The Benchmark Inclusion Subsidy

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Abstract

We study the effects of evaluating asset managers against a benchmark on corporate decisions, e.g., investments, M&A, and IPOs. We introduce asset managers into an otherwise standard model and show that firms inside the benchmark are effectively subsidized by the asset managers. This "benchmark inclusion subsidy" arises because asset managers have incentives to hold some of the equity of firms in the benchmark regardless of their risk characteristics. Due to the benchmark inclusion subsidy, a firm inside the benchmark values an investment project more than the one outside. The same wedge arises for valuing M&A, spinoffs, and IPOs. These findings are in contrast to the standard result in corporate finance that the value of an investment is independent of the entity considering it. We show that the higher the cash-flow risk of an investment and the more correlated the existing and new cash flows are, the larger the subsidy; the subsidy is zero for safe projects. We review a host of empirical evidence that is consistent with the model's implications.

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1 Introduction

The asset management industry is estimated to control more than \$85 trillion worldwide. Most of this money is managed against benchmarks. For instance, S&P Global reports that in 2017, just under \$10 trillion were managed against the S&P 500 alone.[1] Existing research related to benchmarking has largely focused on its asset pricing implications. Instead, we look at the implications of benchmarking for corporate decisions such as investments, M&A, spinoffs, and IPOs. We argue that firms included in a benchmark are effectively subsidized by asset managers and so should evaluate investment opportunities differently.

In standard corporate finance, the value of an investment is independent of the entity considering it. We show that this is generally not the case when asset managers are present and their performance is measured against a benchmark. Instead, the appropriate discount rate for an investment project differs across firms, depending on whether or not they are part of the benchmark. Specifically, when a firm adds risky cash flows, say, because of an acquisition or by investing in a new project, the increase in the stockholder value is larger if the firm is inside the benchmark rather than outside. We call this the "benchmark inclusion subsidy." Because of this subsidy, a firm in the benchmark would accept cash flows with lower mean and/or larger variance than a non-benchmark firm would.

The reason for this result is that benchmarking induces asset managers to hold some shares of firms in their benchmarks regardless of those firms' cash flows characteristics. So when a firm adds risky cash flows, the market demand for claims on these cash flows is higher if the firm is inside the benchmark rather than outside. The firm, therefore, should take this consideration into account in deciding on its investments, acquisitions and spinoffs.

Here is how the model works. We introduce asset managers into an otherwise standard setting. Some investors manage their own portfolios and others use asset managers. An asset manager's compensation depends on performance relative to a benchmark portfolio. This assumption, critical for our argument, is in keeping with prevalent industry practices and academic evidence such as Ma, Tang, and Gómez (forthcoming). We show that the asset manager's optimal portfolio is a combination of the standard mean-variance portfolio and the benchmark portfolio—the latter appearing because of benchmarking in compensation. Specifically, asset managers hold a fixed part of their portfolio in benchmark firms regardless

[1]As of November 2017, Morgan Stanley Capital International reports that \$3.2 trillion was benchmarked against its All Country World Index and \$1.9 trillion was managed against its Europe, Australasia and Far East index. Across various markets, FTSE-Russell reports that at the end of 2016 \$8.6 trillion was benchmarked to its indices. CRSP reports that assets linked to its indices exceed \$1.3 trillion as of September 2018.

of their stock prices and cash-flow characteristics—most importantly, irrespective of cash-flow variance. As a result, the equilibrium stock price of a firm in the benchmark is less adversely affected by the same cash-flow risk than that of an otherwise identical firm that is outside the benchmark.

We show that the same argument applies to risky investments. Specifically, for a firm in the benchmark risky investments are subsidized. The extra variance of its cash flows resulting from the project will be penalized less than that of an identical non-benchmark firm. This occurs because asset managers have a mechanical demand for a fixed number of shares of benchmark firms, regardless of their risk. This mechanical demand benefits riskier projects more. As cash-flow riskiness decreases, so does the benchmark inclusion subsidy, converging to zero for risk-free projects.

Because of this subsidy, a firm in the benchmark is more likely to invest. A benchmark firm would also accept projects with riskier cash flows. We review empirical evidence that confirms both of these implications.

To demonstrate these predictions in the most transparent way, we construct the simplest possible example. In it, there are three firms with uncorrelated cash flows. Absent asset managers, combining firms creates no value. With asset managers, a firm inside the benchmark creates value when it takes over a firm that is outside the benchmark.

We then turn to an extended model with many firms whose cash flows are correlated. Allowing for correlations brings out additional effects and predictions. In particular, we find that investments whose cash flows are positively correlated with those of the benchmark firms are valued more. As in the economy with uncorrelated cash flows, asset managers' excess demand for benchmark stocks raises those stocks' prices. Now, the stock prices of firms whose cash flows are positively correlated with those of the benchmark stocks also rise. This happens because, in seeking exposure to the benchmark's cash-flow risk, investors substitute away from expensive benchmark stocks into stocks that are correlated with them. This same reasoning means that investment projects or acquisitions that are positively correlated with the benchmark are valued more by all firms (relative to an economy without asset managers).

We show that the main mechanism that delivered the benchmark inclusion subsidy in the example generalizes. The extended model also features an additional channel, owing to the correlation of a project's cash flows with those of the firm's assets-in-place. We demonstrate that if this correlation is positive, the value of the new asset to the benchmark firm exceeds the asset's value were it to join the benchmark as a standalone.

We derive a closed-form expression for the benchmark inclusion subsidy, which turns out to be very simple, and study the variables that influence its size. We show that the higher the cash-flow risk of an investment, the larger the benchmark inclusion subsidy. Furthermore, the benchmark inclusion subsidy is increasing in the correlation of a project's cash flows with the existing assets; in particular, the subsidy is the largest for projects that are clones of a firm's existing assets. Finally, the size of the subsidy rises with the size of the asset management sector.

The ability to characterize the exact determinants of the subsidy allows us to predict the situations when benchmarking is the most and least important. To the best of our knowledge, other theories do not deliver such cross-sectional predictions. We are able to tie the size of the subsidy to characteristics of firms, investment opportunities, or potential acquisitions or divestitures.

For M&A, we stress that a merger of a firm in the benchmark with a firm outside creates value for stockholders. There are no cash-flow synergies, but there is a financial synergy that comes from bringing the target into the benchmark. This contrasts the standard corporate finance result that such a merger creates no value. For IPOs, we highlight the implication that an IPO would have a higher value if the stock joins the benchmark. For spinoffs, we demonstrate that if an asset divested by a benchmark firm lands outside the benchmark, it is stripped of the benchmark inclusion subsidy and hence its value drops.

Our analysis implies that a model for the cost of capital that does not account for the presence of asset managers is misspecified. With asset managers, the benchmark becomes a factor that influences driving expected returns. In our model, both the benchmark and the usual market portfolio matter for pricing assets. The right model for the cost of capital in our environment is, therefore, not the capital asset pricing model (CAPM) that is typically used in corporate finance, but its two-factor modification that accounts for the presence of asset managers.

Finally, it is worth noting that our model applies to both active and passive asset management. We show that the benchmark inclusion subsidy is larger when more asset managers are passive rather than active. While active managers in our model invest only a part of their portfolio in the benchmark, passive managers invest their entire portfolio in it, which amplifies the mechanical demand for stocks in the benchmark.

We review existing empirical work that relates to the model's predictions. Past research confirms, to varying degrees, the predictions regarding the propensity to invest and engage in acquisitions for benchmark vs. non-benchmark firms, the factor structure of returns, as well as the size of the benchmark inclusion subsidy increasing in assets-under-management.

The remainder of the paper is organized as follows. In the next section, we explain how our perspective compares to previous work. Section 3 presents the example, and Section 4 analyzes the general model. Section 5 reviews related empirical evidence. Section 6 presents our conclusions and suggestions for future areas of promising research. Omitted proofs are in the appendix.

2 Related Literature

Our paper revisits one of the main tenets of corporate finance that the value of an investment is independent of the entity considering it. This point is emphasized in leading textbooks such as Berk and DeMarzo (2017) (chapter 19), Brealey, Myers, and Allen (2017) (chapter 19) and Grinblatt and Titman (2002) (chapter 13). There is a huge academic and applied literature on capital budgeting (see, e.g. Stulz, 1999, Jacobs and Shivdasani, 2012, Dessaint, Olivier, Otto, and Thesmar, 2018, for a range of representative studies). Finally, there is survey evidence (e.g. Graham and Harvey, 2001) on how managers make such decisions. In this approach, asset managers play no role.

Our analysis shows that the textbook approach's failure to properly account for the effects of asset managers has important consequences for real corporate decisions. The motivation for taking asset managers seriously comes from the index additions and deletions literature. Harris and Gurel (1986) and Shleifer (1986) were the first to document that when stocks are added to the S&P 500 index, their prices rise. Subsequent papers have also shown that firms that are deleted experience a decline in price. The findings have been confirmed across many studies and for many markets, so that financial economists consider these patterns to be stylized facts.[2] The estimated magnitudes of the index effect vary across studies, and typically most of the effect is permanent. For example, Chen, Noronha, and Singal (2004) find the cumulative abnormal returns of stocks added to the S&P 500 during 1989-2000, measured over two months post announcement, to be 6.2%.[3]

Several theories have been used to interpret the index effect. The first is the investor awareness theory of Merton (1987). Merton posits that some investors become aware of

[2]See, e.g, Beneish and Whaley (1996), Lynch and Mendenhall (1997), Wurgler and Zhuravskaya (2002), Chen, Noronha, and Singal (2004), Petajisto (2011), and Hacibedel (2018).

[3]Calomiris, Larrain, Schmukler, and Williams (2018) find an index effect for emerging market corporate bonds. They trace the rise of the JP Morgan Corporate Emerging Market Bond Index and show how firms in countries that became eligible change issuance patterns (to qualify for index inclusion) and receive lower yields on qualifying bond issues.

and invest in a stock only when it gets included in a popular index. It is unclear why investor awareness declines for index deletions, although there is evidence of a decrease in analyst coverage. The second theory posits that index inclusions convey information about a firm's improved prospects. This theory has difficulty explaining the presence of index effects around mechanical index recompositions (see, e.g., Boyer, 2011, among others). The third theory is that index inclusion leads to improved liquidity, and this in turn boosts stock prices. This theory, however, does not explain increased correlations with other index stocks (documented in, e.g., Barberis, Shleifer, and Wurgler, 2005 and Boyer, 2011).

The final theory can be broadly described as the price pressure theory, proposed by Scholes (1972). Scholes' prediction is that prices of included stocks should rise temporarily, to compensate liquidity providers, but should revert back as investors find substitutes for these stocks. Subsequent literature has argued that the price pressure effects could be (more) permanent, driven by changing compositions of investors. Our model is broadly consistent with the price pressure view. Our benchmarked asset managers put permanent upward pressure on prices of stocks as long as they are in the benchmark. Despite any overpricing, the benchmarking creates a fixed demand for these stocks by a particular clientele: the asset managers. Holding a substitute stock is costly for an asset manager because this entails a (risky) deviation from her benchmark.

The index effect literature only considers the average effect of index inclusion. Our theory has a host of cross-sectional predictions that one could potentially test. For example, stocks with larger cash-flow variance should experience a larger index effect. We also stress that what matters for our channel is whether a stock is in the benchmark, not the index, and so our model can be used to separate competing theories by studying stocks that are in the index but not the benchmark (e.g., "sin" stocks, as analyzed in Hong and Kacperczyk, 2009).

Our work is also related to a theoretical literature in asset pricing that explores the effects of benchmarking on stock returns and their comovement. The first paper in this line of research is Brennan (1993), who, like us, derives a two-factor CAPM in an economy with asset managers. Cuoco and Kaniel (2011), Basak and Pavlova (2013), and Buffa, Vayanos, and Woolley (2014) show how benchmarking creates additional demand for stocks included in the benchmark index, generating an index effect. Basak and Pavlova also derive excess comovement of index stocks. Greenwood (2005) considers a model with passive indexers and arbitrageurs (who are like our conventional investors) and shows that an index reconstitution not only lifts prices of stocks added to the index but also those of non-index stocks that are positively correlated with them. This literature focuses on asset prices, taking stocks' cash flows as given, and does not explore the real effects of benchmarking.

Our paper is perhaps most closely related to Stein (1996). He also studies capital budgeting in situations where the CAPM does not correctly describe expected stock returns. He assumes, however, that the deviations are temporary and arise because of investor irrationality. If market participants fail to appreciate risk and allow a firm to issue mispriced equity, he explains why rational managers may want to issue equity and invest, even if the CAPM-based valuation of a project is negative. In our case, the price differences are not due to irrationality; instead, they arise because of fundamental differences in demand from different types of investors. In Stein's setup, the horizon that managers use for making decisions is critical, and those that are short-term oriented will potentially respond to mispricing if it is big enough. In our model, all managers of firms in the benchmark should account for the subsidy (for as long as the firm remains in the benchmark).

Stein's paper led to a number of follow-on studies that look at other potential behavioral effects that could be associated with inclusion in a benchmark (see Baker and Wurgler, 2013 for a survey). These papers contain much of the empirical work that we cite in favor of our model. While we share several predictions with Stein (1996) there are some notable differences. For instance, Stein's model connects managerial time horizons and financial constraints to capital budgeting decisions. Our model has nothing to say about these considerations. However, we also have many implications that are distinct from his. For instance, our closed-form expression for the benchmark inclusion subsidy generates a number of predictions about which factors should lead firms in the benchmark to make different decisions than ones outside. On the whole, we see the behavioral theories and ours complementing each other.

Finally, there is recent literature on mistakes that managers make in project valuation. For example, Krüger, Landier, and Thesmar (2015) document that diversified firms appear to make investment decisions in non-core businesses by using the discount rate from their core business. Interestingly, in our model, the benchmark inclusion subsidy applies to the entire firm so there is a basis for having part of the cost of capital depend on that firm-wide characteristic.

3 Example

To illustrate the main mechanism, we begin with a simple example with three firms with uncorrelated cashflows. We first consider an economy populated by identical investors in these firms who manage their own portfolios. We then modify the economy by introducing another group of investors who hire asset managers to run their portfolios. Asset managers' performance is evaluated relative to a benchmark. We show that the presence of asset managers invalidates the standard approach to corporate valuation.

3.1 Baseline Economy

Consider the following environment. There are two periods, $t = 0, 1$. Investment opportunities are represented by three risky assets denoted by 1, 2, and y, and one risk-free bond. The risky assets are claims to cash flows D_i realized at $t = 1$, where $D_i \sim N(\mu_i, \sigma_i^2),\ i = 1, 2, y$, and these cash flows are uncorrelated. We think of these assets as stocks of all equity firms. The risk-free bond pays an interest rate that is normalized to zero. Each of the risky assets is available in a fixed supply that is normalized to one. The bond is in infinite net supply. Let S_i denote the price of asset $i = 1, 2, y$.

There is measure one of identical agents who invest their own funds. Each investor has a constant absolute risk aversion (CARA) utility function over final wealth W, $U(W) = -e^{-\gamma W}$, where $\gamma > 0$ is the coefficient of absolute risk aversion. All investors are endowed with one share of each stock and no bonds. At $t = 0$, each investor chooses a portfolio of stocks $x = (x_1, x_2, x_y)^\top$ and the bond holdings to maximize his utility, with $W(x) = \sum_{i=1,2,y} S_i + x_i(D_i - S_i)$.

As is well-known in this kind of setup, the demand x_i for risky asset i and the corresponding equilibrium price S_i will be

$$x_i = \frac{\mu_i - S_i}{\gamma \sigma_i^2},$$
$$S_i = \mu_i - \gamma \sigma_i^2$$

for $i = 1, 2, y$, where the second equation follows from setting the number of shares demanded equal to the supply (which is 1).[4]

[4]We omit derivations for this simple example, but the analysis of our main model contains all proofs for the general case.

When firms $i \in \{1,2\}$ and y merge into a single firm, the demand for the combined firm's stock and the corresponding equilibrium stock price are

$$x_i' = \frac{\mu_i + \mu_y - S_i'}{\gamma(\sigma_i^2 + \sigma_y^2)},$$
$$S_i' = \mu_i + \mu_y - \gamma(\sigma_i^2 + \sigma_y^2) = S_i + S_y.$$

Notice that the combined value of either firm is exactly equal to the sum of its initial value plus the value of y. This is a standard corporate valuation result that says that the owner of a firm does not determine its value. Instead, the value arises from the cash flows (and risks) associated with the firm, which are the same regardless of who owns it.

3.2 Adding Asset Managers

Now we extend the example by considering additional investors who hire asset managers to manage their portfolios. There are now three types of agents in the economy, the same investors as before who manage their own portfolios and whom we refer to as "conventional" investors from now on (constituting a fraction λ_C of the population), asset managers (a fraction λ_{AM}), and shareholders who hire those asset managers (a fraction λ_S).[5] All agents have the same preferences (as in the example).

Shareholders can buy the bond directly, but cannot trade stocks; they delegate the selection of their portfolios to asset managers. The asset managers receive compensation w from shareholders. This compensation has three parts: one is a linear payout based on absolute performance of the portfolio x, the second piece depends on the performance relative to the benchmark portfolio, and the third is independent of performance.[6] Suppose that the benchmark is simply the stock of firm 1. Then

$$w = ar_x + b(r_x - r_{\mathbf{b}}) + c = (a+b)r_x - br_{\mathbf{b}} + c, \tag{1}$$

where $a \geq 0$, $b > 0$ and c are constants, $r_x = \sum_{i=1,2,y} x_i(D_i - S_i)$ and $r_{\mathbf{b}} = D_1 - S_1$. For simplicity, we assume that a, b, and c are set exogenously.[7]

[5] We assume that each shareholder employs one asset manager, so that $\lambda_{AM} = \lambda_S$. Furthermore, $\lambda_C + \lambda_{AM} + \lambda_S = 1$.

[6] This part captures features such as a fee linked to initial assets under management.

[7] Kashyap, Kovrijnykh, Li, and Pavlova (2019) endogenize optimal linear contracts for asset managers.

A conventional investor's demand for asset i continues to be

$$x_i^C = \frac{\mu_i - S_i}{\gamma \sigma_i^2}, \quad i = 1, 2, y.$$

An asset manager's demands are

$$\begin{aligned} x_1^{AM} &= \frac{1}{a+b} \frac{\mu_1 - S_1}{\gamma \sigma_1^2} + \frac{b}{a+b}, \\ x_i^{AM} &= \frac{1}{a+b} \frac{\mu_i - S_i}{\gamma \sigma_i^2}, \quad i = 2, y. \end{aligned} \tag{2}$$

As usual, a conventional investor's portfolio is the mean-variance portfolio, scaled by his risk aversion γ. Asset managers' portfolio choices differ from those of the conventional investors in two ways. First, they hold a scaled version of the same mean-variance portfolio as the one held by the conventional investors. The reason for the scaling is that, as we can see from the first term in (1), for each share that the asset manager holds, she gets a fraction $a + b$ of the total return. Thus the asset manager scales her asset holdings by $1/(a + b)$ relative to those of a conventional investor.

Second, and more importantly, the asset managers are penalized by b for underperforming the benchmark. Because of this penalty, the manager always holds $b/(a + b)$ shares of stock 1 (or more generally whatever is in the benchmark). This consideration explains the second term in (2). This mechanical demand for the benchmark will be critical for all of our results. In particular, the asset managers' incentive to hold the benchmark portfolio (regardless of the risk characteristics of its constituents) creates an asymmetry between stocks in the benchmark and all other stocks.

The second implication is very general and extends beyond our model with CARA preferences. Having a relative performance component as part of her compensation exposes the manager to an additional source of risk—fluctuations in the benchmark—which she optimally decides to hedge. The manager would, therefore, hold a hedging portfolio that is (perfectly) correlated with the benchmark, i.e., the benchmark itself.

Given the demands, we can now solve for the equilibrium prices. Using the market-clearing condition for stocks, $\lambda_{AM} x_i^{AM} + \lambda_C x_i^C = 1,\ i = 1, 2, y$, we find

$$S_1 = \mu_1 - \gamma \Lambda \sigma_1^2 \left(1 - \lambda_{AM} \frac{b}{a+b}\right), \tag{3}$$

$$S_2 = \mu_2 - \gamma\Lambda\sigma_2^2, \tag{4}$$

$$S_y = \mu_y - \gamma\Lambda\sigma_y^2, \tag{5}$$

where $\Lambda = [\lambda_{AM}/(a+b) + \lambda_C]^{-1}$ modifies the market's effective risk aversion.

For concreteness, suppose that $\mu_1 = \mu_2$ and $\sigma_1 = \sigma_2$ so that the return and risks of stocks 1 and 2 are identical. Our first noteworthy finding is that the share price of firm 1 that is inside the benchmark is higher than that of its twin that is not. This happens because asset managers automatically tilt their demand towards the benchmark, effectively reducing the supply of this stock by $b/(b+a)$. The lower the supply of the stock (all else equal), the higher must be its equilibrium price. Another way to understand the result is that the asset managers' mechanical demand for the benchmark means that the adverse effects of variance that typically reduce the demand for any stock, are less relevant for the assets in the benchmark.[8]

Next, consider potential mergers. Suppose first that y merges with the non-benchmark firm (firm 2). The new demands of conventional investors and asset managers for the stock of firm 2 are

$$x_2'^{C} = \frac{\mu_2 + \mu_y - S_2'}{\gamma\left(\sigma_2^2 + \sigma_y^2\right)},$$
$$x_2'^{AM} = \frac{1}{a+b}\frac{\mu_2 + \mu_y - S_2'}{\gamma\left(\sigma_2^2 + \sigma_y^2\right)}.$$

The new equilibrium price of firm 2's stock is

$$S_2' = \mu_2 + \mu_y - \gamma\Lambda\left(\sigma_2^2 + \sigma_y^2\right) = S_2 + S_y.$$

As before, the combined value of firm 2, continues to be the sum of the initial value plus the value of y.

Suppose instead that asset y is acquired by firm 1, which is in the benchmark. Renormalizing the combined number of shares of firm 1 to one, the demands for the stock of

[8]Notice that in this model the asymmetry between benchmark and non-benchmark stocks cannot be arbitraged away. The conventional investors are unrestricted in their portfolio choice and therefore can engage in any arbitrage activity. However, as the asset managers *permanently* reduce the supply of the benchmark stock, conventional investors simply reduce their holdings of the benchmark stock and hold more of the non-benchmark stock. These implications are similar to the effects of quantitative easing in bond markets, whereby a central bank buys a significant fraction of outstanding bonds. As long as asset managers represent a meaningful fraction of the market (i.e., λ_{AM} is non-negligible), there are always differences in prices of stocks inside and outside the benchmark.

the combined firm are

$$x_1'^C = \frac{\mu_1 + \mu_y - S_1'}{\gamma\left(\sigma_1^2 + \sigma_y^2\right)},$$
$$x_1'^{AM} = \frac{1}{a+b}\frac{\mu_1 + \mu_y - S_1'}{\gamma\left(\sigma_1^2 + \sigma_y^2\right)} + \frac{b}{a+b}.$$

Our next major finding is that there is a benchmark inclusion subsidy. Specifically, the new price of firm 1's shares is

$$S_1' = \mu_1 + \mu_y - \gamma\Lambda\left(\sigma_1^2 + \sigma_y^2\right)\left(1 - \lambda_{AM}\frac{b}{a+b}\right) = S_1 + S_y + \gamma\Lambda\sigma_y^2\lambda_{AM}\frac{b}{a+b}, \qquad (6)$$

which is strictly larger than the sum of S_1 and S_y. So when a firm inside the benchmark acquires asset y (which had been outside the benchmark), the combined value exceeds the sum of the initial value plus the value of y.[9] We refer to the increment as the benchmark inclusion subsidy. This subsidy exists because asset managers' demand for the benchmark is partially divorced from the risk and return characteristics of the benchmark, and thus, this kind of acquisition raises the value of the target firm. You can see this by noting that the last term in (6) is proportional to the variance of y, σ_y^2. This is because when y is acquired by firm 1, a portion of asset managers' demand for this asset is now inelastic and independent of its variance. Hence, the market penalizes the variance of y's cash flows less when they are inside firm 1 rather than firm 2.

In contrast, notice that if firm y had started out inside the benchmark, then S_1' would be exactly equal to the sum of prices of stocks 1 and y. In that case, the inelastic demand for the stock would already have been embedded in its price before the merger. So, the extra value of acquisition that accrues to firm 1 relative to firm 2 arises from the increase in the price of y when it becomes part of the benchmark.

To put this more formally, let S_y' denote the price of asset y if it were inside the benchmark. Then $S_y' - S_y = \gamma\Lambda\sigma_y^2\lambda_{AM}b/(a+b)$, which is precisely the extra term in equation (6). Notice that this is directly related to the "index effect" estimated in the literature, which is the percentage change in a firm's stock price when it joins the benchmark, and in our model is

$$\frac{S_y' - S_y}{S_y} = \frac{\gamma\Lambda\sigma_y^2}{S_y}\lambda_{AM}\frac{b}{a+b}, \qquad (7)$$

[9] This result does not depend on the firm being entirely equity financed. We assume no debt financing here just for simplicity.

where S_y is given by (5).

Thus, in this simple example, the benchmark inclusion subsidy reduces to the index effect for the target firm. As we will show in the general model in Section 4, if we allow for any correlation between the acquirer's and the target's cash flows, the benchmark inclusion subsidy will have an additional term accounting for the correlation. When the correlation is positive, the subsidy exceeds the index effect for the target firm.

It is worth noting that our model predicts that the index effect is larger for firms with riskier cash flows. This can be seen from equation (7), where the index effect is increasing in σ_y^2, even after controlling for the stock price before the inclusion. The literature so far has focused on estimating the average index effect. In contrast, our model makes cross-sectional implications about how the index effect varies with firms' risk characteristics.

Finally, the impact of asset managers can also work in the other direction, reducing valuations of spinoffs and divestitures. If y had been part of a firm inside the benchmark and is sold to a non-benchmark firm, the value of y would drop when it is transferred.

In the next section, we consider a richer version of the setup that allows us to analyze several additional questions. Based just on this extremely simplified example, however, we already have seen two empirical predictions. First, consistent with the existing literature on index inclusions, we see that there should be an increase in a firm's share price when it is added to the benchmark. We view this as a necessary condition for the existence of the benchmark inclusion subsidy. In our framework, the stock price increase would remain present for as long as the firm is part of the benchmark.

The other prediction is related to acquisitions (and spinoffs) and is the one we would like to stress. If a firm that has not previously been part of the benchmark is acquired by a benchmark firm, its value should go up purely from moving into the benchmark. This breaks the usual valuation result which presumes that an asset purchase that does not alter any cash flows (of either the target or acquirer) should not create any value. Alternatively, if a firm were spun-off so that it moves from being part of the benchmark to no longer belonging to the benchmark, its value should drop even though its cash flows are unchanged.

The results in this section, and all the ones in the following section, depend on the compensation contract having a non-zero value of b. There is both direct empirical evidence and strong intuitive reasons for why this assumption should hold. For instance, since 2005 mutual funds in U.S. have been required to include a "Statement of Additional Information" in the prospectus that describes how portfolio managers are compensated. Ma, Tang, and Gómez (forthcoming) hand collect this information for 4500 mutual funds and find more than three quarters of the funds explicitly base compensation on performance relative to a benchmark (that they are able to identify). Bank for International Settlements (2003) presents survey-based evidence for a sample of other asset managers including sovereign wealth funds and pension funds, and also finds that performance evaluation relative to benchmarks is pervasive. To see why these results are expected, consider any asset manager that runs multiple funds with different characteristics, for instance, a bond fund and an equity fund. To compensate the portfolio managers of each fund, the simple returns cannot be meaningfully compared because of the differences in risk. However, if each fund's performance is adjusted for a benchmark for its type, then the relative performances can be compared. So, it is hardly surprising that the use of benchmarks is so pervasive and our assumption concerning b should not be controversial.

4 The General Model

We now generalize the example studied in Section 3 in several directions. All results from the previous section hold in this richer model. To analyze a new implication for investment, we will assume that y is not traded initially, so that it can be interpreted as a potential project.

We will only describe elements of the environment that differ from those described in the previous section. There are n risky stocks, whose total cash flows $D = (D_1, \ldots, D_n)^\top$ are jointly normally distributed, $D \sim N(\mu, \Sigma)$, where $\mu = (\mu_1, \ldots, \mu_n)^\top$, $\Sigma_{ii} = \text{Var}(D_i) = \sigma_i^2$, and $\Sigma_{ij} = \text{Cov}(D_i, D_j) = \rho_{ij}\sigma_i\sigma_j$. We assume that the matrix Σ is invertible. Stock prices are denoted by $S = (S_1, \ldots, S_n)^\top$. For simplicity of exposition and for easier comparison to Section 3, we normalize the total number of shares of each asset to one. However, for generality, all of our proofs in the appendix are written for the case when asset i's total number of shares is $\bar{x}_i$.

Some stocks are part of a benchmark. We order them so that all shares of the first k stocks are in the benchmark, and none of the remaining $n - k$ stocks are included. Thus, the ith element of the benchmark portfolio equals the total number of shares of asset i times $\mathbf{1}_i$, where $\mathbf{1}_i = 1$ if $i \in \{1, \ldots, k\}$ and $\mathbf{1}_i = 0$ if $i \in \{k+1, \ldots, n\}$. Denote further $\mathbf{1_b} = (\mathbf{1}_1, \ldots, \mathbf{1}_n)^\top = (\underbrace{1, \ldots, 1}_{k}, \underbrace{0, \ldots, 0}_{n-k})^\top$.

We follow the convention in the literature (see, e.g., Buffa, Vayanos, and Woolley, 2014) by defining $r_x = x^\top(D - S)$ to be the performance of portfolio $x = (x_1, \ldots, x_n)^\top$ and

$r_{\mathbf{b}} = \mathbf{1}_{\mathbf{b}}^{\top}(D - S)$ to be the performance of the benchmark portfolio. Then the compensation of an asset manager with contract (a, b, c) is $w = ar_x + b(r_x - r_{\mathbf{b}}) + c$.[10]

Denote by $x^C = (x_1^C, \ldots, x_n^C)^{\top}$ and $x^{AM} = (x_1^{AM}, \ldots, x_n^{AM})^{\top}$ the optimal portfolio choices of a conventional investor and an asset manager, respectively.

Lemma 1 (**Portfolio Choice**). *Given asset prices S, the demands of a conventional investor and an asset manager are given by*

$$x^C = \Sigma^{-1}\frac{\mu - S}{\gamma}, \tag{8}$$

$$x^{AM} = \frac{1}{a+b}\Sigma^{-1}\frac{\mu - S}{\gamma} + \frac{b}{a+b}\mathbf{1}_{\mathbf{b}}. \tag{9}$$

The demands generalize those from the example exactly as would be expected. In particular, the conventional investors opt for the mean-variance portfolio and the asset managers choose a linear combination of that portfolio and the benchmark. The fact that part of the asset managers' portfolio is invested in the benchmark regardless of prices or other characteristics of these assets will again be crucial for our results below.

An extreme form of our asset manager is a passive manager—someone who faces a very high b, which incentivizes her to hold just the benchmark portfolio and severely punishes any deviations from it. We will discuss this special case further in subsection 4.5.

Using (8)–(9) and the market-clearing condition $\lambda_{AM}x^{AM} + \lambda_C x^C = \mathbb{1} \equiv (1, \ldots, 1)^{\top}$, we have:

Lemma 2 (**Asset Prices**). *The equilibrium asset prices are*

$$S = \mu - \gamma\Lambda\Sigma\left(\mathbb{1} - \lambda_{AM}\frac{b}{a+b}\mathbf{1}_{\mathbf{b}}\right). \tag{10}$$

Equation (10) is a generalization of equations (3) and (4). As before, the price of a benchmark firm is higher than it would be for an otherwise identical non-benchmark firm. The reason is that as Lemma 1 shows, asset managers demand a larger amount of the stock in the benchmark.

Importantly, as Lemma 3 below demonstrates, the standard CAPM does not hold in our environment. It applies only in the special case in which no asset managers are present

[10] In Appendix B we repeat all of the analysis for the case where a manager's compensation is tied to the per-dollar return on the benchmark rather than the per-share return (performance) and confirm that our key results continue to hold.

($\lambda_{AM} = 0$ and $\lambda_C = 1$). Otherwise, the stocks' expected returns depend on *two factors*, the usual market portfolio and the benchmark.[11]

Lemma 3 (Two-Factor CAPM). *Asset returns $R_i = D_i/S_i$, $i = 1, \ldots, n$, can be characterized by*[12]

$$E(R_i) - 1 = \beta_i^{\mathbf{m}} \kappa_{\mathbf{m}} - \beta_i^{\mathbf{b}} \kappa_{\mathbf{b}},\ i = 1, \ldots, n, \tag{11}$$

where

$$\beta_i^{\mathbf{m}} = \frac{Cov(R_i, R_{\mathbf{m}})}{Var(R_{\mathbf{m}})},\ \beta_i^{\mathbf{b}} = \frac{Cov(R_i, R_{\mathbf{b}})}{Var(R_{\mathbf{b}})},\ i = 1, \ldots, n,$$

and $\kappa_{\mathbf{m}} > 0$ and $\kappa_{\mathbf{b}} > 0$ are the market and benchmark risk premia, and $R_{\mathbf{m}}$ and $R_{\mathbf{b}}$ are the market and benchmark returns, respectively, reported in Appendix A.

The benchmark portfolio emerges as a factor because asset managers are evaluated relative to it. Stocks that load positively on this factor have lower expected returns because asset managers overinvest in the benchmark, which drives down the expected returns on its components. Stocks outside the benchmark that covary positively with the benchmark also have lower expected returns because conventional investors (as well as asset managers through their mean-variance portfolio) who desire exposure to the benchmark buy these cheaper, non-benchmark stocks instead, pushing up their prices. Lemma 3 demonstrates this formally.

The two-factor CAPM is not intended to be a fully credible asset pricing model. Our model certainly has its limitations because it does not account for the fact that in practice managers are evaluated relative to heterogeneous benchmarks, representing different investment objectives (large cap, value, growth, etc.).[13] Rather, we emphasize that the prevailing corporate finance approach to valuation based on the "asset beta" coupled with the standard CAPM does not apply in our economy. Lemma 3 implies that the cost of capital for firms inside the benchmark is lower than for their identical twins that are outside. Therefore, the usual conclusion that the value of a project is independent of which firm adopts it does not hold.

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[11] This result has been obtained in Brennan (1993).

[12] The left-hand side of equation (11) contains the return in excess of the (gross) return on the risk-free bond, where the latter is normalized to one in our model.

[13] See Cremers, Petajisto, and Zitzewitz (2012) for a multi-benchmark model that explains the cross-section of mutual fund returns.

4.1 Investment

Suppose there is a project with cash flows $Y \sim N(\mu_y, \sigma_y^2)$, and $\text{Corr}(Y, D_i) = \rho_{iy}$ for $i = 1, \ldots, n$. Investing in this project requires spending I. If firm i (whose cash flows are D_i) invests, its cash flows in period 1 become $D_i + Y$. Let $S^{(i)} = \left(S_1^{(i)}, \ldots, S_n^{(i)}\right)^\top$ denote the stock prices if firm i invests in the project. The firm finances investment by issuing equity.[14] That is, we assume that if firm i invests in the project, it issues δ_i additional shares to finance it, where $\delta_i S_i^{(i)} = I$. If firm i is in the benchmark, then the additional shares enter the benchmark.

To proceed, suppose firm i (and only firm i) invests in the project. Then the new cash flows are $D^{(i)} = D + (0, ..., 0, \underbrace{D_y}_{i}, 0, ..., 0)^\top$, distributed according to $N\left(\mu^{(i)}, \Sigma^{(i)}\right)$, where $\mu^{(i)} = \mu + (0, ..., 0, \underbrace{\mu_y}_{i}, 0, ..., 0)^\top$ and

$$\Sigma^{(i)} = \Sigma + \begin{pmatrix} \mathbf{0} & \begin{matrix}\rho_{1y}\sigma_1\sigma_y \\ \vdots\end{matrix} & \mathbf{0} \\ \rho_{1y}\sigma_1\sigma_y \quad \ldots & \sigma_y^2 + 2\rho_{iy}\sigma_i\sigma_y & \ldots \quad \rho_{ny}\sigma_n\sigma_y \\ \mathbf{0} & \begin{matrix}\vdots \\ \rho_{ny}\sigma_n\sigma_y\end{matrix} & \mathbf{0} \end{pmatrix}.$$

Denote $I^{(i)} = (0, ..., 0, \underbrace{I}_{i}, 0, ..., 0)^\top$.

Lemma 4 (Post-Investment Asset Prices). *The equilibrium stock prices when firm i invests in the project are given by*

$$S^{(i)} \quad = \quad \mu^{(i)} - I^{(i)} - \gamma\Lambda\Sigma^{(i)} \left(\mathbb{1} - \lambda_{AM} \frac{b}{a+b} \mathbf{1_b}\right). \tag{12}$$

The change in the stockholder value of the investing firm i, $\Delta S_i \equiv S_i^{(i)} - S_i$, is

$$\begin{aligned} \Delta S_i = \mu_y - I - \gamma\Lambda\left(\sigma_y^2 + \rho_{iy}\sigma_i\sigma_y\right)\left(1 - \lambda_{AM}\frac{b}{a+b}\mathbf{1}_i\right) \\ -\gamma\Lambda\sum_{j=1}^{n}\rho_{jy}\sigma_j\sigma_y\left(1 - \lambda_{AM}\frac{b}{a+b}\mathbf{1}_j\right). \end{aligned} \tag{13}$$

[14] As in the example, the main results that follow hold even if the firm uses some debt financing. For instance, the size of the benchmark inclusion subsidy is literally identical even if the firm has risk-free debt.

The first two terms in the first line of (13) are the expected cash flows of the project net of the cost of investment, and the remaining terms reflect the penalty for risk. It is evident from (13) that this penalty differs if i is part of the benchmark, so it will turn out to be subject to the benchmark inclusion subsidy that we have already seen in the example in Section 3.

Notice that the terms on second line of (13) are the same regardless of the identity of the investing firm. When any firm invests in a project positively correlated with the benchmark, this firm's cash flows become more correlated with the benchmark. As we have seen from the two-factor CAPM, the presence of asset managers makes stocks that covary positively with the benchmark more expensive relative to what they would have been in the economy with only conventional investors. This happens because the investors substitute from the expensive stocks in the benchmark to stocks that are correlated with it. A similar logic applies to new investment projects that are correlated with the benchmark: relative to the economy without asset managers, firms have a higher valuation for projects whose cash flows are positively correlated with cash flows of benchmark firms. This force, not present in Section 3, applies to all firms contemplating investment, including the non-benchmark ones. We comment further on how it affects firms' investment incentives at the end of this subsection.

We are now ready to derive the benchmark inclusion subsidy in this generalized setting. Consider incentives of firm i to invest in a project. It will do so if its stockholder value goes up as a result of the investment, that is, if $\Delta S_i > 0$. Consider two firms, i_{IN} and i_{OUT}, one in the benchmark and the other is not (i.e., $i_{\text{IN}} \leq k$ and $i_{\text{OUT}} > k$). Suppose that their cash flows with and without the project are identical; specifically, $\sigma_{i_{\text{IN}}} = \sigma_{i_{\text{OUT}}} = \sigma$ and $\rho_{i_{\text{IN}}y} = \rho_{i_{\text{OUT}}y} = \rho_y$. The difference in the incremental stockholder value created by the investment for the two firms is

$$\Delta S_{i_{\text{IN}}} - \Delta S_{i_{\text{OUT}}} = \gamma \Lambda (\sigma_y^2 + \rho_y \sigma \sigma_y) \lambda_{AM} \frac{b}{a+b}. \tag{14}$$

The right hand side of (14) is the analytical expression for the benchmark inclusion subsidy.

Assumption 1. $\sigma_y^2 + \rho_y \sigma \sigma_y > 0$.

So long as Assumption 1 holds, the benchmark inclusion subdidy is positive, and the increase in the stockholder value for the firm in the benchmark is larger than that for the firm outside the benchmark.

In practice one would expect Assumption 1 to hold for most investments. A typical project that a firm undertakes is similar to its existing activities. Even if a project is diversifying, it is still typically positively correlated with the firm's original cash flows.

The more general structure that we consider in this section allows us to fully characterize the benchmark inclusion subsidy in (14) and to derive additional implications relative to Section 3. Notice that the subsidy is the sum of two terms. The first term, $\gamma\Lambda\sigma_y^2\lambda_{AM}b/(a+b)$, is the one that we have already seen in Section 3. It essentially captures the "index effect" for project y, since the investment effectively moves y's cash flows in the benchmark.[15] The second term, $\gamma\Lambda\rho_y\sigma\sigma_y\lambda_{AM}b/(a+b)$, is new. It is proportional to the covariance between the existing and new cash flows, $\rho_y\sigma\sigma_y$, and so we refer to this term as the "covariance subsidy." Intuitively, when the existing and new cash flows are positively correlated, the covariance increases the overall variance of post-investment cash flows. And because the cash-flow variance is penalized less for firms that are inside the benchmark, the subsidy increases in the covariance. If ρ_y is positive, the covariance subsidy is positive and hence, the benchmark inclusion subsidy exceeds the index effect. The covariance subsidy is the largest when $\rho_y = 1$, so that y is a clone of the existing assets. Moreover, assuming that the correlation ρ_y is large enough and the variance of existing cash flows exceeds that of the new cash flows, i.e., $\sigma > \sigma_y$ (both are empirically reasonable assumptions), the covariance subsidy exceeds the index effect.

Keeping in mind that the benchmark inclusion subsidy arises from taking a difference-in-differences, we can further explain the terms that comprise it. The asset managers subsidize the variance of a benchmark firm's post-investment cash flow, which is $\sigma_i^2 + \sigma_y^2 + 2\rho_{iy}\sigma_i\sigma_y$. The first term, σ_i^2, washes out of the first difference given by equation (13) because it is present for the benchmark firm pre- and post-investment. Furthermore, the subsidy includes only one covariance term $\rho_{iy}\sigma_i\sigma_y$, not two. This is because any firm, either inside the benchmark or not, receives a subsidy for the covariance with the benchmark (one can see this from the second line of (13), which is the same for all firms). That is, projects with a positive covariance with the benchmark are more valuable, even if a non-benchmark firm undertakes them. The reason is that since prices of benchmark stocks are inflated due to the mechanical demand from asset managers, conventional investors (and asset managers through their mean-variance portfolios) substitute into assets that provide exposure to the

[15] Technically speaking, if firm y were to join the index as a standalone firm and firm k gets pushed out from the index as a result, then the index effect (in absolute terms) for firm y would be $S'_y - S_y = \gamma\Lambda(\sigma_y^2 - \rho_{ky}\sigma_k\sigma_y)\lambda_{AM}(b/a + b)$. (We discuss this in more detail in the context of IPOs in subsection 4.3.) Since in our setup the effective "index inclusion" of y happens as a result of investment or merger, no other firm is removed from the index, and hence the expression for the benchmark inclusion subsidy does not include the $(-\rho_{ky}\sigma_k\sigma_y)$ term.

benchmark without being in the benchmark itself. Consequently, of the two covariances that enter the extra variance, one is subsidized regardless of which firm invests and the other is subsidized only when the investing firm is a benchmark firm. Hence, one of the two covariances drops out from the difference-in-differences.

The presence of the benchmark inclusion subsidy translates into different investment rules for firms inside and outside the benchmark. We formalize this result in Proposition 1 below.

Proposition 1 (Project Valuation). *A firm in the benchmark is more likely to invest in a project than a firm outside the benchmark if and only if Assumption 1 holds. More precisely, all else equal, a firm in the benchmark accepts projects with a lower mean μ_y, larger variance σ_y^2, and/or larger correlation ρ_y than an otherwise identical firm outside the benchmark if and only if Assumption 1 holds.*

Proposition 1 is at odds with the textbook treatment of investment taught in basic corporate finance courses. The usual rule states that a project's value is independent of which firm undertakes it, and is simply given by the project's cash flows discounted at the project-specific (not firm-specific) cost of capital.[16] The usual rule presumes that the correct way to evaluate the riskiness of a project is to use the CAPM. That is not true in our model. In our model, the compensation for risk is described by a two-factor CAPM (Lemma 3), which accounts for the incentives of asset managers.

The reason why a project is worth more to a firm in the benchmark than to one outside it is because when the project is adopted by the benchmark firm, it is incrementally financed by asset managers regardless of its variance. So, the additional overall cash-flow variance that the project generates is penalized less for a firm inside the benchmark. To further understand the importance of the variance, consider a special case where the project is risk free, i.e., $\sigma_y^2 = 0$. Then Assumption 1 fails and we can see that the project would be priced identically by all firms.

Remark 1 (Risk-Free Projects). *If $\sigma_y^2 = 0$, then a firm's valuation of project y is independent of whether this firm is included in the benchmark or not.*

We can build further intuition about the model by considering what happens when the inequality in Assumption 1 is reversed. This happens if the project is sufficiently negatively correlated with the assets, that is, if $\rho_y \leq -\sigma_y/\sigma$. In this case, a project is a hedge because

[16] See for example Jacobs and Shivdasani (2012) or Berk and DeMarzo (2017), chapter 19.

it reduces the variance of a firm's cash flow. Firms inside the benchmark benefit less from this reduction because their cash-flow variance is subsidized by asset managers and they lose some of that subsidy.[17] Consequently, a benchmark firm will value such a project less than a non-benchmark firm would.

Figure 1 uses a numerical example to display the investment regions for a benchmark- and a non-benchmark firm as a function of μ_y, σ_y, and ρ_y (for a fixed σ). In the left panel, ρ_y is held constant, and σ_y and μ_y vary along the axes. On the right panel, σ_y is kept constant, and ρ_y and μ_y vary along the axes.

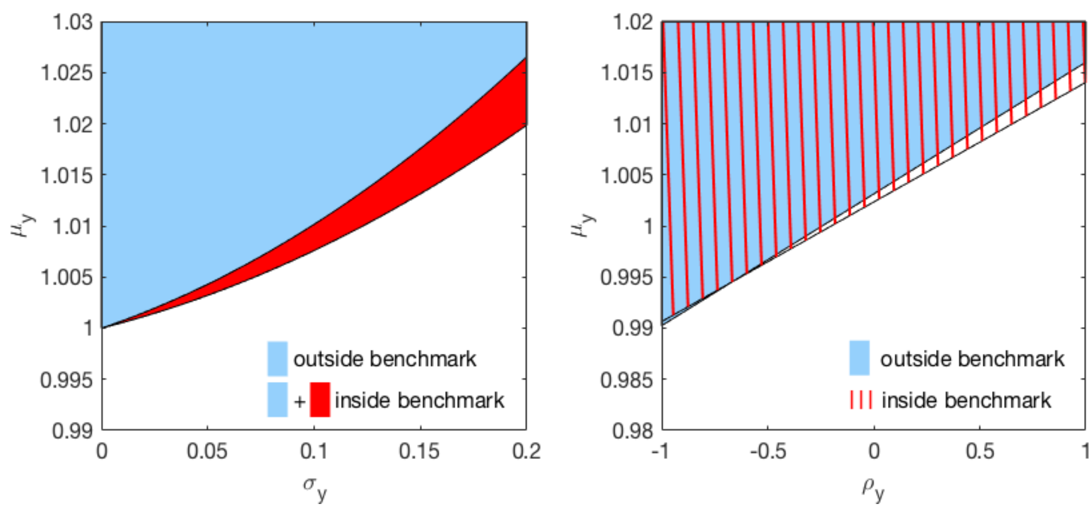

Figure 1: **Investment regions.**

Parameter values: $n = 5$, $k = 3$, $\sigma_i = 0.15$, $\rho_{ij} = 0$, $j \neq i$, $i = 1, \ldots, n$, $\rho_{jy} = 0$ for $j \neq i_{\text{IN}}, i_{\text{OUT}}$, $\gamma = 2$, $\lambda_{AM} = 0.3$, $a = 0.008$, $b = 0.042$. On the left panel, $\rho_y \equiv \rho_{i_{\text{IN}}y} = \rho_{i_{\text{OUT}}y} = 0.75$. On the right panel, $\sigma_y = 0.1$.

From the left panel we can see that holding everything else fixed, a benchmark firm will invest in projects with a lower mean, μ_y, and/or higher variance, σ_y^2, than a non-benchmark firm. The right panel illustrates that compared to a non-benchmark firm, a benchmark firm prefers to invest in projects that are more correlated with its existing cash flows.

Finally, as we mentioned earlier, our model also implies that projects correlated with assets inside and outside the benchmark are valued differently (by firms both inside and outside the benchmark). Projects that are positively correlated with the benchmark provide alternative cheaper exposure to the benchmark firms' cash flows. This is reflected

[17] Notice that when $-\sigma_y/\sigma \leq \rho_y \leq -\sigma_y/(2\sigma)$, although the investment reduces the firm's cash-flow variance, the benchmark inclusion subsidy is positive.

in equation (13), which shows that for any firm, investing in a project that is positively correlated with a component of the benchmark is more beneficial than if the project has the same degree of correlation with an asset outside of the benchmark.

To illustrate this insight graphically, Figure 2 plots the change in the stockholder value ΔS_i given by (13) as a function of ρ_{jy}, where the solid line corresponds to some asset j inside the benchmark, and the dashed line to some j outside the benchmark. In the figure, for concreteness, the investing firm i is in the benchmark. If i were outside the benchmark, these lines would shift down in parallel. The figure shows that the change in stockholder value, ΔS_i, is decreasing in the correlation coefficient ρ_{jy}. If j is in the benchmark, then the downward sloping line is flatter. Moreover, the solid line is above the dashed line for positive correlations and below for negative correlations. This is because positive (negative) correlation of the project with an asset in the benchmark is penalized (rewarded) less than the same correlation with an asset outside the benchmark.

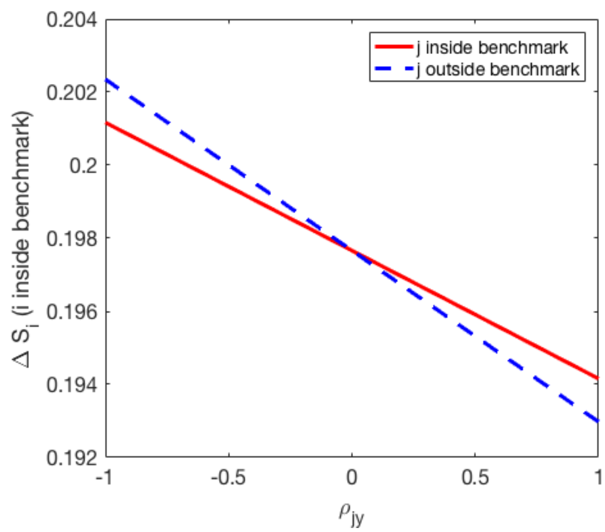

Figure 2: **Change in the stockholder value, ΔS_i, as a function of correlations of project y's cash flows with cash flows of assets inside and outside the benchmark, ρ_{jy} for some $j \leq k$ and some $j > k$.**

Parameter values: $n = 5$, $k = 3$, $\mu_y = 1.2$, $I = 1$, $\sigma_j = 0.15$, $\sigma_y = 0.1$, $\rho_{jy} = 0$ unless it is plotted on the horizontal axis, $\rho_{j\ell} = 0$, $\ell \neq j$, $j = 1, \ldots, n$, $\gamma = 2$, $\lambda_{AM} = 0.3$, $a = 0.008$, $b = 0.042$. Investing firm: $i = 1$. Solid line: $j = 2$. Dashed line: $j = 4$.

Discussions about benchmarking often revolve around the possibility that it leads to more correlation in risk exposures for the people hiring asset managers. Our model points to an additional source of potential correlation generated by benchmarking. Because benchmarking leads to higher valuations of stocks that are correlated with the benchmark, it induces firms—both inside and outside the benchmark—to take on more fundamental risk that is correlated with the benchmark (relative to the economy without benchmarking). Thus, our model predicts that cash flows in the economy with asset managers endogenously become more homogeneous/correlated with each other.

4.2 Mergers and Acquisitions

As we have seen in the example considered in Section 3, the model can also be used to think about mergers and acquisitions.

Proposition 2 **(Mergers and Acquisitions).** *Suppose firm i considers acquiring firm y that is outside the benchmark, and $\sigma_y^2 + \rho_{iy}\sigma_i\sigma_y > 0$. Then firm i is more likely to acquire y if firm i is inside the benchmark than if it is outside.*

The logic behind this statement is identical to the reasoning that leads to the bias in investment. If a benchmark firm acquires y, it gets the benchmark inclusion subsidy. Again, this result is in contrast to the conventional wisdom about the role of financing synergies in the evaluation of potential acquisitions. For example, if a firm has unused debt capacity, it might choose to use more debt financing than otherwise to buy another firm. The usual view is that the discount rate used to value the cash flows of the target firm should not be altered by the availability of the extra debt funding. The case for not adjusting the discount rate is that the same additional debt funding could have been used for any other potential acquisition. So, it would be a mistake to say that any particular target company is a more attractive firm to acquire just because some low-risk debt could be issued to finance the purchase.

In our setup, there is a more fundamental synergy that is responsible for lower financing costs. Because the asset managers will want to purchase part of any stock that is issued to undertake the transaction, those savings should be accounted for. The size of the subsidy will depend on the parameters that appear in Assumption 1. Thus, for example, all else equal, the higher is a correlation of the cash flows of the target firm with the acquiring benchmark firm, the larger will be the financing advantage associated with that acquisition. Conversely, a hedging acquisition by a firm in the benchmark, where the target firm's cash flows are negatively correlated with acquirer's, always comes with a lower subsidy.

Proposition 2 works in reverse for spinoffs and divestitures. Specifically, assuming that the condition $\sigma_y^2 + \rho_{iy}\sigma_i\sigma_y > 0$ is satisfied, a division y is worth more if it is part of a firm inside the benchmark than if it is spun off and trades as a separate entity outside the benchmark or is sold to a firm outside the benchmark.

4.3 IPOs and Incentives to Join the Benchmark

Suppose y is now a standalone firm, which is held privately by conventional investors and is considering an IPO. We demonstrate that y's incentive to go public depends on whether it will be included in the benchmark.

We consider two scenarios. In the first scenario, when firm y becomes public and gets included in the benchmark, no other firm leaves the benchmark. Most of the best known stock indexes in the world have a fixed number of firms. In the second scenario, if y joins the benchmark, then firm k is removed, so that the number of firms in the benchmark remains constant.

Proposition 3 (IPOs and Benchmarks). *Consider a privately-held firm y considering an IPO.*

(i) Firm y is always more likely to proceed with an IPO if it gets included in the benchmark and no other firm leaves the benchmark.

(ii) Firm y is more likely to proceed with an IPO if it gets included in the benchmark and firm k is removed from the benchmark, if and only if $\sigma_y^2 - \rho_{ky}\sigma_k\sigma_y > 0$.

The argument for the result in part (i) is the same as for other results in the paper—firm y gets the benchmark inclusion subsidy if it joins the benchmark. In part (ii) where y pushes another firm out of the benchmark, there is an additional consideration, as firm y loses part of the benchmark subsidy coming from its correlation with that firm. In other words, when firm y is included in the benchmark and firm k is pushed out, firm y's correlation with the benchmark increases by σ_y^2 because firm y is correlated with itself (and it enters the benchmark), and is reduced by $\rho_{ky}\sigma_k\sigma_y$ because firm k drops out of the benchmark. The net subsidy is therefore proportional to $\sigma_y^2 - \rho_{ky}\sigma_k\sigma_y$.

One could apply the above argument to any firm, not just a newly listed one. A firm is worth more inside the benchmark rather than outside. So, there is an added benefit to any corporate action that results in the firm's benchmark inclusion—for example, aimed at increasing the firm's size or meeting other criteria for benchmark inclusion. The costs of taking such action of course have to be outweighed by the benchmark inclusion subsidy, but a clear empirical prediction emerging from this discussion is that firms with good prospects of benchmark inclusion have an incentive to alter their behavior in order to gain membership in the benchmark. Similarly, benchmark firms that are close to the threshold for exclusion, have incentives to engage in potentially costly corporate actions that ensure that they retain their benchmark membership.

4.4 Comparative Statics with respect to λ_{AM}

In this subsection we analyze the benchmark inclusion subsidy as a function of the size of the asset management sector. Consider (14), and rewrite it recognizing that $\Lambda = [\lambda_{AM}/(a+b) + \lambda_C]^{-1}$ and $\lambda_C = 1 - 2\lambda_{AM}$:

$$\Delta S_{i_{\text{IN}}} - \Delta S_{i_{\text{OUT}}} = \gamma \left[1 + \frac{1-2\lambda_{AM}}{\lambda_{AM}}(a+b)\right]^{-1} b(\sigma_y^2 + \rho_y \sigma \sigma_y). \tag{15}$$

Notice that this expression is strictly increasing in λ_{AM}. This means that the effects described in this paper related to the difference in valuations by a firm inside the benchmark relative to a firm outside the benchmark become larger as the size of the asset management sector increases.

If the contract parameters a and b were endogenous, chosen optimally by shareholders, then a and b in (15) would implicitly depend on λ_{AM}. In a companion paper (Kashyap, Kovrijnykh, Li, and Pavlova, 2019), we analyze optimal contracts chosen by shareholders in a similar environment. Deriving analytical results for the contract parameters as a function of λ_{AM} is difficult in general, so we study the relationship numerically, and show that the benchmark inclusion subsidy is increasing in λ_{AM} even if a and b are endogenously determined.

4.5 Passive Asset Management

As we mentioned earlier, in a limiting case of our setup when $b \to \infty$ corresponds to having passive asset managers. In this case, it is easy to see that asset managers hold only the benchmark portfolio, i.e, $x^{AM} = \mathbf{1_b}$. A generalization of our model would be to include both active and passive asset managers. If we denote the fractions of them in the economy by λ_{AM}^A and λ_{AM}^P, then the equilibrium stock prices would be

$$S = \mu - \gamma \Lambda \Sigma \left(\mathbb{1} - \left[\lambda_{AM}^A \frac{b}{a+b} + \lambda_{AM}^P\right] \mathbf{1_b}\right),$$

where $\Lambda = \left[\lambda_{AM}^A/(a+b) + \lambda_C\right]^{-1}$.

All of our results extend to this case. Passive asset managers hold benchmark stocks irrespective of their characteristics, and they invest nothing in the mean-variance portfolio. Therefore, with passive managers the benchmark inclusion subsidy becomes even larger. For example, the additional value from investing for a firm in the benchmark is given by $\gamma\Lambda(\sigma_y^2 + \rho_y\sigma\sigma_y)\left[\lambda_{AM}^A b/(a+b) + \lambda_{AM}^P\right]$, which is a generalization of (14). The following proposition shows that this value is increasing if more asset managers are passive.

Proposition 4 (Passive Asset Managment). *The benchmark inclusion subsidy is larger when $\lambda_{AM}^P/\lambda_{AM}^A$ is larger, holding $\lambda_{AM}^P + \lambda_{AM}^A$ constant.*

As in the previous subsection, we show that the result continues to hold even if the contract parameters a and b are determined endogenously.

5 Related Empirical Evidence

We now turn to the empirical evidence that is related to the predictions of our model. In keeping with the presentation in the last section, we organize the discussion around four predictions of the model. The first implication of our model is that upon inclusion in a benchmark, there should be an increase in a firm's share price. This prediction is not original, but our theory does imply that there is an important distinction between a benchmark and an index. The second one is that firms inside the benchmark should be more prone to invest and to engage in mergers. Third, the subsidy should be higher when there are more assets under management. Finally, there should be a two-factor CAPM that accounts for the fact that correlation with the benchmark should affects stock returns.

5.1 Benchmark Effect

Consistent with the empirical evidence, our model generates an index effect. Stock price changes are symmetric for index additions and deletions and the effect persists for as long as the stock is in the index. We also have a more subtle prediction: the share price response should depend on becoming part of a benchmark and not just because of being added to an index. In most cases, separating the effect of being in the index and benchmark is challenging. One exception arises for firms that operate in so-called "sin" industries, such as alcohol, tobacco and gaming. Large firms in these industries would be included in indices such as the S&P 500, Russell 1000 or FTSE 100, but are deemed odious by some investors.

Hence, there are benchmarks that keep almost all of the firms in the index but exclude these firms.

According to the U.S. Social Investment Forum, as of the beginning of 2018, $12 trillion of assets were managed according to some sort of social screen, and $2.89 trillion were in funds that specifically avoid investing in tobacco. Hence, these kinds of exclusion are common enough to be detectable.

Hong and Kacperczyk (2009) study the returns of these so-called sin firms. Their motivation is behavioral, but the empirical results can also be interpreted as a test of our model. Their headline result is that sin firms earn higher expected returns than comparable firms by about 21 basis points per month—i.e. roughly 2.5% per year. Their matching process controls for four factors commonly thought to determine expected returns—the market portfolio, firm size, the book to market ratio, and a momentum proxy. They also find similar results for a set of sin stocks in Canada, France, Germany, Italy, the Netherlands, Spain, Switzerland and the United Kingdom. In the international sample, the sin stocks outperform their peers about 21 basis points per month.

In addition to these estimates for returns, they also estimate the differences in the levels of stock prices for sin stocks and others. These regressions control for a number of firm characteristics including current and future return on equity, research and development (relative to sales) and membership in the S&P 500. They find differences of between 15 and 20 percent depending on the valuation ratio. They also present a Gordon growth style calculation showing that the return and valuation estimates are mutually consistent. Hong and Kacperczyk's results about sin stocks have also subsequently been confirmed in several studies.[18]

Our model also suggests another subtle implication of Hong and Kacperczyk's results. When firms are dropped from a benchmark, our model predicts that this raises their cost of capital since they are denied the benchmark inclusion subsidy. Part of the benchmark inclusion subsidy is the covariance subsidy that affects how benchmark firms should evaluate alternative additional projects. On the margin, when sin stocks are excluded from a benchmark, their incentives to invest in their own sinful industries are also reduced because they lose the covariance subsidy.

[18]See Fabozzi, Ma, and Oliphant (2008), Statman and Glushkov (2009), and Kim and Venkatachalam (2011) for evidence of superior performance of sin stocks, as well as Blitz and Fabozzi (2017) who caution that the performance of sin stocks can be explained by two new quality factors.

5.2 Changes in Corporate Actions Following Benchmark Inclusion

There are some papers that attempt to assess whether the model predictions regarding investment and mergers hold for benchmark firms versus non-benchmark firms. This is challenging because ideally one wants to control for both the selection into the benchmark and all the other factors that influence these kinds of expenditures.[19]

There are three papers that we are aware of that attempt to measure these effects and all find some evidence in favor of our model's predictions. Massa, Peyer, and Tong (2005) compare 222 firms that were added to the S&P 500 with a control set of firms who prior to the addition were similar with respect to size, market-to-book, the number of analysts following them, and the percentage of stock owned by institutional investors. They treat the benchmark inclusion as an exogenous factor that can be used as an instrument for the firms' cost of capital. They then test for effects of the (instrumented) cost of capital on investment and equity issuance. They find that inclusion is associated with higher levels of equity issuance and more investment, with a substantial portion of the investment coming via increased mergers.

Vijh and Yang (2008) attempt to directly test the idea that firms included in the S&P 500 are more prone to undertake acquisition than firms outside the index. They study all the acquisitions of firms that are tracked by the Center for Research on Securities Prices between 1980 and 2004. They are motivated to test the hypothesis that benchmark inclusion brings more analyst and news coverage and hence, could lead to better governance and decision-making. Nevertheless, the basic statistical analysis can also be used to test the predictions from our model. The main challenge for this type of exercise is that S&P 500 firms are very different than non-index firms, and they also acquire different firms. For instance, the median acquiring firm in the S&P 500 index is about 15 times larger (measured by assets) than the non-index acquirers and has significantly higher levels of cash flows to assets, return on assets, and Tobin's Q. The index firms also tend to acquire larger firms, those with higher value of Tobin's Q, and more profitability. They find that firms in the

[19]For example, consider the evidence in Gutiérrez and Philippon (2017). They document that higher institutional ownership accompanies lower investment. They stress, however, that it is difficult to establish causality without any plausibly exogenous movement in ownership. Their preferred interpretation is that investment is crowded out by higher payouts, though they admit this could be due to a preference by institutional firms to find firms with high payouts. In Gutiérrez and Philippon (2018) they attempt to isolate variation in payout variation that can be ascribed to ownership structure. They find that controlling for cash flow (and other firm specific variables), the higher ownership induced payouts are associated with lower investment. Of course, the sources and uses of funds accounting identity may also lead to this kind of pattern in the data.

S&P500 do undertake significantly more acquisitions, in line with our model's predictions. These findings hold after they account (as much as they can) for observable differences in target and acquirer characteristics, though it is hard to know whether the controls are truly adequate.

The third and perhaps most convincing piece of evidence comes from Bena, Ferreira, Matos, and Pires (2017). They study differences in investment and employment for firms across 30 countries between 2001 and 2010. Their basic regression relates capital expenditures relative to assets (or the number of employees) to institutional ownership by foreign investors and a host of firm-level controls (including sales, Tobin's Q, and cash holdings). Importantly, they instrument for the ownership variable using additions to the MSCI ACWI index. They find a large, statistically significant effect of the benchmark additions on both investment and employment. The results are also present when they restrict the analysis to firms that are close to the cutoff for inclusion in the index and when they estimate the effects of inclusion using a difference-in-difference experimental design.

Bena, Ferreira, Matos, and Pires (2017) also analyze patenting patterns to study how research and development is affected by benchmark inclusion. It is common to use research and development indicators as a proxy for corporate risk-taking (e.g. Coles, Daniel, and Naveen, 2006 and Gormley, Matsa, and Milbourn, 2013). They show that research and development increases significantly with benchmark inclusion and this finding is robust to controlling for a number of factors and is confirmed in their difference-in-differences tests. This evidence is consistent with the model prediction that a firm inside the benchmark would accept riskier projects than an otherwise indentical firm outside the benchmark would.

5.3 Variation in the Subsidy Size

The comparative-statics prediction regarding the effect of λ_{AM} on the subsidy can be inferred from re-interpreting previous work on index inclusion. Our model makes predictions about how λ_{AM} affects both the index effect and the covariance subsidy, although existing studies only inform us about the former. Past research contains two types of relevant evidence.

One type involves the time-series prediction that the size of the index inclusion effect should rise as the asset management sector grows. There are some practical challenges that arise in carrying out this kind of test. It is relatively straightforward to calculate the announcement return (computed over one or two days) associated with the news that a stock will be added or subtracted from an index—typically the S&P 500. However, to infer the permanent effect of that change, a subsequent return must be computed to account for any reversal, and that requires a decision on how long the window should be. See Patel and Welch (2017) for a good discussion of this issue.

Our reading of the evidence is that the announcement effects associated with inclusions and exclusion from the 1980s through the early 2000s were becoming larger, see, e.g., Wurgler and Zhuravskaya (2002). Since the early 2000s there are fewer studies on the S&P 500. Patel and Welch (2017) argue that the announcement effect is smaller and the post-announcement reversal is larger since 2005.

Another confounding problem is that as the inclusion effect has become better known, sophisticated investors (e.g., hedge funds, and, more recently, some ETF and index funds) have started buying a portfolio of stocks shortlisted for index inclusion prior to the announcement day. Such front-running creates a pre-announcement drift in stock prices. For example, Patel and Welch (2017) document that deleted stocks lose 12% of their value in 42 days preceding the announcement. The front-running that precedes additions and deletions also creates a hidden cost of indexing to passive investors who only make substitutions when the additions and deletions go into effect. For example, Petajisto (2011) estimates the hidden costs of rebalancing for index funds as 21–28 bps annually for the S&P 500 and 38–77 bps annually for the Russell 2000. He stresses that these estimates are lower bounds because his measurement window does not fully account for front-running, which is especially relevant in the later part of his sample.

An alternative test that we find quite compelling is a cross-sectional test that becomes possible when a stock moves from being part of one index to another. Studying this kind of change has two advantages relative to the time-series tests. First, this kind of movement is typically not triggered because a corporate action occurred (such as a recent merger), so the index switch is not necessarily associated with changes in the firm's cash-flow properties. Second, these events are not subject to the alternative interpretation offered by Merton (1987), that addition to an index brings increased analyst coverage and other forms of attention. If a stock is already part of one index, then that type of attention should already be at least partially present.

Perhaps the cleanest of these studies uses changes that move a stock around the boundary of being above and below the 1000th largest stock in the U.S. Historically, for firms that move from just below rank 1000 to just above, they move out of the Russell 2000 benchmark and into the Russell 1000 (and vice versa). Chang, Hong, and Liskovich (2015) study these transitions. The interesting thing is that the firm whose fortunes improve, move from the widely-benchmarked Russell 2000 index to the less-benchmarked Russell 1000. Although their fundamentals are improving, the demand by asset managers will have declined. The authors find that despite the improved fundamentals, their share price drops by about 5% from the rebalancing. Conversely, firms that fall into the Russell 2000 see price increases by about 5%.

5.4 Asset Pricing Tests

Finally, an alternative way to assess the existence of the benchmark effect is to see whether inclusion in a benchmark shows up as a factor that helps explain the cross section of stock returns. Two direct tests of the specification equivalent to our two-factor CAPM in Lemma 3 are presented in Gómez and Zapatero (2003) and Brennan, Cheng, and Li (2012). They arrive at conflicting conclusions.

Gómez and Zapatero (2003) use the S&P 500 index as a proxy for the index factor. They test the model on the universe of stocks that were included in the S&P 500 for the entire duration of their sample. They find that the index factor is priced and that the risk premium for the factor is sizeable and of the correct sign. Moreover, the size of the risk premium for the index factor is rising over the sample period, and so is the associated statistical significance. Gómez and Zapatero argue that this trend is consistent with the growth of the asset management industry (growth in λ_{AM} in our model).

Brennan, Cheng, and Li (2012) also use the S&P 500 index as a proxy for the index factor, but expand the universe of risky assets to all CRSP stocks and include size (market cap) as a control in their tests. These two changes to the test end up significantly reducing the risk premium on the index factor, making it very small and virtually undetectable. The index factor comes out both economically and statistically significant only for a subsample of large stocks, consistent with the results of Gómez and Zapatero (2003). Furthermore, when they allow for the possibility of multiple indexes, with the remaining indexes representing value and size indexes and proxied for by the Fama and French (1992) HML and SMB factors, the index factor loses significance even for large stocks.

One challenge for both papers (and any other attempt) to identify an index factor is the presence of multiple benchmarks. The critical consideration governing relative returns in our model is the relative demand for different stocks by all asset managers. So in either of these papers the presence of additional benchmarks (e.g. the FTSE Russell 1000 or 2000) will confound the tests. Cremers, Petajisto, and Zitzewitz (2012) show that a multi-benchmark model fits a cross-section of mutual fund returns well.

6 Concluding Remarks

We have seen that for firms that are part of a benchmark, the inelastic demand for their shares by asset managers lowers their cost of capital for investments, mergers, and IPO decisions. We have specific cross-sectional predictions for the size of this effect. While there is empirical evidence that speaks to some of these predictions, there are others that have yet to be tested. One obvious direction for future work would be to fill in these gaps.

For instance, there are many claims by practitioners (e.g. McKinsey on Finance, 2004) that a strong motive for undertaking an IPO is to become part of a benchmark. We believe no one has tested this hypothesis. Despite the practitioner attention, this implication is not part of the very long list of commonly cited reasons by economists that are usually considered. For example, Celikyurt, Sevilir, and Shivdasani (2010) observe, "in theory, an IPO creates liquidity for the firm's shares, provides an infusion of capital to fund growth, allows insiders to cash out, provides cheaper and ongoing access to capital, facilitates the sale of the company, gives founders the ability to diversify their risk, allows venture capitalists and other early stage investors to exit their investment, and increases the transparency of the firm by subjecting it to capital market discipline." So there would be some novelty value to confirming the model prediction.

More importantly, international differences create variation in IPO incentives that would make it possible to cleanly uncover the predicted effect. Specifically, not only do different exchanges have different requirements about how many shares have to be floated, but the relevance of benchmarks also varies across markets. So, the ease of qualifying for a public listing across markets will differ from the size of the subsidy implied by our theory. This should make it feasible to test the theory.

Our model also predicts that the index effect is larger for firms with riskier cash flows. This can be seen from equation (7), where the index effect is increasing in σ_y^2, even after controlling for the stock price before the inclusion. The literature so far has focused on estimating the average index effect. It would be interesting to see if the index effect varies with firms' risk characteristics, though finding a suitable empirical analog to cash-flow volatility would be tricky.

It would also be interesting to test the model's predictions about how the presence of benchmarks can alter the incentives regarding mergers. We saw that the benchmark inclusion subsidy is larger (smaller) for targets whose cash flows are more positively (negatively) correlated with the acquiring firm. While there is a large literature studying merger patterns, we believe this somewhat unusual prediction of our theory has not been investigated. Furthermore, our model may suggest an alternative explanation for the rise in the industry concentration ("monopolization") in the U.S. over the past 15 years.[20] A significant driving force behind this phenomenon is mergers. According to our model, firms in prominent benchmarks (e.g., the S&P 500) value targets outside the benchmark above their standalone values, and the valuation gap increases with the size of the asset management sector. Perhaps the rapid growth of the asset management industry over the last 15 years has contributed to the increased merger activity.

Another model prediction is that firms—both inside and outside the benchmark—are rewarded for taking on more fundamental risk that is correlated with the benchmark (relative to the economy without benchmarking). Thus, our model predicts that cash flows in the economy with asset managers endogenously become more homogeneous/correlated with each other. This force could eventually subtly change business-cycle dynamics. However, this effect will take time to play out, so finding an empirical strategy to identify it will be challenging.

Finally, it might be important to estimate the size of the benchmark inclusion subsidy. To do that directly, it would be necessary to estimate the model parameters that enter the expression for the benchmark inclusion subsidy. In particular, estimating variances and covariances for the relevant cash flows would require data that is not typically analyzed, so this would not be a simple task.

As we discussed in Section 4, the benchmark inclusion subsidy, e.g., in the case of a merger, is the sum of the index effect for the target firm and the covariance subsidy. Assuming the covariance of the cash flows for the acquirer and target firm is non-negative, the index effect (for the target firm) constitutes a lower bound for the subsidy. Thus, an estimate for the average index effect could help determine this lower bound.

It is not obvious which estimate from the literature is most appropriate for this kind of calculation. One candidate is the 6% estimate for S&P 500 inclusions that is indicative of the findings in the index inclusion literature. Connecting this estimate to our model is tricky. In practice, active asset managers benchmarked against the S&P 500 never hold all 500 stocks, instead they typically hold only about 100 of its largest and most liquid stocks.

[20] David Leonhardt, "The Monopolization of America", *The New York Times*, November 25, 2018, https://www.nytimes.com/2018/11/25/opinion/monopolies-in-the-us.html.

Such a strategy saves on trading costs while delivering a portfolio that is still sufficiently close to the benchmark. New additions to the index are typically smaller and less liquid stocks. Passive managers may buy them on the inclusion date because of their mandates, but active managers are unlikely to do so. The average price increase of 6% estimated in the literature, therefore, only reflects purchases by a subset of asset managers.

An alternative estimate of the valuation impact of benchmark inclusion could be gleaned from the literature on sin stocks, which compares sin stocks to otherwise identical stocks that are not excluded from benchmarks. Recall that Hong and Kacperczyk (2009) find estimates of between 15% and 20%. It is hard to say if these numbers overstate or understate our benchmark inclusion subsidy. On the one hand, their exercise is aimed at determining what happens if some investors completely avoid owning the sin stocks. That would not generally be the case in our model, because in our model, asset managers and conventional investors still hold non-benchmark stocks. This suggests that Hong and Kacperczyk's figures could overstate what we are trying to measure. On the other hand, mandates that exclude sin stocks represent only a fraction of the total money that is managed against benchmarks. Furthermore, their estimates do not account for our covariance-subsidy component of the benchmark inclusion subsidy, which is positive under the natural assumption that cash flows of most traded firms are positively correlated. These latter two considerations imply that Hong and Kacperczyk's estimates understate the figure we care about. We have no way of judging the net effect of these factors. Accordingly, it seems reasonable to consider a range of possible valuation effects of between 6% and 20%.

Given an estimate for the change in value from joining the benchmark, one can use the Gordon growth model to convert that figure into an estimate for the change in the expected return on equity—under the assumptions that the growth rate of dividends after joining the benchmark are unchanged, and that the initial dividend price ratio is known.[21] If we assume that the dividend price ratio is 5% and that dividends grow at about 6% (the recent historical average for the S&P 500 firms), then this suggests a change in expected returns of between about 30 and 100 basis points.[22] This range strikes us as being meaningful,

[21] Formally, $S = D_1/(r_E - g)$ so that $S^{before} = D_1/(r_E^{before} - g^{before})$ and $S^{after} = D_1/(r_E^{after} - g^{after})$, and therefore $D_1/S^{after} \times (S^{after} - S^{before})/S^{before} = r_E^{before} - r_E^{after} - (g^{before} - g^{after})$. Using $D_1/S^{after} = D_0(1+g)/S^{after}$, we can compute $r_E^{before} - r_E^{after}$.

[22] This calculation is similar in spirit to the one offered by Hong, Kubik, and Stein (2008) who attempt to quantify the impact of home bias in the preference for stocks on expected returns. Their home bias estimates suggest price differences of between 5% and 10% for firms in lightly populated and densely populated areas, so their estimate on the difference in expected returns is consistent with the lower end of our range of estimates.

though we acknowledge it rests on many assumptions.

Appendix A

In the main text, for simplicity of exposition we normalize the total supply of shares of each asset to one. Here, to show the generality of our analysis, we suppose that stock i has the total supply of $\bar{x}_i$ shares. The per-share cash flow of asset i is then $D_i/\bar{x}_i$.

Proof of Lemma 1. Denote by $\hat{x}_i^\ell$ the fraction of shares of asset i that agent $\ell \in \{C, AM\}$ holds, i.e., $\hat{x}_i^\ell = x_i^\ell/\bar{x}_i$. Let $\hat{x}^\ell = \left(\hat{x}_1^\ell, \ldots, \hat{x}_n^\ell\right)^\top$, $\ell \in \{C, AM\}$, and $\bar{x} = (\bar{x}_1, \ldots, \bar{x}_n)^\top$. Then the maximization problem of a conventional investor with $\hat{x}^C = z$ is the same as that of an asset manager with $(a+b)\hat{x}^{AM} + b\mathbf{1_b} = z$ and can be written as $\max_z -E\exp\{-\gamma(z(D - \bar{x}\cdot S))\}$, where $\bar{x}\cdot S = (\bar{x}_1 S_1, \ldots, \bar{x}_n S_n)^\top$. It is well known that when asset returns are normally distributed, the optimization of an agent with CARA preferences is equivalent to the following mean-variance problem:

$$\max_z z^\top(\mu - \bar{x}\cdot S) - \frac{\gamma}{2} z^\top \Sigma z.$$

The optimal solution is $z = \Sigma^{-1}(\mu - \bar{x}\cdot S)/\gamma$. Thus we have

$$\hat{x}^C = \Sigma^{-1}\frac{\mu - \bar{x}\cdot S}{\gamma}, \tag{16}$$

$$\hat{x}^{AM} = \frac{1}{a+b}\Sigma^{-1}\frac{\mu - \bar{x}\cdot S}{\gamma} + \frac{b}{a+b}\mathbf{1_b}. \tag{17}$$

When $\bar{x} = \mathbb{1} \equiv (1, \ldots, 1)^\top$, $\hat{x}^\ell = x^\ell$ for $\ell \in \{C, AM\}$ and we have equations (8) and (9). $\square$

Proof of Lemma 2. Using the market-clearing condition $\lambda_{AM}\hat{x}^{AM} + \lambda_C \hat{x}^C = \mathbb{1}$, we have the vector of the total share value of the firms

$$\bar{x}\cdot S = \mu - \gamma\Lambda\Sigma\left(\mathbb{1} - \lambda_{AM}\frac{b}{a+b}\mathbf{1_b}\right), \tag{18}$$

This gives us (10) when $\bar{x} = \mathbb{1}$. $\square$

Proof of Lemma 3. Let $\omega_i^{\mathbf{m}} = \bar{x}_i S_i / \sum_{j=1}^n \bar{x}_j S_j$, $i = 1, \ldots, n$, denote the market portfolio weights and let $\omega_i^{\mathbf{b}} = \mathbf{1}_i \bar{x}_i S_i / \sum_{j=1}^n \mathbf{1}_j \bar{x}_j S_j$, $i = 1, \ldots, n$, denote the benchmark portfolio weights. We have the market and benchmark returns equal to

$$R_{\mathbf{m}} = \sum_{j=1}^{n} \omega_j^{\mathbf{m}} \frac{D_j}{\bar{x}_j S_j} = \frac{\sum_{j=1}^{n} D_j}{\sum_{j=1}^{n} \bar{x}_j S_j},$$

$$R_{\mathbf{b}} = \sum_{j=1}^{n} \omega_j^{\mathbf{b}} \frac{D_j}{\bar{x}_j S_j} = \frac{\sum_{j=1}^{k} D_j}{\sum_{j=1}^{k} \bar{x}_j S_j}.$$

To show (11), recall that $E(R_i) = \mu_i/(\bar{x}_i S_i)$. Take the ith row of (10), divide both sides by $\bar{x}_i S_i$ and rearrange terms to get

$$\begin{aligned}
E(R_i) - 1 &= \gamma\Lambda \sum_{j=1}^{n} \bar{x}_j S_j \mathrm{Cov}(R_i, R_{\mathbf{m}}) - \gamma\Lambda\lambda_{AM} \frac{b}{a+b} \sum_{j=1}^{k} \bar{x}_j S_j \mathrm{Cov}(R_i, R_{\mathbf{b}}) \\
&= \frac{\mathrm{Cov}(R_i, R_{\mathbf{m}})}{\mathrm{Var}(R_{\mathbf{m}})} \mathrm{Var}(R_{\mathbf{m}}) \gamma\Lambda \sum_{j=1}^{n} \bar{x}_j S_j - \frac{\mathrm{Cov}(R_i, R_{\mathbf{b}})}{\mathrm{Var}(R_{\mathbf{b}})} \mathrm{Var}(R_{\mathbf{b}}) \gamma\Lambda\lambda_{AM} \frac{b}{a+b} \sum_{j=1}^{k} \bar{x}_j S_j \\
&= \beta_j^{\mathbf{m}} \kappa_{\mathbf{m}} - \beta_j^{\mathbf{b}} \kappa_{\mathbf{b}},
\end{aligned}$$

where $\kappa_{\mathbf{m}} = \mathrm{Var}(R_{\mathbf{m}}) \gamma\Lambda \sum_{j=1}^{n} \bar{x}_j S_j$, and $\kappa_{\mathbf{b}} = \mathrm{Var}(R_{\mathbf{b}}) \gamma\Lambda \sum_{j=1}^{k} \bar{x}_j S_j \lambda_{AM} b/(a+b)$. $\square$

Proof of Lemma 4. Suppose firm i adopts the project. Then the total number of shares of asset i becomes $\bar{x}_i^{(i)} = \bar{x}_i + \delta_i$, where $\delta_i S_i^{(i)} = I$, and $\bar{x}_j^{(i)} = \bar{x}_j$ for $j \neq i$.

Adopting (18) for this case, we have

$$\bar{x}^{(i)} \cdot S^{(i)} = \mu^{(i)} - \gamma\Lambda\Sigma^{(i)} \left(\mathbb{1} - \lambda_{AM} \frac{b}{a+b} \mathbf{1_b} \right).$$

Finally, using the definition of $\bar{x}^{(i)}$, we have

$$\bar{x} \cdot S^{(i)} = \mu^{(i)} - I^{(i)} - \gamma\Lambda\Sigma^{(i)} \left(\mathbb{1} - \lambda_{AM} \frac{b}{a+b} \mathbf{1_b} \right), \tag{19}$$

which simplifies to (12) when $\bar{x} = \mathbb{1}$.

The ith element of $\bar{x} \cdot S$ is

$$\bar{x}_i S_i = \mu_i - \gamma\Lambda \sum_{j=1}^{n} \rho_{ij} \sigma_i \sigma_j \left(1 - \lambda_{AM} \frac{b}{a+b} \mathbf{1}_j \right) \tag{20}$$

and the ith element of $\bar{x} \cdot S^{(i)}$ is

$$\bar{x}_i S_i^{(i)} = \mu_i + \mu_y - I - \gamma\Lambda\left(\sigma_y^2 + \rho_{iy}\sigma_i\sigma_y\right)\left(1 - \lambda_{AM}\frac{b}{a+b}\mathbf{1}_i\right)$$
$$-\gamma\Lambda\sum_{j=1}^{n}\left[\rho_{ij}\sigma_i\sigma_j + \rho_{jy}\sigma_j\sigma_y\right]\left(1 - \lambda_{AM}\frac{b}{a+b}\mathbf{1}_j\right). \tag{21}$$

Subtracting (20) from (21), obtain

$$\bar{x}_i \Delta S_i = \mu_y - I - \gamma\Lambda\left(\sigma_y^2 + \rho_{iy}\sigma_i\sigma_y\right)\left(1 - \lambda_{AM}\frac{b}{a+b}\mathbf{1}_i\right)$$
$$-\gamma\Lambda\sum_{j=1}^{n}\rho_{jy}\sigma_j\sigma_y\left(1 - \lambda_{AM}\frac{b}{a+b}\mathbf{1}_j\right), \tag{22}$$

which is (13) when $\bar{x}_i = 1$. For $i_{\text{IN}} \in \{1, \ldots, k\}$ and $i_{\text{OUT}} \in \{k+1, \ldots, n\}$ we have

$$\bar{x}_{i_{\text{IN}}}\Delta S_{i_{\text{IN}}} - \bar{x}_{i_{\text{OUT}}}\Delta S_{i_{\text{OUT}}} = \gamma\Lambda(\sigma_y^2 + \rho\sigma\sigma_y)\lambda_{AM}\frac{b}{a+b}. \tag{23}$$

□

Proof of Proposition 1. Follows immediately from (14) (or its analog (23)). □

Proof of Proposition 2. The only essential difference with the proof of Lemma 4 that implies Proposition 1 is that when firm y is traded before the merger, then (20) becomes

$$\bar{x}_i S_i = \mu_i - \gamma\Lambda\left[\sum_{j=1}^{n}\rho_{ij}\sigma_i\sigma_j\left(1 - \lambda_{AM}\frac{b}{a+b}\mathbf{1}_j\right) + \rho_{iy}\sigma_i\sigma_y\right].$$

Subtracting this from (21) (and removing the explicit cost of investment), obtain

$$\begin{aligned}
\bar{x}_i \Delta S_i &= \mu_y - \gamma\Lambda\left[\left(\sigma_y^2 + \rho_{iy}\sigma_i\sigma_y\right)\left(1 - \lambda_{AM}\frac{b}{a+b}\mathbf{1}_i\right) - \rho_{iy}\sigma_i\sigma_y\right] \\
&\quad -\gamma\Lambda\sum_{j=1}^{n}\rho_{jy}\sigma_j\sigma_y\left(1 - \lambda_{AM}\frac{b}{a+b}\mathbf{1}_j\right) \\
&= \mu_y - \gamma\Lambda\sigma_y^2 + \gamma\Lambda\left(\sigma_y^2 + \rho_{iy}\sigma_i\sigma_y\right)\lambda_{AM}\frac{b}{a+b}\mathbf{1}_i \\
&\quad -\gamma\Lambda\sum_{j=1}^{n}\rho_{jy}\sigma_j\sigma_y\left(1 - \lambda_{AM}\frac{b}{a+b}\mathbf{1}_j\right).
\end{aligned}$$

Thus (23) in this case is

$$\bar{x}_{i_{\text{IN}}}\Delta S_{i_{\text{IN}}} - \bar{x}_{i_{\text{OUT}}}\Delta S_{i_{\text{OUT}}} = \gamma\Lambda\left(\sigma_y^2 + \rho_{i_{\text{IN}}y}\sigma_{i_{\text{IN}}}\sigma_y\right)\lambda_{AM}\frac{b}{a+b},$$

and $\bar{x}_{i_{\text{IN}}}\Delta S_{i_{\text{IN}}} > \bar{x}_{i_{\text{OUT}}}\Delta S_{i_{\text{OUT}}} \iff \sigma_y^2 + \rho_{i_{\text{IN}}y}\sigma_{i_{\text{IN}}}\sigma_y > 0$. Notice that unlike in Proposition 1, we do not need to assume that $\sigma_{i_{\text{IN}}} = \sigma_{i_{\text{OUT}}}$ and $\rho_{i_{\text{IN}}y} = \rho_{i_{\text{OUT}}y}$. $\square$

Proof of Proposition 3. (i) Suppose firm y issues $\bar{x}_y$ shares when it goes public (in the main text we normalized $\bar{x}_y$ to one). The stock price of firm y if it does not get included in the benchmark is

$$\bar{x}_y S_y^{\text{OUT}} = \mu_y - \gamma\Lambda\left[\sigma_y^2 + \sum_{i=1}^{n}\rho_{iy}\sigma_i\sigma_y\left(1 - \lambda_{AM}\frac{b}{a+b}\mathbf{1}_i\right)\right].$$

The price of firm y if it enters the benchmark and no other firm leaves it, is

$$\bar{x}_y S_y^{\text{IN}} = \mu_y - \gamma\Lambda\left[\sigma_y^2\left(1 - \lambda_{AM}\frac{b}{a+b}\right) + \sum_{i=1}^{n}\rho_{iy}\sigma_i\sigma_y\left(1 - \lambda_{AM}\frac{b}{a+b}\mathbf{1}_i\right)\right].$$

Taking the difference,

$$\bar{x}_y\left(S_y^{\text{IN}} - S_y^{\text{OUT}}\right) = \gamma\Lambda\sigma_y^2\lambda_{AM}\frac{b}{a+b} > 0.$$

(ii) The price of firm y if it replaces firm k in the benchmark is

$$\bar{x}_y\hat{S}_y^{\text{IN}} = \mu_y - \gamma\Lambda\left[\sigma_y^2\left(1 - \lambda_{AM}\frac{b}{a+b}\right) + \sum_{i=1}^{k-1}\rho_{iy}\sigma_i\sigma_y\left(1 - \lambda_{AM}\frac{b}{a+b}\right) + \sum_{i=k}^{n}\rho_{iy}\sigma_i\sigma_y\right].$$

Taking the difference,

$$\bar{x}_y\left(\hat{S}_y^{\text{IN}} - S_y^{\text{OUT}}\right) = \gamma\Lambda\left(\sigma_y^2 - \rho_{ky}\sigma_k\sigma_y\right)\lambda_{AM}\frac{b}{a+b} > 0.$$

Thus $\hat{S}_y^{\text{IN}} > S_y^{\text{OUT}} \iff \sigma_y^2 - \rho_{ky}\sigma_k\sigma_y > 0$. $\square$

Proof of Proposition 4. The subsidy is

$$\Delta S_{in} - \Delta S_{out} = \gamma\Lambda(\sigma_y^2 + \rho_y\sigma\sigma_y)\left[\lambda_{AM}^A\frac{b}{a+b} + \lambda_{AM}^P\right].$$

Using $\Lambda = \left[\lambda_{AM}^A/(a+b) + \lambda_C\right]^{-1}$ and expressing $\lambda_{AM}^A = (1-\lambda_C)/2 - \lambda_{AM}^P$, the subsidy becomes

$$\begin{aligned}\Delta S_{in} - \Delta S_{out} &= \gamma(\sigma_y^2 + \rho_y \sigma \sigma_y) \frac{\frac{b}{a+b}((1-\lambda_C)/2 - \lambda_{AM}^P) + \lambda_{AM}^P}{\frac{1}{a+b}((1-\lambda_C)/2 - \lambda_{AM}^P) + \lambda_C} \\ &= \gamma(\sigma_y^2 + \rho_y \sigma \sigma_y) \frac{\frac{b}{a+b}(1-\lambda_C)/2 + \left(1 - \frac{b}{a+b}\right)\lambda_{AM}^P}{\frac{1}{a+b}(1-\lambda_C)/2 + \lambda_C - \frac{1}{a+b}\lambda_{AM}^P}.\end{aligned}$$

The numerator is increasing and the denominator is decreasing in λ_{AM}^P while holding λ_C constant. □

Appendix B

In this appendix we explore the robustness of our model to an alternative specification where a manager's compensation is tied to the per-dollar returns on the fund's and benchmark portfolios as opposed to the performance measure used in the main text.

Define $R_i = D_i/(\bar{x}_i S_i)$, $i = 1, \ldots, n$, and let $R = (R_1, \ldots, R_n)^\top$ be the vector of (per-dollar) returns. It is distributed normally with mean $\mu_R = (\mu_1/(\bar{x}_1 S_1), \ldots, \mu_n/(\bar{x}_n S_n))$ and variance Σ_R, where

$$(\Sigma_R)_{ij} = \frac{\rho_{ij}\sigma_i\sigma_j}{\bar{x}_i S_i \bar{x}_j S_j}, \;\; i = 1, \ldots, n, \;\; j = 1, \ldots, n.$$

It is now more convenient to specify investors' portfolio optimization problem in terms of fractions θ_i of wealth under management invested in stock i, $i = 1, \ldots, n$, with the remaining fraction $1 - \sum_{i=1}^n \theta_i$ invested in the bond. Denote $\theta = (\theta_1, \ldots, \theta_n)^\top$.

Let us start by considering the problem of a conventional investor. Let W_0^C denote the initial wealth of each conventional investor. Let $\mathbb{1} = (1, \ldots, 1)^\top$ be the vector of ones. As in main model, CARA preferences with normal returns are equivalent to mean-variance preferences. Then the conventional investor's problem can be written as

$$\max_\theta (\theta^\top \mu_R + 1 - \mathbb{1}^\top \theta) W_0^C - \frac{\gamma}{2} \theta^\top \Sigma_R \theta \left(W_0^C\right)^2.$$

The optimal solution is

$$\theta^C W_0^C = \Sigma_R^{-1} \frac{\mu_R - \mathbb{1}}{\gamma}.$$

Now consider asset managers. Suppose each asset manager is given W_0^{AM} amount of money to manage, which is all or part of the shareholder's initial wealth. The asset manager's compensation is

$$w = [aR_\theta + b(R_\theta - R_{\mathbf{b}})]W_0^{AM} + c,$$

where $R_\theta = \theta^\top R + 1 - \mathbb{1}^\top\theta$ is the return on the asset manager's portfolio, and $R_{\mathbf{b}} = \omega^\top R$ is the benchmark return. The benchmark weights (defined as in the proof of Lemma 3) are

$$\omega_i = \frac{\mathbf{1}_i \bar{x}_i S_i}{\sum_{j=1}^n \mathbf{1}_j \bar{x}_j S_j},$$

and $\omega = (\omega_1, \ldots, \omega_n)^\top$. Then the asset manager's compensation can be written as

$$w = \left[(a+b)(\theta^\top R + 1 - \mathbb{1}^\top\theta) - b\omega^\top R\right] W_0^{AM} + c,$$

and the asset manager's problem is

$$\max_\theta \left[(a+b)(\theta^\top \mu_R + 1 - \mathbb{1}^\top\theta) - b\omega\mu_R\right] W_0^{AM} - \frac{\gamma}{2}[(a+b)\theta - b\omega]^\top \Sigma_R[(a+b)\theta - b\omega] \left(W_0^{AM}\right)^2.$$

The optimal solution is

$$\left[(a+b)\theta^{AM} - b\omega\right] W_0^{AM} = \Sigma_R^{-1} \frac{\mu_R - \mathbb{1}}{\gamma}.$$

Equating total demand with total supply, $\lambda_{AM}\theta^{AM}W_0^{AM} + \lambda_C\theta^C W_0^C = \bar{x}\cdot S$, and rearranging terms, we arrive at the following representation of the stocks' expected returns:

$$\begin{pmatrix} \frac{\mu_1}{\bar{x}_1 S_1} - 1 \\ \vdots \\ \frac{\mu_n}{\bar{x}_n S_n} - 1 \end{pmatrix} = \gamma\Lambda \begin{pmatrix} \frac{\sigma_1^2}{\bar{x}_1^2 S_1^2} & \cdots & \frac{\rho_{1n}\sigma_1\sigma_n}{\bar{x}_1 S_1 \bar{x}_n S_n} \\ \vdots & & \vdots \\ \frac{\rho_{1n}\sigma_1\sigma_n}{\bar{x}_1 S_1 \bar{x}_n S_n} & \cdots & \frac{\sigma_n^2}{\bar{x}_n^2 S_n^2} \end{pmatrix} \left(\begin{pmatrix} \bar{x}_1 S_1 \\ \vdots \\ \bar{x}_n S_n \end{pmatrix} - \lambda_{AM} W_0^{AM} \frac{b}{a+b}\omega \right). \quad (24)$$

Simplifying further, we have

$$\begin{pmatrix} \mu_1 - \bar{x}_1 S_1 \\ \vdots \\ \mu_n - \bar{x}_n S_n \end{pmatrix} = \gamma\Lambda \begin{pmatrix} \frac{\sigma_1^2}{\bar{x}_1 S_1} & \dots & \frac{\rho_{1n}\sigma_1\sigma_n}{\bar{x}_n S_n} \\ \vdots & & \vdots \\ \frac{\rho_{1n}\sigma_1\sigma_n}{\bar{x}_1 S_1} & \dots & \frac{\sigma_n^2}{\bar{x}_n S_n} \end{pmatrix} \left(\begin{pmatrix} \bar{x}_1 S_1 \\ \vdots \\ \bar{x}_n S_n \end{pmatrix} - \lambda_{AM} W_0^{AM} \frac{b}{a+b} \omega \right),$$

which after plugging in

$$\omega = \frac{1}{\sum_{i=1}^n \mathbf{1}_i \bar{x}_i S_i} \begin{pmatrix} \mathbf{1}_1 \bar{x}_1 S_1 \\ \vdots \\ \mathbf{1}_n \bar{x}_n S_n \end{pmatrix}$$

gives us an implicit expression for share values:

$$\bar{x} \cdot S = \mu - \gamma\Lambda\Sigma \left(\mathbb{1} - \lambda_{AM} \frac{b}{a+b} \frac{W_0^{AM}}{\sum_i \mathbf{1}_i \bar{x}_i S_i} \mathbf{1_b} \right). \tag{25}$$

Notice that (25) is identical to our expression for share values (18) in the main model with $\mathbf{1_b} W_0^{AM} / \sum_i \mathbf{1}_i \bar{x}_i S_i$ instead of $\mathbf{1_b}$.

Notice that the value of assets under management, W_0^{AM}, itself depends on asset prices. In general, (25) cannot be solved in closed form. Consider a special case when W_0^{AM} consists only of the benchmark stocks, i.e., $W_0^{AM} = \sum_i \mathbf{1}_i \bar{x}_i S_i$. Then (25) becomes exactly (18).

Lemmas 1–3 from the main text extend straightforwardly to the considered extension. The extension of Lemma 4 is a bit more tricky in general. We perform it in two special cases. In the first case we assume that $W_0^{AM} = \sum_i \mathbf{1}_i \bar{x}_i S_i$ as discussed above. In the second case we assume that the value of asset under management is independent of equilibrium stock prices, which happens, e.g., when the endowment of shareholders is in terms of bonds only. Finally, for simplicity we assume that investment is financed by internal funds (or, equivalently, with the risk-free bond). Then the cost of investment to any firm is I (which is also true in our original model). We discuss briefly at the end what happens if investment is financed by equity instead.

In this case, if firm i invests, we have $\bar{x} \cdot S^{(i)}$ and $\bar{x}_i \Delta S_i$ given exactly by (19) and (22), respectively. So Lemma 4 extends to this case. Performing the same analysis as in the main text, we get Proposition 1. The results about mergers and acquisitions and IPOs extend in the same way.

The second special case is the value of asset under management is fixed, independent of equilibrium stock prices (which happens, e.g., when the endowment of shareholders is in terms of bonds only).

Denote $T = \sum_i \mathbf{1}_i \bar{x}_i S_i$ as the total value of firms that are in the benchmark. Multiplying both sides of (25) by $\mathbf{1}_\mathbf{b}^\top$ and taking the positive root of the resulting quadratic equation

$$T = \mu^\top \mathbf{1}_\mathbf{b} - \gamma\Lambda \mathbf{1}_\mathbf{b}^\top \Sigma \mathbb{1} + \gamma\Lambda \mathbf{1}_\mathbf{b}^\top \Sigma \mathbf{1}_\mathbf{b} \lambda_{AM} \frac{bW_0^{AM}}{(a+b)T},$$

we have

$$T = \frac{\mu^\top \mathbf{1}_\mathbf{b} - \gamma\Lambda \mathbf{1}_\mathbf{b}^\top \Sigma \mathbb{1} + \sqrt{\left(\mu^\top \mathbf{1}_\mathbf{b} - \gamma\Lambda \mathbf{1}_\mathbf{b}^\top \Sigma \mathbb{1}\right)^2 + 4\gamma\Lambda \mathbf{1}_\mathbf{b}^\top \Sigma \mathbf{1}_\mathbf{b} \lambda_{AM} W_0^{AM} b/(a+b)}}{2}.$$

Then we have an explicit expression for asset prices given by

$$\bar{x} \cdot S = \mu - \gamma\Lambda\Sigma \left(\mathbb{1} - \lambda_{AM} \frac{b}{a+b} \frac{W_0^{AM}}{T} \mathbf{1}_\mathbf{b} \right).$$

If firm i invests,

$$\bar{x} \cdot S^{(i)} = \mu^{(i)} - I^{(i)} - \gamma\Lambda\Sigma^{(i)} \left(\mathbb{1} - \lambda_{AM} \frac{b}{a+b} \frac{W_0^{AM}}{T^{(i)}} \mathbf{1}_\mathbf{b} \right).$$

where $T^{(i)} = \sum_j \mathbf{1}_j \bar{x}_j S_j^{(i)}$ is given by the positive root of

$$T^{(i)} = \left(\mu^{(i)} - I^{(i)}\right)^\top \mathbf{1}_\mathbf{b} - \gamma\Lambda \mathbf{1}_\mathbf{b}^\top \Sigma^{(i)} \mathbb{1} + \gamma\Lambda \mathbf{1}_\mathbf{b}^\top \Sigma^{(i)} \mathbf{1}_\mathbf{b} \lambda_{AM} \frac{bW_0^{AM}}{(a+b)T^{(i)}}. \tag{26}$$

The corresponding change in firm i's value is

$$\begin{aligned}
\bar{x}_i \Delta S_i =& \mu_y - I - \gamma\Lambda \sum_{j=1}^{n} \left[\rho_{jy}\sigma_j\sigma_y + (\sigma_y^2 + \rho_{iy}\sigma_i\sigma_y)\mathcal{I}_{j=i}\right] \left(1 - \lambda_{AM} \frac{b}{a+b} \frac{W_0^{AM}}{T} \mathbf{1}_j\right) \\
&- \gamma\Lambda \sum_{j=1}^{n} \left[\rho_{jy}\sigma_j\sigma_y + (\sigma_y^2 + \rho_{iy}\sigma_i\sigma_y)\mathcal{I}_{j=i} + \rho_{ij}\sigma_i\sigma_j\right] \lambda_{AM} \frac{b}{a+b} W_0^{AM} \mathbf{1}_j \left(\frac{1}{T} - \frac{1}{T^{(i)}}\right),
\end{aligned}$$

where $\mathcal{I}_{j=i} = 1$ if $j = i$ and $\mathcal{I}_{j=i} = 0$ otherwise.

Suppose we have two firms i_{IN} and i_{OUT}, $i_{\text{IN}} \in \mathcal{B}$, $i_{\text{OUT}} \notin \mathcal{B}$ that are otherwise symmetric, i.e., $\sigma_{i_{\text{IN}}} = \sigma_{i_{\text{OUT}}} = \sigma$, $\rho_{i_{\text{IN}}y} = \rho_{i_{\text{OUT}}y} = \rho$ and $\rho_{i_{\text{IN}}j} = \rho_{i_{\text{OUT}}j} = \rho_j$ for all $j \neq i_{\text{IN}}, i_{\text{OUT}}$. Then the analog of (14) in the main text is

$$\bar{x}_{i1}\Delta S_{i_{\text{IN}}} - \bar{x}_{i2}\Delta S_{i_{\text{OUT}}} = \left[\sigma_y^2 + \rho\sigma\sigma_y\right]\gamma\Lambda\frac{b}{a+b}\lambda_{AM}\frac{W_0^{AM}}{T^{(i_{\text{IN}})}} - \frac{T^{(i_{\text{IN}})} - T^{(i_{\text{OUT}})}}{T^{(i_{\text{OUT}})}}\gamma\Lambda\frac{b}{a+b}\lambda_{AM}\frac{W_0^{AM}}{T^{(i_{\text{IN}})}}\sum_{j=1}^{n}\left(\rho_{jy}\sigma_j\sigma_y + \rho_j\sigma\sigma_j\right)\mathbf{1}_j. \quad (27)$$

The first term is positive by Assumption 1. The second term comes from the fact that the sum of benchmark weights is different depending on whether the investing firm is inside or outside the benchmark. It captures the fact that by investing, the firm grows and effectively reduces importance of other firms in the benchmark. Notice that $T^{(i_{\text{IN}})} - T^{(i_{\text{OUT}})} = o(T)$ when project y is small relative to T ($T^{(i_{\text{IN}})}$, $T^{(i_{\text{OUT}})}$, and T are all of the same order). So the term $\left(T^{(i_{\text{IN}})} - T^{(i_{\text{OUT}})}\right)/T^{(i_{\text{OUT}})}$ is $O(1/T)$ and $o(1)$. The rest of the second term, $\gamma\Lambda(b/a+b)\lambda_{AM}(W_0^{AM}/T^{(i_{\text{IN}})})\sum_{j=1}^{n}\left(\rho_{jy}\sigma_j\sigma_y + \rho_j\sigma\sigma_j\right)\mathbf{1}_j$, is of the same order as $\bar{x}_{i_{\text{IN}}}S^{(i_{\text{IN}})}$. So the second term is $O(\bar{x}_{i_{\text{IN}}}S^{(i_{\text{IN}})}/T^{(i_{\text{IN}})})$, i.e., of the order of the benchmark weight $\omega_{i_{\text{IN}}}$.

Consider a special case when project y is risk free, i.e., $\sigma_y = 0$. It is easy to show that $T^{(i_{\text{OUT}})} = T$ for $i_{\text{OUT}} \in \{k+1, \ldots, n\}$. Moreover, suppose that $I = \mu_y$ so that there are no arbitrage opportunities. Then $\mu^{(i_{\text{IN}})} - I^{(i_{\text{IN}})} = \mu$ and $\Sigma^{(i_{\text{IN}})} = \Sigma$, and thus $T^{(i_{\text{IN}})} = T$ for $i_{\text{IN}} \in \{1, \ldots, k\}$. As a result, for the risk-free project with $\mu_y = I$ we have $\Delta S_{i_{\text{IN}}} - \Delta S_{i_{\text{OUT}}} = 0$, i.e., both firms evaluate it equally.

Finally, if I is financed by issuing $\delta_i = I/S_i^{(i)}$ additional shares, then instead of $T^{(i)} = \sum_i \mathbf{1}_j x_j S_j^{(i)}$ we have $T^{(i)\prime} = \sum_{j\neq i}\mathbf{1}_j\bar{x}_j S_j^{(i)} + \mathbf{1}_i S_i^{(i)}(\bar{x}_i + \delta_i) = \sum_j \mathbf{1}_j\bar{x}_j S_j^{(i)} + \mathbf{1}_i I$. Then $T^{(i)\prime}$ is the positive root of

$$T^{(i)\prime} = \mu^{(i)\top}\mathbf{1_b} - \gamma\Lambda\mathbf{1_b}^\top\Sigma^{(i)}\mathbb{1} + \gamma\Lambda\mathbf{1_b}^\top\Sigma^{(i)}\mathbf{1_b}\lambda_{AM}\frac{bW_0^{AM}}{(a+b)T^{(i)\prime}}.$$

Comparing this equation to (26), one can see that $T^{(i)\prime} = T^{(i)}$ if i is outside the benchmark, and $T^{(i)\prime} > T^{(i)}$ if i is inside the benchmark. Hence the additional effect coming from the change in the total index value that we have seen in (27) is stronger when the investment is financed by equity.

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