

An Asset-Pricing View of External Adjustment*

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Abstract

Recent literature has argued that conventional measures of external sustainability—the trade balance and current account—are misleading because they omit capital gains on net foreign asset positions. We adjust the definition of the current account to include the capital gains and discuss how this may affect our thinking about external adjustment and sustainability. We do so in the context of a two-country macro-finance model of Pavlova and Rigobon (2008a) that allows to explore the interconnections between equilibrium portfolios and external accounts' dynamics. We calibrate the model and find that it generates several testable implications, some of which have already been validated empirically. First, we establish dynamic properties of the capital-gains adjusted current account and show that they are fundamentally different from those of the conventional current account. Second, we find that capital gains have a stabilizing effect on the trade balance and the current account. Finally, we demonstrate that in response to a shock, the conventional and the capital-gains adjusted current accounts may move in opposite directions.

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1. Introduction

An unprecedented rise in cross-border equity holdings over the past two decades¹ has generated a source of income previously disregarded in the national accounts: capital gains on equity holdings. The current practice incorporates capital gains only after they are redeemed, and this lack of marking to market may result in a significant misrepresentation of the extent of external imbalances worldwide—especially in the US, most of Europe, and Japan. The importance of correctly accounting for capital gains has been at the heart of a recent debate on the sustainability of the US current account deficit. The (conventionally-measured) current account deficits in the US have been unparalleled, indicating the need for a significant correction. However, income from the capital gains could have been financing consumption in the US, and so the imbalances could have been sustainable.² After the 2008 financial crisis some of the original arguments will require certain revision, but one central conclusion is undisputed: the growth of gross asset holdings during the last couple of decades must change significantly our understanding of how measures of sustainability have to be defined, and how the adjustment process needs to take place.

In this paper we respond to the critique of the conventional definition of the current account and define a *capital-gains adjusted current account*—a measure that explicitly accounts for capital gains on net foreign asset positions of a country. We investigate the properties of this measure in the context of a two-country macro-finance model of Pavlova and Rigobon (2008a) and compare it to other measures of external accounts. The model is solved in closed-form, which allows us to examine several analytical properties that link the external accounts and financial asset holdings. Moreover, because asset prices and portfolio holdings are all endogenous, it is possible to study the interconnections between external sustainability and portfolio decisions.

To evaluate the stochastic properties of the external measures of sustainability, we calibrate our model to reflect the current state of the US economy. In particular, through our parameter selection, we attempt to match the magnitudes of the trade balance and current account deficits in the US, home bias in asset holdings, net foreign debt of the US, and average cross-country correlations of consumption expenditures. We choose the parameters assuming that the current situation is one of equilibrium (as in our economy). In this environment, we first analyze separately the two

¹As documented in e.g., Gourinchas and Rey (2007a), Lane and Milesi-Ferretti (2001), Lane and Milesi-Ferretti (2007), Tille (2003), and Tille (2008).

²See, e.g., Caballero, Farhi, and Gourinchas (2008), Hausmann and Sturzenegger (2006), and Gourinchas and Rey (2007b).

elements that are missing from the conventional current account: the *expected* and the *unexpected* capital gains. We show that the former have a stabilizing property, offsetting the fluctuations in the trade balance and the traditional current account. Gourinchas and Rey (2007b) document a similar effect occurring in their dataset. It is the *unexpected* part of the capital gains, however, that is key to the dramatic differences in the dynamic properties of the traditional and the capital-gains adjusted current account in our model. The traditional current account follows a persistent process, while the capital-gains adjusted current account is highly volatile and serially uncorrelated. This is consistent with the evidence presented in Kollmann (2006) and Lane and Shambaugh (2007). In other words, the capital-gains adjusted current account behaves much like asset returns, whose short-term dynamics are also dominated by unexpected capital gains.

In order to understand the role of capital gains (valuation effects) in the external adjustment mechanism, we study impulse responses of our economy. The standard model of external adjustment is the one based on the canonical intertemporal approach to the current account. In that model, when a shock occurs, we first study its implications for output and consumption, and given those implications, we can trace their impact on the trade balance, the current account, the savings decisions, and ultimately on international positions. Our view in this paper is different. It starts by recognizing that agents already have wealth invested internationally. Therefore, the starting point—even before the shock shows up—is to determine the distribution of wealth and how it is invested (i.e, the composition of international portfolios). When a shock takes place, the first step is to track its impact on production and asset prices. Once these impacts are understood, we can track how the net foreign asset positions are going to be affected by the shock. That in turn will allow us to compute a new wealth distribution in the world economy. Agents' wealth will determine their consumption patterns, and given output, we can track the implications for the external accounts. Guided by this view of external adjustment, we do not find it surprising that our impulse responses show that following a shock, the conventional current account and the capital-gains adjusted current account may move in opposite directions.

Our work is related to the growing theoretical macro-finance literature that incorporates portfolio choice and asset pricing into models of open economy macroeconomics. Similarly to our approach here, Devereux and Sutherland (2008), Evans and Hnatkovska (2007), Ghironi, Lee, and Rebucci (2006), Kollmann (2006), and Tille and van Wincoop (2007) all base their analyses of

external accounts on stochastic portfolio models with incomplete markets.³ These papers employ various approximation techniques to study the behavior of their models around their steady states. By contrast, we base our analysis on an exact closed-form characterization of our equilibrium. Moreover, the steady state in our economy is stochastic.

The rest of the paper is organized as follows. Section 2 briefly describes the model. Section 3 defines the capital-gains adjusted current account and explores some links between external accounts and financial asset holdings. Section 4 studies dynamic properties of the capital-gains adjusted current account, contrasting them to those of the conventional current account. Section 5 discusses the external adjustment mechanism in our model. Section 6 offers some concluding remarks and directions for future research. The online appendix gives further theoretical background for our expressions.

2. The Model

2.1. The Economic Setting

For the purposes of our investigation, we adopt the model from Pavlova and Rigobon (2008a). We briefly review it here for completeness.

We work with a pure-exchange finite-horizon continuous-time economy populated by two countries: Home and Foreign. The Home country represents a large industrialized country, while Foreign stands for the rest of the world. Each country is endowed with a Lucas tree producing a strictly positive amount of a country-specific perishable good:

$$dY(t) = \mu_Y(t) Y(t) dt + \sigma_Y(t) Y(t) dw(t) \quad (\text{Home}), \quad (1)$$

$$dY^*(t) = \mu_{Y^*}(t) Y^*(t) dt + \sigma_{Y^*}(t) Y^*(t) dw^*(t) \quad (\text{Foreign}), \quad (2)$$

where w and w^* are the (independent) Brownian motions representing Home and Foreign output shocks, respectively, and $\mu_Y, \mu_{Y^*}, \sigma_Y > 0$, and $\sigma_{Y^*} > 0$ are the mean growth rates and volatilities of output. The prices of the Home and Foreign goods are denoted by p and p^* , respectively. We fix the world numeraire basket to contain $a \in (0, 1)$ units of the Home good and $(1-a)$ units of the

³Also related, but, unlike ours, cast in the context of production economies, are elegant analyses of Devereux and Saito (2006), and Kraay and Ventura (2000). Coeurdacier, Kollmann, and Martin (2008) primarily focus on equity home bias, but do also report implications for NFA dynamics similar to ours. Other important contributions to this literature, but with a different focus than ours, include Engel and Matsumoto (2006) and Mendoza, Quadrini, and Rios-Rull (2007).

Foreign good, and normalize the price of this basket to be equal to unity. The terms of trade, q , are defined as the price of the Home good relative to that of the Foreign good: $q \equiv p/p^*$. Our modeling of financial markets is standard. The Home and Foreign stocks S and S^* , are claims to the Home and Foreign trees, respectively. They are available for trade by all investors and are in fixed supply of one share each. There is also the “world” bond B in zero net supply, which is a money market account locally riskless in units of the numeraire.

The Home country’s representative consumer starts with an endowment of \bar{s}_H^S shares of the Home stock, $\bar{s}_H^{S^*}$ shares of the Foreign stock and a (negative) position in the bond. The Foreign consumer owns the remaining shares of the stocks and an offsetting (positive) position in the bond, denoted by \bar{b} . Later in this paper we calibrate the model so that Home represents the US economy, whose net bond position is large and negative. This is the rationale for giving the countries initial bondholdings. The initial wealth of the Home resident is thus $W_H(0) = \bar{s}_H^S S(0) + \bar{s}_H^{S^*} S^*(0) - \bar{b}$ and that of the Foreign resident is $W_F(0) = S(0) + S^*(0) - W_H(0)$. A representative consumer in each country i , $i \in \{H, F\}$, chooses nonnegative consumption of each good $(C_i(t), C_i^*(t))$ and a portfolio of the available securities $s_i(t) \equiv (s_i^S(t), s_i^{S^*}(t))$, where s_i^j the number of shares of asset j held by consumer i . The dynamic budget constraint of each consumer has the standard form

$$\begin{aligned} dW_i(t) = & s_i^B(t) dB(t) + s_i^S(t) (dS(t) + p(t)Y(t)dt) + s_i^{S^*}(t) (dS^*(t) + p^*(t)Y^*(t)dt) \\ & - p(t)C_i(t) dt - p^*(t)C_i^*(t) dt, \end{aligned} \quad (3)$$

where $W_i(T) \geq 0$, $i \in \{H, F\}$. Preferences of consumer i , are represented by a time-additive utility function defined over consumption of both goods:

$$E \left[\int_0^T e^{-\rho t} u_i(C_i(t), C_i^*(t)) dt \right], \quad \rho > 0, \quad i \in \{H, F\}, \quad (4)$$

where

$$\begin{aligned} u_H(C_H(t), C_H^*(t)) &= \alpha_H(t) \log C_H(t) + \beta_H(t) \log C_H^*(t), \\ u_F(C_F(t), C_F^*(t)) &= \beta_F \log C_F(t) + \alpha_F \log C_F^*(t). \end{aligned}$$

Stochastic processes α_H and β_H in the utility of the Home country represent preference shifts toward the Home and Foreign good, respectively. For generality, the innovations to α_H and β_H are given by a combination of supply shocks w and w^* as well as two independent standard Brownian motions w^α and w^β . Without the last two Brownian motions, financial markets are complete.

But if the preference shifters display nontrivial dependence on w^α and w^β , the existing investment opportunity cannot span the uncertainty in the model, and hence the possibilities of hedging against the preference shifts is impaired and risk sharing in the world economy becomes imperfect. The main focus of this paper is on the Home country, which is why we assume away any preference shifts at Foreign and concentrate on the effects of domestic preference shifts at Home.⁴

The preference shifts or *demand shocks* are important ingredients of our model, for two reasons. First, in the absence of the demand shocks, free trade in goods makes stock prices perfectly correlated and financial markets irrelevant (Helpman and Razin (1978), Cole and Obstfeld (1991), Zapatero (1995)). Second, empirical evidence indicates that demand shocks are important for reproducing the real-world dynamics; supply shocks alone are typically not sufficient. For example, Stockman and Tesar (1995) calibrate preference shocks to be roughly 85% of the size of supply shocks. Pavlova and Rigobon (2007) estimate a model more similar to ours and conclude that demand shocks have approximately the same volatility as supply shocks.

2.2. Characterization of Equilibrium

An equilibrium in this economy is defined in a standard way: it is a collection of goods and asset prices (p, p^*, S, S^*, B) and consumption-investment policies $(C_i(t), C_i^*(t), x_i^S(t), x_i^{S^*}(t))$, $i \in \{H, F\}$ such that (i) each consumer-investor maximizes his utility (4) subject to the budget constraint (3) and (ii) goods, stock, and bond markets clear.

In this incomplete markets economy, the usual construction of a representative agent's (planner's) utility as a weighted sum, with constant weights, of individual utility functions is not possible. However, as we explain in Pavlova and Rigobon (2008a), one can still solve for the equilibrium allocations in this economy using an analogue of the central planning approach in which the weights in the fictitious world representative agent utility are stochastic.⁵ This fictitious representative agent

⁴Solving a model with stochastic preference shifts at Foreign involves an additional technical difficulty, which has to do with the derivation of the expressions for equilibrium stock prices. While we believe that the model with stochastic preference shifts in all countries is still analytically tractable, we leave this extension for future research.

⁵The approach of solving for competitive equilibrium via solving for weights in the representative agent was pioneered by Negishi (1960). It was extended for continuous-time economies by Cuoco and He (1994) and afterwards used extensively in dynamic asset pricing models with financial market frictions (see, among many others, Basak and Cuoco (1998), Detemple and Serrat (2003), and Pavlova and Rigobon (2008b)). A related approach is the extra-state-variable methodology of Kehoe and Perri (2002) and Marcet and Marimon (1999).

maximizes his utility subject to the resource constraints:

$$\max_{\{C_H, C_H^*, C_F, C_F^*\}} E \left[\int_0^T e^{-\rho t} (u_H(C_H(t), C_H^*(t)) + \lambda(t) u_F(C_F(t), C_F^*(t))) dt \right] \quad (5)$$

$$\text{s. t.} \quad C_H(t) + C_F(t) = Y(t), \quad (6)$$

$$C_H^*(t) + C_F^*(t) = Y^*(t), \quad (7)$$

where we have normalized the weight on the Home consumer to be equal to one and assigned the weight λ to the Foreign consumer. The possibly stochastic weighting process λ is to be determined as part of the equilibrium. The consumption allocations and the goods and asset prices supporting these allocations are the same as those in a decentralized competitive equilibrium. In the decentralized equilibrium, λ is simply the ratio of the marginal utilities of either good of the two countries. When markets are complete, this ratio is constant (constant λ). When markets are incomplete, the marginal utilities are no longer proportional, and so λ becomes stochastic. It enters as an additional (endogenous) state variable, which reflects the relative weight the “planner” puts on the Foreign country’s utility.

We do not reproduce the full equilibrium characterization here and just report the expressions that are relevant for our discussion in this paper. We refer the reader to Pavlova and Rigobon for a complete characterization and to the supplementary appendix to this paper for an intuitive derivation of the expressions below.

The equilibrium terms of trade and stock prices are given by:

$$q(t) = \frac{\alpha_H(t) + \lambda(t)\beta_F}{\beta_H(t) + \lambda(t)\alpha_F} \frac{Y^*(t)}{Y(t)} \quad (8)$$

and

$$S(t) = \frac{1 - e^{-\rho(T-t)}}{\rho} \frac{q(t)}{aq(t) + 1 - a} Y(t), \quad (9)$$

$$S^*(t) = \frac{1 - e^{-\rho(T-t)}}{\rho} \frac{1}{aq(t) + 1 - a} Y^*(t) \quad (10)$$

To convey the economic mechanisms behind the formulas, we present the following simple table that summarizes the signs of the directions of responses to movements in the underlying state

variables.

Effects of	Y	Y^*	α_H	β_H	a drop in λ^\dagger
On the terms of trade q	-	+	+	-	+
On the Home stock S	+	+	+	-	+
On the Foreign stock S^*	+	+	-	+	-

A positive output shock at Home (an increase in Y) raises the dividend on the Home stock and so Home's stock price increases. At the same time, it increases the supply of the Home good in the world. As the good becomes less scarce, its price falls relative to that of the Foreign good. Hence Home's terms of trade deteriorate and Foreign's terms of trade improve. The improvement of Foreign terms of trade increases the value of Foreign's output and hence the Foreign stock goes up. The response following a Foreign output shock is analogous.

A preference shift towards the Home good (an increase in α_H) creates an excess demand for the Home good. This pushes up its price relative to that of the Foreign good and therefore improves Home's terms of trade. The value of Home's output (dividend) increases while that of Foreign's decreases. Hence Home's stock market goes up and Foreign's stock market falls.

A negative shift in λ represents an increase of the weight of the Home country's utility in the representative agent. This weight can be linked formally to the wealth distribution in the economy. Indeed, the wealth of Home and Foreign can be represented as (see Appendix)

$$W_H(t) = \frac{\alpha_H(t) + \beta_H(t)}{\alpha_H(t) + \lambda(t)\beta_F} S(t), \quad W_F(t) = \frac{\lambda(t)(\alpha_F + \beta_F)}{\beta_H(t) + \lambda(t)\alpha_F} S^*(t), \quad (11)$$

and hence

$$\lambda(t) = \frac{W_F(t)}{W_H(t)} \frac{\alpha_H(t) + \beta_H(t)}{\alpha_F + \beta_F}. \quad (12)$$

A drop in λ is then akin to a wealth transfer from Foreign to Home. To develop the intuition for the effect of the wealth transfer on the terms of trade it is useful to recall the classic Transfer Problem.⁶

A wealth (income) transfer to Home raises Home's demand for both goods. But in the presence of

[†]The shocks to the first four variables are positive, while the shock to the Foreign's weight λ is negative (for ease of interpretation). Obtaining unambiguous signs of the effect of λ (reported in the Table) requires an additional assumption of home bias in consumption ($\alpha_i > \beta_i$).

⁶The original "Transfer Problem" was the outcome of a debate between Bertil Ohlin and John Maynard Keynes regarding the true value of the burden of reparations payments demanded of Germany after World War I. Keynes argued that the payments would result in a reduction of the demand for German goods and cause a deterioration of the German terms of trade, making the burden on Germany much higher than the actual value of the payments. See Krugman and Obstfeld (2003) for an elaboration and references.

home bias in consumption, demand for the Home good goes up by more. Hence the relative price of the Home good increases, i.e., Home's terms of trade improve, just as in the Transfer Problem. Since Home's terms of trade improve, the value of its output (dividend) goes up, and hence Home's stock price increases. As Foreign's terms of trade deteriorate, the value of that country's output goes down and the price of its stock falls.

All equilibrium quantities can be pinned down as functions of the four exogenous state variables (Y , Y^* , α_H , and β_H) and the endogenous state variable λ . The dynamics of λ are derived as part of the equilibrium characterization.

3. The Current Account

Before we turn to investigating equilibrium behavior of our economy and making a link between portfolio rebalancing and external adjustment mechanisms, we highlight the model's implications for the current account and its dynamics. We first start with the textbook definition of the current account and then consider the definition based on changes in a country's net foreign asset position. In principle, the two definitions should yield similar measures. However, once expected and realized capital gains are accounted for, the differences between the two measures, and especially their dynamic properties, can be striking, as we demonstrate below.

3.1. The Conventional Current Account

Let us concentrate on the Home country. The conventional measure of the current account, employed in international finance textbooks and used in the national accounts, is

$$CA = \text{Trade Balance} + \text{Net Dividend Payments} + \text{Net Interest Payments.}$$

In our economy, the trade balance is given by

$$TB_H(t) = (p(t)(Y(t) - C_H(t)) - p^*(t)C_H^*(t))dt,$$

and the current account by

$$CA_H(t) = TB_H(t) + \left[s_H^{S^*}(t)p^*(t)Y^*(t) - s_F^S(t)p(t)Y(t) + s_H^B(t)B(t)r(t) \right] dt. \quad (13)$$

The second and the third terms in (13) are dividend receipts from foreign assets minus dividend payments to Foreign, and the last term is net interest payments. Recall that each of the above

quantities in our model is defined as a rate (e.g., the export rate, the dividend rate, etc.) and hence need to be scaled by a time increment.

3.2. The Capital-Gains Adjusted Current Account

Defining the current account as the change in the net foreign asset (NFA) position of a country, we have

$$CGCA_H(t) = d \left[s_H^{S^*}(t) S^*(t) - s_F^S(t) S(t) + s_H^B(t) B(t) \right], \quad (14)$$

where the first two terms in the square brackets are Home's investment in the Foreign stock minus Foreign' investment in the Home stock, and the last term is Home's balance on the bond account. The label "CGCA" stands for "capital-gains adjusted current account," the rationale for which will become clear shortly. Note that, by market clearing, $s_F^S(t) = 1 - s_H^S(t)$ and that, by definition, Home's wealth equals its portfolio value, $W_H(t) = s_H^S(t) S(t) + s_H^{S^*}(t) S^*(t) + s_H^B(t) B(t)$. Hence, we can rewrite (14) as

$$CGCA_H(t) = dW_H(t) - dS(t) \quad (15)$$

To simplify the exposition below, let us introduce some new notation. Let us summarize all the Brownian motions driving the economy in the vector $\vec{w} \equiv (w, w^*, w^\alpha, w^\beta)$ and represent the dynamics of the bond and the stocks as follows

$$dB(t) = B(t)r(t)dt, \quad (16)$$

$$dS(t) + p(t)Y(t)dt = S(t)[\mu_S(t)dt + \sigma_S(t)d\vec{w}(t)], \quad (17)$$

$$dS^*(t) + p^*(t)Y^*(t)dt = S^*(t)[\mu_{S^*}(t)dt + \sigma_{S^*}(t)d\vec{w}(t)], \quad (18)$$

where the interest rate r , the stocks expected returns μ_S and μ_{S^*} and their volatilities σ_S and σ_{S^*} are determined in equilibrium (e.g., from (9)–(10)). Armed with this notation, we can rewrite the budget constraint of Home (3) as follows:

$$\begin{aligned} dW_H(t) = & \left[s_H^B(t) B(t) r(t) + s_H^S(t) S(t) \mu_S(t) + s_H^{S^*}(t) S^*(t) \mu_{S^*}(t) \right] dt \\ & + \left[s_H^S(t) S(t) \sigma_S(t) + s_H^{S^*}(t) S^*(t) \sigma_{S^*}(t) \right] d\vec{w}(t) + TB_H(t) - p(t)Y(t)dt. \end{aligned}$$

Substituting it into (15) and then using (17) and the stock market clearing, we arrive at the following

expression:

$$\begin{aligned}
CGCA_H(t) &= TB_H(t) + \left[s_H^{S^*}(t)S^*(t)\mu_{S^*}(t) - s_F^S(t)S(t)\mu_S(t) + s_H^B(t)B(t)r(t) \right] dt \\
&\quad + \left[s_H^{S^*}(t)S^*(t)\sigma_{S^*}(t) - s_F^S(t)S(t)\sigma_S(t) \right] d\vec{w}(t). \tag{19}
\end{aligned}$$

The first difference between the conventional and the capital-gains adjusted current account revealed by equation (19) is the presence of the diffusion ($d\vec{w}$) component in (19) and its absence in (13). This component is the *unexpected* part of the realized capital gains on Home's net foreign assets. A shock $d\vec{w}$ typically has a differential impact on the stock portfolios of Home and Foreign. The capital-gains adjusted current account of Home improves if its return on foreign asset holdings exceeds the return the Foreign country makes on its holdings of Home's assets. Equation (19) assumes that all capital gains are marked to market. This aspect makes *CGCA* different from book-value based measures of current account such as the one in (13), commonly employed in practice.

It is important to note that the two measures of the current account have fundamentally different dynamic properties. The conventional current account is a persistent process, in line with the results in the existing literature. In contrast, the capital-gains adjusted current account features additionally an increment of a random walk ($d\vec{w}$) process, and therefore it bears a closer resemblance to the dynamics of a stock market rather than a persistent macroeconomic series such as dividends or the conventional current account. It is amply documented by the proponents of the efficient markets hypothesis in finance that stocks' capital gains are large, volatile, and serially uncorrelated. And so should be the fluctuations in a NFA position of a country, captured here by *CGCA*.

The second difference between the two measures of the current account stems from the differences in the *expected* (dt) component. Comparing equations (13) and (19), we find two elements that are common to the conventional and the capital-gains adjusted current account measures: the trade balance and net interest payments on the (locally) riskless bond. The remaining terms, due to holdings of the risky stocks, are different. To better highlight the differences, note from (17)–(18) that

$$S(t)\mu_S(t)dt = E_t[dS(t)] + p(t)Y(t)dt \quad \text{and} \quad S^*(t)\mu_{S^*}(t)dt = E_t[dS^*(t)] + p^*(t)Y^*(t)dt,$$

where $E_t[\cdot]$ is the expectation conditional on the information at time t . The first term on the right-hand side of these expressions is the expected capital gains and the second one is the dividends per

share. Denoting the expected capital gains on holding a share of stocks S and S^* by ECG_S and ECG_{S^*} , respectively, we can express the difference between the expected capital-gains adjusted current account and the conventional one as

$$E_t[CGCA_H(t)] - CA_H(t) = -s_F^S(t) ECG_S(t) + s_H^{S^*}(t) ECG_{S^*}(t). \quad (20)$$

This simple formula describes what Hausmann and Sturzenegger (2006) label “the missing dark matter” in the measurement the current account. This “dark matter” accumulates due to the fact that the countries hold risky assets which are issued at home as well as abroad, and these risky assets have different expected capital gains. The expected capital gain on an asset, of course, reflects its risk and return tradeoff. If domestic assets are safer and have lower expected returns than foreign, a country earns a higher expected return on its assets than it owes on its liabilities. In this scenario, the difference between the capital-gains adjusted and the conventional current account is expected to be positive. We expect such pattern to be occurring in the US whose gains on domestic assets have been lower than gains on assets abroad (Gourinchas and Rey (2007b)).

The sum of net expected capital gains and net unexpected capital gains is what has been typically labeled as “valuation effects” in the current account adjustment literature. Empirically, it is very hard to disentangle the two individual components. However, as we will point out below, their stochastic properties are very distinct.

One may wonder why the terms of trade were mentioned nowhere in this discussion. After all, the original arguments highlighting valuation effects stress primarily the fact that the values of a country’s asset and liabilities change in response to fluctuations in the exchange rate (or the terms of trade), and these fluctuations need to be taken into account when evaluating the NFA position of a country. This argument definitely applies in our model—in fact, all intuitions for the behavior of stock prices that we have given above (Section 2.2), are based on the movements in the terms of trade. The terms of trade are thus embedded in the prices of stocks and their dynamics.

It is worth mentioning that in our derivations in this section we did not need to specialize consumer preferences to be log-linear. The expressions for both the current account in equation (13) and the capital-gains adjusted current account in equation (19) are valid for a general utility function. The log-linear form of the preferences is needed only for obtaining a closed-form characterization of the inputs into the formulas, namely the stock prices and the countries’ portfolios.

3.3. The Links between Financial Asset Holdings and External Accounts

With the analytical structure we have established above, we can derive several properties linking external accounts and financial asset holdings in the context of our model. Of most relevance for our discussion in subsequent sections are the following two properties.⁷

Property 1: *The relationship between the Home country’s net debt position and its current account is as follows:*

$$CA_H(t) = \left(r(t) - \frac{\rho}{1 - e^{-\rho(T-t)}} \right) s_H^B(t) B(t) dt. \quad (21)$$

Property 2: *The relationship between the Home country’s net foreign asset position and its trade balance is given by*

$$TB_H(t) = -\frac{\rho}{1 - e^{-\rho(T-t)}} NFA_H(t) dt. \quad (22)$$

Property 1 may appear counter-intuitive: if the term in the parentheses is positive, a debtor country could be running a current account deficit forever! Note, however, that ours is not a bond-only economy and bondholdings constitute only a part of the country’s net external wealth. The country’s net external wealth also includes net equity holdings, and is captured by NFA. Countries with negative NFA positions (“debtor countries”) must run a trade surplus; they clearly cannot have a deficit forever, according to Property 2. This result (Property 2) is very intuitive and in fact is quite similar to the relationship derived in Obstfeld and Rogoff (1996), Chapter 2, with the main difference being that we consider a general equilibrium model with trade in equities. Moreover, having a negative debt position in equilibrium implies that a country (say, Home) is holding a leveraged portfolio. That portfolio is riskier than that of the Foreign country. The Home country is compensated for taking a greater share of world risk by collecting risk premia on its risky asset holdings. These risk premia can be used to finance current account deficits.⁸ In a recent paper Gourinchas and Rey (2007a) argue that the US, whose external liabilities are mainly Treasury bills and bonds (with relatively low expected returns) while its external assets are riskier instruments (with relatively high expected returns), collects additional income from positive risk premia on its NFA positions. Our model allows for a calibration that captures this possibility.

⁷These properties are, of course, model-specific. For a proof of the properties see Pavlova and Rigobon (2008a).

⁸We thank Mick Devereux for suggesting this intuition to us.

4. Dynamic Implications

In this section, we calibrate our model to match several key aspects of the data. The Home country in our calibration represents the US economy and the Foreign country the rest of the world. This exercise allows us to contrast different measures of external accounts and to quantify the extent of discrepancies across the measures. It also enables us to examine valuation effects entailed by our model.

4.1. Calibration

In our choice of parameter values we have tried to match several important statistics of the US economy. Where directly observable, we set the parameters equal to their average values in the data. Where not observable (e.g., the preference shifters), we calibrate the parameters so that the model implies that the Home country (i) runs a trade deficit and a (conventional) current account deficit of the sizes roughly matching those of the US economy, and (ii) so that the countries have positive cross-holdings and their portfolios exhibit a home bias of a reasonable magnitude. Table 1 lists the ensuing parameter values.

Parameter	Value	Parameter	Value	Parameter	Value
$Y(0)$	1	$\alpha_H(0)$	0.7	σ_Y	0.01
$Y^*(0)$	2	α_F	0.7	σ_{Y^*}	0.01
μ_Y	0.02	$\beta_H(0)$	0.3	σ_{α_H}	$(z_1, 0, z_2, 0)$
μ_{Y^*}	0.02	β_F	0.3	σ_{β_H}	$(0, z_1, 0, z_2), \quad z_1 \in [0, 0.0061],$
ρ	0.05	$\lambda(0)$	0.9		$z_2 \in [0.002, 0.0136]$
a	0.5				

Table 1: Parameter choices.

According to this parametrization, the countries' output processes are geometric Brownian motions (since both the drift and diffusion are constant). This is a specification most commonly assumed in the literature. We chose the mean growth rate of 2% and the volatility of 1% for realism, but the results do not change much as we vary them. The literature provides little guidance for the form of preference shifts, and so we simply assume that they are also geometric Brownian motions. Furthermore, motivated by the estimated dynamics of demand shocks in Pavlova and Rigobon (2007), we posit that α_H loads positively on the Home output shock w and β_H loads positively on the Foreign output shock w^* . The presence of demand shocks allows us to attain realistic values for the consumption (expenditure) correlations across countries and for a home bias in

portfolios. Otherwise, as has been well-documented in the literature, the cross-country correlation of consumption is too high, and definitely higher than the output correlation. Moreover, with no demand shocks, the countries' portfolios in our model are indeterminate (for the same reason as in Cole and Obstfeld (1991)), and hence no meaningful matching of international portfolio compositions is possible. As a sensitivity exercise, we vary the relative sizes of the demand and supply shocks (from 0.05 to 1.85) by varying the volatility of the demand shocks and keeping that of the supply shocks fixed, as well as vary the instantaneous correlation between the demand and supply shocks (from 0 to 45 percent).⁹ The ensuing ranges of σ_{α_H} and σ_{β_H} are specified in Table 1. The implied cross-country consumption (expenditure) correlations are mainly sensitive to the parameter controlling the relative sizes of the supply and demand shocks, while the portfolio compositions also to the parameter controlling the correlation of the shocks. The initial value of the Foreign country's weight λ reflects the allocation of initial endowments of shares of stock and the bond. This allocation is necessarily asymmetric since we are trying to match the external asset-liability structure of the US, and hence $\lambda(0)$ is different from one. This $\lambda(0)$ is, of course, endogenous, but we can set the model primitives—initial portfolios—supporting this choice of $\lambda(0)$, for example, equal to those reported in Figure 4.

Our economy is always in a (stochastic) steady state. We solve for the initial steady state as determined by our calibrated parameters and report the values of key variables in that steady state in Figure 4. Unless stated otherwise, all variables are for the Home country. We plot the number of shares of the Home stock held by the Home residents (panel (a)), the number of shares of the Foreign stock held by Home residents (panel (b)), the value of their bondholdings (panel (c)), the annualized trade balance and conventional current account measured as fractions of GDP (panels (d) and (e)), and the capital-gains adjusted current account as a fraction of GDP (panel (f)). In all figures, the x-axis measures the instantaneous correlation between the demand and supply shocks and the y-axis the relative size of the demand and supply shocks, as captured by the ratio of their volatilities.

Our choice of parameters implies a home bias in portfolios, whereby Home holds more than 89 percent of the supply of Home shares, and between 10 to 20 percent of that of Foreign shares. When

⁹The former is formally defined as $\sqrt{\|\sigma_{\alpha_H}\|^2}/\sigma_Y$ and the latter as $\sigma_Y i_1 \sigma_{\alpha_H}^\top / (\sigma_Y \sqrt{\|\sigma_{\alpha_H}\|^2})$, where $i_1 = (1, 0, 0, 0)$. Here, the supply shock is represented by the Home output shock and the demand shock by a preference shift towards the Home good. The analogous quantities corresponding to the Foreign good, namely $\sqrt{\|\sigma_{\beta_H}\|^2}/\sigma_{Y^*}$ and $\sigma_{Y^*} i_1 \sigma_{\beta_H}^\top / (\sigma_{Y^*} \sqrt{\|\sigma_{\beta_H}\|^2})$, vary symmetrically (and simultaneously), within the same range. We bound the volatility of the demand shocks from below because values close to zero produce a lot of variation in the countries' portfolios, with portfolios becoming indeterminate in the limit of zero volatility (Cole and Obstfeld).

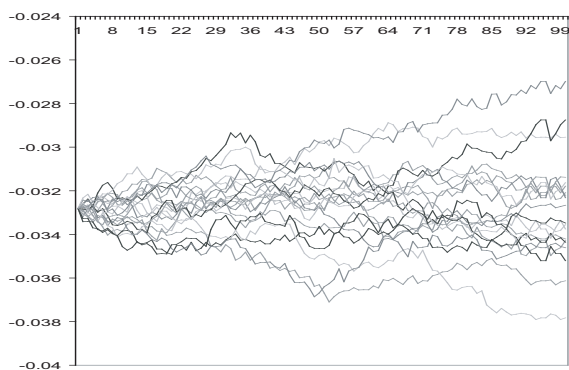
demand and supply shocks are uncorrelated, bond holdings are exactly zero. When the correlation increases, Home’s bondholdings become negative.¹⁰ Home demands more of both the Home and Foreign stock, ending up owning more shares of the Foreign stock than foreigners own abroad, and finances these stock purchases not by selling domestic shares (rebalancing), but by borrowing. The selling of the bond by Home implies a current account deficit, as shown in panel (e) and implied by Property 1. In our steady state, the current account deficit varies from zero up to 11 percent of GDP, while the trade deficit is constant at 3 percent of GDP (panel (d)). Note that CGCA comes out positive, at around 3 percent of GDP (panel (f)). It is worth mentioning that the initial NFA position of Home is also positive. In recent US data, we observe both trade deficits and negative NFA positions. In our model, such a situation cannot be consistent with equilibrium according to Property 2, which says that countries running trade deficits must have positive NFA. So interpreted within our model, recent external imbalances in the US would be deemed unsustainable.

4.2. Unconditional responses

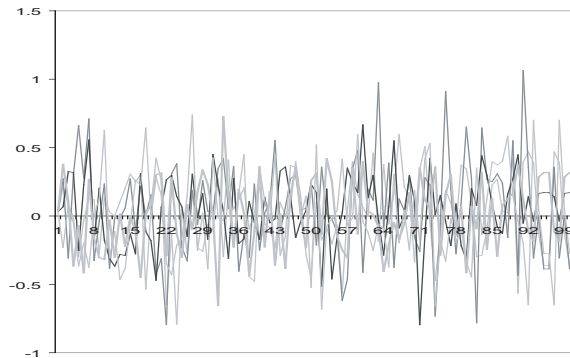
In this simulation we generate 500 histories, 100 years each, by randomly drawing all four shocks. We start by examining the serial correlation of the conventional current account and the serial correlation of the capital-gains adjusted current account.

To provide an illustration, Figure 1 depicts a constellation of sample paths of the conventional and the capital-gains adjusted current accounts emerging from the Monte-Carlo simulations. The difference in the dynamic behavior of the two series is striking. The conventional current account and the trade balance (not reported here for brevity) are highly persistent series, while the capital-gains adjusted current account is not. The main reason why the latter does not exhibit much persistence is the fact that much of its variation is explained by the *unexpected* capital gains, which are not serially correlated. The series in panel (b) looks much like a return on a financial asset: very volatile and largely unpredictable. Table 2 validates these observations and reports the serial correlations of each of the three measures of external accounts. We estimate a simple AR(1) and average the coefficients across all simulations, for the entire parameter space. One can see that irrespectively of the parameters chosen for a simulation, the current account is highly

¹⁰It is important to highlight that whether bondholdings are positive or negative depends on the sign of the correlation between the demand shocks and the supply shocks. In our case, home supply shocks are positively correlated with the agents’ demand for the Home good, and therefore, the Home stock is a good hedge against that correlation.



(a) Conventional current account



(b) Capital-gains adjusted current account

Figure 1: Simulated sample paths of the trade balance, conventional current account, and capital-gains adjusted current account (these series have not been scaled by GDP). Each sample path corresponds to one simulated history.

	AR(1) Serial Correlation			Standard Error
	Average	Max	Min	
CA_H	97.52%	97.53%	97.51%	0.00%
$CGCA_H$	0.87%	1.97%	0.58%	0.35%

Table 2: AR(1) serial correlation of the simulated conventional current account and capital-gains adjusted current account (in fractions of GDP). The average is computed as the simple average of the estimates across our parameter space, and the standard error is for that average.

serially correlated. CGCA, however, does not exhibit much serial correlation.¹¹ Its dynamics closely resemble those of asset returns.

4.3. Valuation Effects: Expected and Unexpected Capital Gains

Capital gains (valuation effects) have been at the source of the debate on the current state of external imbalances in the US and worldwide. Capital gains can be split between expected and unexpected capital gains, which contribute in rather different ways to the international adjustment process. The empirical literature faces a significant challenge of being able to disentangle the expected part of the capital gains from the unexpected part. We build on our theoretical insights and study the properties of capital gains within our simulation.

Recall our decomposition of the current account:

$$CA_H(t) = \left[TB_H(t) + \underbrace{s_H^{S^*}(t)p^*(t)Y^*(t) - s_F^S(t)p(t)Y(t)}_{\text{Net Dividend Payments}} + \underbrace{s_H^B(t)B(t)r(t)}_{\text{Net Interest Payments}} \right] dt$$

and

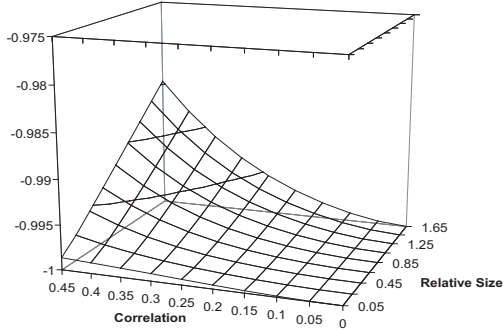
$$CGCA_H(t) = \left[CA_H(t) + \underbrace{s_H^{S^*}(t)ECG_{S^*}(t) - s_F^S(t)ECG_S(t)}_{\text{Net Expected Capital Gains}} \right] dt + \underbrace{\left[s_H^{S^*}(t)S^*(t)\sigma_{S^*}(t) - s_F^S(t)S(t)\sigma_S(t) \right]}_{\text{Net Unexpected Capital Gains}} d\vec{w}(t).$$

In what follows, we'll frequently drop the word “net” when we refer to capital gains, with the understanding that we are alluding to capital gains on the NFA positions. Unless mentioned otherwise, all variables that we discuss below are for the Home country.

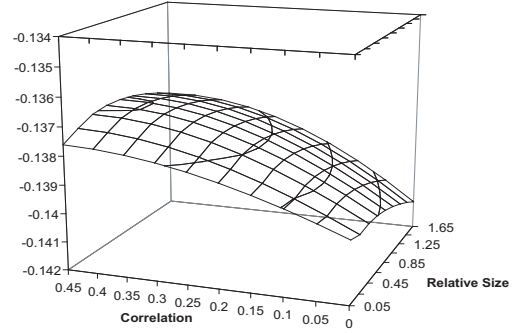
Recent empirical literature has produced an intriguing stylized fact: valuation effects—the sum of expected and unexpected capital gains—are negatively correlated with the trade balance (see Gourinchas and Rey (2007b) and Devereux and Sutherland (2008)), or, in other words, capital gains have a stabilizing effect on the trade balance. As one can see from Figure 2a, indeed in all our simulations expected capital gains are highly negatively correlated with the trade balance (around -90 percent), providing a stabilizing effect. This result is consistent with Gourinchas and Rey. The correlation of the unexpected capital gains with the trade balance is consistently negative as well (Figure 2b), although the magnitudes of the correlations are much smaller (around -13 percent). Almost the same relationship exists for the correlations with the traditional current account (around -90 percent for expected and -11 percent for unexpected capital gains).¹²

¹¹This is consistent with the evidence presented in Kollmann (2006) and Lane and Shambaugh (2007).

¹²One possible explanation for the negative correlations between the current account and expected capital gains



(a) Expected capital gains and TB

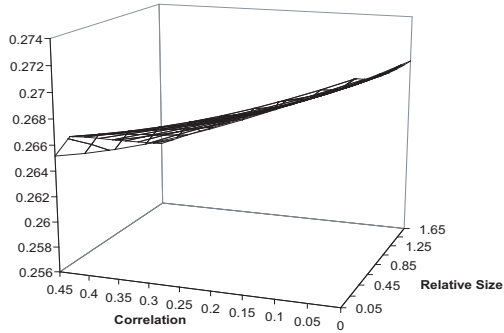


(b) Unexpected capital gains and TB

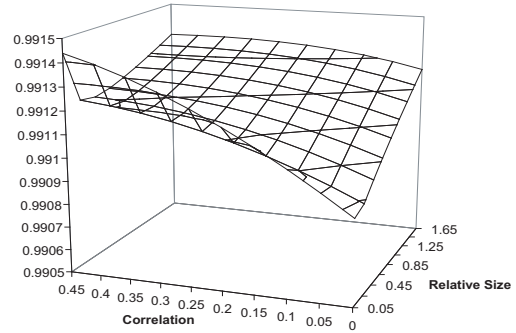
Figure 2: Correlation of expected capital gains (panel (a)) and unexpected capital gains (panel (b)) with the trade balance (measured as a fraction of GDP). The x-axis measures the instantaneous correlation between the demand and supply shocks $\sigma_Y i_1 \sigma_{\alpha_H}^\top / (\sigma_Y \sqrt{\|\sigma_{\alpha_H}\|^2})$, where $i_1 = (1, 0, 0, 0)$. The y-axis measures the ratio of the volatilities of the demand and supply shocks $\sqrt{\|\sigma_{\alpha_H}\|^2} / \sigma_Y$ (relative size of demand and supply shocks).

Note that while the correlations of expected capital gains with the trade balance and with the current account are very high, those of unexpected capital gains are much smaller. However, when we compare the correlations with the change in the net foreign asset positions—the capital-gains adjusted current account—the roles are reversed. The correlation of expected capital gains with the change in the NFA positions is positive but not very high (Figure 3a), but unexpected capital gains are almost perfectly positively correlated (Figure 3b). The reason for the change is that unexpected capital gains completely dominate the share attributable to the current account and expected capital gains in the variation of net foreign assets. This conclusion is very much in line with the evidence reported in, among others, Devereux and Sutherland.

is that expected returns and dividend payments are “mechanically” negatively correlated if the total asset returns are held constant and a fraction of that return is randomly attributed to a capital gain and a fraction to dividend. For example, assume that a country has a total expected return on net foreign assets (expected capital gains plus dividends) of, say, 5 percent. Assume this is constant across all states of the world. If changes in the state variable imply high dividends, then, to keep the total expected return constant, an increase in the dividends has to imply a drop in the expected capital gain. If this is the case, therefore, because the current account has dividends in it but not capital gains, a negative correlation is just an outcome of accounting. We reject this possibility. First, expected capital gains are already negatively correlated with the trade balance. Second, when we compute the correlation between the expected capital gains and dividends, we find that they are perfectly positively correlated, and hence the negative relationship between the current account and expected capital gains is coming from the trade balance and interest payments terms. We thank Pierre-Olivier Gourinchas for motivating us to explore this “accounting” explanation for the observed behavior.



(a) Expected capital gains and *CGCA*



(b) Unexpected capital gains and *CGCA*

Figure 3: Correlation of expected capital gains (panel (a)) and unexpected capital gains (panel (b)) with the capital-gains adjusted current account (measured as a fraction of GDP). The x-axis measures the instantaneous correlation between the demand and supply shocks $\sigma_Y i_1 \sigma_{\alpha_H}^T / (\sigma_Y \sqrt{\|\sigma_{\alpha_H}\|^2})$, where $i_1 = (1, 0, 0, 0)$. The y-axis measures the ratio of the volatilities of the demand and supply shocks $\sqrt{\|\sigma_{\alpha_H}\|^2} / \sigma_Y$ (relative size of demand and supply shocks).

4.4. Standard Macroeconomic (IRBC) Statistics

To relate our work to the international real business cycles (IRBC) literature, we report the standard statistics that this literature targets. We caution, however, that our model does not include many realistic features highlighted in IRBC research, such as capital accumulation, time to build, labor choice, etc. Table 3 reports the cross-country correlations and volatilities of consumption expenditures and GDPs, volatilities of the terms of trade, of the trade balance and of the current account for four scenarios. To generate the scenarios, we again vary the correlation between demand and supply shocks as well as their relative size.

With the caveat that our model has nothing to say about investment—the most important IRBC moment—we proceed to describe our findings. One long-standing puzzle in the literature is the consumption correlation puzzle: optimal risk sharing makes cross-country consumption (expenditures) highly correlated, and more so than the countries' GDPs. We do not observe this pattern in the data. In our model, there are two channels that reduce the cross-country consumption correlation: (i) the presence of demand shocks and (ii) imperfect risk sharing due to market incompleteness. The first channel is especially important in our setup: in the simulations in which demand shocks are large (columns 2 and 4), we observe particularly low correlations. These corre-

Correlation of supply and demand shocks	0.05	0.05	0.4	0.4
Relative size of supply and demand shocks	0.25	1.45	0.25	1.45
<u>Correlations</u>				
Home and Foreign GDPs	0.53	0.23	0.52	0.20
Home and Foreign Consumption Expenditure	0.84	0.52	0.82	0.46
Home Consumption Expenditure and Home GDP	0.91	0.76	0.90	0.75
<u>Standard Deviations (%)</u>				
Home Consumption Expenditure	6.29	6.90	6.34	7.09
Foreign Consumption Expenditure	6.83	7.78	6.77	7.75
Home Consumption of the Home Good	3.57	3.60	3.59	3.65
Home GDP	5.95	6.47	6.03	6.66
Foreign GDP	8.39	9.67	8.30	9.53
Real Output (Home) (Y)	1.00	1.00	1.00	1.00
Real Output (Foreign) (Y^*)	2.00	2.00	2.00	2.00
Terms of Trade	21.96	30.45	18.27	23.86
Trade Balance	3.18	7.84	3.21	7.94
Current Account	0.09	0.50	0.64	3.84
Capital-Gains Adjusted Current Account	23.40	58.06	23.97	59.34

Table 3: IRBC statistics. All series are in log differences and detrended. The consumption expenditure of Home is defined as $pC_H + p^*C_H^*$ and that of Foreign as $pC_F + p^*C_F^*$. Home consumption of the home good is C_H^H . GDP of Home is defined as pY and of Foreign as p^*Y^* , and real output as Y and Y^* , respectively. All external accounts are for Home, and are measured in fractions of Home's GDP. The (instantaneous) correlation between the demand and supply shocks (row 1) is defined as $\sigma_Y i_1 \sigma_{\alpha_H}^\top / (\sigma_Y \sqrt{\|\sigma_{\alpha_H}\|^2})$, where $i_1 = (1, 0, 0, 0)$ and the relative size of demand and supply shocks (row 2) denotes the ratio of the volatilities of the demand and supply shocks $\sqrt{\|\sigma_{\alpha_H}\|^2} / \sigma_Y$.

lations are still higher than the cross-country GDP correlations, and so we do not quite resolve the consumption correlation puzzle, but we are making a step in the right direction. The correlations of Home consumption and GDP are high, and higher than the correlations of Home and Foreign consumption, which is consistent with the data.

We report volatilities of two types of variables: real variables, such as output and consumption (e.g., Y , C_H) and nominal variables such as GDP and consumption expenditure (e.g., pY , $pC_H + p^*C_H^*$). The volatilities of real variables are either somewhat higher (because of demand shocks) or roughly in line with those reported in the standard IRBC models,¹³ but those of nominal quantities are several times higher. This is because prices are volatile in our model, which is necessary for obtaining realistic properties of the financial side of our economy. As one can see from Table 3, we find the terms of trade to be quite volatile, much more so than GDP. This is due to the presence of incomplete markets and demand shocks. In the data, the terms of trade are about three times as volatile as GDP, contrary to predictions of the standard IRBC model, in which the terms of trade are too smooth. Finally, it is worth noting the relatively low volatilities of the trade balance and the conventional current account. The capital-gains adjusted current account is, of course, significantly more volatile in our model (23–60 percent). The numbers are high, but not exorbitant in light of the evidence presented in Kollmann (2006) and Lane and Shambaugh (2007).

5. External Adjustment Process

Properties 1 and 2 establish strong links between a country’s asset positions and its external accounts. For example, Property 2 says that any gains and losses on financial assets comprising the country’s portfolio must be compensated with an offsetting change to the trade balance. As we have highlighted, part of these gains/losses is unexpected. The mechanism through which countries in our economy respond in the event of experiencing a gain (loss) is to increase (reduce) their trade deficits. So the trigger in this trade balance adjustment mechanism is the fluctuations in asset prices—and the trade balance then responds accordingly. This is the chain of reasoning we have in mind when we talk about an “asset-pricing” view of external adjustment. In this section, we discuss how shocks are propagated in our model.

¹³To save space, we just report the volatility of C_H . The volatilities of C_H^* , C_F , and C_F^* are similar.

5.1. Impact responses

We focus our attention on the propagation of two types of shocks: a permanent shift in preferences toward the home good (a demand shock) and a permanent increase in productivity at Home (a supply shock). Our model also allows for an investigation of the effects of a preference shift towards the Foreign good and a productivity shock at Foreign, but for brevity we omit that discussion. We emphasize that the shocks that we study are permanent.¹⁴ That is, we have chosen to close all the intrinsic dynamics and concentrate on the extrinsic dynamics of the model (see Obstfeld and Stockman (1985) for a discussion of these two types of dynamics). Following a shock, our economy moves directly into a new steady state. That is why it is sufficient to look at impact responses to the shocks—impulse responses (over many periods) do not contain any significant information beyond that already captured by the impact responses.

For each shock we present a sequence of figures that capture the impact responses of the real side of the economy and of the financial side of the economy and then highlight the resulting response of the current account of the Home country. To compute the impact responses, we start from our steady state (illustrated in Figure 4) and then introduce one shock (of one standard deviation). Then, for each pertinent variable, we subtract the steady-state series for this variable from the one with the shock, and report the impact of the shock—the resulting change at the time of the shock.

Figure 5 presents impact responses to a shock dw^α . Recall that by construction, such a shock has no effect on the Home or Foreign output, and the only change in the primitives of our model that takes place in response to the shock is a preference shift at Home towards the domestic good (α_H increases). The direction in which the terms of trade and stock prices move following the shock can be easily derived from equations (8)–(10). The economic intuition behind these responses is even simpler. A pure demand shock of the type we are considering here creates an excess demand for the Home good. This pushes its price up and therefore improves Home’s terms of trade. This is why panel (a) of Figure 5 reports an increase in q for all parameter values. Output in both countries stays the same, but because of the movement in the terms of trade, the value of the output increases at Home and decreases at Foreign. Hence the increase in the stock market values at Home (panel (b)) and the decrease at Foreign (panel (c)). It is now easy to understand why the Home country

¹⁴Our focus on permanent shocks is motivated largely by the limitations of the model: while productivity shocks can be modeled as transitory, preference shifts have to be permanent (recall that, for tractability, we have assumed that α_H and β_H are martingales). The analysis of transitory shocks is thus beyond the scope of this paper, and we leave it for future research.

suffers an net unexpected capital loss (panel (d)). Remember that our parameters imply positive steady state holdings of both stocks by both countries. The increase in the stock price at Home thus implies a capital gain for foreigners, while the decline in foreign stock prices implies a loss for Home agents. Both effects act in the same direction, against the Home residents. The capital loss at Home induces the “planner” to reassign weights to the countries in its objective function—in favor of Foreign (panel (e)). Such an outcome, in our case, is equivalent to an (unexpected) wealth transfer from Home to Foreign.¹⁵

As shown in panel (f), the trade balance improves for all parameter values. This improvement of the trade balance is simply the flip side of the deterioration of the NFA position due to the unexpected net capital loss (Property 2). According to our model, the current trade balance has to adjust to absorb the capital loss. For all parameters, except the ones under which the demand and supply shocks are uncorrelated, Home purchases the bond. (For the case of zero correlation, the gross holdings of the bond are zero). These bond purchases decrease the deficit in the current account according to Property 1—and indeed an improvement in the current account is evident from panel (h). Finally, note that the movements in the net unexpected capital gains dominate the dynamics of the NFA position. The response of the capital-gains adjusted current account mimics that of the net unexpected capital gains—CGCA is negative for all parameter values (panel (i)). Note that the direction of the response of the conventional current account is just the opposite (panel (h)).

In Figure 6 we provide the impact responses to a shock to the Home output dw . This case is more complex than the one above because a positive Home output shock also causes an increase in α_H through our choice of parametrization (see Table 1 and the surrounding discussion). Again, the direction in which the terms of trade and stock prices move following the shock can be formally derived from (8)–(10). The intuition behind the responses is as follows. A positive shock to the Home output Y_H increases the value of its stock market (panel (b)). At the same time, the Home good becomes less scarce and hence its price drops relative to that the Foreign good—or in other words, Home’s terms of trade deteriorate. The Foreign’s terms of trade therefore improve, which increases the value of its output and benefits its stock market (panel (c)). These responses are

¹⁵We caution the reader that such a statement requires some qualification. This can be seen from equation (12): $\lambda(t) = \frac{W_F(t)}{W_H(t)} \times \frac{\alpha_H(t) + \beta_H(t)}{\alpha_F + \beta_F}$. Indeed, a part of a change in the relative weight λ coincides with a change in the wealth distribution (a pure wealth transfer), but the remaining part is due to imperfect demand risk sharing: under complete markets, λ does not move in response to any shock, but it does respond to shocks under incomplete markets because the shocks cannot be perfectly hedged. In our case, the wealth transfer effect dominates, but this need not be the case in general.

somewhat mitigated by the increase in α_H that occurs contemporaneously. As we have seen in the previous figure, a positive shock to α_H improves Home’s terms of trade, boosts the stock market at Home, while decreasing the stock market abroad. We can see that this confounding effect becomes more pronounced when the demand shocks have a high loading on the supply shock dw —i.e., when both the correlation between the demand and supply shocks and the relative variance of the demand and supply shocks is high.

Furthermore, we find that Home enjoys a net unexpected capital gain in response to the supply shock (panel (d)). The intuition of this result is complicated because it depends on how prices and portfolio choices interact. A capital gain at Home causes a wealth transfer from Foreign to Home, or, more accurately, the weight λ of Foreign in the “planner’s” problem falls, increasing the allocation to the Home country (panel (e)). Since the NFA position of Home has improved, it can afford to deteriorate its trade balance, which is what we see in panel (f). Part of the financing of the increased purchases of the two stocks comes from selling the bond (panel (g)), which according to our Property 1 deteriorates the conventional current account (panel (h)). The capital-gains adjusted current account, however, moves in the opposite direction—the same direction as the net unexpected capital gains, which are positive here (panel (i)).

One question frequently asked in the literature is that about the directions and magnitudes of responses of real and financial variables that need to accompany a reduction of a current account deficit. In our calibration, Home is the country running a current account deficit, and so let us ask the question of how a reduction of this deficit can be achieved. Figures 5 and 6 illustrate two possible channels: (i) via a fall in Home output (a negative output shock, $dw < 0$) and (ii) via a demand shift towards the home good ($dw^\alpha > 0$). Both these channels have been well established in the literature. However, contrary to the conventional wisdom, in our model the current account correction comes with an improvement of the terms of trade of Home. As we have highlighted above, an improvement in the terms of trade brings about a capital loss on Home’s NFA position. In response to this capital loss, Home is forced to curb its current account deficit.

6. Concluding Remarks

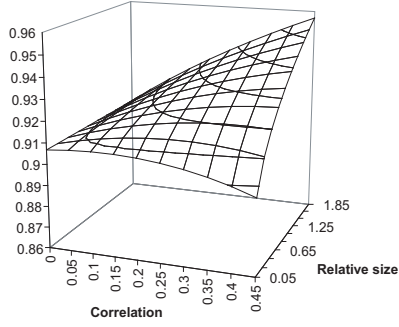
The swelling of international equity holdings in the last two decades has altered the way economists think of external sustainability: new measures of sustainability are being proposed, new channels of external adjustment are being uncovered, and new frameworks are being and will be developed.

In this paper, we investigate properties of an economy which incorporates endogenous dynamics of international equity portfolios into a standard open economy macro model. Solving such a model relies on several important simplifications that future research needs to relax. First, in order to be able to obtain closed-form solutions and introduce asset pricing into our economy, we have chosen to close all the intrinsic dynamics and concentrate on the extrinsic dynamics of the model. In other words, all shocks are permanent and the economy is always in a (stochastic) steady state. Therefore, our definitions of sustainability and the relationship between the external accounts and the financial holdings are all steady-state characterizations. Extending the present framework to allow for intrinsic dynamics is clearly an important next step. Second, we have made a number of simplifying assumptions about the fundamentals of the model: preferences are log-linear, demand shocks come only in the form of expenditure shifts, output is produced by Lucas trees (i.e., there is no investment), and so on. While the model has been able to replicate some key stylized facts documented in the recent literature on net foreign asset dynamics, it is still a very crude depiction of reality. Clearly, our assumptions will have to be relaxed in the future to tackle important questions that afflict practitioners and policymakers.

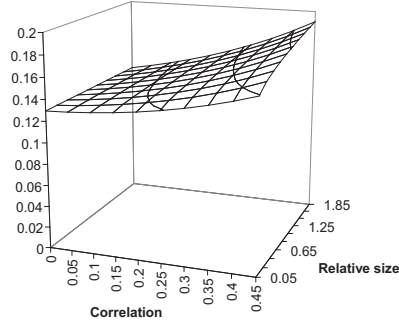
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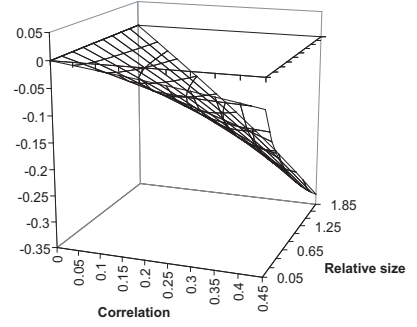
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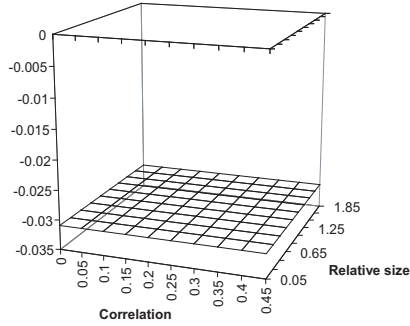
(a) Home's holdings of Home stock s_H^H



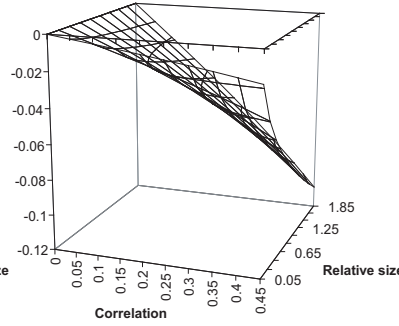
(b) Home's holdings of Foreign stock s_H^F



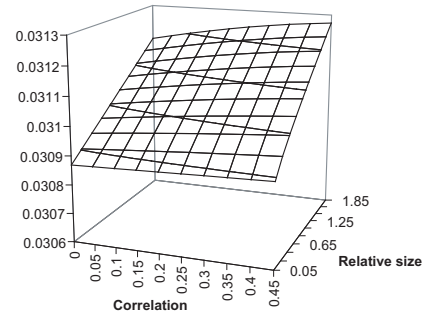
(c) Home's bondholdings (value) $s_H^B B$



(d) Trade balance TB_H

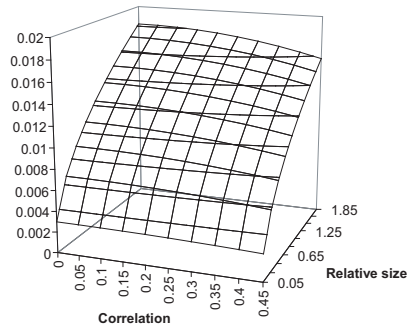


(e) Current account CA_H

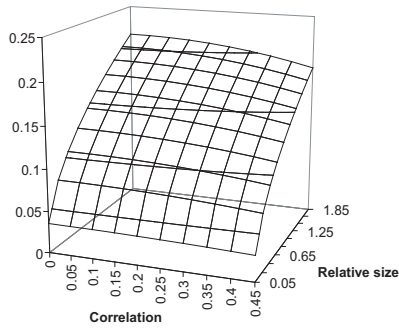


(f) Capital-gains adjusted current account $CGCA_H$

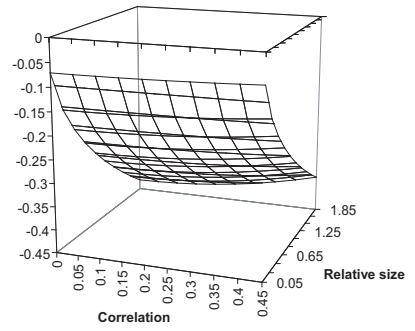
Figure 4: Steady state values. The x-axis measures the instantaneous correlation between the demand and supply shocks $\sigma_Y i_1 \sigma_{\alpha_H}^\top / (\sigma_Y \sqrt{\|\sigma_{\alpha_H}\|^2})$, where $i_1 = (1, 0, 0, 0)$. The y-axis measures the ratio of the volatilities of the demand and supply shocks $\sqrt{\|\sigma_{\alpha_H}\|^2} / \sigma_Y$ (relative size of demand and supply shocks).



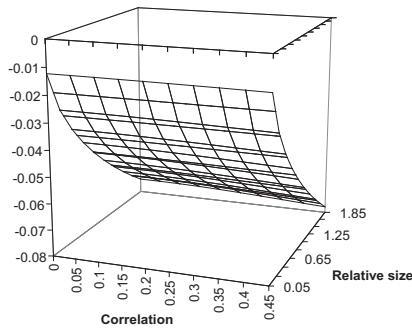
(a) Terms of trade q



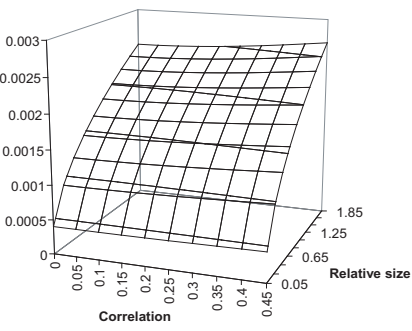
(b) Home stock S



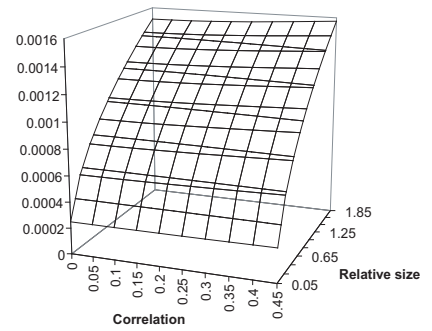
(c) Foreign stock S^*



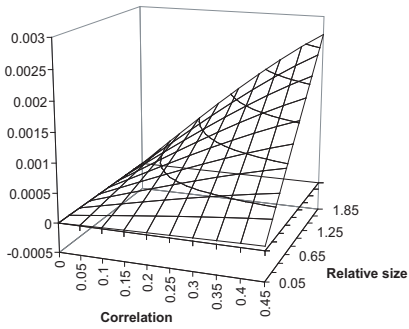
(d) Net unexpected capital gains of Home



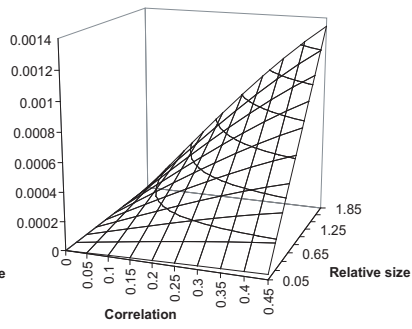
(e) Weight of Foreign λ



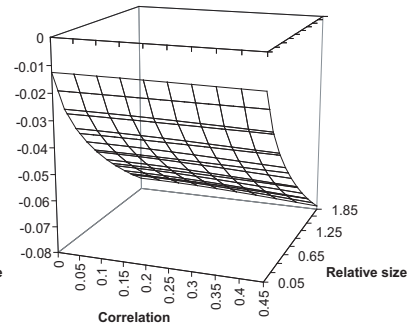
(f) Trade balance TB_H



(g) Home's bondholdings (value)
 $s_H^B B$

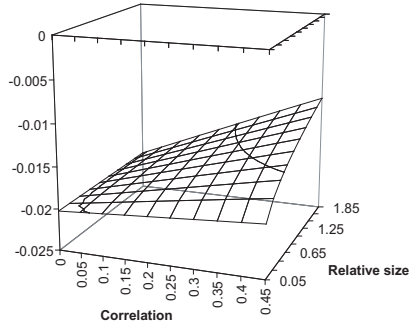


(h) Current account CA_H

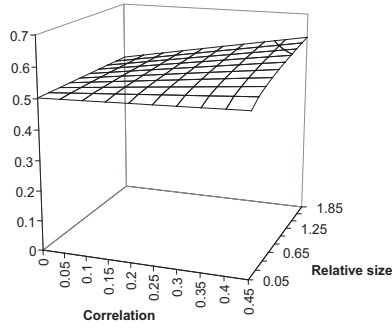


(i) Capital-gains adjusted current account $CGCA_H$

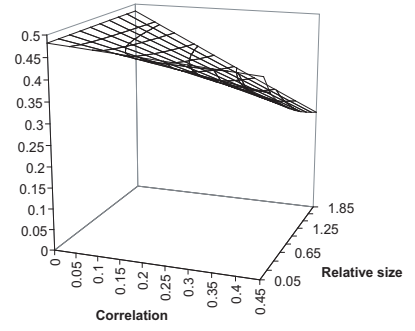
Figure 5: Impact responses to a shock dw^α (a shock to the utility weight α_H). The x-axis measures the instantaneous correlation between the demand and supply shocks $\sigma_Y i_1 \sigma_{\alpha_H}^\top / (\sigma_Y \sqrt{\|\sigma_{\alpha_H}\|^2})$, where $i_1 = (1, 0, 0, 0)$. The y-axis measures the ratio of the volatilities of the demand and supply shocks $\sqrt{\|\sigma_{\alpha_H}\|^2} / \sigma_Y$ (relative size of demand and supply shocks).



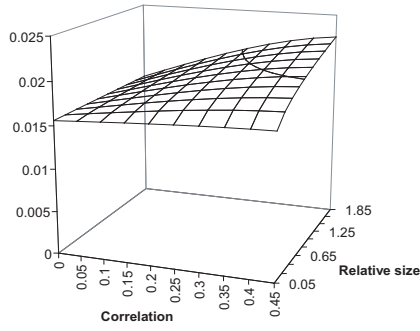
(a) Terms of trade q



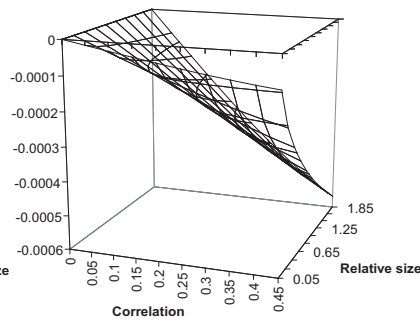
(b) Home stock S



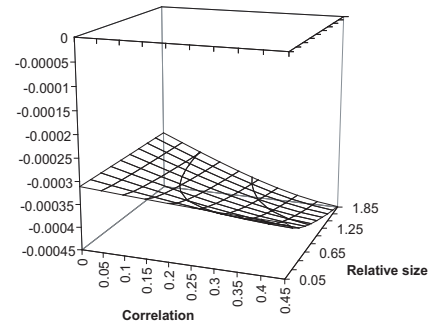
(c) Foreign stock S^*



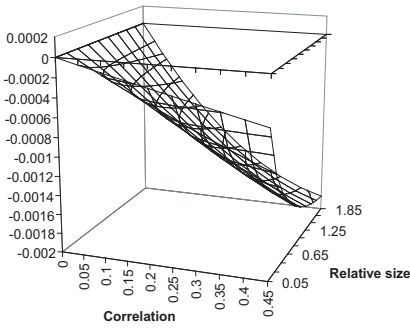
(d) Net unexpected capital gains of Home



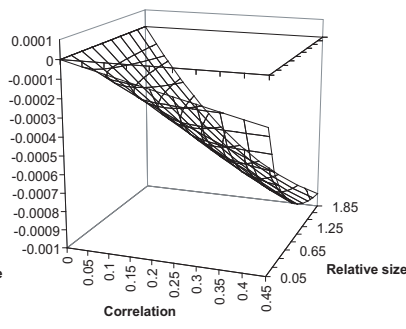
(e) Weight of Foreign λ



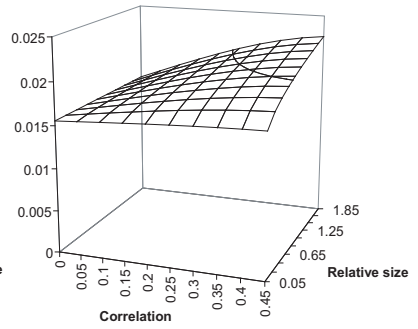
(f) Trade balance TB_H



(g) Home's bondholdings (value)
 $s_H^B B$



(h) Current account CA_H



(i) Capital-gains adjusted current account $CGCA_H$

Figure 6: Impact responses to an output shock at Home dw . The x-axis measures the instantaneous correlation between the demand and supply shocks $\sigma_Y i_1 \sigma_{\alpha_H}^\top / (\sigma_Y \sqrt{\|\sigma_{\alpha_H}\|^2})$, where $i_1 = (1, 0, 0, 0)$. The y-axis measures the ratio of the volatilities of the demand and supply shocks $\sqrt{\|\sigma_{\alpha_H}\|^2} / \sigma_Y$ (relative size of demand and supply shocks).

An Asset Pricing View of External Adjustment Supplementary Appendix (Not for Publication)

Appendix: A Heuristic Derivation of Key Equations

In this appendix we present derivations of our key equations at an intuitive level. A formal treatment can be found in Pavlova and Rigobon (2008a).

We first consider the partial equilibrium optimization problems of the countries. Let us focus on the Home country. The initial step is to reduce the two-good problem to a single-good one. At each time t , we can derive the consumer's demand for the Home and Foreign goods as a function of the overall consumption expenditure at that date, $\mathbb{C}_H(t)$. This is a simple consumer problem under certainty.

$$\begin{aligned} \max_{C_H, C_H^*} \quad & \alpha_H(t) \log C_H(t) + \beta_H(t) \log C_H^*(t) \\ \text{s. t.} \quad & p(t)C_H(t) + p^*(t)C_H^*(t) \leq \mathbb{C}_H(t). \end{aligned}$$

Solving this problem, we obtain the indirect utility function of the form

$$U_H(\mathbb{C}_H(t); p(t), p^*(t)) = (\alpha_H(t) + \beta_H(t)) \log \mathbb{C}_H(t) + \dots$$

The only term that depends on the consumption expenditure is the first one, and so we omit the remaining terms. Since the budget constraint (3) can be easily rewritten in terms of the consumption expenditure, we have reduced the multi-good problem to a single-good one.

The next step is to solve the ensuing single-good optimization problem for the consumption expenditure and the trading strategies. Under complete markets, the problem is standard. Under incomplete markets, the problem is more complex, but one can apply the methods developed in He and Pearson (1991) to tackle it. One can show that the country demands a standard mean-variance portfolio and a hedging portfolio to protect against the fluctuations in the preference shock $\alpha_H + \beta_H$. If $\alpha_H + \beta_H$ is a constant, and in particular, if $\beta_H = 1 - \alpha_H$, the country does not demand a hedging portfolio.

Our next step is to derive equilibrium allocations and prices. From the fictitious social planner's problem (5)–(7), we solve for consumption of the Home country (Foreign consumes the remainder of total output)

$$C_H(t) = \frac{\alpha_H(t)}{\alpha_H(t) + \lambda(t)\beta_F} Y(t), \quad C_H^*(t) = \frac{\beta_H(t)}{\beta_H(t) + \lambda(t)\alpha_F} Y^*(t) \quad (23)$$

The marginal rate of substitution between the Home and the Foreign good gives the terms of trade (8).

The price of a stock is given by the expected present value of its dividends, discounted by the pricing kernel. Any agent's marginal utility acts as a pricing kernel. The marginal utility of the Home country is

$$e^{-\rho t} [\alpha_H(t) + \beta_H(t)] \frac{1}{\mathbb{C}_H(t)}.$$

Because of log preferences,

$$\frac{p(t)C_H(t)}{\mathbb{C}_H(t)} = \frac{\alpha_H(t)}{\alpha_H(t) + \beta_H(t)}. \quad (24)$$

Hence, the price of the Home stock is given by

$$\begin{aligned} S(t) &= \frac{1}{MU(t)} E_t \int_t^T MU(s)p(s)Y(s)ds \\ &= \frac{\mathbb{C}_H(t)}{e^{-\rho t}[\alpha_H(t) + \beta_H(t)]} E_t \int_t^T e^{-\rho s}[\alpha_H(s) + \beta_H(s)] \frac{1}{\mathbb{C}_H(s)} p(s)Y(s)ds \\ &= \frac{p(t)C_H(t)}{e^{-\rho t}\alpha_H(t)} E_t \int_t^T e^{-\rho s} \alpha_H(s) \frac{Y(s)}{C_H(s)} ds, \end{aligned}$$

where MU denotes the marginal utility of Home. Combining this with equation (23), we have

$$S(t) = \frac{p(t)Y(t)}{e^{-\rho t}[\alpha_H(t) + \lambda(t)\beta_F]} E_t \int_t^T e^{-\rho s}[\alpha_H(s) + \lambda(s)\beta_F] ds.$$

By assumption, α_H is a martingale. When markets are complete, λ is a constant; when they are incomplete, λ is a martingale. The former result is technical; see Pavlova and Rigobon (2008a) for a proof. Hence

$$S(t) = \frac{1 - e^{-\rho(T-t)}}{\rho} p(t)Y(t),$$

which is equivalent to (9) given our price normalization $ap(t) + (1-a)p^*(t) = 1$ and the definition of the terms of trade q . The price of the Foreign stock is derived analogously.

To derive the expressions for wealth of the Home country, we note that wealth is the expected present value of future consumption expenditure, discounted at Home's marginal utility:

$$W_H(t) = \frac{1}{MU(t)} E_t \int_t^T MU(s)\mathbb{C}_H(s)ds = \frac{\mathbb{C}_H(t)}{e^{-\rho t}[\alpha_H(t) + \beta_H(t)]} E_t \int_t^T e^{-\rho s}[\alpha_H(s) + \beta_H(s)] ds.$$

A closed-form evaluation of the conditional expectation is possible due to the assumption that both α_H and β_H are martingales. The remainder of the derivation is straightforward (in particular, we use (24) and (23)).