

International Macro-Finance

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Abstract

International macro-finance is a new area of open economy macroeconomics that brings portfolio choice and asset pricing considerations into models of international macroeconomics. The importance of these considerations—typically relegated to Finance and largely overlooked in traditional macroeconomics—for the international macroeconomy have been underscored by a series of recent financial crises and by unprecedented global imbalances. In this paper, we survey recent developments in this area, primarily on the theoretical front. We also suggest several promising directions for future research.

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1. Introduction

Financial markets and their role in international risk sharing have inspired a vast body of theoretical literature. Over the past 40 years, international finance and economics has evolved into a vibrant field spreading from the basic international version of the CAPM to some of the most sophisticated dynamic stochastic general equilibrium (DSGE) models. Interestingly, however, the research effort in Economics has evolved almost in parallel with that in Finance. In Economics, the main focus has been on real quantities and international relative prices such as consumption, investment, current account, terms of trade and exchange rates. International portfolio choice and international equity markets have been largely overlooked. Indeed, the asset structure of these models has been mostly of two types: either the only asset is an international bond and markets are incomplete, or there is a full set of Arrow-Debreu securities and markets are complete. Both approaches have been very useful, but they cannot address many questions pertaining to portfolio problems and to the international equity markets. Finance, on the other hand, has focused more on cross-country portfolio allocations and asset prices. Terms of trade and hence exchange rates have been largely overlooked because the majority of the models featured a single-good framework, in which forces of arbitrage equate terms of trade to unity. Models with endogenous portfolio selection, equity prices and time-varying terms of trade and exchange rates in a single framework have been quite rare. Although we have learned a tremendous amount from this research, in recent years two main phenomena have required a redefinition of the agenda behind the theories of financial markets in the international context: contagion among developed and relatively unconnected countries, and the role that asset prices and exchange rates play in the global imbalances.

Contagion refers to the transmission of crises from one country to another. Prominent examples of this phenomenon include the 1997 Asian crisis, the 1998 Russian crisis, and the subprime mortgage crisis of 2007-2008. Contagion has attracted attention of academics in the aftermath of the 1994 Mexican crisis, and from the very beginning (Gerlach and Smets (1995)), crises were thought to be transmitted through trade relationships, driven by competitive devaluations. Also, contagion appeared to be mostly an emerging markets

problem. In recent years, however, these views have shifted. Today, we tend to think that turmoil is transmitted, most likely, through the financial sector and that contagion is a global problem, affecting all countries, regardless of their level of financial development.

The emphasis on the financial sector as the main transmission mechanism started shortly after the 1997 Asian and the 1998 Russian crises. During these crises shocks propagated through international banks and other financial intermediaries. The transmission had little to do with competitive devaluations and their impact on trade, but rather with the impact of large swings in exchange rates on asset values, balance sheets, and interest rates. For example, Kaminsky and Reinhart (2000) document that Thailand's 1997 currency downfall led to capital losses for Japanese banks, forcing them to curb their lending to other Asian countries. It is thus becoming increasingly apparent to both academics and policymakers that no analysis of contagion is complete without a thorough understanding of how shocks are transmitted internationally through financial markets and intermediaries.

The second significant development that has influenced the literature was the unprecedented rise in external deficits in many developed nations, which sparked a discussion about sustainability and the possible dramatic unraveling of global imbalances (see Roubini and Setser (2004) for a particularly striking prediction). The rise in external deficits, however, came hand-in-hand with the explosion in cross-border risky asset holdings. Before 1985, the US held virtually no foreign equities; nowadays, foreign equities account for a large and growing part of the country's assets. Following influential work of Lane and Milesi-Ferretti (2001) and Gourinchas and Rey (2007), it has become clear that (unrealized) capital gains on these equity positions are missing from national accounts. The alarming current accounts deficits worldwide may then be simply due to this misreported income from equity positions. Today an extremely active literature, both empirical and theoretical, is trying to better understand how the capital gains on foreign equity positions, or the so-called valuation effects, affect our thinking about the current account and the external adjustment process. Unfortunately, most of the existing international macro models are not well-suited for dealing with these issues because they are missing equity markets and portfolio choice. A new and rapidly growing strand of literature, commonly known as International Macro-Finance, is trying to

fill this gap. This new generation of macro models provides a redefinition of the current account (adjusted for capital gains on equity holdings) and modifies the standard theories of the current account. More generally, this research program focuses on the interaction between the financial sector and the real economy, and as such can address a wide spectrum of issues such as contagion, valuation effects, and others.

The literature in international economics and finance can be split along several paradigms: small open economy (partial equilibrium) vs. general equilibrium; pure exchange vs. production economies; single- vs. multiple-good models; complete vs. incomplete markets. It is impossible to summarize the research that has been done in all these areas.¹ In this survey we concentrate on a framework that has become the core of international macro-finance: general equilibrium asset-pricing models with multiple goods. The richness of this framework comes at a cost: most of the macro-finance models are quite complex. This literature can be split into several parts based on their approaches. The first approach relies on traditional approximation methods. Ghironi, Lee, and Rebucci (2006) and Kollmann (2006) compute portfolios and changes in net foreign assets using standard first-order approximations around a deterministic steady state. The second approach makes use of higher-order approximations to analyze countries' portfolios and the evolution of external accounts. These methodologies grew out of Samuelson (1970) and Judd and Guu (2001) and were developed by Engel and Matsumoto (2006), Evans and Hnatkovska (2007), Devereux and Sutherland (2010a), and Tille and van Wincoop (2010). A disadvantage shared by these two approaches is that to this day little is known about the behavior of these economies away from the deterministic steady state, where the underlying volatilities are not small.²

The third strand of the macro-finance literature simplifies the models and seeks to find

¹Even a comprehensive textbook such as Obstfeld and Rogoff (1996) is unable to cover the entire literature. The two strands that are the closest to our subject here are international real business cycles in Economics and international asset pricing in Finance. For the former, see Baxter (1995) and Crucini (2008) for surveys of the international real business cycles literature, whose main focus is on economic fluctuations generated by real general equilibrium production economies and their relationship to the data. For the latter, see Stulz (1995) and Sercu and Uppal (2000) for international asset pricing, whose main focus is on equity prices and portfolios within single-good economies and on the international CAPM.

²More generally, the accuracy and performance of numerical algorithms for solving multi-country DSGE models remain the subject of an ongoing debate. The January 2010 special issue of the *Journal of Economic Dynamics and Control* on numerical methods for multi-country DSGE models (see Den Haan, Judd, and Juillard (2010)) provides an excellent reference for the current state of this debate.

exact solutions. The main advantage of this approach is that the economy can be analyzed away from the steady state, but the disadvantage is that solutions only exist in few special cases. An early work that presents one of such special cases is Helpman and Razin (1978). Their setup has been developed further by Cole and Obstfeld (1991), Zapatero (1995), and Pavlova and Rigobon (2007). These papers consider pure-exchange economies in which a representative agent in each country has log-linear preferences. In recent years the literature has made progress extending the setup beyond log-linear preferences—Coeurdacier (2005) to the logarithm of a CES aggregator and Stathopoulos (2008) to preferences with habit formation.³ The solution is especially simple in complete markets—and we discuss this solution in detail in Section 2—but it is also possible to introduce market frictions (Schornick (2007), Pavlova and Rigobon (2008), and Pavlova and Rigobon (2010)). We believe that models incorporating market incompleteness and institutional frictions are the most promising step towards understanding the phenomena of contagion, world systemic risk, and the proper definition of external sustainability.

In terms of the economic applications, the two questions that the theoretical literature on macro-finance has mainly focused on so far were the (i) composition of international portfolios and (ii) valuation effects and global imbalances. Besides papers already mentioned above, the first strand is represented by, e.g., Coeurdacier (2009), Coeurdacier and Gourinchas (2009), Engel and Matsumoto (2009), and Coeurdacier, Kollmann, and Martin (2010). The second strand includes Mendoza, Quadrini, and Rios-Rull (2007), Caballero, Farhi, and Gourinchas (2008), and Devereux and Sutherland (2010b). Also related, but developed before valuation effects were on the agenda of international macroeconomists, are portfolio balance models of Kraay and Ventura (2000), Kraay and Ventura (2003), and Devereux and Saito (2006). Recently, the *Journal of International Economics* ran a special issue on international macro-finance (see Devereux, Engel, Matsumoto, Rebucci, and Sutherland (2010)). This collection of works gives an excellent overview of the latest contributions to this field and provides further references.

³See also Li and Muzere (2010) for a related model with heterogenous beliefs.

2. The Workhorse Model

In this section, we develop a simple multi-good multi-country general equilibrium asset pricing model that allows for closed-form solutions. While the setup here is certainly simplified, it possesses many elements that form the core of international macro-finance. We believe it is this kind of models that should lead future research in this area.

We consider a discrete-time pure exchange economy along the lines of Lucas (1982). Let time t run from 0 to T . There is a finite number of trading dates, but one can also look at the infinite-horizon version of our economy by adding an appropriate transversality condition and taking the limit as $T \rightarrow \infty$. There are two countries in the world: Home and Foreign. Each country is endowed with a Lucas tree producing a country-specific perishable good. In our baseline model, the uncertainty about the output of the trees is the only source of risk in the economy. The state ω_t , an element of the set Ω_t , is the history of the economy up to time t . This history occurs with probability $\pi(t, \omega_t)$. In our notation hereafter, we suppress the second argument, ω_t , unless necessary for clarity. The state-dependent outputs of the Home and Foreign trees are denoted by $Y(t)$ and $Y^*(t)$, respectively, and the corresponding prices of the goods by $p(t)$ and $p^*(t)$. The terms of trade, ToT , are defined as the price of the Home good relative to that of the Foreign good: $ToT \equiv p/p^*$.⁴ We fix the world numeraire basket to contain $\alpha \in (0, 1)$ units of the Home good and $(1-\alpha)$ units of the Foreign good, and normalize the price of this basket to be equal to unity.

Available for investment are one-period riskless bonds, with prices $B(t)$ and $B^*(t)$, paying out in units of Home and Foreign good, respectively, and claims to the Home and Foreign trees (stocks), with prices $S(t)$ and $S^*(t)$. Additionally, agents can trade a complete set of Arrow-Debreu securities. In what follows, we are particularly interested in determining equilibrium prices of the stocks and bonds. Due to space limitations, we do not examine the

⁴The presence of time-varying terms of trade is what makes this framework different from standard international asset pricing models in Finance. Most of these models feature a single good, and therefore, by construction, the terms of trade and the real exchange rate are equal to unity. Nontrivial implications for terms of trade or exchange rates in single-good models have been obtained by either introducing shipping costs into a real model, or by exogenously specifying a monetary policy and focusing on the nominal exchange rate. For models with shipping costs see Dumas (1992) and a textbook by Sercu and Uppal (2000). See Bakshi and Chen (1997), Basak and Gallmeyer (1999) for monetary general equilibrium models, and Solnik (1974), Adler and Dumas (1983) for the international CAPM.

countries' portfolios, and so we allow for more assets than what is needed to ensure market completeness. The surplus assets will simply be priced by no-arbitrage. In the presence of redundant assets, of course, the countries' portfolios are indeterminate. To make our model a meaningful model of portfolio allocation, we simply need to drop the assumption that the Arrow-Debreu securities are available for trade and instead to (dynamically) complete the markets with the countries' stocks, bonds, and possibly other assets. One, of course, has to ensure that there are at least as many assets as there are states of the world at each node; otherwise, our solution approach needs to be modified to explicitly account for market incompleteness.

Each country i is populated by a representative agent with preferences characterized by the expected utility function

$$E \left[\sum_{t=0}^T \beta^t u_i(C_i(t), C_i^*(t)) \right], \quad i = \{H, F\}, \quad (1)$$

with $0 < \beta < 1$. In particular, the agents in each country derive utility from consuming both the domestic and the foreign good. Since we have assumed that markets are complete, we can write the budget constraints of the agents in their Arrow-Debreu form:

$$E \left[\sum_{t=0}^T q(t)(p(t)C_i(t) + p^*(t)C_i^*(t)) \right] = W_i, \quad i = \{H, F\}, \quad (2)$$

where $q(t, \omega_t)$ is the Arrow-Debreu state price of ω_t divided by the probability of that state $\pi(t, \omega_t)$. The quantity W_i denotes the initial wealth of country i ; here, we have $W_H = S(0)$ and $W_F = S^*(0)$. This form of a budget constraint is widely used in the asset-pricing literature; it is often called a “static” budget constraint, implying that all asset trades take part at time 0 and no trades take place afterwards. The ability to do so, of course, requires market completeness. If markets are incomplete or there are other frictions, one can still write the budget constraint in its static form, but the state prices $q(t)$ entering the budget constraint will be country-specific.

A competitive equilibrium in this economy consists of state-contingent allocations (C_i, C_i^*) , $i \in \{H, F\}$ and prices (ToT, q) such that (i) both countries maximize their utility subject to the budget constraint and (ii) goods and asset markets clear. We find it convenient to solve

for this equilibrium using the Negishi method. We first solve the social planner's problem, for a set of utility weights $\{\lambda_i\}$, and then pin down the weights λ_i 's and the competitive equilibrium prices. The planner's problem is as follows:

$$\max_{\{C_H, C_H^*, C_F, C_F^*\}} E \sum_{t=0}^T \beta^t \left[\lambda u_H(C_H(t), C_H^*(t)) + u_F(C_F(t), C_F^*(t)) \right] \quad (3)$$

with multipliers

$$\text{s. t.} \quad C_H(t) + C_F(t) = Y(t), \quad \eta(t), \quad (4)$$

$$C_H^*(t) + C_F^*(t) = Y^*(t), \quad \eta^*(t), \quad (5)$$

where, without loss of generality, we have normalized the weight on the Foreign country to be equal to one. This problem can be broken down into a series of state-by-state maximizations of the quantity inside the summation, weighted by the probability of the corresponding state $\pi(t, \omega_t)$, subject to the resource constraints (4)–(5) for that state. The multipliers from that maximization allow us to compute the state prices prevailing in the decentralized equilibrium. The multiplier on (4), $\eta(t, \omega_t)$, is the price of one unit of the Home good to be delivered at time t in state ω_t . Therefore, the price of one unit of the numeraire to be delivered in state ω_t , scaled by the probability of that state, is

$$q(t, \omega_t) = \frac{\eta(t, \omega_t)}{\pi(t, \omega_t)p(t, \omega_t)}. \quad (6)$$

In Finance, this quantity q is typically referred to as the state-price deflator. It is, of course, nothing else than the marginal utility of the representative agent evaluated over the consumption index (over the two goods) at the optimum. The terms of trade are simply the marginal rate of substitution between the Home and Foreign goods, or, equivalently, the ratio of the multipliers on the resource constraints:

$$ToT(t) = \frac{\eta(t)}{\eta^*(t)}. \quad (7)$$

The planner's weight λ is determined by substituting the solution to (3)–(5) into either country's budget constraint (2).

We are now ready to price financial assets in our model. By no-arbitrage, the prices of stocks and bonds are given by their state-contingent payoffs, discounted with the state-price

deflator q :

$$S(t) = \frac{1}{q(t)} E_t \left[\sum_{s=t+1}^T q(s)p(s)Y(s) \right], \quad S^*(t) = \frac{1}{q(t)} E_t \left[\sum_{s=t+1}^T q(s)p^*(s)Y^*(s) \right], \quad (8)$$

$$B(t) = \frac{1}{q(t)} E_t [q(t+1)p(t+1)], \quad B^*(t) = \frac{1}{q(t)} E_t [q(t+1)p^*(t+1)], \quad (9)$$

where $E_t[\cdot]$ denotes the expectation conditional on the information available up to time t . In our baseline model, the bonds mature in one period. To price s -period zero-coupon bonds, we obviously need to replace $t+1$ by s in (9).

2.1. Log-Linear Preferences

We now examine a well-known special case of our model which serves as an important benchmark in international finance. In this benchmark, stock prices can be computed in closed form. Suppose that the countries' utilities are log-linear. That is,

$$u_i(C, C^*) = a_i \log C + (1 - a_i) \log C^*, \quad a_i \in (0, 1), \quad i \in \{H, F\} \quad (10)$$

This setup appears in Helpman and Razin (1978) and, more recently, in Cole and Obstfeld (1991) and Zapatero (1995). For this economy, the expressions that we have presented above simplify significantly. Let us first compute the multipliers on the resource constraints of Home and Foreign (equations (4) and (5)). They are

$$\eta(t) = \beta^t \pi(t) \frac{\lambda a_H + a_F}{Y(t)} \quad \text{and} \quad \eta^*(t) = \beta^t \pi(t) \frac{\lambda(1 - a_H) + 1 - a_F}{Y^*(t)}. \quad (11)$$

Hence, the state-price deflator is

$$q(t) = \beta^t \frac{\lambda a_H + a_F}{p(t)Y(t)}. \quad (12)$$

Substituting (11) into (7), we find the terms of trade:

$$ToT(t) = \frac{\lambda a_H + a_F}{\lambda(1 - a_H) + 1 - a_F} \frac{Y^*(t)}{Y(t)}. \quad (13)$$

The reason why it is typically quite difficult to solve for asset prices and portfolios in macro-finance models is that the conditional expectations in (8)–(9) cannot be evaluated

explicitly. Numerical evaluation is not straightforward because future payoffs of the trees and the state prices are endogenously determined in equilibrium. Solving for optimal portfolios, accordingly, is also complicated because portfolio positions depend on the expected returns on the assets. To this day, the main methods that we have in macro-finance for solving these types of problems are the methods involving approximations around a deterministic steady state. We have reviewed this literature in the introduction. The advantage of the setup here is that there is a helpful simplification and the conditional expectations in the expressions for stock prices admit closed-form representations. In particular, the Home stock's price turns out to be

$$\begin{aligned} S(t) &= \frac{1}{q(t)} E_t \left[\sum_{s=t+1}^T q(s)p(s)Y(s) \right] = \frac{1}{q(t)} E_t \left[\sum_{s=t+1}^T \beta^t (a_H + \lambda a_F) \right] \\ &= \frac{\beta(1 - \beta^{T-t})}{1 - \beta} p(t) Y(t) = \frac{\beta(1 - \beta^{T-t})}{1 - \beta} \frac{ToT(t)}{\alpha ToT(t) + 1 - \alpha} Y(t), \end{aligned} \quad (14)$$

where in the last equality we have used our price normalization $\alpha p(t) + (1 - \alpha) p^*(t) = 1$.

Similarly, the Foreign stock's price is

$$S^*(t) = \frac{\beta(1 - \beta^{T-t})}{1 - \beta} \frac{1}{\alpha ToT(t) + 1 - \alpha} Y^*(t). \quad (15)$$

Remark 1. *Single consumption good.* The majority of models of international asset pricing considers single-good economies. Interestingly, multi-good frameworks can be more tractable than single-good ones, despite the fact in single-good economies one does not need to solve for the countries' terms of trade. To see why, consider a variation of our model in which both trees pay off in the same (Home) good and the countries derive utility only from that good. In particular, we can set $a_H = a_F = 1$ in (10). This is the model considered in Cochrane, Longstaff, and Santa-Clara (2008). In this model, $ToT(t) = p(t) = 1$, with the latter equality occurring because the consumption good is the numeraire. The state-price deflator is now

$$q(t) = \frac{1}{(\lambda + 1)(Y(t) + Y^*(t))}. \quad (16)$$

Note that the world's total output of the good is now $Y + Y^*$, and the denominator in (16) reflects this adjustment. Let us again focus on the Home stock; the argument for the Foreign stock is analogous. The price of the Home stock is given by the same formula as in (8), but with $p(t) = 1$:

$$S(t) = \frac{1}{q(t)} E_t \left[\sum_{s=t+1}^T q(s)Y(s) \right] = \frac{1}{q(t)} E_t \left[\sum_{s=t+1}^T \beta^t \frac{Y(s)}{Y(s) + Y^*(s)} \right] \quad (17)$$

In a closed economy, $Y^* = 0$, and so one can again readily evaluate the conditional expectation in (17). In an international setting, the task is far more complex. Even under the most tractable distributional assumptions for the output processes used in asset pricing—lognormality—the expectation in (17) is not straightforward to compute. This is because the sum of lognormally distributed random variables is not itself lognormally distributed. Cochrane, Longstaff, and Santa-Clara are able to evaluate the conditional expectation of the quantity in (16) only (i) under i.i.d. $Y(t)$ and $Y^*(t)$, distributed lognormally, (ii) for specific values of the discount factor and the volatility of output, and (iii) in continuous time.⁵

Remark 2. *Real Exchange Rate.* The real exchange rate, e , is defined as $e(t) = P_H(t)/P_F(t)$, where P_H and P_F are Home and Foreign price indexes, respectively. For the preferences that we consider here, the price indexes are⁶

$$P_H(t) = \left(\frac{p(t)}{a_H} \right)^{a_H} \left(\frac{p^*(t)}{1 - a_H} \right)^{1 - a_H}, \quad P_F(t) = \left(\frac{p(t)}{a_F} \right)^{a_F} \left(\frac{p^*(t)}{1 - a_F} \right)^{1 - a_F}.$$

Hence, the real exchange rate, expressed as a function of the terms of trade, is

$$e(t) = ToT(t)^{a_H - a_F} \frac{(1 - a_F)^{1 - a_F} a_F^{a_F}}{(1 - a_H)^{1 - a_H} a_H^{a_H}}.$$

We find it more convenient to work with the terms of trade rather than the exchange rate. In our analysis, we just keep in mind that there is a one-to-one mapping between the exchange rate and the terms of trade. If we assume further that $a_H - a_F > 0$ —an inequality that will be satisfied once we assume home bias in consumption—the real exchange rate is an increasing function of the terms of trade.

The explicit computation of bond prices and interest rates requires additional distributional assumptions on the output processes Y and Y^* . We do not make these assumptions at this point, because, as it turns out, our setup here is not a good one for studying asset prices. The reason for that is apparent from examining (14)–(15):

$$\frac{S(t)}{S^*(t)} = \frac{\lambda a_H + a_F}{\lambda(1 - a_H) + 1 - a_F}, \tag{18}$$

where the last equality follows from (13). The right-hand side of (18) is constant, and hence the two stock markets are perfectly correlated! Cole and Obstfeld show further that the existence of financial markets does not matter in this model at all: even with no investment opportunities available, the countries are able to reach a Pareto efficient allocation through international trade (in goods) alone; there are no benefits to investing internationally. Even

⁵See Martin (2009) for an important extension involving general utility and N trees.

⁶See Obstfeld and Rogoff (1996) for the details of this construction.

if we restrict the countries' investment opportunities to stocks alone, their portfolios are indeterminate because the two stocks represent the same investment opportunity. This result has had big impact in the international finance literature. While quite stark, however, the result is not robust: it holds only for asset-market economies with log-linear consumer preferences in which all goods are tradable.⁷ Any departure from that setup leads to an economy with regular equilibria. One such departure that maintains the tractability of the setup but breaks the perfect correlation among international stock markets has been suggested by Pavlova and Rigobon (2007), who introduce demand shocks.

2.2. Log-linear Preferences with Demand Shocks

Consider the following modification of the preferences specified in (10):

$$u_i(C, C^*) = \theta_i(a_i \log C + (1 - a_i) \log C^*), \quad i \in \{H, F\}, \quad (19)$$

where θ_i is a state-dependent quantity representing a country's demand shock. We require further that each θ_i is a martingale; that is, $E_t[\theta_i(s)] = \theta_i(t)$. A demand shock creates shifts in the countries' demand schedules which may or may not be related to supply. An example of a demand shock is news about weather. This news is unrelated to supply news, but it does affect agents' demands (e.g., for heating oil). The empirical evidence indicates that demand shocks are important for reproducing the real-world dynamics; supply shocks alone are typically not sufficient. For example, Stockman and Tesar (1995) calibrate preference shocks to be roughly 85% of the size of supply shocks. Pavlova and Rigobon (2007) estimate the model presented here and conclude that demand shocks have approximately the same volatility as supply shocks.

The solution to this model follows similar steps to the ones outlined above. The terms of trade, now reflecting demand shocks, are given by

$$ToT(t) = \frac{\lambda \theta_H(t) a_H + \theta_F(t) a_F}{\lambda \theta_H(t) (1 - a_H) + \theta_F(t) (1 - a_F)} \frac{Y^*(t)}{Y(t)}. \quad (20)$$

⁷One attempt to "break" the perfect stock market correlation result and address the issue of portfolio home bias with the introduction of nontradables has been made by Serrat (2001). However, Kollmann (2006) argues that the claims to the trees producing *tradables* are still perfectly correlated, and so that model is still not suitable for studying portfolio choice.

The stock prices also have a simple closed-form representation, the same one as that presented above:

$$S(t) = \frac{\beta(1 - \beta^{T-t})}{1 - \beta} \frac{ToT(t)}{\alpha ToT(t) + 1 - \alpha} Y(t) \quad \text{and} \quad S^*(t) = \frac{\beta(1 - \beta^{T-t})}{1 - \beta} \frac{1}{\alpha ToT(t) + 1 - \alpha} Y^*(t), \quad (21)$$

Now the ratio $S(t)/S^*(t)$ is stochastic, and so the countries' stock markets are no longer perfectly correlated. Bond prices and interest rates are still difficult to compute in this model, even under additional distributional assumptions. One way to resolve this technical difficulty is to cast the model in continuous time.

To convey the economic mechanisms behind the formulas, we need to make the following assumption:

Assumption 1 (Home Bias in Consumption): $a_H(1 - a_F) - a_F(1 - a_H) > 0$.

For this assumption to be satisfied it is sufficient that $a_H > 1 - a_H$ and $1 - a_F > a_F$, or, in words, the expenditure shares on the domestic good for the Home and Foreign country, respectively, exceed the expenditure shares on the foreign good.

The following simple table summarizes how the terms of trade and the stock respond to movements in the underlying state variables and some important comparative statics. Boldface in the table means that the sign obtains unambiguously; otherwise (also in green, for emphasis), the sign obtains if and only if Assumption A1 is satisfied.

Effects of	Y	Y^*	θ_H	θ_F	λ
On the terms of trade ToT	-	+	+	-	+
On the Home stock S	+	+	+	-	+
On the Foreign stock S^*	+	+	-	+	-

A positive output shock at Home (an increase in Y) raises the dividend on the Home stock and so Home's stock price increases. At the same time, it increases the supply of the Home good in the world. As the good becomes less scarce, its price falls relative to that of the Foreign good. Hence Home's terms of trade deteriorate and Foreign's terms of trade

improve. This effect of an output shock on the terms of trade is known as the *Ricardian effect* (Ricardo (1817)). The improvement of Foreign terms of trade increases the value of Foreign's output and hence the Foreign stock goes up. The response following a Foreign output shock (a shock to Y^*) is analogous. So, the stock markets *always comove* in response to an output shock.

A positive demand shock at Home (an increase in θ_H) creates an excess demand in the world for both goods. Since Home has a preference bias for the domestic good, however, the demand for the Home good goes up by more. This pushes up its price relative to that of the Foreign good and therefore improves Home's terms of trade. This effect is best known as the *dependent economy* effect, highlighted in Dornbusch (1980), Chapter 6. The value of Home's output (dividend) increases while that of Foreign's decreases. Hence Home's stock market goes up and Foreign's stock market falls. Demand shocks thus cause divergence in the international financial markets.

A positive shift in λ represents an increase of the weight of the Home country's utility in the representative agent. This weight reflects the initial wealth distribution in the economy. We leave this as an exercise to the reader to show that $W_H/W_F = \lambda \theta_H(0)/\theta_F(0)$. An increase in λ is then akin to a wealth (income) transfer from Foreign to Home. To develop an intuition for the effect of the wealth transfer on the terms of trade it is useful to recall the classic *Transfer Problem*.⁸ A wealth transfer to Home raises Home's demand for both goods. But in the presence of home bias in consumption, demand for the Home good goes up by more. Hence the relative price of the Home good increases, i.e., Home's terms of trade improve, just as in the Transfer Problem. Since Home's terms of trade improve, the value of its output (dividend) goes up, and hence Home's stock price increases. As Foreign's terms of trade deteriorate, the value of that country's output goes down and the price of its stock falls.

One of the reasons we highlight the effect of a change in the planner's weight λ here is its relevance for models with financial market frictions such as, for example, portfolio constraints

⁸The original "Transfer Problem" was the outcome of a debate between Bertil Ohlin and John Maynard Keynes regarding the true value of the burden of reparations payments demanded of Germany after World War I. Keynes argued that the payments would result in a reduction of the demand for German goods and cause a deterioration of the German terms of trade, making the burden on Germany much higher than the actual value of the payments. See Krugman and Obstfeld (2003) for an elaboration and references.

or incomplete markets. One solution method, proposed by Cuoco and He (2001), involves solving for a competitive equilibrium using a representative agent with stochastic weights. Here, weights in the planner problem are constant and the allocation is Pareto efficient. A device involving stochastic weights is employed for solving for an inefficient allocation, occurring in models with frictions. The solution technique follows much the same steps as the ones in our exposition above, except that a stochastic weight λ emerges in place of the constant λ that we have here. This stochastic λ becomes a new (endogenous) state variable in the model, and much of interesting dynamics are due to movements in this state variable (i.e., endogenous wealth transfers). The final step—solving for λ —is more complex than in complete markets models, but still feasible. For more on this method and for further references, see Pavlova and Rigobon (2010).

3. Next Steps

The models we have referenced so far offer important insights on how elements of international asset pricing, international trade, and open economy macroeconomics can be combined within one framework and how they interact with each other. The literature is evolving in several directions, and future research in this area is going to be active and fruitful. Although we have learned a great deal, many questions remain open. In order to tackle more ambitious questions raised by the data and current events, the existing models certainly require improvements along several dimensions.

First, the discussion of global imbalances, current account sustainability, and, more generally, of international portfolios and risk sharing requires the introduction of market incompleteness into the story. This direction is important not only because markets are generally believed to be incomplete, but also because there is no role for policy under complete markets (an allocation is already Pareto efficient). Having incomplete markets adds a layer of methodological complexity. In his Ohlin lecture, Maurice Obstfeld remarks that “portfolio choice under incomplete markets is largely *terra incognita*.” Developing such models and understanding their workings constitutes frontier research in international macroeconomics these days. In the standard textbook models, market incompleteness is due to the inability

to trade any asset other than an international bond. This is one form of incompleteness that is certainly relevant, but in order to understand portfolio choices and how portfolio income contributes to the current account, we need to consider a broader menu of financial assets. Otherwise, one can no longer address the question “What makes a country’s current account path sustainable?”—the question that continues to be at the core of international macroeconomics for more than 150 years now.

Second, many models that have been developed in international macro-finance so far feature pure-exchange economies. This view of production is too simplistic. The natural next step is to include factors of production into these asset pricing models. Labor market considerations such as effort and unemployment, as well as investment, are important elements through which the real economy and financial markets interact with each other.

Third, our models are missing a full-fledged financial sector, the importance of which has been underscored by a series of recent contagious crises. The first step could be to model the financial inefficiencies stemming from the organizational structure of the financial sector in reduced-form—for example, in the form of financial constraints on certain market participants (margin constraints, VaR considerations), which may prevent them from supplying liquidity at times when it is needed the most. The next step would then be to endogenize these constraints—i.e., to model the agency problems that give rise to the need to restrict traders to take unlimited asset positions and unlimited risk. The fact that constraints on traders that we observe in the real world tend to bind at the same time, normally in bad times, can emerge as one of the leading explanations of contagion and as a channel of propagation of systemic risk. Another set of interesting frictions includes enforcement problems (as in, e.g., Kehoe and Perri (2002)) and financial market deepness (as in Caballero, Farhi, and Gourinchas (2008)). All these frictions are important and complementary, and exploring their role constitutes a promising direction of future research.

Finally, so far closed-form solutions have been obtained only in models in which agents have log-linear preferences. Although some closed-form solutions have been found for the CES case (Coeurdacier (2005)), future research should continue extending the workhorse model to include utility functions that generate more realistic price/dividend ratios, equity

premia, and other asset-pricing moments. Some promising work in this direction is by Stathopoulos (2008). Of course, there is a natural limit to a set of models that admit closed-form solutions; for the remaining, more general, models the literature will need to rely on numerical methods. Problems involving portfolio choice are particularly difficult to analyze because for these problems standard first-order approximation methods cannot deliver desired results (see Devereux and Sutherland (2010a) and Tille and van Wincoop (2010)). The literature is now testing the appropriateness of higher order approximation methods, with the approximations taken around a deterministic steady state. Perhaps even more complex methods (finite-element methods or projection methods) is what is required.

The field of international macro-finance is a new and active area of research. There are many ways in which one can push its frontier. Here we have highlighted just several possible promising directions. We are sure that there are many more.

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