An Asset Pricing View of External Adjustment
Supplementary Appendix (Not for Publication)

Appendix: A Heuristic Derivation of Key Equations

In this appendix we present derivations of our key equations at an intuitive level. A formal treatment can be found in Pavlova and Rigobon (2008a).

We first consider the partial equilibrium optimization problems of the countries. Let us focus on the Home country. The initial step is to reduce the two-good problem to a single-good one. At each time t, we can derive the consumer's demand for the Home and Foreign goods as a function of the overall consumption expenditure at that date, $\mathbb{C}_H(t)$. This is a simple consumer problem under certainty.

$$\max_{C_H, C_H^*} \alpha_H(t) \log C_H(t) + \beta_H(t) \log C_H^*(t)$$

$$\text{s. t. } p(t)C_H(t) + p^*(t)C_H^*(t) \leq \mathbb{C}_H(t).$$

Solving this problem, we obtain the indirect utility function of the form

$$U_H\left(\mathbb{C}_H(t); p(t), p^*(t)\right) = (\alpha_H(t) + \beta_H(t)) \log \mathbb{C}_H(t) + \ldots$$

The only term that depends on the consumption expenditure is the first one, and so we omit the remaining terms. Since the budget constraint (3) can be easily rewritten in terms of the consumption expenditure, we have reduced the multi-good problem to a single-good one.

The next step is to solve the ensuing single-good optimization problem for the consumption expenditure and the trading strategies. Under complete markets, the problem is standard. Under incomplete markets, the problem is more complex, but one can apply the methods developed in He and Pearson (1991) to tackle it. One can show that the country demands a standard mean-variance portfolio and a hedging portfolio to protect against the fluctuations in the preference shock $\alpha_H + \beta_H$. If $\alpha_H + \beta_H$ is a constant, and in particular, if $\beta_H = 1 - \alpha_H$, the country does not demand a hedging portfolio.

Our next step is to derive equilibrium allocations and prices. From the fictitious social planner's problem (5)–(7), we solve for consumption of the Home country (Foreign consumes the remainder of total output)

$$C_H(t) = \frac{\alpha_H(t)}{\alpha_H(t) + \lambda(t)\beta_F} Y(t), \quad C_H^*(t) = \frac{\beta_H(t)}{\beta_H(t) + \lambda(t)\alpha_F} Y^*(t) \tag{23}$$

The marginal rate of substitution between the Home and the Foreign good gives the terms of trade (8).

The price of a stock is given by the expected present value of its dividends, discounted by the pricing kernel. Any agent's marginal utility acts as a pricing kernel. The marginal utility of the Home country is

$$e^{-\rho t}[\alpha_H(t) + \beta_H(t)]\frac{1}{\mathbb{C}_H(t)}.$$

Because of log preferences,

$$\frac{p(t)C_H(t)}{\mathbb{C}_H(t)} = \frac{\alpha_H(t)}{\alpha_H(t) + \beta_H(t)}. \tag{24}$$

Hence, the price of the Home stock is given by

$$S(t) = \frac{1}{MU(t)} E_t \int_t^T MU(s)p(s)Y(s)ds$$

$$= \frac{\mathbb{C}_H(t)}{e^{-\rho t}[\alpha_H(t) + \beta_H(t)]} E_t \int_t^T e^{-\rho s}[\alpha_H(s) + \beta_H(s)]\frac{1}{\mathbb{C}_H(s)}p(s)Y(s)ds$$

$$= \frac{p(t)C_H(t)}{e^{-\rho t}\alpha_H(t)} E_t \int_t^T e^{-\rho s}\alpha_H(s)\frac{Y(s)}{C_H(s)}ds,$$

where MU denotes the marginal utility of Home. Combining this with equation (23), we have

$$S(t) = \frac{p(t)Y(t)}{e^{-\rho t}[\alpha_H(t) + \lambda(t)\beta_F]} E_t \int_t^T e^{-\rho s}[\alpha_H(s) + \lambda(s)\beta_F]ds.$$

By assumption, α_H is a martingale. When markets are complete, λ is a constant; when they are incomplete, λ is a martingale. The former result is technical; see Pavlova and Rigobon (2008a) for a proof. Hence

$$S(t) = \frac{1 - e^{-\rho(T-t)}}{\rho} p(t)Y(t),$$

which is equivalent to (9) given our price normalization $ap(t) + (1-a)p^*(t) = 1$ and the definition of the terms of trade q. The price of the Foreign stock is derived analogously.

To derive the expressions for wealth of the Home country, we note that wealth is the expected present value of future consumption expenditure, discounted at Home's marginal utility:

$$W_H(t) = \frac{1}{MU(t)} E_t \int_t^T MU(s)\mathbb{C}_H(s)ds \quad = \frac{\mathbb{C}_H(t)}{e^{-\rho t}[\alpha_H(t) + \beta_H(t)]} E_t \int_t^T e^{-\rho s}[\alpha_H(s) + \beta_H(s)]ds.$$

A closed-form evaluation of the conditional expectation is possible due to the assumption that both α_H and β_H are martingales. The remainder of the derivation is straightforward (in particular, we use (24) and (23)).