Asset Prices and Institutional Investors*

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Abstract

Empirical evidence indicates that trades by institutional investors have sizable effects on asset prices, generating phenomena such as index effects, asset-class effects and others. It is difficult to explain such phenomena within standard representative-agent asset pricing models. In this paper, we consider an economy populated by institutional investors alongside standard retail investors. Institutions care about their performance relative to a certain index. Our framework is tractable, admitting exact closed-form expressions, and produces the following analytical results. We find that institutions optimally tilt their portfolios towards stocks that comprise their benchmark index. The resulting price pressure boosts index stocks, while leaving nonindex stocks unaffected. By demanding a higher fraction of risky stocks than retail investors, institutions amplify the index stock volatilities and aggregate stock market volatility, and give rise to countercyclical Sharpe ratios. Trades by institutions induce excess correlations among stocks that belong to their benchmark index, generating an asset-class effect.

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1. Introduction

A significant part of the trading volume in financial markets is attributed to institutional investors. Trades by retail investors constitute only a small fraction of the trading volume.[1] In contrast, the standard theories of asset pricing stipulate that prices in financial markets are determined by households (or by the "representative consumer" aggregated over households) who seek to optimize their consumption and investment over their life cycle. This approach leaves no role for important considerations influencing institutional investors' portfolios such as, for instance, compensation-induced incentives or implicit incentives arising from the predictability of inflows of capital into the money management business. This underscores the importance of studying how the incentives of institutional investors may influence the prices of the assets they hold.

In this paper, we take institutional investors to be institutional/professional asset managers. These managers have a mandate to manage a portfolio for a mutual fund, a hedge fund, a pension fund, an endowment, an asset management team in a bank or insurance company, etc. In our analysis, we focus on perhaps the most prominent feature of professional managers' incentives: concern about own performance vis-à-vis some benchmark index (e.g., S&P 500). This characteristic is what induces institutional investors to act differently from retail investors. Relative performance matters because inflows of new money into institutional portfolios and payouts to asset managers at year-end depend on it, or simply because managers care about their standing in the profession (status). Our goal is to demonstrate how, in the presence of such incentives, institutions optimally tilt their portfolios towards the stocks in their benchmark index, influencing the performance of the index, and in the process how they exacerbate leverage in the economy as well as stock market volatility, and boost the correlation among the stocks that are included in the index.

We consider a dynamic general equilibrium model with two classes of investors: "retail" investors with standard logarithmic preferences and "institutional" investors who are concerned not only about their own performance but also about the performance of a benchmark index. The institutional investors have an additional incentive to post a higher return

[1] See, for example, Griffin, Harris, and Topaloglu (2003).

when their benchmark is high than when it is low, in an effort to outdo the benchmark. Formally, their marginal utility of wealth is increasing with the level of their benchmark index. Towards that we take a reduced-form approach in our specification of the institutional investor's objective function that captures the above salient features and admits much tractability. In our model, there are multiple risky stocks, some of which are part of the index, and a riskless bond. The stocks are in positive net supply, while the bond is in zero net supply. The model is designed to capture several important empirical phenomena and to provide the economic mechanisms generating these phenomena. One major advantage of our model is that it delivers exact, closed-form expressions for all quantities which are behind our results described below.

We first examine the tilt in the portfolios of the institutional investors which is caused by the presence of the benchmark indexing. We find that, relative to the retail investor, the institution increases the fraction of index stocks in the portfolio so as not to fall behind when the index does well. To finance this additional demand for index stocks, the institution takes on leverage.[2] So the institutions in our model always end up borrowing funds from the retail sector, to the extent allowed by the size of their assets under management that serve as collateral. As institutions continue to do well and accumulate assets, they increase the overall leverage in the economy, but only up to a certain point determined by the lending capacity of the retail sector.

We next investigate how the presence of institutions influences asset prices. Our first finding is that institutions push up the prices of stocks in the benchmark index. In the economy with institutional investors, the index stock prices are higher both relative to those in the retail-investor-only economy and relative to their (otherwise identical) nonindex counterparts. This is because institutions generate excess demand for the index stocks. This finding is well supported in the data: such an "index effect" occurs in many markets and

[2]Our institutions may be interpreted as mutual funds. Due to regulation, most mutual funds choose to be long-only, although some do use leverage (e.g., the 130/30 funds). Other, less regulated, institutional investors can use leverage (closed-end funds, hedge funds, etc.). We further note that leverage is inevitable in a model with heterogeneous agents and a zero-net supply riskless bond. However, it is not essential for our mechanism; what is essential is that institutions have excess demand for index stocks. In Appendix C, we present a variant of our model without leverage. In that model there are only (positive net supply) risky stocks available for trading and there is no riskless bond. Typically the investors are long in all stocks. They have an excess demand for index stocks and they finance this additional demand by reducing their portfolio weights in nonindex stocks.

countries.[3]

We also find that the price pressure from the institutions boosts the level of the overall stock market in addition to the index. This is because the institutions have a higher demand for risky assets than retail investors. Since the stocks are in fixed supply, the index stocks have to become less attractive for markets to clear. This translates into higher volatilities and lower Sharpe ratios for the index stocks and the overall stock market. The presence of institutions also induces time variation in these quantities; in particular makes Sharpe ratios countercyclical. This is because the institutions are over-weighted in the risky assets. They therefore benefit more from good cash flow news than retail investors, and so become more dominant in the economy. This amplifies the cash flow news and pushes down the Sharpe ratios. As the size of institutions increases, their influence on equilibrium also becomes more pronounced. Therefore, the Sharpe ratios are lower in good times than in bad times. In light of these findings, one can attempt to examine the effects on asset markets of several popular policy recommendations put forward during the 2007-2008 financial crisis. We make no welfare comparisons here; we simply highlight the side effects of some policy recommendations. One such recommendation was to impose leverage caps on institutions, excessive leverage of which had arguably caused the crisis. In our model, when institutions do not control the dominant fraction of wealth in the economy, a leverage cap brings down the riskiness of their portfolios (an intended effect) but it also brings down the level of stock prices, creating an adverse side effect.

Finally, we examine the correlations among stocks included in the index and stocks outside the index. We find that the presence of institutions who care explicitly about their index induces time-varying correlations and generates an "asset-class" effect: returns on stocks belonging to the index are more correlated amongst themselves than with those of otherwise identical stocks outside the index. This asset-class effect is, of course, absent in the retail-

[3]Starting from Harris and Gurel (1986) and Shleifer (1986), a series of papers documents that prices of stocks that are added to the S&P 500 and other indices increase following the announcement and prices of stocks that are deleted drop. For example, Chen, Noronha, and Singal (2004) find that during 1989-2000, the stock price increased by an average of 5.45% on the day of the S&P 500 inclusion announcement and a further 3.45% between the announcement and the actual addition. The corresponding figures for the S&P 500 deletions are -8.46% and -5.97%, respectively. Moreover, the index effect has become stronger after 1989. While there are possible alternative explanations to this phenomenon, the growth of the institutions who benchmark their performance against the index remains a leading one.

investor-only benchmark economy: there, the correlation between any two stocks' returns is determined simply by the correlation of their fundamentals (dividends). The additional correlation among the index stocks is caused by the additional demand of institutions for the index stocks: the institutions hold a hedging portfolio, consisting of only index stocks, that hedges them against fluctuations in the index. Following a good realization of cash flow news, institutions get wealthier and demand more shares of index stocks relative to the retail-investor-only benchmark. This additional price pressure affects all index stocks at the same time, inducing excess correlations among these stocks. Empirical research lends support to our findings; asset-class effects have now been documented in many markets.[4] We get the time-varying correlations in the presence of institutions for the same reasons as for the time-varying volatilities.

It is somewhat surprising that despite extensive empirical work showing that institutions have important effects on asset prices and despite the 2007-2008 financial crisis that has made this point all too obvious, we still have little theoretical work on equilibrium in the presence of professional money management. Brennan (1993) is the first to attempt to introduce institutional investors into an asset pricing model. Brennan considers a static mean-variance setting with constant absolute risk aversion (CARA) utility agents who are compensated based on their performance relative to a benchmark index. He shows that in equilibrium expected returns are given by a two-factor model, with the two factors being the market and the index. More recent related, also static, mean-variance models appear in Gómez and Zapatero (2003), Cornell and Roll (2005), Brennan and Li (2008), Leippold and Rohner (2008), and Petajisto (2009). CARA utility, as is well-known, rules out wealth effects, which play a central role in our paper.

Cuoco and Kaniel (2011) develop a dynamic equilibrium model with constant relative risk aversion (CRRA) agents who explicitly care about an index due to performance-based

[4] For example, Barberis, Shleifer, and Wurgler (2005) show that when a stock is added to the S&P 500 index, its beta with respect to the S&P 500 goes up while its non-S&P 500 "rest of the market" beta falls; and the opposite is true for stocks deleted from the index. Moreover, these effects are stronger in more recent data. Boyer (2011) provides similar evidence for BARRA value and growth indices. He finds that "marginal value" stocks—the stocks that just switched from the growth into the value index—comove significantly more with the value index; the opposite is true for the "marginal growth" stocks. Consistent with the institutional explanation for this phenomenon, Boyer finds that the effect appears only after 1992, which is when BARRA indices were introduced.

fees.[5] In a two-stock economy, Cuoco and Kaniel show that inclusion in an index increases a stock's price and illustrate numerically that it also lowers its unconditional expected return and increases its unconditional volatility. However, in an exercise more closely related to the one we perform in this paper, they show numerically that, in contrast to our work, the conditional volatilities of the index stock and aggregate stock market decrease in the presence of benchmarking. In another closely related paper, He and Krishnamurthy (2012a) consider a dynamic single-stock model with CRRA (logarithmic) institutions, in which institutions are constrained in their portfolio choice due to contracting frictions. They show that in bad states of the world (crises), institutional constraints are particularly severe, causing increases in the stock's Sharpe ratio and conditional volatility and replicating other patterns observed during crises. This literature remains sparse due to the modeling challenges of tractably solving for asset prices in the presence of wealth effects and multiple assets. We overcome this challenge by modeling institutions differently: our model has the tractability of CARA-based models but it additionally features wealth effects. This tractability not only allows us to elucidate the mechanisms through which institutions influence asset prices, but also to extend our setting to multiple risky stocks, permitting an analysis of the "asset-class" effect. The closest theoretical model that exhibits the "asset-class" effect is by Barberis and Shleifer (2003), whose explanation for this phenomenon is behavioral. By providing microfoundations for investors' demand schedules, we can establish a set of primitives that give rise to the asset-class effect and discuss what these primitives imply for other equilibrium quantities (time-varying volatilities, Sharpe ratios, leverage, risk tolerance and others). Moreover, the correlations of stocks within an asset class in our model are time-varying due to wealth effects.

Other related papers that have explored equilibrium effects of delegated portfolio management include He and Krishnamurthy (2012b), in which poor performance of fund managers triggers portfolio outflows due to contracting frictions, Dasgupta and Prat (2008), Dasgupta, Prat, and Verardo (2008), Malliaris and Yan (2010), Vayanos and Woolley (2010), Guerreri and Kondor (2012), in which outflows following poor performance are due to learning about managerial ability, and Vayanos (2004) and Kaniel and Kondor (2012) in which outflows oc-

[5]See also Kapur and Timmermann (2005) and Arora, Ju, and Ou-Yang (2006) for related dynamic models.

cur for exogenous reasons, dependent on fund performance. They show that, similar to our findings, flow-based considerations amplify the effects of exogenous shocks on asset prices. All of these papers model various agency frictions. In our model, we simplify this aspect, but offer a richer model of a securities market. We view these papers as complementary to our work. Our paper is also connected to the literature on relative wealth concerns and asset prices (Galí (1994)). For example, in DeMarzo, Kaniel, and Kremer (2004, 2008), where such concerns arise endogenously, agents care about their relative wealth in the community which causes them to overinvest in stocks held by other members of their community. In our model, the institutional investors end up overinvesting in the index stocks.

Finally, there is a related literature on the effects of fund flows and benchmarking considerations on portfolio choice of fund managers, at a partial equilibrium level. For example, Carpenter (2000), Basak, Pavlova, and Shapiro (2007, 2008), Hodder and Jackwerth (2007), Binsbergen, Brandt, and Koijen (2008), and Chen and Pennacchi (2009) show that future fund flows induce a manager to tilt her portfolio towards stocks that belong to her benchmark. These papers demonstrate that there is a range over which such benchmarking considerations induce her to take more risk. The main difference of our paper from this body of work is that we examine the general equilibrium effects of benchmarking.

The remainder of our paper is organized as follows. Section 2 presents a simplified single-stock version of our model for which we establish a number of our results in the clearest possible way. Section 3 discusses the index effect, institutional risk-taking, wealth effects, and the resulting policy implications. Section 4 presents the general multi-stock version of our model and focuses on the asset-class effect and Section 5 summarizes our key predictions and empirical implications. Appendix A contains all proofs, Appendix B generalizes the analysis to nonzero dividend growth and interest rate, Appendix C presents the stocks-only version of our model, and Appendix D provides an agency-based justification for the institutional objective function.

2. Economy with Institutional Investors

2.1. Economic Setup

We consider a simple and tractable pure-exchange security market economy with a finite horizon. The economy evolves in continuous time and is populated by two types of market participants: retail investors, $\mathcal{R}$, and institutional investors, $\mathcal{I}$. In the general specification of our model, there are N stocks, M of which are included in the index against which the performance of institutions is measured, as well as a riskless bond. In this section, however, we specialize the securities market to feature a single risky stock, henceforth referred to as the stock market *index*, and a riskless bond. The index is exposed to a single source of risk represented by a Brownian motion ω. The main reason for considering the single-stock case is expositional simplicity. It turns out that a number of key insights of this paper can be illustrated within the single-stock economy. We then build on our baseline intuitions and expand them (Section 4) to demonstrate how our economy behaves in the general case in which there are multiple stocks and multiple sources of risk.

Investment opportunities. The stock market index, S, is posited to have dynamics given by

$$dS_t = S_t[\mu_{St}dt + \sigma_{St}d\omega_t], \tag{1}$$

with $\sigma_{St} > 0$. The mean return μ_S and volatility σ_S are determined endogenously in equilibrium (Section 3). The bond is in zero net supply. It pays a riskless interest rate r, which we set to zero without loss of generality.[6] The stock market index is in positive net supply. It is a claim to the terminal payoff (or "dividend") D_T, paid at time T, and hence $S_T = D_T$. This payoff D_T is the terminal value of the process D_t, with dynamics

$$dD_t = D_t[\mu dt + \sigma d\omega_t], \tag{2}$$

where μ and $\sigma > 0$ are constant. The process D_t represents the arrival of news about

[6] This is equivalent to using the riskless bond as the numeraire and denoting all prices in terms of this numeraire. Such an assumption is innocuous because our model does not have intermediate consumption. In other words, there is no intertemporal choice that would pin down the interest rate. Our normalization is commonly employed in models with no intermediate consumption (see e.g., Pastor and Veronesi (2012) for a recent reference). In Appendix B, we incorporate a nonzero (constant) interest rate. This change modifies our formulae, but leaves our economic insights unchanged.

D_T. We refer to it as the *cash flow news.* Equation (2) implies that cash flow news arrives continuously and that D_T is lognormally distributed. The lognormality assumption is made for technical convenience. For expositional purposes, we set $\mu = 0$, as this simplification does not alter any of our economic insights.[7]

Investors. Each type of investor $i = \mathcal{I}, \mathcal{R}$ in this economy dynamically chooses a portfolio process ϕ_i, where ϕ_i denotes the fraction of the portfolio invested in the stock index, or the *risk exposure*, given the initial assets of W_{i0}. The wealth process of investor i, W_i, then follows the dynamics

$$dW_{it} = \phi_{it} W_{it} [\mu_{St} dt + \sigma_{St} d\omega_t]. \tag{3}$$

The (representative) institutional and retail investors are initially endowed with fractions $\lambda \in [0, 1]$ and $(1 - \lambda)$ of the stock market index, providing them with initial assets worth $W_{\mathcal{I}0} = \lambda S_0$ and $W_{\mathcal{R}0} = (1 - \lambda) S_0$, respectively.[8] The parameter λ thus represents the (initial) fraction of the institutional investors in the economy—or equivalently, how large the institutions are relative to the overall economy. It is an important comparative statics parameter in our analysis, which allows us to illustrate how the growth of the financial sector (or more precisely, funds managed by institutions) can influence asset prices.

The retail investor has standard logarithmic preferences over the terminal value of her portfolio:

$$u_{\mathcal{R}}(W_{\mathcal{R}T}) = \log(W_{\mathcal{R}T}). \tag{4}$$

In modeling the institutional investor's objective function, we consider two noteworthy features of the professional money management industry that make institutions behave differently from retail investors. First, institutional investors care about their benchmark index. This can be due to implicit or explicit incentives. The implicit incentives to perform well in relative terms come in the form of inflow/outflow of funds in response to relative performance.

[7] Appendix B generalizes the analysis to incorporate a nonzero dividend growth rate and demonstrates that all our predictions remain equally valid. Moreover, in related analysis (not presented here due to space limitations), we relax the lognormality assumption and show that the bulk of our results remains valid for more general stochastic processes, but the characterization of our economy becomes more complex.

[8] We do not explicitly model households, who delegate their assets to institutions to manage, but simply endow the institutions with an initial portfolio. The households who delegate their money to the institutions can be thought of, for example, as participants in defined benefit pension plans (worth $3.14 trillion in the US as of June 2009 according to official figures and significantly more according to Novy-Marx and Rauh (2010)).

The fee of practically any type of an institutional asset manager includes a fraction of funds under management. Institutional asset managers therefore seek to perform well relative to their peer group so as to attract more new funds than their less successful peers. This positive flows-performance relation is a prevalent finding in asset management, as documented by Chevalier and Ellison (1997) and Sirri and Tufano (1998) for mutual funds, Agarwal, Daniel, and Naik (2004) and Ding, Getmansky, Liang and Wermers (2010) for hedge funds, and Del Guercio and Tkac (2002) for pension funds. For the purposes of highlighting explicit incentives of fund managers, it is important to distinguish between external and internal managers. External managers (working for e.g., large pension funds or endowments) typically have mandates that get reviewed every one to five years by the trustees. These reviews are based to a large extent on their performance relative to passive benchmarks, and these managers win/lose mandates as well as attract inflows based on their relative performance (BIS (2003), p.22). With the exception of hedge funds, internal managers receive bonuses that are linked to performance relative to their benchmarks (e.g., BIS (2003), p.23; Ma, Tang, and Gómez (2012)). Such explicit and implicit incentives make both types of managers care about their relative performance. This discussion leads us to the second feature of professional asset management that we attempt to capture: money managers strive to post a higher return when their benchmark is high than when it is low, in an effort to outdo their benchmark. Putting this formally, their marginal utility of wealth is increasing in the level of their benchmark index. This feature can also be microfounded using an agency-based argument, following the ideas of Holmström (1979, 1982). In Appendix D, we employ the approach of Edmans and Gabaix (2011) to demonstrate it formally. Accordingly, we formulate the institutional investor's objective function over the terminal value of her portfolio as being given by:

$$u_{\mathcal{I}}(W_{\mathcal{I}T}) = (a + bS_T)\log(W_{\mathcal{I}T}), \tag{5}$$

where a, $b > 0$. We set $a = 1$, since by homogeneity only the ratio b/a matters in the ensuing analysis. In this one-stock economy, the manager's benchmark index coincides with the stock market.[9]

[9]The objective function then has another interpretation: the institutional investor has an incentive to perform well during bull markets (high S_T). This is plausible since empirical evidence indicates that during bull markets payouts to money managers are especially high. For example, there are higher money inflows

There are, of course, multiple alternative specifications that are consistent with benchmark indexing, but the empirical literature to date is unclear as to what the exact form of the dependence on the benchmark index should be. An interesting recent attempt to estimate the form of a money manager's objective function is by Koijen (2010). In (5) we have chosen a particularly simple affine specification, which renders tractability to our model. This specification is as tractable as CARA utility, but it behaves like CRRA preferences, inducing wealth effects. In future work, it is certainly desirable to extend our specification to a more general class of functions.

Remark 1 (Alternative specifications for institutional objective). A natural, alternative specification of our institutional objective function is $u_{\mathcal{I}}(W_{\mathcal{I}T}) = (1+bS_T/S_0)\log(W_{\mathcal{I}T})$, which is defined over the return on the index as opposed to the level of the index. Since S_0 is endogenously determined in equilibrium, this specification is more difficult to analyze. Nonetheless, we demonstrate that all our main implications remain valid under this specification, and our model continues to deliver closed-form solutions (see Remark 3).

One may be concerned that our institutional objective function is increasing in the index level, whereas contract theory (see Appendix D) and common sense suggests that it should instead be decreasing. To alleviate this concern, it is important to note that for the purposes of deriving asset pricing implications, what really matters is the marginal utility of wealth. Our objective function can be made decreasing in the index level if we subtract from it a sufficiently increasing function of S_T, such as e.g., $\log S_T$. This transformation does not impact the marginal utility of the institutional investor and hence none of our expressions change.

Finally, we note that our specification of the institutional objective function is not the only one that delivers the property that the marginal utility is increasing in the index level. One alternative specification satisfying this property is $u_{\mathcal{I}}(W_{\mathcal{I}T}) = \log(W_{\mathcal{I}T} - S_T)$, or a variant of this utility that penalizes gains and losses differently. While this is certainly a valid specification to consider, it loses its tractability with multiple stocks and asset classes. For CRRA investors with the objective defined over $\alpha + \beta W_{\mathcal{I}T} + \gamma(W_{\mathcal{I}T} - S_T)$, Cuoco and Kaniel (2011) obtain numerically a subset of our results for a two-stock economy, which is a valuable robustness check for our findings. Under the CARA objective $-e^{a(W_{\mathcal{I}T}-S_T)}$, Brennan (1993) and subsequent literature are able to tackle the multiple-stock case analytically. This

into mutual funds following years when the market has done well (e.g., Karceski (2002)), and so fund managers have an implicit incentive to do well in those years so as to attract a larger fraction of the inflows. Managers guided by these incentives would perform worse in bad times, but fund outflows are typically a lot less responsive to poor relative performance (e.g., Sirri and Tufano (1998)).

The mechanism through which the institutional managers' payouts are computed is unfortunately complex and opaque, but vast anecdotal evidence suggests that bonuses are higher in good years and especially of those managers who have done well in those years. One could also draw inferences from the CEO compensation literature documenting that payouts are positively correlated with the stock market returns (e.g., Gabaix and Landier (2008)).

literature, again, provides an important robustness check, but our implications are richer because of the presence of wealth effects. The key property that unites the objective functions that have been proposed in this related literature is that the marginal utility is increasing in the index level.

Remark 2 (Status interpretation). One may alternatively interpret the objective function of our institutional investor as that of an agent with a preference for status. Building on the ideas of Friedman and Savage (1948) and developing them more formally, Robson (1992) has proposed to model status concerns by introducing an additional argument in the utility function. This additional argument captures the aggregate wealth/consumption of the comparison group. In Finance, a related paper exploring this idea is DeMarzo, Kaniel and Kremers (2004). In particular, similarly to us, these authors argue that the marginal utility of consumption should be increasing with the level of community consumption (p. 1700). DeMarzo et al. discuss how such status concerns may induce agents to overinvest in the assets that are correlated with the status variable. In our model, the institutional investor overinvests in the index—a benchmark for his performance evaluation within the investment management community.

The view that an institutional investor's desire to do well relative to an index may not be entirely due to monetary incentives but is instead driven by social esteem considerations is supported by the experimental/behavioural evidence on status concerns. Ball et al. (2001) present evidence from a lab experiment demonstrating that subjects care about status and that the preference for status affects economic outcomes. They argue further that status has to be publicly observable to influence outcomes. This is one of the reasons why status concerns are particularly important in labor markets, in which one's standing in the profession is easier to observe (as also argued by, e.g., Ellingsen and Johannesson (2007) and Heffetz and Frank (2009)). In professional money management, status is associated with a fund's relative performance, a publically observable characteristic for most funds.

Direct empirical support for the status-based interpretation of our model is provided in Hong, Jiang, and Zhao (2011), who adopt the formulation in this section as a basis for their analysis. Using Chinese data they argue that status concerns of residents of wealthier provinces in China influence their risk exposure and affect asset prices, in directions as predicted by our model.

2.2. Investors' Portfolio Choice

Each type of investor's dynamic portfolio problem is to maximize her expected objective function in (4) or (5), subject to the dynamic budget constraint (3). Lemma 1 presents the investors' optimal portfolios explicitly, in closed form.

Lemma 1. *The institutional and retail investors' portfolios are given by*

$$\phi_{\mathcal{I}t} = \frac{\mu_{St}}{\sigma_{St}^2} + \frac{b\,D_t}{1 + b\,D_t}\,\frac{\sigma}{\sigma_{St}}, \tag{6}$$

$$\phi_{\mathcal{R}t} = \frac{\mu_{St}}{\sigma_{St}^2}. \tag{7}$$

Consequently, the institution invests a higher fraction of wealth in the stock market index than the retail investor does, $\phi_{\mathcal{I}t} > \phi_{\mathcal{R}t}$.

The first term in the expression for the institutional investor's portfolio is the standard (instantaneous) mean-variance efficient portfolio. It is the same mean-variance portfolio that the retail investor holds. The wedge between the portfolio holdings of the two groups of investors is created by the second term in (6): the hedging portfolio. This hedging portfolio arises because the institution has an additional incentive to do well when his benchmark does well, and so the hedging portfolio is positively correlated with cash flow news (D_t). The instrument that allows the institution to achieve a higher correlation with cash flow news is the stock market index itself, and so the institution holds more of it than does the retail investor. This implies that the institution ends up taking on more risk than the retail investor does. We are going to demonstrate shortly (Section 3.2) that in equilibrium, the institution finances its additional demand for the stock by borrowing from the retail investor. So the higher effective risk appetite of the institutional investor induces her to lever up. One can draw parallels with the 2007-2008 financial crisis, in which leverage of financial institutions was one of the key factors contributing to the instability. Excessive leverage has often been ascribed to the bonus structure of market participants. While we do not dispute the conclusion that an option-like compensation function can generate excessive risk taking, we would like to stress that a simple incentive to do well when the stock market index is high, which we model here, also leads to a higher effective risk appetite.

We here note the resemblance of the results in Lemma 1 to those of Brennan (1993). In a static setting, Brennan argues that an investor who is paid based on performance relative to an index has an additional demand for the index portfolio. A similar observation is made in the portfolio choice literature studying the behavior of mutual funds (e.g., Basak, Pavlova, and Shapiro (2007) and Binsbergen, Brandt, and Koijen (2008)). Cuoco and Kaniel (2011) make a related point in the context of a dynamic equilibrium model and provide explicit solutions for the case of investors compensated with fulcrum fees, though their mechanism is different and it does depend on the nature on the fees. In particular, the managers' equilibrium portfolios are buy-and-hold, while our managers in equilibrium buy in response to good cash flow news and grow in the importance in the economy (Figure 2), which is central to our mechanism.

3. Equilibrium in the Presence of Institutional Investors

We are now ready to explore the implications of the presence of institutions in the economy on asset prices and their dynamics. As we have shown in the previous section, institutions have an incentive to take on more risk relative to the retail investors, and hence their presence increases the demand for the risky stock. In this section, we demonstrate how these incentives boost the price and the volatility of the risky stock and how they affect the behavior of all market participants.

Equilibrium in our economy is defined in a standard way: equilibrium portfolios and asset prices are such that (i) both the retail and institutional investors choose their optimal portfolio strategies, and (ii) stock and bond markets clear. We will often make comparisons with equilibrium in a *benchmark economy* in which institutions are not concerned about the index (i.e., $b = 0$) and so behave as retail investors. We refer to this economy as the economy without institutional investors.

3.1. Stock Price, Volatility, and Index Effect

Proposition 1. *In the economy with institutional investors, the equilibrium level of the stock market index is given by*

$$S_t \;=\; \overline{S}_t \frac{1 + b\,D_0 + \lambda\,b(D_t - D_0)}{1 + b\,D_0 + \lambda\,b\,(e^{-\sigma^2(T-t)}D_t - D_0)}, \tag{8}$$

where $\overline{S}_t$ is the equilibrium index level in the benchmark economy with no institutional investors given by

$$\overline{S}_t \;=\; e^{-\sigma^2(T-t)}D_t. \tag{9}$$

Consequently, the stock market index level is increased in the presence of institutional investors, $S_t > \overline{S}_t$. Moreover, it increases with the fraction λ of the institutional investors in the economy.

The presence of institutions generates price pressure on the stock market index. Recall that institutions in our model have a higher demand for the risky stock than retail investors. Therefore, relative to the benchmark economy, there is an excess demand for the stock market index. The stock is in fixed supply, and so its price must be higher. As the fraction of institutional investors goes up (λ increases), there is more price pressure on the index, pushing it up further. This is the simplest way to capture the "index effect" in our model. (This result is generalizable to the multi-stock case. In that case, only the stocks included in the index trade at a premium due to the excess demand for these stocks by the institutions; prices of the nonindex stocks remain unchanged. See Section 4.) Finally, it is worth noting that the expressions for asset prices that we derive here and below are all simple and in closed form. This is a very convenient feature of our framework, which allows us to explore the economic mechanisms in play within our model and comparative statics without resorting to numerical analysis.

Since institutional investors affect the level of the index, it is conceivable that they also influence its volatility. They demand a riskier portfolio relative to that of the retail investors, and so one would expect them to amplify the riskiness of the index. Proposition 2 verifies this conjecture.

Proposition 2. *In the equilibrium with institutional investors, the volatility of the stock market index returns is given by*

$$\sigma_{St} = \overline{\sigma}_{St} + \lambda\, b\, \sigma \frac{\left(1 - e^{-\sigma^2(T-t)}\right)(1 + (1-\lambda) b D_0) D_t}{(1 + (1-\lambda)\, b\, D_0 + \lambda\, b\, e^{-\sigma^2(T-t)} D_t)\,(1 + (1-\lambda)\, b\, D_0 + \lambda\, b\, D_t)}, \tag{10}$$

where $\overline{\sigma}_{St}$ is the equilibrium index volatility in the benchmark economy with no institutions, given by

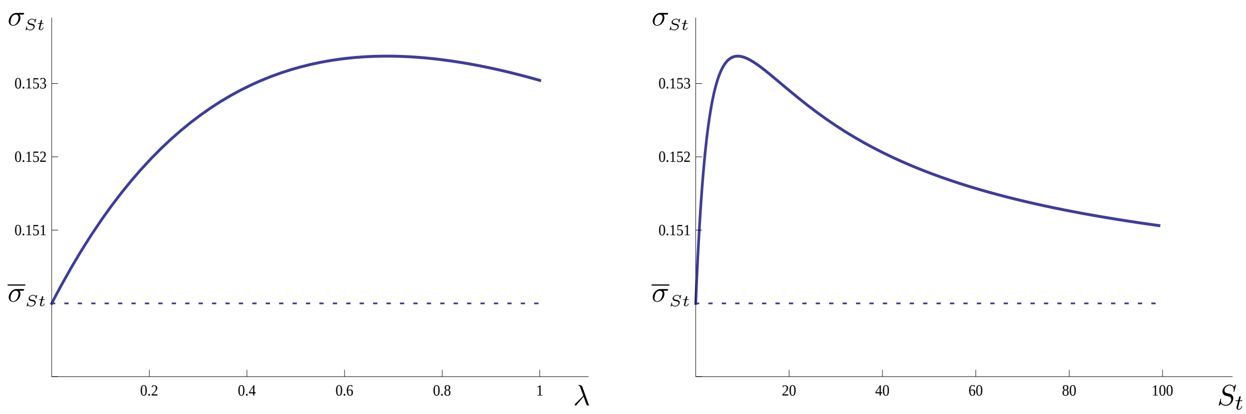
$$\overline{\sigma}_{St} = \sigma.$$

Consequently, the index volatility is increased in the presence of institutions, $\sigma_{St} > \overline{\sigma}_{St}$.

In the benchmark economy with no institutional investors, the index return volatility is simply a constant. In the presence of institutional investors, it becomes stochastic, and in particular, dependent on the cash flow news. It also depends on the fraction of institutional investors in the economy, λ. The notable implication here is that institutional investors make the stock more volatile. In other words, the effects of cash flow news are amplified by institutional investors. This is again due to the institutions' higher risk appetite. The institutions demand a riskier portfolio, but the risky stock market is in fixed supply. Hence, to clear markets, the stock market must become relatively less attractive in the presence of institutions. In our framework, that is achieved by the market volatility increasing relative to the benchmark economy with no institutions.

Figure 1 depicts the equilibrium index volatility as a function of the size of the institutions in the economy (λ) and the stock market level (S_t). As institutions become larger, they constitute a larger fraction of the economy, and hence the risk appetite in the economy increases. Along with that comes an increase in the total leverage taken out by the institutions and an increase in the volatility of index returns. However, the institutions' ability to lever up depends on the lending capacity of retail investors, who in equilibrium provide a counterparty to the institutional investors in the market for borrowing/lending. As the fraction of institutions increases further, there is lesser lending capacity that can be provided by the retail investors. This in turn forces the institutional leverage to go down in equilibrium, pushing down the index volatility along with it. This explains the peak in the volatility in panel (a) of Figure 1. Turning to panel (b) of Figure 1, depicting the behavior of the stock market index volatility as a function of the stock market level, we see that for most values of the stock market, the volatility increases in response to a decreasing stock market. This is consistent with the empirical evidence that the stock market volatility increases in bad times (Schwert (1989), Mele (2007)). We note from both panels of Figure 1 that the magnitudes of our volatility effects are fairly small. This is perhaps not so surprising given that we employ logarithmic preferences.[10]

[10] We conjecture that to obtain larger magnitudes of the stock market volatility in our model, one would need to employ higher levels of risk aversion or add habits to the objective functions (as in, e.g., Campbell and Cochrane (1999)).



(a) Effect of the size of institutions

(b) Effect of the stock market level

Figure 1: **Equilibrium index volatility.** This figure plots the index volatility in the presence of institutions against the fraction of institutions in the economy λ and against the stock market index level S_t. The dotted lines correspond to the equilibrium index volatility in the benchmark economy with no institutions. The plots are typical. The parameter values are: $b = 1$, $D_0 = 1$, $\sigma = 0.15$, $t = 1$, $T = 5$. In panel (a) $D_t = 2$, and in panel (b) $\lambda = 0.2$.

3.2. Risk Taking, Leverage, and Wealth Effects

To further understand the underlying economic mechanisms operating in our model, we look more closely at the investors' portfolios in equilibrium. Towards that, we also look at the investors' portfolios in terms of the number of shares in the risky stock, i.e.,

$$\pi_{\mathcal{I}t} = \phi_{\mathcal{I}t} \frac{W_{\mathcal{I}t}}{S_t}, \quad \pi_{\mathcal{R}t} = \phi_{\mathcal{R}t} \frac{W_{\mathcal{R}t}}{S_t},$$

where as before ϕ_{it} denotes the fraction of investor's wealth invested in the index. This is to enable us to explicitly identify the nature of the wealth effects in the economy, i.e., who buys or sells in response to cash flow news. Proposition 3 reports the investors' equilibrium portfolios, as well as an important property of the institutional portfolio holdings.

Proposition 3. *The institutional and retail investors' portfolios in equilibrium are given by*

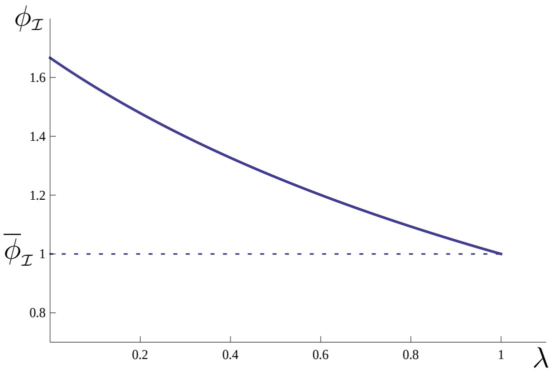
$$\pi_{\mathcal{I}t} = \lambda \frac{1 + b\,D_t}{1 + (1-\lambda)\,b\,D_0 + \lambda\,b\,D_t} \left(1 - \frac{\lambda\,b\,D_t}{1 + (1-\lambda)\,b\,D_0 + \lambda\,b\,D_t} \frac{\sigma}{\sigma_{St}} + \frac{b\,D_t}{1 + b\,D_t} \frac{\sigma}{\sigma_{St}}\right), \quad (11)$$

$$\pi_{\mathcal{R}t} = (1-\lambda) \frac{1 + b\,D_0}{1 + (1-\lambda)\,b\,D_0 + \lambda\,b\,D_t} \left(1 - \frac{\lambda\,b\,D_t}{1 + (1-\lambda)\,b\,D_0 + \lambda\,b\,D_t} \frac{\sigma}{\sigma_{St}}\right), \quad (12)$$

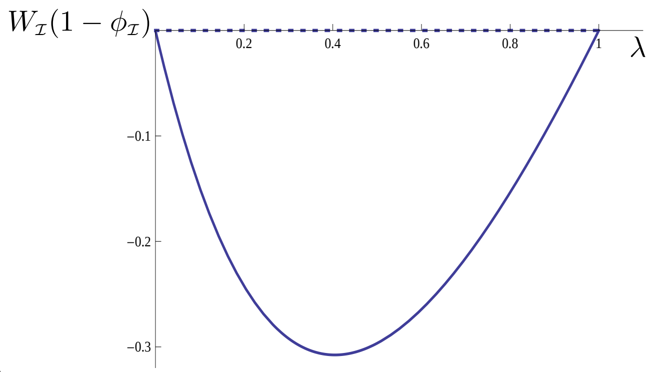
where σ_{St} is as in Proposition 2.
Consequently, for $\lambda \in (0,1)$ the institutional investor is always levered, $W_{It}(1-\phi_{\mathcal{I}t}) < 0$.

To better highlight the results in Proposition 3, Figure 2 plots the institutional investor's equilibrium portfolios against the size of the institution (λ) and cash flow news (D_t). We see that the institution always "tilts" her portfolio towards the index stock, as compared to an otherwise identical benchmark investor who does not directly care about the index (Figure 2(a)). Indeed, in the benchmark economy with no institutions the investor puts all his wealth in the stock market ($\overline{\phi}_{\mathcal{I}} = 1$). Here, the institutional investor holds a higher fraction of his wealth in the stock market. In order to be able to finance this additional demand, the institution borrows from the retail investor and so it always levers up in equilibrium (Figure 2(b)). The bell-shaped plot in Figure 2(b) is an important illustration of how leverage in the economy depends on the size of the institutional sector. One extreme is when the size of the institutional sector is zero ($\lambda = 0$). In that case, all agents in the economy are retail investors with identical preferences, and so no one is willing to take a counterparty position in the market for borrowing and lending (recall that the bond is in zero net supply). The bondholdings of all investors are then equal to zero. The other extreme is when there are no retail investors in the economy ($\lambda = 1$). Again, there is no heterogeneity to induce borrowing and lending in equilibrium, and the bondholdings are zero. In the intermediate range, $0 < \lambda < 1$, the institution borrows from the retail investor, using its initial wealth as collateral. The budget constraint always forces the borrower to repay; the higher the initial wealth, the more leverage the borrower is able take on. This is why we see an increase in the overall leverage as the size of the institutional sector starts to increase (λ increases). At a certain point, however, it peaks and then starts to fall. This is because, as the institutional sector becomes larger, the size of the retail sector $(1-\lambda)$ shrinks, and therefore the lending capacity of the retail sector progressively reduces. This in turn reduces the equilibrium leverage in the economy.

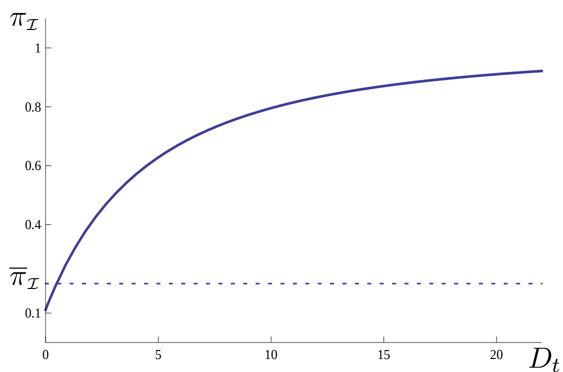
In Figure 2(c) we illustrate the response of the institutional investors' equilibrium portfolios to cash flow news. Rebalancing following positive cash flow news is simply a "wealth effect" (as highlighted by, e.g., Kyle and Xiong (2001)). In equilibrium, both types of in-



(a) Effect of size of institution



(b) Effect of size of institution



(c) Effect of cash flow news

Figure 2: **The institutional investor's portfolio holdings.** Panels (a) and (b) of this figure plot the institution's fraction of wealth invested in the index $\phi_{\mathcal{I}}$ and the bond holdings $W_{\mathcal{I}}(1-\phi_{\mathcal{I}})$ against the size of the institution λ. Panel (c) plots the institution's holdings of the shares of the index $\pi_{\mathcal{I}}$ against cash flow news D_t. The lines for $\overline{\pi}$ correspond to the holdings of an otherwise identical investor in the benchmark economy. The plots are typical. In panels (a) and (b) $D_t = 2$, and in panel (c) $\lambda = 0.2$. The remaining parameter values are as in Figure 1.

vestors have positive holdings of the risky stock, and so good cash flow news translates into higher wealth for each investor. As the investors become wealthier, they want to increase the riskiness of their portfolios, which in this model implies buying more shares of the risky stock. Of course, for the stock market to clear, both investors cannot be buying the stock simultaneously; one of them has to sell. To determine who is buying and who is selling, one can look at the change in the wealth distribution in the economy. In this case, as positive cash flow news arrives (D_t increases), the wealth distribution shifts in favor of the institutional investor. Intuitively, this is because the institutional portfolio is over-weighted in the risky stock relative to that of the retail investor, and so good news about the stock produces a higher return on the institutional portfolio relative to that of the retail investor.[11] Hence, following good cash flow news, the institution buys from the retail investor (Figure 2(c)). This wealth effect is an important part of the economic mechanisms that operate in our model. It is useful to stress at this point that the bulk of the related literature, developed in the framework in which investors have CARA preferences, is not able to capture wealth effects. The assumption of CARA utilities is made, of course, for tractability. In our model, tractability is achieved through alternative channels, which we highlight in this section and the next.

3.3. Sharpe Ratio and Further Discussion

We now explore the behavior of the Sharpe ratio (or market price of risk), stock mean return per unit volatility $\kappa_t \equiv \mu_{St}/\sigma_{St}$, in the presence of institutions in equilibrium. It has been well-documented in the data that this quantity is countercyclical. It is of interest to explore the nature of the time variation in the Sharpe ratios that the presence of institutions may induce.

[11] We show formally that the institution becomes wealthier relative to the retail investor following good cash flow news, i.e., $W_{\mathcal{I}t}/W_{\mathcal{R}t}$ increases with D_t, in the proof of Proposition 3 in Appendix A. In particular, we show that the wealth distribution is given by

$$\frac{W_{\mathcal{I}t}}{W_{\mathcal{R}t}} = \frac{\lambda}{1-\lambda}\,\frac{1+b\,D_t}{1+b\,D_0}.$$

Proposition 4. *In the economy with institutional investors, the Sharpe ratio is given by*

$$\kappa_t = \frac{1 + (1-\lambda)\, b\ D_0}{1 + (1-\lambda)\, b\, D_0 + \lambda\, b\, e^{-\sigma^2(T-t)} D_t}\, \overline{\kappa}, \tag{13}$$

where the benchmark economy Sharpe ratio is $\overline{\kappa} = \sigma$*.*
Consequently, in equilibrium:

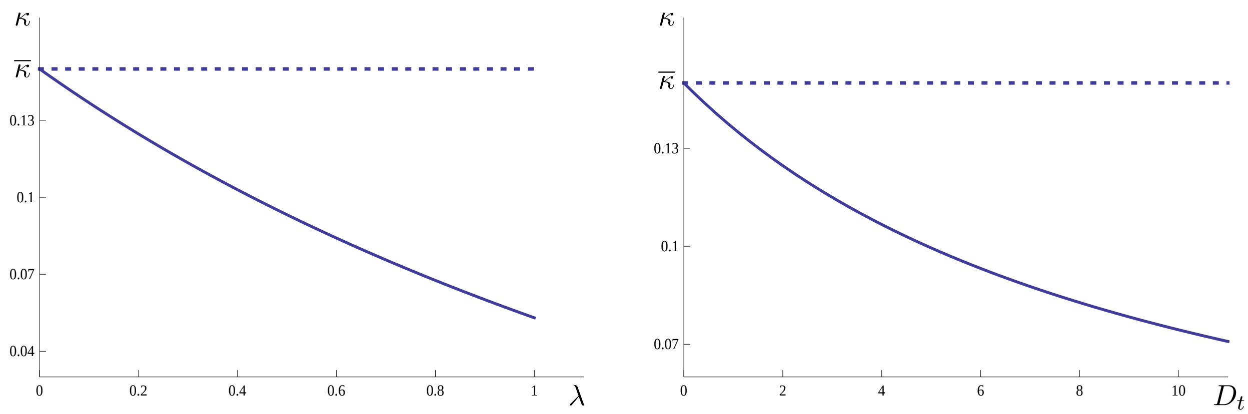
(i) the Sharpe ratio is decreased in the presence of institutions;

(ii) the Sharpe ratio decreases with the fraction λ *of institutional investors in the economy;*

(iii) the Sharpe ratio decreases following good cash flow news.

In the benchmark economy with no institutions, the Sharpe ratio is constant. As revealed by Proposition 4, the presence of institutions causes the Sharpe ratio to decrease and to become countercyclical. As with the volatility effects, this is due to the institutions demanding a riskier portfolio. However, since the risky stock is in fixed supply, it must become less attractive in the presence of institutions to clear markets. So, the stock market Sharpe ratio decreases, and its volatility simultaneously increases, relative to the benchmark economy with no institutions. The decrease in the Sharpe ratio is more pronounced with more institutions in the economy (Figure 3(a) and property (ii) of Proposition 4). The countercyclicality of the Sharpe ratio is due to wealth transfers between institutions and retail investors. Because the institutions are over-weighted in the risky stock relative to the retail investors, good cash flow news always produces a wealth transfer from the retail investors to the institutions (footnote 11). So, the higher the prospects of the economy D_t, the bigger the share of wealth managed by the institutions, and hence the higher is their impact in equilibrium. The Sharpe ratio is therefore decreasing in D_t (Figure 3(b), property (iii) of Proposition 4).

A similar price-pressure intuition applies to the expected (excess) stock market index return in the economy, μ_S. However, the comparative statics for μ_S is more complex. This is because by no arbitrage $\mu_{St} = \kappa_t \sigma_{St}$, and, as we have shown in Propositions 2 and 4, the Sharpe ratio κ always decreases in the presence of institutions while the volatility σ_S always increases. For all reasonable calibrations of our model, the first effect dominates and so the expected stock return behaves analogously to the Sharpe ratio. It is, however, theoretically



(a) Effect of the size of institutions

(b) Effect of cash flow news

Figure 3: **Sharpe ratio.** This figure plots the equilibrium Sharpe ratio in the presence of institutions against the fraction of institutions in the economy λ and against the cash flow news D_t. The dotted lines correspond to the equilibrium Sharpe ratio in the benchmark economy with no institutions. The plots are typical. The parameter values are as in Figure 1.

possible that the volatility effect dominates, and the expected return actually increases in the presence of institutions when the size of the institutional sector is small. This occurs, for example, for $T = 50$—a calibration that does not appear plausible given that the typical length of a mandate of an institutional asset manager is one to five years, depending on the country and industry sector (BIS (2003), p.21).

Regarding stationarity, we would like to recall that our model is cast in finite horizon T. In order to have a stationary framework, we would need to develop a version of our current model with an infinite horizon and intertemporal consumption and to ensure that neither class of investors dominates the economy in the long run. Such a framework appears considerably more complex to solve since now the equilibrium stock price and state price density (defined in Appendix A) processes would need to be simultaneously determined. Consequently, one may need to resort to numerical analysis as much of the tractability of our framework may be lost.

Remark 3 (Alternative specification for institutional objective). As highlighted in Remark 1, a natural alternative for our institutional objective function is $u_{\mathcal{I}}(W_{\mathcal{I}T}) = (1 + bS_T/S_0)\log(W_{\mathcal{I}T})$, where the institution now strives to do well relative to the index return

S_T/S_0. Following the analysis of Section 2.2 and Appendix A, for this objective function, we derive the institutional investor's optimal portfolio to be

$$\phi_{\mathcal{I}t} = \frac{\mu_{St}}{\sigma_{St}^2} + \frac{b\,D_t/S_0}{1 + b\,D_t/S_0}\,\frac{\sigma}{\sigma_{St}}.$$

We again obtain the tilt in the institutional portfolio towards the index, arising due to the institution's desire to perform well as compared to the index return. Moving to equilibrium, similarly to Section 3.1, we determine the equilibrium market index level as

$$S_t = \overline{S}_t \frac{1 + b\,D_0/S_0 + \lambda\,b(D_t/S_0 - D_0/S_0)}{1 + b\,D_0/S_0 + \lambda\,b\,(e^{-\sigma^2(T-t)}D_t/S_0 - D_0/S_0)}, \tag{14}$$

where $\overline{S}_t$ is the equilibrium index level in the benchmark economy as in (9). The endogenous initial index level S_0 solves (14), and it can be shown that the unique strictly positive solution is given by

$$S_0 = D_0 \frac{\sqrt{B^2 - 4b\,e^{-\sigma^2 T}} - B}{2},$$

where $B = b - e^{-\sigma^2 T} + \lambda b(be^{-\sigma^2 T} - 1)$. Comparing with Proposition 1 of the earlier analysis, we see that under this alternative specification, we have a very similar index level expression. The main difference is that the index cash flow news news quantity D is now expressed per unit of the initial index level. More importantly, the price pressure from the institutions increases the stock index level, as before. Similarly, all other results and intuitions, including the index volatility, Sharpe ratio, go through for this alternative specification, with similarly modified expressions.

3.4. Asset Pricing Implications of Popular Policy Measures

The two main policy measures we would like to consider in the context of our model are the effects of deleveraging (a mandate to reduce leverage) and the effects of a transfer of capital to leveraged institutions. These two policy instruments have widely been employed during the 2007-2008 financial crisis. The objective, of course, was to improve the balance sheets of individual institutions in difficulty. But these policy actions, given their size and scope, inevitably had an effect on the overall economy, including asset prices. In this paper, we have nothing to say about the welfare consequences of these policies; in future research it would be interesting to address this question. Our goal here is to simply analyze the spillover effects of the popular policy measures on asset prices in our model.

At this point, we also draw a distinction between long-only institutions ("real money") and leveraged institutions ("leveraged money"). So far we have only dealt with the latter category. We model long-only institutions, $\mathcal{L}$, in a very simple form: these institutions do not solve any optimization problem but simply buy and hold the risky stock they are endowed with. The initial endowments of the retail investors, leveraged institutions, and long-only institutions are now $W_{\mathcal{R}0} = (1-\lambda)S_0$, $W_{\mathcal{I}0} = \lambda\theta S_0$, and $W_{\mathcal{L}0} = \lambda(1-\theta)S_0$, respectively. That is, the endowment of the retail investors is as before, but the endowment of institutions is now divided between the leveraged institutions and long-only institutions in proportions θ and $1-\theta$, respectively. The new parameter $\theta \in (0,\, 1)$ then captures the mass of "leveraged money" as a fraction of funds held (initially) by institutions. By reducing θ we can model a transfer of assets from leveraged institutions to long-only, or deleveraging.

Denote by λ' the endowment held by the leveraged institutional investors relative to the combined endowment of all active investors (the retail and leveraged institutional investors), so that

$$\frac{\lambda'}{1-\lambda'} = \frac{\lambda\theta}{1-\lambda}.$$

Proposition 5 summarizes how asset prices and equilibrium portfolios in our model are affected by the introduction of this new class of institutions.

Proposition 5. *The equilibrium index level, volatility, and institutional portfolio in the presence of long-only and leveraged institutions are given by their counterparts in Propositions 1–3, but with λ replaced by λ'.*

Consequently, the equilibrium stock price and volatility are higher in the presence of institutions and the stock price increases further as the fraction of leveraged institutions, θ, increases.

We again find it useful to highlight the results of the proposition in a figure. Figure 4 plots the bond and stock holdings of the leveraged institution, as well as the equilibrium stock market index and its volatility, as functions of the size of the "leveraged money" sector θ. The figure confirms that the stock price and the stock holdings of the leveraged institution are unambiguously increasing in θ. The effect of θ on bondholdings (leverage), however, is not necessarily unambiguous. It depends on the total size of the institutional investors (both real and leveraged money) relative to that of the retail investors. If there is enough lending capacity in the economy—the mass of retail investors is high—then the total amount of borrowing always increases with the size of the leveraged money sector. If, however, the mass of retail investors is relatively high, then leverage in the economy can peak for some θ and then start decreasing beyond that point. The economic mechanism generating such a bell-shaped pattern is as in Section 3.2, when we discussed the effects of λ on equilibrium leverage. For realistic calibrations of the model, we however find that the relevant scenario is the one in which the equilibrium leverage never reaches its maximum (i.e., there are enough retail investors to provide counterparties in the market for borrowing and lending to the leveraged institutions).

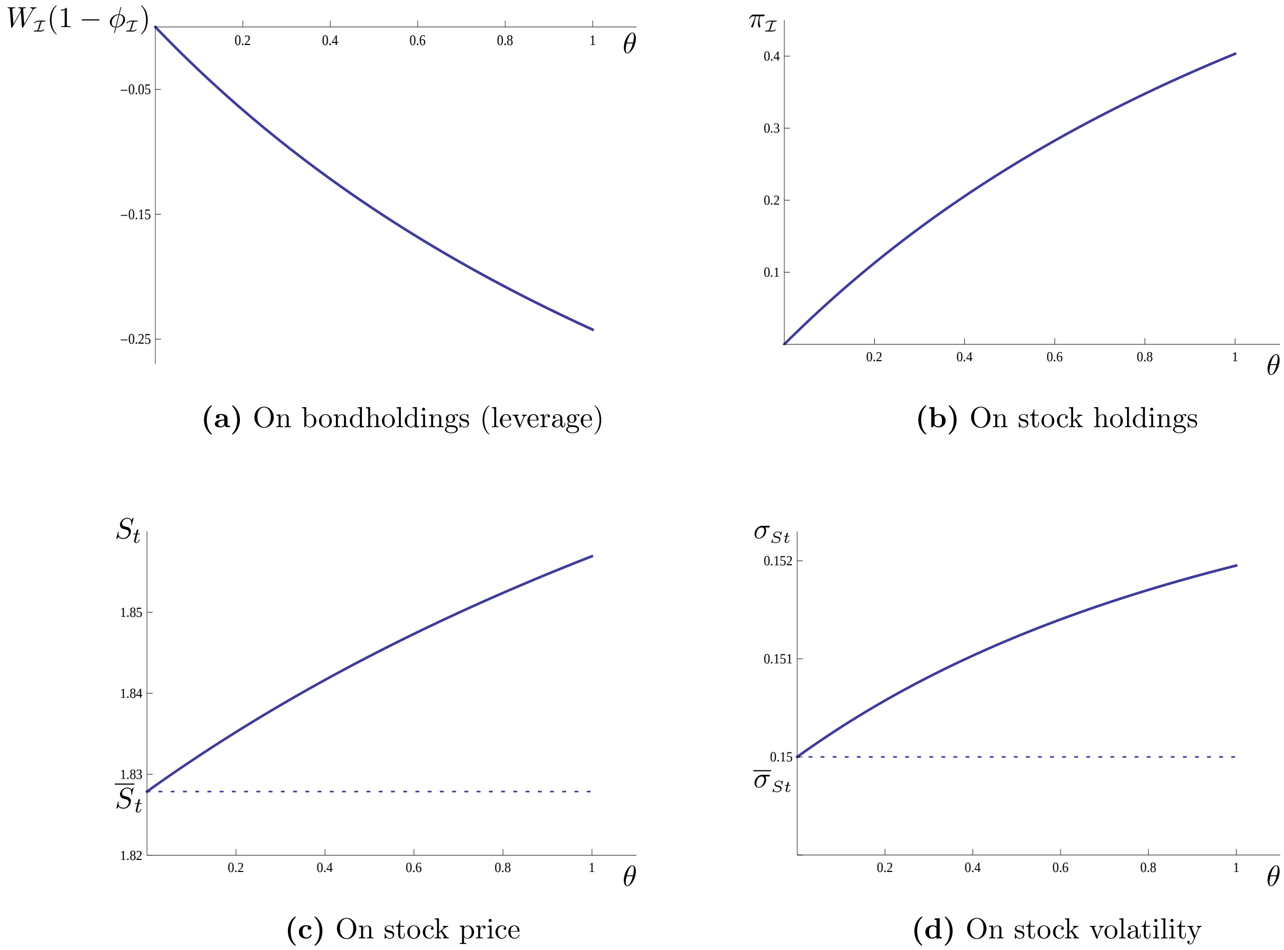


Figure 4: **The effects of the size of the leveraged institutions in the economy.** This figure plots the leveraged institution's holdings of the the bond $W_{\mathcal{I}}(1-\phi_{\mathcal{I}})$ (panel (a)), the leveraged institution's holdings of the shares of the index $\pi_{\mathcal{I}}$ (panel (b)), the stock index (panel (c)), the stock index volatility (panel (d)) against the size of the institution θ. The plots are typical. The parameter values are: $b = 1$, $D_0 = 1$, $\sigma = 0.15$, $t = 1$, $T = 5$. In panel (a) $D_t = 2$, and in panel (b) $\lambda = 0.2$. The remaining parameter values are as in Figure 1.

a. Effects of deleveraging

In our framework, we model deleveraging as a transfer of assets from leveraged institutions to long-only institutions at time 0. This policy can be interpreted as a requirement that a fraction of leveraged institutions must convert into "real money" long-only investors. In our model, we capture this as a reduction in the fraction of leveraged institutions θ.

Figure 4(a) reveals that a reduction in the mass of leveraged institutions indeed decreases the total leverage in the economy, with the total amount of outstanding bondholdings going down. Not being able to finance a risky asset position of the same size as prior to deleveraging, the institutional sector reduces its demand for the risky stock and the stock holdings of the sector fall (Figure 4(b)). While the deleveraging policy does achieve its desired outcome—the riskiness of the institutional portfolios going down—it does, however, come with side effects. The most notable one is that a reduction in the number of leveraged institutions also brings down the stock market index (Figure 4(c)). This effect is a simply a consequence of the drop in demand for the stock index by the institutions.

b. Effects of a capital injection

In our model, a capital injection into leveraged institutions at time 0 is equivalent to an increase in the mass of leveraged institutions θ. So the effects of such an injection are the opposite from those of deleveraging. This policy does boost the stock market index (Figure 4(c)) because a capital injection increases the demand of the institutions for the risky stock and they purchase more shares of it (Figure 4(b)). As a result of the stock price increase, everybody in the economy, including retail investors, becomes wealthier. But along with the run-up in the stock market, comes an increase in the leverage of institutional investors (Figure 4(a)). When the institutions do not control a dominant fraction of the financial wealth in the economy ($\theta \ll 1$), the stock price volatility also increases (Figure 4(d)). These side effects could be undesirable.

4. Multiple Stocks, Asset Classes, and Correlations

Our analysis has so far been presented in the context of a single-stock economy. Our goal in this section is to demonstrate how our results generalize in a multi-stock economy and to examine the correlations between stock returns. For the latter, we aim to demonstrate how institutional investors in our model generate an "asset-class" effect—i.e., how they make returns of assets belonging to an index to be more correlated amongst themselves than with those of otherwise identical assets outside the index.

4.1. Economic Setup

The general version of our economy features N risky stocks and N sources of risk, generated by a standard N-dimensional Brownian motion $\omega = (\omega_1, \ldots, \omega_N)^\top$, as well as a riskless bond.[12] Each stock price, S_j, $j = 1, \ldots, N$, is posited to have dynamics

$$dS_{jt} = S_{jt}[\mu_{S_j t} dt + \sigma_{S_j t} d\omega_t], \tag{15}$$

where the vector of stock mean returns $\mu_S \equiv (\mu_{S_1}, \ldots, \mu_{S_N})^\top$ and the stock volatility matrix $\sigma_S \equiv \{\sigma_{S_{j\ell}}, j, \ell = 1, \ldots, N\}$ are to be determined in equilibrium. The (instantaneous) correlation between stock j and ℓ returns, $\rho_{j\ell t} \equiv \sigma_{S_j t}^\top \sigma_{S_\ell t} / \sqrt{||\sigma_{S_j t}||^2 ||\sigma_{S_\ell t}||^2}$, is also to be endogenously determined.[13] The value of the equity *market portfolio*, S_{MKT}, is the sum of the risky stock prices:

$$S_{MKT\,t} = \sum_{j=1}^{N} S_{jt}, \tag{16}$$

with posited dynamics

$$dS_{MKT\,t} = S_{MKT\,t}[\mu_{MKT\,t}\, dt + \sigma_{MKT\,t}\, d\omega_t]. \tag{17}$$

[12] We include the bond to keep the discipline of a standard asset pricing framework, which serves as our benchmark. However, our analysis is equally valid without the bond present and the investment opportunities represented only by risky stocks. Such a variant of our model is perhaps more appropriate for modeling funds, whose investments are typically restricted to a single asset class, e.g., equities. We present the analysis for the stocks-only economy in Appendix C.

[13] The notation $||z||$ denotes the dot product $z \cdot z$.

Additionally, there is a value-weighted *index* (in terms of returns) made up of the first M stocks in the economy:

$$S_{It} = \sum_{i=1}^{M} S_{jt}.$$

This stock index S_I represents a specific asset class in the economy, and we will refer to the first M stocks as "index stocks" and the remainder $N - M$ stocks as the nonindex stocks.

Each stock is in positive net supply of one share. Its terminal payoff (or dividend) D_{jT}, due at time T, is determined by the process

$$dD_{jt} = D_{jt}\sigma_j d\omega_t, \tag{18}$$

where $\sigma_j > 0$ is constant for all stocks except for the last ones in the index and the market (the M^{th} and N^{th} stocks).[14] The process D_{jt} represents the cash flow news about the terminal stock dividend D_{jT}, and $S_{jT} = D_{jT}$. For expositional clarity and the thought experiment that we are going to undertake in this section, assume that the stocks' fundamentals (dividends) are independent. We thus assume that only the jth element of σ_j in (18) is nonzero, while all other elements are zero, so that the volatility matrix of cash flow news is diagonal. This implies zero correlation among all stocks' cash flow news, $\sigma_j^\top \sigma_\ell = 0$ for all $j \neq \ell$.

The stock market has a terminal payoff $S_{MKT\,T} = D_T$, given by the terminal value of the process

$$dD_t = D_t\sigma d\omega_t, \tag{19}$$

where $\sigma > 0$ is constant. Similarly, the index has a terminal value $S_{IT} = I_T$, determined by the process

$$dI_t = I_t\sigma_I d\omega_t, \tag{20}$$

with $\sigma_I > 0$ constant and having its first M components non-zero and the remainder $N - M$ components zero. The latter assumption is to make σ_I consistent with our assumptions

[14] That is, we do not explicitly specify the process of the cash flow news for the last stock in the index and in the market; but, in what follows, we specify processes for the sums of all stocks in the index and in the market. This modeling device is inspired by Menzly, Santos, and Veronesi (2004). It allows us to assume that the stock market and the index cash flow news follow geometric Brownian motion processes (equations (19) and (20)), which improves the tractability of the model considerably. In related analysis, we find that one may alternatively not assume a geometric Brownian motion process for the index cash flow news, but instead assume that stock M's dividend follows a geometric Brownian motion process. In that case, the analogs of the expressions that we report below are less elegant, and several results can be obtained only numerically.

about the individual stocks' cash flow news processes. Accordingly, while the index stocks' cash flow news have positive correlation with that of the index, $\sigma_j^\top \sigma_I > 0$, $j = 1, \dots, M$, the cash flow news of the nonindex stocks have zero correlation, $\sigma_k^\top \sigma_I = 0$, $k = M+1, \dots, N$.

Each type of investor $i = \mathcal{I}, \mathcal{R}$ now dynamically chooses a multi-dimensional portfolio process ϕ_i, where $\phi_i = (\phi_{i1}, \dots, \phi_{iN})^\top$ denotes the portfolio weights in each risky stock. The portfolio value W_i then has the dynamics

$$dW_{it} = W_{it}\phi_{it}^\top[\mu_{St}dt + \sigma_{St}d\omega_t]. \tag{21}$$

The retail investor is initially endowed with $1-\lambda$ fraction of the stock market, providing initial assets $W_{\mathcal{R}0} = (1-\lambda)S_{MKT0}$, and has the same objective function as in the single-stock case: $u_{\mathcal{R}}(W_{\mathcal{R}T}) = \log(W_{\mathcal{R}T})$. The institutional investor is initially endowed with λ fraction of the stock market and hence has initial assets worth $W_{\mathcal{I}0} = \lambda S_{MKT0}$. In this multi-stock version of our economy, the objective function of the institution is given by

$$u_{\mathcal{I}}(W_{\mathcal{I}T}) = (1 + bI_T)\log(W_{\mathcal{I}T}), \tag{22}$$

where $b > 0$ and $I_T = S_{IT}$ is the terminal value of the index (composed of the first M stocks in the economy). Here, the institutional investor has a benchmark that is distinct from the overall stock market. He now strives to perform particularly well when a specific asset class, represented by the index S_I, does well. One can think of this asset class as value stocks, technology stocks, or the stocks included in the S&P 500 index.

4.2. Investors' Portfolio Choice

We are now ready to examine how the results derived in the earlier analysis extend to the multi-stock case. We start with Lemma 2, which reports the investors' optimal portfolios in closed form.

Lemma 2. *The institutional and retail investors' optimal portfolio processes are given by*

$$\phi_{\mathcal{I}t} = (\sigma_{St}\sigma_{St}^\top)^{-1}\mu_{St} + \frac{b\,I_t}{1+b\,I_t}(\sigma_{St}^\top)^{-1}\sigma_I, \tag{23}$$

$$\phi_{\mathcal{R}t} = (\sigma_{St}\sigma_{St}^\top)^{-1}\mu_{St}. \tag{24}$$

Moreover,

(i) The institutional investor's hedging portfolio, the second term in (23), has positive portfolio weights in the index stocks $j = 1, \ldots, N-1$, but zero weights in the nonindex stocks in equilibrium;

(ii) The institutional investor invests a higher fraction of wealth in the index stocks $j = 1, \ldots, N-1$ than the retail investor, while holding the same fractions in the nonindex stocks as the retail investor.

The investors' portfolios in (23)–(24) are natural multi-stock generalizations of the single-stock case. Again, the institutional investor holds the mean-variance efficient portfolio plus an additional portfolio hedging her against fluctuations in her index. In our single-stock economy, the hedging demand of the institutional investor generates a tilt in her portfolio towards the risky stock, as compared to the retail investor. The multi-stock economy refines this implication. It is not the case that the institutional investor simply desires to take on more risk; rather, she demands a portfolio that is highly correlated with her index. This is why she has the same demand for the nonindex stocks as the retail investor, but demands additional holdings of index stocks, so as to not fall behind when the index does well. As we will see shortly, this excess demand for index stocks by the institution is the key driver of the index effect in our model.

From Lemma 2, we also see that the institution's optimal portfolio satisfies a three-fund separation property, with the three funds being the mean-variance efficient portfolio, the intertemporal hedging portfolio, and the riskless bond. The importance of this decomposition will become apparent later, when we discuss the asset-class effect in Section 4.4. For now, we just note that the hedging portfolio has positive portfolio weights in the index stocks, and when the institution gets wealthier—following for example, good cash flow news—she demands more shares of the index stocks (a wealth effect). This additional price pressure (beyond the standard increase in demand for the mean-variance portfolio) is applied to all index stocks simultaneously. There is no additional demand for the nonindex stocks.

Our implications for the higher risk-taking by institutions, who take on leverage in order to finance the hedging portfolio, remain the same as in our earlier analysis. We do not repeat them here and proceed to exploring the additional insights that a multiple stock environment is able to offer.

4.3. Stock Prices and Index Effect

Proposition 6 reports the equilibrium stock prices in closed form and highlights the effects of institutions on stock prices.

Proposition 6. *In the economy with institutional investors and multiple risky stocks, the equilibrium prices of the market portfolio, index stocks $j = 1, \ldots, M-1$ and nonindex stocks $k = M+1, \ldots, N-1$ are given by*

$$S_{MKT\,t} = \overline{S}_{MKT\,t} \frac{1 + b\,I_0 + \lambda\,b\,(I_t - I_0)}{1 + b\,I_0 + \lambda\,b\,(e^{-\sigma_I^\top \sigma (T-t)} I_t - I_0)}\,, \tag{25}$$

$$S_{jt} = \overline{S}_{jt} \frac{1 + b\,I_0 + \lambda\,b\,(e^{(-\sigma_I^\top \sigma + \sigma_j^\top \sigma_I)(T-t)} I_t - I_0)}{1 + b\,I_0 + \lambda\,b\,(e^{-\sigma_I^\top \sigma (T-t)} I_t - I_0)}\,, \tag{26}$$

$$S_{kt} = \overline{S}_{kt}, \tag{27}$$

where $\overline{S}_{MKT\,t}$, $\overline{S}_{jt}$, and $\overline{S}_{kt}$ are the equilibrium prices of the market portfolio, index and nonindex stocks, respectively, in the benchmark economy with no institutions, given by

$$\overline{S}_{MKT\,t} = e^{-||\sigma||^2 (T-t)} D_t, \quad \overline{S}_{jt} = e^{-\sigma_j^\top \sigma (T-t)} D_{jt}, \quad \overline{S}_{kt} = e^{-\sigma_k^\top \sigma (T-t)} D_{kt}. \tag{28}$$

Consequently, the market portfolio and index stock prices are increased in the presence of institutional investors, while nonindex stock prices are unaffected.

Proposition 6 generalizes our earlier discussion in the single-stock case and underscores the index effect occurring in our model. The direction of the effect is as before—the price pressure from the institutions raises the level of the index relative to that in the economy with no institutions. But now we can also make cross-sectional statements. If a stock j is added to the index I and a stock k is dropped, the price of stock j gets a boost, while that of stock k falls.[15] This is precisely the empirical regularity that is robustly documented in the data. In our model, however, we cannot make finer predictions which separate announcement-date returns and inclusion-date returns; our results concern only the announcement date. To generate inclusion-date abnormal returns, one could introduce passive indexers who buy at the inclusion date.

[15] Barberis and Shleifer (2003) obtain a similar implication within a behavioural model in which investors categorize risky assets into different styles and move funds among these styles according to certain (exogenously specified) rules. In a two-stock economy, Cuoco and Kaniel (2011) numerically obtain similar implications within a rational model for the case of managers being compensated with fulcrum fees. They also provide numerical results for the effect of benchmarking on the conditional volatilities of an index and a nonindex stock—the quantities that we consider in the next section—but because the mechanisms are different, our models differ in their implications.

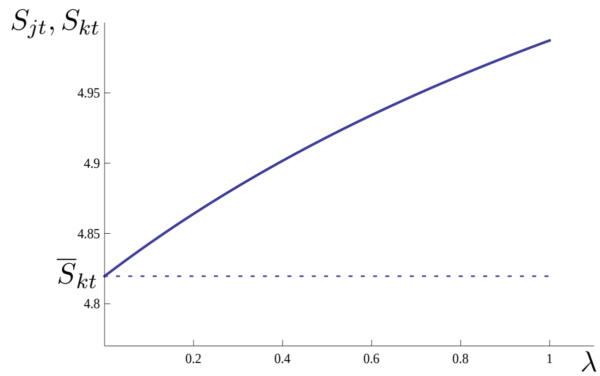


Figure 5: **An index effect.** This figure plots the prices of an index stock S_j (solid line) and a nonindex stock S_k (dotted line) in the presence of institutions against the fraction of institutions in the economy λ. The plot is typical. The parameter values are: $M = 3$, $N = 6$, $j = 1$, $k = 4$, $\sigma_j = 0.15\, i_j$, where i_j is an N-dimensional unit vector with the j^{th} element equal to 1 and the remaining elements equal to 0, $\sigma_k = 0.15\, i_k$, $\sigma_I = 0.15 \sum_{j=1}^{M} i_j/\sqrt{M}$, $\sigma = 0.15 \times \mathbf{1}/\sqrt{N}$, where $\mathbf{1}$ is an N-dimensional vector of ones, $I_0 = 1$, $I_t = 2$, and $D_t = 5$. The normalizations by $\sqrt{M}$ and $\sqrt{N}$ are adopted so as to keep $||\sigma_I||$ and $||\sigma||$ constant as we vary the number of stocks. The remaining parameters are as in Figure 1.

Figure 5 presents a plot of the price of an index stock relative to that of an otherwise identical nonindex stock. The plot is drawn as a function of the size of institutions λ. As expected, we see that the stock price is increasing with λ. This is due to the additional price pressure on index stocks as the institutional sector becomes larger. We also observe that the magnitudes are reasonable for our calibration. Chen, Noronha, and Singal (2004) find that during 1989-2000, a stock's price increases by an average of 5.45% on the day of the S&P 500 inclusion announcement and a further 3.45% between the announcement and the actual addition. The effects that we find are smaller, but roughly in line with these figures.

4.4. Stock Volatilities, Correlations, and Asset-class Effects

We now turn to examining the implications of our model for stock return volatilities and correlations. We report them in the following proposition in closed form.

Proposition 7. *In the economy with institutional investors and multiple risky stocks, the equilibrium volatilities of the market portfolio, index stocks $j = 1, \ldots, M-1$, and nonindex* *stocks $k = M+1, \dots, N-1$ are given by*

$$\sigma_{MKT\,t} = \overline{\sigma}_{MKT\,t} + \lambda\, b\, \sigma_I \frac{\left(1 - e^{-\sigma^\top \sigma_I (T-t)}\right)(1 + (1-\lambda)\, b\, I_0) I_t}{\left(1 + (1-\lambda)\, b\, I_0 + \lambda\, b\, e^{-\sigma_I^\top \sigma (T-t)} I_t\right)(1 + (1-\lambda)\, b\, I_0 + \lambda\, b\, I_t)}, \quad (29)$$

$$\sigma_{S_j t} = \overline{\sigma}_{S_j t} + \lambda\, b\, \sigma_I$$
$$\times \frac{\left(1 - e^{-\sigma_j^\top \sigma_I (T-t)}\right)(1 + (1-\lambda)\, b\, I_0) e^{(-\sigma_I^\top \sigma + \sigma_j^\top \sigma_I)(T-t)} I_t}{\left(1 + (1-\lambda)\, b\, I_0 + \lambda\, b\, e^{-\sigma_I^\top \sigma (T-t)} I_t\right)\left(1 + (1-\lambda)\, b\, I_0 + \lambda\, b\, e^{(-\sigma_I^\top \sigma + \sigma_j^\top \sigma_I)(T-t)} I_t\right)}, \quad (30)$$

$$\sigma_{S_k t} = \overline{\sigma}_{S_k t}, \quad (31)$$

where $\overline{\sigma}_{MKTt}$, $\overline{\sigma}_{S_j t}$, and $\overline{\sigma}_{S_k t}$ are the equilibrium market portfolio, index stock, and nonindex stock volatilities, respectively, in the benchmark economy with no institutions, given by

$$\overline{\sigma}_{MKTt} = \sigma, \quad \overline{\sigma}_{S_j t} = \sigma_j, \quad \overline{\sigma}_{S_k t} = \sigma_k. \quad (32)$$

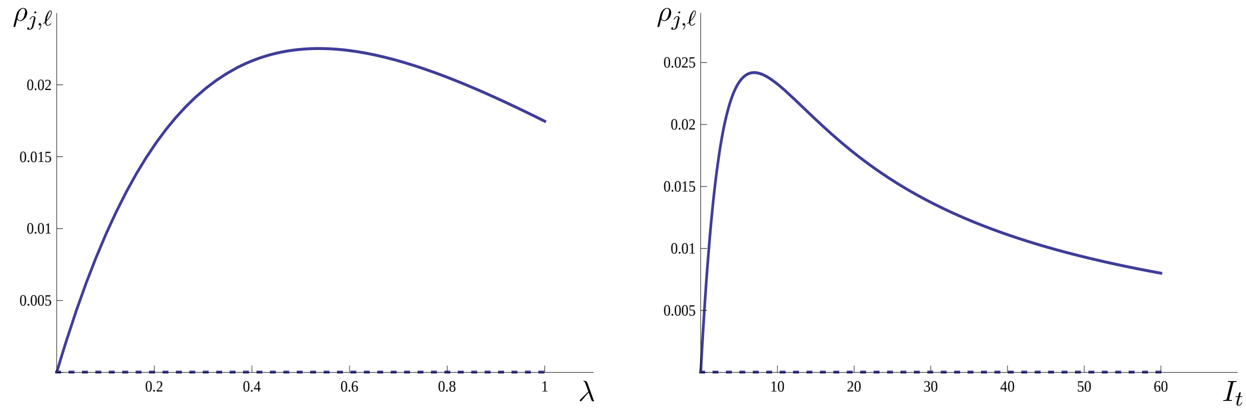
Consequently, in equilibrium:

(i) The market portfolio and index stock volatilities are increased in the presence of institutional investors, while nonindex stock volatilities are unaffected;

(ii) The correlations between index stocks are increased in the presence of institutional investors, while the correlations between nonindex stocks and between index and nonindex stocks are unaffected.

As one could expect from our earlier analysis, only the volatilities of the index stocks change in the presence of institutions; the volatilities of the nonindex stocks remain unchanged. The index stocks become riskier for the same reason as in the single-stock economy: the risk appetite of in the economy is higher in the presence of institutional investors.

The multiple stock formulation offers additional insights, allowing us to explore how the presence of institutions affects the correlations of stock returns. These results, based on fully analytical closed-form expressions, are reported in Proposition 7. Consistent with the empirical evidence on asset-class effects, we find that the presence of institutions increases the correlations among the stocks included in their index. The intuition is as follows. In the benchmark retail-investor-only economy, the cash flow news on all stocks are independent, and the stock returns turn out to be independent as well. Now consider the economy with



(a) Effect of the size of institutions

(b) Effect of index cash flow news

Figure 6: **An asset-class effect.** This figure plots the correlation between two index stock returns (solid plot) and the correlation between two nonindex stock returns (dashed line) in the presence of institutions against the fraction of institutions in the economy λ and against index cash flow news I_t. The two index stocks are stocks 1 and 2 and the two nonindex stocks are stocks $M+1$ and $M+2$. The plots are typical. All these four stocks j we set $\sigma_j = 0.15 i_j$. In panel (a) $I_t = 2$, and in panel (b) $\lambda = 0.2$. The remaining parameter values are as in Figure 5.

institutions. As we have established in the single-stock case in Section 3.2, following a good realization of cash flow news, institutions demand more shares of the index. This is simply a wealth effect. In the multi-stock case, the institutions demand more shares of *all* index stocks. This is a consequence of the three-fund separation property, discussed in the context of Lemma 2. It is important to keep in mind that the additional price pressure affects all index stocks, but not the nonindex stocks because the third fund, the hedging portfolio, consists only of index stocks. Hence, as compared to the retail-investor-only benchmark, following good cash flow news, all index stocks get an additional boost and following bad news, they all suffer from an additional selling pressure. This mechanism induces the comovement between index stocks, absent in the retail-investor-only benchmark. The correlation between the nonindex stocks is still zero, as in the benchmark, because these stocks are not part of the hedging portfolio of institutions, and so there is no additional buying or selling pressure on these stocks relative to the benchmark. The same is true for the correlations between the index and nonindex stocks. Figure 6 illustrates these effects. So summing this up, consistent with the empirical evidence, the returns of stocks belonging to an index to be more correlated amongst themselves than with those of otherwise identical stocks outside the index.

Figure 6(b) depicts the time-variation in the index stock correlations. The pattern here is similar to the one observed for the conditional volatilities of the stocks (Figure 1). The institutions are over-weighted in the index stocks, and therefore good index cash flow news create a wealth transfer from the retail investors to the institutions. In good states of the world (high I_t), the institutions dominate the economy and in bad states (low I_t) the retail investors control a larger share of total wealth. The correlations peak when the investor heterogeneity is the highest. To the right of the peak, the correlations decline, which resemble their behavior in the data.

5. Summary of Key Predictions and Empirical Implications

Institutions and the incentives they face feature prominently in models of corporate finance and banking, but they have largely been ignored in the standard asset pricing theory. In this paper, we develop an asset-pricing model that incorporates incentives of institutional investors to do well relative to their index. We demonstrate that this simple ingredient has profound implications for asset prices. In particular, it generates index effects and creates excess correlations among stocks belonging to an index (an asset-class effect). It also increases volatilities of index stocks and the overall market volatility. Moreover, the presence of institutions decreases the market Sharpe ratio, making it countercyclical.

It is difficult to obtain differential predictions for index and nonindex stocks within (heterogeneous-agent) consumption-based asset pricing models. For a standard model to deliver similar results, one would require that either (i) risk aversions "with respect to different stocks" are different, or (ii) capital constraints are more stringent for nonindex stocks than for index stocks, or (iii) agents have more optimistic beliefs about index stocks than nonindex stocks. Within a standard asset pricing model, these assumptions appear somewhat contrived, and we believe that benchmarking considerations represent a more plausible explanation.

Empirical literature lends support to all our key predictions. The extensive literature on index effects growing out of Harris and Gurel (1986) and Shleifer (1986) has now confirmed in many countries and many markets that stocks rise when added to an index (e.g., Wurgler and Zhuravskaya (2002), Chen, Noronha, and Singal (2004) and Greenwood (2005)). The presence of asset-class effects has been documented by e.g., Barberis, Shleifer, and Wurgler (2005) and Boyer (2011) for stocks, Rigobon (2002) for sovereign bonds, Tan and Xiong (2011) for commodity futures. There is a large literature documenting that institutional ownership increases stock volatility (e.g., Bushee and Noe (2000), Sias (2006)). Recently, Greenwood and Thesmar (2012) linked this finding to the asset-class effect. Gabaix et al. (2006) argue that institutions amplify volatility by examining trades of very large institutional investors. Our finding that the Sharpe ratio is countercyclical is also well-documented in the literature (see, e.g., Lettau and Ludvigson (2010)). Our model also predicts that the Sharpe ratios of index stocks are lower than those of nonindex stocks—this is a testable implications that future research might explore. Finally, Asparouhova et al. (2010) provide experimental support in favor of our model by documenting significant effects of money managers on asset prices in a large-scale experimental setting.

We left for future research several unexplored implications and potential extensions of our model. The presence of institutions may generate momentum of stock returns. Recently, the link between institutional fund flows and momentum has been established theoretically by Vayanos and Woolley (2010) and empirically by Lou (2009). The explanation in Vayanos and Woolley relies additionally on delayed reaction of traders; it would be interesting to see whether our model can also generate momentum and whether one needs to assume further that traders cannot immediately rebalance. Another fruitful avenue to explore is to endogenize the reward for good performance relative to an index (as in, for example, Berk and Green (2004)).

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Online Appendix for "Asset Prices and Institutional Investors"

Appendix A: Proofs

Proof of Lemma 1. Since the securities market in our setup is dynamically complete, it is well known that there exists a state price density process, ξ, such that the time-t value of a payoff C_T at time T is given by $E_t\big[\xi_T C_T\big]/\xi_t$. In our setting, the state price density is a martingale and follows the dynamics

$$d\xi_t = -\xi_t \kappa_t d\omega_t, \tag{A1}$$

where $\kappa_t \equiv \mu_{St}/\sigma_{St}$ is the Sharpe ratio process. Accordingly, investor i's dynamic budget constraint (3) can be restated as

$$E_t\big[\xi_T W_{iT}\big] = \xi_t W_{it}. \tag{A2}$$

Maximizing the institutional investor's expected objective function (5) subject to (A2) evaluated at time $t = 0$ leads to the institution's optimal terminal wealth as

$$W_{\mathcal{I}T} = \frac{1 + bD_T}{y_{\mathcal{I}} \xi_T},$$

where $1/y_{\mathcal{I}}$ solves (A2) evaluated at $t = 0$. Using the fact that D_t is lognormally distributed for all t, we obtain

$$\frac{1}{y_{\mathcal{I}}} = \frac{\lambda \xi_0 S_0}{1 + bD_0}.$$

Consequently, the institution's optimal terminal wealth is given by

$$W_{\mathcal{I}T} = \frac{\lambda \xi_0 S_0}{\xi_T} \frac{1 + bD_T}{1 + b\, D_0}, \tag{A3}$$

and from (A2) its optimal time-t wealth by

$$\xi_t W_{\mathcal{I}t} = \lambda \xi_0 S_0 \frac{1 + b\, D_t}{1 + b\, D_0}. \tag{A4}$$

Applying Itô's lemma to both sides of (A4), and using (3) and (A1), leads to

$$\xi_t W_{\mathcal{I}t}(\phi_{\mathcal{I}t}\sigma_{St} - \kappa_t) dw_t = \lambda \xi_0 S_0 \frac{bD_t}{1 + bD_0} \sigma d\omega_t,$$

which after matching the diffusion terms and rearranging gives the institutional investor's optimal portfolio (6). Similarly, we obtain the retail investor's optimal terminal and time-t wealth as

$$W_{\mathcal{R}T} = \frac{(1-\lambda)\xi_0 S_0}{\xi_T}, \tag{A5}$$

$$\xi_t W_{\mathcal{R}t} = (1-\lambda)\xi_0 S_0. \tag{A6}$$

Application of Itô's lemma leads to the standard retail investor's optimal portfolio in (7). *Q.E.D.*

Proof of Proposition 1. By no arbitrage, the stock market price in this complete market setup is given by

$$S_t = \frac{E_t\big[\xi_T D_T\big]}{\xi_t}. \tag{A7}$$

We proceed by first determining the equilibrium state price density process ξ. Imposing the market clearing condition $W_{\mathcal{R}T} + W_{\mathcal{I}T} = D_T$, and substituting (A3) and (A5) yields

$$\left(\lambda \frac{1+bD_T}{1+bD_0} + (1-\lambda)\right) \frac{\xi_0 S_0}{\xi_T} = D_T, \tag{A8}$$

which after rearranging leads to the equilibrium terminal state price density:

$$\xi_T = \frac{\xi_0 S_0}{1+b\,D_0} \frac{1}{D_T} \left(1 + b\,D_0 + \lambda b(D_T - D_0)\right). \tag{A9}$$

Consequently, the equilibrium state price density at time t is given by

$$\begin{aligned}
\xi_t &= E_t\big[\xi_T\big] \\
&= \frac{\xi_0 S_0}{1+b\,D_0} E_t\big[1/D_T\big] \left(1 + b\,D_0 + \lambda b\left(1/E_t[1/D_T] - D_0\right)\right) \\
&= \frac{\xi_0 S_0}{1+b\,D_0} \frac{e^{\sigma^2(T-t)}}{D_t} \left(1 + b\,D_0 + \lambda b\left(e^{-\sigma^2(T-t)} D_t - D_0\right)\right),
\end{aligned} \tag{A10}$$

where the last equality employs the fact that D_t is lognormally distributed.

Finally, to determine the equilibrium stock market level, we substitute (A9)–(A10) into (A7) and manipulate to obtain the stated expression (8). The stock market level $\overline{S}$ in the benchmark economy with no institutions (9) follows by considering the special case of $b = 0$ in (8). The property that the stock market is higher in the presence of institutions follows from the fact that the factor multiplying $\overline{S}_t$ in expression (8) is strictly positive, and being increasing in λ from the fact that the numerator in that factor is increasing at a faster rate than the denominator does in λ. *Q.E.D.*

Proof of Proposition 2. We write the equilibrium stock price in (8) as

$$S_t = \overline{S}_t \frac{X_t}{Z_t}, \tag{A11}$$

where

$$\overline{S}_t = e^{-\sigma^2(T-t)} D_t,$$

$$X_t = 1 + b\, D_0 + \lambda b \left(D_t - D_0\right),$$

$$Z_t = 1 + b\, D_0 + \lambda b \left(e^{-\sigma^2(T-t)} D_t - D_0\right).$$

Applying Itô's lemma to (A11) we obtain

$$\sigma_{St} = \sigma + \sigma_{Xt} - \sigma_{Zt}, \tag{A12}$$

where

$$\sigma_{Xt} = \frac{\lambda b D_t}{1 + b\, D_0 + \lambda b \left(D_t - D_0\right)} \sigma,$$

$$\sigma_{Zt} = \frac{\lambda b e^{-\sigma^2(T-t)} D_t}{1 + b\, D_0 + \lambda b \left(e^{-\sigma^2(T-t)} D_t - D_0\right)} \sigma.$$

We note that $X_t \sigma_{Xt} = \lambda b D_t \sigma$, $Z_t \sigma_{Zt} = \lambda b e^{-\sigma^2(T-t)} D_t \sigma$, and so $X_t \sigma_{Xt} = Z_t \sigma_{Zt} e^{-\sigma^2(T-t)}$. Hence, we have

$$X_t \sigma_{Xt} Z_t - Z_t \sigma_{Zt} X_t = X_t \sigma_{Xt} (1 - e^{-\sigma^2(T-t)}) \left(1 + (1-\lambda) b D_0\right). \tag{A13}$$

Substituting (A13) into the expression $\sigma_{Xt} - \sigma_{Zt} = (X_t \sigma_{Xt} Z_t - Z_t \sigma_{Zt} X_t)/X_t Z_t$, and then into (A12) leads to the equilibrium stock index volatility expression in (10). The property that the stock volatility is higher than the volatility in the benchmark with no institutions is immediate since $\sigma_{Xt} - \sigma_{Zt} > 0$. *Q.E.D.*

Proof of Proposition 3. We first determine the investors' equilibrium fractions of wealth invested in the stock index, ϕ_{it}, $i = \mathcal{I}, \mathcal{R}$. From (A7) and (A9) we have

$$\begin{aligned} \xi_t S_t &= E_t\left[\xi_T D_T\right] \\ &= \frac{\xi_0 S_0}{1 + b D_0} (1 + (1-\lambda) b D_0 + \lambda b D_t). \end{aligned} \tag{A14}$$

Applying Itô's lemma to both sides of (A14), we obtain

$$\sigma_{St} - \kappa_t = \frac{\lambda b D_t}{1 + (1-\lambda) b D_0 + \lambda b D_t} \sigma,$$

or

$$\frac{\kappa_t}{\sigma_{St}} = 1 - \frac{\lambda b D_t}{1 + (1-\lambda) b D_0 + \lambda b D_t} \frac{\sigma}{\sigma_{St}}, \tag{A15}$$

where σ_{St} is as given in Proposition 2. Substituting (A15) into the investors' optimal portfolios (6)–(7) in Lemma 1 yields their equilibrium portfolios ϕ_{it}.

Next, we determine the investors' wealth per unit of the stock market level, W_{it}/S_t, in equilibrium. Substituting the deflated time-t wealth of investors, (A4) and (A6), along with the deflated stock market level (A14), we obtain

$$\frac{W_{\mathcal{I}t}}{S_t} = \frac{\xi_t W_{\mathcal{I}t}}{\xi_t S_t} = \lambda \frac{1 + bD_t}{1 + bD_0 + \lambda b(D_t - D_0)}, \tag{A16}$$

$$\frac{W_{\mathcal{R}t}}{S_t} = \frac{\xi_t W_{\mathcal{R}t}}{\xi_t S_t} = (1-\lambda) \frac{1 + bD_0}{1 + bD_0 + \lambda b(D_t - D_0)}. \tag{A17}$$

As a remark, we here note that the ratio of the two investors' wealth in equilibrium is given by substituting (A16) in (A17):

$$\frac{W_{\mathcal{I}t}}{W_{\mathcal{R}t}} = \frac{\lambda}{1-\lambda} \frac{1 + bD_t}{1 + bD_0}, \tag{A18}$$

as highlighted in footnote (11). Finally, the investors' equilibrium portfolio weights ϕ_{it} above, along with their equilibrium per unit of stock index level leads to their equilibrium strategies in units of shares π_{it}, as given by (11)–(12) in Proposition 3.

The leverage property follows by substituting (A15) into (6) and rearranging to get the fraction of wealth invested in the riskless bond as

$$1 - \phi_{\mathcal{I}t} = \frac{\lambda b D_t}{1 + (1-\lambda) b D_0 + \lambda b D_t} \frac{\sigma}{\sigma_{St}} - \frac{b D_t}{1 + b\, D_t} \frac{\sigma}{\sigma_{St}} < 0.$$

Q.E.D.

Proof of Proposition 4. Applying Itô's Lemma to both sides of (A10) and manipulating leads to the equilibrium Sharpe ratio expression (13). The benchmark Sharpe ratio with no institutions is obtained by considering the special case of $b = 0$ in (13). The properties reported are straightforward to derive from the expression in (13). *Q.E.D.*

Proof of Proposition 5. We first consider the investors' optimal choices in partial equilibrium. The retail investor's optimal terminal wealth and time-t wealth are as in the proof of Lemma 1, given by (A5)–(A6). The "leveraged" institutional investor with initial wealth $W_{\mathcal{I}0} = \theta\lambda S_0$ now chooses its optimal terminal wealth and time-t wealth as

$$W_{\mathcal{I}T} = \frac{\theta\lambda\xi_0 S_0}{\xi_T} \frac{1 + b\, D_T}{1 + b\, D_0} \quad \text{and} \quad \xi_t W_{\mathcal{I}t} = \theta\lambda\xi_0 S_0 \frac{1 + b\, D_t}{1 + b\, D_0}, \tag{A19}$$

which are the same as in the baseline economy but with $\theta\lambda$ replacing λ. Both the levered institutional and retail investors' optimal portfolios are as before, given by (6)–(7) in Lemma 1.

Moving to general equilibrium, we first note that in the presence of the additional buy-and-hold institutional investor with initial assets $W_{\mathcal{L}0} = (1-\theta)\lambda S_0$, the market clearing condition is now:

$$W_{\mathcal{I}T} + W_{\mathcal{R}T} = (1-(1-\theta)\lambda)D_T. \tag{A20}$$

Substituting the investors' optimal terminal wealth (A5) and (A19) into (A20), we have

$$\left(\lambda\theta\frac{1+b\,D_T}{1+b\,D_0} + (1-\lambda)\right)\frac{\xi_0 S_0}{\xi_T} = (1-(1-\theta)\lambda)D_T.$$

Manipulating, we obtain the equilibrium terminal state price density as

$$\xi_T = \frac{\xi_0 S_0}{1+b\,D_0}\frac{1}{(1-(1-\theta)\lambda)D_T}\left((1-\lambda)(1+bD_0) + \theta\lambda(1+b\,D_T)\right). \tag{A21}$$

and the time-t state price density as

$$\xi_t = E_t\big[\xi_T\big] = \frac{\xi_0 S_0}{1+b\,D_0}\frac{e^{\sigma^2(T-t)}}{(1-(1-\theta)\lambda)D_t}\Big((1-\lambda)(1+b\,D_0) + \theta\lambda(1+be^{-\sigma^2(T-t)}D_t)\Big). \tag{A22}$$

The stock price is then given by

$$S_t = \frac{E_t\left[\xi_T D_T\right]}{\xi_t} = e^{-\sigma^2(T-t)}D_t\frac{1+b\,D_0+\frac{\theta\lambda}{1-\lambda}(1+b\,D_t)}{1+b\,D_0+\frac{\theta\lambda}{1-\lambda}(1+b\,e^{-\sigma^2(T-t)}D_t)}, \tag{A23}$$

which is the same formula as in the baseline economy (Proposition 1) but with $\lambda/(1-\lambda)$ replaced by $\lambda'/(1-\lambda')$. It then follows immediately that the volatility σ_{St} is the same as in Proposition 2, but with λ replaced by λ'. The same transformation also applies to the investors' portfolios. One can see this from the proof of Proposition 3, which gets modified analogously. The comparative statics results parallel those in Propositions 1-3 because λ' increases in θ. *Q.E.D.*

Proof of Lemma 2. The securities market is still dynamically complete in this multi-stock setup with N risky stocks and N sources of risk. Hence, there exists a state price density process, ξ, which is a martingale and follows the dynamics

$$d\xi_t = -\xi_t \kappa_t^T d\omega_t, \tag{A24}$$

where $\kappa_t \equiv \sigma_{St}^{-1}\mu_{St}$ is the N-dimensional Sharpe ratio process.

Following the same steps as in the proof of Lemma 1, the single-stock case, and using the fact that the index cash flow news I is lognormally distributed, we obtain the institutional investor's optimal terminal wealth and time-t wealth as

$$W_{\mathcal{I}T} = \frac{\lambda\xi_0 S_{MKT\,0}}{\xi_T}\frac{1+b\,I_T}{1+bI_0}, \tag{A25}$$

$$\xi_t W_{\mathcal{I}t} = \lambda \xi_0 S_{MKT\,0} \frac{1 + b\, I_t}{1 + bI_0}. \tag{A26}$$

Applying Itô's lemma to (A26) leads to

$$\xi_t W_{\mathcal{I}t}(\phi_{\mathcal{I}t}^T \sigma_{St} - \kappa_t^T) dw_t = \lambda \xi_0 S_{MKT\,0} \frac{bI_t}{1 + bI_0} \sigma_I d\omega_t, \tag{A27}$$

which after matching coefficients yields the institutional optimal portfolio as reported in (23). The retail investor's optimal terminal wealth and time-t wealth are as in the single-stock case given by (A5) and (A6), which leads to the optimal portfolio in (24).

To prove property (i), we first note from (A27) that

$$\sigma_{St}^T \phi_{\mathcal{I}t} = \kappa_t + A_t \sigma_I^T, \tag{A28}$$

where the scalar $A_t \equiv bI_t/(1 + bI_t)$. This is a system of N equations in N unknowns (ϕ_{It}). We represent its solution in the form

$$\phi_{\mathcal{I}t} \equiv \phi_{\mathcal{R}t} + \phi_{Ht},$$

where $\phi_{\mathcal{R}t}$ is the mean-variance portfolio and ϕ_{Ht} denotes the hedging portfolio. The mean-variance portfolio is given by (24), and together with (A24) satisfies

$$\sigma_{St}^T \phi_{\mathcal{R}t} = \kappa_t. \tag{A29}$$

The hedging portfolio is well-known to be a portfolio that has the maximal correlation with the state variable I_t (e.g., Ingersoll (1987)). Here the securities market is dynamically complete, and so the perfect correlation of 1 can be achieved. Let us now consider an auxiliary securities market in which we replace any of the stocks, say the first stock, by the index S_I itself. In equilibrium, S_I and σ_{S_I} are given by (26) and (30) in Propositions 6 and 7, respectively, with the subscript j replaced by the subscript I. The index value S_I is therefore driven by a single state variable I_t, and hence by investing in the index S_I one can achieve a unit correlation with I_t. So, we can conclude that the hedging portfolio ϕ_H is of the form $\phi_{Ht} = C_t(1, 0, \ldots, 0)^T$, where C_t is a scalar, satisfying

$$\sigma_{St}^T \phi_{Ht} = A_t \sigma_{It}^T, \tag{A30}$$

which together with (A29) satisfy (A28). The relation (A30) holds because the first row of σ_{St} in the auxiliary economy, $\sigma_{S_{It}}$, and σ_I are collinear. This is nothing else but the three-fund separation property, with the funds being the mean-variance efficient portfolio, the index, and the riskless bond. Moreover, $\phi_{Ht} > 0$ since $\sigma_{S_I} > 0$ and $A_t, C_t > 0$. In this auxiliary economy, therefore, the optimal hedging portfolio puts zero weights on all securities but the index. Mapping the auxiliary economy back into the original economy and recognizing that the index S_I is a portfolio of one share in each of the index stocks, we arrive at property (i). Property (ii) then follows immediately. *Q.E.D.*

Proof of Proposition 6. We first determine the equilibrium state price density process. Imposing the market clearing condition $W_{\mathcal{I}T} + W_{\mathcal{R}T} = D_T$, substituting (A25) and (A5), and manipulating yields the terminal equilibrium state price density as

$$\xi_T = \frac{\xi_0 S_{MKT\,0}}{1 + bI_0} \frac{1}{D_T} \left(1 + b\, I_0 + \lambda b(I_T - I_0)\right). \tag{A31}$$

To obtain the time-t equilibrium state price density, we use the properties of lognormal distribution $E_t\big[1/D_T\big] = e^{||\sigma||^2(T-t)}/D_t$, $E_t\big[I_T/D_T\big] = e^{(||\sigma||^2 - \sigma_I^T\sigma)(T-t)} I_t/D_t$, which along with (A31) and some manipulations we get

$$\xi_t = \frac{\xi_0 S_{MKT\,0}}{1 + bI_0} \frac{e^{||\sigma||^2(T-t)}}{D_t} \Big(1 + bI_0 + \lambda b(e^{-\sigma_I^T\sigma(T-t)} I_t - I_0)\Big). \tag{A32}$$

To determine the equilibrium market portfolio price, we first compute its deflated process from (A31) as, after some manipulation

$$\begin{aligned} \xi_t S_{MKT\,t} &= E_t\big[\xi_T D_T\big] \\ &= \frac{\xi_0 S_{MKT\,0}}{1 + bI_0} \left(1 + b\, I_0 + \lambda b(I_t - I_0)\right). \end{aligned} \tag{A33}$$

Substituting (A32) into (A33) yields the market portfolio level as reported in (25). The price in the benchmark economy with no institution is obtained as a special case by setting $b = 0$.

To determine the equilibrium price of an index stock $j = 1, ..., M-1$, we first find its deflated process:

$$\xi_t S_{jt} = E_t\big[\xi_T D_{jT}\big]. \tag{A34}$$

From (A31), we have

$$\xi_T D_{jT} = \frac{\xi_0 S_{MKT\,0}}{1 + b\, I_0} \frac{D_{jT}}{D_T} \left(1 + b\, I_0 + \lambda b(I_T - I_0)\right). \tag{A35}$$

After some manipulations and substitution of the properties of the properties of lognormally distributed processes

$$E_t\left[\frac{D_{jT}}{D_T}\right] = e^{(||\sigma||^2 - \sigma_j^T\sigma)(T-t)} \frac{D_{jt}}{D_t},$$

$$E_t\left[\frac{D_{jT} I_T}{D_T}\right] = e^{(\sigma_j^T\sigma_I + ||\sigma||^2 - \sigma_I^T\sigma - \sigma_j^T\sigma)(T-t)} \frac{D_{jt} I_t}{D_t},$$

we obtain

$$\begin{aligned} E_t\left[\xi_T D_{jT}\right] = &\frac{\xi_0 S_{MKT\,0}}{1 + b\, I_0} e^{(||\sigma||^2 - \sigma_j^T\sigma)(T-t)} \frac{D_{jt}}{D_t} \\ &\times \Big(1 + b\, I_0 + \lambda b(e^{(-\sigma_I^T\sigma + \sigma_j^T\sigma_I)(T-t)} I_t - I_0)\Big). \end{aligned} \tag{A36}$$

Finally, substituting (A32) and (A36) into (A34), we obtain the equilibrium price of an index stock as reported in (26) of Proposition 5. The index stock price in the benchmark economy is obtained as a special case by setting $b = 0$.

To determine the equilibrium price of a nonindex stock $k = M+1, ..., N-1$, we proceed as in the index stock case and obtain the same stock price equation (26) but now with the correlation with the index $\sigma_k^T \sigma_I = 0$ substituted in. With this zero correlation, the nonindex stock price collapses to its value in the benchmark economy with no institutions. The stated property of higher market portfolio and index stock prices is immediate from the expressions (25)–(26). *Q.E.D.*

Proof of Proposition 7. To determine the equilibrium volatilities in this multi-stock case, we proceed as in Proposition 3. For the market portfolio, we express its equilibrium price as $S_{MKT,t} \equiv \overline{S}_{MKT,t} X_t / Z_t$ and apply Itô's lemma to obtain

$$\sigma_{MKT\,t} = \sigma + \sigma_{Xt} - \sigma_{Zt},$$

where

$$\sigma_{Xt} = \frac{\lambda b I_t}{1 + b\, I_0 + \lambda b\,(I_t - I_0)} \sigma_I,$$

$$\sigma_{Zt} = \frac{\lambda b e^{-\sigma_I^T \sigma (T-t)} I_t}{1 + b\, I_0 + \lambda b (e^{-\sigma_I^T \sigma (T-t)} I_t - I_0)} \sigma_I.$$

So we have $\sigma_{Zt} Z_t = \sigma_{Xt} X_t e^{-\sigma_I^T \sigma (T-t)}$, implying after some manipulation that

$$(\sigma_{Xt} - \sigma_{Zt}) X_t Z_t = \lambda b (1 - e^{-\sigma_I^T \sigma (T-t)})\,(1 + (1-\lambda) b I_0)\, I_t \sigma_I,$$

leading to the market portfolio volatility as reported in (29).

For the index stock volatility, analogously we express the equilibrium price of an index stock $j = 1, ..., M-1$ as $S_{jt} \equiv \overline{S}_{jt} X_{jt} / Z_{jt}$. Applying Itô's lemma we obtain

$$\sigma_{S_j t} = \sigma_j + \sigma_{X_j t} - \sigma_{Z_j t},$$

where

$$\sigma_{X_j t} = \frac{\lambda b e^{(-\sigma_I^T \sigma + \sigma_j^T \sigma_I)(T-t)} I_t}{1 + (1-\lambda) b I_0 + \lambda b e^{(-\sigma_I^T \sigma + \sigma_j^T \sigma_I)(T-t)} I_t} \sigma_I,$$

$$\sigma_{Z_j t} = \frac{\lambda b e^{-\sigma_I^T \sigma (T-t)} I_t}{1 + (1-\lambda) b I_0 + \lambda b e^{-\sigma_I^T \sigma (T-t)} I_t} \sigma_I.$$

hence, we have $\sigma_{Z_j t} Z_{jt} = \sigma_{X_j t} X_{jt} e^{-\sigma_j^T \sigma_I (T-t)}$, implying

$$(\sigma_{X_j t} - \sigma_{Z_j t}) X_{jt} Z_{jt} = \lambda b (1 - e^{-\sigma_j^T \sigma_I (T-t)})\,(1 + (1-\lambda) b I_0)\, e^{(-\sigma_I^T \sigma + \sigma_j^T \sigma_I)(T-t)} I_t \sigma_I,$$

leading to the market portfolio volatility as reported in (30).

The implications that the market portfolio and index stock volatilities are higher follow immediately from the expressions (29)–(30). As for the higher correlation property (ii) amongst index stocks, we need to show that for two index stocks j and l

$$\frac{\sigma_{S_jt}^T\sigma_{S_lt}}{\sqrt{||\sigma_{S_jt}||^2||\sigma_{S_lt}||^2}} > \frac{\overline{\sigma}_{S_jt}^T\overline{\sigma}_{S_lt}}{\sqrt{||\overline{\sigma}_{S_jt}||^2||\overline{\sigma}_{S_lt}||^2}}.$$

Since $\overline{\sigma}_{S_jt}^T\overline{\sigma}_{S_lt} = \overline{\sigma}_j^T\overline{\sigma}_l = 0$, above is equivalent to showing $\sigma_{S_jt}^T\sigma_{S_lt} > 0$. From (25), for an index stock we have

$$\sigma_{S_jt} = \sigma_j + f_j(I_t)\sigma_I,$$

where f_j is some strictly positive function of I_t specific to stock j. Consequently, we have

$$\sigma_{S_jt}^T\sigma_{S_lt} = \sigma_j^T\sigma_l + f_j\sigma_j^T\sigma_I + f_l\sigma_l^T\sigma_I + f_jf_l||\sigma_I||^2 > 0, \tag{A37}$$

proving the desired result. The correlation property regarding the nonindex stocks is obvious.

Q.E.D.

Appendix B: Generalization to Nonzero Dividend Growth and Interest Rate

In this appendix, we generalize our setup to additionally feature a nonzero growth rate for the stock dividend and a nonzero riskless rate for the bond. This setting turns out to be equally tractable, leading to closed-form expressions for all quantities, as demonstrated below. Importantly, however, all our previous conclusions and intuitions remain robust to this generalization.

The economic setup is as in Section 2.1, but now the stock market payoff (the "dividend") D_T is the terminal value of the process D_t with dynamics

$$dD_t = D_t[\mu dt + \sigma d\omega_t], \tag{B1}$$

where the growth rate μ and $\sigma > 0$ are constant. Consequently, D_t is lognormally distributed, as before. Moreover, the zero-net supply bond now pays a nonzero, riskless interest at a constant rate r. As becomes evident from the analysis below, when $\mu \neq 0$, the expressions in the text and the appendices remain the same, replacing D_t by $D'_t = e^{\mu(T-t)}D_t$ (and, consequently, replacing D_0 by $D'_0 = e^{\mu T}D_0$).

Given the dynamically complete market, there exists a state price density process, ξ, which is no longer a martingale and follows the modified dynamics

$$d\xi_t = -\xi_t r dt - \xi_t\kappa_t d\omega_t, \tag{B2}$$

where $\kappa_t \equiv (\mu_{St} - r)/\sigma_{St}$ is the modified Sharpe ratio process. Accordingly, investor i's dynamic budget constraint (3) can again be restated as

$$E_t\big[\xi_T W_{iT}\big] = \xi_t W_{it}. \tag{B3}$$

We first determine the investors' optimal portfolios. Maximizing the institutional investor's objective function (5) subject to (B3) evaluated at time $t = 0$ leads to the institution's optimal terminal wealth as

$$W_{\mathcal{I}T} = \frac{1 + bD_T}{y_{\mathcal{I}}\xi_T},$$

where $1/y_{\mathcal{I}}$ solves (B3) evaluated at $t = 0$, and with D_t lognormally distributed, we obtain

$$\frac{1}{y_{\mathcal{I}}} = \frac{\lambda \xi_0 S_0}{1 + be^{\mu T} D_0}.$$

Consequently, the institution's optimal terminal wealth is given by

$$W_{\mathcal{I}T} = \frac{\lambda \xi_0 S_0}{\xi_T} \frac{1 + bD_T}{1 + b\, e^{\mu T} D_0}, \tag{B4}$$

and from (B3) its optimal time-t wealth by

$$\xi_t W_{\mathcal{I}t} = \lambda \xi_0 S_0 \frac{1 + b\, e^{\mu(T-t)} D_t}{1 + b\, e^{\mu T} D_0}. \tag{B5}$$

Applying Itô's lemma to both sides of (B5), and using (3) and (B2), leads to

$$\xi_t W_{\mathcal{I}t}(\phi_{\mathcal{I}t}\sigma_{St} - \kappa_t) dw_t = \lambda \xi_0 S_0 \frac{be^{\mu(T-t)} D_t}{1 + be^{\mu T} D_0} \sigma d\omega_t,$$

which after matching the diffusion terms and rearranging gives the institutional investor's optimal portfolio below. The retail investor's optimal portfolio is obtained similarly.

Lemma B1. *The institutional and retail investors' portfolios are given by*

$$\phi_{\mathcal{I}t} = \frac{\mu_{St} - r}{\sigma_{St}^2} + \frac{b\, e^{\mu(T-t)} D_t}{1 + b\, e^{\mu(T-t)} D_t} \frac{\sigma}{\sigma_{St}}, \tag{B6}$$

$$\phi_{\mathcal{R}t} = \frac{\mu_{St} - r}{\sigma_{St}^2}. \tag{B7}$$

As in Lemma 1, the institution demands a higher fraction of wealth in the stock market index than the retail investor, due to the hedging portfolio in (B6), and the same intuition holds.

We next turn to the equilibrium asset pricing implications of the presence of institutional investors. To determine the equilibrium state price density process ξ, we impose the market clearing condition $W_{\mathcal{R}T} + W_{\mathcal{I}T} = D_T$, and substitute (B4) and (A5) to obtain

$$\left(\lambda \frac{1 + bD_T}{1 + be^{\mu T} D_0} + (1 - \lambda)\right) \frac{\xi_0 S_0}{\xi_T} = D_T, \tag{B8}$$

which after rearranging leads to the equilibrium terminal state price density:

$$\xi_T = \frac{\xi_0 S_0}{1 + be^{\mu T} D_0} \frac{1}{D_T} \left(1 + be^{\mu T} D_0 + \lambda b(D_T - e^{\mu T} D_0)\right). \tag{B9}$$

From (B2) we have

$$\begin{aligned}
\xi_T &= \xi_t e^{-r(T-t) - \frac{1}{2}\int_t^T \kappa_s^2 ds - \int_t^T \kappa_s d\omega_s} \\
&= \xi_t e^{-r(T-t)} \eta_T / \eta_t,
\end{aligned} \tag{B10}$$

where η is the exponential martingale defined by $\eta_t = e^{-\frac{1}{2}\int_0^t \kappa_s^2 ds - \int_0^t \kappa_s d\omega_s}$. Taking expectations on both sides of (B10) leads to

$$E_t\left[\xi_T\right] = \xi_t e^{-r(T-t)}.$$

Consequently, the equilibrium state price density at time t is given by

$$\begin{aligned}
\xi_t &= e^{r(T-t)} E_t\big[\xi_T\big] \\
&= \frac{\xi_0 S_0 e^{r(T-t)}}{1 + be^{\mu T} D_0} E_t\big[1/D_T\big] \left(1 + be^{\mu T} D_0 + \lambda b \left(1/E_t[1/D_T] - e^{\mu T} D_0\right)\right) \\
&= \frac{\xi_0 S_0}{1 + be^{\mu T} D_0} \frac{e^{(r-\mu+\sigma^2)(T-t)}}{D_t} \left(1 + be^{\mu T} D_0 + \lambda b \left(e^{(\mu-\sigma^2)(T-t)} D_t - e^{\mu T} D_0\right)\right),
\end{aligned} \tag{B11}$$

where the last equality employs the fact that D_t is lognormally distributed. To determine the equilibrium stock market level, we substitute (B9)–(B11) into (A7) and manipulate to obtain the stated expression (B12) below. The stock market level $\overline{S}$ in the benchmark economy with no institutions follows by considering the special case of $b = 0$ in (B12).

Proposition B1. *In the economy with institutional investors, the equilibrium level of the stock market index is given by*

$$S_t = \overline{S}_t \frac{1 + b\, e^{\mu T} D_0 + \lambda\, b(e^{\mu(T-t)} D_t - e^{\mu T} D_0)}{1 + b\, e^{\mu T} D_0 + \lambda\, b\, (e^{(\mu-\sigma^2)(T-t)} D_t - e^{\mu T} D_0)}, \tag{B12}$$

where $\overline{S}_t$ is the equilibrium index level in the benchmark economy with no institutional investors given by

$$\overline{S}_t = e^{(\mu-r-\sigma^2)(T-t)} D_t.$$

As in Section 3.1, the stock market index level is increased in the presence of institutional investors, $S_t > \overline{S}_t$, with identical price pressure intuition.

To derive the stock market volatility, we write the equilibrium stock price in (B12) as

$$S_t = \overline{S}_t \frac{X_t}{Z_t}, \tag{B13}$$

where

$$X_t = 1 + be^{\mu T} D_0 + \lambda b \left(e^{\mu(T-t)} D_t - e^{\mu T} D_0 \right),$$

$$Z_t = 1 + be^{\mu T} D_0 + \lambda b \left(e^{(\mu-\sigma^2)(T-t)} D_t - e^{\mu T} D_0 \right).$$

Applying Itô's lemma to (B13) and following the same steps as in the proof of Proposition 2 in Appendix A we obtain the following.

Proposition B2. *In the equilibrium with institutional investors, the volatility of the stock market index returns is given by*

$$\sigma_{St} = \overline{\sigma}_{St} + \lambda\, b\, \sigma \frac{\left(1 - e^{-\sigma^2(T-t)}\right)(1 + (1-\lambda) b e^{\mu T} D_0) e^{\mu(T-t)} D_t}{(1 + (1-\lambda)\, b\, e^{\mu T} D_0 + \lambda\, b\, e^{(\mu-\sigma^2)(T-t)} D_t)\,(1 + (1-\lambda)\, b\, e^{\mu T} D_0 + \lambda\, b\, e^{\mu(T-t)} D_t)},$$

where $\overline{\sigma}_{St}$ is the equilibrium index volatility in the benchmark economy with no institutions, given by

$$\overline{\sigma}_{St} = \sigma.$$

Consequently, the index volatility is increased in the presence of institutions, $\sigma_{St} > \overline{\sigma}_{St}$, as in the analysis of Section 3.1.

Finally, we determine the investors' equilibrium portfolios following identical steps as in the proof of Proposition 3 in Appendix A and obtain the following, with the same implications as in the analysis of Section 3.2.

Proposition B3. *The institutional and retail investors' portfolios in equilibrium in terms of shares in the stock index are given by*

$$\pi_{\mathcal{I}t} = \lambda \frac{1 + b\, e^{\mu(T-t)} D_t}{1 + (1-\lambda)\, b\, e^{\mu T} D_0 + \lambda\, b\, e^{\mu(T-t)} D_t}$$
$$\times \left(1 - \frac{\lambda\, b\, e^{\mu(T-t)} D_t}{1 + (1-\lambda)\, b\, e^{\mu T} D_0 + \lambda\, b\, e^{\mu(T-t)} D_t} \frac{\sigma}{\sigma_{St}} + \frac{b\, e^{\mu(T-t)} D_t}{1 + b\, e^{\mu(T-t)} D_t} \frac{\sigma}{\sigma_{St}} \right),$$

$$\pi_{\mathcal{R}t} = (1-\lambda) \frac{1 + b\, e^{\mu T} D_0}{1 + (1-\lambda)\, b\, e^{\mu T} D_0 + \lambda\, b\, e^{\mu(T-t)} D_t} \left(1 - \frac{\lambda\, b\, e^{\mu(T-t)} D_t}{1 + (1-\lambda)\, b\, e^{\mu T} D_0 + \lambda\, b\, e^{\mu(T-t)} D_t} \frac{\sigma}{\sigma_{St}} \right),$$

where σ_{St} is as in Proposition B2.

Consequently, the institutional investor is always levered, $W_{It}(1 - \phi_{\mathcal{I}t}) < 0$.

The results of Section 4 with multiple stocks generalize analogously, and all our economic insights obtained in that section remain exactly the same.

Appendix C: Stocks-Only Economy

This appendix presents a variant of our multi-stock economy in Section 4 in which there are only risky stocks available for trading and there is no riskless bond. Such a variant is perhaps more appropriate for modeling institutional investors for whom portfolios are typically restricted to a single asset class, e.g., equities, and do not normally involve leverage. We first extend our earlier analysis to such a stocks-only setting and show that our main implications presented in Section 4 remain valid. The main difference here is that instead of borrowing through the riskless bond to finance the additional demand for index stocks, the institutional investors reduce their positions in nonindex stocks to fund this additional demand. This model, however, is less tractable because unlike in Section 4, we have only been able to demonstrate such portfolio implications numerically.

The economic setup is as follows. The securities market is driven by N sources of risk represented by the N-dimensional Brownian motion $\omega = (\omega_1, \ldots, \omega_N)^T$, but now features $N+1$ risky stocks and no riskless bond. As in Section 4, each stock is in positive net supply of one share and is a claim against a terminal payoff D_{jT} at time T. Each stock price, S_j, $j = 1, \ldots, N+1$, is then posited to have dynamics

$$dS_{jt} = S_{jt}[\mu_{S_j t} dt + \sigma_{S_j t} d\omega_t],$$

where the vector of stock mean returns $\mu_S \equiv (\mu_{S_1}, \ldots \mu_{S_{N+1}})^T$ and the stock volatility matrix $\sigma_S \equiv \{\sigma_{S_{j\ell}}, \, j = 1, \ldots, N+1, \, \ell = 1, \ldots, N\}$, now with dimensions $(N+1)\times 1$ and $(N+1)\times N$, respectively, are determined in equilibrium. The stock market is again the sum of all the stocks in the economy with the terminal payoff $S_{MKTT} = D_T$, while the stock index is made up of the first M stocks with the terminal payoff $S_{IT} = I_T$. The primary difference here from the setup in Section 4 is the presence of the additional stock, $N+1$, and the absence of the riskless bond. By dropping the riskless bond, we are departing from the typical investment opportunity set featured in the canonical asset pricing model. We note, however, that in this dynamically complete-markets setting such a bond can be synthetically replicated using the $N+1$ risky stocks.

We first examine the investors' optimal portfolios. In this stocks-only economy, each investor type $i = \mathcal{I}, \mathcal{R}$ now chooses an $(N+1)$-dimensional portfolio process $\phi_i \equiv (\phi_{i1}, \ldots, \phi_{iN+1})^T$, where ϕ_i denotes the portfolio weights in each risky stock. The investor's investment portfolio value W_i then follows the dynamics

$$dW_{it} = W_{it}\phi_{it}^\top [\mu_{St} dt + \sigma_{St} d\omega_t].$$

Following the analysis of Section 4, and particularly the same steps as in the proof of Lemma 2, we obtain the same equations (A24)–(A28) in determining the institution's optimal portfolio. In particular, we still have that

$$\sigma_{St}^T \phi_{\mathcal{I}t} = \kappa_t + A_t \sigma_I^T, \tag{C1}$$

where $A_t \equiv bI_t/(1 + bI_t)$. The only difference now is that this is a system of N equations in $N+1$ unknowns $(\phi_{\mathcal{I}t})$. The last equation that is needed to determine the optimal portfolio process is that the portfolio weights add up to one:

$$\mathbf{1}^T \phi_{\mathcal{I}t} = 1, \tag{C2}$$

where $\mathbf{1}$ is an $(N+1)\times 1$ vector of 1's. Equations (C1) and (C2) together fully determine the institution's portfolio.

To derive the analogue of Lemma 2, we define the following augmented volatility matrix and vectors:

$$\tilde{\sigma}_{St} \equiv \begin{bmatrix} & 1 \\ \sigma_{St} & \vdots \\ & 1 \end{bmatrix}, \quad \tilde{\kappa}_t \equiv \begin{bmatrix} \kappa_t \\ 1 \end{bmatrix}, \quad \tilde{\sigma}_I \equiv \begin{bmatrix} \sigma_I \\ 0 \end{bmatrix},$$

where we have added the $N+1$st column of $\mathbf{1}$ to the σ_{St} matrix, appended "1" to the Sharpe ratio vector κ and "0" to the σ_I vector. We then obtain from (C1) and (C2) that

$$\phi_{\mathcal{I}t} = (\tilde{\sigma}_{St}^T)^{-1}\tilde{\kappa}_t + \frac{bI_t}{1+bI_t}(\tilde{\sigma}_{St}^T)^{-1}\tilde{\sigma}_I. \tag{C3}$$

The retail investor's optimal portfolio is determined by setting $b=0$:

$$\phi_{\mathcal{R}t} = (\tilde{\sigma}_{St}^T)^{-1}\tilde{\kappa}_t\,. \tag{C4}$$

Substituting $\tilde{\mu}_{St} \equiv (\tilde{\sigma}_{St}^T)^{-1}\tilde{\kappa}_t = \mu_{St} + \mathbf{1}$, we obtain the following.

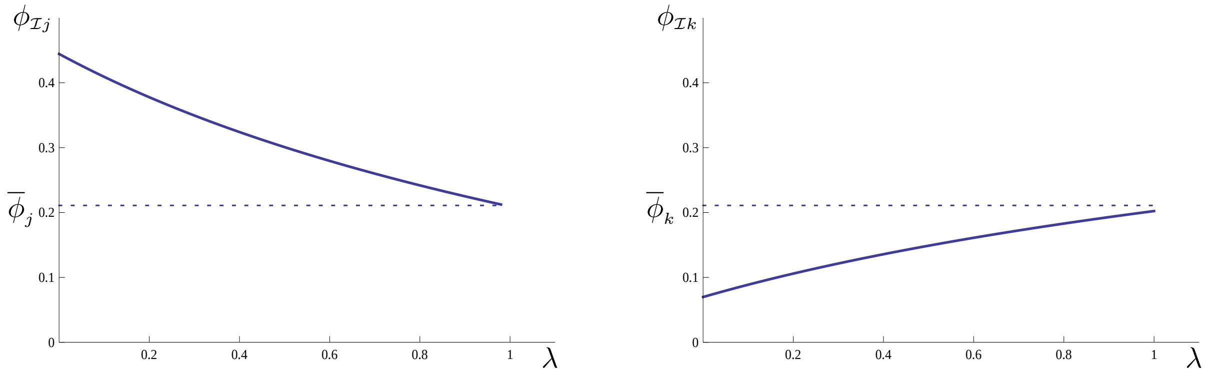
Lemma 2′. *The institutional and retail investors' optimal portfolio processes in the stocks-only economy are given by*

$$\phi_{\mathcal{I}t} = (\tilde{\sigma}_{St}\tilde{\sigma}_{St}^T)^{-1}\tilde{\kappa}_t + \frac{bI_t}{1+bI_t}(\tilde{\sigma}_{St}^T)^{-1}\tilde{\sigma}_I\,,$$

$$\phi_{\mathcal{R}t} = (\tilde{\sigma}_{St}\tilde{\sigma}_{St}^T)^{-1}\tilde{\mu}_{St}\,.$$

The structure of the optimal portfolios here closely resembles that presented in Lemma 2. The main difference is that the hedging portfolio of the institutional investor is not collinear with the index because of the last element of $\tilde{\sigma}_I$. This breaks the simple three-fund separation property that we have relied on in deriving our implications in Lemma 2. In particular, it is no longer the case that the hedging portfolio consists of index stocks only. It turns out that this portfolio has positive portfolio weights in index stocks and negative portfolio weights nonindex stocks. That is, the institution has a positive tilt in the index stocks and a negative tilt in nonindex stocks.[16] We have not, however, been able to prove this implication analytically, unlike the implications in Lemma 2. Our numerical analysis, consistent with out intuitions, confirms that this implication is true for a wide range of parameters. We depict the typical institutional portfolio in Figure 7. We further note that the counterparty to the institutional investor, the retail investor, ends up tilting his portfolio in the opposite direction. One may reinterpret the retail investor in our model as another institution, but one that is not benchmarked to the same index—for example, a hedge fund, whose performance is evaluated relative to a different benchmark, can be the counterparty to our institutional investor.

[16]The additional demand for index and nonindex relative to our earlier analysis is effectively demand for a portfolio replicating a riskless bond.



(a) Portfolio weights in index stocks **(b)** Portfolio weights in nonindex stocks

Figure 7: **The institutional investor's portfolio weights.** Panels (a) and (b) of this figure plot the institution's portfolio weights in an index stock j and a nonindex stock k, respectively, against the size of the institution λ. The lines for $\overline{\phi}$ correspond to the portfolio weights of an otherwise identical investor in the benchmark economy. The plots are typical. The remaining parameter values are as in Figure 6.

The implications reported in Propositions 6 and 7 go through in the stocks-only setting.

Proposition 6′. *In the stocks-only economy with $N + 1$ risky stocks, the equilibrium prices of the market portfolio, index stocks $j = 1, \ldots, M-1$ and nonindex stocks $k = M+1, \ldots, N$ are the same as those reported in Proposition 6 (in equations (26)–(27)). Consequently, all properties reported in Proposition 6 remain valid.*

Proposition 7′. *In the stocks-only economy with $N + 1$ risky stocks, the equilibrium volatilities of the market portfolio, index stocks $j = 1, \ldots, M-1$, and nonindex stocks $k = M+1, \ldots, N$ are the same as those reported in Proposition 7 (in equations (29)–(31)). Consequently, properties (i) and (ii) of the equilibrium stock volatilities and correlations remain as in Proposition 7.*

We find differences in portfolio holdings from the analysis in Section 4, due to the fact that the market structure of the available securities has changed. However, all other primitives of the model, including the objective functions and the terminal payoffs of the stock market D_T and the stock index I_T, have remained the same. Therefore, the equilibrium valuation of index and nonindex stocks, as presented in Proposition 6, and their ensuing equilibrium volatilities and correlations, as presented in Proposition 7, remain exactly the same. Hence, our asset-pricing implications are unchanged in the stocks-only economy.

In the typical case of our numerical analysis, no investor in the model takes on leverage. In rare cases one can obtain a scenario in which the negative tilt in the nonindex stocks is so large that it counterbalances the positive weights of these stocks in the mean-variance portfolio and the overall portfolio weights of nonindex stocks are negative. Such portfolios, featuring leverage, resemble portfolios held by 130/30 mutual funds who are short in some stocks to finance the increase in their exposure to other stocks in their portfolio. While this is theoretically possible in our model, this situation rarely occurs in our numerical analysis.

Appendix D: An Agency Justification for Benchmarking

In this appendix, we provide possible microfoundations for the two key properties of our institutional investor's objective function, namely that (i) it depends on the index level I_T and that (ii) the marginal utility of wealth is increasing in the index level I_T. The institutional investor can be thought of as an agent working for a principal, whom we have not explicitly specified in the body of the paper but will specify in this appendix. The agency problem is due to moral hazard.[17] The value of the managed portfolio is not observed perfectly by the principal, and the agent may take an unobservable action that reduces the portfolio value. For the former, see e.g., Getmansky, Lo, and Makarov (2004) for evidence that hedge funds report returns that are smoother than true returns. For the latter see e.g., Lakonishok et al. (1991) for evidence on "window dressing" by money managers—managers engage in unnecessary trades from the viewpoint of fund investors and incur unwarranted trading costs. See also Zitzewitz (2006) for evidence on value-destroying late trading activity by mutual funds, and Mahoney (2004) for an overview article on the hidden costs of investor-manager conflicts in mutual funds. The key property (i) of the institutional objective is relatively straightforward to obtain in this context. The argument that it may be beneficial for the principal to make the agent's contract depend on an index dates back to Holmström (1979). Since the index I_T is correlated with the (unobserved) portfolio value, it adds valuable information. Therefore for different contingencies signalled by I_T, the agent should receive different remunerations. To support key property (ii), one needs to characterize the optimal contract. For a dynamic model with a continuous state space like ours, it is well-known that this task is highly complex. To make it manageable, we adopt the tractable contracting setup recently proposed by Edmans and Gabaix (2011), which imposes few restrictions on preferences and distributions.[18]

[17] A related recent work modelling portfolio delegation under moral hazard is Dybvig, Farnsworth, and Carpenter (2010) who show that under certain conditions, benchmarking the manager against the index emerges as an optimal compensation contract.

[18] An alternative setup that one may employ is the moral-hazard-based relative performance model of Bolton and Dewatripont (2005, Chapter 8). Bolton and Dewatripont consider a setting with CARA agents and normally distributed shocks and focus on linear contracts. "Output" (portfolio value, in our context) produced by each agent is affected by a common shock. In that model, if individual outputs are not independent, it is optimal to make the contract contingent on the other agent's output. Moreover, if the two outputs are positively correlated, the other agent's output should enter negatively, so as to filter out the common shock. As also stressed by Holmström (1982), such a contract exposes the agent to less risk. It is easy to see from the analysis of Bolton and Dewatripont that the agent's marginal utility (under the optimal contract) increases in the output of his rival. Bolton and Dewatripont's model simplifies in our setting because instead of the rival's output we have an exogenous benchmark. The contract obtained in that setting will satisfy all three properties stated in Proposition D1 below, providing a valuable robustness check for our key insights in this appendix.

Following Edmans and Gabaix, we specify the institutional investor's objective as

$$E\left[u\big(v(c_T) + g(p_T)\big)\right], \tag{D1}$$

where $u(\cdot)$ is his utility function and $v(\cdot)$ is the felicity function that denotes the agent's utility from the cash compensation c_T. The agent, the "manager," reports at time T the value of his portfolio $\hat{W}_T \equiv (1 - p_T)W_T$, where $p_T \in (0, 1)$ is the fraction of portfolio value that gets diverted by the manager at time T (by engaging in value-destroying activities, an extreme version of which is stealing). The manager derives a private benefit $g(p_T)$ from this diversion activity. Only the agent observes the true state of the world, which in our setting is captured by the state price density ξ_T. The principal can thus offer the manager a compensation contract contingent on $\hat{W}_T$ but not on ξ_T. We also allow the manager to include the index level I_T in the compensation contract, noting that I_T is correlated with ξ_T (although not perfectly, as in our setup of Section 4). The principal knows the distribution of ξ_T conditional on I_T.

Borrowing from Edmans and Gabaix, we make two important assumptions. First, the manager "takes action after noise"—i.e., decides on the diversion policy p_T after observing the state of the world ξ_T. Second, the principal wishes to implement the action $p_T = p^*$—i.e., we do not solve for the optimal action but specify it exogenously and solve for the contract that implements it. This assumption can be justified within a setting in which the marginal benefit to the principal of reducing diversion far exceeds the benefit of diversion to the manager, and so the level of diversion should be set equal to its lower bound p^*, as specified, e.g., by the trustees of the fund.

The manager chooses his optimal terminal wealth W_T and the action p_T to maximize (D1) subject to the following constraint:

$$E[\xi_T W_T] = W_0, \tag{D2}$$

where $c_T = c(\hat{W}_T, I_T)$. This constraint is the budget constraint written in a static form (see equation (A2)), which allows us replace the problem of solving for the optimal portfolio ϕ by the (simpler) problem of solving for the optimal W_T as a function of the state variable ξ_T (Cox and Huang (1989)). It is then easy to recover the portfolio ϕ that implements the optimal W_T.

The formal solution to this problem can be found in Edmans and Gabaix. Here we provide a heuristic derivation assuming that the functions u, v and g satisfy all necessary regularity conditions. The first-order conditions to the manager's problem with respect to W_T and p_T, respectively, are:

$$u'(v(c_T) + g(p_T))v'(c_T)c_{\hat{W}}(\hat{W}_T, I_T)(1 - p_T) = y_M \xi_T, \tag{D3}$$

$$u'(v(c_T) + g(p_T))(v'(c_T)c_{\hat{W}}(\hat{W}_T, I_T)(-W_T) + g'(p_T)) = 0, \tag{D4}$$

where y_M is the Lagrange multiplier on the manager's static budget constraint (D2). From (D4) evaluated at $p_T = p^*$, we derive that

$$v'(c_T)c_{\hat{W}}(\hat{W}_T, I_T) = \frac{1-p^*}{\hat{W}_T} g'(p^*),$$

which after integrating over $\hat{W}_T$ yields the optimal contract of the form

$$c(\hat{W}_T, I_T) = v^{-1}\left((1-p^*)g'(p^*)\log(\hat{W}_T) + K(I_T)\right). \tag{D5}$$

The function $K(I_T)$ is chosen by the principal so as to maximize his expected utility subject to the manager's participation constraint $E[u\big(v(c_T) + g(p_T)\big)] \geq \underline{u}$, where $\underline{u}$ is the manager's reservation utility. This is the same contract as in Edmans and Gabaix (see especially their Appendix C).

The principal is a fund investor, who is left unmodelled in the body of the paper. This investor delegates all his money to the manager. For simplicity, we assume that this investor is risk-neutral (this assumption can be relaxed in future work). He chooses the function $K(I_T)$ to minimize the expected cost of the contract in (D5) subject to the manager's participation constraint:

$$\min_{K(\cdot)} E\, v^{-1}\left((1-p^*)g'(p^*)\log(\hat{W}_T) + K(I_T)\right) \tag{D6}$$

$$\text{s.t. } E\left[u\Big((1-p^*)g'(p^*)\log(\hat{W}_T) + K(I_T) + g(p^*)\Big)\right] \geq \underline{u}. \tag{D7}$$

We now specialize the manager's preferences to $u(x) = e^{(1-\gamma)x}/(1-\gamma)$, $\gamma > 1$, and $v(x) = \log x$. This specification has been adopted by, e.g., Edmans and Gabaix (2011) and Edmans et al. (2012). Under this specification, one can use equations (D3) and (D2) to compute the optimal terminal wealth of the agent (after diversion) in closed form:

$$\hat{W}_T = \xi_T^{-\frac{1}{1-Z_2}} \frac{W_0(1-p^*)e^{M_2\,K(I_T)}}{E\left[\xi_T^R e^{M_2\,K(I_T)}\right]}$$

and reduce the principal's problem (D6)–(D7) to

$$\min_{K(\cdot)} E\left[e^{M_1\,K(I_T)}\xi_T^{R/(1-\gamma)}\right]\left(E\left[e^{M_2\,K(I_T)}\xi_T^R\right]\right)^{Z_1} \tag{D8}$$

$$\text{s.t. } E\left[e^{M_1\,K(I_T)}\xi_T^R\right] \geq Q\left(E\left[e^{M_2\,K(I_T)}\xi_T^R\right]\right)^{Z_2}, \tag{D9}$$

where

$$Z_1 = -(1-p^*)g'(p^*), \quad Z_2 = (1-\gamma)(1-p^*)g'(p^*), \quad Q = \frac{(1-\gamma)\underline{u}}{(W_0(1-p^*))^{Z_2}\, e^{g(p^*)}},$$

$$M_1 = \frac{1-\gamma(1-\gamma)(1-p^*)g'(p^*)}{1-(1-\gamma)(1-p^*)g'(p^*)}, \quad M_2 = \frac{1-\gamma}{1-(1-\gamma)(1-p^*)g'(p^*)}, \quad R = Z_1 M_2.$$

Denote the optimal function $K(I_T)$ by $\overline{K}(I_T)$ and consider a small perturbation $\varepsilon L(I_T)$, where ε is a scalar and $L(I_T)$ is a function of I_T. Replacing $K(I_T)$ by $\overline{K}(I_T) + \varepsilon L(I_T)$ in the principal's problem (D8)–(D9) and considering only terms of up to order 1 in ε, we arrive at

$$\min_{L(I_T)} E\left[e^{M_1\overline{K}(I_T)}\xi_T^{R/(1-\gamma)}\right]\left(E\left[e^{M_2\overline{K}(I_T)}\xi_T^R\right]\right)^{Z_1} + E\left[\left(Y_1 e^{M_1\overline{K}(I_T)}\xi_T^{R/(1-\gamma)} + Y_2 e^{M_2\overline{K}(I_T)}\xi_T^R\right)\varepsilon L(I_T)\right] \tag{D10}$$

s.t.

$$E\left[e^{M_1\overline{K}(I_T)}\xi_T^R\right] - Q\left(E\left[e^{M_2\overline{K}(I_T)}\xi_T^R\right]\right)^{Z_2} + E\left[M_1 e^{M_1\overline{K}(I_T)}\xi_T^R\varepsilon L(I_T)\right] \geq E\left[Y_3 e^{M_2\overline{K}(I_T)}\xi_T^R\varepsilon L(I_T)\right], \tag{D11}$$

where

$$Y_1 = \left(E\left[e^{M_2\overline{K}(I_T)}\xi_T^R\right]\right)^{Z_1} M_1, \quad Y_2 = Z_1 M_2 E\left[e^{M_1\overline{K}(I_T)}\xi_T^{R/(1-\gamma)}\right]\left(E\left[e^{M_2\overline{K}(I_T)}\xi_T^R\right]\right)^{Z_1-1},$$
$$Y_3 = Q Z_2 M_2\left(E\left[e^{M_2 K(I_T)}\xi_T^R\right]\right)^{Z_2-1}.$$

The terms in $\overline{K}(I_T)$ can be dropped from the objective because the maximization is with respect to L_T. They can be dropped from the constraint because it is satisfied for $\overline{K}(I_T)$. Hence, the principal's problem reduces to

$$\min_{L(I_T)} E\left[\left(Y_1 e^{M_1\overline{K}(I_T)}\xi_T^{R/(1-\gamma)} + Y_2 e^{M_2\overline{K}(I_T)}\xi_T^R\right) L(I_T)\right] \tag{D12}$$

$$\text{s.t. } E\left[\left(M_1 e^{M_1\overline{K}(I_T)}\xi_T^R - Y_3 e^{M_2\overline{K}(I_T)}\xi_T^R\right) L(I_T)\right] \geq 0. \tag{D13}$$

Since $\overline{K}(I_T)$ is optimal, this minimum must be zero. Therefore, the terms in the parentheses of the objective (D12) and of the constraint (D13) have to be the same, state-by-state, up to a multiplicative constant. Otherwise, it would be possible to find a function $L(I_T)$ which renders the objective negative without violating the constraint. We therefore have

$$\begin{aligned} &Y_1 e^{M_1\overline{K}(I_T)} E\left[\xi_T^{R/(1-\gamma)}\middle| I_T\right] + Y_2 e^{M_2\overline{K}(I_T)} E\left[\xi_T^R\middle| I_T\right] \\ &\qquad = \psi\left(M_1 e^{M_1\overline{K}(I_T)} E\left[\xi_T^R\middle| I_T\right] - Y_3 e^{M_2\overline{K}(I_T)} E\left[\xi_T^R\middle| I_T\right]\right), \end{aligned}$$

where the expectations are conditional on I_T because the principal observes I_T but not ξ_T, and ψ is a constant. Solving this equation for $\overline{K}(I_T)$ and simplifying, we obtain

$$\overline{K}(I_T) = \frac{1}{\gamma}\log\left(\frac{E\left[\xi_T^R\middle| I_T\right](\psi Y_3 - Y_2)}{\psi M_1 E\left[\xi_T^R\middle| I_T\right] + Y_1 E\left[\xi_T^{R/(1-\gamma)}\middle| I_T\right]}\right), \tag{D14}$$

where the constant ψ is such that the participation constraint (D9) binds with equality. Equation (D14) reveals that if the index I_T and the state price density ξ_T are independent,

$K(I_T)$ is a constant and hence the optimal contract does not depend on the index I_T. In the language of Holmström (1979), in that case the signal I_T does not carry any valuable information about the state of the world ξ_T. Only if the two random variables are correlated, the signal becomes valuable, and so it is beneficial for the principal to include I_T in the compensation contract.

To derive the relevant properties of the optimal contract, we need to evaluate the conditional expectations in (D14). Towards this, we impose distributional assumptions on the processes ξ and I. We note that the results derived below hold under milder assumptions, but at the expense of expositional clarity. We assume that

$$dI_t = \sigma_I I_t d\omega_{1t}, \tag{D15}$$

$$d\xi_t = -\kappa_1 \xi_t d\omega_{1t} - \kappa_2 \xi_t \omega_{2t}, \tag{D16}$$

where the Brownian motions ω_1 and ω_2 are independent and where $\kappa_1,\ \kappa_2 \geq 0$ are constant. As in Section 4, the index cash flow news process I_t loads on a subset of Brownian motions driving the economy, while the state price density process ξ_t loads on all of them. By observing I_T the principal learns of the realization of ω_{1T} but not of ω_{2T}. We can now compute the conditional expectations in (D14) as follows:

$$E\left[\xi_T^R \middle| I_T\right] = \xi_0^R e^{\left(\frac{(R\kappa_2)^2}{2} - R\frac{||\kappa||^2}{2}\right)T} e^{-R\kappa_1 f(I_T)}$$

$$E\left[\xi_T^{R/(1-\gamma)} \middle| I_T\right] = \xi_0^{R/(1-\gamma)} e^{\left(\frac{(R/(1-\gamma)\kappa_2)^2}{2} - \frac{R}{1-\gamma}\frac{||\kappa||^2}{2}\right)T} e^{-\frac{R}{1-\gamma}\kappa_1 f(I_T)},$$

where $f(I_T) = \left(\log\left(I_T/I_0\right) + \sigma_I^2 T/2\right)/\sigma_I$ and $\kappa = (\kappa_1, \kappa_2)$. It follows from our definitions that $R < 0$ and $\gamma > 1$. It is then straightforward to show that

$$\frac{\partial E\left[\xi_T^R \middle| I_T\right]}{\partial I_T} < 0 \quad \text{and} \quad \frac{\partial E\left[\xi_T^{R/(1-\gamma)} \middle| I_T\right]}{\partial I_T} > 0.$$

The economic intuition for these results is that good states of the world (low ξ_T states) are more likely to occur when the index I_T is high.

Under our assumptions, the (indirect) utility of the manager is given by

$$u_{\mathcal{I}}(\hat{W}_T, I_T) = \frac{e^{(1-\gamma)\left((1-p^*)g'(p^*)\log \hat{W}_T + \overline{K}(I_T) + g(p^*)\right)}}{1-\gamma},$$

where we have substituted the optimal contract. Taking the pertinent derivatives and signing them, one can prove the following result.

Proposition D1. *As long as the index I_T and the state price density ξ_T are correlated ($\kappa_1 \neq 0$),* *(i) the manager's optimal compensation contract is contingent on I_T;*

(ii) the manager's compensation decreases in the level of the index $\left(\frac{\partial c(\hat{W}_T, I_T)}{\partial I_T} < 0\right)$;

(iii) the marginal utility of wealth of the manager increases in the level of the index $\left(\frac{\partial^2 u_{\mathcal{I}}(\hat{W}_T, I_T)}{\partial \hat{W}_T \partial I_T} > 0\right)$.

In our discussion of the optimal contract we have already highlighted property (i) of Proposition D1. This property does not rely on any distributional assumptions on ξ_T and I_T. The intuition for property (ii) can be adapted from Holmström (1979): the manager should not be excessively penalized for poor performance (beyond optimal risk sharing) if his index has also performed poorly; on the contrary, if the index has done well, the manager's poor performance could be an indication of a high level of cash flow diversion, and so the manager should be penalized. Note that the objective function in the body of the paper does not satisfy property (ii). However, as discussed in Remark 1, the institutional investor's objective function can be made decreasing in I_T without any change to our results (by, e.g., subtracting from it a sufficiently increasing function of I_T). Since the contract explicitly penalizes the manager for underperformance relative to his index, his marginal utility of wealth is especially high in the states in which the index has done well. This intuition is formalized in property (iii) of Proposition D1. Property (iii) plays an important role in our results reported in the body of the paper. As our analysis in this appendix demonstrates, an agency problem in money management is a channel through which it may arise.

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