International Capital Markets and Wealth Transfers*

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Abstract

In periods of global stress there are large movements in exchange rates and assets prices. Currencies of developed economies appreciate, with the US dollar appreciating the most. Global stock markets fall, but the fall is smaller for the US market than other markets. Richer countries have riskier external balance sheets and thus their net foreign assets fall, but this effect is overturned by currency appreciations, resulting in a wealth transfer to richer countries. We build a general equilibrium model that helps us understand these facts. Specifically, we consider a model with time-varying risk appetites that produces asymmetric portfolios. Richer countries effectively have more appetite for risk, levering up their external portfolios by borrowing from poorer countries. As a result, their net foreign assets fall in periods of stress, yet there is a wealth transfer from poor to rich countries due to currency appreciations. The model delivers realistic currency risk premia and matches key asset pricing moments, while reproducing asset positions in stress and tranquil periods.

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JEL Classification: E43, F31, G12, G15.
I. Introduction

As capital markets around the world become more integrated, chances that a shock originating in one market propagates to the rest of the world intensify. During times of global stress, financial markets worldwide suffer losses. However, not all markets are affected alike – some fall more than others. Periods of global stress are also accompanied by large movements in the exchange rates and the US dollar (USD) tends to appreciate during these periods. The turmoil in the financial markets accompanied by the USD appreciation is typically associated with alarmingly large drops in the US net foreign assets (NFA) position. This is because the US external balance sheet is disproportionally exposed to equities while US foreign liabilities consist mostly of USD denominated debt. During normal times, the US exploits its “exorbitant privilege” and earns a risk premium on its NFA position, as documented by Gourinchas and Rey (2007) in their influential work. As argued by Gourinchas et al. (2017), however, the flip side of the exorbitant privilege is that the US carries an “exorbitant duty” to provide insurance to the rest of the world that pays off during times of global stress. Arguably, the US then transfers wealth to the rest of the world in bad times.

In this paper we revisit the influential hypothesis of Gourinchas et al. and argue that the wealth transfers may go the opposite way. While the NFA indeed fall, the US investor actually benefits from the USD appreciation. This is because they exhibit a home bias in their portfolios\(^1\) and US asset values fall less than asset values in the rest of the world because of the USD appreciation. We develop a model that illustrates this mechanism. Specifically, we consider a model with time-varying risk appetites that produces asymmetric portfolios. Richer countries effectively have more appetite for risk, levering up their external portfolios by borrowing from poorer countries. This leads to asymmetric responses of macroeconomic variables to local and global shocks. The model is solved in closed form, which allows us to illustrate the economic mechanisms in a transparent way. In addition, the model is able to jointly match the key asset pricing moments and deliver realistic currency risk premia. Notably,

\(^1\)A large literature documents a portfolio home bias (see, e.g., Coeurdacier and Rey, 2013).
the model captures the reversal in currency risk premia over different horizons, which has been challenging to explain within equilibrium models as pointed out by Engel (2016).

In the economy there are multiple countries, each producing their own good. All goods are tradable and consumers derive utility from consuming all of the goods produced in the economy, albeit with a preference for their domestically produced good. Risk appetites are time varying, which we capture with preferences that exhibit deep habits.\textsuperscript{2} A higher consumption of a particular good in the current period makes consumers more willing to buy that good in the future due to the force of habit. In contrast to standard habit formation, deep habits give rise to more realistic exchange rates properties; we show that they are able to generate both exchange rate appreciation in rich countries during times of stress and realistic currency risk premia.

We solve for equilibrium in our dynamic model in closed form and show that a rich country like the US behaves as more risk tolerant and holds a riskier portfolio than the rest of the world. This is implemented by borrowing from poorer countries to invest in stocks (i.e., the richer countries hold a leveraged position in the stock market). Yet, because of a greater appetite for the domestically produced consumption good,\textsuperscript{3} consumers in a rich country (like the US) do not scale back consumption of the domestic good as much as the rest of the world. They are able to achieve this by buying insurance from the rest of the world, which pays off during periods of stress in the domestic market. This channel is new to our model, and it creates a more nuanced response to shocks – rich countries can act as more averse than the rest of the world to domestic shocks but at the same time be more tolerant in response to global shocks.

The asymmetric responses of richer and poorer countries to shocks drives the dynamics of net exports, NFA, and cross-country wealth transfers. Consider a rich country like the US. Because US consumers have stronger habits with respect to the domestic good, they are reluctant to reduce their consumption of the domestic good and therefore the US exports less

\textsuperscript{2}In contrast to standard habit formation, as in for instance Campbell and Cochrane (1999), deep habits assume that the agents have habit formation with respect to each good rather than over the aggregate consumption basket.

\textsuperscript{3}This is an outcome of a consumption home bias and deep habits.
of it in response to a negative shock to the domestic production. At the same time, the lower supply of the domestic good rises their relative price and makes imports relatively cheaper for domestic consumers. Consequently, their expenditure on imported goods declines. The country effectively becomes a more closed economy in response to adverse domestic shocks. As the domestic good relative price rises, the same country enjoys an appreciation of its real exchange rate and its stock market falls less than foreign stock markets. This benefits the domestic investors as most of their wealth is invested in domestic assets due to their consumption home bias. Their wealth share goes up in times of domestic stress. In other words, they receive a wealth transfer from the rest of the world. This result is in contrast to Gourinchas et al. (2017), who argue that the US has an exorbitant duty and transfers wealth to the rest of world (provides insurance) at times of stress. Yet, this does not contradict the finding that the NFA of the US fall during times of stress. This is because their external portfolio is a leveraged position on foreign stock markets. Hence, our model simultaneously implies lower NFA and a wealth transfer to the rich country in periods of domestic stress.

We calibrate our model to the G10 countries and show that the model does a good job in matching exchange rate moments such as the excess volatility and the failure of the uncovered interest rate parity (UIP). In addition, the model delivers high risk premia in the stock markets. Hence, our model captures the unconditional moments of the stock and foreign exchange (FX) markets. To examine the model’s ability to capture the dynamics during periods of stress, we compare the model-implied moments to the data using NBER recession dates. We show that the model reproduces the facts that (i) the US stock market falls less than foreign stock markets, (ii) the USD appreciates during times of stress, and (iii) the model generates the fall in the US NFA during such times (yet, the wealth of the US relative to the rest of the world increases).

Finally, we investigate the currency risk premium implied by the model in more detail. We verify that our model can reproduce the well-established deviation from UIP and the ability of the real exchange rate to predict currency returns. Specifically, the model is rich enough to reproduce the predictability reversal documented by Bacchetta and Van Wincoop (2010) and
Engel (2016), which is challenging for leading models in the literature. Engel argues that this is difficult to match in a frictionless model and suggests an explanation based on liquidity risk. Our model is frictionless and the additional feature we rely on is stochastic volatility of the output growth process (calibrated to the real GDP growth of the G10 countries). The variation in risk appetite together with the stochastic output volatility—both plausible features—are enough to resolve the predictability reversal. In conclusion, our model can match the properties of the NFA, create rich wealth dynamics, and jointly match the dynamics of stocks and currencies.

Related literature

Our paper is related to the literature documenting the importance of valuation changes in cross-country portfolio holdings, starting from Lane and Milesi-Ferretti (2001) and Gourinchas and Rey (2007). They highlight the asymmetric nature of the US external portfolio, which consists of a levered position in the stock market of foreign countries with USD denominated liabilities. Gourinchas and Rey (2007) argue that the US earns a risk premium on its external portfolio. The flip side is that the US provides insurance to the rest of the world that pays off during times of global stress, resulting in a wealth transfer from the US to the rest of the world in those times (Gourinchas et al., 2017). We argue that the USD appreciation that coincides with times of global stress is a boon to the US stock market. This effect can be large enough to reverse the direction of the wealth transfer.

Other related papers studying global imbalances, the NFA, and the exorbitant privilege include Dou and Verdelhan (2015), Maggiori (2017), and Jiang et al. (2020). These papers also study the effects of portfolio asymmetries.4 None of them however argue that the US receives a wealth transfer from the rest of the world in times of global stress.

Our paper is also related to the macro-finance literature studying asset prices and exchange rates. The closest to our paper are Moore and Roche (2008), Verdelhan (2010), Evans (2014), Heyerdahl-Larsen (2014), and Stathopoulos (2017), who use time-varying risk appetite driven

4Another important paper studying cross-country asymmetries is Hassan (2013).
by habit formation to generate realistic asset pricing and exchange rate moments. Other approaches include models with long-run risk (e.g., Colacito and Croce, 2011, Bansal and Shaliastovich, 2013) and rare disasters (Farhi and Gabaix, 2016). While part of our goal is also to match asset prices and exchange rate moments, our primary focus is on asymmetric portfolio positions, NFA, and cross-country wealth transfers.

Finally, our paper is related to the literature on the currency risk premium and its properties. It is well established that the UIP does not hold and that the interest rate differential predicts currency excess returns (high interest rate currencies appreciate over time, resulting in high returns). Fama (1984) shows that under rational expectations this implies tight conditions on the currency risk premium. More recently, Bacchetta and Van Wincoop (2010) and Engel (2016) find that UIP deviations reverse over the horizon. Dahlquist and Pénasse (2021) argue that this reversal relates to the predictability of currency returns by the real exchange rate. Chernov and Creal (2021) study this in a no-arbitrage model of the real exchange rate. Our model reproduces the reversal in general equilibrium.

II. Empirical motivation

In this section, we highlight empirical patterns that help us formulate the theoretical model. We first show that the USD appreciates during periods of stress and that both the USD and the US equity market are less vulnerable to volatility shocks. We then discuss that the US has a riskier external balance sheet and that its NFA falls during periods of stress. Finally, we argue that in spite of NFA falling the US becomes relatively richer than the rest of the world during periods of stress. The Data Appendix contains a more detailed description of the data.

Figure 1 shows the behavior of FX and equity markets during periods of stress. We measure periods of stress as NBER recessions and by equity volatility. The top panel shows the log real USD against a basket of nineteen currencies (in blue) together with the S&P 500 realized volatility (in black). The exchange rate is expressed in USD per unit of foreign currency basket, which means that an increase in the exchange rate is a USD depreciation against the foreign
currencies. Shaded regions are NBER recessions. The USD appreciates on average by about 3.1% per year during NBER recessions. We view this as a lower bound as the USD tends to revert before a recession ends. Stock market volatility also captures periods of stress. Increases in equity realized volatility coincide with USD appreciations. We next test for the statistical significance of such an association.

The bottom left panel shows regressions results for changes in the real exchange rate on realized volatility (in blue) and the VIX (in red). We report volatility beta coefficients and 90% confidence bands from panel regressions with currency fixed effects. All real exchange rates are expressed in domestic currency per unit of foreign currency basket. The figure shows results for the USD, G10 currencies (excluding the USD), and emerging market currencies. The USD has a significantly negative volatility beta, confirming that the USD appreciates in periods of stress. The volatility betas for non-USD G10 and emerging market currencies are not statistically significantly different from zero.

It is well established that the US stock return volatility increases in periods of stress (see, e.g., Brandt and Kang, 2004; Mele, 2007). The bottom right panel extends the evidence to international equity markets. The figure shows volatility betas on excess returns for the US, G10 countries (excluding the US), and emerging markets. All equity markets exhibit negative volatility betas (i.e., they fall in periods of stress as measured by US equity volatility increases). Notably, the US equity market falls less than the G10 markets, which in turn fall less than emerging markets. We present the results for excess returns expressed in USD, but the monotonic pattern is also present for excess returns in local currencies. In sum, the figure shows that the USD appreciates during periods of stress, and that both the USD and the US equity market are less exposed to increases in volatility.

We reproduce in Figure 2 important facts about US external positions as documented by Gourinchas and Rey (2015) and Gourinchas et al. (2017). The left panels show the gross equity and debt positions as fractions of GDP. US foreign assets predominantly consist of equities, while US foreign liabilities mostly consist of debt. It means that NFA are disproportionally exposed to equities and earn an equity premium. This results in the exorbitant privilege to the
US dollar and equity volatility

The top figure shows the log real USD against a basket of nineteen currencies (in blue) together with the S&P 500 realized volatility (in black). Shaded regions are NBER recessions. The bottom left figure shows panel regression results for changes in the real exchange rate on changes in realized volatility (in blue) and changes in the VIX (in red); the bottom right figure shows the panel regression results for equity excess returns (expressed in USD). Volatility beta coefficients and 90% confidence bands are depicted by circles and error bars. The bottom figures show results for the US, the G10 countries (excluding the US) and ten emerging currencies. All real exchange rates are expressed in domestic currency per unit of foreign currency basket.

US. The right panel shows NFA over GDP together with the equity volatility. NFA are defined as US net equity positions plus US net debt positions. The NFA fall during periods of stress, a consequence of the US net equity position and of the USD appreciation. This results in the exorbitant duty to the US. The great financial crisis was a case in point – between 2007:Q4 and 2009:Q1 NFA fell by 14.5 percentage points. Gourinchas et al. (2017) find a similar decline (19 percentage points) using a broader definition of NFA.

The fall in NFA during periods of stress reflects a wealth transfer from the US to the rest of the world. However, this transfer does not necessarily imply that the US is relatively worse off in periods of stress, as the US relative position depends not only on NFA but also on domestic
wealth. Indeed, the results presented in Figure 1 exert an opposing force on US relative wealth. While the USD appreciation tends to have a negative impact on US NFA, it has a positive impact on US domestic wealth. The lower vulnerability of the US equity market also makes the US relatively richer in periods of stress. Which force dominates thus depends on the relative change of NFA and domestic wealth.

Figure 3 compares US total wealth and the NFA from the previous figure. The US wealth is calculated by Piketty and Zucman (2014) and measured as the market-value national wealth. Over the postwar period, the US total wealth ranged between 3 and 4 times US GDP. NFA today equal around 30% of GDP (in absolute values), which is about ten times smaller. This means that to offset the effect of a one percent USD appreciation on domestic wealth, NFA would have to fall by about ten percent. NFA are insufficiently volatile for this to be likely. For instance, NFA fell by 14.5 percentage points of GDP during the great financial crisis, a 69.7% decline. While substantial, the NFA decline coincides with a 18.1% USD appreciation over the same period. Hence, a USD appreciation is generally sufficient to overcome the fall of

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5The magnitudes remain the same using broader definitions of NFA, such as the one in Piketty and Zucman (2014).
NFA. In other words, the US becomes relatively richer in times of stress.

![Figure 3: US wealth and net foreign assets](image)

This figure shows the US market-value national wealth and NFA as fractions of GDP.

We remark that the wealth estimates of Piketty and Zucman (2014) only include measurable measures of wealth and do not include, for instance, human capital. We can infer an alternative estimate based on the price-dividend ratio. Over long periods, it seems reasonable that the claims to aggregate consumption grow at the same rate as dividends. If, in addition, these claims bear the same risk as dividends, the wealth-consumption and the price-dividend ratio should be equal over the long run. In our sample, the average price-dividend ratio is around 37. Over the same period, the ratio of consumption over GDP was around 65%, which suggests an estimate of the wealth over GDP ratio of $37 \times 65\% \approx 24$. This back-of-the-envelope calculation, which is an order of magnitude larger than the one we used previously, remains conservative as it assumes that consumption claims are as risky as dividends. Lustig et al. (2013) do not impose that assumption and find that consumption claims are considerably safer, which translates into a higher average wealth-consumption ratio of 83 (see Shiller, 1995, for a similar argument). Taken together, these alternative calculations imply a much larger effect of the USD appreciation on total wealth, supporting the argument that the US becomes relatively
III. Model

In this section, we present an $N$-country economy, which rationalizes the empirical facts in Section II and provides additional predictions. The key ingredient of the model is time-varying risk appetites, as in Campbell and Cochrane (1999), which are high during good times but reduce drastically during periods of stress. Another important feature is that we allow countries to be asymmetric in terms of their size, which leads to asymmetric external portfolios and wealth transfers across countries in good and bad times. In Section V.E, we show further that the model delivers realistic currency risk premia and matches key asset pricing moments.

A. Output

There are $N$ countries in the world economy, each producing its own perishable consumption good. We refer to country one as the home country and the other $N - 1$ countries as foreign. The output of country $i$ is given by

$$Y_{i,t} = Y_t X_{i,t},$$

where $Y_t$ is a global factor and $X_{i,t}$ is a country-specific factor. The dynamics of the global factor is

$$dY_t = Y_t (\mu_Y dt + \sigma_Y dw_{Y,t}),$$

where $w_{Y,t}$ is a standard Brownian motion. To model the country-specific factor, $X_{i,t}$, we define $x_{i,t} = \log (X_{i,t})$, where

$$dx_{i,t} = \kappa_{x,i} (\bar{x}_i - x_{i,t}) dt + \sigma'_{x,i} dw_{X,t}.$$  

The shock, $w_{X,t}$, is a $N$ dimensional standard Brownian motion. The country-specific factors are mean-reverting and consequently the quantities of each good, $Y_{i,t}$, are co-integrated with each other. We stack together the $N + 1$ shocks in the vector $w_t = (w_{Y,t}, w_{X,t})$. We assume
that the local volatility is \( \sigma_{x_{i,t}} = \nu_t \sigma_{x_i} \), where \( \nu_t \) is a common stochastic volatility component, which can be driven by other shocks than \( w_t \). We specify the dynamics of \( \nu_t \) when calibrating the model in Section V, as it is not critical for the derivation of the equilibrium.

B. Preferences

We introduce time-varying risk appetites of investors by adopting habit-based preferences, which is one of the workhorse preferences in finance (e.g., Campbell and Cochrane, 1999). Risk appetites are high when investors’ consumption significantly exceeds their habits; the opposite is true when consumption is close to the habit. Specifically, each country is populated by a representative investor with preferences given by

\[
U^j(C;H) = \mathbb{E} \left[ \int_0^\infty e^{-\rho t} \sum_{i=1}^N \log (C_{i,t}^j - H_{i,t}^j) dt \right],
\]

where \( C_{i,t}^j \) and \( H_{i,t}^j \) are the consumption and habit of good \( i \) for the investor in country \( j \), respectively. We assume that the habit for each good is external (i.e., the investor does not take into account how consumption today effect the habit level in the future). Moreover, we model habit as good-specific, that is, “deep habits” as proposed by Ravn et al. (2006).\(^6\) Hence, there is a separate habit for each good. With deep habits, as opposed to habit defined over the aggregate consumption basket, the relative prices are volatile even with smooth consumption of the individual goods.

Assumption 1: We assume that the habit of good \( i \) for the investor in country \( j \) is \( H_{i,t}^j = \phi_i^j H_i^t \) with \( \sum_{i=1}^N \phi_i^j = 1, \sum_{j=1}^N \phi_i^j = 1 \) and \( \phi_i^j \geq 0 \) for \( i, j = 1, \ldots, N \), where \( H_{i,t} < Y_{i,t} \) for all \( t \).

Assumption 1 implies that all investors benchmark their consumption against the same habit level for each good, \( H_{i,t} \), but attach different importance of the habit governed by \( \phi_i^j \).

Note that except from the importance attached to the habit level, \( \phi_i^j \), there is no heterogeneity in the preferences.

\(^6\)van Binsbergen (2016) also considers preferences with deep habits to study asset prices in a production economy.
Another important feature of agents’ preferences is consumption home bias.

Assumption 2: *Agents exhibit consumption home bias, that is,* \( \phi_j > \phi_i \) for \( i \neq j \).

Assumption 2 implies that the weights on the domestically produced good in the consumption basket is higher than foreign produced consumption goods. This common assumption is a robust feature of the data and can be microfounded by explicitly modeling no-traded goods.

Generalizing Campbell and Cochrane (1999) to an environment with multiple goods, we define the aggregate surplus consumption ratio for good \( i \) as

\[
s_{i,t} = \frac{Y_{i,t} - H_{i,t}}{Y_{i,t}}. \tag{5}
\]

A low surplus consumption ratio for good \( i \) corresponds to bad times for consumers of good \( i \) and more so for consumers of country \( i \) as they exhibit a home bias in consumption. As suggested by Menzly et al. (2004), it is convenient to work with the inverse surplus consumption ratio

\[
R_{i,t} \equiv \frac{1}{s_{i,t}}. \tag{6}
\]

We make the following assumption about its dynamics.

Assumption 3: *The inverse surplus consumption ratio of good* \( i \), \( R_{i,t} \), *has a two-factor structure and can be decomposed as* \( R_{i,t} = G_t G_{i,t} \) *where*

\[
dG_t = \kappa (\bar{G} - G_t) dt - \alpha (G_t - \lambda) \sigma_Y dw_{Y,t} \tag{7}
\]

\[
dG_{i,t} = \kappa_i (\bar{G}_i - G_{i,t}) dt - \alpha_i (G_{i,t} - \lambda_i) \sigma_{x_{i,t}}' dw_{X,t}. \tag{8}
\]

The inverse surplus consumption ratio \( R_{i,t} \) is driven by two independent processes \( G_t \) and \( G_{i,t} \). Both of these processes have the same form as the inverse surplus consumption ratio in Menzly et al. (2004). One can show that the two-factor structure also emerges if investors have standard habit formation as well as deep habits. In this case, \( G_t \) represents the inverse surplus consumption ratio of the standard habit and \( G_{i,t} \) represents the inverse surplus consumption
ratios of the good-specific habits. Alternatively, one can interpret it as shocks to the curvature of the utility for a specific good, where one part is due to the global factor and the other part is unique to that good. Importantly, we follow Menzly et al. (2004) and impose the same restrictions on the parameters of $G_t$ and $G_{i,t}$. These restrictions imply that the inverse surplus consumption ratio is high after a series of negative shocks to the global factor $Y_t$ or the local factor $X_{i,t}$ and therefore the curvature of the utility function is high in these states. The variations in the inverse surplus consumption ratios drive the variation in the risk appetite in our model.

C. Goods prices and exchange rates

We denote prices of each good as $p_{i,t}$ for $i = 1, \ldots, N$, to be determined in equilibrium. The dynamics of $p_{i,t}$ are posited as

$$\frac{dp_{i,t}}{p_{i,t}} = \mu_{p_{i,t}} dt + \sigma_{p_{i,t}}' dw_t. \quad (9)$$

Defining the real exchange rate requires a definition of the countries’ consumption price indexes. In a model with deep habits there is no natural concept of an aggregate consumption price index or aggregate consumption. We follow Ravn et al. (2012) and use an arithmetic average-based consumption basket. As discussed in Ravn et al. (2012), a constant weight arithmetic average mimics the construction of price indices in most developed countries. Specifically, we define the price of the consumption basket in country $j$ as

$$P^j_t = \sum_{i=1}^{N} h^j_i p_{i,t}, \quad (10)$$

where $h^j_i$ represents the importance of good $i$ in the basket of country $j$. We discuss how we set the weights when we calibrate the model in Section V.

In what follows, we are primarily concerned with exchange rates faced by investors in the the home country. For each foreign country $j$, we can define the real exchange rate as the ratio
of the price of country \( j \)'s consumption basket to the price of the home consumption basket, that is,

\[
Q^j_t = \frac{P^j_t}{P^1_t}.
\]  

(11)

D. Investment opportunities

In the model there are \( N + 1 \) priced shocks and therefore we need \( N + 2 \) assets to complete the market. To do so, we introduce \( N \) stocks, a “global insurance contract” and a locally risk-free bond paying out in the numeraire basket of country one. All stock markets are in fixed net supply of one share each and the bond and the global insurance contract are in zero net supply. The dynamics of the bond are

\[
\frac{dB_t}{B_t} = r_t dt.
\]  

(12)

Each country’s stock market is the claim to the aggregate output produced in that country. The value of the aggregate stock market of country \( i = 1, \ldots, N \) in units of numeraire is \( S_{i,t} \), with instantaneous returns given by

\[
dR_{i,t} = \frac{dS_{i,t} + p_{i,t}Y_{i,t} dt}{S_{i,t}} = \mu_{R_{i,t}} dt + \sigma_{R_{i,t}} dw_t.
\]  

(13)

The bond and stock dynamics are determined in equilibrium.

The global insurance contract hedges the shock to \( Y_t \) which is common for all countries. The dynamic of the global insurance contract is

\[
dR_{0,t} = \mu_{0,t} dt - \sigma_0 dw_t.
\]  

(14)

where \( \sigma_0 = (1, 0) \) with 0 being an \( N \)-vector of zeros. The parameter \( \sigma_0 \) is exogenously specified and \( \mu_{0,t} \) is determined in equilibrium. The global insurance contract is introduced for purely technical reasons to complete the market.

Since we study the violation of the UIP and the currency risk premium it is also convenient to introduce \( N \) bonds, each paying out in the respective country’s numeraire basket. The bond
of country one corresponds to Equation (12), i.e., $B_1^t = B_t$ and $r_1^t = r_t$. The rest of the bonds are redundant, but we price them using no-arbitrage. The dynamics of the bonds are

$$\frac{dB_j^t}{B_j^t} = r_j^t dt.$$  \hspace{1cm} (15)

### E. Individual optimization

Investors maximize their lifetime utility in Equation (4), subject to the dynamic budget constraint

$$dW_i^j = \varphi_i^j \frac{dB_1^t}{B_1^t} + \pi_{0,t}^j dR_{0,t} + \sum_{i=1}^N \pi_i^j dR_{i,t} - \sum_{i=1}^N p_{i,t} C_{i,t}^j dt,$$  \hspace{1cm} (16)

for $j = 1, \ldots, N$, where $\pi_i^j$ represents the amount held in stock $i$ by investor $j$, $\varphi_i^j$ is the amount held in the money market account (bond) of country one, $\pi_{0,t}^j$ is the amount held in the global insurance contract, and $p_{i,t}$ is the price of good $i$ in units of the numeraire.

We assume that $W_0^j = \sum_{i=1}^N \pi_{i,0}^j$ (i.e., the investors are endowed with shares in the stocks). We set the initial wealth of all investors to be sufficiently high to support consumption levels that exceed the habit levels.\(^7\)

### IV. Equilibrium

In this section we solve and characterize the equilibrium. Specifically, we solve for the consumption, wealth, and asset prices in closed form. We show how different shocks impact the NFA and wealth shares of the countries, and relate the results to Gourinchas et al. (2017). We start with the definition of the equilibrium.

Definition 1: Equilibrium is a collection of allocations $\left( C_{i,t}^j, \varphi_i^j, \pi_{0,t}^j, \pi_i^j \right)$ for $i, j = 1, \ldots, N$, and a price system $\left( \mu_{R_i,t}, \mu_{p_i,t}, \mu_{0,t}, r_1^t, \sigma_{R_i,t}, \sigma_{p_i} \right)$ such that the allocations solve the investors’ optimization problems and all markets clear, i.e., $\sum_{j=1}^N C_{i,t}^j = Y_{i,t}$, $\sum_{j=1}^N \pi_{i,t}^j = S_{i,t}$, $\sum_{j=1}^N \pi_{0,t}^j = 0$.

\(^7\) Note that given the external habit formation, the initial wealth has to be high enough to support a consumption that exceeds the habit. Hence, the initial allocations cannot be chosen independently from the habit parameters $\phi_i^j$. A sufficient condition for this is to choose $\pi_i^j = \phi_i^j S_{i,0}$. 

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and \( \sum_{j=1}^{N} \varphi^j_t = 0 \) for all \( t \) and \( i = 1, \ldots, N \).

Since the financial markets are complete in our economy, equilibrium consumption allocations coincide with those in the central planner’s problem. We therefore consider the planner’s problem first and then solve for prices that prevail in the decentralized equilibrium. Let \( a^j \) denote the Pareto weight of investor \( j \) normalized such that \( \sum_{j=1}^{N} a^j = 1 \), then the planner’s problem is

\[
U(Y_t; H_t, a) = \max_{C_i} \sum_{j=1}^{N} a^j \sum_{i=1}^{N} \log \left( C^{j}_{i,t} - H^{j}_{i,t} \right) \quad \text{s.t.} \quad \sum_{j=1}^{N} C^{j}_{i,t} = Y_{i,t} \quad i = 1, \ldots, N, \tag{17}
\]

where \( Y_t = (Y_{1,t}, \ldots, Y_{N,t}) \), \( H_t = (H_{1,t}, \ldots, H_{N,t}) \) and \( a = (a^1, \ldots, a^N) \) for all \( t \). The higher the initial wealth of a country, the higher is its Pareto weight.\(^8\) We often consider the home country being a rich country (e.g., the US), which is captured by a high Pareto weight \( a^1 \). Problem (17) is easy to solve because it reduces to a sequence of simple state-by-state optimization problems.

### A. Risk sharing and time-varying risk appetites

The first order conditions (FOC) of the central planner’s problem in Equation (17) are

\[
\frac{a^j}{C^{j}_{i,t} - H^{j}_{i,t}} = \eta_{i,t}, \quad i, j = 1, \ldots, N, \tag{18}
\]

where \( \eta_t = (\eta_{1,t}, \ldots, \eta_{N,t}) \) are the Lagrange multipliers on the resource constraint in (17). Hence, we can relate the consumption of investor \( j \) and \( k \) through \( \frac{a^j}{C^{j}_{i,t} - H^{j}_{i,t}} = \frac{a^k}{C^{k}_{i,t} - H^{k}_{i,t}} \). Imposing the market clearing in each consumption good and using \( \sum_{j=1}^{N} \phi^j_i = 1 \), we have that

\[
(C^{j}_{i,t} - H^{j}_{i,t}) = a^j (Y_{i,t} - H_{i,t}), \tag{19}
\]

Hence, the optimal habit-adjusted consumption of every investor in each good, \( C^{j}_{i,t} - H^{j}_{i,t} \), is

---

\(^8\)The Pareto weights are determined by matching the consumption allocations in the competitive equilibrium to those in the planner’s problem.
proportional to the aggregate habit-adjusted output, $Y_{i,t} - H_{i,t}$. Importantly, the habit-adjusted consumption does not depend on the individual habit directly, but only the aggregate habit in the economy.\footnote{See Heyerdahl-Larsen (2014) for a discussion.} However, the actual level of consumption, $C_{i,t}^j$, depends on how we model the individual habit $C_{i,t}^j = \phi^j_i H_{i,t} + \alpha^j (Y_{i,t} - H_{i,t})$. Given the above, we have the following:

**Proposition 1:** The optimal consumption of investor $j$ of good $i$ is

$$C_{i,t}^j = f_{i,t}^j Y_{i,t}$$

where $f_{i,t}^j$ is the consumption share given by

$$f_{i,t}^j = \phi^j_i + (\alpha^j - \phi^j_i) s_{i,t},$$

where $s_{i,t} = \frac{Y_{i,t} - H_{i,t}}{Y_{i,t}}$ is the aggregate surplus-consumption ratio for good $i$.

From Proposition 1, we see that the consumption share is linear in the surplus consumption ratio. We assume that the aggregate habit levels are never negative and never exceeds the aggregate output of the same good. Specifically, we have that for good $i$, $s_i \in (0, s_i^{max}]$. Given the above, we have the following proposition.

**Proposition 2:** The consumption share of investor $j$ of good $i$ is bounded in the intervals

$$(\alpha^j s_i^{max} + \phi^j_i (1 - s_i^{max}), \phi^j_i)$$

if $\phi^j_i > \alpha^j$ and

$$(\phi^j_i, \alpha^j s_i^{max} + \phi^j_i (1 - s_i^{max}))$$

if $\phi^j_i < \alpha^j$. Moreover, $\frac{\partial f_{i,t}^j}{\partial s_{i,t}} = \alpha^j - \phi^j_i$.

From Proposition 2 we see that the response of the consumption share to the surplus consumption ratio depends on the sign of the difference between country $j$’s Pareto weight and its habit sensitivity, $\alpha^j - \phi^j_i$. As a higher surplus consumption ratio implies a better state of the world, the consumption share is pro-cyclical for countries with a low sensitivity to the habit, $\phi^j_i$, or high wealth as measured by the Pareto weight, $\alpha^j$.

In a setting with multiple goods the concept of risk aversion is not well defined. Still, we
can make a comparison to Campbell and Cochrane (1999). To do so, consider the marginal utility of investor $j$ with respect to consumption of good $i$ is

$$\frac{1}{C_{i,t}^j - H_{i,t}^j} = \frac{1}{C_{i,t}^j s_{i,t}^j},$$

(22)

where $s_{i,t}^j = \frac{C_{i,t}^j - H_{i,t}^j}{C_{i,t}^j}$ is the surplus consumption ratio of good $i$ for investor $j$. The aggregate surplus consumption ratio for good $i$, $s_{i,t}$, defined in Equation (5) is in general different to the individual-specific surplus consumption ratio, $s_{i,t}^j$, as different agents have different sensitivities to the aggregate habit represented by the weights $\phi^j$. As in Campbell and Cochrane (1999), time variation in risk premiums depends crucially on the surplus consumption ratios, $s_{i,t}^j$. The surplus consumption ratios, $s_{i,t}^j$, shrinks in bad times when the consumption approaches the habit. In Campbell and Cochrane (1999), the (effective) risk aversion goes up when the surplus consumption ratio is low. In our paper, the mechanism is essentially the same except that there is a surplus consumption ratio for each good. We will refer to the states in which the surplus consumption ratio for good $i$ is high as times when investors have high risk appetite towards risk originating in country $i$.

It is convenient to consider the inverse of the surplus consumption ratio $R_{i,t}^j \equiv \frac{1}{s_{i,t}^j}$, where a high value of $R_{i,t}^j$ corresponds to bad times. These are times when the marginal utility of the consumption of good $i$ by investor $j$ is high. We will refer to this as the local curvature of investor $j$’s utility for good $i$. Using Eq. (19)-(21), we have that

$$R_{i,t}^j = 1 + (R_{i,t} - 1) \phi_{i}^j a_j^j,$$

(23)

From Equation (23) we see that the local curvature of the utility function, $R_{i,t}^j$, depends on the ratio $\phi_{i}^j a_j^j$. investors with a high risk appetite insures those with low risk appetite in equilibrium, leading to a pro-cyclical consumption share as the investor with high risk appetite take larger losses in bad times but also gain more in good times.

The procyclical consumption share of investors with higher risk appetites in the model with
habit formation is similar to that in a model with heterogeneous risk aversion. However, in contrast to a model with heterogeneous risk aversion, a higher wealth effectively increases risk appetite in our model due to the non-homothetic preferences.

Moreover, unlike a model with heterogeneous risk aversion, investors in our model all have less appetite for risk in bad states. Finally, and maybe most importantly, given the multiple good setting with a different habit sensitivities, \( \phi^j_i \), for each good, the investors have different utility curvatures for different goods. However, the initial wealth, represented by the Pareto weight \( a^j_i \), directly impact the overall risk appetite as it changes the local utility curvature of all goods.

**B. Exchange rates, asset prices, and net foreign assets**

In this subsection we solve for exchange rates, stock prices and country wealths, and use them to study the NFA. Central to the equilibrium is the stochastic discount factor (SDF). The next proposition shows the equilibrium SDF.

**Proposition 3**: The stochastic discount factor in country \( j \) is

\[
M^j_t = \sum_{i=1}^{N} h^j_i M_{i,t} \tag{24}
\]

where

\[
M_{i,t} = e^{-\rho t} \frac{1}{Y_{i,t} - H_{i,t}} \tag{25}
\]

where \( h^j_i \) is the weight of good \( i \) in the basket of country \( j \).

The stochastic discount factor in Proposition 3 is a multi-good analog of the stochastic discount factor with external habit. Equation (25) is the familiar expression from Campbell and Cochrane (1999). The stochastic discount factor in Equation (24) takes into account that all payoffs are defined in terms of the basket we use as numeraire.

The marginal utility in (22) is proportional to the discount factor, \( M_{i,t} \), the prices a cash flow in units of good \( i \). It follows from the no-arbitrage condition that the price at time \( t < s \) of a
claim to an asset with payoff $D_{i,s}$ in terms of good $i$ at time $s$ is $E_t \left( \frac{M_{i,s}}{M_{i,t}} D_{i,s} \right) = E_t \left( \frac{M_{i,s}}{M_{i,t}} \frac{p_{i,s}}{p_{i,t}} D_{i,s} \right)$.

In complete market it follows that

$$p_{i,t} = \frac{M_{i,t}}{M_{i,t}^t}$$

(26)

As the exchange rate between country $j$ and 1 is the ratio of the price of the basket of country $j$ to country 1 we have the following

$$Q_j^t = \frac{M_j^t}{M_i^t}$$

(27)

Given the expression for the relative prices, the equilibrium characterization are given in the next proposition.

Proposition 4: The equilibrium price of good $i$ in terms of the consumption basket of country 1 is

$$p_{i,t} = \left( \frac{G_{i,t}}{Y_{i,t}} \right) \left( \frac{1}{\sum_{l=1}^{N} h_l \frac{G_{l,t}}{Y_{l,t}}} \right).$$

$$Q_j^t = \frac{\sum_{l=1}^{N} h_j \frac{G_{l,t}}{Y_{l,t}}}{\sum_{l=1}^{N} h_l \frac{G_{l,t}}{Y_{l,t}}}.$$ 

(28)

(29)

From Equation (29), we see that the real exchange rate not only depends on the relative output but also on the good specific risk appetite through the inverse surplus consumption ratios. As we show below, most of the variation in the real exchange rates come from the variation in the risk appetite due to changes in the surplus consumption ratios. Also note that it is only the local habits, $G_{i,t}$, the impact the exchange rates. The global habit, $G_t$, does not as this effectively works as a level factor. Still, the global habit is important in understanding the risk sharing and hence the NFA.

As we show below, the NFA of a country is tightly linked to net exports. Using the equilibrium consumption from Proposition 1 and the relative prices from Proposition 4 we can
calculate the net export. To do so, note that the net exports of country 1, \( N X_t \), is given by

\[
N X_t = p_{1,t} \left( Y_{1,t} - C_{1,t}^1 \right) - \sum_{i=2}^{N} p_{i,t} C_{i,t}^1
\]

(30)

Using the optimal consumption from Proposition 1 and the relative price in Proposition 4 we get the following.

**Proposition 5:** The net export, \( N X_t \), of country one is

\[
N X_t = p_{1,t} Y_{1,t} \left( 1 - \sum_{i=1}^{N} \left( \frac{G_{i,t}}{G_{1,t}} \right) \left( \phi_i + (a^1 - \phi_i) s_{i,t} \right) \right),
\]

(31)

where \( p_{1,t} \) is given in Equation (28).

It is useful to consider the net export as a fraction of the home production value

\[
\frac{N X_t}{p_{1,t} Y_t} = \left( 1 - \left( \phi_1 + (a^1 - \phi_1) s_{1,t} \right) \right) - \sum_{i=2}^{N} \left( \frac{G_{i,t}}{G_{1,t}} \right) \left( \phi_i + (a^1 - \phi_i) s_{i,t} \right)
\]

(32)

As we see from Equation (32), the net export as a fraction of the home production value only depends on the risk appetite and not the level of the output. However, in contrast to the real exchange rate, the relative net export also depends on the global risk appetite, \( G_t \).

Figure 4 plots the import, export and net exports as a fraction of the home production value as we change the US local risk appetite (left plot) and the global risk appetite (right plot). From the left plot we see that both import and exports are increasing in the US surplus consumption ratio, \( 1/G_{US} \), but at different rates. Specifically, the imports increase at a much faster rate than the exports, leading to a decrease in net exports. The reason for this is the consumption home bias. Due to consumption home bias, US has a higher utility curvature with respect to the domestic good relative to the rest of the world. Hence, when the US surplus consumption ratio is high, the US consumers exports more of the domestically
produced good. Note that since we measure all quantities relative to the home production value, the exchange rate has no effect on the exports in the figure. In contrast, the imports are affected by the exchange rate, and since USD is depreciating in good times, i.e., when the US surplus consumption ratio is high, the imports become more expensive. Still, the US investor does not change the amount imported (the consumption share stays the same), and therefore the value of the imports relative to the home production value increases. The exchange rate effect on the imports is large and therefore net export is decreasing as the US surplus consumption ratio increases.

Turning to the right hand plot, we see that the net exports are also decreasing as a function of the global surplus consumption ratio, $1/G_t$ (i.e., net exports are low in good times as reflected by a high global surplus consumption ratio). In contrast to the the effect from the US surplus consumption ratio, there is no impact on the exchange rate when the global surplus consumption ratio is changing. Hence, the mechanism is quite different from that in Panel A. First, exports are increasing due to the consumption home bias as in Panel A. However, the imports increase is due to risk sharing and not to relative prices. This is because the US has a lower utility curvature for foreign goods. In good times when the global surplus consumption ratio is high, the US consumes a larger fraction of foreign goods so that imports increase. In other words, the decreasing net exports are due to changes in the quantities of imported goods and not exchange rate changes.

As we seen above, the net exports are decreasing in good times. The dynamics of net exports is important for the NFA as we show below since as they are linked through the net present value formula.

The NFA, $NFA_j^t$, is defined as the difference between the country $j$‘s ownership of foreign assets minus the ownership of foreign investors of assets in country $j$. Stated formally,

$$NFA_j^t = \sum_{i=1, i\neq j}^{N} \pi_{i,t}^j - \sum_{k=1, k\neq j}^{N} \pi_{k,t}^j + \varphi_t^j + \pi_{0,t}^j,$$

where the first term is country $j$‘s ownership of foreign stocks, the second term is the amount
Figure 4: Net exports
The figure plots the net exports (top row), exports and imports (middle row) and the distribution of the surplus consumption ratios. The left column plots the net exports, exports and imports as a function of the home surplus consumption ratio ($1/G_{US}$). The bottom left histogram plots the distribution of the home surplus consumption ratio. For all figures in the left column we keep the global inverse habit level at the long-run mean ($G = \bar{G}$). The right column plots the the net exports, exports and imports as a function of the global surplus consumption ratio ($1/G$). The bottom right figure plots the distribution of the global surplus consumption ratio. For all figures in the right column, we keep the home surplus consumption ration at the steady state level ($G_{US} = \bar{G}_{US}$). The parameters are the same as the parameters used in the calibration section (see Table I and III).
of country $j$’s stock market held by foreign countries, and the last two terms reflect country $j$’s balance on the bond accounts. Country $j$’s time-$t$ wealth is $W_t^j = \sum_{i=1}^{N} \pi_{i,t}^j + \phi_t^j + \pi_{0,t}^j$. Together with equation (33) and market clearing (see Definition 1), this implies that $NFA_t^j = W_t^j - S_{j,t}$.

We have the following immediate relation between net exports and NFA

$$NFA_t^j = W_t^j - S_{j,t} = -E_t \left( \int_t^\infty \frac{M_s}{M_t}NX_s^j ds \right). \quad (34)$$

If we endow the country with the claim to their own output at time zero, then we trivially have that $NFA_0^j = 0$ and the budget condition implies that the net present value of future net exports are zero.

To calculate the NFA, we need to know the value of the stock markets in each country and the wealth of each country. We consider the stock markets in each country to be the claim to the aggregate output of the good in that country. The next proposition characterizes the stock prices.

Proposition 6: Then the stock price of country $i = 1, \ldots, N$ is

$$S_{i,t} = p_{i,t}Y_{i,t}PD_{i,t}, \quad (35)$$

where $PD_{i,t}$ is the price-dividend ratio and is given by

$$PD_{i,t} = \frac{1}{R_{i,t}}F_{i,t}, \quad (36)$$

where

$$F_{i,t} = \frac{R_i}{\rho} + \frac{\bar{G}(G_{i,t} - \bar{G}_i)}{\rho + \kappa_i} + \frac{\bar{G}_i(G_t - \bar{G})}{\rho + \kappa} + \frac{(G_t - \bar{G})(G_{i,t} - \bar{G}_i)}{\rho + \kappa + \kappa_i} \quad (37)$$

where $R_i = \bar{G}\bar{G}_i$. 

24
Given the optimal consumption in Proposition 1, we can calculate the wealth of each investors in equilibrium.

Proposition 7: The wealth of investor \( j = 1, \ldots, N \) is

\[
W^j_t = \sum_{i=1}^{N} \phi^j_i S_{i,t} + \frac{b^j}{e^{\rho t} M_t},
\]

where

\[
b^j = \frac{1}{\rho} \left( N a^j - \sum_{i=1}^{N} \phi^j_i \right).
\]

From Proposition 7, we see that the wealth can be decomposed in the following way:

\[
W^j_t = \underbrace{\sum_{i=1}^{N} \phi^j_i S_{i,t}}_{\text{Buy and hold portfolio}} + \underbrace{\frac{b^j}{e^{\rho t} M_t}}_{\text{Dynamic portfolio}}
\]

Hence, a key in understanding the portfolio positions is the coefficients \( b^j \). Note that \( b^j \) is high when the Pareto weights \( a^j \) is high or if the habit coefficients \( \phi^j_i \) are low (i.e., if the investor is wealthy or risk tolerant). The next Proposition considers a special case when there will be no trade in risky assets in equilibrium with the exception of an initial trade at the beginning of the economy.

Corollary 1: If \( b^j = 0 \) for all \( j = 1, \ldots, N \) then the optimal portfolio of investor \( j \) is a buy-and-hold portfolio with weights \( \phi^j_i \) and there is no trade is the risk-free asset.

Corollary 1 highlights the case when there is no lending or borrowing in equilibrium. For instance, this would be the case if we consider a symmetric economy (i.e., \( \phi^j_j = \phi^k_k, \phi^j_i = \phi^k_i \) for \( i \neq j, k \) and \( a^j = \frac{1}{N} \)). To decentralize the economy, the Pareto weights has to be mapped into initial wealth. Assume that the all surplus consumption ratios are in the steady state at time zero and that each country is initially endowed with the claim to their own endowment stream (the stock market), then we have the following.

Proposition 8: If each country is initially endowed with the claim to their own output, i.e.,
$W_0^j = S_{j,t}$, and the surplus consumption ratios are in the steady state at time 0, i.e., $\mathcal{R}_{i,0} = \bar{\mathcal{R}}_i$ for $i = 1, \ldots, N$, then the Pareto weights, $a^j$, are

$$a^j = \frac{1}{N} \left( \sum_{i=1}^{N} \phi_i^j + \left( \bar{\mathcal{R}}_j - \sum_{i=1}^{N} \phi_i^j \bar{\mathcal{R}}_i \right) \right). \quad (41)$$

There are several things to note from Proposition 8. First, if $\bar{\mathcal{R}}_i = \bar{\mathcal{R}}$ for $i = 1, \ldots, N$, then $a^j = \frac{1}{N} \left( \bar{\mathcal{R}} - (\bar{\mathcal{R}} - 1) \sum_{i=1}^{N} \phi_i^j \right)$ and with $\bar{\mathcal{R}} > 1$ the “average risk aversion” measured by $\sum_{i=1}^{N} \phi_i^j$ of any investor cannot be too big as the investor cannot afford a consumption that exceed the habit level. Moreover, the Pareto weight is decreasing in the average risk aversion. This follows from the fact that a higher average risk aversion implies a higher marginal utility and therefore the central planner puts less weight on that investor through a lower Pareto weight. Second, a higher steady state level of the inverse surplus consumption ratio, $\mathcal{R}_i$, implies a higher wealth for that country. The reason is that a good with a high steady state inverse surplus consumption ratio, $\mathcal{R}_i$, is a valuable good as investors are very reluctant to scale back on it. Therefore the country that is endowed with such a good is also a rich country.

It is convenient to split expression for NFA of country $j$ in equation (42) into two components

$$NFA_t^j = W_t^j - S_{j,t} = \sum_{i=1, i \neq j}^{N} \phi_i^j S_{i,t} - (1 - \phi_j^j) S_{j,t} + \frac{b^j}{e^{rt}} M_t. \quad (42)$$

We refer to the first component as the portfolio home bias intensity and the second as the exorbitant duty. The conventional wisdom is that the rich country takes a leveraged position in the external portfolio and therefore experience higher losses during bad times (exorbitant duty), but in good times earns a higher return (exorbitant privilege) due to the risk premium in risky assets. This mechanism is also in our model as the next proposition shows.

Proposition 9: Let $ED_t^j = \frac{b^j}{e^{rt} M_t}$ be the exorbitant duty term for country $j$. Consider a wealthy
country, i.e., $a^j > \frac{1}{N}$. Then we have the following:

$$\frac{\partial ED^j}{\partial R_{i,t}} < 0$$

(43)

for all $i = 1, \ldots, N$.

From Proposition 9 we see that the exorbitant duty component of the NFA of rich countries is decreasing in the inverse surplus consumption ratios, $R_i$, and consequently in bad times the contribution of the exorbitant duty term to the NFA is negative. Effectively, rich countries behave as less risk averse investors and provide insurance to the rest of the world.

Rather than working with the NFA in levels, it is convenient to normalize the NFA by total world wealth. Hence, NFA can be expressed as the difference between the wealth share, $f_{j,t}^W$ and the stock market share $f_{j,t}^S$.

$$nfa^j_t = f_{j,t}^W - f_{j,t}^S$$

(44)

In Figure 5 we plot $nfa^{US}$ as function of the local US surplus consumption ratio $s_{US} = \frac{1}{G_{US}}$. Consistent with the discussion above, the figure shows that the NFA of the US is decreasing in bad states (i.e., for low levels of the US surplus consumption ratio). Moreover, the US stock market share is decreasing in the the surplus consumption ratio. The reason is the effect of the USD appreciation is strong enough to make the foreign stock market fall more than the US stock market when measured in USD. The wealth share of US, $f_{US}^W$ is the sum of the normalized NFA and the stock market share. However, since most of the wealth of the US investors are in local US assets, the effect from the relative increase in the US stock market dominates the decrease in the wealth share due to the NFA. In conclusion, we see that the response of the NFA to a bad times as measured by a high $G_{US}$ is negative. Still, the wealth share of the US is increasing.

If we instead consider the effect of the global habit on the NFA and wealth share of the US the mechanism is different. First, consistent with Proposition 9 the NFA of US drops. In addition, as the global shock works almost like a level shock there is no appreciation of the
domestic exchange rate and therefore the wealth share of the US is decreasing.

To summarize, the discussion above illustrates that the US NFA is decreasing in bad states, regardless of the nature of the shock. However, the US wealth share is sensitive to the source of the shock. If the shock is affecting the local risk appetite for the US good, the US exchange rate appreciates causing a wealth transfer to the US. If instead it is due to the global risk appetite, there is no effect on the exchange rate and the wealth share of the US declines.

![Figure 5: Net foreign assets](image)

The figure plots the NFA scaled by world wealth for the US (top row) and the distribution of the surplus consumption ratios (bottom row). The left column plots NFA scaled by world wealth as a function of the home surplus consumption ratio ($1/G_{US}$). The bottom left histogram plots the distribution of the home surplus consumption ratio. For the figure in the left column we keep the global inverse habit level at the long-run mean ($G = \bar{G}$). The right column plots the NFA scaled by world wealth as a function of the global surplus consumption ratio ($1/G$). The bottom right figure plots the distribution of the global surplus consumption ratio. For the figure in the right column, we keep the home surplus consumption ration at the steady state level ($G_{US} = \bar{G}_{US}$). The parameters are the same as the parameters used in the calibration section (see Table I and III).
C. Dynamics

The previous section focused on the “level” of the equilibrium quantities such as the exchange rate, optimal consumption, net exports, wealth, stock market valuations and NFA. However, we did not derive the dynamics of these quantities. In this section we focus on the dynamics.

We start with the dynamics of the SDF from Proposition 10. This pins down the equilibrium market prices of risk and the real risk-free rates.

Proposition 10: The dynamics of the state price density, $M_t^j$, is

$$\frac{dM_t^j}{M_t^j} = -r_t^j dt - (\theta_t^j)' dw_t, \quad (45)$$

where

$$r_t^j = \sum_{i=1}^{N} \omega_{i,t}^j r_{i,t} \quad (46)$$

and

$$\theta_t^j = \sum_{i=1}^{N} \omega_{i,t}^j \theta_{i,t} \quad (47)$$

where

$$\omega_{i,t}^j = \frac{h_{i,t}^j M_{i,t}}{\sum_{l=1}^{N} h_{i,t}^l M_{i,t}} \quad (48)$$

and where

$$r_{i,t} = \rho + \mu_{Y,i,t} - \sigma_{Y,i,t}^T \sigma_{Y,i,t}$$

$$+ \kappa \left(1 - \bar{G}_i\right) - \alpha \left(1 - \frac{\lambda}{G_t}\right) \sigma_{Y}^2$$

$$+ \kappa_i \left(1 - \frac{\bar{G}_i}{G_i,t}\right) - \alpha_i \left(1 - \frac{\lambda_i}{G_i,t}\right) \sigma_{x_i,t}^2 \sigma_{x_i,t}' \sigma_{x_i,t} \quad (49)$$

and and the market prices of risk are

$$\theta_{t} = (\theta_{Y,t}, \theta_{x,t}) \quad , \quad (50)$$
where

\[
\theta_{Y,t} = \sigma_Y \left(1 + \alpha \left(1 - \frac{\lambda}{G_t}\right)\right)
\]  
\[
\theta_{x_i,t} = \sigma_{x_i,t} \left(1 + \alpha_i \left(1 - \frac{\lambda_i}{G_{i,t}}\right)\right)
\]

(51)  
(52)

where \(\theta_{Y,t} \in \mathbb{R}\) and \(\theta_{x_i,t} \in \mathbb{R}^N\).

Proposition IV.C highlights several features of the model. First, as illustrated by Proposition 10, country \(j\)'s SDF is a weighted average of the good-specific stochastic discount factors. This is inherited by both the risk-free rate and the market prices of risk. They are, just as the SDFs, weighted average of the corresponding risk-free rates and market prices of risk of the good specific interest rates and market prices of risk. However, in contrast to the level of the SDFs, the risk-free rates and market prices of risk have weights that are stochastic themselves as illustrated by the weights in Equation (48).

Given the state price densities in each country, we can derive the dynamics of the real exchange rate (taking the home country/country one) as the numeraire.

Proposition 11: The dynamics of the real exchange rate of country \(j\) relative to country 1 is

\[
\frac{dQ_j^i}{Q_j^i} = \mu_{Q_j^i} dt + \sigma_{Q_j^i} dw_t,
\]

(53)

where

\[
\sigma_{Q_j^i} = \theta_t - \theta_{j}^i,
\]

(54)

and

\[
\mu_{Q_j^i} = r_t - r_j^i + \theta_j^i \sigma_{Q_j^i}
\]

(55)

where we drop the superscript on home country quantities, i.e., \(r_t = r_1^i\) and \(\theta_t = \theta_1^1\).

Note that the drift, \(\mu_{Q_j^i}\), is equal to instantaneous expected depreciation rate, i.e., \(E_t \left(\frac{dQ_j^i}{Q_j^i}\right) / dt = \)
\( \mu_{Q_i} \). In other words, the expected excess return (i.e., the currency risk premium), is equal to

\[
E_t \left( \frac{dQ^j_t}{Q^j_t} / dt \right) + r^j_t - r_t = \theta'_t \sigma_{Q^j_t}.
\]  

(56)

Consequently, the drift of the real exchange rate, \( \mu_{Q_i} \), depends on the interest differential and the risk premium. The uncovered interest rate parity (UIP) would hold whenever the risk premium, \( \theta'_t \sigma_{Q^j_t} \), is zero.

As we will show in Section V.E, the model can replicate the failure of the UIP in the data. In addition, the model also matches the predictability reversal as shown in Bacchetta and Van Wincoop (2010), Engel (2016) and Dahlquist and Pénasse (2021). To analyze the mechanism, it is useful to consider a special case when \( h^j_t = 1 \), that is each country’s price index is putting all weight on the domestically produced consumption good.\(^{10}\) In addition, we will assume for expositional reasons that the parameters for the two countries are the same and that \( x_{i,t} \) and \( x_{j,t} \) are uncorrelated. Given the above special case, we will drop subscripts and write \( \alpha_i = \alpha, \sigma_{x_{i,t}} = \sigma_{x,t} \) etc.

In this case we can write the drift of the real exchange rate as

\[
\mu_{Q_i} = \left( \alpha \lambda \sigma^2_{x,t} - \kappa \bar{G} \right) (s_{1,t} - s_{j,t}) + \theta^2_{x_{1,t}}
\]

(57)

It is easy to show that \( \theta_{x_{1,t}} \) is decreasing the the surplus consumption ratio, and it immediately follows that the currency risk premium is also decreasing in the surplus consumption ratio. As shown by Fama (1984), to match a negative UIP slope, the risk premium has to be negatively correlated with the interest rate differential and the risk premium must be more volatile than the expected depreciation rate. Therefore, to match the UIP, the interest rate differential has to be increasing in the surplus consumption ratio.

From (57), one can see that the interest rate differential, \( r^1_t - r^j_t \), is increasing in the surplus consumption ratio of country 1, \( s_{1,t} \), if \( (\alpha \lambda \sigma^2_{x,t} - \kappa \bar{G}) > 0 \). Note that this condition implies

\(^{10}\)This implies a PPI based real exchange rate.
that a positive shock to country-specific factor, $X_{1,t}$, increases the interest rate differential, which we refer to as a pro-cyclical interest rate differential. This corresponds to the condition in Verdelhan (2010) where a pro-cyclical interest rate differential is necessary to match the failure of the UIP. Note that due to the common component in interest rates due to the global habits, the pro-cyclical interest rate differential does not imply a negative slope of the real term structure.

So how can the model explain a predictability reversal? To examine this, note that unlike Verdelhan (2010), the coefficient on the difference in the surplus consumption ratios, $(\alpha \lambda \sigma^2_{x,t} - \kappa \bar{G})$, is not constant as it depends on the volatility of the output processes. In times of high volatility, the pro-cyclicality of the interest rate differential is stronger. As the volatility drops, the cyclicality changes from pro- to counter-cyclical. In addition, higher output volatility implies a higher risk premium and therefore the cyclicality of the interest rate differential and the currency risk premium are positively correlated. In our calibration in Section V.E, the volatility is less persistent than the local habits and this implies different long-run and short run predictability.

Given the stock prices in Proposition 6, an application of Itô’s lemma gives the following.

Proposition 12: The instantaneous return on the stock market in country $i$ is

$$dR_{i,t} = \mu_{R_{i,t}} dt + \sigma'_{R_{i,t}} dw_t,$$

where the expected return is

$$\mu_{R_{i,t}} = r_t + \sigma'_{R_{i,t}} \theta_t,$$

with $\sigma_{R_{i,t}} = (\sigma^Y_{R_{i,t}}, \sigma^X_{R_{i,t}})$ and

$$\sigma^Y_{R_{i,t}} = \sigma_Y + \theta_{Y,t} - \frac{1}{PD_{i,t}} \left( \frac{\bar{G}_i}{\rho + \kappa} + \frac{G_{i,t} - \bar{G}_i}{\rho + \kappa + \kappa_i} \right) \alpha (G_t - \lambda) \sigma_Y$$

$$\sigma^X_{R_{i,t}} = \sigma_{x,t} + \theta_{x,t} - \frac{1}{PD_{i,t}} \left( \frac{\bar{G}}{\rho + \kappa_i} + \frac{G_t - \bar{G}}{\rho + \kappa + \kappa_i} \right) \alpha_i (G_{i,t} - \lambda_n) \sigma_{x,t},$$
where \( \theta_t = (\theta_{Y,t}, \theta_{X,t}) \) contains the market prices of risk with respect to \( w_{Y,t} \) and \( w_{X,t} \) as in Proposition 10.

Given the dynamic budget condition in Equation (16), applying Ito’s lemma to the optimal wealth in Proposition 7 we can solve for the optimal portfolio positions.

Proposition 13: Let \( \pi^j_t = (\pi^j_{0,t}, \pi^j_{1,t}, \ldots, \pi^j_{N,t}) \) and define the return diffusion matrix as \( \sigma_{R,t} \in \mathbb{R}^{N+1 \times N+1} \) where element \((i,j)\) is the loading on asset \(i\) on Brownian motion \(j\). Then the optimal dollar amount invested in each of the risky assets by investor \(j\) is

\[
\pi^j_t = I_{S,t} \phi^j + \frac{b^j}{e^{\rho_t M_t}} (\sigma'_{R,t})^{-1} \theta_t. \tag{62}
\]

The optimal dollar amount invested in the risk-free asset is

\[
\varphi^j_t = \frac{b^j}{e^{\rho_t M_t}} \left(1 - 1'(\sigma'_{R,t})^{-1} \theta_t\right) \tag{63}
\]

where \( I_{S,t} \) is a matrix with \( S_{i,t} \) in element \((i,i)\) and zero everywhere else and \( 1 \) is a \( N + 1 \) dimensional vector of ones.

The optimal amount invested in the risky assets can be decomposed into two parts. The first, \( I_{S,t} \phi^j \), is the static buy-and-hold composition of the portfolio. In a symmetric economy with \( a^j = \frac{1}{N} \) this is the only term. This gives us the following Corollary.

Corollary 1: Consider a symmetric economy with \( \phi^j = \phi^H > \phi^F = \phi^j_i \) for \( i \neq j \). Let the surplus consumption ratios be in their steady states, then there is both consumption and portfolio home bias in equilibrium.

The second part, \( \frac{b^j}{e^{\rho_t M_t}} (\sigma'_{R,t})^{-1} \theta_t \), is the dynamic component. First note that this part is proportional to the growth optimal portfolio, \( (\sigma'_{R,t})^{-1} \theta_t \) (i.e., the optimal portfolio of a log utility investor). The sign of the proportionality factor, \( \frac{b^j}{e^{\rho_t M_t}} \), is determined by \( b^j \). As \( b^j \) is positive for countries that are rich or have a low average risk aversion, these are also the countries that increase their exposure to the risky assets in equilibrium relative to the buy-
and-hold component. However, this part of their portfolio does not have any portfolio home bias, but is simply based on the growth optimal portfolio. Countries that have a high risk aversion or are poor as measured by a negative $b^j$ reduces their exposure to the risky assets relative to the static buy-and-hold component. From the optimal position in the bond in Proposition 13 we see that the the rich or risk tolerant countries fund their increased exposure to the risky assets by borrowing.

V. Calibration

In this section we calibrate the model to the G10 countries.\footnote{G10 consists of Australia, Canada, Switzerland, Germany, the United Kingdom, Japan, Norway, New Zealand, Sweden, and the United States.} We consider an asymmetric calibration with USA being wealthier than the average country. For the rest of countries, we assume symmetry.\footnote{Relatedly, Hassan (2013) studies the connection between country size and the level of interest rates.}

A. Output

We calibrate the output processes to real GDP data. For our theoretical results we did not need to specify the dynamics of the volatilities for the country specific part of the endowment process, $\sigma_{x_{i,t}}$. However, to simulate the model we need the dynamics. We assume the shocks to US impact all other countries, but not the other way around. Specifically, for US we assume that $\sigma_{x_{1,t},1} = \nu_t$ and $\sigma_{x_{1,t},k} = 0$ for $k = 2, \ldots, 10$. For the other countries we assume that $\sigma_{x_{i,t},1} = \rho \nu_t$ and $\sigma_{x_{i,t},i} = \sqrt{1 - \rho^2} \nu_t$ and zero for the rest. We define $V_t = \log (\nu_t)$ and assume the following dynamics

$$dV_t = \kappa_V (\bar{V} - V_t) + \sigma_V dz_{V,t}$$

(64)

where $z_{V,t}$ is a Brownian motion independent of all other shocks. Table I presents the parameter values for the output processes.

Table II reports the moments of the GDP in the data and the model. The data is based...
Table I: Parameters – output

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected GDP growth rate</td>
<td>( \mu_Y )</td>
<td>0.0200</td>
</tr>
<tr>
<td>Volatility of world trend growth</td>
<td>( \sigma_Y )</td>
<td>0.0080</td>
</tr>
<tr>
<td>Speed of mean reversion in GDP growth</td>
<td>( \kappa_x )</td>
<td>0.0326</td>
</tr>
<tr>
<td>Long run mean country specific factor</td>
<td>( \bar{x} )</td>
<td>0</td>
</tr>
<tr>
<td>Speed of mean reversion in volatility</td>
<td>( \kappa_v )</td>
<td>0.1583</td>
</tr>
<tr>
<td>Long run mean of volatility</td>
<td>( \bar{V} )</td>
<td>-4.1366</td>
</tr>
<tr>
<td>Volatility of volatility</td>
<td>( \sigma_V )</td>
<td>0.1842</td>
</tr>
<tr>
<td>US and ROW correlation</td>
<td>( \rho )</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The table shows the parameters for the output processes (Equations (1)–(3) and (64)).

Table II: Moments – output

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP growth</td>
<td>3.0</td>
<td>2.0</td>
</tr>
<tr>
<td>GDP growth (ROW)</td>
<td>2.7</td>
<td>2.0</td>
</tr>
<tr>
<td>Stdev GDP growth</td>
<td>2.7</td>
<td>2.0</td>
</tr>
<tr>
<td>Stdev GDP growth (ROW)</td>
<td>1.6</td>
<td>2.0</td>
</tr>
<tr>
<td>Average GDP correlation</td>
<td>25.5</td>
<td>25.0</td>
</tr>
<tr>
<td>Average GDP correlation (ROW)</td>
<td>19.4</td>
<td>18.3</td>
</tr>
<tr>
<td>GDP growth predictability</td>
<td>-0.08</td>
<td>-0.08</td>
</tr>
</tbody>
</table>

The table compares the population moments from the model with the data. The population moments are based on 10,000 years of monthly observations. The quarterly real GDP growth data were downloaded from the OECD. The rest of the world consists of G10 countries (excluding the US): Australia, Canada, Switzerland, Germany, the UK, Japan, Norway, New Zealand, and Sweden. Means and standard deviations are annualized.
real GDP of the G10 countries, with ROW referring to the average excluding US. In our data the average real GDP growth is 3\% in the US and 2.7\% in the ROW, which is a higher than comparable numbers reported in the literature. Therefore we use a GDP growth of 2.0\% in the model. The standard deviation of the GDP growth in the US is lower than the ROW in the data. In the model we set the standard deviation of the real GDP growth to be 2\% in both the US and the ROW. US real GDP has a higher correlation with the real GDP growth than the ROW with a value of 25.5\% compared to an average correlation among the ROW of 19.4\%. We match the correlation in the data closely with values of 25.0\% and 18.3\%, respectively. We use the parameter \( \rho \), to match the average correlation between US vs the ROW and within the ROW.

In our model, GDP growth is predictable as \( x_{i,t} \) is a mean reverting process. To calibrate the speed of mean reversion in GDP growth we calculate the real GDP shares, \( s_{i,t}^{GDP} = \frac{Y_{i,t}}{\sum_{l=1}^{N} Y_{l,t}} \), and use the GDP shares to predict GDP growth. As Table II illustrates, the GDP predictability is similar in the data and the model.

### B. Endowments and preferences

In addition to the output processes, we need to specify the parameters of the endowments and preferences. Table III shows the parameters. We set the weight, \( a^{US} \), to 0.3, to capture that the endowment of the US is higher than that of the average country. The remaining countries have equal endowments, i.e., we set \( a^j = 0.7/9 \) for \( j = 2, \ldots, 10 \). The preference parameter that govern the degree on consumption home bias, \( \phi^j \), is set to 0.95 and we set the weight on all other goods to be homogeneous, i.e., \( \phi^j = 0.05/9 \) for \( j \neq i \). These are also the weights used for the consumption baskets, i.e., \( h^j_i = \phi^j_i \). The remaining parameters are set to match asset pricing and exchange rate moments.

Specifically, the parameters that govern the global habit, \( (\kappa, \bar{G}, \alpha, \lambda) \), are mainly impacting the moments of asset prices and the risk-free rates. They do not affect the exchange rate moments. Hence, we set the parameters of the global habit to target a realistic equity premia.
and low risk-free rates. As the global habit is a common "level" factor in the return on all stocks markets, making the variation in returns too dependent on the global habit will cause an excessive correlation among stocks.

The parameters that govern the local habits, \((\kappa_i, \bar{G}_i, \alpha_i, \lambda_i)\), are set to target the exchange rate dynamics. Unlike the global habit, the local habits impact both asset pricing and exchange rate moments. We maintain as much symmetry as possible with the only parameter that differs across countries is that long-run mean, \(\bar{G}_i\) which is higher for the US. A higher long-run mean leads to a higher price of the US good relative to goods produced in other countries and therefore making the US stock market a larger than the stock markets in other countries on average. Second, we set the parameters to match interest rate dynamics and exchange rate volatility in such a way that the model replicates the failure of the uncovered interest rate parity (UIP). We discuss the UIP in detail in a later section.

### Table III: Parameters – endowments and preferences

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Secondary Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight US</td>
<td>(a_{US})</td>
<td>0.3</td>
<td>NFA dynamics</td>
<td>US is net borrower</td>
</tr>
<tr>
<td>Weights in other countries</td>
<td>(\frac{1-a_{US}}{N-1})</td>
<td>0.3/9</td>
<td>NFA dynamics</td>
<td>US is net borrower</td>
</tr>
<tr>
<td>Weight on home good</td>
<td>(\phi_j)</td>
<td>0.95</td>
<td>consumption home bias</td>
<td>portfolio home bias</td>
</tr>
<tr>
<td>Time preference</td>
<td>(\rho)</td>
<td>0.02</td>
<td>risk-free rates</td>
<td>price-dividend ratios</td>
</tr>
<tr>
<td>Speed of mean reversion global habit</td>
<td>(\kappa)</td>
<td>0.15</td>
<td>equity premium</td>
<td>risk-free rates</td>
</tr>
<tr>
<td>Long-run mean global habit</td>
<td>(\bar{G})</td>
<td>8</td>
<td>equity premium</td>
<td>risk-free rates</td>
</tr>
<tr>
<td>Global habit sensitivity</td>
<td>(\alpha)</td>
<td>60</td>
<td>equity premium</td>
<td>risk-free rates</td>
</tr>
<tr>
<td>Minimum global habit</td>
<td>(\lambda)</td>
<td>5</td>
<td>equity premium</td>
<td>risk-free rates</td>
</tr>
<tr>
<td>Speed of mean reversion local habit</td>
<td>(\kappa_i)</td>
<td>0.015</td>
<td>exchange rate dynamics</td>
<td>risk-free rates</td>
</tr>
<tr>
<td>Long-run mean local habit US</td>
<td>(\bar{G}_{US})</td>
<td>6.7</td>
<td>exchange rate dynamics</td>
<td>risk-free rates</td>
</tr>
<tr>
<td>Long-run mean local habit (ROW)</td>
<td>(\bar{G}_i)</td>
<td>6.5</td>
<td>exchange rate dynamics</td>
<td>risk-free rates</td>
</tr>
<tr>
<td>Local habit sensitivity</td>
<td>(\alpha_i)</td>
<td>81.6</td>
<td>exchange rate dynamics</td>
<td>risk-free rates</td>
</tr>
<tr>
<td>Minimum local habit</td>
<td>(\lambda_i)</td>
<td>5</td>
<td>exchange rate dynamics</td>
<td>risk-free rates</td>
</tr>
</tbody>
</table>

The table shows the preference parameters and the initial allocations for the US and ROW. Target and Secondary Target shows the data moments we try to match.
C. Unconditional moments

Table IV summarizes the key unconditional moments. The model generates an equity premium of 3.1% in the US and 3.7% in the ROW which are somewhat lower than the corresponding values in the data. The higher equity premium of the ROW is due to the fact the returns are measured in USD and therefore impacted by the exchange rate volatility. The model matches closely the volatility of both the US stock market and the ROW, where the ROW is more volatile than the US stock market. The model implies an average correlation between the stock markets of 84%, which is higher than in the data. As the table illustrates the model generates a reasonable level for the risk-free rates with a slightly high volatility. The exchange rate has a volatility of 12.3% which is slightly higher than the data equivalent of 9.1%.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity premium</td>
<td>6.5</td>
<td>3.1</td>
</tr>
<tr>
<td>Excess premium (ROW)</td>
<td>6.0</td>
<td>3.7</td>
</tr>
<tr>
<td>Return volatility</td>
<td>14.4</td>
<td>14.5</td>
</tr>
<tr>
<td>Return volatility (ROW)</td>
<td>18.5</td>
<td>18.8</td>
</tr>
<tr>
<td>Average return correlation</td>
<td>0.65</td>
<td>0.84</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>0.7</td>
<td>1.9</td>
</tr>
<tr>
<td>Risk-free rate (ROW)</td>
<td>1.5</td>
<td>2.0</td>
</tr>
<tr>
<td>Risk-free rate volatility</td>
<td>2.2</td>
<td>3.6</td>
</tr>
<tr>
<td>Risk-free rate volatility (ROW)</td>
<td>2.1</td>
<td>3.6</td>
</tr>
<tr>
<td>$P/D$</td>
<td>42.6</td>
<td>61.4</td>
</tr>
<tr>
<td>$P/D$ (ROW)</td>
<td>33.8</td>
<td>60.5</td>
</tr>
<tr>
<td>Stdev $P/D$</td>
<td>17.7</td>
<td>12.5</td>
</tr>
<tr>
<td>Stdev $P/D$ (ROW)</td>
<td>8.5</td>
<td>12.5</td>
</tr>
<tr>
<td>Real exchange rate volatility</td>
<td>9.1</td>
<td>12.3</td>
</tr>
</tbody>
</table>

The table compares the population moments from the model with the data. The population moments are based on 10,000 years of monthly observations. All variables are sampled monthly (Mar-1976–May-2018) from GFD. We present data for the US and averaged across rest of the world (ROW) countries: Australia, Canada, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, and the UK. Real risk-free rates are constructed following Schorfheide et al. (2018). Means and standard deviations are annualized.
D. Good versus bad times

In this section we discuss the business cycle variation implications of the model. In the data we calculate averages based on the NBER recession indicator in the US. Over the sample period 88.8% corresponds to expansion periods and the remaining 11.2% are recessions. To create a similar measure in the model we calculate the the 88.8% percentile of the inverse surplus consumption ratio for the US, $R_{US}$. We report the averages conditional on being below or above the recession threshold. Table V presents the results. From table V we see that the realized returns are lower in recessions for the ROW compared to the US both in the data and the model. Moreover, the model captures the increase in volatilities and correlations from expansions to recessions. The level of the real exchange rate drops in recessions both in the data and the model, implying a strengthening of the USD in recessions. The NFA decreases and the net export increases, consistent with the analysis in section IV.B. Although the model is not able to replicate all the changes quantitatively, it is able to replicate the sign of the change from expansions to recessions.
Table V: Business cycle properties

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Expansions</th>
<th>Contractions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>$P/D$</td>
<td>42.6</td>
<td>61.4</td>
<td>1.0</td>
</tr>
<tr>
<td>$P/D$ (ROW)</td>
<td>33.8</td>
<td>60.5</td>
<td>0.9</td>
</tr>
<tr>
<td>Excess return</td>
<td>6.5</td>
<td>3.1</td>
<td>1.9</td>
</tr>
<tr>
<td>Excess return (ROW)</td>
<td>6.0</td>
<td>3.7</td>
<td>3.3</td>
</tr>
<tr>
<td>Return volatility</td>
<td>14.4</td>
<td>14.5</td>
<td>-1.0</td>
</tr>
<tr>
<td>Return volatility (ROW)</td>
<td>18.5</td>
<td>18.8</td>
<td>-1.0</td>
</tr>
<tr>
<td>Return correlation</td>
<td>65</td>
<td>84</td>
<td>-5</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>1.3</td>
<td>1.02</td>
<td>0.0</td>
</tr>
<tr>
<td>Exchange rate volatility</td>
<td>9.1</td>
<td>12.3</td>
<td>-0.3</td>
</tr>
<tr>
<td>$NFA/MV$ (%)</td>
<td>-1.2</td>
<td>4.4</td>
<td>1.3</td>
</tr>
<tr>
<td>$NX/GDP$ (%)</td>
<td>-0.0</td>
<td>-5.8</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

The table show the unconditional moments (all), and the conditional moments in expansions and contractions. The expansion and contraction columns report the difference from the unconditional moments. A positive numbers in the Expansion (Contraction) column implies that the variable is higher in expansions (contractions) than the unconditional value. For the simulated data from the model, we define contractions as the 11.2% highest values for the US inverse surplus consumption ration ($R$). All variables except NFA and Net Exports are sampled monthly (Mar-1976–May-2018) from GFD. We present data for for the US and averaged across rest of the world (ROW) countries: Australia, Canada, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, and the UK. Net foreign assets and Net Export series are detrended by removing a five-year moving average. Net foreign assets are constructed as net equity assets less net debt assets (source: Fed Board) and is scaled by US stock market cap, obtained from GFD. The series is available over the period Q4-1999–Q1-2018. Net Export (NX) are obtained from the OECD and are scaled by GDP over the period Q1-1965–Q4-2018. In the data, contractions are defined by the NBER recessions indicator.
E. Currency risk premia

In this section we take a closer look at the currency risk premium. We verify that our model can reproduce the well-established deviation from UIP and the ability of the real exchange rate to predict currency returns. Under rational expectation, currency return predictability implies that UIP does not hold and that there is a time-varying currency risk premium. Table VI shows predictability regressions of the currency excess return on the interest rate differential, often referred to as a Fama (1984) regression. The table presents results using data for both nominal and real interest rate differentials, and the population equivalent in real terms from the model. The regressions use currency fixed effects and standard errors are clustered by month.

In the regressions of the currency return on the interest rate differential (the foreign minus the US interest rate), the coefficient estimates are positive in the data and in the model. A nonzero coefficient indicates that currency returns are predictable (i.e., UIP does not hold and the currency risk premium is time-varying). Moreover, consistent with the literature, the estimates are above one, which means that a foreign currency tends to appreciate when its interest rate is higher than the US interest rate. The model reproduces these patterns.

Dahlquist and Pénasse (2021) argue in a present-value model under purchasing power parity that the Fama regression should be augmented by the real exchange rate. Table VI shows in such augmented regressions that the coefficient associated with the real exchange rate is negative in the data and in the model. This means that when the USD price of a foreign currency is high, the subsequent currency return tends to be low, and vice versa. The model reproduces this fact as the currency risk premium is lower when the USD is appreciated.

We next ask if the model can reproduce the predictability reversal documented by Bacchetta and Van Wincoop (2010) and Engel (2016). In particular, Engel (2016) shows that the positive relationship between the expected currency return and the interest rate differential reverses

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13 We estimate real interest rate differentials as the fitted value of a projection of the realized real interest rate differential on the current nominal interest rate differential and the inflation differential over the previous month, in the spirit of Schorfheide et al. (2018).
over the horizon, a feature that rational expectation models of the currency risk premium have difficulty reproducing and considered to be a puzzle. Following Dahlquist and Pénasse (2021), we regress one month expected excess currency returns $j$ periods ahead on the interest rate differential and real exchange rate. Expectations for horizons above one month are based on a VAR model of the currency return, the interest rate differential, and the real exchange rate. Figure 6 shows that the model can replicate the pattern in the data. In the short-run, the coefficient exceeds one, consistent with the Fama regression results. However, in the medium-run, the coefficient becomes negative. In the long-run, the coefficient converges to zero (i.e., there is no relation between the currency risk premium and the interest rate differential).

![Figure 6: Predictability reversal puzzle](image)

Our model can match the deviations from UIP and the predictability reversal because of stochastic volatility of the country-specific factors ($\nu_t$). The stochastic volatility creates an
additional source of risk, which makes the relationship between the interest rate differential and the currency premium dependent of the volatility. When the volatility is low, risk premia are low and precautionary savings are weak, leading to a negative relationship between the interest rate differential and the currency risk premium. However, when the volatility is high, risk premia are high and precautionary savings are strong, leading to a positive relationship between the interest rate differential and the risk premium. In the short-run, the effect of the high volatility states dominates and the coefficient in the Fama regression is positive. As the volatility persistence is fairly low, the effect of the low volatility states dominate in the medium run and the coefficient becomes negative. In the long-run, the effect of the volatility dies out and there are no deviations from the UIP.

Table VI: Fama regression

<table>
<thead>
<tr>
<th></th>
<th>Data (nominal)</th>
<th>Data (real)</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate differential</td>
<td>1.429***</td>
<td>1.500***</td>
<td>2.715***</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
<td>(0.20)</td>
<td>(0.44)</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>–</td>
<td>–0.011***</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>(0.00)</td>
<td>–</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.027</td>
<td>0.032</td>
<td>0.020</td>
</tr>
<tr>
<td># obs.</td>
<td>3,834</td>
<td>3,834</td>
<td>3,834</td>
</tr>
</tbody>
</table>

This table shows estimation results for panel regressions of the log currency return on the log interest rate differential (the foreign minus the US interest rate) and the log real exchange rate. The dataset covers G10 currencies and is sampled monthly during the period April 1976 to May 2018. The regression includes currency fixed effects; standard errors are clustered by month. *** p < 0.01, ** p < 0.05, * p < 0.1. The model sample is constructed from averaging across 1000 paths of 20,000 months as a proxy for the population equivalent.

VI. Conclusion

This paper explores joint dynamics of cross-country wealth transfers and asset prices in periods of stress and tranquility. We show that in periods of stress the USD appreciates and therefore the US stock market does not fall as much as other stock markets. We propose a model in which US investors have a home bias in their portfolios, which dampens the shock to US wealth.
due to the USD appreciation. This effect is large enough to overturn the loss due to falling
NFA and the wealth share of the US in the world economy increases. In addition, the model
can reproduce patterns for the currency risk premium that were challenging for earlier general
equilibrium models.

We see at least three avenues for future research. First, our calibration focuses on G10
countries. However, emerging markets are important holders of US debt, and the bulk of
wealth transfers to the US in periods of stress comes from emerging markets. As our motivating
empirical facts suggest, there are differences in the exposure to global shocks between the G10
countries and the emerging markets. It would be interesting to take these differences into
account in a calibration of the model. This could help in better understanding the dynamics
of wealth transfers between emerging and developed markets.

Second, in contrast to many papers in the international macro-finance literature, we con-
sider an asymmetric calibration in which the US differs from all other countries in the world
economy. Yet, as we have a tractable model, one could easily allow for a three region cali-
brations – the US, developed markets, and emerging markets. This could allow for a more
nuanced exploration of the NFA dynamics and the currency risk premia around the world.

Third, we assume that markets are complete. This lets us solve the model in closed form,
which gives transparency and tractability. However, there are important frictions that we do
not model. One possible extension would be to incorporate realistic frictions affecting cross-
border asset positions.
Data Appendix

We retrieve monthly spot and one-month forward exchange rates from Barclays Bank International and Reuters (via Datastream) for the period January 1976 to May 2020. We consider the G10 currencies: the Australian dollar (AUD), Canadian dollar (CAD), euro (EUR), Japanese yen (JPY), Norwegian krona (NOK), New Zealand dollar (NZD), Swedish krona (SEK), Swiss franc (CHF), pound sterling (GBP), and US dollar (USD). We let the USD be the domestic currency and express all exchange rates in USD per unit of the foreign currency. For the CAD, EUR (spliced with the German mark before 1999), JPY, NOK, SEK, CHF, and GBP, the sample begins in January 1976; for the AUD and NZD, data availability makes the sample start in January 1985.\textsuperscript{14} We also collect data for emerging currencies: Colombia (COP), Indonesia (IDR), Japan, Republic of Korea (KRW), Malaysia (MYR), Mexico (MXN), New Zealand, Norway, Philippines (PHP), Singapore (SGD), Sri Lanka (LKR), Taiwan (TWD), Turkey (TRY).

Log real USD are computed as $q_t = s_t + p^*_t - p_t$, where $p^*_t$ and $p_t$ are log consumer price indices obtained from the Organization for Economic Co-operation and Development (OECD). The statistical agencies in Australia and New Zealand release price indices on a quarterly basis. We therefore forward fill the price indices for the AUD and NZD in the months until the next quarter. This creates stale prices but avoids introducing future information into the economist’s information set.

We compute implied one-month interest rate differentials using the covered interest rate parity (CIP): $i^*_t - i_t = s_t - f_t$, where $s_t$ and $f_t$ denote the log spot and forward exchange rates, respectively. Log excess returns for a US investor going long in a foreign currency are computed as $r x_{t+1} = s_{t+1} - s_t + i^*_t - i_t$.

Our stock return data come from Global Financial Data (GFD). We compute excess returns in USD by subtracting the US 3-month T-Bill rate (also obtained from GFD) from total equity dollar returns (in logs).

\textsuperscript{14}For the JPY up to 1978 we use data obtained from the Financial Times as in Hsieh (1984).
Realized volatility in a given country $i$ and month $t$ is computed using daily data as

$$\text{Vol}_{i,t} = \sqrt{\frac{K_{i,t}}{2} \sum_{k=1}^{K_{i,t}} \frac{|x_{i,k}|}{\sqrt{K_{i,t}}}},$$

where $K_{i,t}$ is the number of observations in a given country/month. The daily excess return $x_{i,k}$ is approximated by index price changes. We use S&P 500 data for the US, obtained from GFD; for non-US countries, we use MSCI indices when available, and GFD indices otherwise.

To parallel the model, our NFA data include only equity and debt positions (and thus does not include FDI). Our main US series come from the Board of Governors of the Federal Reserve System, downloaded from FRED. We use Equity assets (ticker ROWCEAQ027S), Debt assets (DODFFSWCMI), Equity liabilities: (ROWCESQ027S), and Debt liabilities: (DODFFSWCMI). These data are quarterly and are available since 1945.Q3.

The US market-value national wealth over GDP was initially calculated by Piketty and Zucman (2014); we downloaded data up to 2015 from the World Inequality Database. The database also includes estimates of the US NFA. Both series are sampled annually.
Proofs

Proof of Proposition 1. As we assume complete markets, we can solve the equilibrium by solving the corresponding central planner problem, Equation (17) in the paper, repeated here for convenience:

$$U(Y_t; H_t, a) = \max_{C_{i,t}} \sum_{j=1}^{N} a^j \sum_{i=1}^{N} \log \left( \frac{C_{i,t} - H_{i,t}}{a^j} \right) \quad \text{s.t.} \quad \sum_{j=1}^{N} C_{i,t}^j = Y_{i,t} \quad i = 1, \ldots, N,$$

where $Y_t = (Y_{1,t}, \ldots, Y_{N,t})$, $H_t = (H_{1,t}, \ldots, H_{N,t})$ and $a = (a^1, \ldots, a^N)$ for all $t$. The FOCs of the central planner problem in Equation (65)

$$\frac{a^j}{C_{i,t}^j - H_{i,t}^j} = \eta_{i,t}, \quad i, j = 1, \ldots, N,$$

where $\eta_t = (\eta_{1,t}, \ldots, \eta_{N,t})$ are the Lagrange multipliers on the resource constraint in (65). Hence, we can relate the consumption of investor $j$ and $k$ through $\frac{a^j}{C_{i,t}^j - H_{i,t}^j} = \frac{a^k}{C_{i,t}^k - H_{i,t}^k}$. Imposing the market clearing in each consumption good and using $\sum_{j=1}^{N} \phi_j^i = 1$, we have that

$$(C_{i,t}^j - H_{i,t}^j) = a^j (Y_{i,t} - H_{i,t})$$

Next, we have that

$$f_{i,t}^j = \frac{C_{i,t}^j}{Y_{i,t}} = a^j \left( 1 - \frac{H_{i,t}}{Y_{i,t}} \right) + \phi_j^i H_{i,t} Y_{i,t}$$

$$= a^j + (\phi_j^i - a^j) \frac{H_{i,t}}{Y_{i,t}}$$

$$= a^j + (\phi_j^i - a^j) (1 - s_{i,t})$$

$$= \phi_j^i + (a^j - \phi_j^i) s_{i,t}$$

Proof of Proposition 2. We have from Proposition 1 that $f_{i,t}^j = \phi_j^i + (a^j - \phi_j^i) s_{i,t}$. Consider
first the case of $\phi^j_i > a^j$. It immediately follows that $\frac{\partial f^j_i}{\partial s_i} < 0$ and therefore the minimum is attained for $s_i = s^\text{max}_i$. For the maximum we have that if $s_i \to 0$ then $f^j_i \to \phi^j_i$. The case of $\phi^j_i < a^j$ follows from a similar argument.

**Proof of Proposition 24.** From the FOCs of the individual optimization we have that

$$\frac{e^{-\rho t}}{C^j_i - H^j_{i,t}} = \kappa^j M_{i,t},$$

where $\kappa^j$ is the Lagrange multiplier associated with

$$U^j(C^j_t) = \max_{C^j_t} \left[ \int_0^\infty e^{-\rho t} \sum_{i=0}^N \log \left( C^j_{i,t} - H^j_{i,t} \right) dt \right]$$

s.t

$$E \left[ \int_0^\infty \sum_{i=0}^N M_{i,t} C^j_{i,t} dt \right] \leq W^j_0$$

Using the optimal consumption in Proposition 1 and noting that $\kappa^j = \frac{1}{a^j}$ we have

$$M_{i,t} = e^{-\rho t} \frac{1}{Y_{i,t} - H_{i,t}}.$$  

(72)

The stochastic discount factor in country $j$ is the discount factor that prices a basket with $h^j_i$ units of good $i = 1, \ldots, N$, i.e.,

$$M^j_t = \sum_{i=1}^N h^j_i M_{i,t}$$

(73)

**Proof of Proposition 4.** This follows from noting that the price of good $i$ in terms of units of the consumption basket of country 1 is $p_{i,t} = \frac{M^j_t}{M^1_t}$. Using the expressions for $M_{i,t}$ and $M^j_t$ we
\[ p_{i,t} = \frac{M_{i,t}}{M_{i,t}} = \frac{M_{i,t}}{\sum_{i=1}^{N} h_i^{j} M_{i,t} e^{-\rho t} \frac{1}{Y_{i,t} - H_{i,t}}} = \frac{\sum_{i=1}^{N} h_i^{j} e^{-\rho t} \frac{1}{Y_{i,t} - H_{i,t}}}{\sum_{i=1}^{N} h_i^{j} e^{-\rho t} \frac{1}{Y_{i,t} - H_{i,t}}} = \left( \frac{G_{i,t}}{Y_{i,t}} \right) \frac{1}{\sum_{i=1}^{N} h_i^{j} \left( \frac{G_{i,t}}{Y_{i,t}} \right)} \] (74)

and it follows that the real exchange rate of country \( j \) relative to country 1 is

\[ Q_{j} = \frac{\sum_{i=1}^{N} h_i^{j} \left( \frac{G_{i,t}}{Y_{i,t}} \right)}{\sum_{i=1}^{N} h_i^{j} \left( \frac{G_{i,t}}{Y_{i,t}} \right)} \] (75)

**Proof of Proposition 5.** The net export of country 1 is the

\[ NX_t = p_{1,t} (Y_{1,t} - C_{1,t}^1) - \sum_{i=2}^{N} p_{i,t} C_{i,t}^1. \] (76)

Using the optimal consumption shares from Proposition 1 we have

\[ NX_t = p_{1,t} (Y_{1,t} - f_{1,t} Y_{1,t}) - \sum_{i=2}^{N} p_{i,t} f_{i,t} Y_{i,t} \]

\[ = p_{1,t} Y_{1,t} (1 - f_{1,t} Y_{1,t}) - \sum_{i=2}^{N} p_{i,t} Y_{i,t} f_{i,t} \]

\[ = p_{1,t} Y_{1,t} \left( 1 - f_{1,t} - \sum_{i=2}^{N} G_{i,t} f_{i,t} \right) \]

\[ = p_{1,t} Y_{1,t} \left( 1 - \sum_{i=1}^{N} \frac{G_{i,t}}{G_{1,t}} f_{i,t} \right) \]

\[ = p_{1,t} Y_{1,t} \left( 1 - \sum_{i=1}^{N} \frac{G_{i,t}}{G_{1,t}} \left( \phi_i^1 + (a^1 - \phi_i^1) s_{i,t} \right) \right) \] (77)
Proof of Proposition 6. We have that the stock price of country $i$ is

$$S_{i,t} = p_{i,t} E_t \left( \int_t^\infty \frac{M_{i,u}}{M_{i,t}} Y_{i,u} du \right)$$

$$= p_{i,t} Y_{i,t} PD_{i,t},$$

where

$$PD_{i,t} = E_t \left( \int_t^\infty \frac{M_{i,u}}{M_{i,t}} Y_{i,u} Y_{i,t} du \right)$$

We can write the price-dividend ratio, $PD_{i,t}$, as

$$PD_{i,t} = E_t \left( \int_t^\infty \frac{M_{i,u}}{M_{i,t}} Y_{i,u} du \right)$$

$$= E_t \left( \int_t^\infty \frac{R_{i,u}}{R_{i,t}} du \right)$$

$$= \frac{1}{R_{i,t}} F_{i,t}$$

where $F_{i,t} = E_t \left( \int_t^\infty R_{i,u} du \right)$ and therefore we have

$$F_{i,t} = E_t \left( \int_t^\infty R_{i,u} du \right)$$

$$= \int_t^\infty E_t (R_{i,u}) du$$

$$= \int_t^\infty E_t (G_{i,u}) E_t (G_u) du,$$

where the last line follows from the independence of $G_{i,t}$ and $G_t$. We have

$$E_t (G_{i,u}) = G_{i,t} e^{-\kappa (u-t)} + \bar{G}_i \left( 1 - e^{-\kappa (u-t)} \right),$$

and

$$E_t (G_u) = G_t e^{-\kappa (u-t)} + \bar{G}_i \left( 1 - e^{-\kappa (u-t)} \right)$$
Inserting Equations (82) and (83) into Equation (81) and solving the integral yields the result.

\[ W_t^j = E_t \left( \int_t^\infty \sum_{i=1}^N \frac{M_u}{M_t} p_{i,t} \frac{C_{i,u}^j}{C_{i,t}^j} du \right) \]

\[
= \sum_{i=1}^N p_{i,t} C_{i,t}^j \int_t^\infty E_t \left( \frac{M_{i,u}}{M_{i,t}} \frac{C_{i,u}^j}{C_{i,t}^j} \right) du \\
= \sum_{i=1}^N p_{i,t} C_{i,t}^j \int_t^\infty E_t \left( e^{-\rho(t-u)} \frac{R_{i,u}}{R_{i,t}} \left( \frac{Y_{i,t}}{Y_{i,u}} \right) \left( \frac{f_{i,u}^j}{f_{i,t}^j} \right) \right) du \\
= \sum_{i=1}^N p_{i,t} Y_{i,t} \int_t^\infty E_t \left( e^{-\rho(t-u)} \frac{R_{i,u}}{R_{i,t}} \left( \phi_i^j + \left( a_i^j - \phi_i^j \right) \frac{1}{R_{i,u}} \right) \right) du \\
= \sum_{i=1}^N p_{i,t} Y_{i,t} \frac{F_{i,t}}{R_{i,t}} \phi_i^j + \sum_{i=1}^N \frac{p_{i,t} Y_{i,t} 1}{R_{i,t}} \left( \frac{a_i^j - \phi_i^j}{\rho} \right) \\
= \sum_{i=1}^N \phi_i^j S_{i,t} + \frac{b_i}{e^{\rho t} M_t} \tag{84} \]

Proof of Proposition 7. The wealth of investor \( j \) is

Proof of Proposition 8. From Proposition 7 we have that

\[ W_t^j = \sum_{i=1}^N \phi_i^j S_{i,t} + \frac{b_i}{e^{\rho t} M_t}, \tag{85} \]

If each country is initially endowed with the claim to their own output we have that \( W_t^j = S_{j,t} \).

Using this together with the fact that \( b_j = \frac{1}{\rho} \left( N a_j - \sum_{i=1}^N \phi_i^j \right) \) and \( R_{i,0} = \bar{R}_i \) for \( i = 1, \ldots, N \), then solving for \( a_j \) yields the result.

Proof of Proposition 9. We can write \( ED_t^j \) as

\[ ED_t^j = \frac{b_j}{e^{\rho t} M_t} = b_j \left( \sum_{i=1}^N h_i^j \frac{R_{i,t}}{Y_{i,t}} \right)^{-1}. \tag{86} \]
Note that for \( a^j > \frac{1}{N} \) we have that \( b^j > 0 \). Moreover, we have

\[
\frac{\partial}{\partial R_{i,t}} \sum_{i=1}^{N} h_i^j \frac{R_{i,t}}{Y_{i,t}} = -\frac{1}{\left( \sum_{i=1}^{N} h_i^j \frac{R_{i,t}}{Y_{i,t}} \right)^2} h_i^j \frac{1}{Y_{i,t}} < 0, \tag{87}
\]

hence \( \frac{\partial ED^j}{\partial R_{i,t}} < 0 \) if \( b^j > 0 \). \( \square \)

**Proof of Proposition 10.** The stochastic discount factor of country \( j \) follows

\[
\frac{dM^j_t}{M^j_t} = -r^j_t dt - \left( \theta^j_t \right)^{'} dw_t. \tag{88}
\]

Applying Ito’s lemma to \( M^j_t = e^{-\rho t} \sum_{i=1}^{N} h_i^j \frac{1}{Y_{i,t}-R_{i,t}} \) and equating the drift and diffusion coefficients with the drift and diffusion coefficient from Equation (88) yields the result. \( \square \)

**Proof of Proposition 11.** The real exchange rate can be written as the ratio of SDF

\[
Q^j_t = \frac{M^j_t}{M^1_t}. \tag{89}
\]

Applying Ito’s lemma to Equation (89) yields the result. \( \square \)

**Proof of Proposition 12.** The instantaneous return on stock \( i \) is

\[
dR_{i,t} = \frac{dS_{i,t} + Y_{i,t} dt}{S_{i,t}} = \mu_{R_{i,t}} dt + \sigma_{R_{i,t}}^{'} dw_t. \tag{90}
\]

By no-arbitrage \( \mu_{R_{i,t}} = r_t + \sigma_{R_{i,t}}^{'} \theta_t \). Applying Ito’s lemma to the stock price in Proposition 6 to calculate the diffusion coefficients \( \sigma_{R_{i,t}} \) yields the result. \( \square \)

**Proof of Proposition 13.** By applying Ito’s lemma to the wealth in Proposition 7 and matching the drift and diffusion coefficients with the drift and diffusion coefficients in the dynamics budget constrain in Equation (16) yields the result. \( \square \)
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