

# The Role of Portfolio Constraints in the International Propagation of Shocks

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## Abstract

We study the comovement among stock prices and among exchange rates in a three-good three-country Center-Periphery dynamic equilibrium model in which the Center's agents face portfolio constraints. We characterize equilibrium in closed form for a broad class of portfolio constraints, solving for stock prices, terms of trade, and portfolio holdings. We show that portfolio constraints generate wealth transfers between the Periphery countries and the Center, which increase the comovement of the stock prices across the Periphery. We associate this excess comovement caused by portfolio constraints with the phenomenon known as contagion. The model generates predictions consistent with other important empirical results such as amplification and flight-to-quality effects.

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# 1. Introduction

Even a casual observer of stock markets would recall many recent episodes when adverse shocks originating in one country triggered financial “contagion.” For instance, financial markets worldwide suffered severe losses in the wake of the 1997 Asian collapse and the 1998 Russian default. A closer look at the data reveals that these episodes reflect a widespread phenomenon: the cross-country comovement of stock returns and of exchange rates tend to increase in periods of turmoil, particularly in emerging economies (Table 1). This excess comovement during crises, known as contagion, is an important economic phenomenon that merits a thorough theoretical investigation.

	<u>Stock market returns</u>		<u>Exchange rate returns</u>	
	Tranquil times	Crises	Tranquil times	Crises
Emerging economies	27.0%	46.5%	5.6%	20.4%
Developed economies	37.4%	42.9%	54.9%	57.9%

Table 1: Cross-country correlations of stock market and of exchange rate returns during tranquil times and during crises. Source: authors’ calculations.<sup>1</sup>

Existing literature has highlighted two main channels through which financial markets can comove. The first one, put forward by the international economics literature, is the terms of trade channel. A shock to a country affects its terms of trade and therefore affects stock prices in the countries it is trading with. The second one, emphasized in the international asset pricing literature, is the common discount factor channel.<sup>2</sup> When investors are diversified internationally and financial markets are frictionless, the cash flows of all assets are discounted with the same state prices (common discount factor). Changes in state prices then cause asset prices to comove even if their cash flows are uncorrelated. However, neither of these channels can account for the full extent of international comovement in the data. Direct measures of the terms of trade channel fail to explain contagion during crises.<sup>3</sup> For example, only 0.2% of Brazil’s exports go to Russia, and yet Brazil was one of the worst-affected countries during the 1998 Russian crisis, with its equity prices plummeting more than 20%. Testing for the common discount factor channel is much

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<sup>1</sup>Appendix B describes our sample and the details of the estimation. The correlations are adjusted for heteroskedasticity as in Forbes and Rigobon (2002). The magnitudes of the crisis-induced increases in the correlations in our sample are consistent with the recent literature (e.g., Fostel (2006) and Pan and Singleton (2005)).

<sup>2</sup>See Ammer and Mei (1996), Cochrane, Longstaff, and Santa-Clara (2008), Dumas, Harvey, and Ruiz (2003), Kodres and Pritsker (2002), and Kyle and Xiong (2001). These papers are all cast in a single-good framework, and hence highlight exclusively propagation through the common discount factor (or attribute the cross-stock spillovers to changes in investors’ risk aversion, which is equivalent in this framework).

<sup>3</sup>See Kaminsky, Reinhart, and Végh (2003) for a survey.

harder as such tests require an asset pricing model. Yet the extensive evidence from reduced-form specifications suggests that a single common factor cannot explain contagion.<sup>4</sup> Moreover, neither theory can explain the sizeable shifts in cross-country correlations of sovereign bond yields following credit rating changes—events that do not reveal any economic news.<sup>5</sup> For instance, when Mexican debt was upgraded from non-investment grade to investment grade in March of 2000, its correlation with bonds of other Latin American countries dropped by 30 percentage points.

The shortfalls of the standard theories inspired a search for alternative channels of propagation. For example, Calvo (2002) argues that Wall Street was the carrier of the “Russian virus” in the fall of 1998 when binding margin constraints forced leveraged investors (most notably LTCM) to curtail their exposure to all emerging markets. Kaminsky and Reinhart (2000) document that Thailand’s 1997 currency crisis led to capital losses for Japanese banks, forcing them to curb their lending to other Asian countries. This newer literature highlights the prominent role that financial frictions, and in particular institutionally- or government-imposed portfolio constraints, play in the propagation of turmoil via financial centers.

In this paper, we analyze formally the effects of portfolio constraints on asset prices within a unified framework that encompasses both the terms of trade and the common discount factor channels. Our general-equilibrium model allows us to disentangle the effects of portfolio constraints from those of the traditional channels and define precisely the notion of contagion as the comovement in excess of that in the unconstrained economy. Within our model, we can understand why portfolio constraints amplify stock price fluctuations, and how changes in their tightness contribute to excess comovement. From a methodological viewpoint, this paper offers a tractable model, solved in closed form, that can be used to study the impact of a broad class of constraints on the terms of trade and stock prices.

Our starting point is a canonical Arrow-Debreu economy, which we generalize to include portfolio constraints.<sup>6</sup> There are three countries—one Center and two Periphery countries—each popu-

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<sup>4</sup>For instance, principal component estimates show that the weight on the first principal component changes around crises and type of country (see e.g., Calvo and Reinhart (1996) and Kaminsky and Reinhart (2007)). For evidence based on reduced-form linear latent factor models, see e.g., Corsetti, Pericoli, and Sbracia (2005), Dungey, Fry, González-Hermosillo, and Martin (2005), and Rigobon (2003), and for that based on the copula approach, see e.g., Hartmann, Straetmans, and de Vries (2004) and Rodriguez (2007).

<sup>5</sup>See Kaminsky and Schmukler (2002) and Rigobon (2002).

<sup>6</sup>See Cass, Siconolfi, and Villanacci (2001) for an analysis of a general financial equilibrium model with portfolio constraints. They show existence and finite local uniqueness of equilibrium. For more applied analyses, see also Boyer, Kumagai, and Yuan (2006), Geanakoplos (2003), Gromb and Vayanos (2002), Mendoza and Smith (2002), and Yuan (2005). This recent literature is rapidly growing and has already produced many important insights; however, to deal with the technical difficulties involved in solving dynamic models with multiple risky assets, it has either resorted to

lated by a representative consumer-investor. We think of the Center as a large developed economy and of the Periphery countries as small emerging markets. Each country is endowed with a Lucas tree, producing a country-specific good. Each tree's output is driven by its own supply shock, and stocks are claims to the Lucas trees. There is also a riskless bond in zero net supply. Each agent has log-linear preferences over all three goods, with a preference bias for the home good. The goods markets are frictionless, but financial markets are imperfect in that the Center's agent faces constraints limiting his portfolio choice. We assume a general form for the constraints, that nests, among others, portfolio concentration constraints, VaR constraints, margin requirements, and collateral constraints.

Absent portfolio constraints, all comovement in our model is due to the common discount factor and the terms of trade channels. The former works through changes in the investors' aversion to risk. Consider, for instance, a negative shock to the Center's stock.<sup>7</sup> Because of their log-linear preferences, all investors hold identical mean-variance efficient portfolios, which include positive amounts of the Center's stock. Therefore, the negative shock implies a negative return on their portfolios. As the investors become poorer, they become more averse to risk and lower their demand for the stocks. However, they cannot all sell. Hence, prices of *all three stocks* must fall. Since all investors hold the same portfolios, shocks leave wealth distribution unchanged. This mechanism illustrates the propagation of shocks across stock markets due to the investors' desire to diversify internationally. An alternative intuition for the same phenomenon makes use of the terms of trade channel. The terms of trade improve for the country experiencing a negative shock and therefore deteriorate for all others, which in turn reduces their stock market prices (see Section 2).

To understand the additional effects that constraints introduce, let us now impose portfolio constraints on the Center's agent. While the two Periphery countries continue to hold identical portfolios, the constraints force the Center's agent to hold a different one. As a result, the returns on portfolios of the Center and the Periphery countries implied by a common shock now differ. Suppose, for example, that the constrained portfolio is such that the Periphery countries lose more. This implies a change in wealth distribution, which we call a *wealth transfer* (from the Periphery to the Center). Wealth transfers, absent in the frictionless economy, work much like income transfers in the classic *Transfer Problem* (also known as the Keynes effect).<sup>8</sup> That is, as the Periphery

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a static or a partial equilibrium framework, relied on behavioral assumptions, or abstracted away from international trade and other cross-country linkages intrinsic to the phenomenon of international financial contagion.

<sup>7</sup>Considering a shock directly to an endogenously-determined stock price rather than a supply shock here and below is a shortcut allowing us to bypass some technical steps of our formal analysis while conveying correct intuitions.

<sup>8</sup>The Transfer Problem stems from the argument made originally by Keynes that in a world with a home bias in

countries' wealth drops, so does their demand for all three goods, but because of their preference biases toward their own goods, the demand for the Periphery goods suffers more. Hence, both Periphery countries see their goods' prices fall relative to that of the Center's good, deteriorating their terms of trade. Consequently, the value of the dividends on the Periphery countries' trees falls, depressing the prices of their stocks. The Center's terms of trade improve and its stock market rises. Hence, the Keynes effect (i) increases the comovement of the stock markets and of the terms of trade across the Periphery and (ii) decreases the comovement between those of the Periphery and the Center, relative to that in the unconstrained model. We note that for these results to hold, portfolio constraints in the Center must be binding, which we associate with periods of financial crises.

The Keynes effect may also give rise to important phenomena such as “amplification” and “flight to quality” effects.<sup>9</sup> Consider a constraint that limits the fraction of wealth the Center may invest in the stock markets of the two Periphery countries. Relative to the frictionless case, the constraint forces the Center to decrease its holdings of the Periphery stocks and to increase its holdings of the Center's stock. First, consider a negative shock to the Center's stock. In that case, all agents experience a negative return, as in the frictionless model. However, the Center's portfolio has a lower return, as it is now over-weighted in the Center's stock relative to the Periphery countries. This implies a wealth transfer from the Center to the Periphery, which in turn causes a deterioration of the Center's terms of trade and a further decline of its stock market. Hence, the portfolio constraint leads to an *amplification* of negative shocks to the Center's stock market—consistent with the literature arguing that portfolio constraints exacerbate market volatility. Now, consider a negative shock to one of the Periphery countries's stocks. The Periphery countries are over-weighted in their stocks, and lose more than the Center, which generates a wealth transfer away from the Periphery. This further depresses both Periphery stocks, but has an incrementally positive effect on the Center's stock—a phenomenon often referred to as a *flight to quality*.

To illustrate both the model's workings and its fit in the context of recent crises, we consider two examples of constraints: a concentration constraint and a market share constraint. The former puts a ceiling on the fraction of the Periphery stocks in the Center's portfolio in absolute terms, while the latter specifies it in proportion to the market share of the Periphery in the world. Both

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consumption, an income transfer from one country to another will deteriorate the terms of trade of the sender of the transfer.

<sup>9</sup>See e.g., Bernanke, Gertler, and Gilchrist (1996), Eichengreen, Hale, and Mody (2001), and Vayanos (2004). The definitions of these effects differ somewhat depending on the application.

constraints produce similar dynamic behavior of prices, but they have different implications for capital flows. The impact on flows is driven by how the tightness of a constraint changes following a given shock. For instance, in the case of the market share constraint, a negative shock in the Periphery reduces the market capitalization of the Periphery, tightening the constraint.<sup>10</sup> This tightening generates capital withdrawals and large price drops across the Periphery through two mechanisms. First, the Periphery agents became poorer and hence want to downsize their stock market positions. Second, the Center’s agent who would have chosen to invest more in the Periphery has to curb his position to respect his portfolio constraint. Therefore, the change in the tightening of the constraint generates an additional source of price comovement across the Periphery. To quantify these effects, we parameterize the model and find that the correlation between the stock returns of the two Periphery countries goes up by 15–20% while the correlation between the Center and the Periphery decreases by 5–8%.

The literature closest to our work is the two-good two-country asset-pricing models of Helpman and Razin (1978), Cole and Obstfeld (1991), Pavlova and Rigobon (2007), and Zapatero (1995), which feature both the terms of trade and the common discount factor channels of international propagation, but no portfolio constraints. Also related is the literature on portfolio constraints in asset pricing. Basak and Croitoru (2000), Basak and Cuoco (1998), Detemple and Murthy (1997), Detemple and Serrat (2003), Gallmeyer and Hollifield (2004), Shapiro (2002), among others, all consider the effects of portfolio constraints on asset prices in dynamic economies.<sup>11</sup> Their solution methodology is similar to ours, however, they consider single-good closed economies and hence have no implications for the terms of trade. We employ techniques developed in Cvitanić and Karatzas (1992) to solve the (partial equilibrium) dynamic optimization problem of an investor who is facing portfolio constraints. Our paper illustrates the usefulness of these techniques in solving dynamic equilibrium models with portfolio constraints.

The rest of the paper is organized as follows. Section 2 describes the model and characterizes its equilibrium in the benchmark unconstrained case. Section 3 investigates the equilibrium in the economy with portfolio constraints. Section 4 argues that our results hold even if the Periphery countries do not trade with each other. Section 5 specializes the model to examine the effect of

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<sup>10</sup>The concentration constraint works in exactly the opposite direction. After a negative shock to the Periphery, the constraint becomes looser. The market share constraint provides a better fit for the stylized facts that have emerged after recent financial crises, most notably the Russian default in 1998.

<sup>11</sup>For earlier literature examining the effects of constraints on cross-country holdings in a static setting, see e.g., Errunza and Losq (1985) and Errunza and Losq (1989).

two particular portfolio constraints. Section 6 discusses caveats and possible extensions, and the appendices contain all proofs and other supplementary material.

## 2. The Model

We now present the formal model and, before turning to portfolio constraints, analyze its solution in the unconstrained case.

### 2.1. The Economic Setting

We consider a continuous-time pure-exchange world economy with a finite horizon,  $[0, T]$ , in the spirit of Lucas (1982). The main advantage of using continuous time is tractability—an analogous discrete-time model does not admit a closed-form solution. Uncertainty represented by a filtered probability space  $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$ , on which is defined a standard three-dimensional Brownian motion  $w(t) = (w^0(t), w^1(t), w^2(t))^\top$ ,  $t \in [0, T]$ . All stochastic processes are assumed adapted to  $\{\mathcal{F}_t; t \in [0, T]\}$ , the augmented filtration generated by  $w$ . All stated (in)equalities involving random variables hold  $P$ -almost surely. In what follows, given our focus, we assume all processes introduced to be well-defined, without explicitly stating regularity conditions ensuring this.

There are three countries in the world economy, indexed by  $j \in \{0, 1, 2\}$ . Country 0 represents a large Center country (e.g., an industrialized economy) and countries 1 and 2 smaller Periphery countries (e.g., emerging economies). Each country  $j$  produces its own perishable good via a strictly positive output process modeled as a Lucas tree:

$$dY^j(t) = \mu_{Y^j}(t) Y^j(t) dt + \sigma_{Y^j}(t) Y^j(t) dw^j(t), \quad j \in \{0, 1, 2\}, \quad (1)$$

where  $\mu_{Y^j}$  and  $\sigma_{Y^j} > 0$  are arbitrary adapted processes.<sup>12</sup> The price of the good produced by country  $j$  is denoted by  $p^j$ . Since prices are not pinned down in a real model such as ours, we need to adopt a numeraire. We fix a basket containing  $\beta \in (0, 1)$  units of the good produced in country 0 and  $(1 - \beta)/2$  units of each of the remaining two goods and normalize the price of this basket to be equal to unity. We think of  $\beta$  as the size of the (large) Center country relative to the

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<sup>12</sup>For generality, we allow the parameters of the output processes to depend on all information generated up to time  $t$ , including that generated abroad. This may be useful for a potential extension of our model in which, unlike here, these parameters are unobserved and agents estimate them using all available information (see Detemple and Murthy (1994) for a detailed analysis of such an economy). For the purposes of the current model, however, we may instead assume that the parameters are either constant or depend only on the history generated by the domestic output processes.

world economy. It is convenient to define the terms of trade from the viewpoint the Center country (country 0):  $q^1 \equiv p^1/p^0$  and  $q^2 \equiv p^2/p^0$  are the terms of trade of the Periphery countries 1 and 2, respectively, with the Center country.

Each country is endowed with a stock  $S^j$ , a claim to its output stream. All stocks are in unit supply. Additionally, we assume that there is a riskless bond so that markets are complete. The bond market and the stock markets of the three countries follow

$$dB(t) = B(t)r(t)dt, \quad B(0) = 1, \quad (2)$$

$$dS^j(t) + p^j(t)Y^j(t)dt = S^j(t)(\mu^j(t)dt + \sigma^j(t)dw(t)), \quad j = 0, 1, 2, \quad (3)$$

respectively, where the interest rate  $r$ , the stocks' expected returns  $\mu \equiv (\mu^0, \mu^1, \mu^2)^\top$  and the volatility matrix of stock returns  $\sigma \equiv [\sigma^j; j = 0, 1, 2]$  are to be determined in equilibrium.

A representative consumer-investor of each country is endowed at time 0 with a total supply of the stock market of his country; the initial wealth of agent  $i$  is denoted by  $W_i(0)$ . Each consumer  $i$  chooses nonnegative consumption of each good  $(C_i^0(t), C_i^1(t), C_i^2(t))$ ,  $i \in \{0, 1, 2\}$ , and a portfolio of the available risky securities  $x_i(t) \equiv (x_i^{s^0}(t), x_i^{s^1}(t), x_i^{s^2}(t))^\top$ , where  $x_i^j$  denotes a fraction of wealth  $W_i$  invested in security  $j$ , so as to maximize his time-additive utility

$$E \left[ \int_0^T u_i(C_i^0(t), C_i^1(t), C_i^2(t)) dt \right],$$

with

$$\begin{aligned} u_0(C_0^0(t), C_0^1(t), C_0^2(t)) &= \alpha_0 \log C_0^0(t) + \frac{1 - \alpha_0}{2} \log C_0^1(t) + \frac{1 - \alpha_0}{2} \log C_0^2(t), \\ u_1(C_1^0(t), C_1^1(t), C_1^2(t)) &= \frac{1 - \alpha_1(t)}{2} \log C_1^0(t) + \alpha_1(t) \log C_1^1(t) + \frac{1 - \alpha_1(t)}{2} \log C_1^2(t), \\ u_2(C_2^0(t), C_2^1(t), C_2^2(t)) &= \frac{1 - \alpha_2(t)}{2} \log C_2^0(t) + \frac{1 - \alpha_2(t)}{2} \log C_2^1(t) + \alpha_2(t) \log C_2^2(t). \end{aligned}$$

subject to the budget constraint

$$\begin{aligned} \frac{dW_i(t)}{W_i(t)} &= x_i^{s^0}(t) \frac{dS^0(t) + p^0(t)Y^0(t)dt}{S^0(t)} + x_i^{s^1}(t) \frac{dS^1(t) + p^1(t)Y^1(t)dt}{S^1(t)} + x_i^{s^2}(t) \frac{dS^2(t) + p^2(t)Y^2(t)dt}{S^2(t)} \\ &\quad + (1 - x_i^{s^0}(t) - x_i^{s^1}(t) - x_i^{s^2}(t)) \frac{dB(t)}{B(t)} - \frac{1}{W_i(t)} (p^0(t)C_i^0(t) + p^1(t)C_i^1(t) + p^2(t)C_i^2(t)) dt, \quad (4) \end{aligned}$$

with  $W_i(T) \geq 0$ , where the subscript  $i$ ,  $i = 0, 1, 2$ , indexes the consumer in countries 0, 1, and 2, respectively. Two features of the preferences specification are noteworthy. First, we assume

that each  $\alpha_i$  is between  $1/3$  and  $1$ , or in other words, that there is a home bias in preferences.<sup>13</sup> Second,  $\alpha_i$ ,  $i = 1, 2$  are stochastic, i.e., we allow for demand shifts modeled along the lines of Dornbusch, Fischer, and Samuelson (1977). This assumption is useful because in the absence of demand uncertainty free trade in goods may imply excessively high correlation of stock market returns and irrelevancy of a financial market structure.<sup>14</sup> Moreover, empirical evidence indicates that demand uncertainty is of the same order of magnitude as supply uncertainty (see Pavlova and Rigobon (2007)). Formally, we assume that each  $\alpha_i$  is a martingale (i.e.,  $E[\alpha_i(s)|\mathcal{F}_t] = \alpha_i(t)$ ,  $s > t$ ), and hence can be represented as

$$d\alpha_1(t) = \sigma_{\alpha_1}(t)^\top dw(t), \quad d\alpha_2(t) = \sigma_{\alpha_2}(t)^\top dw(t),$$

where  $\sigma_{\alpha_1}(t)$  and  $\sigma_{\alpha_2}(t)$  are such that our restriction that  $\alpha_1$  and  $\alpha_2$  take values between  $1/3$  and  $1$  is satisfied.<sup>15</sup> Since our primary focus is on the Periphery countries, for expositional clarity, we keep the preference parameter of the Center country,  $\alpha_0$ , fixed. The log-linear specification of the preferences is adopted for tractability.

While investment policies of the Periphery countries are unconstrained, the Center faces a portfolio constraint that we allow to take the most general form, suggested by Cvitanić and Karatzas (1992). Namely, portfolio values  $x_0$  are constrained to lie in a closed, convex, non-empty subset  $K \in \mathbb{R}^3$ . Moreover, the deterministic subset  $K$  may be replaced by a family of stochastic constraints, so that

$$x_0(t, \omega) \in \{K_t(\omega); (t, \omega) \in [0, T] \times \Omega\}.$$

Making the constraint set stochastic, and in particular dependent on exogenous variables in the Center's optimization problem (e.g.,  $S^i$ ,  $p^i$ ,  $Y^i$ ,  $i = 0, 1, 2$ ), allows for more flexibility in specifying

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<sup>13</sup>This assumption is quite common in open economy macroeconomics. It may be replaced by explicitly accounting for the demand of nontradables and assuming that the nontradables are produced using domestically produced inputs, or by explicitly modeling transport costs. The implications of these richer models for the properties of the terms of trade and other pertinent quantities are known to be very similar (see, e.g., Obstfeld and Rogoff (2000) and Samuelson (1954)), and we hence adopted the more parsimonious specification. Furthermore, note that the purpose of the assumption is to generate a home bias in *consumption*, and not in *portfolios*.

<sup>14</sup>As established by Helpman and Razin (1978), Cole and Obstfeld (1991), and Zapatero (1995). For other recent attempts to break the financial market structure irrelevancy result, see Engel and Matsumoto (2006), Ghironi, Lee, and Rebucci (2006), Pavlova and Rigobon (2007), Serrat (2001), and Soumare and Wang (2007).

<sup>15</sup>In our specification, the demand shifts are driven by the same Brownian motions as output shocks. We do so to reduce the dimensionality of uncertainty in the model and hence to require fewer assets for market completeness. It is, however, possible to extend our model to the case where there are independent demand shifts but the number of assets remains the same (incomplete markets). While many of our results, and in particular Propositions 1 and 2, remain unchanged, the explicit expressions in Proposition 3 require some modification. This extension may be useful for future applications that would focus more closely on demand shifts. Finally, we comment on the bounds we impose on  $\alpha_i$ ,  $i = 1, 2$ . An example of a martingale process that does not exit the interval  $(1/3, 1)$  is  $\alpha_i(t) = E[\alpha_i(T)|\mathcal{F}_t]$ , with  $\alpha_i(T) \in (1/3, 1)$ . We thank Mark Loewenstein for this example.

constraints, which we exploit in Section 5.<sup>16</sup> Examples of portfolio constraints belonging to this class include prohibitions to trade certain stocks or some less severe provisions such as limits on the fraction of the portfolio that could be invested in the emerging markets  $S^1$  and  $S^2$ . This specification can also capture constraints on borrowing, VaR constraints, margin requirements, collateral constraints, etc. A body of empirical literature has argued that regulation or risk-management practices adopted in financial centers were largely responsible for financial contagion in emerging markets in the past two decades.<sup>17</sup> This motivates our choice of imposing the constraint on the Center. In this paper, we do not provide a model supporting the economic rationale behind imposing portfolio constraints. Typically, such constraints are either government-imposed or arise in response to an agency problem in institutional money management.<sup>18</sup>

## 2.2. Countries' Optimization: The Solution Method and Main Results

In this section we sketch the method for solving the (partial-equilibrium) optimization problems of the Center and the Periphery countries. All technical details are provided in Appendix A.

The consumption-portfolio problem of the Periphery countries is standard because both countries face complete markets (see e.g., Duffie (2001)). The only difference here is that we work in a multi-good as opposed to a single-good framework typically adopted in the finance literature. The problem faced by the Center country, on the other hand, is non-standard because of the presence of portfolio constraints. To solve it, we rely on Cvitanić and Karatzas (1992) who show that the optimization problem of an investor subject to portfolio constraints is formally equivalent to an auxiliary problem with no constraints but the investor facing a modified investment opportunity set

$$\begin{aligned} dB(t) &= B(t)(r(t) + \delta(\nu(t)))dt, \quad B(0) = 1, \\ dS^j(t) + p^j(t)Y^j(t)dt &= S^j(t)[(\mu^j(t) + \nu^j(t) + \delta(\nu(t)))dt + \sigma^j(t)dw(t)], \quad j = 0, 1, 2, \end{aligned}$$

where the function  $\delta(\cdot)$  and an endogenously determined stochastic process  $\nu \equiv (\nu^0, \nu^1, \nu^2)^\top$  are defined in Appendix A. There are two key differences between this investment opportunity set and the one faced by the unconstrained countries (2)–(3): (i) the “effective” expected returns

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<sup>16</sup>See Cvitanić and Karatzas for (minor) regularity conditions imposed on the constraint set.

<sup>17</sup>See Calvo (2002), Kaminsky and Reinhart (2000), and Van Rijckeghem and Weder (2003).

<sup>18</sup>For the latter, see, for example, recent papers by Basak, Pavlova, and Shapiro (2006) and Dybvig, Farnsworth, and Carpenter (2006). Such constraints are particularly prevalent in developed countries, where risk management practices are more sophisticated.

on the stocks and (ii) the interest rate on the bond are tilted away from the values faced by the unconstrained investors. One may think of the process  $\nu$  as the Lagrange multiplier associated with the set of portfolio constraints. For example, if a portfolio constraint imposes an upper bound on investment in stock  $S^1$ , the corresponding  $\nu^1$  is going to be negative, making the expected return on this stock less favorable from the viewpoint of the constrained investor and hence convincing him to invest less in this stock so that the constraint is satisfied. This is the fundamental idea behind Lagrange multipliers. In Appendix A, we specify the minimization problem that this process  $\nu$  has to solve and then derive its solution,  $\nu^*$ , for each example of the portfolio constraints we consider in this paper (Section 5).

Assuming that the process  $\nu^*$  has been determined, optimal consumption and portfolios of a constrained investor have the same characterizations as that of an unconstrained except that, in each instance, the interest rate  $r$  is replaced by the “effective” interest rate  $r(t) + \delta(\nu^*)$  and the expected returns on the stocks by (in vector notation)  $\mu + \nu^* + \delta(\nu)\bar{1}$ , where  $\bar{1} = (1, 1, 1)^\top$ . In particular, we can claim the following result and its corollary.

**Lemma 1.** *The optimal consumption allocations and wealth are linked as follows:*

$$\begin{aligned} \begin{pmatrix} C_0^0(t) \\ C_1^0(t) \\ C_2^0(t) \end{pmatrix} &= \frac{1}{p^0(t)(T-t)} \begin{pmatrix} \alpha_0 W_0(t) \\ \frac{1-\alpha_1(t)}{2} W_1(t) \\ \frac{1-\alpha_2(t)}{2} W_2(t) \end{pmatrix}, & \begin{pmatrix} C_0^1(t) \\ C_1^1(t) \\ C_2^1(t) \end{pmatrix} &= \frac{1}{p^1(t)(T-t)} \begin{pmatrix} \frac{1-\alpha_0}{2} W_0(t) \\ \alpha_1(t) W_1(t) \\ \frac{1-\alpha_2(t)}{2} W_2(t) \end{pmatrix}, \\ \begin{pmatrix} C_0^2(t) \\ C_1^2(t) \\ C_2^2(t) \end{pmatrix} &= \frac{1}{p^2(t)(T-t)} \begin{pmatrix} \frac{1-\alpha_0}{2} W_0(t) \\ \frac{1-\alpha_1(t)}{2} W_1(t) \\ \alpha_2(t) W_2(t) \end{pmatrix}. \end{aligned}$$

As is to be expected in a model with log-linear preferences, the consumption expenditure on each good is proportional to wealth. However, in our economy the marginal propensity to consume out of wealth is stochastic, due to possible demand shifts. Lemma 1 allows us to easily generalize the standard implication of the single-good models that logarithmic agents follow myopic trading strategies, holding only the Merton (1971) mean-variance efficient portfolio.

**Corollary 1.** *The countries’ portfolios of risky assets are given by*

$$x_0(t) = (\sigma(t)\sigma(t)^\top)^{-1}(\mu(t) + \nu^*(t) - r(t)\bar{1}), \quad x_i(t) = (\sigma(t)\sigma(t)^\top)^{-1}(\mu(t) - r(t)\bar{1}), \quad i \in \{1, 2\}.$$

Note that the portfolio of the investor in the Center generally differs from those chosen by the investors in the Periphery because his investment opportunity set is augmented by the portfolio

constraint through the multiplier  $\nu^*$ . Only when the constraint is absent or at times when it is not binding ( $\nu^*(t) = 0$ ) all investors in the world economy hold the same portfolio.<sup>19</sup>

### 2.3. Benchmark Unconstrained Equilibrium

To establish a benchmark, we solve for an equilibrium in an economy with no portfolio constraints. In equilibrium, agents optimize and stock, bond, and goods markets clear. The derivation of equilibrium consumption allocations and the terms of trade in this economy is standard. It is usually convenient to consider the planner's problem. The planner's utility is

$$U(C^0, C^1, C^2; \lambda_1, \lambda_2) = E \left[ \int_0^T u(C^0(t), C^1(t), C^2(t); \lambda_1, \lambda_2) dt \right],$$

with

$$u(C^0, C^1, C^2; \lambda_1, \lambda_2) = \max_{\sum_{i=0}^2 C_i^j = C^j, \text{ all } j} u_0(C_0^0, C_0^1, C_0^2) + \lambda_1 u_1(C_1^0, C_1^1, C_1^2) + \lambda_2 u_2(C_2^0, C_2^1, C_2^2),$$

where  $\lambda_i > 0$ ,  $i = 1, 2$ , are (constant) weights on consumers 1, and 2, respectively, reflecting the value of their endowments. The planner is endowed with the aggregate supply of all assets and consumes the aggregate output, i.e.,  $C^j(t) = Y^j(t)$ , all  $j$ .

In frictionless pure-exchange models, the problem of solving for optimal allocations in this dynamic economy reduces to a static maximization problem (see e.g., Backus, Kehoe, and Kydland (1994)). The sharing rules for aggregate endowment are given by

$$\begin{pmatrix} C_0^0(t) \\ C_1^0(t) \\ C_2^0(t) \end{pmatrix} = \frac{Y^0(t)}{\alpha_0 + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \frac{1-\alpha_2(t)}{2}} \begin{pmatrix} \alpha_0 \\ \lambda_1 \frac{1-\alpha_1(t)}{2} \\ \lambda_2 \frac{1-\alpha_2(t)}{2} \end{pmatrix}, \quad (5)$$

$$\begin{pmatrix} C_0^1(t) \\ C_1^1(t) \\ C_2^1(t) \end{pmatrix} = \frac{Y^1(t)}{\frac{1-\alpha_0}{2} + \lambda_1 \alpha_1(t) + \lambda_2 \frac{1-\alpha_2(t)}{2}} \begin{pmatrix} \frac{1-\alpha_0}{2} \\ \lambda_1 \alpha_1(t) \\ \lambda_2 \frac{1-\alpha_2(t)}{2} \end{pmatrix}, \quad (6)$$

$$\begin{pmatrix} C_0^2(t) \\ C_1^2(t) \\ C_2^2(t) \end{pmatrix} = \frac{Y^2(t)}{\frac{1-\alpha_0}{2} + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \alpha_2(t)} \begin{pmatrix} \frac{1-\alpha_0}{2} \\ \lambda_1 \frac{1-\alpha_1(t)}{2} \\ \lambda_2 \alpha_2(t) \end{pmatrix}. \quad (7)$$

---

<sup>19</sup>This result may appear surprising because the investors in our model are heterogenous and, in particular, have a home bias in consumption. However, it follows from Lemma 1 that their total consumption expenditures constitute the same fraction of wealth. Thus the investors trade assets to achieve the maximal possible consumption expenditure (which requires the same portfolios), and then allocate this expenditure among goods through importing/exporting.

These consumption allocations are similar to familiar sharing rules arising in equilibrium models with logarithmic preferences. In the benchmark economy with perfect risk sharing, the correlation between consumption of a particular good and its aggregate output would have been perfect if not for the demand shifts.

Since consuming the aggregate output must be optimal for the representative agent, the terms of trade are given by the pertinent marginal rates of substitution processes

$$q^1(t) = \frac{u_{C^1}(Y^0(t), Y^1(t), Y^2(t); \lambda_1, \lambda_2)}{u_{C^0}(Y^0(t), Y^1(t), Y^2(t); \lambda_1, \lambda_2)} = \frac{\frac{1-\alpha_0}{2} + \lambda_1 \alpha_1(t) + \lambda_2 \frac{1-\alpha_2(t)}{2}}{\alpha_0 + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \frac{1-\alpha_2(t)}{2}} \left( \frac{Y^0(t)}{Y^1(t)} \right), \quad (8)$$

$$q^2(t) = \frac{u_{C^2}(Y^0(t), Y^1(t), Y^2(t); \lambda_1, \lambda_2)}{u_{C^0}(Y^0(t), Y^1(t), Y^2(t); \lambda_1, \lambda_2)} = \frac{\frac{1-\alpha_0}{2} + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \alpha_2(t)}{\alpha_0 + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \frac{1-\alpha_2(t)}{2}} \left( \frac{Y^0(t)}{Y^2(t)} \right). \quad (9)$$

Since in our model the terms of trade would play a central role in linking together the countries' stock markets, we structure our benchmark economy so as to be able to capture some of their most important properties highlighted in international economics. First, the terms of trade of the Periphery countries with the Center decrease in their domestic output and increase in the Center's output. This is a standard feature of Ricardian models of international trade: terms of trade move against countries experiencing an increase in productivity or output as their goods become relatively less scarce.<sup>20</sup> Second, we attempt to capture the "dependent economy" effects highlighted in open economy macroeconomics: the terms of trade improve for a country,  $i$ , that has experienced a positive demand shift (an increase in  $\alpha_i$ ). The intuition for this result is that a higher demand for domestic goods increases the price of domestic relative to foreign goods, improving the terms of trade.

The key to the tractability of our model is that the stock prices can be computed in closed form. We report the resulting expressions in the following lemma.

**Lemma 2.** *The prices of the stocks of the Center and the Periphery countries are given by*

$$S^0(t) = \frac{1}{\beta + \frac{1-\beta}{2}q^1(t) + \frac{1-\beta}{2}q^2(t)} Y^0(t)(T-t), \quad (10)$$

$$S^1(t) = \frac{q^1(t)}{\beta + \frac{1-\beta}{2}q^1(t) + \frac{1-\beta}{2}q^2(t)} Y^1(t)(T-t), \quad (11)$$

$$S^2(t) = \frac{q^2(t)}{\beta + \frac{1-\beta}{2}q^1(t) + \frac{1-\beta}{2}q^2(t)} Y^2(t)(T-t). \quad (12)$$

---

<sup>20</sup>This result is independent of the wealth distribution and the consumption shares.

Equations (5)–(12) summarize the prices and allocations which would prevail in the competitive equilibrium in our economy. The expressions for all of these quantities are explicit but involve (endogenous) weights  $\lambda_1$  and  $\lambda_2$ . It turns out that in our model these weights can also be computed in closed form; we report them in Appendix A.

At this point it is important to note the expressions for neither the prices nor allocations feature the wealth distribution in the economy as a state variable. This is because wealth distribution is constant, determined by the weights in the planner’s problem:

$$\frac{W_1(t)}{W_0(t)} = \lambda_1 \quad \text{and} \quad \frac{W_2(t)}{W_0(t)} = \lambda_2. \quad (13)$$

The equalities in (13) follow from, for example, (5) combined with Lemma 1. This is a convenient feature of our benchmark equilibrium, allowing us to easily disentangle the effects of the time-varying wealth distribution in the economy with portfolio constraints, presented in the next section.

To facilitate the comparison with the economy with portfolio constraints, we need the following proposition.

**Proposition 1.** *(i) The joint dynamics of the terms of trade and the three stock markets in the benchmark unconstrained economy are given by*

$$\begin{bmatrix} \frac{dq^1(t)}{q^1(t)} \\ \frac{dq^2(t)}{q^2(t)} \\ \frac{dS^0(t)}{S^0(t)} \\ \frac{dS^1(t)}{S^1(t)} \\ \frac{dS^2(t)}{S^2(t)} \end{bmatrix} = I(t)dt + \underbrace{\begin{bmatrix} a(t) & b(t) & 1 & -1 & 0 \\ \tilde{a}(t) & \tilde{b}(t) & 1 & 0 & -1 \\ -X_{\alpha_1}(t) & -X_{\alpha_2}(t) & \beta M(t) & \frac{1-\beta}{2}M(t)q^1(t) & \frac{1-\beta}{2}M(t)q^2(t) \\ a(t) - X_{\alpha_1}(t) & b(t) - X_{\alpha_2}(t) & \beta M(t) & \frac{1-\beta}{2}M(t)q^1(t) & \frac{1-\beta}{2}M(t)q^2(t) \\ \tilde{a}(t) - X_{\alpha_1}(t) & \tilde{b}(t) - X_{\alpha_2}(t) & \beta M(t) & \frac{1-\beta}{2}M(t)q^1(t) & \frac{1-\beta}{2}M(t)q^2(t) \end{bmatrix}}_{\equiv \Theta_u(t)} \begin{bmatrix} d\alpha_1(t) \\ d\alpha_2(t) \\ \sigma_{Y^0}(t)dw^0(t) \\ \sigma_{Y^1}(t)dw^1(t) \\ \sigma_{Y^2}(t)dw^2(t) \end{bmatrix}.$$

The drift term  $I$  and quantities  $X_{\alpha_1}$ ,  $X_{\alpha_2}$ ,  $M$ ,  $a$ ,  $\tilde{a}$ ,  $b$ , and  $\tilde{b}$  are defined in Appendix A.

Proposition 1 decomposes stock and commodity markets returns into responses to five underlying factors: demand shifts in Periphery countries 1 and 2 and output (supply) shocks in all three countries. These responses are captured in matrix  $\Theta_u$ , henceforth referred to as the *unconstrained dynamics*. The exact form of the elements of  $\Theta_u$  need not concern us at this point—we are primarily interested in their signs.

Understanding the responses of the terms of trade to the shocks is key to understanding the transmission of the shocks to the remaining quantities. The directions of transmission of supply

Variable/ Effects of	$d\alpha_1(t)$	$d\alpha_2(t)$	$dw^0(t)$	$dw^1(t)$	$dw^2(t)$
$\frac{dq^1(t)}{q^1(t)}$	+	$-\mathbb{C}_1$	+	-	0
$\frac{dq^2(t)}{q^2(t)}$	$-\mathbb{C}_1$	+	+	0	-
$\frac{dS^0(t)}{S^0(t)}$	$-\mathbb{C}_2$	$-\mathbb{C}_2$	+	+	+
$\frac{dS^1(t)}{S^1(t)}$	$+\mathbb{C}_1$	$-\mathbb{C}_2$	+	+	+
$\frac{dS^2(t)}{S^2(t)}$	$-\mathbb{C}_2$	$+\mathbb{C}_1$	+	+	+

Table 2: Terms of trade and stock returns in the benchmark unconstrained economy. Where a sign is ambiguous, we specify a sufficient or a necessary and sufficient condition for the sign to obtain:  $\mathbb{C}_1$  stands for the “small country” condition  $\mathbb{C}1$ , and  $\mathbb{C}_2$  stands for the “similar country” condition  $\mathbb{C}2$ .

shocks are unambiguous and easy to sign. On the other hand, those of the demand shifts depend on the relative sizes of the countries involved. Our leading interpretation of the economy involves a large Center country (a developed economy) and two small and relatively similar Periphery countries (emerging markets). Such interpretation allows us to get sharper predictions for the signs of the responses of the terms of trade and therefore the stock market prices to the demand shocks. It justifies considering the following conditions:<sup>21</sup>

**Condition  $\mathbb{C}1$ .** *The Periphery countries are small relative to the Center.*

$$\lambda_2 < \frac{3\alpha_0 - 1}{3\alpha_2(t) - 1}$$

$$\lambda_1 < \frac{3\alpha_0 - 1}{3\alpha_1(t) - 1}$$

**Condition  $\mathbb{C}2$ .** *The Periphery countries are similar.*

$$\frac{3\alpha_0 - 1}{3\alpha_0 + 1} < \frac{Y^2(t)}{Y^1(t)} < \frac{3\alpha_0 + 1}{3\alpha_0 - 1}$$

Let us now discuss the details of the transmission mechanisms in our model and relate them to the literature. Table 2 summarizes the patterns of responses of the terms of trade and stock prices to the underlying shocks.<sup>22</sup> One immediate implication of Table 2 is that supply shocks create

<sup>21</sup>In the sequel, we always specify whether a sign is unambiguous or occurs under a specific condition. Condition  $\mathbb{C}1$  is necessary and sufficient and Condition  $\mathbb{C}2$  is sufficient. Condition  $\mathbb{C}2$  is imposed on exogenous quantities, while Condition  $\mathbb{C}1$  involves the distribution of wealth, endogenously determined within the model. Further discussion of these conditions is presented in Appendix A. The conditions affect none of the derivations, they are used only for presenting the directions of responses of the stocks and the terms of trade to the underlying shocks.

<sup>22</sup>In our specification, demand and supply shocks are correlated. We nonetheless find it useful to report their effects separately in Table 2. We do so because the implications of the supply shocks for the stock market comovement are of the opposite nature as those of the demand shocks, and disentangling the effects of the two types of shocks is useful for understanding the mechanism behind international propagation.

comovement among stock market prices worldwide. The comovement is generated by two channels of international transmission: the terms of trade and the common worldwide discount factors for cash flows (common state prices). To illustrate the workings of the former channel, consider a positive supply shock in country  $j$ . Such a shock has a direct (positive) effect on country  $j$ 's stock market. Additionally, it has an indirect (also positive) effect on the remaining stock markets through the terms of trade. Indeed, as discussed earlier, a supply shock in country  $j$  creates an excess supply of good  $j$ , and hence causes a drop in its price relative to the rest of the goods. This implies that the prices of all the other goods increase relative to that of good  $j$ , boosting the value of the dividends and therefore the stock markets in the rest of the world. This explanation of the transmission of shocks across countries appears to be solely based on goods markets clearing, where the terms of trade act as a propagation channel. This channel, however, is not unrelated to the second transmission vehicle: the well-functioning financial markets creating the common discount factor for all financial assets. Indeed, in our model, clearing in good markets implies clearing in stock and bond markets as well, and hence the above intuition could be restated in terms of equilibrium responses of the stock market prices. Such intuition for “financial contagion” was highlighted by Kyle and Xiong (2001), who see contagion as a wealth effect (see also Cochrane, Longstaff, and Santa-Clara (2008)). An output shock in one of the countries always increases its stock market price and hence each agent’s wealth (because all agents have positive positions in each stock market). At a partial equilibrium level, a wealth increase triggers portfolio rebalancing. In particular, it is easy to show that, for diversification reasons, our agents demand more of all stocks. At an equilibrium level, of course, no rebalancing takes place because the agents have identical portfolios and they must jointly hold the entire supply of each market. Therefore, prices of all stocks move upwards to counteract the incentive to rebalance. So, the two transmission channels—the terms of trade and the common discount factor—interact and may potentially be substitutes for each other. Note that none of these arguments makes any assumption about the correlation of output shocks across countries—in fact, in our model they are independent. The existing literature would identify the phenomenon we described here as “contagion” (the comovement in stock markets in excess of the comovement in fundamentals). In our personal views, this comovement is not contagion—we view it as nothing else but a simple consequence of market clearing and hence a natural propagation that is to be expected in any international general equilibrium model. Our definition of contagion is the comovement in excess of the natural propagation described above.

While supply shocks induce comovement among the countries’ stock markets, demand shocks

potentially introduce divergence. Consider, for example, a positive demand shift occurring in country 1. Country 1 now demands more of the domestically-produced good and less of the foreign goods, which unambiguously increases the price of the domestic good. The direction of the response of the other Periphery country's terms of trade depends on its wealth relative to the Center,  $\lambda_2$ . If the country is small (Condition C1), it suffers disproportionately more due to a drop in demand for its good, and its terms of trade with the Center deteriorate. The impact on the stock markets, however, requires a more detailed discussion. We can represent the stock market prices of the countries in the following form:  $S^0(t) = p^0(t)Y^0(t)(T - t)$ ,  $S^1(t) = q^1(t)p^0(t)Y^1(t)(T - t)$ , and  $S^2(t) = q^2(t)p^0(t)Y^2(t)(T - t)$ . A demand shift in country 1 improves its relative price  $q^1$  and deteriorates the other Periphery countries relative price  $q^2$ , pushing  $S^1$  up and  $S^2$  down—this is the direct effect. However, there is also an indirect effect due to a fall in the price level in the Center country. The conditions of similar and small Periphery countries ensure that the impact of these demand shocks on the Center price  $p^0$  are small, forcing the terms of trade effect to dominate. However small, there is a drop in the price of the Center's good  $p^0$ , and hence the stock price of the Center falls.

### 3. Equilibrium in the Economy with Portfolio Constraints

Having established a reference point by examining a frictionless economy, we are now ready to explore the role of portfolio constraints in propagation and amplification of shocks. We stress the importance of addressing this question within a general equilibrium framework, which highlights the critical role of wealth redistribution in the transmission mechanism.

#### 3.1. The Solution Method

We now briefly sketch our solution method, both to provide a roadmap for our formal analysis and to highlight its generality. The benchmark unconstrained model is solved by first considering the planner's problem, with weights  $\lambda_i$ ,  $i = 1, 2$ , and deriving consumption sharing rules as functions of these weights (5)–(7). The terms of trade prevailing in the unconstrained equilibrium are then identified from the consumers' marginal rates of substitution (8)–(9), and the weights in the planner's problem from their budget constraints. This solution method is standard. The main new element that we bring over much of the international economics literature is our ability to solve in closed form for the countries' portfolios and for stock prices (see also Pavlova and Rigobon (2007)).

This has been possible because of three modeling features: (i) log-linear preferences over multiple goods, (ii) endowments given by shares in Lucas trees, and (iii) demand shocks.

Solving for equilibrium in the economy with portfolio constraints is a more formidable task because we are dealing with a general equilibrium model with investor heterogeneity, market frictions, and multiple risky assets. But again, the three key elements (i)–(iii) highlighted above render the model tractable. To characterize equilibrium with portfolio constraints, we first solve for the investors’ portfolios at a partial equilibrium level (Section 2.2). The resulting portfolios of constrained investors incorporate the multipliers on the portfolio constraints, which we can pin down when we specialize to a particular constraint. Then, building on recent literature in asset pricing theory, we again consider a “planner’s problem,” but with the planner’s utility now featuring *stochastic* weights  $\lambda_i$ ,  $i = 1, 2$ . These weights are again endogenous. We then proceed as in the unconstrained benchmark and derive the consumption sharing rules and the terms of trade, as functions of exogenous variables and endogenous weights  $\lambda_i$ . We also show that stock prices are given by the same expressions as in the unconstrained benchmark (Lemma 2), but with the weights  $\lambda_i$  now stochastic. We can thus isolate the effects of the portfolio constraints on the stock prices by looking at the incremental contribution brought about by a change in  $\lambda$ ’s. The economics behind a (positive) change in  $\lambda_i$  is that it is a wealth transfer from the Center to the Periphery country  $i$ , since  $\lambda_i(t) = W_i(t)/W_0(t)$ . Finally, we use our characterization of optimal policies and prices as a function of  $\lambda$ ’s to pin down the processes  $\lambda_1$  and  $\lambda_2$ . This is done in Section 5, for two particular portfolio constraints.

This solution technique applies more generally than just to the economic phenomena we highlight in this paper (e.g., excess comovement). In particular, it does not depend on the number of countries, the specific form of the portfolio constraints, the probability distribution of output, or market completeness. We comment on possible generalizations of our model throughout the paper, and especially in Section 6.

### 3.2. The Common Factor due to Constraints

In the economy with financial markets imperfections the equilibrium allocation would not be Pareto optimal, and hence the usual construction of a representative agent’s utility as a weighted sum (with constant weights) of individual utility functions is not possible. Instead, we are going to employ a representative agent with stochastic weights (introduced in an important contribution by Cuoco

and He (1994)), with these stochastic weights capturing the effects of market frictions.<sup>23</sup> This representative agent has utility function

$$U(C^0, C^1, C^2; \lambda_1, \lambda_2) = E \left[ \int_0^T u(C^0(t), C^1(t), C^2(t); \lambda_1(t), \lambda_2(t)) dt \right],$$

with

$$u(C^0, C^1, C^2; \lambda_1, \lambda_2) = \max_{\sum_{i=0}^2 C_i^j = C^j, \text{ all } j} u_0(C_0^0, C_0^1, C_0^2) + \lambda_1 u_1(C_1^0, C_1^1, C_1^2) + \lambda_2 u_2(C_2^0, C_2^1, C_2^2),$$

where  $\lambda_i > 0$ ,  $i = 1, 2$  are (yet to be determined) weighting processes, which may be stochastic. The advantage of employing this approach is that a bulk of the analysis of the previous section can be directly imported to this section. In particular, the only required modification to equations (5)–(9) is that the constant weights  $\lambda_1$  and  $\lambda_2$  are now replaced by their stochastic counterparts. The expressions for stock market prices (10)–(12) also continue to hold in the constrained economy (see the proof of Lemma 2 in Appendix A). Furthermore, as a consequence of the consumption sharing rules and Lemma 1, we again conclude that  $\lambda_1(t) = W_1(t)/W_0(t)$  and  $\lambda_2(t) = W_2(t)/W_0(t)$ . So in the constrained economy the wealth distribution, captured by the quantities  $\lambda_1$  and  $\lambda_2$ , becomes a new state variable. Finally, in the constrained economy, we also have an analog of Proposition 1, except now the weighting processes  $\lambda_1$  and  $\lambda_2$  enter as additional factors. These factors capture the effects of the portfolio constraint imposed on the Center’s consumer.

**Proposition 2.** (i) *In an equilibrium with the portfolio constraint, the weighting processes  $\lambda_1$  and  $\lambda_2$  are the same up to a multiplicative constant.*

(ii) *When such equilibrium exists, the joint dynamics of the terms of trade and three stock markets in the economy with the portfolio constraint are given by*

$$\begin{bmatrix} \frac{dq^1(t)}{q^1(t)} \\ \frac{dq^2(t)}{q^2(t)} \\ \frac{dS^0(t)}{S^0(t)} \\ \frac{dS^1(t)}{S^1(t)} \\ \frac{dS^2(t)}{S^2(t)} \end{bmatrix} = I_c(t)dt + \begin{bmatrix} A(t) \\ \tilde{A}(t) \\ -X_\lambda(t) \\ A(t) - X_\lambda(t) \\ \tilde{A}(t) - X_\lambda(t) \end{bmatrix} \Theta_u(t) \begin{bmatrix} \frac{d\lambda(t)}{\lambda(t)} \\ d\alpha_1(t) \\ d\alpha_2(t) \\ \sigma_{Y^0}(t)dw^0(t) \\ \sigma_{Y^1}(t)dw^1(t) \\ \sigma_{Y^2}(t)dw^2(t) \end{bmatrix}$$

where  $\lambda(t) \equiv \lambda_1(t)$ ,  $I_c$  and  $X_\lambda$  are reported in Appendix A, and where the unconstrained dynamics

<sup>23</sup>The construction of a representative agent with stochastic weights has been employed extensively in dynamic asset pricing. See, for example, Basak and Croitoru (2000), Basak and Cuoco (1998), and Shapiro (2002). A related approach is the extra-state-variable methodology of Kehoe and Perri (2002). For the original solution method utilizing weights in the representative agent, see Negishi (1960).

matrix  $\Theta_u(t)$  is as defined in Proposition 1.<sup>24</sup>

Proposition 2 reveals that the same transmission channels underlying the benchmark economy are present in the economy with portfolio constraints. Ceteris paribus, the sensitivities of the terms of trade and stock prices to the demand and supply shocks are exactly the same as in Proposition 1. The only difference from the benchmark economy comes in the first,  $d\lambda/\lambda$ , term. This term summarizes the dynamics of the two stochastic weighting processes  $\lambda_1$  and  $\lambda_2$ , which end up being proportional in equilibrium, and hence represent a single common factor we labeled  $\lambda$ .<sup>25</sup> Thus, the process  $\lambda$  should be viewed as an additional factor in stock prices and the terms of trade dynamics, arising as a consequence of the portfolio constraints.

One can already note the cross-market effect of portfolio constraints: the constraint affects not only the Center's stock market, but also Periphery stocks, as well as the terms of trade. This finding is, of course, to be expected in a general equilibrium model. The effects of constraints in financial markets get transmitted to all other (stock, bond, and commodity) markets via pertinent market clearing equations. Our contribution is to fully characterize these spillover effects and identify their direction. The signs of responses to the supply and demand shocks are, of course, the same as in the benchmark unconstrained equilibrium. Additionally, we can sign the responses of all markets to innovations in the new factor; some signs are unambiguous, and some obtain under the following condition:

**Condition C3.** *The effect of the portfolio constraint on  $p^0$  is small.*<sup>26</sup>

$$\frac{1-\beta}{2} q^2(t)(\tilde{A}(t) - A(t)) < \beta A(t), \quad (14)$$

$$\frac{1-\beta}{2} q^1(t)(A(t) - \tilde{A}(t)) < \beta \tilde{A}(t). \quad (15)$$

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<sup>24</sup>Existence of equilibrium can be shown for the case in which the portfolio constraint does not bind (the unconstrained benchmark) and for the case of specific constraints considered in our examples in Section 5, but would be very difficult to show for the general specification of the constraint considered in this section. Still, we feel that our analysis in this section is important, as it characterizes properties of equilibrium that obtain for *any* constraint imposed on the Center country.

<sup>25</sup>This finding depends on the fact that the two Periphery countries face the same investment opportunity set: here, they are both unconstrained. If these two countries faced heterogeneous constraints, in general, one would not expect their weighting processes to be proportional, and hence both  $\lambda_1$  and  $\lambda_2$  would enter as relevant factors. Furthermore, in our specification the new factor  $\lambda$  is not independent from the existing factors—for example, an innovation to any underlying Brownian motion affects the distribution of wealth and hence  $\lambda$ . However, for the purposes of separating the incremental effect of the portfolio constraint relative to the dynamics occurring in the unconstrained benchmark we find it useful to treat  $\lambda$  as an additional factor.

<sup>26</sup>The condition is necessary and sufficient. It is likely to be satisfied under the leading interpretation of the Center country being big. In Appendix A we investigate this condition further, representing it as a combination of two effects: (i) the impact of a change in  $\lambda$  (the implied wealth transfer) on the demand for good 0 and (ii) the cross-country demand reallocation in the Periphery countries. The condition affects none of the derivations, it is used only for presenting the directions of responses of the Periphery countries' stocks to innovations in  $\lambda$ .

Variable/ Effects of	$\frac{d\lambda(t)}{\lambda(t)}$	$d\alpha_1(t)$	$d\alpha_2(t)$	$dw^0(t)$	$dw^1(t)$	$dw^2(t)$
$\frac{dq^1(t)}{q^1(t)}$	+	+	$-\mathbb{C}_1$	+	-	0
$\frac{dq^2(t)}{q^2(t)}$	+	$-\mathbb{C}_1$	+	+	0	-
$\frac{dS^0(t)}{S^0(t)}$	-	$-\mathbb{C}_2$	$-\mathbb{C}_2$	+	+	+
$\frac{dS^1(t)}{S^1(t)}$	$+\mathbb{C}_3$	$+\mathbb{C}_1$	$-\mathbb{C}_2$	+	+	+
$\frac{dS^2(t)}{S^2(t)}$	$+\mathbb{C}_3$	$-\mathbb{C}_2$	$+\mathbb{C}_1$	+	+	+

Table 3: Terms of trade and stock returns in the economy with portfolio constraints. Where a sign is ambiguous, we specify a sufficient or a necessary and sufficient condition for the sign to obtain:  $\mathbb{C}_1$  stands for the “small country” condition  $\mathbb{C}_1$ ,  $\mathbb{C}_2$  for the “similar country” condition  $\mathbb{C}_2$ , and  $\mathbb{C}_3$  for the “small effect on  $p^0$ ” condition  $\mathbb{C}_3$ .

Table 3 reveals the contribution of portfolio constraints to international comovement. The first striking implication is that the terms of trade faced by both Periphery countries move in the *same direction* in response to an innovation in the  $\lambda$  factor. A movement in  $\lambda$  should be viewed in our model as a tightening or a loosening of the portfolio constraint. Given the definition of  $\lambda$ , such innovation reflects a wealth redistribution in the world economy *to* or *away* from the Periphery countries. Parallels may be drawn to the literature studying the effects of wealth transfers on the terms of trade. It is well-known from the classic “Transfer Problem” of the international economics literature that an income (wealth) transfer from one country to another improves the terms of trade of the recipient. As wealth of the recipient of the transfer goes up, his total demand increases, but because of the preference bias for his own good, the demand for the domestic good increases disproportionately more. Hence the price of the home good rises relative to those of the foreign goods, improving the terms of trade of the recipient. This effect is also known as the Keynes effect.<sup>27</sup> In our model, a decrease in the factor  $\lambda$  is interpreted as a wealth transfer to the Center country. Just like in the Transfer Problem, it results in an improvement of its terms of trade against the world and hence a deterioration of the terms of trade of both Periphery countries—the reverse for an increase in  $\lambda$ . The main difference between our work and the Transfer Problem literature is that

<sup>27</sup>The original “Transfer Problem” was the outcome of a debate between Bertil Ohlin and John Maynard Keynes regarding the true value of the burden of reparations payments demanded of Germany after World War I. Keynes argued that the payments would result in a reduction of the demand for German goods and cause a deterioration of the German terms of trade, making the burden on Germany much higher than the actual value of the payments. On the other hand, Ohlin’s view was that the shift in demand would have no impact on relative prices. This implication would be correct if all countries have the exact same demands (in our model this requires an assumption that  $\alpha_i = 1/3$ ,  $i = 0, 1, 2$ .) See Krugman and Obstfeld (2003) for an elaboration and references.

the latter considers exogenous wealth transfers, while wealth transfers are generated endogenously in our model as a result of a tightening of the portfolio constraint. The direction of such a transfer (to or from the Periphery countries) in response to a tightening or a loosening of the constraint depends on the form of a constraint. We determine whether a particular constraint tightens or loosens in response to a shock in Section 5, in which we consider two specific examples of portfolio constraints.

The intuition behind the occurrence of the wealth transfers in our model is simple. Assume for a moment that there is no constraint. Then each country holds the same portfolio. When a (binding) constraint is imposed on the investors in the Center, their portfolio has to deviate from the benchmark, and now the portfolios of the Center and Periphery investors differ. This means that stock market price movements will have differential effects on the investors' wealth. The movements of wealth obviously depend on the type of the constraint. For *any* constraint that binds, however, one can say that the distribution of wealth will fluctuate, becoming the additional transmission vehicle. Moreover, since the Periphery countries hold identical portfolios, their wealth shares move in tandem. That is, the resolution of uncertainty always affects the Periphery countries in the same way: they either both become poorer or both become richer relative to the Center.

The portfolio constraint also generally induces the comovement between the stock markets of the Periphery countries. This comovement may be partially confounded by the Center good price effect, which is of the same nature as the one encountered in the case of the demand shifts in the benchmark model (see Section 2). Consider, for example, a response to a positive shock in  $\lambda$ . While the improving terms of trade effect boosts the Periphery stock markets, the associated downward move in  $p^0$  may potentially offset this. However, under our Condition C3, the terms of trade effect dominates. If we were to quote stock market prices of the Periphery in terms of the production basket of Center, rather than the world consumption basket, the two Periphery markets would always comove in response to a tightening or a loosening of the portfolio constraint. On the other hand, the response of the stock market of the Center is unambiguous and goes in opposite direction of  $\lambda$ , reflecting the effects of an implicit wealth transfer to or from the Center. So, in summary, the implicit wealth transfers due to the portfolio constraint create an additional comovement among the terms of trade of the Periphery countries, as well as their stock market prices, while reducing the comovement between the Center and the Periphery stock markets. This implication resembles the phenomenon known as “contagion”—the comovement in excess of that

occurring in the unconstrained economy.

## 4. Contagion without Trade

In the previous section we have considered a model in which each Periphery country allocates equal expenditure shares to the two goods it imports. This may appear unrealistic in the context of our leading interpretation, where the Center country represents a large developed economy and the Periphery countries two emerging markets, because emerging economies trade with industrialized economies much more than amongst themselves. Moreover, recent empirical studies of emerging markets have cast doubt on the ability of trade relationships to generate international comovement of observed magnitudes and have documented that contagion exists even among countries with insignificant trade relationships. Since the movements in the terms of trade is an essential ingredient of the contagion mechanism in our model, it is natural to ask whether our results still hold under alternative assumptions regarding the extent of trade (in goods) between the Periphery countries. In this section we take our setting to the limit and show that even when Periphery countries do not trade with each other at all, their stock markets comove as described in the baseline analysis.

To examine this scenario, we modify the countries' preferences as follows:

$$\begin{aligned} u_0(C_0^0(t), C_0^1(t), C_0^2(t)) &= \log C_0^0(t), \\ u_1(C_1^0(t), C_1^1(t), C_1^2(t)) &= (1 - \alpha_1(t)) \log C_1^0(t) + \alpha_1(t) \log C_1^1(t), \\ u_2(C_2^0(t), C_2^1(t), C_2^2(t)) &= (1 - \alpha_2(t)) \log C_2^0(t) + \alpha_2(t) \log C_2^2(t). \end{aligned}$$

That is, we assume that the goods produced by the Periphery countries are nontraded, and the only trade occurring in the model is that between each Periphery country and the Center. We continue to assume that there is a home bias in consumption by restricting  $\alpha_i$  to be a martingale lying between 1/2 and 1. As before, the Center country's portfolios are constrained to lie in a closed, convex, non-empty subset  $\{K_t(\omega); (t, \omega) \in [0, T] \times \Omega\}$ .

Under this specification, the terms of the trade of each Periphery country with the Center are

$$q^j(t) = \frac{\alpha_j(t)\lambda_j(t)}{1 + \lambda_1(t)(1 - \alpha_1(t)) + \lambda_2(t)(1 - \alpha_2(t))} \left( \frac{Y^0(t)}{Y^j(t)} \right), \quad j \in \{1, 2\}, \quad (16)$$

where the relative weights  $\lambda_1$  and  $\lambda_2$  are possibly stochastic. It is straightforward to show that the expressions for the stock prices remain the same, given by (10)–(12).

Variable/ Effects of	$\frac{d\lambda(t)}{\lambda(t)}$	$d\alpha_1(t)$	$d\alpha_2(t)$	$dw^0(t)$	$dw^1(t)$	$dw^2(t)$
$\frac{dq^1(t)}{q^1(t)}$	+	+	+	+	-	0
$\frac{dq^2(t)}{q^2(t)}$	+	+	+	+	0	-
$\frac{dS^0(t)}{S^0(t)}$	-	-	-	+	+	+
$\frac{dS^1(t)}{S^1(t)}$	+	+	A	+	+	+
$\frac{dS^2(t)}{S^2(t)}$	+	A	+	+	+	+

Table 4: Terms of trade and stock returns in the economy with portfolio constraints and no trade between the Periphery countries. “A” stands for “ambiguous.”

In the interest of space, we do not provide the dynamics of the terms of trade and stock prices in this economy; we just present a table (Table 4) that mimics Table 3 of Section 3. In contrast to Table 3, only two signs in Table 4 are ambiguous (related, again, to demand shifts); the remaining implications do not require any further conditions. The effects of the demand shocks on the terms of trade are now clear-cut because a demand shift in a Periphery country 1 not only increases the world demand for good 1 relative to all other goods (as before), but also decreases the demand for good 0, while leaving the demand for good 2 unchanged. Therefore, the price of good 0 drops relative to that of both goods 1 and 2. Another set of signs that becomes unambiguous is that for the effects of the innovation in the wealth shares of the Periphery countries captured by  $\lambda$  on the stock prices in the Periphery.

Within this economy it is easy to derive the real exchange rates faced by the Periphery countries.

**Remark 1 (Real Exchange Rates).** The price indexes in each country, derived from the countries’ preferences, are given by

$$P^0(t) = p^0(t), \quad P^1(t) = \left( \frac{p^0(t)}{1 - \alpha_1(t)} \right)^{1 - \alpha_1(t)} \left( \frac{p^1(t)}{\alpha_1(t)} \right)^{\alpha_1(t)}, \quad P^2(t) = \left( \frac{p^0(t)}{1 - \alpha_2(t)} \right)^{1 - \alpha_2(t)} \left( \frac{p^1(t)}{\alpha_2(t)} \right)^{\alpha_2(t)}.$$

The real exchange rates, expressed as functions of the terms of trade, are then

$$e^j(t) = \frac{P^j(t)}{P^0(t)} = (1 - \alpha_j(t))^{\alpha_j(t) - 1} \alpha_j(t)^{-\alpha_j(t)} \left( q^j(t) \right)^{\alpha_j(t)}, \quad j \in \{1, 2\}.$$

Our primary concern is the incremental effect of a change in  $\lambda$  on the real exchange rates, as in the first column of Table 4. Since the utility weights  $\alpha_j$  are positive, the real exchange rates respond to a change in  $\lambda$  in the same direction the terms of trade do. This implies that the excess comovement in the terms of trade due to the portfolio constraint translates into the excess comovement of the real exchange rates of the Periphery countries.

## 5. Examples of Portfolio Constraints

The purpose of this section is to illustrate the applicability of our general framework to studying specific portfolio constraints. Under a specific constraint, we can fully characterize the countries' portfolios and hence identify the direction of the constraint-necessitated wealth transfers. This will allow us to address questions of the following nature: "Does a positive shock in the Center entail a wealth transfer to the Center?", "How does the origin of a shock affect stock returns worldwide?", "Does the constraint amplify the shocks?"

### 5.1. Pure Wealth Transfers: A Portfolio Concentration Constraint

Here, we return to our workhorse model presented in Section 2 and specialize the constraint set  $K$  to represent a portfolio concentration constraint. That is, the resident of the Center country now faces a constraint permitting him to invest no more than a certain fraction of his wealth  $\gamma$  into the stock markets of Periphery countries 1 and 2:

$$x_0^{S^1}(t) + x_0^{S^2}(t) \leq \gamma, \quad \gamma \in \mathbb{R}. \quad (17)$$

While this constraint is prevalent in practice, we do not intend to argue that such a constraint is necessarily behind the patterns of correlations observed in reality. Our goal is to merely illustrate the workings of our model. We feel that (17) is particularly well-suited for this purpose since its impact on the portfolio composition and hence the entailed wealth transfers are easy to understand.<sup>28</sup>

For the concentration constraint, we can fully characterize the process  $\lambda$  and hence the remaining equilibrium quantities. Note that the consumption allocations, terms and trade, and stock prices all depend on the primitives of the model and the unknown stochastic weights. Therefore, once the process  $\lambda$  and the constants  $\lambda_1(0)$  and  $\lambda_2(0)$  are determined, we would be able to pin down all of these equilibrium quantities. This step is inevitably somewhat technical. A reader interested primarily in economic mechanisms and intuitions may skip directly to Subsection 5.1.1. The set of equations, required to fully close the model in the economy with portfolio constraints, is presented in the following two propositions.

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<sup>28</sup>We concede that other constraints, especially government-imposed, may be more economically relevant, but in this section we consider only two possible constraints. Another set of restrictions absent from the model is those on the Periphery countries. We believe that the model possesses sufficient flexibility to accommodate these alternative constraints (see our discussion in Section 6), but we leave this analysis, as well as a formal calibration, for future applications.

**Proposition 3.** *When equilibrium exists, the wealth distribution  $\lambda$  follows*

$$d\lambda(t) = \lambda(t)[r(t) - r_0(t) + m(t)^\top (m_0(t) - m(t))]dt - \lambda(t)(m_0(t) - m(t))^\top dw(t), \quad (18)$$

where  $r_0$  and  $m_0$  are interpreted as the interest rate and the market price of risk faced by the Center and  $m$  is the market price of risk faced by the Periphery countries.<sup>29</sup> The quantities  $m_0$  and  $m$  are related as follows:

$$\text{When } (i_1 + i_2)^\top (\sigma(t)^\top)^{-1} m(t) \leq \gamma,$$

$$m_0(t) = m(t), \quad \psi(t) = 0, \quad (\text{Constraint not binding}), \quad (19)$$

otherwise,

$$m_0(t) = m(t) - (\sigma(t))^{-1} (i_1 + i_2) \psi(t), \quad (20)$$

$$\psi(t) = -\frac{\gamma - (i_1 + i_2)^\top (\sigma(t)^\top)^{-1} m(t)}{(i_1 + i_2)^\top (\sigma(t) \sigma(t)^\top)^{-1} (i_1 + i_2)} > 0, \quad (\text{Constraint binding}),$$

where  $i_0 \equiv (1, 0, 0)^\top$ ,  $i_1 \equiv (0, 1, 0)^\top$ ,  $i_2 \equiv (0, 0, 1)^\top$ , and the volatility matrix  $\sigma$  is provided in Appendix A. Furthermore,

$$\begin{aligned} \sigma_{Y^0}(t) i_0 &= \frac{\left( \lambda_1(t) \frac{1-\alpha_1(t)}{2} + \lambda_2(t) \frac{1-\alpha_2(t)}{2} \right) (m(t) - m_0(t)) - \frac{\lambda_1(t)}{2} \sigma_{\alpha_1}(t) - \frac{\lambda_2(t)}{2} \sigma_{\alpha_2}(t)}{\alpha_0 + \lambda_1(t) \frac{1-\alpha_1(t)}{2} + \lambda_2(t) \frac{1-\alpha_2(t)}{2}} \\ &= X_\lambda(t)(m(t) - m_0(t)) + X_{\alpha_1}(t) \sigma_{\alpha_1}(t) + X_{\alpha_2}(t) \sigma_{\alpha_2}(t) + \frac{1-\beta}{2} M(t) (q^1(t) + q^2(t)) \sigma_{Y^0}(t) i_0 \\ &\quad - \frac{1-\beta}{2} M(t) q^1(t) \sigma_{Y^1}(t) i_1 - \frac{1-\beta}{2} M(t) q^2(t) \sigma_{Y^2}(t) i_2 + m_0(t). \end{aligned} \quad (21)$$

Equations (19)–(20) are the complementary slackness conditions coming from the constrained portfolio optimization of the resident of the Center. At times when the constraint is not binding, the market price of risk faced by the Center coincides with that faced by the Periphery. Therefore, the portfolio of the Center is given by the same equation as the unconstrained portfolios. When the constraint is binding, however, there is a wedge between the market prices of risk faced by the Center and the Periphery (20). The quantity  $\psi$  is to be interpreted as the multiplier on the portfolio constraint specified in (17). Equation (21) is the direct consequence of market clearing in

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<sup>29</sup>For more discussion of the market price of risk and interest rate processes faced by the Center and the Periphery, see the beginning of Appendix A. Proving existence consists of showing existence of a solution to algebraic equations (19)–(21) given our state variables, and then showing that this solution implies existence and uniqueness of a solution to the stochastic differential equation (18). The first step requires showing the invertibility of matrix  $\sigma$ . There is a possibility that this matrix may not be invertible in our model, which happens when there are no demand shifts. However, existence for that case has been established in the previous literature (Cass and Pavlova (2004), Zapatero (1995)). To highlight and characterize the behavior of asset prices in our model, in the following subsections we compute the solution to our model for specific parameter values. In all of the examples the matrix  $\sigma$  was always invertible. The second step amounts to verifying that Lipschitz and growth conditions (see, for example, Øksendal (2003) Theorem 5.2.1) are satisfied for the drift and diffusion terms in equation (18).

the consumption goods—it is a consumption-CAPM-type equation. Together, (19)–(21) allow us to pin down the equilibrium market prices of risk of Center and Periphery, and hence the responses of all three stock markets to innovations in the underlying Brownian motions  $w^0$ ,  $w^1$ , and  $w^2$ , as functions of the state variables in the economy. Once the market prices of risk processes  $m_0$  and  $m$  are determined, it is straightforward to compute the effective interest rate differential faced by the Center country (Proposition 4), which completes our description of the dynamics of the process  $\lambda$  in (18). This, together with the countries’ portfolio holdings reported in Corollary 1, concludes the full characterization of the economy.

**Proposition 4.** *When equilibrium exists, the differential between the interest rates faced by countries 1 and 2 and that effectively faced by country 0 is given by*

$$r(t) - r_0(t) = \frac{\gamma - (i_1 + i_2)^\top (\sigma(t)^\top)^{-1} m(t)}{(i_1 + i_2)^\top (\sigma(t) \sigma(t)^\top)^{-1} (i_1 + i_2)} \gamma. \quad (22)$$

From (19)–(20) and (22), one can easily show that the interest rate differential is always nonpositive. That is, the interest rate effectively faced by the constrained country is higher than the world (unconstrained) interest rate. This accounts for the effects of the portfolio constraints. Recall from Section 2.2 that the optimization problem of the Center subject to a portfolio constraint is formally equivalent to an auxiliary problem with no constraints but the Center facing a fictitious investment opportunity set in which the bond and the Center’s stock (the unrestricted investments) are made more attractive relative to the original market, and the stocks of the Periphery countries (the restricted investments) are made relatively less attractive. In this fictitious market, the Center optimally invests more in the bond and in the Center’s stock relative to the original market, and less in the Periphery countries’ stocks.

### 5.1.1. Analysis of Equilibrium with the Portfolio Concentration Constraint

The dynamics of our model implied by Proposition 3 are best illustrated by means of plots. The parameters used in the analysis are summarized in Table 5. All time-dependent variables in Table 5 are the state variables in our model. In the interest of space, in our figures we fix all of them but the wealth shares of the Periphery countries  $\lambda_1(t)$  and  $\lambda_2(t)$ . These stochastic wealth shares are behind the additional common factor driving the stock prices and terms of trade that we identify in our model, and it is of interest to highlight the dependence of the prices and portfolios in our model on these wealth shares. Hence, the horizontal axes in all the figures measure  $\lambda_1$  and  $\lambda_2$ .

$\beta$	0.9	$\gamma$	0.35		
$\alpha_0$	0.75	$Y^0(t)$	1.0	$\sigma_{Y^0}(t)$	0.3
$\alpha_1$	0.75	$Y^1(t)$	0.3	$\sigma_{Y^1}(t)$	0.1
$\alpha_2$	0.75	$Y^2(t)$	0.3	$\sigma_{Y^2}(t)$	0.3
$\lambda_1(t) \in [0.05, 0.35]$					
$\lambda_2(t) \in [0.05, 0.35]$					
$\sigma_{\alpha_1}(t)$	(0, 0.2, 0)				
$\sigma_{\alpha_2}(t)$	(0, 0, 0.2)				

Table 5: Parameter choices

The reasoning behind the choice of our parametrization is the following. In our leading interpretation, the Periphery countries are small, so for the choice of the numeraire consumption basket we decided that they represented 5 percent of the world. We have chosen 75 percent as the share of expenditures on the domestic good, which is a conservative estimate, given the share of the service sector in GDP. In terms of output, the Periphery countries are one tenth of the Center, and three times as volatile. We assume that the wealth ratios relative to the Center for both Periphery countries may range from 0.05 to 0.35. Finally, we need to specify the parameters of the demand shocks. Recall that in our model there are only three primitive sources of uncertainty—the Brownian motions  $w^0$ ,  $w^1$ , and  $w^2$ —and so the supply and demand shocks are necessarily correlated. In Pavlova and Rigobon (2007) we find that in the data demand shocks are positively correlated with domestic supply innovations. Therefore, we assume that a demand shift in country  $j$  has a positive loading on  $w^j$  and zero loadings on the remaining Brownian motions.<sup>30</sup>

To develop initial insight into the solution we examine the region where the constraint is binding. The tightness of the constraint is measured by the multiplier  $\psi$  from equations (19) and (20). As is evident from Figure 1, for small wealth shares of the Periphery countries, the portfolio constraint is not binding, and the multiplier is zero. As their wealth shares increase, the constraint tightens: the multiplier is increasing in both  $\lambda$ 's. In the unconstrained economy, larger  $\lambda$ 's imply that Periphery countries constitute a larger fraction of world market capitalization, and hence, they command a larger share of the investors' portfolios. Therefore, given the same upper bound constraint on the investment in the Periphery countries, the larger these countries are, the tighter the constraint.

[Figure 1 about here]

Let us now concentrate on how the portfolio constraint affects portfolio decisions by the Center's

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<sup>30</sup>We have repeated the analysis using different parameterizations and have found that the main message remained unaltered.

investor. In our parametrization, the Periphery countries are symmetric, and therefore we only show figures for one of the Periphery countries. Figure 2 depicts the changes in portfolio weights relative to the unconstrained economy: the “excess” weight in the Center country’s stock is shown in panel (a), and the “excess” weight in the Periphery country 1’s stock in panel (b). For the range of  $\lambda$ ’s where the constraint is not binding, the portfolio holdings are identical to those in the unconstrained equilibrium. For the range where it becomes binding, the investor in the Center is forced to decrease his holdings of the Periphery markets. The freed-up assets get invested in the stock market of the Center country and the bond, making the Center country over-weighted in the Center’s stock market relative to its desired unconstrained position. Of course, the Periphery countries take the offsetting position so that the asset markets clear. In other words, the portfolio constraint forces a “home bias” on the Center’s and the Periphery’s investors. As we will demonstrate, this “home bias” implies that the wealth of the investor in the Center is more sensitive to shocks to the Center’s stock market, while the wealth of the Periphery investors is relatively more susceptible to shocks to the Periphery.

[Figure 2 about here]

### 5.1.2. Amplification, Flight to Quality, and Excess Comovement

The next goal is to analyze how the distribution of wealth evolves in response to shocks in the three countries. From equation (18) we have computed the diffusion term in the evolution of the wealth shares  $\lambda_1 (= \lambda)$  and  $\lambda_2$ , which appears in Figure 3. (Recall that the two wealth shares are perfectly correlated.) Panel (a) depicts the move in these wealth shares when the Center receives a negative shock, and panel (b) shows what happens to it when a negative shock originates in one of the Periphery countries. Again, because of symmetry we only consider one of the Periphery countries. The response of the wealth share of the Periphery countries clearly depends on the origin of the shock: a negative shock in the Center increases the share (a wealth transfer from the Center to the Periphery), while a negative shock in the Periphery decreases it (a wealth transfer from the Periphery to the Center). To understand this effect, consider the representation of the evolution of  $\lambda$  in terms of the countries’ portfolios:

$$\frac{d\lambda(t)}{\lambda(t)} = \text{Drift terms } dt + \left( \frac{dS^0(t)}{S^0(t)}, \frac{dS^1(t)}{S^1(t)}, \frac{dS^2(t)}{S^2(t)} \right) (x_i(t) - x_0(t)), \quad i = 1, 2, \quad (23)$$

which follows from (18) and Corollary 1. The portfolios are the same over a range where the constraint is not binding, and hence no wealth transfers take place. In the constraint-binding

range, the first component of the vector  $x_i(t) - x_0(t)$ ,  $i = 1, 2$ , is negative, while the last two are positive. This is because the investor in the Center (Periphery) is over-weighted (under-weighted) in the Center's stock market and under-weighted (over-weighted) in the Periphery stock markets. One can verify that, although country-specific shocks spread internationally inducing comovement, the effect of a shock on own stock market is bigger than on the remaining stocks (because of divergence induced by the demand shocks).

[Figure 3 about here]

A tighter constraint implies larger transfers, a looser constraint smaller transfers, and in the limit when the constraint is not binding, there are no wealth transfers taking place. Consequently, the effects of the transfers on the terms of trade and the stock prices become larger when the constraint is tighter. For brevity, we here omit a figure depicting the effects of the supply shocks on the terms of trade, which simply confirms the intuition we gathered from the Transfer Problem.

The incremental effect on the stock prices, brought about by the portfolio constraint, mimics the effects on the terms of trade. A country experiencing an improvement of its terms of trade enjoys an increase in its stock market, and that experiencing a deterioration sees its stock drop. Now we can fully address the issue of the comovement among the stock markets that the portfolio constraint induces. These results are presented in Figures 4. Panel (a) demonstrates the impact that a negative shock to the Center has on the Center's stock market, beyond the already negative effect that takes place in the unconstrained economy. In the region where the constraint is not binding, the effect is zero, but it is negative over the remainder of the state space. That is, the negative effect of a shock to the Center is *amplified* in the presence of the constraint. Furthermore, the magnitude of the effect is increasing with  $\lambda$ , which is to be expected because the higher the wealth shares of the Periphery countries are, the tighter the constraint. The exact same intuition applies to the effects of the shocks in the Periphery on domestic stock prices (panel (d)).

[Figure 4 about here]

The transmission of shocks across countries is depicted in panels (b), (c), and (e). The impact of a productivity shock in the Center on the Periphery stock prices is shown in panel (b), that of a shock in a Periphery country on the Center in panel (c) and, finally, that of a shock in one Periphery country on the other Periphery country in panel (e). Again, these are incremental effects due to the constraint, net of the comovement implied by the unconstrained model. The emerging pattern

is consistent with the flight to quality and contagion effects, observed in the data. The *flight to quality* and contagion refer to a transmission pattern where a negative shock to one of the Periphery countries (emerging markets) depresses stocks of other countries in the Periphery, but boosts the Center country’s stock market (an industrialized economy). Panels (c)–(e) demonstrate that in our model a negative shock to one of the Periphery countries reduces its stock price, decreases the stock price of the other Periphery country (contagion), and increases the stock market price in the Center (flight to quality). A similar pattern occurs if the Center receives a positive shock.

Finally, we examine the extent of excess comovement of the Periphery stock markets that our model generates. To do so, we compute the (instantaneous) variance-covariance matrix  $\sigma(t)\sigma(t)^\top$  from Proposition 3 in the constrained and unconstrained economies and evaluate the instantaneous cross-country correlations of stock market returns in the constrained economy in excess of those in the unconstrained (Figure 5). One can see that the correlations increase in magnitude as wealth shares of the Periphery countries increase and hence the constraint in the Center becomes tighter. As the theory predicts, the comovement between the Center and the Periphery decreases, while the comovement across the Periphery countries goes up. This effect is also sizable: the decrease in the correlation of the stock returns of the Center and the Periphery can be around 4–7% and the increase in the correlation between the Periphery markets returns around 10–15%.

[Figure 5 about here]

## 5.2. Varying Restrictiveness: A Market Share Constraint

The previous constraint is one of the simplest that can be studied within our framework. However, it generates some counterfactual implications. For instance, a negative shock to the Periphery relaxes the constraint, instead of tightening it.<sup>31</sup> We therefore consider a constraint of a different nature, a *market share constraint*, which becomes more restrictive when the market share of the Periphery countries in the world drops:

$$x_0^{S^1}(t) + x_0^{S^2}(t) \leq \gamma F\left(\frac{S^1(t) + S^2(t)}{S^0(t) + S^1(t) + S^2(t)}\right), \quad \gamma \in \mathbb{R}, \quad (24)$$

where  $F$  is an arbitrary increasing function. This constraint is very similar to the concentration constraint, with the only difference that the upper bound on the investment in the Periphery is specified not in absolute but in relative terms, reflecting the market capitalization of the Periphery.

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<sup>31</sup>It has been argued in the empirical literature that recent contagious crises in emerging markets may have been caused by the tightenings of constraints in developed countries in response to a crisis in one emerging market.

The characterization of the equilibrium quantities of interest in the economy with the market share constraint (24) is as before, with the only difference that each entry of  $\gamma$  in Propositions 3–4 gets replaced with the term on the right-hand side of equation (24). This is due to the fact that logarithmic preferences induce myopic behavior, and hence the investor in the Center do not hedge against changes in the restrictiveness of their portfolio constraint.

We again describe the effects of the constraint on the economy by means of plots. We have tried several linear and polynomial functions  $F$ , and they all produce very similar patterns. In fact, the qualitative implications are identical. Figure 6 depicts the multiplier on the market share constraint. One can easily see that in contrast to the case of the concentration constraint, presented in Figure 1, the multiplier is zero when the wealth shares of the Periphery countries are large. As the wealth shares of the Periphery countries in the world fall, the constraint starts to bind, becoming more and more restrictive as the wealth shares drop. The tilt in the portfolio of the Center country reflects the restrictiveness of the constraint: the highest tilt occurs when the wealth shares of the Periphery countries are small (Figure 7). The sign of the tilt in the asset allocation of the Center is the same as before: the Center is over-weighted in the Center’s stock market and under-weighted in the Periphery stock markets, relative to the unconstrained economy. Like the concentration constraint, the market share constraint restricts the investment in the Periphery, causing wealth transfers to/from the Periphery in response to a shock in the Center or the Periphery (Figure 8). However, unlike in the case of the concentration constraint, the restrictiveness of the constraint changes in response to a wealth transfer. For example, a wealth transfer from the Periphery to the Center makes the constraint more restrictive as the market share of the Periphery falls. These two effects—(i) a wealth transfer and (ii) a change in the restrictiveness of the constraint—interact in our model, producing rich variations in the pattern of capital flows. Now the Center withdraws funds from the Periphery when it receives a wealth transfer, because the constraint becomes more restrictive. Therefore, the flight to quality pattern emerging in Figure 9, where a negative shock to one of the Periphery depresses stock prices in the other Periphery country (panel (e)), while boosting the stock market in the Center (panel (c)), is accompanied by a capital flight from the Periphery towards the Center. That is, in response to a negative shock in a Periphery country, the Center becomes more constrained, causing it to sell shares in the Periphery and invest domestically, as well as invest in the bond. This pattern represents a more realistic model of the world, as it is consistent with recent crises in which some developed countries have been forced to withdraw funds from emerging markets in order to meet tightened constraints at home. Accordingly, in response

to the negative shock, the correlation between the Periphery country stock returns increases and the correlation between the Periphery and the Center drops (Figure 10); the magnitude of these effects is roughly twice of that for the concentration constraint case. A market share constraint is just one example of a constraint that would generate such richer patterns of transmission. We leave for future research analysis of other, perhaps more prevalent and realistic, constraints that could create this, as well as other, more sophisticated, transmission patterns.

[Figures 6-10 about here]

## 6. Discussion and Future Research

In this section, we would like to discuss the robustness of our main results as well as make suggestions for possible extensions or our framework. One important implication of our model is that portfolio constraints give rise to wealth transfers, which—through the Keynes effect—can produce amplification of shocks and excess comovement among the Periphery countries’ stocks. One may wonder, however, whether the same Keynes effect can take place in an environment with no financial frictions but non-identical (e.g., home-biased) portfolios held by the countries. Indeed, in such a model, wealth distribution is no longer constant and hence wealth transfers occur even in the absence of frictions. One way to extend our model so that it produces non-identical portfolios in the unconstrained benchmark, is to generalize the expenditure shares in the utility functions of the agents. Such a model still admits a closed-form solution, but, in contrast to our model, wealth shares of the Periphery countries no longer coincide with the weights in the planner’s problem. Therefore, one can disentangle the effect of a time-varying wealth distribution from that of stochastic weights in the “planner’s” problem. Interestingly, in the world with perfect risk sharing, the Keynes effect does not operate even when the portfolios exhibit a home bias. This happens because agents can perfectly hedge fluctuations in wealth distribution—and hence neither consumption nor the terms of trade are affected by these fluctuations. In contrast, under imperfect risk sharing, the terms of trade are affected by wealth transfers through their effects on the stochastic weights  $\lambda_1$  and  $\lambda_2$ , which in turn cause the response predicted by the Transfer Problem (the Keynes effect).

Throughout the paper we have stressed the importance of portfolio constraints in generating an asymmetric pattern of cross-country correlations depending on whether they are binding or not. Portfolio constraints also have implications for the persistence of negative shocks. Consider, for example, a market share constraint. As soon as it starts to bind, it forces a capital outflow from the

Periphery and reduces the wealth shares of the Periphery. *Ceteris paribus*, these reduced wealth shares make the constraint even more binding in the future, thereby increasing the persistence of negative shocks to the Periphery. We leave a thorough investigation of this, as well as other feedback mechanisms for future research.

Furthermore, in this paper we have maintained the assumption that only one agent is constrained in his portfolio choice. Motivated by the contagion literature, we have imposed such a constraint on the (large) Center country. This leaves out an important case in which more than one investor face constraints. For example, a realistic scenario could be one in which only a small fraction of the population of Periphery country 1 is permitted to hold shares of Periphery country 2 and vice versa. Our conjecture is that the effects of imposing a constraint on the Center are going to be qualitatively the same. For example, a constraint limiting the Center's investment in the Periphery is still going to force the Center to hold less of the Periphery and more of the Center—relative to the *new benchmark* in which the Periphery countries hold less of each other's stock. In response, just like in our model, the unconstrained residents of the Periphery countries will be persuaded to hold more of both Periphery countries' stocks, for diversification reasons. The resulting biases in the countries' portfolios relative to the (new) benchmark are of the same sign as in our model, and hence the main implications that we describe in Section 5 should hold. Verifying this conjecture is possible but not straightforward, and we leave it for future research.<sup>32</sup>

One counterfactual implication of our framework is that the interest rate in the Center (a developed economy) is higher than the interest rate in the Periphery (emerging economies). Clearly, this is not supported by the data. However, we have not included important determinants of interest rates such as default or expropriation risk. One could model expropriation as a random variable affecting the cash flows of the Periphery countries' stocks or bonds. This might be an interesting extension to pursue.

For simplicity, our specification does not allow for demand shocks to be independent of supply shocks. This assumption guarantees that financial markets are complete. It is possible, however, to extend the model to the case where demand shocks are driven by independent Brownian motions and hence markets are incomplete.<sup>33</sup> While the analysis of such an economy becomes more complicated,

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<sup>32</sup>Two interesting recent attempts to examine the effects of multiple constraints are Schornick (2007) and Soumare and Wang (2007).

<sup>33</sup>A recent work by Pavlova and Rigobon (2008) considers a generalization of our model in which demand shocks are independent of supply shocks and financial markets are incomplete.

the main results in Sections 2–4 remain unchanged. The examples in Section 5 are still tractable, but the formulas we report in Proposition 3 need to be adjusted for the increased dimensionality of uncertainty. In this paper we do not pursue this extension and treat the demand shocks primarily as a modeling device that ensures that the stock markets are not perfectly correlated. This rules out an investigation of several potentially interesting questions. For example, one may ask whether demand shocks get amplified in the presence of portfolio constraints or whether demand shocks magnify the correlation of the Periphery stock markets when portfolio constraints are binding.

Finally, it would be interesting to take the model to the data and try to address the following issues. In recent financial crises, how much of the comovement could be attributed to the common discount factor, and how much to the fact that financial institutions were forced to curb their positions due to the tightening of their constraints? Such questions are very difficult to answer without an underlying theoretical framework. Our model offers one such framework.

## Appendix A

**Countries' Optimization: Details of the Analysis** Our solution method relies on replacing the dynamic Radner-style budget constraint (4) in each country's optimization problem by a static Arrow-Debreu-style budget constraint. This operation is routine if investors in a country are unconstrained and face complete markets, but more complicated if a country's investment opportunity set is constrained.

We start with the optimization of the Periphery countries 1 and 2 that are unconstrained. Dynamic market completeness implies existence of a common state price density process  $\xi$ , consistent with no arbitrage, known to admit the following parametrization:<sup>34</sup>

$$d\xi(t) = -\xi(t)(r(t)dt + m(t)^\top dw(t)), \quad (\text{A.1})$$

where  $m(t) \equiv \sigma^{-1}(t)(\mu(t) - r(t)\bar{1})$  is the market price of risk process associated with the Brownian motions  $w^0$ ,  $w^1$ , and  $w^2$ . The quantity  $\xi(t, \omega)$  is interpreted as the Arrow-Debreu price per unit probability  $P$  of one unit of the numeraire delivered in state  $\omega \in \Omega$  at time  $t$ . We can now convert each country's dynamic optimization problem into a static problem (see Cox and Huang (1989) or Karatzas, Lehoczky, and Shreve (1987)):

$$\max_{C_i^0, C_i^1, C_i^2} E \left[ \int_0^T u_i(C_i^0(t), C_i^1(t), C_i^2(t)) dt \right] \quad (\text{A.2})$$

$$\text{subject to} \quad E \left[ \int_0^T \xi(t) (p^0(t)C_i^0(t) + p^1(t)C_i^1(t) + p^2(t)C_i^2(t)) dt \right] \leq W_i(0), \quad i = 1, 2. \quad (\text{A.3})$$

Its first-order conditions are given by

$$\frac{\partial u_i(C_i^0(t), C_i^1(t), C_i^2(t))}{\partial C_i^j(t)} = y_i p^j(t) \xi(t), \quad i = 1, 2, \quad j = 0, 1, 2. \quad (\text{A.4})$$

where  $y_i$  the (scalar) Lagrange multiplier that makes the budget constraint (A.3) hold with equality.

We now turn to the optimization problem of the Center country, which faces a portfolio constraint. Following Cvitanić and Karatzas (1992) we define the function  $\delta(\cdot)$  in the modified investment opportunity set of the Center to be  $\delta(z) \equiv \sup_{x_0 \in K} (-x_0^\top z)$ . Furthermore,  $\nu$  is specified to be a stochastic process taking values in  $\tilde{K} = \{z \in \mathbb{R}^3; \delta(z) < \infty\}$ . A state price density,  $\xi^\nu$ , implied by this modified investment opportunity (i.e., that incorporates the modified interest rate and expected returns on the stocks) set is given by

$$d\xi^\nu(t) = -\xi^\nu(t)[(r(t) + \delta(\nu(t)))dt + (m(t) + \sigma(t)^{-1}\nu(t))^\top dw(t)]. \quad (\text{A.5})$$

The quantities  $r_0 \equiv r + \delta(\nu)$  and  $m_0 \equiv m + \sigma(t)^{-1}\nu$  are interpreted as the interest rate and the market price of risk effectively faced by the Center country. Under the assumption of logarithmic

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<sup>34</sup>Strictly speaking, we can only claim that markets are potentially complete because it is not guaranteed that in equilibrium all assets are non-redundant. To ensure the validity of our solution method, this non-redundancy needs to be verified in the equilibrium we construct. For the representation of the state price density under complete markets see Duffie (2001, Ch. 6).

preferences, Cvitanić and Karatzas establish that for a particular  $\nu^* \in \tilde{K}$ , namely<sup>35</sup>

$$\nu^*(t) = \arg \min_{\nu \in \tilde{K}} (2\delta(\nu) + \|m(t) + \sigma(t)^{-1}\nu\|^2), \quad (\text{A.6})$$

the optimization problem of the constrained consumer can also be stated in a static form, just as in the unconstrained case, but with the personalized state price density  $\xi_0 \equiv \xi^{\nu^*}$  replacing  $\xi$  in (A.3):

$$\begin{aligned} & \max_{C_0^0, C_0^1, C_0^2} E \left[ \int_0^T u_0(C_0^0(t), C_0^1(t), C_0^2(t)) dt \right] \\ \text{subject to} \quad & E \left[ \int_0^T \xi_0(t) (p^0(t)C_0^0(t) + p^1(t)C_0^1(t) + p^2(t)C_0^2(t)) dt \right] \leq W_0(0). \end{aligned} \quad (\text{A.7})$$

Note that the problem of solving for the stochastic process  $\nu^*$  determining  $\xi_0$  reduces to a simple pointwise minimization of a quadratic form in (A.6).<sup>36</sup> The rest of the characterization is routine. The first-order conditions for the above problem are given by

$$\frac{\partial u_0(C_0^0(t), C_0^1(t), C_0^2(t))}{\partial C_0^j(t)} = y_0 p^j(t) \xi_0(t), \quad j = 0, 1, 2. \quad (\text{A.8})$$

where  $y_0$  is the Lagrange multiplier that makes the budget constraint (A.7) hold with equality.

**Proof of Lemma 1.** It follows from the existing literature (Cvitanić and Karatzas (1992), Karatzas and Shreve (1998)) that  $W_0(t)$  and  $W_i(t)$ ,  $i = 1, 2$ , have representations

$$\begin{aligned} W_0(t) &= \frac{1}{\xi_0(t)} E \left[ \int_t^T \xi_0(s) (p^0(s)C_0^0(s) + p^1(s)C_0^1(s) + p^2(s)C_0^2(s)) ds \middle| \mathcal{F}_t \right] \\ W_i(t) &= \frac{1}{\xi_i(t)} E \left[ \int_t^T \xi_i(s) (p^0(s)C_i^0(s) + p^1(s)C_i^1(s) + p^2(s)C_i^2(s)) ds \middle| \mathcal{F}_t \right], \quad i = 1, 2. \end{aligned}$$

These expressions, combined with equations (A.4) and (A.8), yield

$$W_0(t) = \frac{T-t}{y_0 \xi_0(t)}, \quad W_i(t) = \frac{T-t}{y_i \xi_i(t)}, \quad i = 1, 2.$$

Making use of the first-order conditions (A.4) and (A.8), we arrive at the statement of the lemma.

**Proof of Corollary 1.** This is a standard result for logarithmic preferences over a single good (e.g., Karatzas and Shreve (1998, Ch.6, Example 4.2)). The modification of the standard argument

<sup>35</sup>The notation  $\|z\|^2$  stands for the dot product  $z \cdot z$ .

<sup>36</sup>See Karatzas and Shreve (1998, Ch.6) for a detailed discussion of constrained portfolio optimization problems (at a partial equilibrium level) in general and a proof of this result using the duality approach in particular. The duality approach involves converting the original constrained consumer/investor's optimization with respect to consumption into the dual optimization with respect to  $\nu$  such that  $E \int_0^\infty \|\nu^2(t)\| dt < \infty$  so as to determine the optimal Lagrange multiplier process  $\nu^*$  reflecting the impact of the portfolio constraints first and then back out the optimal consumption and portfolio processes. See Karatzas and Shreve for further necessary regularity conditions on  $\xi^\nu$ .

for the case of multiple goods is simple thanks to Lemma 1. In particular, we can equivalently represent the objective function of country 0 in the form

$$\begin{aligned} & E \int_0^T \left[ \alpha_0 \log \left( \frac{W_0(t)}{p^0(t)(T-t)} \right) + \frac{1-\alpha_0}{2} \log \left( \frac{W_0(t)}{p^1(t)(T-t)} \right) + \frac{1-\alpha_0}{2} \log \left( \frac{W_0(t)}{p^2(t)(T-t)} \right) \right] dt \\ &= E \int_0^T \left[ \log W_0(t) - \alpha_0 \log(p^0(t)(T-t)) - \frac{1-\alpha_0}{2} \log(p^1(t)(T-t)) - \frac{1-\alpha_0}{2} \log(p^2(t)(T-t)) \right] dt. \end{aligned}$$

Since the investor of country 0 takes prices in the good markets  $p^j$ ,  $j = 0, 1, 2$  as given, and hence from his viewpoint the last three terms in the integrand are exogenous, this objective function belongs to the family considered by Karatzas and Shreve. A similar argument applies to investors 1 and 2. *Q.E.D.*

**Weights in the Planner's Problem.** There are many ways to determine the values of  $\lambda_1$  and  $\lambda_2$ , all of which give the same answer. Consider, for example, the expressions for  $C_0^0(0)$  and  $C_1^1(0)$  provided in Lemma 1 and combine them with the corresponding ones from the sharing rules (5)–(6). Recalling the assumption that the initial endowments of countries 0 and 1 are given by  $W_i(0) = S^i(0)$ ,  $i = 0, 1$ , and substituting the expressions for  $S^0(0)$  and  $S^1(0)$  from (10)–(11), we arrive at a system of two equations in two unknowns. The system admits an explicit solution  $\lambda_1 = (1 - \alpha_0)/(1 - \alpha_1(0))$  and  $\lambda_2 = (1 - \alpha_0)/(1 - \alpha_2(0))$ .

**Proof of Lemma 2.** It is easy to verify that an equivalent representation of the expressions in Lemma 2 is

$$S^0(t) = p^0(t)Y^0(t)(T-t), \quad S^1(t) = p^1(t)Y^1(t)(T-t), \quad \text{and} \quad S^2(t) = p^2(t)Y^2(t)(T-t).$$

To value the stocks, we employ the no-arbitrage conditions of the (unconstrained) Periphery countries. The requirement that the representative agent holds all available shares of stocks, and hence that his wealth is the sum of stock prices, implies that each stock's price is equal to its fundamental value (the present value of its dividends):

$$S^j(t) = \frac{1}{\xi(t)} E \left[ \int_t^T \xi(s) p^j(s) Y^j(s) ds \middle| \mathcal{F}_t \right], \quad j = 0, 1, 2. \quad (\text{A.9})$$

It follows from (A.4) and (5) that

$$\frac{\frac{1-\alpha_1(t)}{2}}{y_1 p^0(t) \xi(t)} = \frac{\frac{1-\alpha_1(t)}{2} \lambda_1(t) Y^0(t)}{\alpha_0 + \lambda_1(t) \frac{1-\alpha_1(t)}{2} + \lambda_2(t) \frac{1-\alpha_2(t)}{2}}, \quad (\text{A.10})$$

where  $\lambda_1$  and  $\lambda_2$  are constant weights in the unconstrained economy of Section 2 and stochastic in

the constrained economy of Section 3. Hence, in equilibrium

$$\begin{aligned}
p^0(t)\xi(t) &= \frac{\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}}{y_1\lambda_1(t)Y^0(t)} \\
&= \frac{1}{y_1Y^0(t)} \left( \alpha_0 \frac{1}{\lambda_1(t)} + \frac{1-\alpha_1(t)}{2} + \frac{1-\alpha_2(t)}{2} \frac{\lambda_2(t)}{\lambda_1(t)} \right) \\
&= \frac{1}{y_1Y^0(t)} \left( \alpha_0 \frac{1}{\lambda_1(t)} + \frac{1-\alpha_1(t)}{2} + \frac{1-\alpha_2(t)}{2} \frac{y_1}{y_2} \right)
\end{aligned} \tag{A.11}$$

To derive the last equality we use (A.4), (A.8), and Lemma 1 to show that

$$\lambda_1(t) = \frac{y_0\xi_0(t)}{y_1\xi(t)} \quad \text{and} \quad \lambda_2(t) = \frac{y_0\xi_0(t)}{y_2\xi(t)}. \tag{A.12}$$

Analogous steps can be used to derive that

$$p^1(t)\xi(t) = \frac{1}{y_1Y^1(t)} \left( \frac{1-\alpha_0}{2} \frac{1}{\lambda_1(t)} + \alpha_1(t) + \frac{1-\alpha_2(t)}{2} \frac{y_1}{y_2} \right) \tag{A.13}$$

$$p^2(t)\xi(t) = \frac{1}{y_1Y^2(t)} \left( \frac{1-\alpha_0}{2} \frac{1}{\lambda_1(t)} + \frac{1-\alpha_1(t)}{2} + \alpha_2(t) \frac{y_1}{y_2} \right) \tag{A.14}$$

Making use of the the assumption that  $\alpha_1$  and  $\alpha_2$  are martingales, from (A.9)–(A.11) we obtain

$$\begin{aligned}
S^0(t) &= \frac{p^0(t)y_1\lambda_1(t)Y^0(t)}{\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}} E \left[ \int_t^T \frac{1}{y_1} \left( \alpha_0 \frac{1}{\lambda_1(s)} + \frac{1-\alpha_1(s)}{2} + \frac{1-\alpha_2(s)}{2} \frac{y_1}{y_2} \right) ds \middle| \mathcal{F}_t \right] \\
&= \frac{p^0(t)\lambda_1(t)Y^0(t)}{\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}} \left( \alpha_0 \frac{1}{\lambda_1(t)} + \frac{1-\alpha_1(t)}{2} + \frac{1-\alpha_2(t)}{2} \frac{y_1}{y_2} \right) (T-t) \\
&\quad + \frac{p^0(t)\lambda_1(t)Y^0(t)}{\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}} \left( E \left[ \int_t^T \alpha_0 \frac{1}{\lambda_1(s)} ds \middle| \mathcal{F}_t \right] - \alpha_0 \frac{1}{\lambda_1(t)} (T-t) \right) \\
&= p^0(t)Y^0(t)(T-t) + \frac{\alpha_0 p^0(t)\lambda_1(t)Y^0(t)}{\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}} \left( E \left[ \int_t^T \frac{1}{\lambda_1(s)} ds \middle| \mathcal{F}_t \right] - \frac{1}{\lambda_1(t)} (T-t) \right).
\end{aligned}$$

An analogous argument can be used to show that

$$\begin{aligned}
S^1(t) &= p^1(t)Y^1(t)(T-t) + \frac{\frac{1-\alpha_0}{2} p^1(t)\lambda_1(t)Y^1(t)}{\frac{1-\alpha_0}{2} + \lambda_1(t)\alpha_1(t) + \lambda_2(t)\frac{1-\alpha_2(t)}{2}} \left( E \left[ \int_t^T \frac{1}{\lambda_1(s)} ds \middle| \mathcal{F}_t \right] - \frac{1}{\lambda_1(t)} (T-t) \right), \\
S^2(t) &= p^2(t)Y^2(t)(T-t) + \frac{\frac{1-\alpha_0}{2} p^2(t)\lambda_1(t)Y^2(t)}{\frac{1-\alpha_0}{2} + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\alpha_2(t)} \left( E \left[ \int_t^T \frac{1}{\lambda_1(s)} ds \middle| \mathcal{F}_t \right] - \frac{1}{\lambda_1(t)} (T-t) \right).
\end{aligned}$$

Note that the term  $E \left[ \int_t^T \frac{1}{\lambda_1(s)} ds \middle| \mathcal{F}_t \right] - \frac{1}{\lambda_1(t)} (T-t)$  enters the expression for each stock symmetrically. Therefore, at any time  $t$ , the prices of all stocks in the economy are either above or below the value of their dividends, augmented by the factor  $T-t$ :

$$\begin{aligned}
S^j(t) \leq p^j(t)Y^j(t)(T-t) \quad &\text{if} \quad E \left[ \int_t^T \frac{1}{\lambda_1(s)} ds \middle| \mathcal{F}_t \right] - \frac{1}{\lambda_1(t)} (T-t) \leq 0, \\
S^j(t) \geq p^j(t)Y^j(t)(T-t) \quad &\text{if} \quad E \left[ \int_t^T \frac{1}{\lambda_1(s)} ds \middle| \mathcal{F}_t \right] - \frac{1}{\lambda_1(t)} (T-t) \geq 0, \quad j = 0, 1, 2,
\end{aligned} \tag{A.15}$$

where we have used the restrictions that  $0 < \alpha_i < 1/3$ ,  $\lambda_i > 0$ , and  $Y^i > 0$ ,  $i = 0, 1, 2$ , at all times.

On the other hand, from bond market clearing we have that

$$W_0(t) + W_1(t) + W_2(t) = S^0(t) + S^1(t) + S^2(t) \quad (\text{A.16})$$

and from Lemma 1 and market clearing for goods 0, 1 and 2 that

$$\frac{1}{p^0(t)} \left( \frac{\alpha_0 W_0(t)}{T-t} + \frac{\frac{1-\alpha_1(t)}{2} W_1(t)}{T-t} + \frac{\frac{1-\alpha_2(t)}{2} W_2(t)}{T-t} \right) = Y^0(t), \quad (\text{A.17})$$

$$\frac{1}{p^1(t)} \left( \frac{\frac{1-\alpha_0}{2} W_0(t)}{T-t} + \frac{\alpha_1(t) W_1(t)}{T-t} + \frac{\frac{1-\alpha_2(t)}{2} W_2(t)}{T-t} \right) = Y^1(t), \quad (\text{A.18})$$

$$\frac{1}{p^2(t)} \left( \frac{\frac{1-\alpha_0}{2} W_0(t)}{T-t} + \frac{\frac{1-\alpha_1(t)}{2} W_1(t)}{T-t} + \frac{\alpha_2(t) W_2(t)}{T-t} \right) = Y^2(t). \quad (\text{A.19})$$

Hence, by multiplying (A.17), (A.18) and (A.19) by  $p^0(t)$ ,  $p^1(t)$  and  $p^2(t)$ , respectively, and adding them up, we can show that

$$W_0(t) + W_1(t) + W_2(t) = p^0(t)Y^0(t)(T-t) + p^1(t)Y^1(t)(T-t) + p^2(t)Y^2(t)(T-t).$$

This, together with (A.15) yields the required result. *Q.E.D.*

**Proof of Propositions 1 and 2.** Since our proofs of the two propositions follow analogous steps, we present them together.

We first report the quantities  $A(t)$ ,  $\tilde{A}(t)$ ,  $a(t)$ ,  $\tilde{a}(t)$ ,  $b(t)$ ,  $\tilde{b}(t)$ ,  $M(t)$ ,  $X_\lambda$ ,  $X_{\alpha_1}$ , and  $X_{\alpha_2}$  omitted in the body Propositions 1 and 2:

$$A(t) \equiv \frac{\left( \alpha_0 \alpha_1(t) - \frac{1-\alpha_0}{2} \frac{1-\alpha_1(t)}{2} \right) \lambda_1(t) + \frac{1-\alpha_2(t)}{2} \frac{3\alpha_0-1}{2} \lambda_2(t)}{\left( \alpha_0 + \lambda_1(t) \frac{1-\alpha_1(t)}{2} + \lambda_2(t) \frac{1-\alpha_2(t)}{2} \right) \left( \frac{1-\alpha_0}{2} + \lambda_1(t) \alpha_1(t) + \lambda_2(t) \frac{1-\alpha_2(t)}{2} \right)}, \quad (\text{A.20})$$

$$\tilde{A}(t) \equiv \frac{\left( \alpha_0 \alpha_2(t) - \frac{1-\alpha_0}{2} \frac{1-\alpha_2(t)}{2} \right) \lambda_2(t) + \frac{1-\alpha_1(t)}{2} \frac{3\alpha_0-1}{2} \lambda_1(t)}{\left( \alpha_0 + \lambda_1(t) \frac{1-\alpha_1(t)}{2} + \lambda_2(t) \frac{1-\alpha_2(t)}{2} \right) \left( \frac{1-\alpha_0}{2} + \lambda_1(t) \frac{1-\alpha_1(t)}{2} + \lambda_2(t) \alpha_2(t) \right)}, \quad (\text{A.21})$$

$$a(t) \equiv \frac{\frac{\lambda_1(t)}{2}}{\alpha_0 + \lambda_1(t) \frac{1-\alpha_1(t)}{2} + \lambda_2(t) \frac{1-\alpha_2(t)}{2}} + \frac{\lambda_1(t)}{\frac{1-\alpha_0}{2} + \lambda_1(t) \alpha_1(t) + \lambda_2(t) \frac{1-\alpha_2(t)}{2}}, \quad (\text{A.22})$$

$$\tilde{a}(t) \equiv \frac{\lambda_1(t)}{2} \frac{\frac{1-3\alpha_0}{2} - \frac{1-3\alpha_2(t)}{2} \lambda_2(t)}{\left( \alpha_0 + \lambda_1(t) \frac{1-\alpha_1(t)}{2} + \lambda_2(t) \frac{1-\alpha_2(t)}{2} \right) \left( \frac{1-\alpha_0}{2} + \lambda_1(t) \frac{1-\alpha_1(t)}{2} + \lambda_2(t) \alpha_2(t) \right)}, \quad (\text{A.23})$$

$$b(t) \equiv \frac{\lambda_2(t)}{2} \frac{\frac{1-3\alpha_0}{2} - \frac{1-3\alpha_1(t)}{2} \lambda_1(t)}{\left( \alpha_0 + \lambda_1(t) \frac{1-\alpha_2(t)}{2} + \lambda_2(t) \frac{1-\alpha_2(t)}{2} \right) \left( \frac{1-\alpha_0}{2} + \lambda_1(t) \alpha_1(t) + \lambda_2(t) \frac{1-\alpha_2(t)}{2} \right)}, \quad (\text{A.24})$$

$$\tilde{b}(t) \equiv \frac{\frac{\lambda_2(t)}{2}}{\alpha_0 + \lambda_1(t) \frac{1-\alpha_2(t)}{2} + \lambda_2(t) \frac{1-\alpha_2(t)}{2}} + \frac{\lambda_2(t)}{\frac{1-\alpha_0}{2} + \lambda_1(t) \alpha_1(t) + \lambda_2(t) \frac{1-\alpha_2(t)}{2}} \quad (\text{A.25})$$

$$M(t) \equiv \frac{1}{\beta + \frac{1-\beta}{2}q^1(t) + \frac{1-\beta}{2}q^2(t)}, \quad X_\lambda(t) \equiv \frac{1-\beta}{2}M(t)(q^1(t)A(t) + q^2(t)\tilde{A}(t)), \quad (\text{A.26})$$

$$X_{\alpha_1} \equiv \frac{1-\beta}{2}M(t)(a(t)q^1(t) + \tilde{a}(t)q^2(t)), \quad X_{\alpha_2} \equiv \frac{1-\beta}{2}M(t)(b(t)q^1(t) + \tilde{b}(t)q^2(t)) \quad (\text{A.27})$$

These expressions are the same across Propositions 1 and 2, except that in Proposition 1  $\lambda_i(t)$  are constant weights.

To demonstrate that in Proposition 2  $\lambda_1(t)$  and  $\lambda_2(t)$  are the same up to a multiplicative constant, we use (A.12) together with the observation that  $y_1$  and  $y_2$  are constants.

Taking logs in (8) we obtain

$$\log q^1(t) = \log \frac{\frac{1-\alpha_0}{2} + \lambda_1(t)\alpha_1(t) + \lambda_2(t)\frac{1-\alpha_2(t)}{2}}{\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}} + \log Y^0(t) - \log Y^1(t).$$

Applying Itô's lemma to both sides and simplifying, we have

$$\begin{aligned} \frac{dq^1(t)}{q^1(t)} = & \text{Itô terms } dt + \frac{1}{\frac{1-\alpha_0}{2} + \lambda_1(t)\alpha_1(t) + \lambda_2(t)\frac{1-\alpha_2(t)}{2}} \left( \lambda_1(t)\alpha_1(t) \frac{d\lambda_1(t)}{\lambda_1(t)} + \lambda_1(t) d\alpha_1(t) - \frac{\lambda_2(t)}{2} d\alpha_2(t) \right. \\ & \left. + \lambda_2(t) \frac{1-\alpha_2(t)}{2} \frac{d\lambda_2(t)}{\lambda_2(t)} \right) - \frac{1}{\alpha_0 + \lambda_1(t)\frac{1-\alpha_1(t)}{2} + \lambda_2(t)\frac{1-\alpha_2(t)}{2}} \left( \lambda_1(t) \frac{1-\alpha_1(t)}{2} \frac{d\lambda_1(t)}{\lambda_1(t)} \right. \\ & \left. - \frac{\lambda_1(t)}{2} d\alpha_1(t) + \lambda_2(t) \frac{1-\alpha_2(t)}{2} \frac{d\lambda_2(t)}{\lambda_2(t)} - \frac{\lambda_2(t)}{2} d\alpha_2(t) \right) + \frac{dY^0(t)}{Y^0(t)} - \frac{dY^1(t)}{Y^1(t)}. \end{aligned}$$

Substituting  $\frac{d\lambda_1(t)}{\lambda_1(t)} = \frac{d\lambda_2(t)}{\lambda_2(t)} = \frac{d\lambda(t)}{\lambda(t)}$  in the expression above, simplifying, and making use of (1) and the definitions in (A.20)–(A.27), we arrive at the statement in the propositions. Of course, in Proposition 1,  $d\lambda_1(t) = d\lambda_2(t) = 0$ , and hence the terms involving  $d\lambda_1(t)$  and  $d\lambda_2(t)$  drop out. The dynamics of  $q^2$  are derived analogously.

To derive the dynamics of  $S^0$ , we restate (10)–(12) as

$$\log S^0(t) = -\log \left( \beta + \frac{1-\beta}{2}q^1(t) + \frac{1-\beta}{2}q^2(t) \right) + \log Y^0(t) + \log(T-t), \quad (\text{A.28})$$

$$\log S^j(t) = \log q^j(t) - \log \left( \beta + \frac{1-\beta}{2}q^1(t) + \frac{1-\beta}{2}q^2(t) \right) + \log Y^j(t) + \log(T-t). \quad (\text{A.29})$$

Applying Itô's lemma to both sides of (A.28)–(A.29), we arrive at

$$\begin{aligned} \frac{dS^0(t)}{S^0(t)} = & \text{Drift terms } dt - \frac{\frac{1-\beta}{2}}{\beta + \frac{1-\beta}{2}q^1(t) + \frac{1-\beta}{2}q^2(t)} \left( q^1(t) \frac{dq^1(t)}{q^1(t)} + q^2(t) \frac{dq^2(t)}{q^2(t)} \right) + \frac{dY^0(t)}{Y^0(t)}, \\ \frac{dS^j(t)}{S^j(t)} = & \text{Drift terms } dt + \frac{dq^j(t)}{q^j(t)} - \frac{\frac{1-\beta}{2}}{\beta + \frac{1-\beta}{2}q^1(t) + \frac{1-\beta}{2}q^2(t)} \left( q^1(t) \frac{dq^1(t)}{q^1(t)} + q^2(t) \frac{dq^2(t)}{q^2(t)} \right) + \frac{dY^j(t)}{Y^j(t)}. \end{aligned}$$

Substituting the dynamics of  $q^1$  and  $q^2$  derived above and making use of the definitions in (A.20)–(A.27) we arrive at the statement in the propositions.

Computation of the drift term  $I_c(t) \equiv (I_{c_1}, I_{c_2}, I_{c_3}, I_{c_4}, I_{c_5})$  is straightforward but tedious, so in the interest of space we report just the end result.

$$\begin{aligned}
I_{c_1}(t) &= \mu_{Y^0}(t) - \mu_{Y^1}(t) + \sigma_{Y^0}(t)^2 + \sigma_{Y^1}(t)^2 + A(t)\sigma_{Y^0}(t)\sigma_\lambda(t)^\top i_0 - A(t)\sigma_{Y^1}(t)\sigma_\lambda(t)^\top i_1 \\
&\quad + a(t)\sigma_{Y^0}(t)\sigma_{\alpha_1}(t)^\top i_0 - a(t)\sigma_{Y^1}(t)\sigma_{\alpha_1}(t)^\top i_1 + b(t)\sigma_{Y^0}(t)\sigma_{\alpha_2}(t)^\top i_0 - b(t)\sigma_{Y^1}(t)\sigma_{\alpha_2}(t)^\top i_1, \\
I_{c_2}(t) &= \mu_{Y^0}(t) - \mu_{Y^2}(t) + \sigma_{Y^0}(t)^2 + \sigma_{Y^2}(t)^2 + \tilde{A}(t)\sigma_{Y^0}(t)\sigma_\lambda(t)^\top i_0 - \tilde{A}(t)\sigma_{Y^2}(t)\sigma_\lambda(t)^\top i_2 \\
&\quad + \tilde{a}(t)\sigma_{Y^0}(t)\sigma_{\alpha_1}(t)^\top i_0 - \tilde{a}(t)\sigma_{Y^2}(t)\sigma_{\alpha_1}(t)^\top i_2 + \tilde{b}(t)\sigma_{Y^0}(t)\sigma_{\alpha_2}(t)^\top i_0 - \tilde{b}(t)\sigma_{Y^2}(t)\sigma_{\alpha_2}(t)^\top i_2, \\
I_{c_3}(t) &= \mu_{Y^0}(t) - \frac{1-\beta}{2}M(t)q^1(t)D(t) - \frac{1-\beta}{2}M(t)q^2(t)\tilde{D}(t) - \frac{1}{T-t}, \\
I_{c_4}(t) &= \mu_{Y^1}(t) + \left(\beta + \frac{1-\beta}{2}q^2(t)\right)M(t)G(t) - \frac{1-\beta}{2}M(t)q^2(t)\tilde{G}(t) - \frac{1}{T-t}, \\
I_{c_5}(t) &= \mu_{Y^2}(t) + \left(\beta + \frac{1-\beta}{2}q^1(t)\right)M(t)\tilde{H}(t) - \frac{1-\beta}{2}M(t)q^1(t)H(t) - \frac{1}{T-t},
\end{aligned}$$

where

$$\begin{aligned}
D(t) &\equiv I_{c_1}(t) + A(t)\sigma_{Y^0}(t)\sigma_\lambda(t)^\top i_0 + a(t)\sigma_{Y^0}(t)\sigma_{\alpha_1}(t)^\top i_0 + b(t)\sigma_{Y^0}(t)\sigma_{\alpha_2}(t)^\top i_0 + \sigma_{Y^0}(t)^2, \\
\tilde{D}(t) &\equiv I_{c_2}(t) + \tilde{A}(t)\sigma_{Y^0}(t)\sigma_\lambda(t)^\top i_0 + \tilde{a}(t)\sigma_{Y^0}(t)\sigma_{\alpha_1}(t)^\top i_0 + \tilde{b}(t)\sigma_{Y^0}(t)\sigma_{\alpha_2}(t)^\top i_0 + \sigma_{Y^0}(t)^2, \\
G(t) &\equiv I_{c_1}(t) + A(t)\sigma_{Y^1}(t)\sigma_\lambda(t)^\top i_1 + a(t)\sigma_{Y^1}(t)\sigma_{\alpha_1}(t)^\top i_1 + b(t)\sigma_{Y^1}(t)\sigma_{\alpha_2}(t)^\top i_1 - \sigma_{Y^1}(t)^2, \\
\tilde{G}(t) &\equiv I_{c_2}(t) + \tilde{A}(t)\sigma_{Y^1}(t)\sigma_\lambda(t)^\top i_1 + \tilde{a}(t)\sigma_{Y^1}(t)\sigma_{\alpha_1}(t)^\top i_1 + \tilde{b}(t)\sigma_{Y^1}(t)\sigma_{\alpha_2}(t)^\top i_1 - \sigma_{Y^1}(t)^2, \\
H(t) &\equiv I_{c_1}(t) + A(t)\sigma_{Y^2}(t)\sigma_\lambda(t)^\top i_2 + a(t)\sigma_{Y^2}(t)\sigma_{\alpha_1}(t)^\top i_2 + b(t)\sigma_{Y^2}(t)\sigma_{\alpha_2}(t)^\top i_2 - \sigma_{Y^2}(t)^2, \\
\tilde{H}(t) &\equiv I_{c_2}(t) + \tilde{A}(t)\sigma_{Y^2}(t)\sigma_\lambda(t)^\top i_2 + \tilde{a}(t)\sigma_{Y^2}(t)\sigma_{\alpha_1}(t)^\top i_2 + \tilde{b}(t)\sigma_{Y^2}(t)\sigma_{\alpha_2}(t)^\top i_2 - \sigma_{Y^2}(t)^2,
\end{aligned}$$

and  $\sigma_\lambda(t) \equiv -\lambda(t)(m_0(t) - m(t))$ ,  $i_0 \equiv (1, 0, 0)^\top$ ,  $i_1 \equiv (0, 1, 0)^\top$ , and  $i_2 \equiv (0, 0, 1)^\top$ . In Proposition 1, the weights  $\lambda_1$  and  $\lambda_2$  are constant, and hence the drift term  $I$  is a special case of  $I_c$  in which  $\sigma_\lambda(t) = 0$ . *Q.E.D.*

**Proof of Proposition 3.** Recall that due to the portfolio constraint, agents in the Center and the Periphery countries face different state price densities,  $\xi_0$  and  $\xi$ , respectively. In particular, the (constrained) Center country's effective interest rate and the market price of risk,  $r_0(t)$  and  $m_0(t)$ , do not coincide with  $r(t)$  and  $m(t)$  (faced by the Periphery countries) when the constraint is binding. It follows from Proposition 2 and (A.12) that  $\lambda(t) = \frac{y_0\xi_0(t)}{y_1\xi(t)}$ . Applying Itô's lemma to this expression and using the definitions of  $\xi$  and  $\xi_0$  from (A.1) and (A.5), we obtain (18).

Substituting (18) into the expressions in Proposition 2, we have the following representation for  $\sigma$ :

$$\sigma(t) = \begin{bmatrix} -X_\lambda(t) & -X_{\alpha_1}(t) & -X_{\alpha_2}(t) & \beta & \frac{1-\beta}{2} & \frac{1-\beta}{2} \\ A(t) - X_\lambda(t) & a(t) - X_{\alpha_1}(t) & b(t) - X_{\alpha_2}(t) & \beta & \frac{1-\beta}{2} & \frac{1-\beta}{2} \\ \tilde{A}(t) - X_\lambda(t) & \tilde{a}(t) - X_{\alpha_1}(t) & \tilde{b}(t) - X_{\alpha_2}(t) & \beta & \frac{1-\beta}{2} & \frac{1-\beta}{2} \end{bmatrix} \begin{bmatrix} (m(t) - m_0(t))^\top \\ \sigma_{\alpha_1}(t)^\top \\ \sigma_{\alpha_2}(t)^\top \\ M(t)\sigma_{Y^0}(t) i_0^\top \\ M(t)q^1(t)\sigma_{Y^1}(t) i_2^\top \\ M(t)q^2(t)\sigma_{Y^2}(t) i_2^\top \end{bmatrix}.$$

The  $3 \times 3$  matrix  $\sigma$  represents the loadings on the three underlying Brownian motions  $w^0$ ,  $w^1$ , and  $w^2$  of the three stocks:  $S^0$  (captured by the the first row of  $\sigma$ ),  $S^1$  (the second row), and  $S^2$  (the third row). In the benchmark unconstrained economy or at times when the constraint is not binding, all countries face the same state price density, and hence the market price of risk  $m_0(t)$  coincides with  $m(t)$ , and the matrix  $\sigma$  coincides with its counterpart in the benchmark unconstrained economy.

Equations (19) and (20) are derived at a partial equilibrium level. We follow the steps outlined in Section 2.2 and the beginning of this appendix to solve for the modified investment opportunity set of Center.<sup>37</sup> We have

$$K = \left\{ x_0 \in \mathbb{R}^3 : (i_1 + i_2)^\top x_0 \leq \gamma \right\},$$

and hence

$$\delta(z) = \begin{cases} -\gamma \bar{z} & \text{if } z = \bar{z}(i_1 + i_2) \text{ for some } \bar{z} \leq 0, \\ \infty & \text{otherwise,} \end{cases}$$

$$\tilde{K} = \{ z \in \mathbb{R}^3 : z = \bar{z}(i_1 + i_2) \text{ for some } \bar{z} \leq 0 \}.$$

This establishes that the process  $\nu$  we are looking for must be of the form  $\nu(t) = \bar{\nu}(t)(i_1 + i_2)$  for some  $\bar{\nu}(t) \leq 0$ . To solve for  $\bar{\nu}(t)$  and therefore for the process  $\nu^*$ , we make use of (A.6):

$$\min_{\bar{\nu}(t) < 0} (-2\gamma\bar{\nu}(t) + \|m(t) + \sigma(t)^{-1}\bar{\nu}(t)(i_1 + i_2)\|^2). \quad (\text{A.30})$$

This optimization is straightforward and results in the expressions reported in (19)–(20), where for expositional reasons we have replaced the (negative) solution  $\bar{\nu}^*(t)$  to (A.30) by  $\psi(t) = -\bar{\nu}^*(t)$ . The quantity  $\psi(t)$  can be interpreted as the multiplier on the portfolio constraint (17) at time  $t$  in the maximization of the Center's country utility over the budget set and the constrained investment opportunity set.

Equation (21) follows from market clearing, coupled with the investors' first-order conditions. It follows from, for example, (A.4) and (5) that

$$\frac{\alpha_0}{y_0 p^0(t) \xi_0(t)} = \frac{\alpha_0 Y^0(t)}{\alpha_0 + \lambda_1(t) \frac{1-\alpha_1(t)}{2} + \lambda_2(t) \frac{1-\alpha_2(t)}{2}}. \quad (\text{A.31})$$

Applying Itô's lemma to both sides of (A.31) and equating the ensuing diffusion terms, we arrive at the statement in the proposition. *Q.E.D.*

<sup>37</sup>For a similar derivation, but with different portfolio constraints, see Teplá (2000).

**Proof of Proposition 4.** We have  $r_0(t) = r(t) + \delta(\nu^*(t))$ . Substituting the expressions for  $\delta(z)$  and  $\nu^*$  from the proof of Proposition 3 we arrive at the expression in the corollary. *Q.E.D.*

**Sign Implications in Tables 1 and 2.** Due to the restrictions  $\alpha_i \in (1/3, 1)$  and  $\beta \in (0, 1)$ , the quantities  $A(t)$ ,  $\tilde{A}(t)$ ,  $a(t)$ ,  $\tilde{b}(t)$ ,  $M(t)$ ,  $X_\lambda(t)$ ,  $X_{\alpha_1}(t)$ , and  $X_{\alpha_2}(t)$  are all unambiguously positive. We also have

$$\begin{aligned}\tilde{a}(t) < 0 & \quad \text{iff} \quad \frac{1-\alpha_0}{2} - \alpha_0 < \left( \frac{1-\alpha_2(t)}{2} - \alpha_2(t) \right) \lambda_2(t) \\ b(t) < 0 & \quad \text{iff} \quad \frac{1-\alpha_0}{2} - \alpha_0 < \left( \frac{1-\alpha_1(t)}{2} - \alpha_1(t) \right) \lambda_1(t),\end{aligned}$$

It follows from Propositions 1 and 2 that the effect of demand shift in country 1 (2) on the terms of trade in country 2 (1) is negative iff  $\tilde{a}(t) < 0$  ( $b(t) < 0$ ), which are the conditions in our Condition C1. Deriving the signs of the responses of the stock prices to the demand shifts is then straightforward, given the characterization in Propositions 1 and 2. Condition C2, the rationale for which is given in the body of Section 2, provides a sufficient condition for the effects to result in the signs reported in Tables 1 and 2.

The effects of a change in  $\lambda$  on the terms of trade of each Periphery country are positive because  $A(t) > 0$  and  $\tilde{A}(t) > 0$ —guaranteed by the assumption that  $\alpha_i \in (1/3, 1)$  (home bias in consumption).

We now derive an alternative form of Condition C3. It follows from Proposition 2 that the impact of the constraint (the effect of a change in  $\lambda$ ) on the stock prices of the Periphery countries is positive iff (14)–(15) hold. Notice that if  $A(t) < \tilde{A}(t)$ , condition (15) is trivially satisfied, and if  $A(t) > \tilde{A}(t)$ , (14) is the one that is satisfied. This means that, in general, only one of the conditions needs to be checked. If we assume that  $A(t) < \tilde{A}(t)$ , then a sufficient condition for both effects to be positive is that (14) is satisfied. The condition guaranteeing that  $A(t) < \tilde{A}(t)$  is

$$\frac{3\alpha_1(t) - 1}{3\alpha_2(t) - 1} < \frac{\lambda_2(t)}{\lambda_1(t)}. \quad (\text{A.32})$$

After some algebra and using the fact that  $\alpha_1(t) > 1/3$  one can show that (14) is satisfied when

$$\alpha_1(t)\lambda_1(t) + \frac{1 - \alpha_2(t)}{2}\lambda_2(t) > \frac{1 - \beta}{\beta} \frac{Y_0(t)}{Y_2(t)} \frac{1}{2} \left( \frac{3\alpha_2(t) - 1}{3\alpha_0 - 1} \lambda_2(t) - \frac{3\alpha_1(t) - 1}{3\alpha_0 - 1} \lambda_1(t) \right). \quad (\text{A.33})$$

This is a sufficient condition guaranteeing that Condition C3 is satisfied. The left-hand-side of equation (A.33) represents the direct effect of lambda (the wealth transfer, as explained in Section 3) on the relative price of good 1. The terms on the right-hand side represent the two indirect effects: (i) the impact of the drop in the demand for good 0, and (ii) the impact of the cross-country demand reallocation in the Periphery countries. We discuss these effects in detail in the NBER working paper version of this paper.

## Appendix B

The sample includes the Mexico 1994, Thailand 1997, Hong Kong 1997, Korea 1997–98, Asia 1997–98, Russia 1998, Brazil 1999, Turkey 2001, and Argentina 2002 crises. The crisis windows are

as follows: Mexico: 1994-12-19 until 1995-02-15; Argentina: 2002-01-11 until 2002-03-01; Brazil: 1999-01-13 until 1999-03-01; Asia: 1997-07-02 until 1998-03-31 (this window includes the Thailand, Hong Kong and Korea's crises); Turkey: 2001-02-22 until 2001-03-31, and Russia 1998-08-01 until 1998-09-31. The crises analyzed are all pooled together. All correlations represent bilateral correlations of all stated countries within the group. Our sample of emerging economies consists of Argentina, Brazil, Chile, China, Colombia, Czech Republic, Hong Kong, Hungary, India, Indonesia, Korea, Malaysia, Mexico, the Philippines, Poland, Russia, Singapore, Taiwan, Thailand, Turkey, and Venezuela. Our sample of developed economies consists of Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the U.K., and the U.S. All exchange rates are measured against the U.S., and hence the U.S. is dropped from the correlation measures for developed economies.

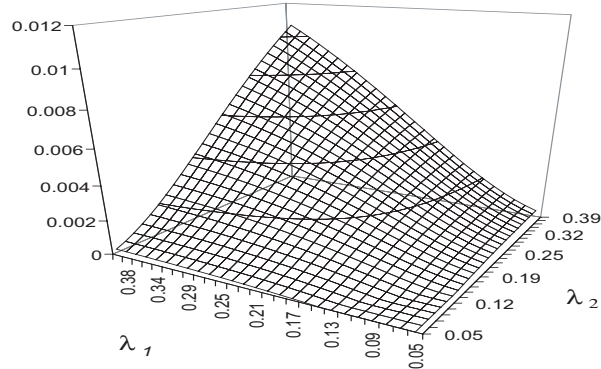
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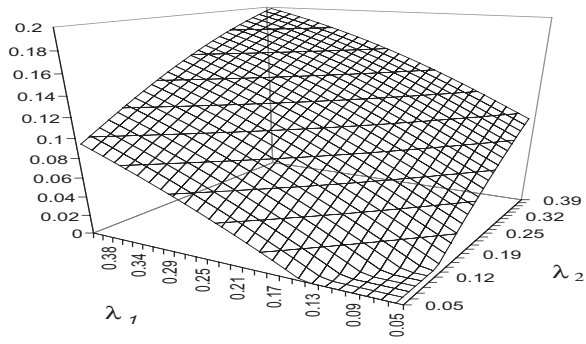
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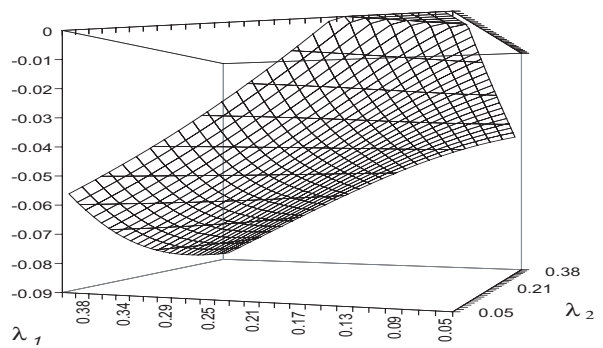
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**Figure 1:** Value of the multiplier on the portfolio concentration constraint  $\psi$ .

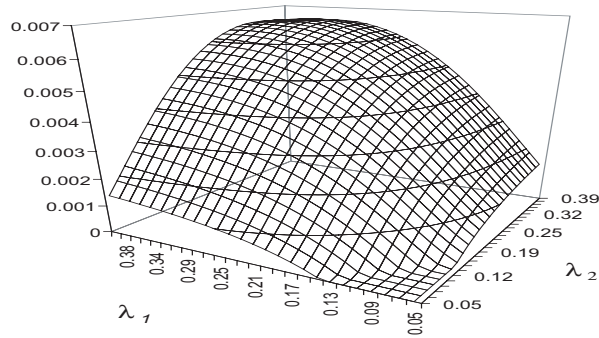


(a) tilt in the investment in the Center's stock

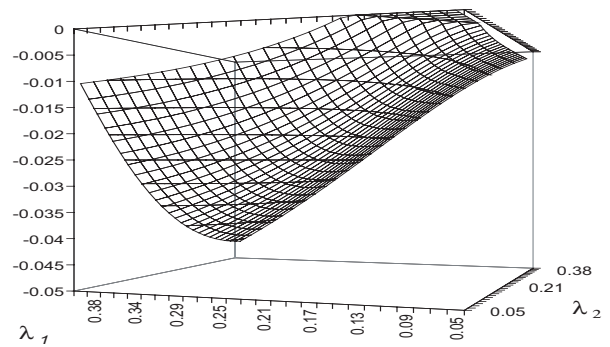


(b) tilt in the investment in a Periphery country's stock

**Figure 2:** Tilt in the asset allocation of the Center relative to the unconstrained allocation due to the portfolio concentration constraint.

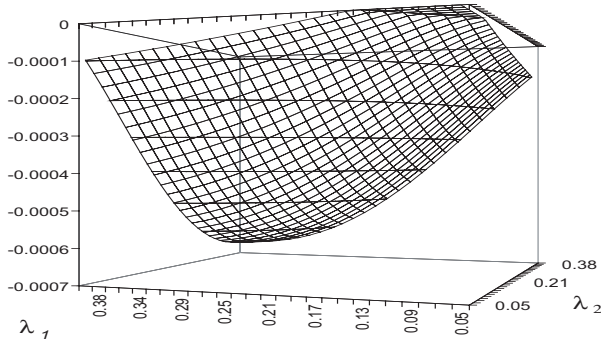


(a) of a negative shock in the Center  $dw^0$

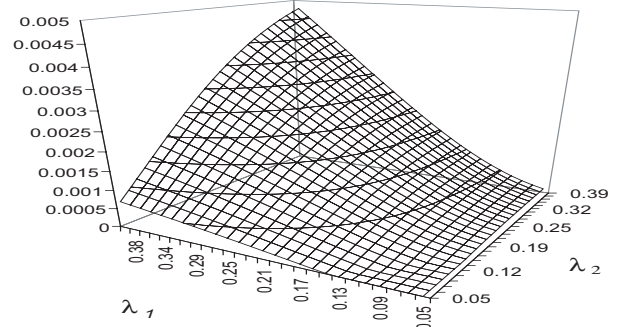


(b) of a negative shock in the Periphery  $dw^1$

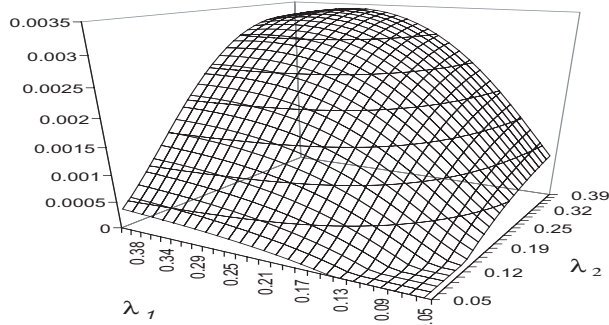
**Figure 3:** The effects of supply shocks on wealth distribution,  $\frac{d\lambda}{\lambda}$ , in the presence of the portfolio concentration constraint.



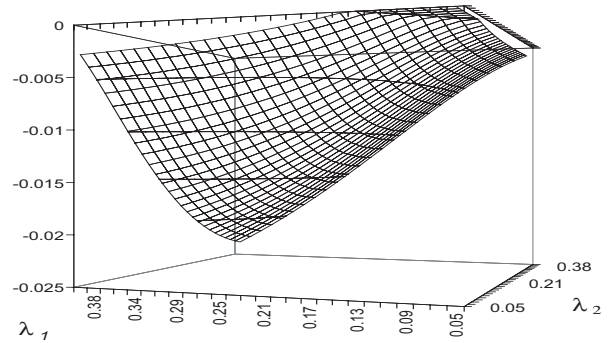
(a) of a negative shock in the Center,  $dw^0$ , on the Center's stock return,  $\frac{dS^0}{S^0}$



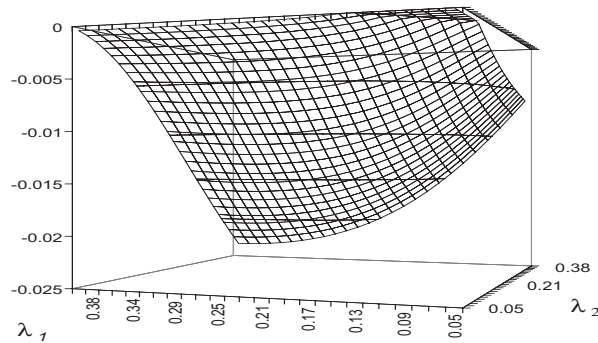
(b) of a negative shock in the Center,  $dw^0$ , on a Periphery country's stock return,  $\frac{dS^1}{S^1}$



(c) of a negative shock in Periphery country 1,  $dw^1$ , on the Center's stock return,  $\frac{dS^0}{S^0}$

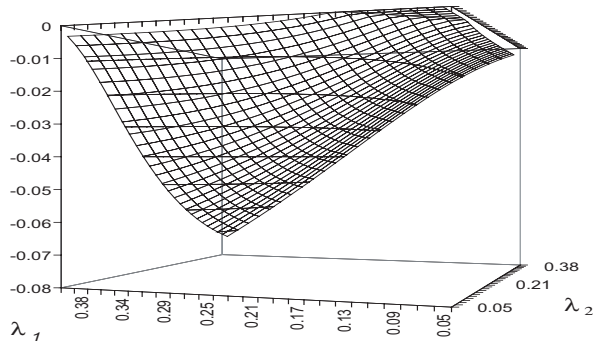


(d) of a negative shock in Periphery country 1,  $dw^1$ , on the Periphery country's stock return,  $\frac{dS^1}{S^1}$

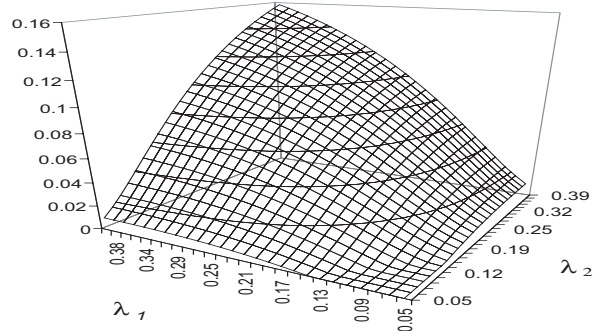


(e) of a negative shock in Periphery country 1,  $dw^1$ , on the other Periphery country's stock return,  $\frac{dS^2}{S^2}$

**Figure 4:** Incremental effects of supply shocks on stock prices in the presence of the portfolio concentration constraint.

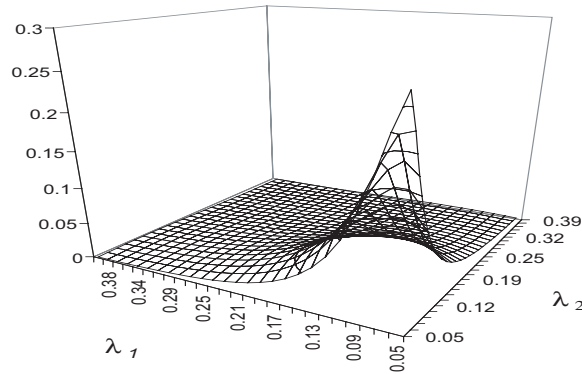


(a) of the Center and Periphery country 1

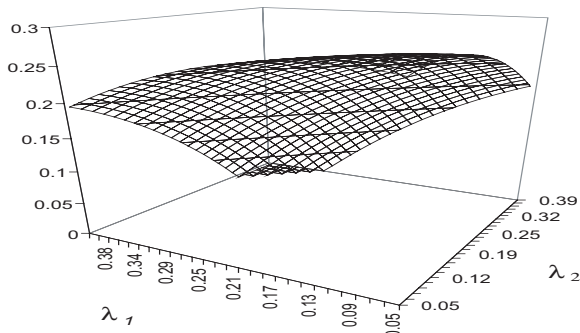


(b) of Periphery countries 1 and 2

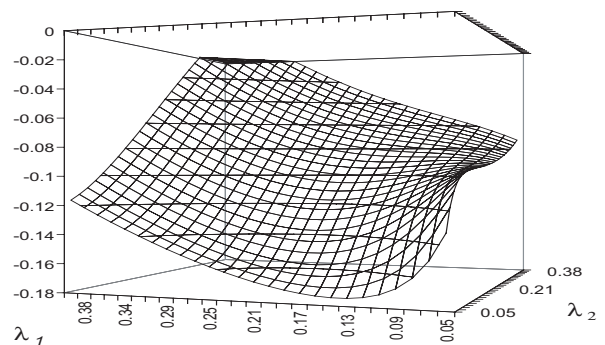
**Figure 5:** Excess correlations of stock market returns due to the portfolio concentration constraint.



**Figure 6:** Value of the multiplier on the market share constraint  $\psi$ .

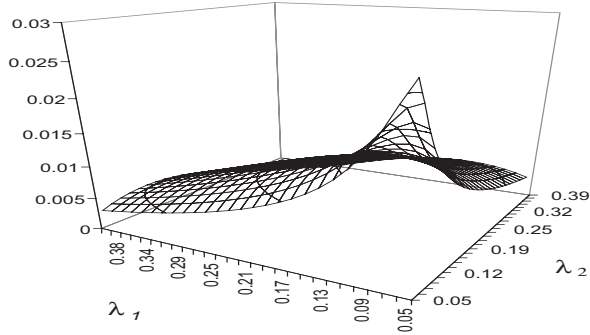


(a) tilt in the investment in the Center's stock

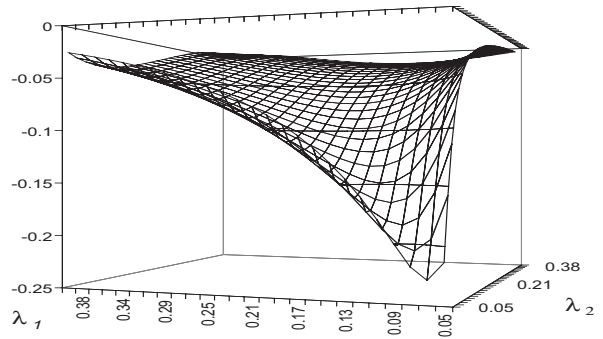


(b) tilt in the investment in a Periphery country's stock

**Figure 7:** Tilt in the asset allocation of the Center relative to the unconstrained allocation due to the market share constraint.

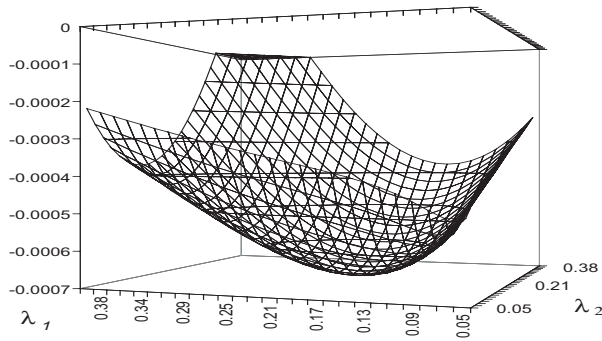


(a) of a negative shock in the Center  $dw^0$

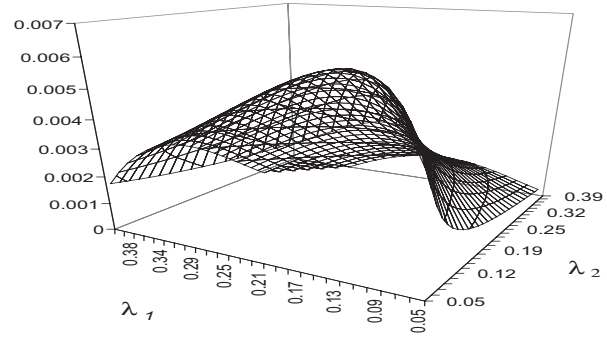


(b) of a negative shock in the Periphery  $dw^1$

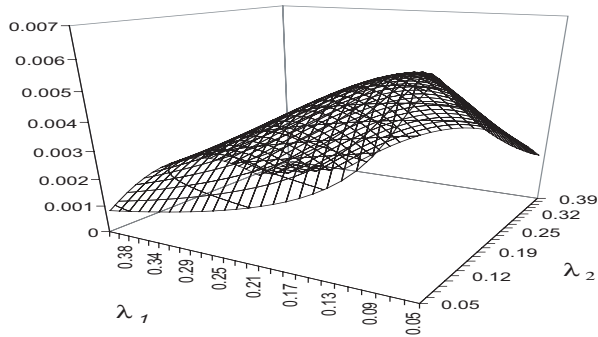
**Figure 8:** The effects of supply shocks on wealth distribution,  $\frac{d\lambda}{\lambda}$ , in the presence of the market share constraint.



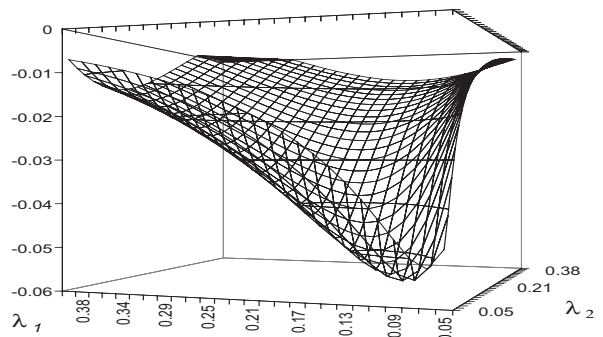
(a) of a negative shock in the Center,  $dw^0$ , on the Center's stock return,  $\frac{dS^0}{S^0}$



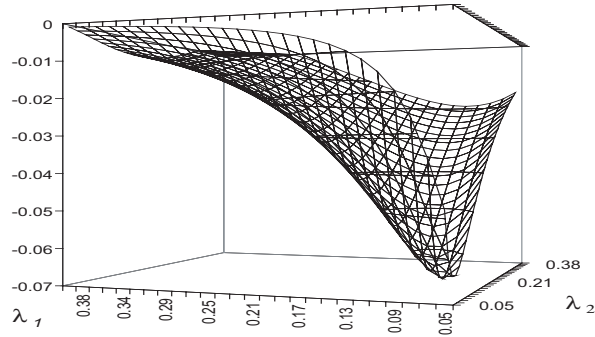
(b) of a negative shock in the Center,  $dw^0$ , on a Periphery country's stock return,  $\frac{dS^1}{S^1}$



(c) of a negative shock in Periphery country 1,  $dw^1$ , on the Center's stock return,  $\frac{dS^0}{S^0}$

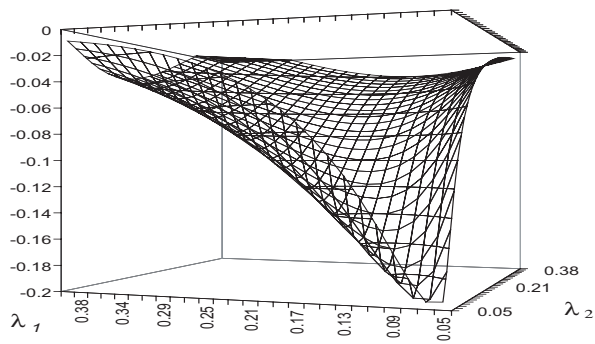


(d) of a negative shock in Periphery country 1,  $dw^1$ , on the Periphery country's stock return,  $\frac{dS^1}{S^1}$

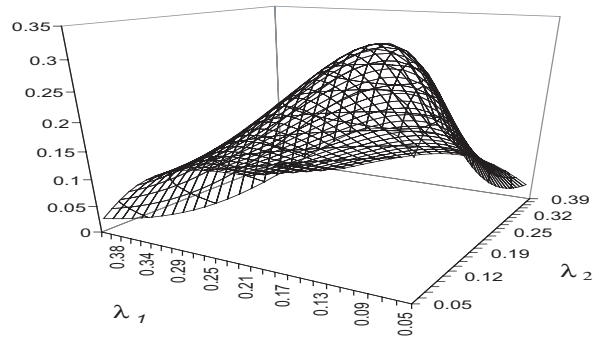


(e) of a negative shock in Periphery country 1,  $dw^1$ , on the other Periphery country's stock return,  $\frac{dS^2}{S^2}$

**Figure 9:** Incremental effects of supply shocks on stock prices in the presence of the market share constraint.



(a) of the Center and Periphery country 1



(b) of Periphery countries 1 and 2

**Figure 10:** Excess correlations of stock market returns due to the market share constraint.