Contracting With Synergies

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Most compensation models consider a single agent (e.g. CEO). But most work is conducted in teams.

Team members’ actions are synergistic (effort by one agent reduces colleague’s cost of effort). Structure of synergies is complex:

- Asymmetric: CEO has greater impact on VP than vice-versa
- Number of synergistic relationships varies across agents

Broad framework of contracting under synergies where:

- Effort is continuous (bi-level contracting problem)
- Synergies are asymmetric
- Number of synergistic relationships differs

Solve for optimal effort and wages (absolute and relative); total wage bill
Introduction (cont’d)

Purpose of model:
- Address questions that cannot be explored in a single-agent framework, e.g. XS pay and effort differences, team composition
- Explain real-life practices that contradict single-agent models, e.g. high incentives given to employees with little direct effect on output

Existing literature on multi-agent principal-agent problems:
- Free-rider problem, efforts are perfect substitutes: Holmstrom (1982)
- Contracting with externalities:
  * Kandel and Lazear (1992): peer pressure; effort affects others’ utility
  * Segal (1999): agents impact others’ utilities rather than costs. Action is participation not effort: no output
  * Bernstein and Winter (2012) extend to heterogeneous externalities
Roadmap

- General model illustrating synergy concept and effect of changing $i$’s influence on his effort and pay
- Specific model allows additional study on $j$’s effort and pay
  - Two-agent model
  - Three-agent model
- Extensions
  - Negative synergies
  - First-best analysis
  - Production complementarities
The General Model

- RN principal ("firm") and $N$ agents ("workers"). LL, $w = 0$
- Output is $r \in \{0, 1\}$, contractible. Production function:
  $$\text{Pr}(r = 1) = P(p_1, \ldots, p_N) = P(p) \text{ with } P_i > 0, P_{ii} < 0$$
- Effort is $p_i \in [0, 1]$
- $i$’s overall cost function is $C^i(p) = k_i g_i(p_i) \prod_{j \neq i} h_{ji}(p_j)$
  - $g_i(p_i)$ is $i$’s individual cost function with $g'_i > 0, g''_i > 0$
  - $h_{ji}(p_j)$, with $h'_{ji}(\cdot) < 0$, represents $j$’s influence on $i$
    * Affects marginal cost, not just total cost
    * Cost reduction or private benefit enhancement
  - $k_i = 1$ initially
- $P$ and $C^i$ are common knowledge before contracting. Agents choose efforts $p_i$ simultaneously in a Nash equilibrium
Agent $i$ is paid 0 upon failure and $w_i$ upon success; principal must choose optimal $w_i$

- $w_i$ captures both *level* and *sensitivity* of pay

Agent $i$ solves $\max_{p_i} w_i P(p) - C_i(p)$. Assuming FOA, $w_i = \frac{C_i}{P_i}$

Principal solves $\max_{\{p_i\},\{w_i\}} P(p_1, \ldots, p_N) (1 - \sum_i w_i) = PM$. 
Analysis (cont’d)

- FOC wrt $p_i$ is

$$0 = P_i \left[ 1 - \sum_i w_i \right]$$

- $P_i \frac{g''(p_i^*) \prod_{j \neq i} h_{ji}(p_j^*) P_i - g'(p_i^*) \prod_{j \neq i} h_{ji}(p_j^*) P_{ii}}{P_i^2}$

- $P \sum_{j \neq i} g'_j (p_j^*) \frac{h'_{ij}(p_i^*) \prod_{m \neq i, j} h_{mij}(p_m^*) P_j - \prod_{j \neq i} h_{ij}(p_i^*) P_{ij}}{P_j^2}$

- First term: increased production multiplied by principal’s share
- Second term: increased wage required to induce a higher $p_i$
- Third term results from complementarities
  - First part: effect of $p_i$ on colleagues’ cost (influence)
  - Second part: effect of $p_i$ on colleagues’ productivity
Effect of Increasing 1’s Influence

- \( h_{1j} (p_j) \) shifts to \( \tilde{h}_{1j} (p_j) \), where \( \tilde{h}_{1j} \leq h_{1j} \) and \( \tilde{h}'_{1j} \leq h'_{1j} \ \forall \ j \neq 1 \), with \( \tilde{h}_{1j} < h_{1j} \) and \( \tilde{h}'_{1j} < h'_{1j} \) for at least one \( j \). Two effects:

1. Reduces \( j \)'s cost \( C^j \) and \( \text{MC} \ C^j \) to

   \[
   C^j (p) = g_j (p_j) \tilde{h}_{1j} (p_1) \prod_{m \neq 1,j} h_{mj} (p_m)
   \]

   \[
   C^j_1 (p) = g'_j (p_j) \tilde{h}'_{1j} (p_1) \prod_{m \neq 1,j} h_{mj} (p_m)
   \]

   Reduction would occur even if \( p_1 \) held constant at \( p_1^* \). Could be achieved by reducing \( k_j \), so present in a model w/o synergies

2. Reduces cross-partial \( C^j_{1j} = g'_j (p_j) \tilde{h}'_{1j} (p_1) \prod_{m \neq 1,j} h_{mj} (p_m) < 0 \). Specific to a model of synergy
We thus shift 1’s influence to reduce $C_{1j}^j$, but keep $C_j^j$ and $C_{j}^j$ unchanged at $p_1^*$. Set $k_j = \frac{h_{1j}(p_1^*)}{\tilde{h}_{1j}(p_1^*)}$, so j’s new cost function is:

$$\tilde{C}_j^j(p) = \frac{h_{1j}(p_1^*)}{\tilde{h}_{1j}(p_1^*)} g_j(p_j) \tilde{h}_{1j}(p_1) \prod_{m \neq 1, j} h_{mj}(p_m).$$

Note that $C_j^j(p_1^*, p_2, ..., p_N) = \tilde{C}_j^j(p_1^*, p_2, ..., p_N)$

Principal’s new objective function is $P\tilde{M}$
Effect of Increasing 1’s Influence: Results

- \( \frac{\partial}{\partial p_j} |_{p^*} P \tilde{M} = 0 \), so \( \frac{\partial}{\partial p_1} |_{p^*} P \tilde{M} > 0 \) is sufficient for \( \tilde{p}_1^* > p_1^* \)

\[
\frac{\partial}{\partial p_1} |_{p^*} P \tilde{M} = P_1 \left[ 1 - \frac{C_1}{P_1} - \sum_{j \neq 1} \frac{\tilde{C}_j}{P_j} \right] \\
+ P \left[ - \frac{C_{11}^1 P_i - C_1^1 P_{11}}{P_1^2} - \sum_{j \neq 1} \frac{\tilde{C}_{j1} P_j - \tilde{C}_j^1 P_{j1}}{P_j^2} \right] \\
= \frac{\partial}{\partial p_1} |_{p^*} PM + \sum_{j \neq 1} \frac{P}{P_j} \left( C_{j1}^j - \tilde{C}_{j1}^j \right) \\
= \sum P \prod_{i \neq 1} \frac{P_j}{P_i} h_{ij}(p_i^*) g_j^i(p_j^*) \left[ h_{1j}^j(p_1^*) - k_j \tilde{h}_{1j}^j(p_1^*) \right]
\]

- No \( h_{ij} \) terms (taken care of by rescaling), only \( h_{ij}' \) terms
- \( P_{j1} \) drops out: already taken care of in original maximization
Effect of Increasing 1’s Influence: Results (cont’d)

- Since $\tilde{h}'_{1j} \leq h'_{1j} < 0$ and $k_j \geq 1$, $\frac{\partial}{\partial p_1} |_{p^*} P\tilde{M} > 0$ so $\tilde{p}^*_1 > p^*_1$
  - Robust to any $P$, $g$, $h_{ij}$. All we need is $\tilde{h}'_{1j} (p^*_i) < h'_{1j} (p^*_i)$

- If $P_{1j} = 0 \, \forall \, j$ and $h_{j1} (\cdot) = 1 \, \forall \, j$, $\tilde{w}^*_i > w^*_i$
  - Not automatic that $\uparrow p \rightarrow \uparrow w$. In single-agent models with RN and LL, $w^* = \frac{1}{2}$, independent of $p$ (and thus productivity and cost)

- Here, wage unambiguously increases in influence
  - Influence has no direct effect on output; only affects colleague’s cost
  - 1 does not consider this effect, as he takes $j$’s effort as given
  - Sharper contract causes him to internalize the externality. 1 is “over-incentivized” compared to a model without synergy
The Two-Agent Model

The Model

- $\Pr(r = 1) = \frac{p_1 + p_2}{2}$
- $g_i(p_i) = \frac{1}{4} p_i^2$
- $h_{ji}(p_j) = 1 - h_{ji} p_j$ where $h_{ij} \geq 0$

is an influence parameter. Affects marginal cost, not just total cost

- Thus, $C_i(p) = \frac{1}{4} p_i^2 (1 - h_{ji} p_j)$

**Synergy** is the sum of the influence parameters: $s = h_{12} + h_{21}$
Results

- (i) For nonzero synergy, efforts are equal: \( p_1^*(s) = p_2^*(s) \equiv p^*(s) \). Exists a critical synergy level \( s^* > 0 \) s.t.:

\[
p^*(s) = \begin{cases} 
  \frac{2 - \sqrt{4 - 3s}}{3s} & s \in (0, s^*) \\
  1 & s \geq s^*.
\end{cases}
\]

- Seems that more influential agent should exert higher effort. But this decreases colleague’s cost, inducing greater effort
  - Equal effort is model-specific. But, idea that \( p_i^* \) depends on \( i \)’s influence and \( j \)’s influence (not just \( i \)’s individual production and cost functions) is general
  - While influence parameters are individual and asymmetric, the synergy is common to both agents; an “echo”. A sufficient statistic for effort

- Effort \( p^*(s) \) is increasing in synergy \( s \)
  - Direct effect on output unchanged, but reduces colleague’s cost
(ii) Total wages, and expected total wages, are both increasing in $s$ on $(0, s^*)$

- Again, not automatic: even though $p^*$ is higher, cost of effort is lower

Suggests high equity incentives should be given to low-level employees, if large indirect effect (e.g. efficient analyst reduces cost of director going to a meeting)

- Synergies will be particularly high in small and young firms (blurred job descriptions, frequent interactions, flat hierarchy)
- Hochberg and Lindsey (2010): broad-based option plans
Results (cont’d)

- (iii) $\uparrow$ in either influence parameter raises effort and pay
  - A rise in influence raises $s$, and thus effort (i) and wages (ii). But, no independent effect other than through $s$
  - Common synergy, not individual influence parameters, determines $p^*$
  - That $s$ is a sufficient statistic is model-specific, but idea that effort and pay depends on your influence and colleague’s influence is general

- (iv) The more influential agent receives higher pay
  - Holds even though both agents exert same effort (so $\uparrow w$ is not a compensating differential) and have same direct productivity
  - A greater externality to internalize
  - More influential agents should receive higher pay, even if perform same tasks (e.g. senior vs. junior faculty)
Results (cont’d)

- (v) \( \uparrow i \)’s relative influence (i.e. \( \uparrow h_{ij} \) and \( \downarrow h_{ji} \) so that \( s \) is unchanged) → \( \uparrow \) relative and absolute \( w \)

- (vi) \( \uparrow h_{ij} \rightarrow \uparrow w_j^* \) iff \( h_{ji} > \frac{1}{6p^*(s)} \)
  - Optimal to reinforce effect of \( h_{ij} \) by incentivizing \( j \) more iff \( j \) is sufficiently influential

- (vii) The more influential agent receives higher utility
The Model

- \( \text{Pr}(r = 1) = \frac{p_1 + p_2 + p_3}{3} \)
- \( C_i(p) = \frac{1}{6} p_i^2 \left( 1 - \sum_{j \neq i} h_{ji} p_j \right) \)
- Define:
  \[
  A = h_{12} + h_{21} \quad B = h_{13} + h_{31} \quad C = h_{23} + h_{32}.
  \]
- **Synergy profile** \( s \) is the vector \((A, B, C)\).
  - In two-agent model, this was a scalar
- \( A, B \) and \( C \) are the **synergy components** of the synergy profile
- **Size** of \( s \) is \( s = \| (A, B, C) \| \).
Results
Results: Interior Effort

- If $s$ small and synergy component $> \sum$ of the other two, effort profile $p^*(s)$ is interior
  - Rise in $s$ augments effort profile

- Simplex fixes $A + B + C$ at $K$ and changes relative components

- Middle triangle studies interior effort profile
  - All three synergy components matter for the relative size of the individual effort levels: iff $B > C$, $p_1 > p_2$.
  - $p_1$ vs $p_2$ depends on total synergy with agent 3 (B vs C), not just $h_{13}$ vs $h_{23}$. $h_{31}$ doesn’t affect 1’s productivity, but does affect his cost. As in 2-agent case, $p^*$ depends on common synergy, not individual influence

- Agents no longer exert same effort. More synergistic agents work harder
  - In 2-agent case, there is only one synergy component
  - Here, 3 SCs, allowing for asymmetry in effort levels
Results: Boundary Effort

- If one synergy component > sum of other two, model collapses to 2-agent model
  - Third agent is ignored, despite having same direct productivity. His participation depends on extrinsic parameters. Implications for composition of team / boundaries of firm
  - Only the largest synergy component matters for optimal effort profile
    * For interior solution, $p_1 > p_2$ iff $B > C$
    * Here, $p_1 = p_2$ regardless of $B$ vs. $C$. Synergy between 1 and 2 is so important that individual synergies with 3 are irrelevant
  - As in 2-agent model, wages depend on relative influence of each agent
If a single synergy component is close to zero, the two non-synergistic agents can be aggregated into a single agent and model again collapses to 2-agent model

- Third agent exerts same effort as others combined; their efforts depend on relative size of synergies with third agent
- Applies to CEO and divisional managers

* Bebchuk, Cremers, and Peyer (2011): high pay to CEO is inefficient rent extraction
* Rising pay over time consistent with improving communication technologies (cf. Garicano and Rossi-Hansberg (2006))
* Relevant measure of firm size (e.g. in Gabaix and Landier (2008), Terviö (2008)) is scope and depth of synergies, rather than assets or profits. CEO of holding company should be paid less than CEO of small focused firm
Results: Wages

- Total wages depend on total synergy
- Relative wages depend on influence parameters: an increase in one agent’s influence parameter increases his wage in both absolute and relative terms
- If influence parameters are symmetric across a pair of agents, relative wages within this pair are the same. Entire wage profile can be fully solved; ratios of optimal wages coincide with ratios of optimal effort
Extensions

- Negative influence parameters: if \( s > 0 \), results hold; if \( s < 0 \), collapses to a single-agent model

- Complementary effort: \( \Pr(r = 1) = \min(p_1, p_2, \ldots, p_N) \). Results remain robust: effort and wages increase in synergy

- First-best analysis: more influential agent exerts less effort
  - Hence, moral hazard problem \( \rightarrow \uparrow \) relative effort of the more influential agent
  - First-best: wages are compensation for disutility (quadratic total costs)
    * Cost reduction due to 1’s influence is \( h_{12}p_1p_2^2 \), so \( \uparrow h_{12} \) causes principal to \( \uparrow p_2 \) more than \( p_1 \)
  - Second-best: wages are incentives for effort (linear marginal costs)
Production Synergies

- $\Pr (r = 1) = a \frac{p_1 + p_2}{2} + b \sqrt{p_1 p_2}$
- $c_i (p) = \frac{1}{4} p_i^2$
- $p^*_1 = p^*_2 = \frac{a + b}{4}$
- To implement $(p_1, p_2)$, must offer $w_1 = \frac{p_1}{a + b \sqrt{\frac{p_2}{p_1}}}$ and $w_2 = \frac{p_2}{a + b \sqrt{\frac{p_1}{p_2}}}$
- $w^*_1 = w^*_2 = \frac{1}{4}$ and is independent of $a$ or $b$
- $a$ and $b$ enter symmetrically: both increase agent $i$’s impact on output
- Production function and cost function are not isomorphic
  - Contracts are contingent upon output but cannot be contingent upon cost of effort. Thus, agent internalizes effect on production but not on colleague’s costs
  - Cost synergies are true externalities, unlike production synergies
With production synergies, $i$’s FOC changes from

$$w_i P_i(p_i, p_j) - g'_i(p_i) \prod_{j \neq i} h_{ji}(p_j)$$

to

$$w_i \tilde{P}_i(p_i, p_j) - g'_i(p_i) \prod_{j \neq i} \tilde{h}_{ji}(p_j),$$

and so the synergy is internalized. With cost synergies, $P_i$ is unchanged. $h_{ij}$ changes to $h_{ij}$, but this does not show up in $i$’s FOC so is not internalized.
Conclusion

- Paper studies effect of synergies on optimal effort and wages
- General model:
  - Additional term in effort determination equation
  - $i$’s influence $\rightarrow p_i^*$
- 2-agent framework
  - Wages differ, even if same effort and direct productivity
  - Total wages increase with synergy, consistent with high equity incentives in start-ups
  - Effort depends not only on individual production and cost functions, but your influence and colleague’s influence
- 3-agent framework:
  - Efforts differ and depend on synergies, not influence
  - If synergies between two agents are strong, third agent is excluded
  - Agents that exert synergies over more colleagues are paid more
Complementary Effort

- Production function:
  \[ \Pr(r = 1) = \min (p_1, p_2, \ldots, p_N). \]

- Cost function:
  \[ h_i(p_i) = \frac{\kappa_i}{2} p_i^2. \]

- FOCs:
  \[ p_1 = p_2 = \ldots = p_N \equiv p \]
  \[ w_i(p) = \kappa_i p \left( 1 - \sum_{j \neq i} h_{ji} p \right) \]

- All agents exert same effort (as perfect complementarities)
- Agents with more difficult tasks (higher \( \kappa_i \)) receive higher wages.
- **Synergy** is sum of each agent's total influence: \( s = \sum_i (\sum_{j \neq i} h_{ij}\kappa_j) \)
- **Difficulty** is sum of the cost parameters, \( \kappa \equiv \sum_i \kappa_i \).
Complementary Effort: Results

(i) ∃ a critical synergy level \( s^* (\kappa) > 0 \) s.t.:

\[
p^*(s) = \begin{cases} \frac{\kappa - \sqrt{\kappa - 3s}}{3s} & s \in [0, s^* (\kappa)) \\ 1 & s \geq s^* (\kappa) \end{cases}
\]

Optimal effort \( p^*(s) \) is strictly increasing on \([0, s^* (\kappa)]\).

(ii) Total wages, and expected total wages, are both strictly increasing on \([0, s^* (\kappa)]\).

(iii) An increase in any influence parameter of any agent raises effort and wages

(iv) Suppose agent \( i \)'s relative influence increases (total influence increases but total synergy is constant), then his relative and absolute wealth increases
General Model: Wages

\[
\begin{align*}
w_1^* &= \frac{C_1^1(p_1^*)}{P_1(p^*)} = \frac{g'_1(p_1^*) \Pi_{j \neq i} h_{ji}(p_j)}{P_1(p^*)} \\
\tilde{w}_1^* &= \frac{C_1^1'(\tilde{p}_1^*)}{P_1(\tilde{p}^*)} = \frac{g'_1(\tilde{p}_1^*) \Pi_{j \neq i} h_{ji}(\tilde{p}_j)}{P_1(\tilde{p}^*)}.
\end{align*}
\]

- \(g'_1(\tilde{p}_1^*) > g'_1(p_1^*)\), since \(\tilde{p}_1^* > p_1^*\) and costs are convex
- \(P_1(\tilde{p}_1^*, \cdot) < P_1(p_1^*, \cdot)\), since \(\tilde{p}_1^* > p_1^*\) and prod. fn. is concave
- Potentially confounding effects:
  - Even though \(P_1(\tilde{p}_1^*, \cdot) < P_1(p_1^*, \cdot)\), \(P_1\) may not have decreased because it depends on \((p_2^*, \ldots, p_N^*)\). If \(\uparrow p_j^*\) and \(P_{1j} > 0\), or \(\downarrow p_j^*\) and \(P_{1j} < 0\), this tends to \(\rightarrow \uparrow P_1\) and \(\downarrow w_1^*\)
  - If \(\uparrow p_j^*, h_{j1}(\tilde{p}_j^*) < h_{j1}(p_j^*)\)
  - These two effects disappear if \(P_{1j} = 0 \ \forall \ j\) (so \(P_1(\tilde{p}^*) < P_1(p^*)\)) and \(h_{j1}(\cdot) = 1 \ \forall \ j\)