

Asset Prices and Exchange Rates*

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Abstract

In this paper we study the implications of introducing demand shocks and trade in goods into an otherwise standard international asset pricing model. Trade in goods gives rise to an additional channel of international propagation—through the terms of trade—absent in traditional single-good asset pricing models. The inclusion of demand shocks helps overturn many unrealistic implications of existing international finance models in which productivity shocks are the sole source of uncertainty. Our model generates a rich set of implications on how stock, bond, and foreign exchange markets co-move. We solve the model in closed-form, which yields a system of equations that can be readily estimated empirically. Our estimation validates the main predictions of the theory.

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1. Introduction

The financial press has long asserted that stock prices and exchange rates are closely intertwined. In the early 2000s, the depreciation of the dollar against the euro and other major currencies has, it is argued, put pressure on investor sentiment and the US stock markets. Similarly, the preceding decade, associated with high productivity gains and a stock market boom in the US, was accompanied by an exchange rate appreciation. Surprisingly, these connections have rarely been highlighted in workhorse models of exchange rate determination. Our main contribution is to develop a tractable two-country, two-good asset pricing model in which the terms of trade (and hence the exchange rate) play an important role in determining the dynamics of countries' stock and bond markets, thus introducing elements of international trade into an otherwise standard international asset pricing framework.

By adopting a single-good framework, the overwhelming majority of international asset pricing models abstracts from international trade in goods; agents only trade financial assets. Therefore, by construction, the terms of trade and the real exchange rate have to be equal to unity. Nontrivial implications for exchange rates in such a framework have been obtained by either introducing shipping costs into a real model, or by exogenously specifying a monetary policy and focusing on the nominal exchange rate.¹ The notable exceptions are Helpman and Razin (1978), Cole and Obstfeld (1991), and Zapatero (1995). All these are tractable asset pricing models like ours. In contrast to our work, however, the three papers share an implication that movements in the terms of trade perfectly offset output shocks and hence in equilibrium dividends are the same (up to a multiplicative constant) across all stocks. Consequently, stock markets worldwide are perfectly correlated, and there are no benefits to investing internationally. In fact, all three papers recognize the abnormal properties of equilibrium and admit that they only hold in their knife-edge case.² They also call for a variation on the model that possesses “normal” equilibrium behavior. Our paper is trying to fill this gap. In our model, we overcome the above problems by introducing what we identify as “demand shocks”. We therefore can talk about non-trivial dynamics and spillovers across international financial markets, while maintaining tractability.

¹For models with shipping costs see Dumas (1992), Sercu and Uppal (2000). See Bakshi and Chen (1997), Basak and Gallmeyer (1999) for monetary general equilibrium models, and Solnik (1974), Adler and Dumas (1983) for the international CAPM.

²Cass and Pavlova (2004) label such equilibrium a “peculiar financial equilibrium” and offer an extensive discussion of its atypical properties.

In our model, the two countries comprising the economy specialize in producing their own good. The stock market in each country is a claim to the country’s output. Bonds, whose interest rates are determined endogenously within the model, provide further opportunities for international borrowing and lending. A representative agent in each country consumes both goods, albeit with a stronger preference toward the home good, and invests in the stock and bond markets. Uncertainty in the economy is due to output shocks in each country and the consumers’ demand shocks. While the former are very common in models of international macroeconomics and especially international real business cycles, the latter have received considerably less attention. For the bulk of our analysis, we adopt a very general, yet sensible, specification of these demand shocks, imposing more structure later to gain additional insight. We distinguish between the special cases in which (i) there are no demand shocks, (ii) demand shocks are due to pure consumer sentiment, (iii) demand shocks are due to preferences encompassing a “catching up with the Joneses” feature, and (iv) preferences are state-independent and demand shocks are driven by differences of opinion. It is ultimately an empirical question—addressed in the second part of the paper—as to which of these specifications is the most plausible.

One desirable feature of our modeling framework is that it nests a number of models belonging to separate strands of the international economics and finance literature, thus unifying their implications within one economic setting and producing a richer description of the propagation mechanism. For instance, in our framework, *all* stock markets move in the same direction in response to a supply shock in *one* of the countries. A positive output shock leads to a positive return on the domestic stock market (consistent with the asset pricing literature); however, it has to be accompanied by a deterioration of the terms of trade (in line with the comparative advantages theory of the international trade literature). The latter implies an increase in the relative prices of foreign goods, leading to a rise in the value of the foreign countries’ output, thereby boosting their stock markets. Hence, supply shocks in our model generate positive co-movement among stock markets, and the foreign exchange market acts as a channel through which shocks are propagated internationally. This mechanism was behind the perfect stock-market co-movement results of Helpman and Razin, Cole and Obstfeld, and Zapatero.

The dynamics induced by demand shocks is completely different. Demand shocks produce cross-market spillovers that we call “divergence”. A positive shift in domestic demand causes an improvement of the country’s terms of trade with the foreigners (consistent with the open economy

macroeconomics literature). This provides a boost to the domestic stock and bond markets, while asset prices abroad move in the *opposite* direction; hence the term “divergence.” In addition, the demand shocks may help alleviate two important puzzles of the international real business cycles literature: (i) they may increase (or decrease) the variance of the terms of trade without requiring an increase in the variance of output, and (ii) despite perfect risk sharing, they cause divergence in the countries’ consumption indexes, significantly reducing their correlation for a plausible calibration of our model. Finally, we solve for the agents’ portfolios and find that there is an additional hedging portfolio due to the demand shocks that biases their holdings towards domestic assets.

Put together, these implications deepen our understanding of the interconnections among the stock, bond and foreign exchange markets. Existing international asset pricing literature has focused on one channel of international financial markets’ co-movement: the common worldwide discount factor. By introducing the terms of trade into a standard model we highlight the role of an additional channel: the relative goods prices. The two channels interact and could potentially be substitutes for each other.³

Our theory implies that the dynamics of asset prices and exchange rates are described by a latent factor model. In the empirical section, we estimate the model and establish the following results. First, we find that demand shocks are twice as important as supply shocks in describing the behavior of asset prices and exchange rates. Second, we characterize the properties of the demand shocks, and find that the data reject the hypotheses that our demand shocks are generated by either pure sentiment or “catching up with the Joneses”-type behavior. Rather, our results support the view that the demand shocks are likely to represent differences in opinion or a form of consumer confidence. Finally, we investigate the out-of-sample performance of the supply and demand factors in predicting several macroeconomic variables. Overall, we find strong support for the dynamic implications of our theory.⁴

The rest of the paper is organized as follows. Section 1 describes the economy and characterizes

³Dumas, Harvey, and Ruiz (2003) have highlighted the importance of the first channel. See also Cochrane, Longstaff, and Santa-Clara (2005). The second channel has long been emphasized by the international real business cycles literature. See Backus, Kehoe, and Kydland (1992) and Stockman and Tesar (1995) for seminal contributions and Baxter (1995) for a thorough survey. Other papers in this literature that are related are Baxter and Jermann (1997) and Kraay and Ventura (2000), which also look at stock market returns but they specify them exogenously. See also Canova and De Nicolo (1995). We relate our model to this literature in Section 2.4.

⁴Other papers that have looked empirically and/or theoretically at the relationship between stock markets and exchange rates are Stulz (1987), Brandt, Cochrane, and Santa-Clara (2006), and Hau and Rey (2006). Also in the spirit of our model is Serrat (2001), who focuses on the home bias in portfolios induced by nontradable goods, and a recent paper by Aydemir (2005), who looks at cross-industry stock return correlations.

equilibrium. Section 2 presents the empirical analysis and explores the relevance of our model for explaining international business cycles puzzles. Section 3 discusses caveats and avenues for future research. Section 4 concludes, and the appendices provide the proofs and other material omitted in the body of the paper.

2. The Model

2.1. The Economic Setting

We consider a continuous-time pure-exchange world economy along the lines of Lucas (1982). The economy has a finite horizon, $[0, T]$, with uncertainty represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$, on which is defined a standard three-dimensional Brownian motion $\vec{w}(t) = (w(t), w^*(t), w^\theta(t))^\top$, $t \in [0, T]$. All stochastic processes are assumed adapted to $\{\mathcal{F}_t; t \in [0, T]\}$, the augmented filtration generated by \vec{w} . All stated (in)equalities involving random variables hold P -almost surely. In what follows, given our focus, we assume all processes introduced to be well-defined, without explicitly stating regularity conditions ensuring this.

There are two countries in the world economy: Home and Foreign. Each country produces its own perishable good via a strictly positive output process modeled as a Lucas tree:

$$dY(t) = \mu_Y(t)Y(t)dt + \sigma_Y(t)Y(t)dw(t) \quad (\text{Home}), \quad (1)$$

$$dY^*(t) = \mu_Y^*(t)Y^*(t)dt + \sigma_Y^*(t)Y^*(t)dw^*(t) \quad (\text{Foreign}), \quad (2)$$

where $\mu_Y, \mu_Y^*, \sigma_Y > 0$, and $\sigma_Y^* > 0$ are arbitrary adapted processes. Note that the country-specific Brownian motions w and w^* are independent.⁵ The prices of the Home and Foreign goods are denoted by p and p^* , respectively. We fix the world numeraire basket to contain $\alpha \in (0, 1)$ units of the Home good and $(1-\alpha)$ units of the Foreign good, and normalize the price of this basket to be equal to unity.

Investment opportunities are represented by four securities. Located in the Home country are a bond B , in zero net supply, and a risky stock S , in unit supply. Analogously, Foreign issues a bond B^* and a stock S^* . The bonds B and B^* are money market accounts instantaneously riskless in the local good, and the stocks S and S^* are claims to the local output. The terms of trade, q , are defined as the price of the Home good relative to that of the Foreign good: $q \equiv p/p^*$. The terms of

⁵It is straightforward to extend the model to the case where shocks to the countries' outputs are multi-dimensional and correlated. Our equilibrium characterization of the stock prices and the exchange rate would be the same.

trade are positively related to the real and nominal exchange rates; however, we delay making the exact identification until Section 2, where we map nominal quantities available in the data into the variables employed in the model.

A representative consumer-investor of each country is endowed at time 0 with a total supply of the stock market of his country. Thus, the initial wealth of the Home resident is $W_H(0) = S(0)$ and that of the Foreign resident is $W_F(0) = S^*(0)$. Each consumer i , $i \in \{H, F\}$, chooses nonnegative consumption of each good ($C_i(t)$, $C_i^*(t)$) and a portfolio of the available securities ($x_i^S(t)$, $x_i^{S^*}(t)$, $x_i^B(t)$, $x_i^{B^*}(t)$), where x_i^j denotes a fraction of wealth W_i invested in security j . The dynamic budget constraint of each consumer takes the standard form

$$\begin{aligned} \frac{dW_i(t)}{W_i(t)} &= x_i^B(t) \frac{dB(t)}{B(t)} + x_i^{B^*}(t) \frac{dB^*(t)}{B^*(t)} + x_i^S(t) \frac{dS(t) + p(t)Y(t)dt}{S(t)} + x_i^{S^*}(t) \frac{dS^*(t) + p^*(t)Y^*(t)dt}{S^*(t)} \\ &\quad - \frac{1}{W_i(t)} (p(t)C_i(t) + p^*(t)C_i^*(t)) dt, \quad i \in \{H, F\}, \end{aligned} \quad (3)$$

with $W_i(T) \geq 0$. Both representative consumers derive utility from the Home and Foreign goods

$$E \left[\int_0^T e^{-\rho t} \theta_H(t) [a_H \log(C_H(t)) + (1 - a_H) \log(C_H^*(t))] dt \right] \quad (\text{Home country}), \quad (4)$$

$$E \left[\int_0^T e^{-\rho t} \theta_F(t) [a_F \log(C_F(t)) + (1 - a_F) \log(C_F^*(t))] dt \right] \quad (\text{Foreign country}), \quad (5)$$

where $a_H, a_F > 0$ are the weights on the Home good in the utility function of each country and $\rho > 0$. The objective of making a_H and a_F country-specific is to capture the possible home bias in the countries' consumption baskets. This home bias may in part be due to the presence of nontradable goods, and by imposing an assumption that $a_H > a_F$, we would model it in a reduced form.⁶ Heterogeneity in consumer tastes is required for most of our implications; otherwise demand shocks would have no effect on the real exchange rate or the terms of trade.⁷ The “demand shocks,” θ_H and θ_F , are arbitrary positive adapted stochastic processes driven by \vec{w} , with $\theta_H(0) = 1$ and $\theta_F(0) = 1$. The only requirement we impose on these processes is that they be martingales. That is, $E_t[\theta_i(s)] = \theta_i(t)$, $s > t$, $i \in \{H, F\}$. This specification is very general. In the special cases we consider later in this section, we put more structure on these processes depending on the interpretation we adopt. Note that the presence of the stochastic components θ_H and θ_F

⁶We elaborate on this in Appendix B. Modeling a preference for the home good in this form is quite common in the international economics literature (see, e.g., Backus, Kehoe, and Kydland (1994b)).

⁷In our model, movements in the real exchange rate are driven solely by movements in the terms of trade—the relative price of *tradable* goods. Prior literature has argued that part of movements in the exchange rate could be due to fluctuations in the relative prices of *nontradable* goods. Our model does not explicitly incorporate this channel. However, in a recent article, Engel (1999) documents that relative prices of nontradable goods account for almost none of the movement of US real exchange rates.

in (4)–(5) does not necessarily imply that the countries’ preferences are state-dependent. As we discuss in one of the interpretations below, the countries may disagree on probability measures they use to compute expectations. Then, state-independent utilities under their own probability measures give rise to an equivalent representation (4)–(5) of the countries’ expected utilities under the true measure. Each representative consumer maximizes his expected utility subject to his budget constraint (3), taking prices as given.

2.2. Characterization of World Equilibrium

An equilibrium in our model is defined in a standard way. It is a collection of goods, stock, and bond prices (p, p^*, S, S^*, B, B^*) and associated optimal consumption-investment policies $(C_i(t), C_i^*(t), x_i^S(t), x_i^{S^*}(t), x_i^B(t), x_i^{B^*}(t))$, $i \in \{H, F\}$, such that goods, stock, and bond markets clear.

Financial markets in the economy are potentially dynamically complete since there are three independent sources of uncertainty and four securities available for investment. Since endowments are specified in terms of share portfolios, however, this is not sufficient to guarantee that markets are indeed complete in equilibrium, as demonstrated by Cass and Pavlova (2004) for a special case of our economy where θ_H and θ_F are deterministic. Nevertheless, one can still obtain a competitive equilibrium allocation by solving the world social planner’s problem because Pareto optimality is preserved even under market incompleteness.⁸ The planner chooses countries’ consumption so as to maximize the weighted sum of countries’ utilities, with weights λ_H and λ_F (reported in Appendix A), subject to the resource constraints:

$$\begin{aligned} \max_{\{C_H, C_H^*, C_F, C_F^*\}} \quad & E \left[\int_0^T e^{-\rho t} \left\{ \lambda_H \theta_H(t) [a_H \log(C_H(t)) + (1 - a_H) \log(C_H^*(t))] \right. \right. \\ & \left. \left. + \lambda_F \theta_F(t) [a_F \log(C_F(t)) + (1 - a_F) \log(C_F^*(t))] \right\} dt \right] \\ & \text{with multipliers} \\ \text{s. t.} \quad & C_H(t) + C_F(t) = Y(t), \quad \eta(t), \quad (6) \\ & C_H^*(t) + C_F^*(t) = Y^*(t), \quad \eta^*(t). \quad (7) \end{aligned}$$

⁸For a special case of our economy where θ_H and θ_F are deterministic, Cass and Pavlova prove that any equilibrium in the economy must be Pareto optimal, and thus the allocation is a solution to the planner’s problem. When θ_H and θ_F are stochastic, we verify that the allocation is Pareto optimal (Appendix A).

Solving the planner's optimization problem, we obtain the sharing rules

$$C_H(t) = \frac{\lambda_H \theta_H(t) a_H}{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F} Y(t), \quad C_H^*(t) = \frac{\lambda_H \theta_H(t) (1 - a_H)}{\lambda_H \theta_H(t) (1 - a_H) + \lambda_F \theta_F(t) (1 - a_F)} Y^*(t), \quad (8)$$

$$C_F(t) = \frac{\lambda_F \theta_F(t) a_F}{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F} Y(t), \quad C_F^*(t) = \frac{\lambda_F \theta_F(t) (1 - a_F)}{\lambda_H \theta_H(t) (1 - a_H) + \lambda_F \theta_F(t) (1 - a_F)} Y^*(t). \quad (9)$$

These consumption allocations are similar to familiar sharing rules arising in equilibrium models with logarithmic preferences. However, despite perfect risk sharing, the correlation between consumption of a particular good and its aggregate output is not perfect due to demand shocks. To identify the competitive equilibrium state prices, consider the Lagrange multipliers associated with the resource constraints. The multiplier on (6), $\eta(t, \omega)$, is the price of one unit of the Home good to be delivered at time t in state ω . Similarly, $\eta^*(t, \omega)$, the multiplier on (7), is the price of one unit of the Foreign good to be delivered at time t in state ω . We find it useful to represent these quantities as products of two components: the state price and the spot good price. The former is the Arrow-Debreu state price, denoted by $\hat{\eta}$, of one unit of the numeraire to be delivered at (t, ω) and the latter is either p (for the Home good) or p^* (for the Foreign good). The state price density—the Arrow-Debreu state price $\hat{\eta}$ per unit probability P —required for stock valuation below is denoted by ξ .⁹ The equilibrium terms of trade are then simply the ratio of $\eta(t, \omega)$ and $\eta^*(t, \omega)$, which is, of course, the same as the ratio of either country's marginal utilities of the Home and Foreign goods:

$$q(t) = \frac{\eta(t)}{\eta^*(t)} = \frac{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F}{\lambda_H \theta_H(t) (1 - a_H) + \lambda_F \theta_F(t) (1 - a_F)} \frac{Y^*(t)}{Y(t)}. \quad (10)$$

The terms of trade increase in the Foreign and decrease in the Home output. When the Home output increases, all else equal, the terms of trade deteriorate as the Home good becomes relatively less scarce. Analogously, the terms of trade improve when Foreign's output increases. This is the standard terms of trade effect that takes place in Ricardian models of trade (see Ricardo (1817) and Dornbusch, Fischer, and Samuelson (1977) for seminal contributions). So far, most of the standard asset pricing literature has ignored it by assuming a single good or multiple goods that are perfect substitutes for each other.

⁹It is worth mentioning at this point that the presence of demand shocks can potentially produce state-price densities which are relatively more volatile than consumption, even for preferences displaying low risk aversion. To see this, identify the marginal utility of country i with respect to, for example, consumption of the Home good with the state-price density times the price of the Home good: $\lambda_i e^{-\rho t} \theta_i(t) \frac{\partial}{\partial C_i} [\theta_i(t) (a_i \log(C_i(t)) + (1 - a_i) \log(C_i^*(t)))] = p(t) \xi(t)$, $i \in \{H, F\}$ (see Appendix A for details). It is typically argued that the right-hand side of this equality is substantially more volatile than the left-hand side, unless the curvature of the utility function is very high. In our model, the relative risk aversion coefficient is equal to unity. However, the left-hand side can be volatile due to volatile demand shocks. In the empirical section that follows, we indeed find that our demand shocks are substantially more volatile than the supply shocks.

The terms of trade also depend on the relative weight of the two countries in the planner's problem as well as the countries' demand shocks, but only through their ratio θ_H/θ_F . If we make an additional assumption that each country has a preference bias for the local good, then a positive relative demand shock improves Home's terms of trade: $\text{sign}(\partial q/\partial(\theta_H/\theta_F)) = \text{sign}(a_H - a_F) > 0$. The presence of this effect relates our model to the open economy macroeconomics literature. In the "dependent economy" model (see Salter (1959), Swan (1960), and Dornbusch (1980), Chapter 6), a demand shift biased toward the domestic good raises the price of the Home good relative to that of the Foreign, thus appreciating the exchange rate. In our model, if a_H is larger than a_F then the relative demand shock does indeed represent a demand shift biased toward the domestic good.

Finally, we determine stock market prices in the economy. Using the no-arbitrage valuation principle, we obtain

$$S(t) = E_t \int_t^T \frac{\xi(s)}{\xi(t)} p(s) Y(s) ds \quad \text{and} \quad S^*(t) = E_t \int_t^T \frac{\xi(s)}{\xi(t)} p^*(s) Y^*(s) ds. \quad (11)$$

Explicit evaluation of these integrals, the details of which are relegated to Appendix A, yields

$$S(t) = \frac{1 - e^{-\rho(T-t)}}{\rho} \frac{q(t)}{\alpha q(t) + 1 - \alpha} Y(t), \quad (12)$$

$$S^*(t) = \frac{1 - e^{-\rho(T-t)}}{\rho} \frac{1}{\alpha q(t) + 1 - \alpha} Y^*(t), \quad (13)$$

Consistent with insights from the asset pricing literature, each country's stock price is positively related to the national output the stock is a claim to.¹⁰ Recall that the innovations to the processes driving the Home and Foreign outputs are independent. This, however, does not imply that international stock markets are contemporaneously uncorrelated. The correlation is induced through the terms of trade present in (12)–(13). One can combine (12) and (13) to establish a simple relationship tying together the stock prices in the two countries and the prevailing terms of trade:

$$S^*(t) = \frac{1}{q(t)} \frac{Y^*(t)}{Y(t)} S(t). \quad (14)$$

The dynamics of the terms of trade and the international financial markets are jointly determined within our model. Proposition 1 characterizes these dynamics as a function of three sources of uncertainty: two country-specific shocks and the relative demand shock.

¹⁰Note that in our two-good framework, the ratio of the two stocks converges to neither zero nor infinity over time. This is a desirable feature for a study of long-run properties of our economy. Also, the limit of each individual stock's price as $T \rightarrow \infty$ is well-defined. One unrealistic implication of (12)–(13) is that stock price-dividend ratios are deterministic functions of time. This is due to the log-linear preferences.

Proposition 1. *The dynamics of the Home and Foreign stock and bond markets and the terms of trade are given by*

$$\begin{bmatrix} \frac{dS(t)}{S(t)} \\ \frac{dS^*(t)}{S^*(t)} \\ \frac{dB(t)}{B(t)} \\ \frac{dB^*(t)}{B^*(t)} \\ \frac{dq(t)}{q(t)} \end{bmatrix} = \begin{bmatrix} I_1(t) \\ I_2(t) \\ I_3(t) \\ I_4(t) \\ I_5(t) \end{bmatrix} dt + \begin{bmatrix} \frac{1-\alpha}{\alpha q(t)+1-\alpha} A(t) & \frac{\alpha q(t)}{\alpha q(t)+1-\alpha} \sigma_Y(t) & \frac{1-\alpha}{\alpha q(t)+1-\alpha} \sigma_Y^*(t) \\ -\frac{\alpha q(t)}{\alpha q(t)+1-\alpha} A(t) & \frac{\alpha q(t)}{\alpha q(t)+1-\alpha} \sigma_Y(t) & \frac{1-\alpha}{\alpha q(t)+1-\alpha} \sigma_Y^*(t) \\ \frac{1-\alpha}{\alpha q(t)+1-\alpha} A(t) & -\frac{1-\alpha}{\alpha q(t)+1-\alpha} \sigma_Y(t) & \frac{1-\alpha}{\alpha q(t)+1-\alpha} \sigma_Y^*(t) \\ -\frac{\alpha q(t)}{\alpha q(t)+1-\alpha} A(t) & \frac{\alpha q(t)}{\alpha q(t)+1-\alpha} \sigma_Y(t) & -\frac{\alpha q(t)}{\alpha q(t)+1-\alpha} \sigma_Y^*(t) \\ A(t) & -\sigma_Y(t) & \sigma_Y^*(t) \end{bmatrix} \begin{bmatrix} d\theta(t) \\ dw(t) \\ dw^*(t) \end{bmatrix} \quad (15)$$

where $\theta(t) \equiv \theta_H(t)/\theta_F(t)$, $A(t) \equiv \lambda_H \lambda_F (a_H - a_F) / [(\lambda_H \theta(t) a_H + \lambda_F a_F)(\lambda_H \theta(t)(1 - a_H) + \lambda_F(1 - a_F))]$, and $I_j(t)$, $j = 1, \dots, 5$ are reported in Appendix A. Furthermore, if $a_H > a_F$, the diffusion coefficients of the dynamics of the Home and Foreign stock markets and the terms of trade have the following signs:

Variable/ Effects of	$d\theta(t)$	$dw(t)$	$dw^*(t)$
$\frac{dS(t)}{S(t)}$	+	+	+
$\frac{dS^*(t)}{S^*(t)}$	-	+	+
$\frac{dB(t)}{B(t)}$	+	-	+
$\frac{dB^*(t)}{B^*(t)}$	-	+	-
$\frac{dq(t)}{q(t)}$	+	-	+

(16)

Proposition 1 identifies some important interconnections between the financial and real markets in the world economy. Under the home bias assumption, it entails unambiguous directions of contemporaneous responses of all markets to innovations in supply and demand, summarized in (16). Our results in (16) nest some fundamental implications from various strands of international economics, which we highlighted in our earlier discussion. Our goal is to unify them within a simple model and focus on the interactions.

One such interaction sheds light on the determinants of financial co-movement, which in our model is a natural response to an output shock in one of the countries. *Ceteris paribus*, a positive output shock say in Home causes a positive return on the domestic stock market. At the same time, however, it initiates a Ricardian response of the terms of trade: the terms of trade move against the country experiencing a productivity increase. A flip side of the deterioration of the terms of trade in Home is an improvement of those in Foreign. Hence, the value of Foreign's output has to rise, thereby providing a boost to Foreign's stock market. Note that nothing in this argument relies on the correlation between the countries' output processes. In fact, in a special case of our model in which there are no demand shocks (discussed below), we obtain a perfect co-movement of the stock markets despite independence of the countries' output innovations.¹¹ Bond markets

¹¹This observation relates our results to the literature on international financial contagion—a puzzling tendency of

certainly also react to changes in productivity: a positive output shock in Home lowers bond prices in Home and lifts those in Foreign.

While supply shocks move the countries' stock prices in the same direction, demand shocks act in the opposite way. We call this phenomenon "divergence." Thus, a demand shock say in Home causes a relative demand shift biased toward the Home good, thereby boosting Home's terms of trade. Improved terms of trade increase the value of Home's output, and hence lift Home's stock market, while decreasing that of Foreign's output and lowering Foreign's stock market. Similarly, since our bonds are real bonds, a strengthening of Home's terms of trade increases the value of the Home bond and decreases that of the Foreign. A world economy without supply shocks is thus an example of perfect divergence: asset markets of different countries always move in opposite directions.

Finally, Proposition 1 provides analytical characterization of the sensitivities of each market's responses to the supply and demand shocks, and also establishes cross-equation restrictions on how these sensitivities measure up to each other. The system of simultaneous equations describing the joint dynamics of the five markets (15) establishes a basis for our empirical analysis, carried out in Section 2.

Our model also entails implications for the dynamics of consumption, which can be obtained via an application of Itô's lemma to (8)–(9). Since our main focus is on financial markets, for brevity we report just the signs of responses of the variables to the demand and supply shocks:

Variable/ Effects of	$d\theta(t)$	$dw(t)$	$dw^*(t)$	
$\frac{dC_H(t)}{C_H(t)}$	+	+	0	
$\frac{dC_H^*(t)}{C_H^*(t)}$	+	0	+	(17)
$\frac{dC_F(t)}{C_F(t)}$	–	+	0	
$\frac{dC_F^*(t)}{C_F^*(t)}$	–	0	+	

Note that supply shocks create positive co-movement across the countries' consumption of a good, while demand shocks make them move in the opposite directions. We come back to this point

stock markets across the world to exhibit "excessive" co-movement (for a recent detailed account of this phenomenon, see Kaminsky, Reinhart, and Végh (2003)). Co-movement in stock prices is commonly defined to be excessive if it exceeds the co-movement in the underlying output processes. In our views, there is a need for a more refined definition of contagion that encompasses not only the correlation implied by the correlation of output, but also the natural propagation through prices that takes place in general equilibrium. Dumas, Harvey, and Ruiz (2003) make a similar argument.

later (Section 2.4), when we put our model in the context of the international real business cycles literature.

In this analysis we have measured stock and bond prices in the same numeraire—or, loosely speaking, in a single international currency. Similar conclusions are obtained if we define prices in terms of an equivalent domestic currency. In this model a domestic currency should reflect the price of the basket of consumption (of both local and foreign-produced goods) by nationals. It is important to keep in mind that the domestic currency is the price of the consumption basket and not the price of the production basket (local good), and therefore the terms of trade effect on the stock markets will be present regardless of whether prices are quoted in the equivalent domestic currency or the international currency. We come back to these issues in Section 2.1 where we make the mapping from the terms of trade to the real exchange rate.

2.3. Special Cases and Interpretations of θ_H and θ_F

In Proposition 1 we did not commit to specifying the dynamics of the demand shocks θ_H and θ_F . For the empirical analysis to follow, this is advantageous because the relative demand shock $\theta = \theta_H/\theta_F$, in general, depends on the underlying Brownian motions w and w^* inducing a correlation in the error structure that we do not have to impose ex-ante. In this section, we put more structure on the martingales θ_H and θ_F as we consider various economic interpretations of these processes:

$$d\theta_H(t) = \vec{\kappa}_H(t)^\top \theta_H(t) d\vec{w}(t), \quad d\theta_F(t) = \vec{\kappa}_F(t)^\top \theta_F(t) d\vec{w}(t). \quad (18)$$

A. Deterministic Preference Parameters

It is instructive to consider a special case in which θ_H and θ_F are deterministic processes, i.e., $\vec{\kappa}_H$ and $\vec{\kappa}_F$ are zeros. By substituting equation (10) into equation (14), we find that

$$S^*(t) = \frac{\lambda_H \theta_H(0) a_H + \lambda_F \theta_F(0) a_F}{\lambda_H \theta_H(0) (1 - a_H) + \lambda_F \theta_F(0) (1 - a_F)} S(t).$$

As the multiplier term is non-stochastic, the two stock markets must be perfectly correlated. Moreover, to add to the undesirable properties already mentioned in the introduction, it turns out that since there are fewer non-redundant stocks than sources of uncertainty, financial markets become incomplete. Yet, the equilibrium allocation is still Pareto optimal, because the missing markets end up being immaterial for risk sharing purposes. These implications are shared by models of Helpman and Razin (1978), Cole and Obstfeld (1991), Zapatero (1995), and Cass and Pavlova (2004). The

latter article offers extensive critique of equilibrium in such an economy, demonstrating that it violates many fundamental principles of general financial equilibrium theory. Therefore, a departure from this economic setting is important not only to match the data, but also to understand the interconnections among markets. In what follows, we force θ_H and θ_F to be stochastic processes, which would provoke the divergence effect and hence guarantee imperfect correlation between the Home and Foreign stock markets.

B. Preference Shocks or Pure Sentiment

We now consider a special case in which θ_H and θ_F are driven by the Brownian motion w^θ , independent of w and w^* . In this case, $\vec{\kappa}_H = (0, 0, \kappa_H)^\top$ and $\vec{\kappa}_F = (0, 0, \kappa_F)^\top$, where $\kappa_H \neq \kappa_F$ are arbitrary adapted processes. Then θ has dynamics

$$d\theta(t) = (\kappa_F(t)^2 - \kappa_H(t)\kappa_F(t))\theta(t)dt + (\kappa_H(t) - \kappa_F(t))\theta(t)dw^\theta(t).$$

The process θ can then be interpreted as a relative preference shock or a shock to consumer sentiment. There are other possible interpretations that are not connected to sentiment. For example, a demand shock may capture news about weather. This news is unrelated to supply news, but it does affect agents' demands (e.g., for heating oil).¹²

C. “Catching up with the Joneses” / Consumer Confidence

Consider the case where the processes θ_H and θ_F are driven by the country-specific Brownian motions w and w^* , respectively: $\vec{\kappa}_H = (\kappa_H, 0, 0)^\top$ and $\vec{\kappa}_F = (0, \kappa_F, 0)^\top$, where $\kappa_H, \kappa_F < 0$ may depend on w and w^* .¹³ Home and Foreign consumers' preferences then display “catching up with the Joneses” behavior (Abel (1990)), captured here in a very reduced form. The “benchmark” levels of Home and Foreign consumers, θ_H and θ_F , are negatively correlated with aggregate consumption of their country, Y and Y^* , respectively. Positive news to local aggregate consumption reduces satisfaction of the local consumer, as his consumption bundle becomes less attractive in an improved domestic economic environment. The implication of this interpretation for the relative demand

¹²We thank Dave Cass for this example.

¹³The labels κ_H and κ_F are used just to save on notation; they need not be the same as in interpretation **B**. Note that under this interpretation $\vec{\kappa}_H$ and $\vec{\kappa}_F$ do not depend on the Brownian motion w^θ . This specification makes w^θ a sunspot in the sense of Cass and Shell (1983): it affects neither preferences nor aggregate endowments. There is then a potential problem with employing the planner's problem in solving for equilibrium allocations because it only identifies nonsunspot equilibria. Since markets are dynamically complete in our economy, however, the sunspot immunity argument of Cass and Shell goes through.

shock, θ , is that

$$d\theta(t) = \kappa_F(t)^2 \theta(t) dt + \kappa_H(t) \theta(t) dw(t) - \kappa_F(t) \theta(t) dw^*(t), \quad \kappa_H(t), \kappa_F(t) < 0.$$

That is, an innovation to θ is negatively correlated with the Home output shock and positively correlated with the Foreign shock.

An alternative kind of dependence of agents' preferences on the country-specific output shock is a form of consumer confidence, a positive consumption "externality." The idea is that agents get more enthusiastic when their local economy is doing well and hence demand more consumption. Formally, preferences exhibiting consumer confidence are the same as those under catching up with the Joneses, except that $\kappa_H(t), \kappa_F(t) > 0$.

D. Density Processes Reflecting Heterogeneous Beliefs

The previous two special cases assume that consumer preferences are state-dependent. State-dependence of preferences, however, is not necessarily implied by our specification (4)–(5). Under the current interpretation, the countries have state-independent preferences, but they differ in their assessment of uncertainty underlying the world economy. For example, Home residents may believe that the uncertainty is driven by the (vector) Brownian motion $\vec{w}_H \equiv (w_H, w_H^*, w_H^\theta)^\top$ and Foreign residents believe that it is driven by $\vec{w}_F \equiv (w_F, w_F^*, w_F^\theta)^\top$. All we require is that the "true" probability measure P and the country-specific measures H (Home) and F (Foreign) are equivalent; that is, they all agree on the zero probability events. Under their respective measures, the agents' expected utilities are given by

$$E^i \left[\int_0^T e^{-\rho t} [a_i \log(C_i(t)) + (1 - a_i) \log(C_i^*(t))] dt \right], \quad i \in \{H, F\},$$

where E^i denotes the expectation taken under the information set of agent i . Thanks to Girsanov's theorem, the above expectations can be equivalently restated under the true probability measure in the form of (4)–(5). The multiples θ_H and θ_F , appearing in the expressions as a result of the change of measure, are the so-called density processes associated with the Radon-Nikodym derivatives of H with respect to P , $(\frac{dH}{dP})$, and F with respect to P , $(\frac{dF}{dP})$, respectively.

The densities θ_H and θ_F may reflect various economic scenarios. One is the case of incomplete information: the consumers do not observe the parameters of the output processes, and need to estimate them. While the diffusion coefficients $\sigma_Y(t)$ and $\sigma_Y^*(t)$ may be estimated by computing quadratic variations of $Y(t)$ and $Y^*(t)$, estimation of mean growth rates of output is nontrivial.

First, the consumers may be assumed to be Bayesian, endowed with some priors of the growth rates. Then they will be updating their priors each instant as new information arrives, through solving a filtering problem.¹⁴ Second, the consumers may use some updating method other than Bayesian. For example, they may be systematically overly optimistic or pessimistic about the mean growth rates of output. Finally, they may explicitly account for model uncertainty in their decision-making. All these special cases result in consumers' employing a probability measure different from the true one. These differences of opinion can be succinctly represented by some density processes $\theta_H(t)$ and $\theta_F(t)$. Under any of these interpretations, we can no longer assume that θ in Proposition 1 is uncorrelated with either w or w^* , which we have to take into account in our econometric tests.

2.4. Home Bias in Portfolios and International CAPM

We now explore the implications of our model for international portfolio compositions. One of the challenges of theoretical research in international finance has long been to explain a home bias in portfolios; it is thus interesting to investigate whether such bias can occur in our model. We compute the countries' portfolios in closed form and report them in the following proposition.

Proposition 2. (i) *Countries' portfolio positions in stocks S and S^* are given by*

$$x_H^S(t) = \frac{(1 - a_F)(\lambda_H a_H \theta(t) + \lambda_F a_F)}{\lambda_H \theta(t)(a_H - a_F)}, \quad x_H^{S^*}(t) = 1 - x_H^S(t). \quad (19)$$

$$x_F^S(t) = -\frac{(1 - a_H)(\lambda_H a_H \theta(t) + \lambda_F a_F)}{\lambda_F (a_H - a_F)}, \quad x_F^{S^*}(t) = 1 - x_F^S(t). \quad (20)$$

None of the bonds is held by either country.

(ii) *Each country's portfolio exhibits a home bias if and only if $a_H > a_F$.*

(iii) *The international CAPM has the form*

$$\begin{aligned} \frac{E_t(dS^j(t))}{S^j(t)} - r(t) &= Cov_t \left(\frac{dS^j(t)}{S^j(t)}, \frac{dW(t)}{W(t)} \right) - Cov_t \left(\frac{dS^j(t)}{S^j(t)}, \frac{\lambda_H}{\lambda_H \theta_H(t) + \lambda_F \theta_F(t)} d\theta_H(t) \right) \\ &\quad - Cov_t \left(\frac{dS^j(t)}{S^j(t)}, \frac{\lambda_F}{\lambda_H \theta_H(t) + \lambda_F \theta_F(t)} d\theta_F(t) \right), \quad S^j \in \{S, S^*\}, \end{aligned} \quad (21)$$

¹⁴There is a large literature that adopts such economic setting (see Basak (2005) for a recent survey); the only difference in our case is that there are multiple goods, and hence multiple output dynamics need to be inferred. We provide details of the ensuing filtering problems in Appendix A. We consider the case in which the agents believe there are only two independent innovation processes, w_i and w_i^* , and the case in which, additionally, there is the third one, w_i^θ . The first two innovations drive the output processes Y and Y^* , respectively. Following Detemple and Murthy (1994), we impose additional regularity conditions on $\sigma_Y(t)$ and $\sigma_Y^*(t)$ to make the filtering problem tractable: both processes are bounded and are of the form $\sigma_Y(Y(t), t)$ and $\sigma_Y^*(Y^*(t), t)$, respectively. When the agents perceive that the parameters they are estimating also depend on the third innovation process, which can represent either intrinsic of extrinsic uncertainty (a sunspot), they make use of a public signal carrying information about w_i^θ . Finally, we note that the case of incomplete information that we have in mind here is rather different from the case of asymmetric information (see Detemple (2002) for a related model which incorporates asymmetric information).

where $W \equiv W_H + W_F$ is the aggregate wealth and r is the interest rate on the locally riskless world bond, reported in Appendix A.

In our economy, the fact that the countries have logarithmic preferences does not imply that their investment behavior is myopic. Their trading strategies involve holding the standard mean-variance portfolio along with a hedging one. Although the countries do not hedge against changes in the investment opportunity set (which is standard for logarithmic preferences), they do hedge against their respective demand shocks. This explains why the portfolios of the two countries are distinctly different. The implication regarding the home bias in portfolios is immediate from the closed-form expressions (19)–(20). It turns out that in our model, a home bias in portfolios is simply a consequence of a home bias in consumption and the presence of demand shocks. Since our home bias in consumption could be understood as a reduced-form for capturing nontradable goods, this implication is consistent with the literature arguing that the presence of nontradables leads to a home bias in portfolios (see, for example, Tesar (1993)).

The presence of the hedging portfolios optimally held by the agents clearly rules out the traditional one-factor CAPM. In addition to the standard market (aggregate wealth) factor, our model identifies two further factors, the Home and Foreign demand shocks, that affect the risk premia on the stocks. The expression in (21) also differs from the international CAPM (see Solnik (1974), Adler and Dumas (1983)). As is standard for logarithmic preferences, the exchange rate (or the terms of trade) does not appear explicitly in (21); however, its determinants—the demand shocks—do. (One would expect the exchange rate to appear as an additional factor if we relaxed the assumption of log-linear preferences.) The presence of the demand shocks points to a possible misspecification of the version of the CAPM widely tested empirically. Our alternative formulation might then provide an improvement over the standard specification used in the international finance literature, which further research might explore.

3. Empirical analysis

In this section we examine the empirical implications of the model. We use monthly macro data and daily financial data for the US vis-à-vis the UK. All data are from DataStream. Unless explicitly specified otherwise, we use two-day returns in our analysis (instead of daily) so as to address the issue of nonsynchronous trading in the US and the UK. The macroeconomic variables we employ are described later in the section. The financial variables needed for our estimation are stock and

bond returns and the dollar-pound exchange rate. As proxies for our riskless bonds, we use data on the three-month zero-coupon government bonds for each country. The US and UK stock market indexes are taken to represent the countries' stock prices. The dollar-pound exchange rate is used for identifying the real exchange rates and the terms of trade via a procedure described below. For the purposes of calibrating the model, we make a strong assumption that the two countries we analyze represent the world economy. The data are from 1988 until the end of 2002. Figure 1 depicts the evolution of the exchange rate along with stock and bond markets values for the US vis-à-vis the UK. Even a casual observation of the figure suggests that the bond prices do not bear too much relationship to the exchange rate. This observation is strongly supported by a regression analysis, not reported here, whose results may be interpreted as a failure of uncovered interest rate parity, well-documented in the literature and also found in our sample. On the other hand, the amount of co-movement between the stock indexes and the exchange rate is quite striking.

3.1. Real and Nominal Quantities

Before we proceed with our formal empirical analysis, we need to establish a mapping between the quantities employed in our model and those in the data available to us—the data are in nominal terms, while our model is real. The main issue is to identify the terms of trade from the available data on nominal exchange rates. To do so, we first compute the real exchange rate implied by our model. The real exchange rate, e , is defined as $e(t) = P_H(t)/P_F(t)$, where P_H and P_F are Home and Foreign price indexes, respectively. In practice, a variety of methods have been used for computing a country's price level. We have chosen to adopt geometric average price indexes, which address the criticism pertaining to the substitution between goods bias, and are consistent with the countries' homothetic preferences:¹⁵

$$P_H(t) = \left(\frac{p(t)}{a_H}\right)^{a_H} \left(\frac{p^*(t)}{1-a_H}\right)^{1-a_H}, \quad P_F(t) = \left(\frac{p(t)}{a_F}\right)^{a_F} \left(\frac{p^*(t)}{1-a_F}\right)^{1-a_F}.$$

The real exchange rate, expressed as a function of the terms of trade, is then

$$e(t) = q(t)^{a_H-a_F} \frac{(1-a_F)^{1-a_F} a_F^{a_F}}{(1-a_H)^{1-a_H} a_H^{a_H}}.$$

Finally, to obtain the nominal exchange rate, we need to adjust the real exchange rate for the inflation differential. Unfortunately, daily data on inflation, required for our estimation, are not available for the sample under study. However, our data span a relatively short period of time when

¹⁵See Obstfeld and Rogoff (1996) for the details of this construction.

international inflation rates were very low, and very similar across the two countries. Consequently, as argued by Mussa (1979), our real rates are closely related to the nominal ones. We thus assume that inflation differentials are negligibly small and simply make a level adjustment to the real rate to back out the nominal exchange rate, ε : $\varepsilon(t) = \bar{\varepsilon}e(t)$, where $\bar{\varepsilon}$ is the average nominal exchange rate in the sample.

For our empirical investigation of the dynamics reported in Proposition 1, we use the variable

$$q(t) = \left(\frac{\varepsilon(t)}{\bar{\varepsilon}} \frac{(1 - a_H)^{1-a_H} a_H^{a_H}}{(1 - a_F)^{1-a_F} a_F^{a_F}} \right)^{1/(a_H - a_F)} \quad (22)$$

as a proxy for the terms of trade.

Taking logs and applying Itô's lemma, we obtain an equation to replace the last row of (15) in Proposition 1:

$$\frac{d\varepsilon(t)}{\varepsilon(t)} = I_6(t)dt + (a_H - a_F)A(t)d\theta(t) - (a_H - a_F)\sigma_Y(t)dw(t) + (a_H - a_F)\sigma_Y^*(t)dw^*(t),$$

where $I_6(t) = (a_H - a_F)I_5(t) + \frac{1}{2}(a_H - a_F)(a_H - a_F - 1)|\sigma_q(t)|^2$. Under our assumption of no inflation over the time period we are considering, the remaining equations in Proposition 1, (15), are unchanged when prices are expressed in nominal terms. Finally, since the nominal exchange rate, ε , is monotonically increasing with the terms of trade q , the signs of the effects of the supply and demand shocks on $\frac{d\varepsilon(t)}{\varepsilon(t)}$ are the same as those for $\frac{dq(t)}{q(t)}$ reported in (16).

3.2. Latent Factor Model and Demand Shocks

Our model implies that the dynamic behavior of asset prices and exchange rates — summarized in Proposition 1 — can be represented as a factor model. The most direct approach to testing our model would be to take these dynamics literally and estimate specification (15). However, we do not need all five asset returns series to estimate the three factors. Therefore, we only use stock and exchange rate returns to extract the factors, and then evaluate their predictive power for bond returns. Additionally, it follows from our model that our factors are linked to macroeconomic innovations (supply and demand shocks); we therefore compare them with several macroeconomic variables that are available at monthly frequency.

In particular, we estimate the following specification:

$$\begin{bmatrix} \frac{dS(t)}{S(t)} \\ \frac{dS^*(t)}{S^*(t)} \\ \frac{d\varepsilon(t)}{\varepsilon(t)} \end{bmatrix} = \vec{I} + \underbrace{\begin{bmatrix} b(t) & 1-b(t) & b(t) \\ -1+b(t) & 1-b(t) & b(t) \\ (a_H - a_F) & -(a_H - a_F) & (a_H - a_F) \end{bmatrix}}_{\mathcal{B}} \begin{bmatrix} f_\theta(t) \\ f_w(t) \\ f_{w^*}(t) \end{bmatrix}, \quad (23)$$

where \vec{I} is a (vector) intercept term and

$$\begin{bmatrix} f_\theta(t) \\ f_w(t) \\ f_{w^*}(t) \end{bmatrix} \equiv \begin{bmatrix} A(t) d\theta(t) \\ \sigma_Y(t) dw(t) \\ \sigma_Y^*(t) dw^*(t) \end{bmatrix},$$

$$b(t) \equiv \frac{1 - \alpha}{\alpha q(t) + 1 - \alpha}, \quad \text{with} \quad q(t) = \left(\frac{\varepsilon(t)}{\bar{\varepsilon}} \frac{(1 - a_H)^{1-a_H} a_H^{a_H}}{(1 - a_F)^{1-a_F} a_F^{a_F}} \right)^{1/(a_H - a_F)}.$$

In our estimation we treat the innovations f_θ , f_w , and f_{w^*} as latent factors. The first factor, f_θ , captures the relative demand shock, while the remaining two, f_w and f_{w^*} , represent Home and Foreign supply shocks, respectively.¹⁶ We first run a VAR to clean the data for serial correlation of returns and recover the residuals. We use five lags of all the returns series in all of our specifications, but the results are unaffected by increasing the number of lags. This allows an unrestricted modeling of the serial correlation of the structural shocks, while preserving the contemporaneous relationship derived in Proposition 1. Then, at every t we construct the matrix \mathcal{B} and solve for the factors given the residuals from the previous step. In estimation of most latent factor models, one usually needs to impose some restrictions on the factors. A common choice is to set the variances of the factors to be equal to one and/or assume that they are uncorrelated. The estimation then uncovers the relative weights of the factors. The model we are estimating is different in that it contains more structure. We take advantage of this structure and require that the weights on the factors are as specified in the matrix \mathcal{B} . Then we can estimate the latent factors without imposing restrictions on neither their correlation structure, nor their variances. Effectively, all we need to do is to invert the matrix \mathcal{B} to solve for the series on the right-hand side of (23).

The matrix \mathcal{B} is determined by the constant parameters α , a_H , and a_F , which we calibrate, and the terms of trade series q , the proxy for which is given by (22). The parameter α represents the weight of the US-produced goods on the world consumption basket. To calibrate it, we compute

¹⁶There is a slight abuse of terminology in this section; earlier we referred to $d\theta$, dw , and dw^* as the demand and supply shocks.

the ratio of the real GDP of the US in dollars and the sum of the US and UK's real GDP's in dollars. We set α equal to the average of this ratio for 1988–2002. The ratio fluctuates between 85.9 and 87.5 percent with the average of 86.7 percent. The parameters a_H and $1 - a_H$ in our model are equal to the shares of domestic and foreign goods in the total consumption expenditure of Home (this identification is standard for log-linear preferences). To measure the share of foreign goods in the US expenditure, we compute the ratio of the country's imports of goods and services and the GDP minus exports of goods and services. The latter quantity is a measure of the imports as a share of domestic aggregate demand. The US is a relatively close economy. During 1988-2002, imports share of GDP ranges from 11.6 to 15.1 percent with the average of 13.2, while the exports share fluctuates from 9.7 to 11.6 with the average of 10.8. The share of imports in aggregate demand, therefore, moves from 12.9 to 17.0 percent with the average of 14.7. Hence, we set $1 - a_H$ to equal to 14.7. The UK is a more open economy, and its imports share moves from 27.2 to 30.1 with the average of 28.9, while exports fluctuate between 26.2 and 29.3 with the average of 27.5 in our sample. The imports share in the UK's aggregate demand computed year by year fluctuates between 37.0 and 42.2 with the average of 39.9. We calibrate, consequently, the Foreign's imports share in its total consumption expenditure, a_F , to be 39.9. We are now ready to extract our factors.

f_θ	f_w	f_{w^*}
13.309	9.087	13.932

(a) Standard deviations

f_θ	f_w	f_{w^*}
1	0.367	-0.359
	1	0.321
		1

(b) Correlation matrix

Table 1: Estimates of the model parameters: the standard deviations and correlations of factors. The model is specified in (23). f_θ is the factor associated with the relative demand shock, and f_w and f_{w^*} with Home (US) and Foreign (UK) supply shocks, respectively. All estimates are annualized and are specified in percentage terms. All estimates are significant at the 5% level.

The six estimates reported in panel (a) of Table 1 summarize the covariance matrix of the three latent factors. Observe that the demand shocks are very important both in magnitude and significance, rejecting our interpretation **A**, in which we assume that only the supply shocks drive the economy. The variance of the demand shocks is twice as large as that of the supply shocks in the US. These results are robust to estimating the model using weekly instead of two-day returns, and also robust to including the bond returns to estimate the factors.¹⁷

¹⁷Our conclusions are in line with the macroeconomics literature. For example, Clarida and Gali (1994) also find

Panel (b) reveals that the estimates of the correlations constructed from the covariance matrix indicate that the relative demand shock is positively correlated with the US’s supply shock, and negatively correlated with the UK’s. The estimates are statistically significant, strongly rejecting the pure sentiment interpretation **B**. Furthermore, the catching up with the Joneses interpretation **C**, under which the the relative demand shock should negatively co-vary with the US supply shock and positively with the UK’s, is rejected, too. The story the data is telling is the reverse: instead of becoming relatively unhappy when their country’s aggregate consumption increases, agents appear to get enthusiastic when their economy is doing well. This speaks in favor of our consumer confidence interpretation **C**. Equally likely is the differences in opinion interpretation **D**. The correlation between the US and UK supply shocks, which is zero in our model, turns out to be positive in the data. When discussing the sign implications derived in Proposition 1, we were examining each shock in isolation. In light of the empirical finding that the supply and demand shocks are correlated, it follows that the overall effect of one shock has to include the contemporaneous effects on the others.

3.3. Explaining Macroeconomic Variables

The model we have developed implies that the latent factors extracted from asset prices have an economic interpretation, and therefore, we should expect that they have *predictive power* for several macroeconomic variables. Furthermore, our model has *sign implications* on how the factors affect production, employment, certain expectations, and external variables such as exports. We evaluate the performance of the factors along these two dimensions. Given the span of our data, we concentrate on the macro-variables available at monthly frequency. From DataStream we obtain industrial production of the US and the UK, the trade balance, consumer confidence and business trend measures (from the surveys). The survey variables require further description. The consumer confidence variables include the standard one that most market participants concentrate on, plus two of its components: the consumer confidence on the current situation and consumer expectations. The business trend surveys ask business managers about the prospects of exports, employment, and production.

Before discussing our results it is important to mention that there are several papers that have attempted to estimate the macroeconomic content embedded in financial markets fluctuations,

 that demand shocks are large relative to supply shocks using a very different identification strategy and low frequency data.

finding mixed evidence (see, for example, Campbell and Clarida (1987) and Stock and Watson (2003)). In general, the coefficients are unstable, changing their signs and significance depending on the period and country under analysis. Given our theory, this result should have been expected. If the relative importance of the factors changes through the sample, the reduced-form coefficients in the regression of changes in macro-variables on asset returns have to change sign and significance as well.

Consider a simple example suggested by our model. Suppose one were examining a relationship between the exchange rate and domestic stock returns. In our model, the underlying uncertainty consists of demand and supply shocks that both increase domestic stock prices, but have different effects on the exchange rate. Therefore, in periods when the supply shocks dominate the sample, the correlation between exchange rate and stock returns has the opposite sign compared to the periods when demand shocks are relatively more important. In Figure 2(a), we present simple 20-day rolling variances of the factors. The figure demonstrates the relative importance of the shocks varies considerably through the sample. Demand shocks are relatively more important in the first half of the sample, while relatively less important in the second half. Figure 2(b) plots the 20-day rolling correlations between the US-UK exchange rate and US and UK stock returns. The correlations fluctuate significantly in our sample and frequently change their signs. This means that estimating an OLS regression of the exchange rate on the stock returns will imply unstable parameters, even if the structural coefficients are stable. The key difference of our paper is that we impose the structure of our model to attempt to disentangle the underlying innovations, which results in a much more stable regression.

To determine the predictive content of our factors, we first convert the factors from two-day to monthly frequency by accumulating the two-day changes, and then estimate the following simple regression for every macro-variable M :

$$dM(t) = a_M + \sum_{i=\{1,L\}} b_{M,i} f_{\theta}(t-i) + \sum_{i=\{1,L\}} c_{M,i} f_w(t-i) + \sum_{i=\{1,L\}} d_{M,i} f_{w^*}(t-i) + u_M(t), \quad (24)$$

where $u_M(t)$ is the error term. We allow for six lags of the factors to capture slow responses of the macro-variables. The objective is to evaluate if the factors derived from the estimation can predict the macroeconomic variables six months ahead. Notice that in this specification we are predicting changes in the macroeconomic variables with changes in the factors, and we are doing so out of sample—meaning that we have not used the macroeconomic series we are attempting to forecast in the estimation of our model. The results are presented in Table 2.

Dependent variable M	R^2	f_θ	f_w	f_{w^*}
Industrial Production (US)	28.8%	-0.0009 (9.0%)	0.0131 (0.0%)	0.0003 (17.4 %)
Industrial Production (UK)	12.0%	-0.0006 (34.0%)	0.0017 (6.9%)	-0.0004 (27.1%)
Consumer Confidence Total (US)	24.9%	0.0113 (1.9%)	0.0095 (31.3%)	0.0124 (1.0%)
Consumer Confidence Current (US)	17.7%	0.0862 (21.4%)	0.0060 (45.8%)	0.0163 (0.1%)
Consumer Confidence Expectations (US)	24.8%	0.0155 (0.8%)	0.0061 (50.3%)	0.0137 (1.4%)
Business Trend - Exports (US)	25.8%	-0.0411 (0.0%)	0.1496 (0.0%)	0.0065 (14.2%)
Business Trend - Employment (US)	19.6%	-0.0236 (0.0%)	0.0632 (0.0%)	-0.0032 (36.4%)
Business Trend - Production (US)	16.5%	-0.0135 (9.1%)	0.0507 (0.8%)	0.0183 (1.1%)
Bond Prices (US)	14.1%	0.0141 (24.6%)	0.0443 (17.2%)	0.0299 (6.6%)
Bond Prices (UK)	21.8%	0.0696 (11.2%)	0.0969 (14.3%)	-0.0646 (1.9%)

Table 2: Regressions of macro-variables on the factors. The first column reports the R^2 of the regressions specified in (24). The last three columns report the maximum of an impulse response to a one standard deviation shock to each of the factors. The p-values for the significance of the impulse responses are in the parentheses. f_θ is the factor associated with relative demand shock, and f_w and f_{w^*} with Home (US) and Foreign (UK) supply shocks, respectively.

The first column reports the explained variance of a macro-variable from the specification (24). It is clear that the latent factors we are collecting from daily changes in stock prices and the exchange rate have significant explanatory power for the macro variables. For example, as evident from the second column, our factors explain almost 29 percent of the fluctuations of the US industrial production and 12 percent of the fluctuations of the UK production. Our factors explain 25 percent of consumer confidence, almost 18 percent of the consumer confidence on the current situation, and more than 25 percent of the consumer expectations indexes. They are also able to explain nearly 26 percent of the variation in the perceptions about exports, and close to 20 percent of the other business trend survey indexes.

The last three columns in Table 2 report the magnitudes of the six month impulse responses to a (permanent) shock to each of the factors, and their significance. Through this procedure we

can evaluate both the sign implications and the relevance of the factors at the same time.¹⁸ Our theory predicts that the demand factor f_θ should be positively correlated with consumer confidence measures, and those signs are correct: Consumer Confidence Total, Current and Expectations, all have positive impulse responses to a shock in f_θ . Two of these responses are also highly statistically significant (the p-values are below 2%). The domestic supply shocks f_ω are positively associated with the US Industrial Production, and the impulse response is highly statistically significant. However, not all of the estimated signs are consistent with our model. The impulse response of UK's industrial production to the foreign supply shock f_{ω^*} is virtually zero, and the p-values associated with responses of bond prices (all but one) are very weak.

In our model, some signs of impulse responses are either ambiguous or zero. For instance, the theory has no predictions on the effect of f_θ on output and we obtain a negative coefficient. This negative effect is also present in the case of Business Trend - Employment and Production. So, even though the theory is mute regarding the sign of this relationship, the results are consistent across the different measures of output. Another example is the response of exports to a demand shock. In our model, the volume of Home exports decreases, but the price of exports increases — on balance, the direction of the effect is ambiguous. Our estimation suggests that the former effect dominates: the response of Business Trend - Exports to a demand shock is negative and highly significant. As a robustness check, we have run the regressions in (24) including several lags of the dependent variables. The R^2 have gone up, but the overall message regarding the signs and the significance of the impulse responses remained the same.¹⁹

We have also examined stability of our impulse responses across subsamples of our data. In that respect, our factors perform significantly better in the forecasting regressions than raw financial market fluctuations.²⁰ Parameter instability is lower and the forecasts of macro-variables are much less noisy, indicating that our procedure helped to disentangle the confounding influences of demand and supply shocks embedded in financial market fluctuations. Of course, the latent factor method is not the only possible approach to estimating our model. While overall the results are encouraging and we find support for our theoretical implications, more work attempting to isolate the underlying primitive shocks and examine their effects on asset prices and exchange rates is clearly needed.

¹⁸These responses were computed assuming that the shock to each factor is idiosyncratic. That is, in our computation of the responses we did not take into account the contemporaneous correlation across factors.

¹⁹We have also performed tests evaluating the economic significance of introducing the factors into the regression after having controlled for the lagged dependent variables. Except for business surveys, the factors contribute considerably to the explanatory power of the regression.

²⁰The details of the analysis are available from our websites.

3.4. Relation to Real Business Cycles Literature Puzzles

Our model is related to the international real business cycles (IRBC) literature in that we also consider international transmission of shocks in a dynamic general equilibrium framework. Our model does not include many realistic features highlighted in this literature, such as capital accumulation, time to build, labor choice, etc. However, our approach does not rely on numerical analysis, providing an opportunity to derive several qualitative implications. Furthermore, it allows us to investigate the dynamics of stock markets, rarely considered in the IRBC literature. The primary focus of the IRBC literature is on the ability of theoretical models to produce realistic business cycle properties. The literature has been successful in closely matching many moments observed in the data, but several puzzles remain. We explore the ability of our model to shed light on some properties of the data. In particular, we focus on two major puzzles, as highlighted by Backus, Kehoe, and Kydland (1994a): the cross-country consumption correlation puzzle and the terms-of-trade variability puzzle. Then we consider the correlations of the returns on the stock markets. To be consistent with the real business cycle literature, we take logarithms of all processes we focus on, take first differences, and remove the drift terms. Thus, we effectively take out the time-varying mean growth rates of the processes—we can separate these (predictable) components perfectly within the model (as we do in, e.g., Proposition 1)—and compute the moments implied by the unpredictable components of the processes. As in the real business cycles literature, we do not intend to explain growth.

The consumption correlation puzzle refers to the tendency of IRBC models to predict excessively high correlations of consumption across countries, much higher than the correlations of output. This result is driven by the agents' desire to share risk. In the data, however, we see the reverse. To understand this puzzle within our model, consider the consumption sharing rules (8)–(9). In the absence of demand shocks, the correlation of (log) consumption of each good across countries is indeed excessively high — it is equal to one even when the countries' outputs are not correlated. This implication of pure-exchange models is well-known and has motivated a switch to more complex models with capital accumulation (see, e.g., Backus, Kehoe, and Kydland (1992)) and incomplete markets (Kollmann (1996)). In our case, we introduce demand shocks. This implies that the correlation of each good's consumption is no longer perfect, as can be easily seen from (8)–(9). This simple intuition is useful for understanding the correlations of aggregate consumption across countries. To construct a measure of aggregate consumption, we combine consumption of the two

goods in each country into consumption indexes and take logarithms. The resulting expressions are $a_H \log C_H(t) + (1 - a_H) \log C_H^*(t)$ for Home and $a_F \log C_F(t) + (1 - a_F) \log C_F^*(t)$ for Foreign. First, consider the case where there is no preference bias towards the home good ($a_H = a_F$) — the specification most frequently examined in the IRBC literature. One can easily see that the countries consumption indexes are perfectly correlated when there are no demand shocks. Hence, demand shocks always decrease the consumption correlation. In contrast, for the case where $a_H \neq a_F$, the heterogeneity in preferences alone reduces the cross-country correlation of consumption because the countries' consumption indexes are no longer of the same form. As is apparent from (8)–(9), demand shocks in this case may increase or decrease the consumption correlation, depending on the covariance between the demand and supply shocks.

We next assess quantitatively the size and the direction of the effects of the demand shocks in our model. Using our estimated latent factors, we construct the implied consumption and output series, separate the unpredictable components as indicated above, and compute their correlations. The results are reported in Table 3, which reveals that the cross-country consumption correlation drops in the presence of the demand shocks and this drop is quite significant. This conclusion is in agreement with Stockman and Tesar (1995) who advocate including taste shocks in the standard real business cycles formulation. However, the consumption correlation is still higher than the output correlation, so the puzzle is not fully resolved. To evaluate the robustness of the results we report the sensitivity of the correlation to different assumptions on a_H and a_F . Note that it is always the case that the correlation of consumption comes down when the demand shocks are introduced.

The terms-of-trade variability puzzle refers to the inability of IRBC models to generate realistic variability of the terms of trade. Absent demand shocks, our framework is very similar to the pure-exchange setting of Backus, Kehoe, and Kydland (1994b), and hence all shortcomings related to the behavior of the terms of trade discussed in that work also apply to our economy. In contrast, once demand shocks are introduced into the model, the terms of trade are no longer fully determined by the output ratio Y^*/Y , and there is an additional source of variability coming from the variability of the demand shocks. As can be easily deduced from our closed-form expression for the terms of trade (10), however, the ultimate effect of the demand shocks on the variability of terms of trade depends not only on the the variability of the demand shocks but also on the covariance between the relative demand shock and the output ratio ($Cov_t(d\theta(t), d(Y^*(t)/Y(t)))$). If this

Preference parameter a_H	0.853	0.853	0.853	0.753	0.953
Preference parameter a_F	0.298	0.398	0.498	0.398	0.398
<i>Without Demand Shocks</i>					
StDev(Terms of Trade) / StDev(Output)	1.158	1.215	1.296	1.313	1.147
Corr(Home Consumption, Foreign Consumption)	0.736	0.782	0.825	0.848	0.722
Corr(Home Output, Foreign Output)	0.366	0.321	0.260	0.225	0.383
Ratio of Consumption and Output Correlations	2.013	2.435	3.176	3.772	1.884
Stock Returns Correlations	1	1	1	1	1
<i>With Demand Shocks</i>					
StDev(Terms of Trade) / StDev(Output)	0.928	1.066	1.242	1.259	0.920
Corr(Home Consumption, Foreign Consumption)	0.636	0.678	0.708	0.617	0.702
Corr(Home Output, Foreign Output)	0.366	0.321	0.260	0.225	0.383
Ratio of Consumption and Output Correlations	1.741	2.112	2.726	2.742	1.831
Stock Markets Correlations	0.345	0.282	0.191	0.174	0.353

Table 3: Terms of trade variability and cross-country consumption correlation. The variance-covariance matrix of shocks is as estimated in Table 1. All series are in logarithms, in differences, and detrended by removing the drift terms entailed by the model. StDev(Output) is the average of the standard deviations of each country’s output.

covariance is zero or positive—e.g., under our pure sentiment and catching up with the Joneses interpretations—demand shocks increase the variability of the terms of trade. If, however, the covariance is negative—e.g., under the consumer confidence interpretation—the terms of trade variability may increase or decrease. The latter occurs for the estimated demand shocks in our sample. Therefore, as seen in Table 4, the variance of the terms of trade is lower than that in the data (on average, around 2.9, as reported by Backus, Kehoe, and Kydland (1994a)). Stockman and Tesar argue that demand shocks greatly improve the conformity of the IRBC models with the data by boosting the variability of the terms of trade. Our analysis shows that this result needs to be qualified. In Stockman and Tesar, demand shocks are pure sentiment shocks, and this will indeed increase the variability of the terms of trade. However, recall that in Section 3 we reject the pure sentiment specification. So, while demand shocks are a powerful device in bringing theoretical models in conformity with the data, more work identifying the structure of these shocks is needed.

It is of interest to highlight that while the moments delivered by our model are roughly consistent with those obtained in the real business cycles literature, our approach to calibration is quite different. Instead of using quarterly data on the real economy, we use high-frequency financial data. This approach might provide a useful alternative, especially for calibrating demand shocks,

which are far more difficult to identify than better-understood supply or technology shocks.

Finally, we explore the implications of our model for the correlation of the stock returns across countries.²¹ In the model with no demand shocks, this correlation is perfect. However, the demand shocks are able to reduce it substantially. In our simulation, the correlation is between 17 and 35%, depending on the parameters, which is in line with the number obtained in our sample after we control for the lagged dependent variables (by running a VAR as the first step of our analysis in Section 3).

In summary, our simple pure-exchange model offers tractability useful for understanding IRBC moments and produces implications consistent with several features of the data.²² Our model certainly lacks many realistic features that we have left out. We have learned, however, that enhancing the basic international real business cycles models with demand shocks may help to address some of their well-documented shortcomings, and that asset prices offer a vehicle through which those demand shocks can be estimated.

4. Caveats, Robustness, and Future Research

Our framework is certainly very stylized and several assumptions require further exploration. First, our model is real; both stocks and bonds are claims to streams of goods. While perhaps this is a reasonable way to model stocks, in real life, bonds are IOU's specified in nominal terms. A natural next step is to introduce money formally into the model, which would allow for a distinction between real and nominal assets, and derive parallel implications for nominal asset prices and exchange rates. We believe that the main mechanism will survive this extension—due to trade in goods and in asset markets, real exchange rates will continue to be determined by the same factors that drive real stock market returns.

Second, we have assumed that all goods are traded, while the literature on PPP and the real exchange rate determination has demonstrated the importance of the nontradable goods sector. Appendix B offers an extension of our model that explicitly incorporates nontradables. However, the appendix leaves out a potentially interesting investigation of the differing implications of productivity shocks to the tradable versus nontradable sectors on the stock prices of these sectors and

²¹See Aydemir (2005) for a related analysis in the context of a richer, multi-industry, model.

²²Of course, since ours is a pure-exchange model, the menu of comparisons we can draw with the real business cycles literature is somewhat limited. We have nothing to say about other interesting moments; for example, those associated with physical investment.

the exchange rate.

Third, we have employed a log-linear utility in our model. This specification has an advantage of allowing us to compute stock prices in closed form. As a result, we obtain a parsimonious structural model with only three latent factors, making the problem of its estimation manageable. We are aware of few models in the asset pricing literature capable of producing closed-form expressions for stock prices in an economy with multiple risky stocks. An apparent drawback of the log-linear specification is, of course, the constant price-dividend ratio it induces. It would be desirable to examine alternative preferences specifications. This extension is likely to entail the cost of imposing further assumptions on the parameters of the countries' output processes.

We have attempted to relate the supply and demand shocks, treated as latent factors in our estimation, to actual output innovations in the data by moving to lower frequencies at which macroeconomic data are available. A different approach would be to use survey data and macroeconomic announcements as proxies for productivity and demand innovations, and study the direct effects of these innovations on asset prices and exchange rates along the lines of Andersen, Bollerslev, Diebold, and Vega (2003), Rigobon and Sack (2003), and Rigobon and Sack (2004). Also left for future research are tests of our model's implications for the CAPM. Finally, a worthy extension of our model, in our opinion, would be to introduce market segmentation and various constraints on the international asset cross-holdings.²³ These considerations have been shown to be important in explaining the short-run behavior of exchange rates.

To provide an alternative test of our model, we moved to monthly data, and included the dynamics of industrial production (1)–(2) into the estimation of our latent factors. The innovations to industrial production are a direct measure of the supply shocks. To compute the factors we estimated the following specification:

$$\begin{bmatrix} \frac{dS(t)}{S(t)} \\ \frac{dS^*(t)}{S^*(t)} \\ \frac{dB(t)}{B(t)} \\ \frac{dB^*(t)}{B^*(t)} \\ \frac{d\varepsilon(t)}{\varepsilon(t)} \\ \frac{dY(t)}{Y(t)} \\ \frac{dY^*(t)}{Y^*(t)} \end{bmatrix} = \vec{I}_1 + \begin{bmatrix} b(t) & 1-b(t) & b(t) \\ -1+b(t) & 1-b(t) & b(t) \\ b(t) & -b(t) & b(t) \\ -1+b(t) & 1-b(t) & -1+b(t) \\ (a_H - a_F) & -(a_H - a_F) & (a_H - a_F) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_\theta(t) \\ f_w(t) \\ f_{w^*}(t) \end{bmatrix},$$

²³Our setting can accommodate an inclusion of exogenously-imposed restrictions on international asset cross-holdings without compromising the model's tractability (Pavlova and Rigobon (2005)).

We constructed the matrix and estimated the factors using the same procedure as in Section 3, but at monthly frequency. In this model, all the implications derived from the daily data estimation are confirmed. Most notably, demand shocks remain twice as important as the US supply shocks, and the correlation between the demand and supply shocks is consistent with the consumer confidence interpretation.

To confirm the validity of our empirical results, we performed several robustness checks and obtained very similar conclusions. (i) We estimated the model using weekly returns instead of two-day. (ii) In our predictive regressions, we used quarterly GDP instead of industrial production and got qualitatively the same results. (iii) We also estimated the latent factors including additionally bond returns, and the same patterns held. Finally, we estimated the model using alternative econometric methods. In particular, we tested the sign implications of the model using “identification through heteroskedasticity”. Several of these robustness checks are detailed in the NBER working paper version of this manuscript.

5. Conclusions

The introduction of trade in goods and demand shocks into an otherwise standard international asset pricing model has allowed us to deepen our understanding of the channels behind international transmission of shocks. We highlight two main channels: the worldwide financial state prices (emphasized by the asset pricing literature) and the terms of trade (stressed by the IRBC and the open economy macroeconomics literatures). We show that these channels interact with each other. We are able to fully characterize the transmission mechanism and the ensuing dynamic behavior of asset prices and exchange rates.

Our theoretical contribution is to develop a tractable two-country two-good model in which the stock market, bond prices, and exchange rates are all governed by the same set of factors. This renders a new perspective on both asset prices and exchange rates. For the former, we show that aspects of international economics—such as fluctuations in the terms of trade—have strong influence on their behavior. For the later, we show that the same factors that move stock and bond prices also affect the exchange rates. We take the model to the data and find support for it. First, we find that demand shocks are crucial for explaining the dynamics of asset prices. Second, we estimate the latent factors implied by the model and show that they have predictive power for explaining pertinent macroeconomic variables, excluded from the estimation. Finally, we examine

the extent to which the estimated latent factors help in alleviating two of the most important puzzles raised by the IRBC literature. Although we do not fully resolve the puzzles, we demonstrate that the introduction of demand shocks into an asset pricing framework helps ameliorate them.

While our model is largely supported by the data, and its implications are consistent with different strands of the literature, we are aware of its limitations. We have made several simplifying, and sometimes unrealistic, assumptions that future research should relax. We feel that extending our stylized framework may lead to further fruitful insights.

Appendix A: Proofs

We first identify the state-price density (or, more precisely, the state-price deflator), ξ , that prevails in a competitive equilibrium of our economy.²⁴ Given dynamic market completeness (which we verify later in this appendix), the state-price density is unique in our model and can be derived from

$$\lambda_H e^{-\rho t} \frac{\partial u_H(C_H(t), C_H^*(t))}{\partial C_H(t)} = \lambda_F e^{-\rho t} \frac{\partial u_F(C_F(t), C_F^*(t))}{\partial C_F(t)} = p(t) \xi(t), \quad (\text{A.1})$$

$$\lambda_H e^{-\rho t} \frac{\partial u_H(C_H(t), C_H^*(t))}{\partial C_H^*(t)} = \lambda_F e^{-\rho t} \frac{\partial u_F(C_F(t), C_F^*(t))}{\partial C_F^*(t)} = p^*(t) \xi(t), \quad (\text{A.2})$$

where $u_i(C(t), C^*(t)) \equiv \theta_i(t) [a_i \log(C_i(t)) + (1 - a_i) \log(C_i^*(t))]$, $i \in \{H, F\}$ and where the consumption allocations are the ones resulting from the planner's optimization. In the familiar single-good asset-pricing framework, the consumption good's price is normalized to one and the agents' marginal utilities are proportional to the state-price density. Here, we make use of a straightforward generalization of the standard argument to multiple-good economies. Equations (A.1)–(A.2) together with the sharing rules (8)–(9) yield the following convenient representations:

$$\xi(t) = e^{-\rho t} \frac{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F}{p(t) Y(t)} = e^{-\rho t} \frac{\lambda_H \theta_H(t) (1 - a_H) + \lambda_F \theta_F(t) (1 - a_F)}{p^*(t) Y^*(t)}. \quad (\text{A.3})$$

Substituting out consumption goods prices, we obtain

$$\begin{aligned} \xi(t) &= \alpha \xi(t) p(t) + (1 - \alpha) \xi(t) p^*(t) \\ &= e^{-\rho t} \left[\alpha \frac{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F}{Y(t)} + (1 - \alpha) \frac{\lambda_H \theta_H(t) (1 - a_H) + \lambda_F \theta_F(t) (1 - a_F)}{Y^*(t)} \right], \end{aligned} \quad (\text{A.4})$$

where the first equality follows from our price normalization and the second from (A.3).

Derivation of Stock Prices. The price of the Home stock is

$$S(t) = E_t \left[\int_t^T \frac{1}{\xi(s)} \xi(s) p(s) Y(s) ds \right] = \frac{1 - e^{-\rho(T-t)}}{\rho} \frac{q(t)}{\alpha q(t) + 1 - \alpha} Y(t),$$

where the last equality makes use of (A.3) and the fact that θ_H and θ_F are martingales. The Foreign stock price is determined through an analogous procedure.

Weights in the Planner's Problem. To conform with the competitive equilibrium allocation, the weights λ_H and λ_F in the planner's problem are chosen to reflect the countries' initial endowments. In particular, they are identified with the reciprocals of Lagrange multipliers associated with each country's Arrow-Debreu (static) budget constraint. Since in equilibrium these multipliers,

²⁴The definition of the state-price density in our model is $\xi(t)/\xi(0)$, but we will often loosely refer to $\xi(t)$ as the state-price density. The quantity $\xi(t)/\xi(0)$ is, of course, equal to the respective Arrow-Debreu state prices $\hat{\eta}(t)/\hat{\eta}(0)$ per unit probability P .

and hence the weights, cannot be individually determined, we adopt a normalization $a_H\theta_H(0)\lambda_H + a_F\theta_F(0)\lambda_F = 1$. To pin down λ_H , note that $W_H(0) = \frac{1}{\xi(0)}E \left[\int_0^T \xi(t)[p(s)C_H(t) + p^*(s)C_H^*(t)]dt \right] = \frac{1-e^{-\rho T}}{\rho} \frac{1}{\xi(0)}\lambda_H\theta_H(0)$, where we used (8)–(9) and (A.3). On the other hand, $W_H(0) = S(0)$. This together with (12) yields $\lambda_H = 1/\theta_H(0)$. Consequently, $\lambda_F = (1 - a_H)/(a_F\theta_F(0))$.

Proof of Proposition 1. Equation (10) can be equivalently restated as

$$\log q(t) = \log \left(\frac{\lambda_H\theta(t)a_H + \lambda_F a_F}{\lambda_H\theta(t)(1 - a_H) + \lambda_F(1 - a_F)} \right) + \log Y^*(t) - \log Y(t).$$

Applying Itô's lemma to both sides and simplifying, we obtain

$$\begin{aligned} \frac{dq(t)}{q(t)} &= \text{Itô terms } dt + \frac{\lambda_H a_H}{\lambda_H\theta(t)a_H + \lambda_F a_F} d\theta(t) - \frac{\lambda_H(1 - a_H)}{\lambda_H\theta(t)(1 - a_H) + \lambda_F(1 - a_F)} d\theta(t) \\ &\quad + \frac{dY^*(t)}{Y^*(t)} - \frac{dY(t)}{Y(t)} \\ &= \text{Itô terms } dt + A(t)d\theta(t) - \frac{dY(t)}{Y(t)} + \frac{dY^*(t)}{Y^*(t)}. \end{aligned}$$

Equations (12)–(13) are equivalent to

$$\log S(t) = \log q(t) - \log(\alpha q(t) + 1 - \alpha) + \log Y(t) + \log \left(\frac{1 - e^{-\rho(T-t)}}{\rho} \right), \quad (\text{A.5})$$

$$\log S^*(t) = -\log(\alpha q(t) + 1 - \alpha) + \log Y^*(t) + \log \left(\frac{1 - e^{-\rho(T-t)}}{\rho} \right), \quad (\text{A.6})$$

respectively. Applying Itô's lemma to both sides of (A.5) and (A.6), we have

$$\begin{aligned} \frac{dS(t)}{S(t)} &= \text{Itô terms } dt + \left(1 - \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha} \right) \frac{dq(t)}{q(t)} + \frac{dY(t)}{Y(t)} \\ &= \text{Itô terms } dt + \frac{1 - \alpha}{\alpha q(t) + 1 - \alpha} A(t) d\theta(t) + \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha} \frac{dY(t)}{Y(t)} + \frac{1 - \alpha}{\alpha q(t) + 1 - \alpha} \frac{dY^*(t)}{Y^*(t)} \\ \frac{dS^*(t)}{S^*(t)} &= \text{Itô terms } dt - \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha} \frac{dq(t)}{q(t)} + \frac{dY^*(t)}{Y^*(t)} \\ &= \text{Itô terms } dt - \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha} A(t) d\theta(t) + \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha} \frac{dY(t)}{Y(t)} + \frac{1 - \alpha}{\alpha q(t) + 1 - \alpha} \frac{dY^*(t)}{Y^*(t)} \end{aligned}$$

Substituting the dynamics of Y and Y^* from (1)–(2), we arrive at the expressions in Proposition 1. The drift terms (mean growth rates) appearing in the above equations enter nowhere in our estimation procedure. Computation of these terms is straightforward but tedious, so in the interest of space we report just the end result: $I_5(t) = -\mu_Y(t) + \mu_Y^*(t) + \sigma_Y(t)^2 + \sigma_Y^*(t)^2 + A(t)\sigma_Y^*(t) \frac{d[\theta(t), w^*(t)]}{dt} - A(t)\sigma_Y(t) \frac{d[\theta(t), w(t)]}{dt}$, $I_1(t) = \mu_Y(t) + \frac{1-\alpha}{\alpha q(t)+1-\alpha} \left(I_5(t) + \sigma_Y(t)A(t) \frac{d[\theta(t), w(t)]}{dt} - \sigma_Y(t)^2 \right) - \frac{\rho e^{-\rho(T-t)}}{1-e^{-\rho(T-t)}}$ and $I_2(t) = \mu_Y^*(t) - \frac{\alpha q(t)}{\alpha q(t)+1-\alpha} \left(I_5(t) + \sigma_Y^*(t)A(t) \frac{d[\theta(t), w^*(t)]}{dt} + \sigma_Y^*(t)^2 \right) - \frac{\rho e^{-\rho(T-t)}}{1-e^{-\rho(T-t)}}$, where $[\theta(t), w(t)]$

and $[\theta(t), w^*(t)]$ are quadratic covariations of $\theta(t)$ with $w(t)$ and $w^*(t)$, respectively. Further simplification of the quadratic covariation terms requires committing to an interpretation of θ (see Section 2.3).

Home and Foreign bonds are riskless in terms of the local good. That is,

$$dB^i(t) = r^i(t)B^i(t)dt, \quad i \in \{H, F\},$$

where B^i is in units of good i . Here, r^i is the local money market rate, the exact form of which need not concern us in this proposition (the rates r^H and r^F differ across our interpretations of the demand shocks; we report them in the NBER working paper version of this manuscript. Converted into the common numeraire,

$$B(t) = p(t)B^H(t) = \frac{q(t)}{\alpha q(t) + 1 - \alpha} B^H(t), \quad B^*(t) = p^*(t)B^F(t) = \frac{1}{\alpha q(t) + 1 - \alpha} B^F(t).$$

Taking logs and then applying Itô's lemma leads to the required expressions. Again, we report the mean growth terms without providing the details of the computations: $I_3(t) = r^H(t) + \frac{1-\alpha}{\alpha q(t)+1-\alpha} I_5(t)$ and $I_4(t) = r^F(t) - \frac{\alpha q(t)}{\alpha q(t)+1-\alpha} I_5(t)$.

The sign implications follow from observing that $\text{sign}(A(t)) = \text{sign}(a_H - a_F) > 0$, $0 \leq \alpha \leq 1$ and $q(t), \sigma_Y(t), \sigma_Y^*(t) > 0$. *Q.E.D.*

More on the Form of Demand Shocks under Interpretation D. Our analysis until now, as well as our empirical tests, have not required a complete specification of information sets of the agents, all the necessary information has been captured by the density processes θ_H and θ_F . There densities may come from various economic settings; here, for brevity, we consider only the case of incomplete information where the agents observe all economic processes defined in Section 2, but do not have complete information about their dynamics. The description of the setting is intentionally dense, and we refer the reader to Basak (2005) for more detail. We consider two subcases: **D1** and **D2**. Under interpretation **D1**, the agents believe that there are only two independent innovations, w_i and w_i^* , $i \in \{H, F\}$. The innovation process w_i of each agent i is such that given his perceived mean growth of the Home output process, μ_i , the observed Home output process has the dynamics $dY(t) = \mu_i(t)Y(t)dt + \sigma_Y(t)Y(t)dw_i(t)$. Similarly, the innovation w_i^* is such that given μ_i^* the Foreign output has dynamics $dY^*(t) = \mu_i^*(t)Y^*(t)dt + \sigma_Y(t)Y^*(t)dw_i^*(t)$. The differences in opinion pertaining to the underlying innovation process are induced by differences in the agents' priors, which they may or may not update as new information arrives. The case of Bayesian updating is an example of a filtering problem, the details of which need not concern us here since the the optimization problem is well known to be independent of the inferencing problem. The outcome of the inferencing problem is the country-specific drift processes μ_i and μ_i^* , which we treat as exogenous. Note that the agents differ in their assessment of the underlying innovations and the drift terms, but agree on the volatilities, which they may deduce from the quadratic variations of the observed processes. The sub-interpretation **D2** is similar to **D1**, except that now the agents believe that there are three independent innovations, w_i, w_i^* and w_i^θ , $i \in \{H, F\}$, that the parameters they estimate depend on. The dynamics of the Home and Foreign output processes

are perceived to be as above. Additionally, the agents observe a public signal, s , perceived to be driven by the third innovation process, following $ds(t) = \mu_{si}(t)s(t)dt + \sigma_s(t)s(t)dw_i^\theta(t)$, $i \in \{H, F\}$, the drift component of which is not observable.

The innovation processes of agent i bear the following relationship to the “true” underlying Brownian motions: $dw_i(t) = dw(t) + \frac{\mu_Y(t) - \mu_i(t)}{\sigma_Y(t)}dt$, $dw_i^*(t) = dw^*(t) + \frac{\mu_Y^*(t) - \mu_i^*(t)}{\sigma_Y^*(t)}dt$ (interpretations **D1-D2**), and $dw_i^\theta(t) = dw^\theta(t) + \frac{\mu_s(t) - \mu_{si}(t)}{\sigma_s(t)}dt$ (interpretation **D2**). By Girsanov’s theorem, the innovation process $\vec{w}_i \equiv (w_i, w_i^*, w_i^\theta)^\top$ (the last component is absent under interpretation **D1**) is a Brownian motion under the probability measure i , $i \in \{H, F\}$. We can then identify $\vec{\kappa}_H(t)$ in the representation of the density of the Radon-Nikodym derivatives of H with respect to P , $\theta_H(t)$, to be $-\left(\frac{\mu_Y(t) - \mu_H(t)}{\sigma_Y(t)}, \frac{\mu_Y^*(t) - \mu_H^*(t)}{\sigma_Y^*(t)}, 0\right)^\top$ (interpretation **D1**), $-\left(\frac{\mu_Y(t) - \mu_H(t)}{\sigma_Y(t)}, \frac{\mu_Y^*(t) - \mu_H^*(t)}{\sigma_Y^*(t)}, \frac{\mu_s(t) - \mu_{sH}(t)}{\sigma_s(t)}\right)^\top$ (interpretation **D2**). Analogously, $\vec{\kappa}_F = -\left(\frac{\mu_Y(t) - \mu_F(t)}{\sigma_Y(t)}, \frac{\mu_Y^*(t) - \mu_F^*(t)}{\sigma_Y^*(t)}, 0\right)^\top$ (interpretation **D1**), $\vec{\kappa}_F = -\left(\frac{\mu_Y(t) - \mu_F(t)}{\sigma_Y(t)}, \frac{\mu_Y^*(t) - \mu_F^*(t)}{\sigma_Y^*(t)}, \frac{\mu_s(t) - \mu_{sF}(t)}{\sigma_s(t)}\right)^\top$ (interpretation **D2**). Hence, under the true measure P , $d\theta_i(t) = \vec{\kappa}_i(t)^\top \theta_i(t) d\vec{w}_i(t)$, $i \in \{H, F\}$.

Note finally that over a finite horizon the disagreement among the agents is going to persist (see e.g., Liptser and Shiryaev (2001)).

Proof of Proposition 2. We first derive the volatility matrix σ , required to compute the investors’ portfolios. We also determine the number of non-redundant risky assets under each interpretation of the demand shocks and specify the number of elements of the vector x_i , $i \in \{H, F\}$ —the vector of fractions of wealth invested in each non-redundant risky asset. These numbers are interpretation-specific, because the number of non-redundant securities varies across our interpretations. To facilitate comparison with the literature, we consider an additional security: a “world” bond B^W , locally riskless in the numeraire. This bond does not, of course, introduce any new investment opportunities in the economy; it is just a portfolio of α shares of the Home bond and $(1-\alpha)$ shares of the Foreign bond. Let r denote the interest rate on this bond. For the exposition below, we will also require vectors $i_1 \equiv (1, 0, 0)$ and $i_2 \equiv (0, 1, 0)$.

- (i) Under interpretation **A**, one can only determine the investment in the composite (world) stock market, and not in individual stock markets. The volatility matrix σ is not invertible. We do not provide details for this interpretation, and refer the reader to Zapatero (1995) or Cass and Pavlova (2004).
- (ii) Under interpretation **B**, four independent investment opportunities are required to dynamically complete financial markets. We take these to be represented by the Home stock S , the Foreign stock S^* , the Home bond B and the world bond B^W . Hence, x_i has three components: $x_i = (x_i^S, x_i^{S^*}, x_i^B)^\top$. From the dynamics of S , S^* and B reported in (15) and the definitions of θ_H and θ_F , we identify the matrix of the investment opportunity set:

$$\sigma(t) = \frac{1}{\alpha q(t) + 1 - \alpha} \begin{pmatrix} [(1 - \alpha)A(t)\theta(t)(\vec{\kappa}_H(t) - \vec{\kappa}_F(t))^\top + \alpha q(t)\sigma_Y(t)i_1 + (1 - \alpha)\sigma_Y^*(t)i_2] \mathcal{I} \\ [-\alpha q(t)A(t)\theta(t)(\vec{\kappa}_H(t) - \vec{\kappa}_F(t))^\top + \alpha q(t)\sigma_Y(t)i_1 + (1 - \alpha)\sigma_Y^*(t)i_2] \mathcal{I} \\ [(1 - \alpha)A(t)\theta(t)(\vec{\kappa}_H(t) - \vec{\kappa}_F(t))^\top + (\alpha - 1)\sigma_Y(t)i_1 + (1 - \alpha)\sigma_Y^*(t)i_2] \mathcal{I} \end{pmatrix} \quad (\text{A.7})$$

where under this interpretation, \mathcal{I} is a 3×3 identity matrix, the role of which will become clear in the sequel.

- (iii) Under interpretation **C**, there are only two independent sources of uncertainty, and hence three securities are sufficient to dynamically complete financial markets. We take them to be S , S^* and B^W . Hence, x_i has two components: $x_i = (x_i^S, x_i^{S^*})^\top$. From (15) and the definitions of θ_H and θ_F we identify the matrix of the investment opportunity set:

$$\sigma(t) = \frac{1}{\alpha q(t) + 1 - \alpha} \begin{pmatrix} [(1 - \alpha)A(t)\theta(t)(\vec{\kappa}_H(t) - \vec{\kappa}_F(t))^\top + \alpha q(t)\sigma_Y(t)i_1 + (1 - \alpha)\sigma_Y^*(t)i_2] \mathcal{I} \\ [-\alpha q(t)A(t)\theta(t)(\vec{\kappa}_H(t) - \vec{\kappa}_F(t))^\top + \alpha q(t)\sigma_Y(t)i_1 + (1 - \alpha)\sigma_Y^*(t)i_2] \mathcal{I} \end{pmatrix} \quad (\text{A.8})$$

where under this interpretation $\mathcal{I} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$. The role of matrix \mathcal{I} is to modify the dimension of the vectors i_1 , i_2 , and $\vec{\kappa}_i$ so that the volatility matrix σ is a square matrix.

- (iv) To address interpretation **D**, we again consider subcases **D1** and **D2**. Under interpretation **D1**, three securities dynamically complete markets: for instance, S , S^* and B^W , and hence x_i has two components: $x_i = (x_i^S, x_i^{S^*})^\top$. Under interpretation **D2**, there is an additional nonredundant security, B , and hence x_i has three components: $x_i = (x_i^S, x_i^{S^*}, x_i^B)^\top$. The matrix σ is then obtained using the first two rows in (15) under interpretation **D1** to yield an expression in (A.8) and the first three rows in (15) under interpretation **D2** to yield an expression in (A.7). Of course, $\vec{\kappa}_H$ and $\vec{\kappa}_F$ used in the expressions for σ are interpretation-**D**-specific (reported earlier in this appendix).

From now on we focus solely on interpretations **B–D**. Before we proceed to portfolios, we derive the interest rate r on the world bond B^W , and the market price of risk m . Both are identified from the representation of the state price density ξ , $d\xi(t) = -r(t)\xi(t)dt - m(t)^\top \xi(t)d\vec{w}(t)$. Applying Itô's lemma to the closed-form expression for ξ in (A.4) and recovering the drift and diffusion terms, we obtain $m(t) = \left[-\alpha \frac{\lambda_H a_H \vec{\kappa}_H(t) \theta_H(t) + \lambda_F a_F \vec{\kappa}_F(t) \theta_F(t)}{Y(t)} - (1 - \alpha) \frac{\lambda_H (1 - a_H) \vec{\kappa}_H(t) \theta_H(t) + \lambda_F (1 - a_F) \vec{\kappa}_F(t) \theta_F(t)}{Y^*(t)} \right. \\ \left. + \alpha \frac{\lambda_H a_H \theta_H(t) + \lambda_F a_F \theta_F(t)}{Y(t)} (\sigma_Y(t), 0, 0)^\top + (1 - \alpha) \frac{\lambda_H (1 - a_H) \theta_H(t) + \lambda_F (1 - a_F) \theta_F(t)}{Y^*(t)} (0, \sigma_Y^*(t), 0)^\top \right] e^{-\rho t} / \xi(t)$ and $r(t) = \rho + \left\{ \alpha \frac{\lambda_H a_H \theta_H(t) + \lambda_F a_F \theta_F(t)}{Y(t)} \mu_Y(t) + (1 - \alpha) \frac{\lambda_H (1 - a_H) \theta_H(t) + \lambda_F (1 - a_F) \theta_F(t)}{Y^*(t)} \mu_Y^*(t) + \alpha \frac{(\sigma_Y(t), 0, 0) [\lambda_H a_H \vec{\kappa}_H(t) \theta_H(t) + \lambda_F a_F \vec{\kappa}_F(t) \theta_F(t)]}{Y(t)} - (1 - \alpha) \frac{(0, \sigma_Y^*(t), 0) [\lambda_H (1 - a_H) \vec{\kappa}_H(t) \theta_H(t) + \lambda_F (1 - a_F) \vec{\kappa}_F(t) \theta_F(t)]}{Y^*(t)} \right. \\ \left. - \alpha \frac{\lambda_H a_H \theta_H(t) + \lambda_F a_F \theta_F(t)}{Y(t)} \sigma_Y(t)^2 + (1 - \alpha) \frac{\lambda_H (1 - a_H) \theta_H(t) + \lambda_F (1 - a_F) \theta_F(t)}{Y^*(t)} \sigma_Y^*(t)^2 \right\} e^{-\rho t} / \xi(t).$

(i) *Portfolios.* It is straightforward to verify that matrix σ is invertible under interpretations **B–D**. Hence, markets are dynamically complete and the equilibrium allocation is Pareto optimal. Under market completeness, we can appeal to the martingale representation methodology (Cox and Huang (1989), Karatzas, Lehoczky, and Shreve (1987)) to transform the dynamic optimization problem of Home (Foreign) country to a static problem of maximizing the objective in (4) (in (5))

subject to the static budget constraint $E[\int_0^T \xi(t)[p(t)C_i(t) + p^*(t)C_i^*(t)]dt] = W_i(0)$, $i = H$ ($i = F$). The first-order conditions for this optimization are

$$e^{-\rho t} \frac{\theta_i(t)a_i}{C_i(t)} = \frac{1}{\lambda_i} \xi(t)p(t), \quad e^{-\rho t} \frac{\theta_i(t)(1-a_i)}{C_i^*(t)} = \frac{1}{\lambda_i} \xi(t)p^*(t), \quad i \in \{H, F\}. \quad (\text{A.9})$$

Note that the multipliers on the static countries' budget constraints are the reciprocals of the planner's weights λ_H and λ_F .

The optimal trading strategy of agent i is identified from the stochastic integral representation $M_i(t) = M_i(0) + \int_0^t \psi_i(s)^\top d\vec{w}(s)$ of the martingale $M_i(t) \equiv E_t \left[\int_0^T \xi(s)[p(s)C_i(s) + p^*(s)C_i^*(s)]ds \right]$ by modifying the standard argument to account for multiple goods (see, e.g., Karatzas and Shreve (1998), Theorem 7.3):

$$\sigma(t)^\top x_i(t) = \mathcal{I}^\top m(t) + \mathcal{I}^\top \frac{\psi_i(t)}{\xi(t)W_i(t)}, \quad (\text{A.10})$$

where $W_i(t)$ is time- t optimal wealth of agent i , $W_i(t) = \frac{1}{\xi(t)} E_t \left[\int_t^T \xi(s)[p(s)C_i(s) + p^*(s)C_i^*(s)]ds \right]$, and \mathcal{I} and σ are interpretation-specific, as described above. In our model, $M_i(t) = E_t \left[\int_0^T \xi(s)[p(s)C_i(s) + p^*(s)C_i^*(s)]ds \right] = E_t \left[\int_0^T e^{-\rho s} (\lambda_i \theta_i(s)a_i + \lambda_i \theta_i(s)(1-a_i))ds \right] = \int_0^t e^{-\rho s} \lambda_i \theta_i(s)ds + \frac{1-e^{-\rho(T-t)}}{\rho} \lambda_i \theta_i(t)$, and hence $dM_i(t) = \frac{1-e^{-\rho(T-t)}}{\rho} \lambda_i \vec{\kappa}_i(t) \theta_i(t) d\vec{w}(t)$. The diffusion coefficient of $M_i(t)$, $\frac{1-e^{-\rho(T-t)}}{\rho} \lambda_i \vec{\kappa}_i(t) \theta_i(t)$, is identified with $\psi_i(t)$. Note also that

$$\xi(t)W_i(t) = E_t \left[\int_t^T \xi(s)[p(s)C_i(s) + p^*(s)C_i^*(s)]ds \right] = \frac{1-e^{-\rho(T-t)}}{\rho} \lambda_i \theta_i(t). \quad (\text{A.11})$$

Plugging this together with $\psi_i(t)$ into (A.10) we arrive at

$$x_i(t) = \underbrace{(\sigma(t)^\top)^{-1} \mathcal{I} m(t)}_{\text{mean-variance portfolio}} + \underbrace{(\sigma(t)^\top)^{-1} \mathcal{I} \vec{\kappa}_i(t)}_{\text{hedging portfolio}}, \quad i \in \{H, F\}. \quad (\text{A.12})$$

where the compositions of the vector of the fractions of wealth invested in risky assets, x_i are different across interpretations **A–D**, and are reported above. The remaining fraction of wealth, $1 - x_i^\top \vec{1}$, where $\vec{1} = (1, \dots, 1)^\top$, is invested in the world bond B^W . By substituting the expressions for σ and m , specified above, into (A.12) it is tedious but straightforward to verify that the trading strategies are given by (19)–(20).

(ii) *Home bias.* The statement regarding the home bias follows immediately from examining the closed-form expressions (19)–(20).

(iii) *International CAPM.* In an arbitrage-free market, the risk premium on stock j is related to the market price of risk in the following way (e.g., Karatzas and Shreve (1998), Theorem 4.2):

$$\frac{E_t(dS^j(t)/dt)}{S^j(t)} - r(t) = \sum_{k=1}^3 \sigma_k^j(t) m_k(t),$$

where the diffusion coefficients σ_1^j , σ_2^j and σ_3^j are the loadings of stock S^j on Brownian motions w , w^* and w^θ , respectively, and m_k are components of the market price of risk vector m . On the other hand,

$$\text{Cov}_t \left(\frac{dS^j(t)}{S^j(t)}, \frac{d\xi(t)}{\xi(t)} \right) = \frac{\text{Cov}_t(dS^j(t), d\xi(t))}{S^j(t)\xi(t)} = - \sum_{k=1}^3 \sigma_k^j(t) m_k(t) dt.$$

Hence,

$$\frac{E_t(dS^j(t))}{S^j(t)} - r(t)dt = -Cov_t \left(\frac{dS^j(t)}{S^j(t)}, \frac{d\xi(t)}{\xi(t)} \right) \quad (\text{A.13})$$

From (A.11), $W_i(t) = \frac{1-e^{-\rho(T-t)}}{\rho} \lambda_i \theta_i(t) / \xi(t)$, $i \in \{H, F\}$. Hence, the aggregate wealth is

$$W(t) = W_H(t) + W_F(t) = \frac{1 - e^{-\rho(T-t)}}{\rho} \xi(t) [\lambda_H \theta_H(t) + \lambda_F \theta_F(t)].$$

Taking logs, applying Itô's lemma and rearranging, we have

$$\frac{d\xi(t)}{\xi(t)} = -\frac{dW(t)}{W(t)} + \frac{\lambda_H d\theta_H(t) + \lambda_F d\theta_F(t)}{\lambda_H \theta_H(t) + \lambda_F \theta_F(t)} + dt \text{ terms}. \quad (\text{A.14})$$

Then,

$$\begin{aligned} Cov_t \left(\frac{dS^j(t)}{S^j(t)}, \frac{d\xi(t)}{\xi(t)} \right) &= -Cov_t \left(\frac{dS^j(t)}{S^j(t)}, \frac{dW(t)}{W(t)} \right) + Cov_t \left(\frac{dS^j(t)}{S^j(t)}, \frac{\lambda_H d\theta_H(t)}{\lambda_H \theta_H(t) + \lambda_F \theta_F(t)} \right) \\ &\quad + Cov_t \left(\frac{dS^j(t)}{S^j(t)}, \frac{\lambda_F d\theta_F(t)}{\lambda_H \theta_H(t) + \lambda_F \theta_F(t)} \right), \quad S^j \in \{S, S^*\}. \end{aligned}$$

Combining this with (A.13), we obtain the required expression. *Q.E.D.*

Appendix B: Nontradables and Preference for Home Goods

Our baseline specification for the utility function captures in reduced form a more general formulation involving nontradable goods. Here we present one possible extension of our economy that explicitly accounts for the presence of nontradables. In this simple extension we assume that consumers have homogeneous preferences, but one of the goods they consume is not traded.

There are two types of goods in the economy: final and intermediate. Each country produces its own intermediate good, via a Lucas-tree type stochastic technology, just like in our baseline specification. We denote their output by \bar{y} and \bar{y}^* and prices by p and p^* , respectively. These intermediate goods are then used to produce two final goods in each country—a final tradable good and a final nontradable—via constant returns to scale technologies.

$$Y(t) = Ay(t), \quad Y^{NT}(t) = A^{NT}y^{NT}(t), \quad (\text{Home})$$

$$Y^*(t) = A^*y^*(t), \quad Y^{*NT}(t) = A^{*NT}y^{*NT}(t), \quad (\text{Foreign})$$

where Y^{NT} and Y^{*NT} denote output of nontradables in Home and Foreign, respectively, and $y(t) + y^{NT}(t) = \bar{y}(t)$ and $y^*(t) + y^{*NT}(t) = \bar{y}^*(t)$ so that markets for the intermediate goods clear. Note that we assume that Home nontradables use Home intermediates, and Foreign nontradables use Foreign intermediates. In international economics this assumption is standard, but with the factor of production most commonly identified with domestic labor, rather than an intermediate good. For simplicity, we assume that the technology parameters in our economy are constant.

Similarly to our baseline specification, stocks in this economy are represented by claims on the output of the Lucas trees. In particular, there is one claim (stock) for each of the two trees producing the intermediate goods. Due to the constant returns to scale assumption, profits in the final goods sector are zero, and we therefore do not consider stocks of the final-goods producing firms.

The consumers in each country derive utility from final tradable goods, as in our baseline specification, and, additionally, from a final nontradable good produced in the country. Formally, the preferences of Home and Foreign are as follows

$$E \left[\int_0^T e^{-\rho t} \theta_H(t) [a_1 \log(C_H(t)) + a_2 \log(C_H^{NT}(t)) + (1 - a_1 - a_2) \log(C_H^*(t))] dt \right],$$

$$E \left[\int_0^T e^{-\rho t} \theta_F(t) [a_1 \log(C_F(t)) + a_2 \log(C_F^{NT}(t)) + (1 - a_1 - a_2) \log(C_F^*(t))] dt \right].$$

Note that the countries' preferences are homogeneous in the sense that both countries assign the exact same weights (or expenditure shares) to the three goods they consume. (The demand shocks, of course, continue to differ across countries.) However, since Home consumes Home nontradables and Foreign consumes Foreign nontradables, the resulting consumer demands in each country are not the same.

We again consider the social planner's problem, but we now recast it in terms of the intermediate inputs rather than final consumption. Such representation is equivalent because a consumption allocation process $(C_i, C_i^*, C_i^{NT}, i \in \{H, F\})$ can be mapped into a corresponding intermediate input allocation $(y_i, y_i^*, y_i^{NT}, i \in \{H, F\})$, representing country i 's implied demands for the intermediate goods employed in production of the Home tradable, Foreign tradable, and domestic nontradable goods, respectively. The mapping involves a simple scaling by an appropriate technology parameter (for example, $C_H(t) = A y_H(t)$). Dropping unimportant terms and simplifying, we arrive at the following representation of the planner's problem:

$$\begin{aligned} \max_{\substack{y_H, y_H^*, y_H^{NT}, \\ y_F, y_F^*, y_F^{NT}}} & E \left[\int_0^T e^{-\rho t} \left\{ \lambda_H \theta_H(t) [a_1 \log(y_H(t)) + a_2 \log(y_H^{NT}(t)) + (1 - a_1 - a_2) \log(y_H^*(t))] \right. \right. \\ & \left. \left. + \lambda_F \theta_F(t) [a_1 \log(y_F(t)) + a_2 \log(y_F^{NT}(t)) + (1 - a_1 - a_2) \log(y_F^*(t))] \right\} dt \right], \\ \text{s. t.} & \quad y_H(t) + y_F(t) + y_H^{NT}(t) = \bar{y}(t), \\ & \quad y_H^*(t) + y_F^*(t) + y_F^{NT}(t) = \bar{y}^*(t). \end{aligned}$$

From the first-order conditions it is easy to show that

$$y_H^{NT}(t) = \frac{a_2}{a_1} y_H(t)$$

$$y_F^{*NT}(t) = \frac{a_2}{1 - a_1 - a_2} y_F^*(t)$$

Substituting these expressions back into the utility function and the resource constraints, we can reduce the set of choice variables in the planner's problem, recasting it solely in terms of the

intermediate inputs employed in the tradable sector:

$$\begin{aligned} \max_{y_i, y_i^*, i \in \{H, F\}} \quad & E \left[\int_0^T e^{-\rho t} \left\{ \lambda_H \theta_H(t) [(a_1 + a_2) \log(y_H(t)) + (1 - a_1 - a_2) \log(y_H^*(t))] \right. \right. \\ & \left. \left. + \lambda_F \theta_F(t) [a_1 \log(y_F(t)) + (1 - a_1) \log(y_F^*(t))] \right\} dt \right] \\ \text{s. t.} \quad & \frac{a_1 + a_2}{a_1} y_H(t) + y_F(t) = \bar{y}(t), \\ & y_H^*(t) + \frac{1 - a_1}{1 - a_1 - a_2} y_F^*(t) = \bar{y}^*(t). \end{aligned}$$

The problem starts looking quite similar to that derived in the context of our baseline model.

The next step is to solve for the sharing rules for each intermediate good, and then, from the marginal utilities, to back out relative prices of the two intermediate goods that would prevail in a market equilibrium. This relative price of the intermediates in Home and Foreign, p/p^* , is the relevant measure of the terms of trade.²⁵ In this economy, this relative price is given by

$$q(t) = \frac{\lambda_H \theta_H(t) (a_1 + a_2) + \lambda_F \theta_F(t) a_1}{\lambda_H \theta_H(t) (1 - a_1 - a_2) + \lambda_F \theta_F(t) (1 - a_1)} \frac{\bar{y}^*(t)}{\bar{y}(t)},$$

which is identical to (10) if $a_H \equiv a_1 + a_2 > a_F \equiv a_1$. Hence, the assumption of a home bias in consumption in our reduced-form baseline model ($a_H > a_F$) is directly related to an assumption that each country assigns a positive weight to nontradables ($a_2 > 0$) in the utility function.

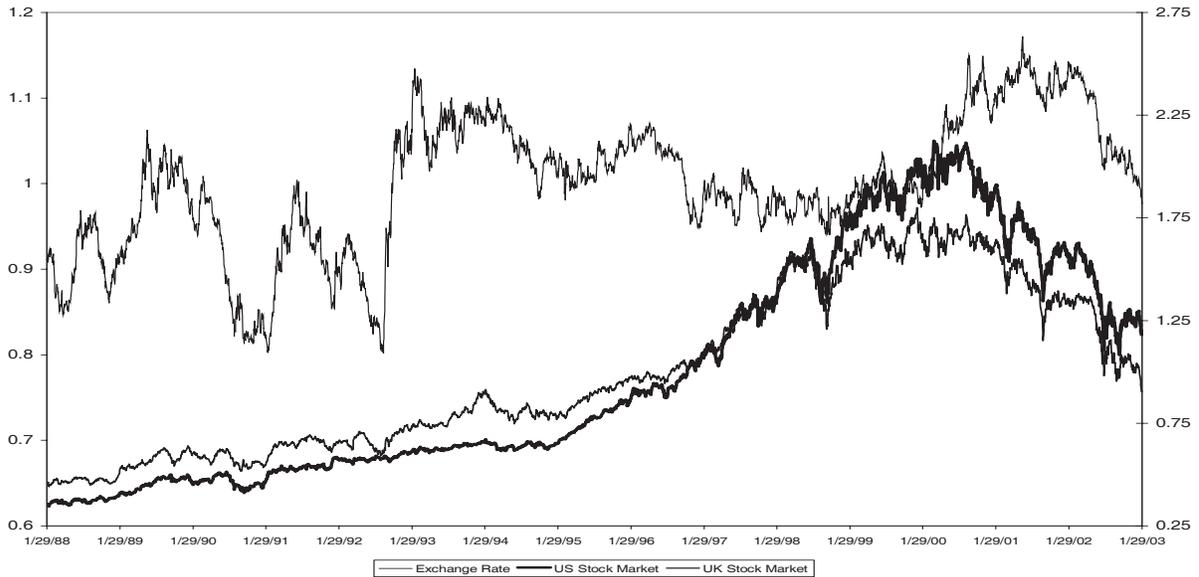
²⁵In the open economy macroeconomics literature, the terms of trade (exchange rates) are typically identified with the relative wage (see, e.g. Dornbusch, Fischer, and Samuelson (1977)). This would also be the case in our economy if we interpret our factors of production (the intermediate goods) as labor.

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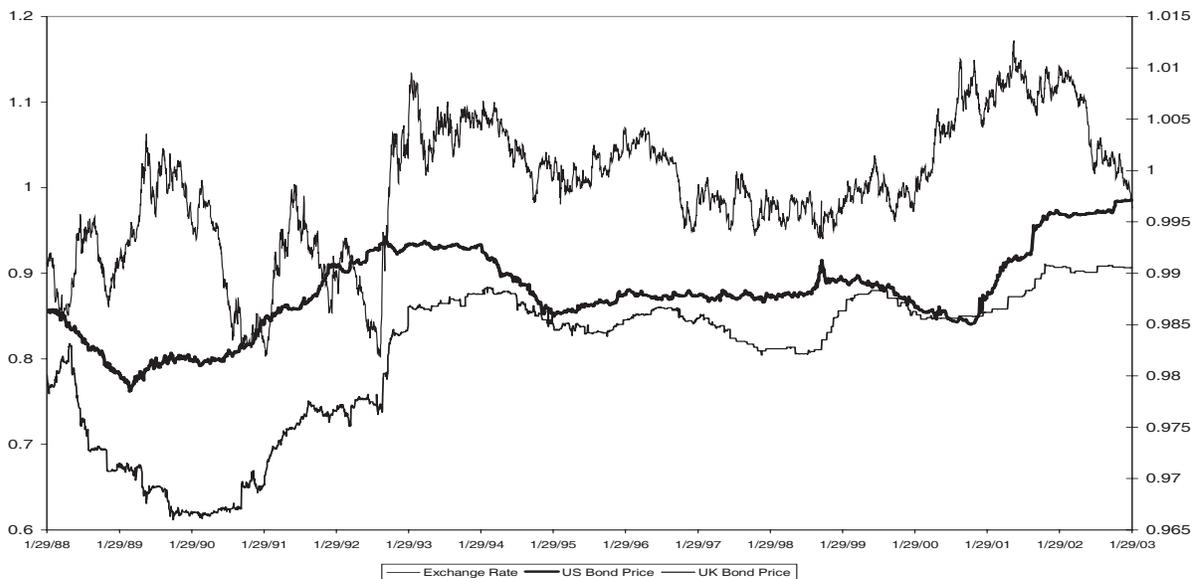
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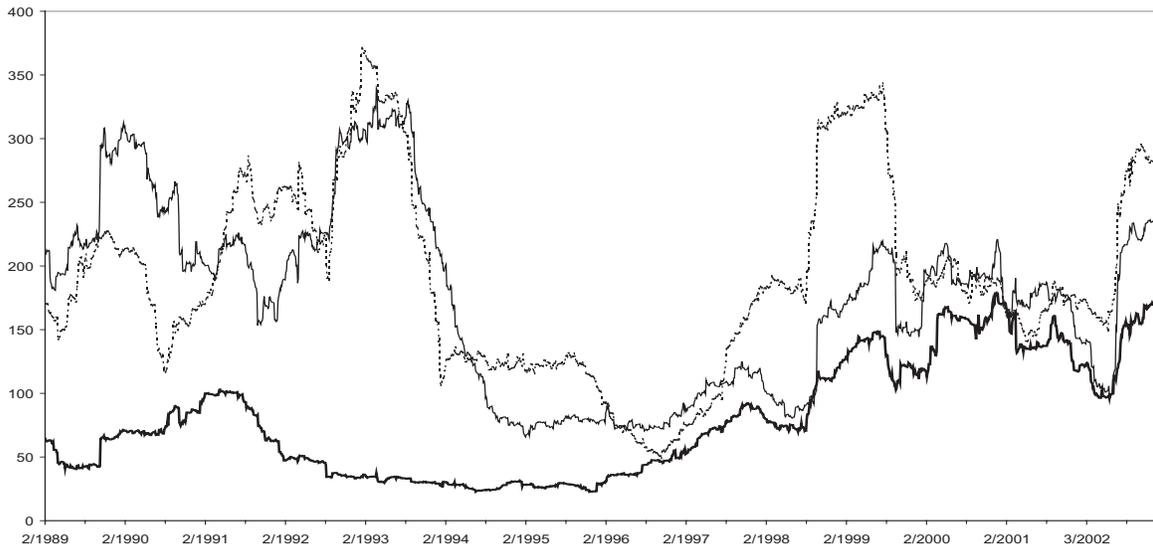


(a) The US and the UK stock market indexes and the dollar-pound exchange rate

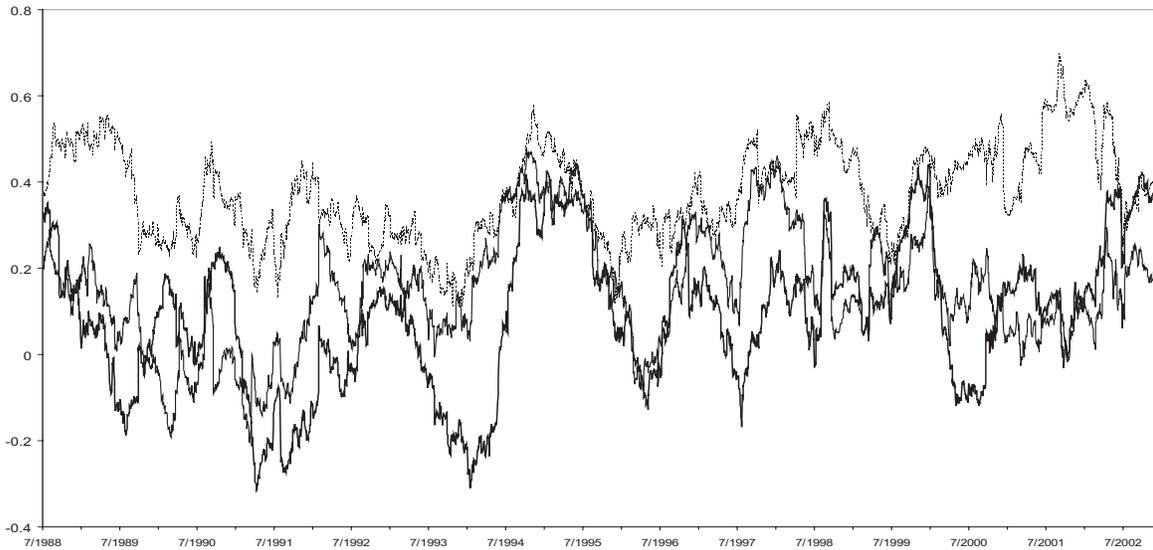


(b) The US and the UK three-month zero-coupon government bond prices and the dollar-pound exchange rate

Figure 1: Asset prices and exchange rates. The exchange rate is measured in the left axis. The right axis measures the US and the UK stock market indexes (panel (a)) and the bond prices (panel (b)). For demonstrative purposes, all prices are normalized so that the average exchange rate is equal to one.



(a) The 20-day rolling variances of estimated supply and demand shocks



(b) The 20-day rolling correlations between the returns on the US and UK stock market indexes and the dollar-pound exchange rate.

Figure 2: Rolling variances and correlations. The model used for extracting the shocks is specified in (23). Panel (a): The 20-day rolling variances of the US supply shock (thick solid plot), the UK supply shock (thin solid plot), and the relative demand shock (dashed plot). Panel (b): The 20-day rolling correlations of the returns on the US stock market index and the dollar-pound exchange rate (thick solid plot), the returns on the UK stock market index and the dollar-pound exchange rate (thin solid plot), and the correlation of the returns on the US and UK stock market indexes (dashed plot).