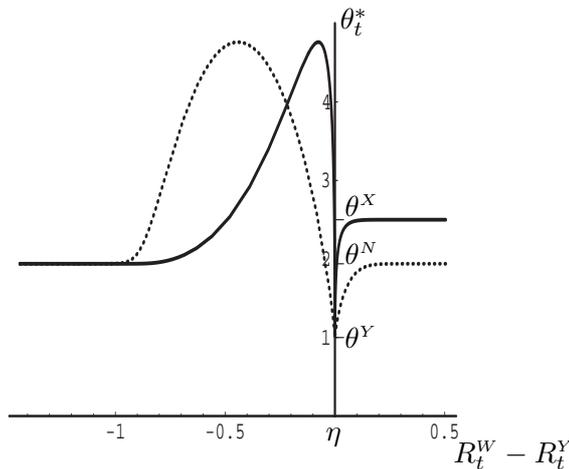
The benchmarking restriction effectively limits the size of the gamble that the manager would be willing to take. Although the constraint is imposed at the terminal date T, at no earlier date $t < T$ the manager would be willing to allow the return on her portfolio to drop more than the allowed shortfall ε below the benchmark. Otherwise, there is a positive probability that the benchmarking restriction may be violated at the terminal date. Moreover, as the manager's underperformance progressively worsens and she approaches the limit she is unwilling to violate, $R_T^X + \varepsilon$, her investment policy becomes increasingly more conservative. The manager abandons her otherwise optimal policy and instead closely follows the benchmark. This pattern is evident in Figure 1: a manager with a sufficiently poor performance "locks in" her relative return by holding the benchmark (choosing the risk exposure θ^X). On the other hand, an underperforming manager with a better relative performance deviates from holding the benchmark only, but not to the same extent as she would if unrestricted. Finally, in the overperformance region, the benchmarking restriction has little effect on a manager's behavior: the policy there resembles the unrestricted policy ($\varepsilon = -\infty$). Hence, the benchmarking restriction in this economic setting is most effective in the underperformance states – precisely the states in which the manager is tempted to engage in excessive risk taking.

The conflict of interest due to fund-flow considerations is not the only one. There may be another conflict of interest arising due to the differences in attitudes towards risk of the manager and the investors. The manager manages assets in line with her own appetite for risk, which need

not coincide with those of fund investors. The benchmarking restriction can be very effective in aligning those incentives as well. Absent implicit incentives, the general rule is very simple: the manager's risk exposure decreases if she is benchmarked to a portfolio X that is less risky than her normal policy, otherwise increases if benchmarked to X that is riskier than her normal policy. The overall effect of the benchmarking restriction on mitigating the conflicts of interest due to fund-flow considerations and differing risk aversions reflects the interaction of the two mechanisms described above. We assess it quantitatively in the following section, and discuss the cost-benefit implications for the investor.

The expressions for the optimal terminal portfolio value revealed by Proposition 1 make the distinction between the subcases we discussed above. The parameter space is subdivided into two (in $a1$) to four (in $a4$) regions of distinct behavior of the manager, as a function of the state-price density ξ representing economic conditions. Although the expressions for the subcases offer additional insights into the subtleties of the manager's economic behavior, we do not present the details here in the interest of preserving space.

So far, we have focused on a benchmark that is safer than both the manager's normal policy and the index, $\theta^X \leq \min\{\theta^N, \theta^Y\}$. In Figure 2 we examine a scenario in which the benchmark is riskier than both the normal policy and the index, $\theta^X > \max\{\theta^N, \theta^Y\}$. The contrast with Figure 1 is striking. The risk-taking incentives are not reduced, on the contrary, the risk exposure is amplified as the manager tilts her portfolio towards the riskier benchmark X. This illustrates the undesired effects of too-aggressive a benchmark when fund-flow considerations come to play and how complicated fine-tuning the manager's adverse behavior may be.

Figure 2. The effects of the benchmarking restriction with $\theta^X > \max\{\theta^N, \theta^Y\}$. The solid plots are for the risk exposure of the manager facing a benchmarking restriction, and the dotted plots are for the unrestricted manager. We let $\delta = 2.5$, $\varepsilon = -0.25$, $\eta = 0$; the remaining parameter values are as in Figure 1.

4. Cost-Benefit Implications of Benchmarking

In this section, we calibrate our economy in order to assess the economic significance of the manager's adverse behavior. To establish a basis for comparison, we consider an investment policy associated with the manager acting in the best interest of fund investors, not accounting for her incentives. A hypothetical fund investor is assumed to have CRRA preferences, $u_I(W_T) = \frac{W_T^{1-\gamma_I}}{1-\gamma_I}$, $\gamma_I > 0$, over the horizon wealth W_T. The investor is passive in that he delegates all his initial wealth, W_0, to the manager to invest. The decision to delegate is exogenous. It captures in a reduced form the choice to abstain from active investing due to various frictions associated with money management such as participation and information costs, time required to implement a dynamic trading strategy, transaction costs, behavioral limitations. Alternatively, the investor may simply believe that the manager has better information or ability. The manager's investment policy that maximizes the investor's utility is given by $\theta^I = \frac{1}{\gamma_I}\frac{\mu-r}{\sigma^2}$.

We first consider an unrestricted manager ($\varepsilon = -\infty$). In order to evaluate the economic significance of the manager's incentives, we compute the utility loss to the investor of the manager's deviating from the policy θ^I. Following Cole and Obstfeld (1991), we define a cost-benefit measure, $\hat{\lambda}$, reflecting the investor's gain/loss quantified in units of his initial wealth:

$$V^I((1+\hat{\lambda})W_0) = \hat{V}(W_0),$$

where $V^I(\cdot)$ denotes the investor's indirect utility under the policy absent incentives θ^I, and $\hat{V}(\cdot)$ his indirect utility under the optimal policy accounting for incentives $\hat{\theta}$. In other words, V^I is the investor's indirect utility assuming he could choose the risk exposure θ^I himself, while $\hat{V}$ represents his indirect utility when the manager chooses her optimal risk exposure $\hat{\theta}$ without benchmarking (Proposition 1 with $\varepsilon = -\infty$). The first figure reported in each cell of Table 2 is the cost due to the manager's adverse incentives, $\hat{\lambda}$. We present the values both for economies (a) with a relatively risk tolerant manager $\theta^N > \theta^Y$, and economies (b) with a relatively risk averse manager $\theta^N < \theta^Y$. For most of our parameter values, the utility loss to the hypothetical investor ranges from 1.4% to 2.4% of his initial wealth in economies (a), and ranges from 7 to 8% in economies (b).[10]

To quantify the effects of imposing a benchmarking restriction, we now define a measure of an incremental increase in the hypothetical investor's utility due to restraining the manager, λ^*:

$$V^I((1+\lambda^*)(1+\hat{\lambda})W_0) = V^*(W_0),$$

---

[10]The market parameters in economies (a) are based on historical estimates. For low level of managerial risk aversion (around 2), these parameter values imply that the manager holds a leveraged portfolio. This is counterfactual, but is no different from the standard prescription of the Merton (1971) asset allocation rule. This observation is related to the discussion whether very high historical equity premium can be reconciled with a typical agent's preferences initiated by Mehra and Prescott (1985). The market parameters in economies (b) represent "unfavorable" market conditions, in which the risky stock has a lower expected return and a higher volatility than in economies (a). Under such market conditions, the normal policy of the manager is more realistic, and in particular, does not entail leverage even for relatively low levels of risk aversion.

where $\hat{\lambda}$ is the utility loss to the investor absent the benchmarking restriction, and $V^*(\cdot)$ is the indirect utility of the investor under managerial incentives and benchmarking. In other words, V^* is the same as $\hat{V}$ but subject to the benchmarking restriction ($\varepsilon > -\infty$). A positive λ^* means that the benchmarking restriction benefits the investor. In Table 2, we report the corresponding gains in terms of the loss recouped (as a fraction of the investor's initial wealth) by benchmarking the manager, λ, given by $\lambda = -(1 + \hat{\lambda})\lambda^*/\hat{\lambda}$ (the second figure in each cell).[11]

At the outset, one rarely thinks of investment restrictions as being beneficial. Restricting the manager would certainly be impairing if the manager were acting in the investor's best interest. However, in the context of active money management, risk management restrictions can be economically justified. Consider, for example, the case of a highly risk averse investor (more precisely, consider the case of $\theta^I < \min\{\theta^N, \theta^Y\}$). Suppose now that we benchmark the fund manager to a low-risk portfolio X, along the lines of that discussed in Section 3. As one can infer from Figure 1, by tightening the benchmarking restriction (increasing ε), the investor or the manager's superior can effectively reduce the risk exposure of the manager, bringing her policy closer to that optimal for the investor. Indeed, the corresponding gains, in terms of the loss recouped, reported in Table 2 for this scenario can be considerable in magnitude: for example, in the top left entry, 93% (most of the loss is recouped) in economies (a) and 73% in economies (b). Most of the loss recouped estimates fall into the 57–77% range in economies (a) and into the 75-82% range in economies (b), which constitute sizable gains to the investor.

The surprising result is that even a risk tolerant investor may benefit from benchmarking a less risk tolerant manager to a safer portfolio. One could argue that such an investor would simply desire to increase the manager's risk exposure, as the latter is normally below the investor's desired policy, by benchmarking the manager to a riskier portfolio. Instead, Table 2 illustrates that the reverse can be true. In Table 2, the benchmark portfolio X is safer than the optimal risk exposure of both the investor and the manager, but nevertheless all entries for the benefit measure λ (including those in which the manager is less risk tolerant than the investor, $\gamma > \gamma_I$) are positive. These results show that the simple argument in favor of a riskier benchmark fails in the context of real-life fund managers whose policies may be driven by implicit incentives to a larger degree than by their attitudes towards risk.

Once we have demonstrated that a benchmarking restriction reduces the cost of active management, the natural next step is to ask how such a restriction may be designed for the highest benefit to the investor. A guideline can be inferred from Table 2. There are two parameters of the restriction one can adjust: the risk exposure of the benchmark θ^X and the allowed shortfall ε. Table 2 shows a desirable quantity for both.[12] For example, in economies (a), the benchmarking restriction

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[11]In particular, the incremental gain due to benchmarking is $(1 + \lambda^*)(1 + \hat{\lambda})W_0 - (1 + \hat{\lambda})W_0 = (1 + \hat{\lambda})\lambda^* W_0$, which when divided by the loss due to flows-induced incentives $= -\hat{\lambda}W_0$, leads to the expression $\lambda = -(1 + \hat{\lambda})\lambda^*/\hat{\lambda}$.

[12]We note that this discussion is necessarily partial in nature since only the implicit incentives are considered and explicit incentives induced by an optimal contract are abstracted away from.

calls for selecting the risk exposure of around 0.3 (40% stock/ 60% money market) and the allowed shortfall of about 0.1. In these economies, it is beneficial to the investor to benchmark the manager to a relatively safe portfolio. The benchmarking restriction is quite loose in economies (a) and very tight, close to the upper bound on ε, in economies (b). Of course, this discussion is necessarily incomplete because the goal of imposing a benchmarking restriction is to limit the downside risk while allowing the manager to pursue active money management. Hence, making the restriction too tight (setting ε to be relatively high) would force the manager to stay close to the benchmark in most states of the world, effectively making her asset allocation policy passive. Exploring such a trade-off is beyond the scope of this paper, and we leave it for future research.

5.   Conclusion

In this paper, we have demonstrated how benchmarking an active money manager to a safer portfolio and imposing limits on her allowed shortfall can be beneficial. This is because the incentives induced by the compensation package may tempt the manager to not act in the best interest of investors and in particular take on excessive risk. The benchmarking restriction we primarily focus on in this paper benefits fund investors. However, as our analysis demonstrates, not all benchmarks necessarily create value for investors – for example, a too-aggressive a benchmark may in fact destroy value. Hence, it is important to take into account of investors' risk tolerance when selecting benchmarks for money managers.

Our study leaves aside many possible constraints that may also be beneficial. We believe that endogenizing investment restrictions in the context of active money management is a fruitful area for future research. It would also be of interest to endogenize within our model the fund-flows to relative-performance relationship that we have taken as given.

Appendix

Proof of Proposition 1. In states in which the benchmarking restriction in (4) is binding, the manager's optimal terminal portfolio value is given by $W_T^* = e^\varepsilon X_T$, where under the geometric Brownian motion dynamics of X_T and ξ_T, the benchmark level X_T is given by $X_T = Ce^{-\varepsilon}\xi_T^{-\delta\sigma/\kappa}$, where C is as in the proposition. In states in which the restriction is not binding, the Lagrange multiplier associated with (4) is zero, and hence the terminal portfolio value coincides with that of the unrestricted manager $W_T^* = \hat{W}_T(y) = \frac{1}{f_H}J\left(\frac{y}{f_H}\xi_T\right)1_{\{\xi_T<\xi_{YH}\}} + e^Y Y_T 1_{\{\xi_{YH}\leq\xi_T<\xi_a\}} + \frac{1}{f_L}J\left(\frac{y}{f_L}\xi_T\right)1_{\{\xi_a\leq\xi_T\}}$ (Basak, Pavlova, and Shapiro, 2007), where y is the Lagrange multiplier of the restricted manager's static budget constraint and ξ_{YH}, ξ_a are as in the proposition. Therefore, the optimal terminal portfolio value is given by $W_T^* = \max\{\hat{W}_T, e^\varepsilon X_T\}$. For $\varepsilon = -\infty$, we have $W_T^* = \hat{W}_T$, while as ε increases to zero, the maximum operator generates the economies $a1$–$a4$ reported in the proposition. To establish these 4 economies, we find the intersection point of $e^\varepsilon X_T(\xi_T)$ and unrestricted policy $\hat{W}_T(\xi_T)$. Because $\hat{W}_T(\xi_T)$ is monotonic and $\theta^X \leq \min\{\theta^Y, \theta^N\}$, there is only one intersection point. Let ξ_{XH} and ξ_{XL} denote the values of ξ_T at which $e^\varepsilon X_T$ crosses the optimal unrestricted terminal portfolio value corresponding to high flow f_H and low flow f_L, respectively, e.g., ξ_{XH} solves $e^\varepsilon X_T(\xi_{XH}) = \frac{1}{f_H}J\left(\frac{y}{f_H}\xi_{XH}\right)$. Similarly, let ξ_{XY} denote the point $e^\varepsilon X_T$ crosses $e^\eta Y_T$. The relationship between ξ_{XH}, ξ_{XL}, ξ_{XY}, ξ_{YH}, ξ_a determines which subeconomy $a1$–$a4$ obtains. For example, $\xi_{YH} < \xi_{XY} \leq \xi_a$ means that $e^\varepsilon X_T$ crosses $e^\eta Y_T$ in the intermediate region and hence we are in subeconomy $a2$. Similar straightforward reasoning leads to conditions: $\xi_{XH} < \xi_{YH}$ for economy $a1$, $\xi_{XL} < \xi_a \leq \xi_{XY}$ for economy $a3$, $\xi_a < \xi_{XL}$ for economy $a4$. We then rearrange these conditions in ξ_T-space in terms of conditions on ε, as reported in Table 1 of the proposition.

Since $W_t^*\xi_t$ is a martingale (given the dynamics of W_t^* and ξ_t), the time-t portfolio value is obtained by evaluating the conditional expectation of $W_T^*\xi_T$ over the relevant regions of ξ_T, yielding:

$$
\begin{aligned}
W_t^* &= \mathcal{N}(d(\gamma,\tilde{\xi}))f_H^{(1/\gamma-1)}Z(\gamma)(y\xi_t)^{-1/\gamma} + \left[\mathcal{N}(d(\hat{\kappa},\check{\xi})) - \mathcal{N}(d(\hat{\kappa},\xi_{YH}))\right]B\,Z(\hat{\kappa})\xi_t^{-1/\hat{\kappa}}1_{\{a2,a3,a4\}} \\
&\quad + \left[\mathcal{N}(d(\gamma,\check{\xi})) - \mathcal{N}(d(\gamma,\xi_a))\right]f_L^{(1/\gamma-1)}Z(\gamma)(y\xi_t)^{-1/\gamma}1_{\{a4\}} + \mathcal{N}(-d(\check{\kappa},\check{\xi}))C\,Z(\check{\kappa})\xi_t^{-1/\check{\kappa}}\quad\text{(A1)}
\end{aligned}
$$

To obtain the risk exposure expression in the proposition, note that from (2), the diffusion term of the manager's optimal value process is $\theta_t^*\sigma W_t^*$. Equating the latter term with the diffusion term obtained by applying Itô's lemma to (A1) yields the expression for θ_t^*. ∎

Analysis of Economies with a Relatively Risk Averse Manager $\theta^{\mathbf{N}} < \theta^{\mathbf{Y}}$

Proposition 2. *The optimal risk exposure and terminal portfolio value of a fund manager facing implicit incentives and a benchmarking restriction in economies with $\theta^N < \theta^Y$ are given by:*

$$
\begin{aligned}
\theta_t^* &= \theta^N + \left[\mathcal{N}(d(\hat{\kappa},\check{\xi})) - \mathcal{N}(d(\hat{\kappa},\xi_b))\right](\gamma/\hat{\kappa} - 1)B\,\theta^N Z(\hat{\kappa})\xi_t^{-1/\hat{\kappa}}/W_t^*\,1_{\{b2,b3,b4,b5\}} \\
&\quad + \left[\left(\mathcal{N}(d(\check{\kappa},\xi_b)) - \mathcal{N}(d(\check{\kappa},\xi_{XL}))\right)1_{\{b2,b3\}} + \mathcal{N}(-d(\check{\kappa},\check{\xi}))\right](\gamma/\check{\kappa} - 1)C\,\theta^N Z(\check{\kappa})\xi_t^{-1/\check{\kappa}}/W_t^*
\end{aligned}
$$

$$+\left\{\left[\phi(d(\hat{\kappa},\breve{\xi}))-\phi(d(\hat{\kappa},\xi_b))\right]B\,Z(\hat{\kappa})\xi_t^{-1/\hat{\kappa}}1_{\{b2,b3,b4,b5\}}\right.$$

$$+\left[\phi(d(\gamma,\tilde{\xi}))f_L^{(1/\gamma-1)}+\left(\phi(d(\gamma,\breve{\xi}))-\phi(d(\gamma,\check{\xi}))\right)f_H^{(1/\gamma-1)}1_{\{b3,b5\}}\right]Z(\gamma)(y\xi_t)^{-1/\gamma}$$

$$+\left.\left[\left(\phi(d(\check{\kappa},\xi_b))-\phi(d(\check{\kappa},\xi_{XL}))\right)1_{\{b2,b3\}}-\phi(d(\check{\kappa},\check{\xi}))\right]C\,Z(\check{\kappa})\xi_t^{-1/\check{\kappa}}\right\}\gamma\theta^N/(\kappa\sqrt{T-t}\,W_t^*),$$

$$
\begin{aligned}
W_T^* \;=\; & \tfrac{1}{f_L}J_M\left(\tfrac{y}{f_L}\xi_T\right)1_{\{\xi_T<\tilde{\xi}\}}+e^\eta Y_T1_{\{\xi_b\le\xi_T<\check{\xi};\,b2,b3,b4,b5\}}+\tfrac{1}{f_H}J_M\left(\tfrac{y}{f_H}\xi_T\right)1_{\{\check{\xi}\le\xi_T<\breve{\xi};\,b3,b5\}}\\
& +e^\varepsilon X_T1_{\{(\xi_{XL}\le\xi_T<\xi_b;\,b2,b3)\ or\ (\breve{\xi}\le\xi_T)\}},
\end{aligned}
$$

where y solves $E[\xi_T W_T^*]=W_0$ and $\xi_b<\xi_{YH}$ satisfies $g(\xi_b)=0$, with $J(\cdot)$, $\mathcal{N}(\cdot)$, $\phi(\cdot)$, $g(\cdot)$, $Z(\cdot)$, $d(\cdot)$, ξ_{YH}, ξ_{XL}, ξ_{XH}, ξ_{XY}, B, C, $\hat{\kappa}$, $\check{\kappa}$ as given in Proposition 1.

Economies (b) have five subeconomies, as summarized in Table 3, with $\tilde{\xi}=\xi_{XL}$ in b1, b2, b3 and $\tilde{\xi}=\xi_b$ otherwise; $\check{\xi}=\xi_{XY}$ in b2, b4 and $\check{\xi}=\xi_{YH}$ in b3, b5; $\breve{\xi}=\xi_{XL}$ in b1, $\breve{\xi}=\xi_{XY}$ in b2, b4, $\breve{\xi}=\xi_{XH}$ in b3, b5.

The proof follows the same steps as in the proof of Proposition 1, and hence is omitted.

Table 1
Conditions for occurrence of subeconomies in economies (a)

There are four possible subeconomies in economies (a) with $\theta^N > \theta^Y$. The following table summarizes the conditions on ε in economies (a).

Subeconomies	
a1	$\varepsilon > \varepsilon_{a1}$
a2	$\varepsilon_{a1} \geq \varepsilon > \varepsilon_{a2}$
a3	$\varepsilon_{a2} \geq \varepsilon > \varepsilon_{a3}$
a4	$\varepsilon_{a3} > \varepsilon$

where

$$
\begin{aligned}
\varepsilon_{a1} &= \frac{1}{\gamma}\left[\left(\frac{\gamma}{\breve{\kappa}}-1\right)\ln \xi_{YH} - \ln y - \gamma \ln W_0 + (1-\gamma)\ln f_H\right] \\
&\quad - \left[(1-\delta)r + \delta\left(\mu - \delta\frac{\sigma^2}{2} - \left(r + \frac{\kappa^2}{2}\right)\frac{\sigma}{\kappa}\right)\right]T, \\
\varepsilon_{a2} - \varepsilon_{a1} &= \left(\frac{1}{\hat{\kappa}} - \frac{1}{\breve{\kappa}}\right)\ln\left(\frac{\xi_{YH}}{\xi_a}\right) < 0, \\
\varepsilon_{a3} - \varepsilon_{a1} &= \left(\frac{1}{\gamma} - \frac{1}{\breve{\kappa}}\right)\ln\left(\frac{\xi_{YH}}{\xi_a}\right) + \left(\frac{1}{\gamma} - 1\right)\ln\left(\frac{f_L}{f_H}\right) < 0.
\end{aligned}
$$

Table 2a

Costs and benefits of benchmarking to fund investors in economies (a)

The gain/loss quantified in units of a hypothetical investor's initial wealth, $\hat{\lambda}$, solves $V^I((1+\hat{\lambda})W_0) = \hat{V}(W_0)$, where $V^I(\cdot)$ denotes the investor's indirect utility under his desired policy absent incentives θ^I, and $\hat{V}(\cdot)$ his indirect utility under the manager's incentives. The incremental increase in the investor's utility due to restraining the manager, λ^*, solves $V^I((1+\lambda^*)(1+\hat{\lambda})W_0) = V^*(W_0)$, where $V^*(\cdot)$ is the indirect utility of the investor in the presence of benchmarking. The percentage loss recouped, λ, by benchmarking the manager is given by $\lambda = -(1+\hat{\lambda})\lambda^*/\hat{\lambda}$. The fixed parameter values are (where applicable) $\gamma = 4$, $\gamma_I = 4$, $\delta = 0.2$, $\varepsilon = -0.1$, $f_L = 0.8$, $f_H = 1.5$, $f_L + f_H = 2.3$, $\beta = 0.5$, $\eta = 0.05$, $\mu = 0.09$, $r = 0.01$, $\sigma = 0.16$, $W_0 = 1$, $T = 1$.

Effects of		Cost-benefit measures (%) $\hat{\lambda}, \lambda$				
Managerial risk	γ	2	3	4	5	6
aversion		-6.12, 93.45	-2.82, 85.80	-1.58, 74.60	-0.96, 58.31	-0.62, 35.04
Implicit reward	f_H-f_L	0.5	0.6	0.7	0.8	0.9
for outperformance		-1.44, 72.13	-1.52, 73.59	-1.58, 74.60	-1.62, 75.30	-1.66, 75.81
Risk exposure	θ^Y	0.25	0.40	0.50	0.60	0.75
of the index		-1.45, 56.76	-1.44, 70.89	-1.58, 74.60	-1.82, 74.28	-2.39, 70.16
Flow threshold	η	0.02	0.04	0.06	0.08	0.10
		-0.78, 64.97	-1.33, 74.97	-1.81, 73.08	-2.19, 68.38	-2.47, 62.37
Risk exposure	θ^X	0.05	0.10	0.20	0.30	0.40
of the benchmark		-1.58, 63.20	-1.58, 68.01	-1.58, 74.60	-1.58, 77.47	-1.58, 77.00
Allowed shortfall	ε	-0.25	-0.20	-0.15	-0.10	-0.05
		-1.58, 28.95	-1.58, 50.44	-1.58, 66.49	-1.58, 74.60	-1.58, 67.07

Table 2b
Costs and benefits of benchmarking to the investor in economies (b)

The gain/loss quantified in units of a hypothetical investor's initial wealth, $\hat{\lambda}$, solves $V^I((1+\hat{\lambda})W_0) = \hat{V}(W_0)$, where $V^I(\cdot)$ denotes the investor's indirect utility under his desired policy absent incentives θ^I, and $\hat{V}(\cdot)$ his indirect utility under the manager's incentives. The incremental increase in the investor's utility due to restraining the manager, λ^*, solves $V^I((1+\lambda^*)(1+\hat{\lambda})W_0) = V^*(W_0)$, where $V^*(\cdot)$ is the indirect utility of the investor in the presence of benchmarking. The percentage loss recouped, λ, by benchmarking the manager is given by $\lambda = -(1+\hat{\lambda})\lambda^*/\hat{\lambda}$. The fixed parameter values are (where applicable) $\gamma = 4$, $\gamma_I = 4$, $\delta = 0.05$, $\varepsilon = -0.1$, $f_L = 0.8$, $f_H = 1.5$, $f_L + f_H = 2.3$, $\beta = 1$, $\eta = 0.05$, $\mu = 0.06$, $r = 0.02$, $\sigma = 0.29$, $W_0 = 1$, $T = 1$.

Effects of		Cost-benefit measures (%) $\hat{\lambda}, \lambda$				
Managerial risk	γ	2	3	4	5	6
aversion		-9.08, 72.67	-7.96, 74.38	-7.33, 79.07	-6.97, 81.40	-6.75, 82.70
Implicit reward	f_H-f_L	0.5	0.6	0.7	0.8	0.9
for outperformance		-8.73, 83.26	-8.46, 82.24	-7.33, 79.07	-7.99, 80.49	-8.44, 81.31
Risk exposure	θ^Y	0.50	0.75	1.00	1.25	1.50
of the index		-1.75, 28.55	-4.35, 67.54	-7.33, 79.07	-8.81, 81.77	-9.46, 82.58
Flow threshold	η	0.02	0.04	0.06	0.08	0.10
		-7.71, 78.91	-7.54, 79.29	-7.08, 78.65	-6.52, 77.36	-5.95, 75.59
Risk exposure	θ^X	0.02	0.04	0.06	0.08	0.10
of the benchmark		-7.33, 80.94	-7.33, 79.73	-7.33, 78.37	-7.33, 76.88	-7.33, 75.25
Allowed shortfall	ε	-0.25	-0.20	-0.15	-0.10	-0.05
		-7.33, 39.74	-7.33, 53.48	-7.33, 67.18	-7.33, 79.07	-7.33, 86.39

Table 3
Conditions for occurrence of subeconomies in economies (b)

There are five possible subeconomies in economies (b) with $\theta^N < \theta^Y$. However, for each set of exogenous parameters only four subcases can occur in (b) as ε changes. The following table summarizes the conditions on ε in economies (b).

Subeconomies	$\gamma \geq 1$	$\gamma < 1$	
		$\varepsilon_{b2} > \varepsilon_{b3}$	$\varepsilon_{b2} < \varepsilon_{b3}$
$b1$	$\varepsilon > \varepsilon_{b1}$	$\varepsilon > \varepsilon_{b1}$	$\varepsilon > \varepsilon_{b1}$
$b2$	$\varepsilon_{b1} \geq \varepsilon > \varepsilon_{b2}$	$\varepsilon_{b1} \geq \varepsilon > \varepsilon_{b2}$	$\varepsilon_{b1} \geq \varepsilon > \varepsilon_{b3}$
$b3$	N/A	N/A	$\varepsilon_{b3} \geq \varepsilon > \varepsilon_{b2}$
$b4$	$\varepsilon_{b2} \geq \varepsilon > \varepsilon_{b3}$	$\varepsilon_{b2} \geq \varepsilon > \varepsilon_{b3}$	N/A
$b5$	$\varepsilon_{b3} > \varepsilon$	$\varepsilon_{b3} > \varepsilon$	$\varepsilon_{b2} > \varepsilon$

where

$$\varepsilon_{b1} = \varepsilon_{a1} + \left(\frac{1}{\hat{\kappa}} - \frac{1}{\check{\kappa}}\right) \ln\left(\frac{\xi_{YH}}{\xi_b}\right),$$

$$\varepsilon_{b2} - \varepsilon_{b1} = \left(\frac{1}{\gamma} - \frac{1}{\hat{\kappa}}\right) \ln\left(\frac{\xi_{YH}}{\xi_b}\right) + \left(\frac{1}{\gamma} - 1\right) \ln\left(\frac{f_L}{f_H}\right) < 0,$$

$$\varepsilon_{b3} - \varepsilon_{b1} = -\left(\frac{1}{\hat{\kappa}} - \frac{1}{\check{\kappa}}\right) \ln\left(\frac{\xi_{YH}}{\xi_b}\right) < 0,$$

and ε_{a1} is an in Table 1.

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