Online Appendix for "The Effect of Diversification on Price Informativeness and Asset Values"

B Endogenous Asset Value: Full Analysis

This section analyzes the endogenous asset value model of Section 3 in full and also contains the proofs of Lemma 5 and Proposition 4 in the main text.

B.1 Exit Under Concentration

Lemma 6 characterizes all the thresholds that emerge in any equilibrium under concentration.

Lemma 6 *(Concentration, endogenous asset values): In any equilibrium under concentration with endogenous asset values, the unique threshold for each manager,* $c^*_{con,end}$*, is given by the solution of* $c^* = \phi_{end}(F(c^*))$ *where*

$$\phi_{end}(\tau) = \Delta - \frac{\omega\Delta}{\tau + \frac{1-\tau}{\beta}}. \tag{23}$$

Prices and trading strategies are characterized by Lemma 1, where τ *is given by* $\tau^*_{con,end} \equiv F\left(c^*_{con,end}\right)$*.*

B.2 Exit Under Diversification

Lemma 7 below characterizes the most efficient equilibrium under diversification.

Lemma 7 *(Diversification, endogenous asset values): The working threshold under the most efficient equilibrium is given by*

$$c^{**}_{div,end} = \begin{cases} \min\{\Delta, F^{-1}(1 - \frac{L/n}{\underline{v}})\} & \text{if } L/n \leq \underline{v}\left(1 - \tau^{**}_{ii,end}\right) \\ c^{**}_{ii,end} \equiv \text{the largest solution of } c^* = \zeta_{end}\left(F\left(c^*\right)\right) & \text{if } \underline{v}\left(1 - \tau^{**}_{ii,end}\right) < L/n < \underline{v} \\ c^*_{con,end} & \text{if } \underline{v} \leq L/n, \end{cases} \tag{24}$$

where $\tau^{**}_{ii,end} \equiv F\left(c^{**}_{ii,end}\right)$ *and*

$$\zeta_{end}(\tau) = \Delta - \frac{\omega\Delta}{\frac{\tau}{1-\beta} + \frac{1-\tau}{\beta}}. \tag{25}$$

Prices and trading strategies are characterized by Lemma 2.

Lemma 7 leads to Proposition 4 in the main text. While Proposition 7 focuses on the most efficient equilibrium, Proposition 5 we consider all equilibria and show that, if β is sufficiently high, *any* equilibrium under diversification is weakly more efficient than the concentration benchmark. If β and L are sufficiently low, there exist equilibria that are less efficient than the benchmark. This can occur under the small-shock equilibria (low L), where the seller never sells good firms. Therefore, the price upon selling is $\underline{v}$, and so there are equilibria in which she does not sell bad firms. This reduces the punishment for shirking, and also the reward for working by lowering the price of a retained firm below $\overline{v}$. This disadvantage is more pronounced, the greater the frequency with which a bad firm is retained. This frequency is greater if β is low, since a bad firm is always retained upon no shock, and if L is low, since a smaller shock allows the investor to retain more bad firms upon a shock. Indeed, as shown in Proposition 1, if γ is sufficiently high and β and L are sufficiently low, price informativeness is lower under diversification, and it is price informativeness that matters for exit (while voice depends on payoff precision, which is always weakly higher under diversification). However, under the efficiency criterion, the most efficient equilibrium will be chosen and so price informativeness and thus governance is always weakly stronger under diversification.

Proposition 5 *(Comparison of equilibria, endogenous asset values): For any* $L > 0$*, there is a unique* $\beta^*(L) \in [0, 1)$ *s.t.:*

- (i) *If* $\beta \geq \beta^*(L)$*, any equilibrium under diversification is weakly more efficient than the concentration benchmark.*
- (ii) *If* $\beta < \beta^*(L)$*, there is an equilibrium under diversification that is strictly less efficient than the concentration benchmark. The least efficient equilibrium is type-(i), where the threshold is given by the smallest solution of* $c^* = q(F(c^*))$ *where*

$$q(\tau) = \Delta - \frac{\omega\Delta}{\frac{\tau}{\gamma^*(\tau)} + 1 - \tau} \tag{26}$$

and $\gamma^*(\tau) = 1 - \beta \frac{L/n}{\underline{v}(1-\tau)}$.

(*iii*) $\beta^*(L)$ *decreases with* L, *where* $\beta^*(0) = 1$ *and* $L/n \geq \underline{v}(1 - F(\Delta)) \Rightarrow \beta^*(L) = 0$.

B.3 Proofs

Proof of Lemma 5. Suppose in equilibrium under ownership structure $\chi \in \{con, div\}$ the market believes that the manager works w.p. τ^*_χ. From (14), if the manager chooses $v_i = \overline{v}$ his expected utility is $(1-\omega)\overline{v} + \omega P_\chi(\overline{v}, \tau^*_\chi) - \widetilde{c}_i$, and if he chooses $v_i = \underline{v}$ his expected utility is $(1-\omega)\underline{v} + \omega P_\chi(\underline{v}, \tau^*_\chi)$. Therefore, he chooses $v_i = \overline{v}$ if and only if $\widetilde{c}_i \leq c^* \equiv (1-\omega)\Delta + \omega\left[P_\chi(\overline{v}, \tau^*_\chi) - P_\chi(\underline{v}, \tau^*_\chi)\right]$. ■

Proof of Lemma 6. In equilibrium, the market and the investor believe the manager follows threshold c^*. Given (3) and (4), the manager expects the price to be $P_{con}(v_i, F(c^*))$ if he chooses v_i. Therefore, he chooses $v_i = \overline{v}$ if and only if

$$(1-\omega)\overline{v} + \omega P_{so}(\overline{v}, F(c^*)) - \widetilde{c}_i \geq (1-\omega)\underline{v} + \omega P_{so}(\underline{v}, F(c^*)), \tag{27}$$

where $P_{so}(v_i, \tau)$ is explicitly given in the proof of Proposition 1. In equilibrium, c^* must solve (49). Using the explicit formulation of $P_{so}(v_i, \tau)$, it follows that (49) is equivalent to $c^* = \phi_{end}(F(c^*))$. Note that $\phi_{end}(\tau)$ is decreasing in τ and is bounded from above and below. Therefore, a solution always exists and is unique, as required. The equilibrium is characterized by Proposition 1, where τ is given by $\tau^*_{con,end}$.

For the comparative statics, note that $\phi_{end}(\tau)$ is strictly decreasing in τ, and so there is a unique equilibrium. Furthermore, the derivative of $\phi_{end}(\tau)$ with respect to a given parameter has the same sign as the response of c^* to that parameter. Therefore, the threshold increases with Δ, ω, and Δ. We also have

$$\frac{\partial \phi_{end}(\tau)}{\partial \beta} = -\omega\Delta \frac{1-\tau}{(\beta\tau + 1 - \tau)^2} < 0,$$

implying that the threshold decreases in β.

Finally, consider two distributions $F_G(\cdot) > F_B(\cdot)$ and their respective equilibrium cutoffs

c^*_G and c^*_B. Then, we have

$$c^*_B = \phi_{end}(F_B(c^*_B)) > \phi_{end}(F_G(c^*_B)). \tag{28}$$

With $\phi_{end}(\tau)$ decreasing in τ, this implies that $c^*_G < c^*_B$. Furthermore, since

$$c^*_B = \phi_{end}(F_B(c^*_B)) < \phi_{end}(F_G(c^*_G)) = c^*_G, \tag{29}$$

we must have $\tau^*_G > \tau^*_B$. ■

Proof of Lemma 7.

First, suppose $\underline{v} \leq L/n$. Based on Proposition 2, any equilibrium is type-(iii). Therefore, $c^{**}_{div,end} = c^*_{con,end}$ in this range. Similar to the proof of Proposition 2 part (iii) and Proposition 6, such an equilibrium indeed exists.

Second, suppose $\underline{v}\left(1 - \tau^{**}_{ii,end}\right) < L/n < \underline{v}$. Based on Proposition 2, any equilibrium is either type-(ii) or type-(iii). Consider a type-(ii) equilibrium. The manager has incentives to choose $v_i = \overline{v}$ if and only if

$$(1-\omega)\,\overline{v} + \omega\left[\beta\overline{p}_{co}\left(\tau^*\right) + (1-\beta)\,\overline{v}\right] - \widetilde{c}_i \geq (1-\omega)\,\underline{v} + \omega\left[\beta\underline{v} + (1-\beta)\,\overline{p}_{co}\left(\tau^*\right)\right]. \tag{30}$$

Using $\overline{p}_{co}\left(\tau\right) = \underline{v} + \Delta\frac{\beta\tau}{\beta\tau+(1-\beta)(1-\tau)}$, we obtain $v_i = \overline{v} \Leftrightarrow \widetilde{c}_i \leq \zeta_{end}(\tau^*)$. Therefore, c^* must solve $c^* = \zeta_{end}\left(F\left(c^*\right)\right)$. Similar to the proof of Proposition 2 part (ii), if $\tau = \tau^{**}_{ii,end}$ then indeed an equilibrium with these properties indeed exists. By definition of $c^{**}_{ii,end}$, such an equilibrium is more efficient than any other type-(ii) equilibrium. Moreover, simple algebra shows that $\zeta_{end}\left(\tau\right) > \phi_{end}\left(\tau\right)$, and so $c^{**}_{ii,end} > c^*_{con,end}$. That is, an equilibrium with $\tau = \tau^{**}_{ii,end}$ is more efficient than any equilibrium with the properties of part (iii). Finally, to show that an equilibrium with $\tau = \tau^{**}_{ii,end}$ is more efficient than any type-(i) equilibrium, note that based on part (i) of Proposition 2, the threshold of the alternative equilibrium must satisfy $L/n \leq \underline{v}\left(1-\tau^*\right)$. However, since by assumption $\underline{v}\left(1 - \tau^{**}_{ii,end}\right) < L/n$, it follows that $\tau^* < \tau^{**}_{ii,end}$, that is, the alternative equilibrium must be less efficient.

Third, suppose $L/n \leq \underline{v}\left(1 - \tau^{**}_{ii,end}\right)$. We argue that a type-(i) equilibrium exists. If true, then this equilibrium is more efficient than any type-(ii) or type-(iii) equilibrium. We argue that the following strategies are an equilibrium: the manager's working threshold is

$c^{**} = \min\{\Delta, F^{-1}(1 - \frac{L/n}{\underline{v}})\}$, the investor's trading strategy is

$$x^*(v_i, \theta) = \begin{cases} 0 & \text{if } v_i = \overline{v} \\ 1 \text{ w.p. } 1 - \eta^* \text{ and } 0 \text{ otherwise} & \text{if } v_i = \underline{v} \text{ and } \theta = 0 \\ 1 & \text{if } v_i = \underline{v} \text{ and } \theta = L, \end{cases} \tag{31}$$

where

$$\eta^* = \begin{cases} 0 & \text{if } \Delta \leq F^{-1}(1 - \frac{L/n}{\underline{v}}) \\ \frac{1}{1-\beta} \frac{1 - \frac{L/n}{\underline{v}}}{\frac{\omega\Delta}{\Delta + \omega\Delta - F^{-1}(1 - \frac{L/n}{\underline{v}})} - \frac{L/n}{\underline{v}}} & \text{otherwise} \end{cases}, \tag{32}$$

and prices are

$$p_i^*(x_i) = \begin{cases} \underline{v} + \Delta \frac{F(c^{**})}{F(c^{**}) + (1-\beta)\eta^*(1 - F(c^{**}))} & \textit{if } x_i = 0 \\ \underline{v} & \textit{if } x_i > 0. \end{cases} \tag{33}$$

If the above equilibrium indeed exists, note that it is must be the most efficient equilibrium among all type-(i) equilibria, and hence the most efficient equilibrium. To understand why, first note that if $c^{**} = \Delta$ then we have first-best and so no other equilibrium is strictly more efficient. However if $c^{**} = F^{-1}(1 - \frac{L/n}{\underline{v}})$ then $\underline{v}(1 - F(c^{**})) = L/n$. Therefore, any other type-(i) equilibrium must satisfy $L/n \leq \underline{v}(1 - \tau^*)$, and hence, $\tau^* \leq F(c^{**})$, which implies that threshold c^{**} is more efficient.

We now prove that the above equilibrium indeed exists. First note that the prices in this equilibrium follow from the investor's trading strategy and the application of Bayes' rule. Second, given these prices, the investor's trading strategy is optimal. Indeed, note that $L/n \leq \underline{v}(1 - F(c^{**}))$, and so the investor can satisfy her liquidity needs by selling only bad firms. Since $x_i > 0 \Rightarrow p_i^* = \underline{v}$, the investor has strict incentives to fully retain good firm, and weak incentives to sell bad firms. The manger works if and only if

$$\begin{aligned} \overline{v} + \omega p_i^*(0) - \widetilde{c}_i &\geq \underline{v} + \omega\left[\beta\underline{v} + (1-\beta)\left(\eta^* p_i^*(0) + (1-\eta^*)\underline{v}\right)\right] \Leftrightarrow \\ \Delta + \omega\left(1 - (1-\beta)\eta^*\right)\left(p_i^*(0) - \underline{v}\right) &\geq \widetilde{c}_i \end{aligned}$$

Using the explicit form of $p_i^*(0)$ as given above, the manager works if and only if

$$\begin{aligned} \Delta+\omega\left(1-(1-\beta)\eta^*\right)\Delta\frac{F\left(c^{**}\right)}{F\left(c^{**}\right)+(1-\beta)\eta^*\left(1-F\left(c^{**}\right)\right)} &\geq \widetilde{c}_i \Leftrightarrow \\ H\left(\eta^*, c^{**}\right) &\geq \widetilde{c}_i \end{aligned}$$

where

$$H(\eta, c)=\Delta+\omega\Delta\left(1-\frac{1}{\frac{F(c)}{1-\beta}\frac{1}{\eta}+1-F(c)}\right)$$

is a continuous function of η and c, and it strictly decreases in η, when $\eta>0$. There are two cases to consider. First, if $\Delta+\Delta\omega \leq F^{-1}(1-\frac{L/n}{\underline{v}})$ then $H\left(0, c^{**}\right)=\Delta$, as required. Second, suppose $\Delta>F^{-1}(1-\frac{L/n}{\underline{v}})$ and note that $H(1, c)<\zeta_{end}(F(c))$ for all c. Recall, $L/n \leq \underline{v}\left(1-\tau_{ii,end}^{**}\right) \Rightarrow c_{ii,end}^{**} \leq F^{-1}(1-\frac{L/n}{\underline{v}})$, where $c_{ii,end}^{**}$ is the largest solution of $c_{ii,end}^{**}=\zeta_{end}\left(F\left(c_{ii,end}^{**}\right)\right)$. Therefore,

$$\begin{aligned} \zeta_{end}\left(F\left(F^{-1}(1-\frac{L/n}{\underline{v}})\right)\right) &< F^{-1}(1-\frac{L/n}{\underline{v}}) \Rightarrow \\ H\left(1, F^{-1}(1-\frac{L/n}{\underline{v}})\right) &< \zeta_{end}\left(F\left(F^{-1}(1-\frac{L/n}{\underline{v}})\right)\right) \Rightarrow \\ H\left(1, F^{-1}(1-\frac{L/n}{\underline{v}})\right) &< F^{-1}(1-\frac{L/n}{\underline{v}}). \end{aligned}$$

Also note that

$$H\left(0, F^{-1}(1-\frac{L/n}{\underline{v}})\right)=\Delta>F^{-1}(1-\frac{L/n}{\underline{v}}).$$

Therefore,

$$H\left(1, F^{-1}(1-\frac{L/n}{\underline{v}})\right)<F^{-1}(1-\frac{L/n}{\underline{v}})<H\left(0, F^{-1}(1-\frac{L/n}{\underline{v}})\right),$$

and by the intermediate value theorem, there is $\hat{\eta} \in(0,1)$ such that $H\left(\hat{\eta}, F^{-1}(1-\frac{L/n}{\underline{v}})\right)=F^{-1}(1-\frac{L/n}{\underline{v}})$. Since $H(\eta, c)$ is decreasing in η, $\hat{\eta}$ is uniquely defined. Finally, one can verify that

$$H\left(\frac{1}{1-\beta}\frac{1-\frac{L/n}{\underline{v}}}{\frac{\omega\Delta}{\Delta+\omega\Delta-F^{-1}(1-\frac{L/n}{\underline{v}})}-\frac{L/n}{\underline{v}}}, F^{-1}(1-\frac{L/n}{\underline{v}})\right)=F^{-1}(1-\frac{L/n}{\underline{v}}),$$

implying that $\eta^* \in (0,1)$ as required. ■

Proof of Proposition 4. First, for $L/n \leq \underline{v}(1-\tau^{**}_{ii,end})$, $c^{**}_{div,end} = \min\{\Delta, F^{-1}\left(1-\frac{L/n}{\underline{v}}\right)\}$, which is decreasing in L/n. Note also at $L/n = \underline{v}(1-\tau^{**}_{ii,end})$, this implies that $c^{**}_{div,end} = F^{-1}\left(1-\frac{L/n}{\underline{v}}\right) = F^{-1}(\tau^{**}_{ii,end}) = c^{**}_{ii,end}$. Furthermore, $c^{**}_{ii,end}$ is not dependent on L/n. Altogether, this implies that $c^{**}_{div,end}$ is continuous and weakly decreasing in L/n for $L/n < \underline{v}$. Note that for $\underline{v} \leq L/n$, $c^{**}_{div,end} = c^{*}_{con,end}$, which is constant in L/n. Finally, since $\zeta_{end}(\tau) > \phi_{end}(\tau)$ for all τ, $c^{**}_{div,end} = \zeta(F(c^{**}_{div,end})) > \phi_{end}(F(c^{**}_{div,end}))$. Since $\phi_{end}(F(c))$ is decreasing in c, it implies that $c^{*}_{con,end}$ such that $\phi_{end}(F(c^{*}_{con,end})) = c^{*}_{con,end}$ is strictly less than $c^{**}_{div,end}$. This confirms that $c^{**}_{div,end}$ is decreasing in L/n, and furthermore that $c^{**}_{div,end} > c^{*}_{con,end}$ for $\underline{v} < L/n$. That $c^{**}_{div,end} = c^{*}_{con,end}$ for $\underline{v} \leq L/n$ trivially shows that the cutoff is identical under concentration and diversification for such values of L/n. ■

To prove Proposition 5, we start with an auxiliary lemma.

Lemma 8 *There is an equilibrium under diversification and endogenous asset values in which the working threshold satisfies* $L/n \leq \underline{v}\,(1-F\,(c^*))$ *if and only if* $L/n \leq \underline{v}\left(1-F\left(\underline{c}^*_{div,end}\right)\right)$*, where* $\underline{c}^*_{div,end}$ *is the smallest solution of* $c^* = q\,(F\,(c^*))$ *where* $q\,(\tau)$ *is given by (26).*

Proof of Lemma 8. Suppose $L/n \leq \underline{v}\left(1-F\left(\underline{c}^*_{div,end}\right)\right)$. We argue that there exists an equilibrium where the manager's working threshold is $\underline{c}^*_{div,end}$ and the investor's selling strategy is

$$x^*\,(v_i,\theta) = \begin{cases} 0 \text{ w.p. } \eta^* \text{ and } 1 \text{ w.p. } 1-\eta^* & \textit{if } v_i = \underline{v} \textit{ and } \theta = L \\ 0 & \textit{otherwise}, \end{cases}$$

where

$$\eta^* = 1 - \frac{L/n}{\underline{v}(1-F\left(\underline{c}^*_{div,end}\right))} \in [0,1]\,.$$

Indeed, following Bayes' rule, prices are given by

$$p\,(x_i) = \underline{v} + \mathbf{1}_{\{x_i=0\}} \times \Delta \frac{F\left(\underline{c}^*_{div,end}\right)}{F\left(\underline{c}^*_{div,end}\right) + (1-\beta+\beta\eta^*)\left(1-F\left(\underline{c}^*_{div,end}\right)\right)},$$

and so the manager works if and only if

$$\begin{aligned}(1-\omega)\,\underline{v}+\omega\left(\beta\left[\eta^{*}p_{i}\left(0\right)+\left(1-\eta^{*}\right)\underline{v}\right]+\left(1-\beta\right)p_{i}\left(0\right)\right) &< \left(1-\omega\right)\overline{v}+\omega p_{i}\left(0\right)-\widetilde{c}_{i}\Leftrightarrow\\ q\left(F\left(\underline{c}^{*}_{div,end}\right)\right) &> \widetilde{c}_{i}.\end{aligned}$$

Note that a solution for $c^{*}=q\left(F\left(c^{*}\right)\right)$ always exists since $q(F(0))=\Delta>0$ and $\lim_{c^{*}\to\infty}q(F(c^{*}))<\Delta$. Since $L/n\leq\underline{v}\left(1-F\left(\underline{c}^{*}_{div,end}\right)\right)$ and, given prices, the investor is indifferent between selling and retaining a bad firm, the strategy is weakly optimal. Thus this equilibrium exists, as required.

Next, suppose $L/n>\underline{v}\left(1-F\left(\underline{c}^{*}_{div,end}\right)\right)$. We argue that in any equilibrium, $L/n>\underline{v}\left(1-F\left(c^{*}\right)\right)$. Suppose on the contrary there is an equilibrium where $L/n\leq\underline{v}\left(1-F\left(c^{*}\right)\right)$, and let $\tau^{*}=F\left(c^{*}\right)$. The equilibrium must be type-(i) and the following must hold:

- There is $\eta\in\left[0,1-\frac{L/n}{\underline{v}(1-\tau^{*})}\right]$ s.t. $x^{*}\left(\underline{v},L\right)\in\left[\frac{L/n}{\underline{v}}\frac{1}{(1-\eta)(1-\tau^{*})},1\right]$ w.p. $1-\eta$ and $x^{*}\left(\underline{v},L\right)=0$ w.p. η.[26]
- There is $\varphi\in[0,1]$ s.t. $x^{*}\left(\underline{v},0\right)>0$ w.p. $1-\varphi$ and $x^{*}\left(\underline{v},0\right)=0$ w.p. φ.
- $p_{i}^{*}\left(0\right)=\underline{v}+\Delta\frac{\tau^{*}}{\tau^{*}+[(1-\beta)\varphi+\beta\eta](1-\tau^{*})}$
- $c^{*}=\hat{q}\left(F\left(c^{*}\right),\varphi,\eta\right)$ where

$$\hat{q}\left(\tau,\varphi,\eta\right)=\Delta-\frac{\omega\Delta}{1-\tau+\frac{\tau}{(1-\beta)\varphi+\beta\eta}}\tag{34}$$

Indeed, if $\theta=L$, the investor sells at least $\frac{L/n}{\underline{v}(1-\tau^{*})}\in(0,1]$ from the bad firms in aggregate, to raise at least L. Let η be the probability that she retains a bad firm when $\theta=L$. Note that $\eta\in\left[0,1-\frac{L/n}{\underline{v}(1-\tau^{*})}\right]$ to ensure that at least L is raised. If $\theta=0$, she is indifferent between selling and not selling a bad firm , since $p_{i}\left(x_{i}\right)=\underline{v}$ for $x_{i}>0$. Let φ be the probability that she retains a bad firm when $\theta=0$. Then, by Bayes' rule,

$$p_{i}\left(0\right)=\underline{v}+\Delta\frac{\tau^{*}}{\tau^{*}+\left[\left(1-\beta\right)\varphi+\beta\eta\right]\left(1-\tau^{*}\right)}$$

[26] In fact, the investor can mix between more than two quantities. However, since any positive quantity leads to the same price $\underline{v}$, without loss of generality we can assume that she mixes between only two quantities, one of which is zero.

and the manager works if and only if

$$\begin{aligned}(1-\omega)\,\underline{v}+\omega\left(\beta\left[\eta p_i\left(0\right)+\left(1-\eta\right)\underline{v}\right]+\left(1-\beta\right)\left[\varphi p_i\left(0\right)+\left(1-\varphi\right)\underline{v}\right]\right) &< (1-\omega)\,\overline{v}+\omega p_i\left(0\right)-\widetilde{c}_i \Leftrightarrow \\ q\left(\tau^*,\varphi,\eta\right) &> \widetilde{c}_i,\end{aligned}$$

as required. Note that

$$q\left(\tau\right)\leq\hat{q}\left(\tau,\varphi,\eta\right)\Leftrightarrow\eta\leq\frac{1-\left(1-\beta\right)\varphi}{\beta}-\frac{L/n}{\underline{v}(1-\tau)}$$

Note that $\frac{1-(1-\beta)\varphi}{\beta}\geq 1$, and hence, $\eta\leq 1-\frac{L/n}{\underline{v}(1-\tau)}\Rightarrow\eta\leq\frac{1-(1-\beta)\varphi}{\beta}-\frac{L/n}{\underline{v}(1-\tau)}$. This implies that any solution of $c^*=\hat{q}\left(F\left(c^*\right),\varphi,\eta\right)$ is weakly larger than $\underline{c}^*_{div,end}$. But since $L/n>\underline{v}\left(1-F\left(\underline{c}^*_{div,end}\right)\right)$, then it must be $L/n>\underline{v}\left(1-F\left(c^*\right)\right)$ as well. ■

The full proof of Proposition 5 now follows.

Proof of Proposition 5. Suppose $L/n\geq\underline{v}(1-F(\Delta))$. Since the threshold in every equilibrium satisfies $c^*>\Delta$, $L/n\geq\underline{v}(1-F(\Delta))$ implies that any equilibrium must involve selling good firms. In this case, as in the arguments in the proof of Proposition 7, the cutoff rule under diversification satisfies either $c^*=\phi_{end}(F(c^*))$ or $c^*=\zeta_{end}(F(c^*))$. Since $c^*_{con,end}=\phi_{end}\left(F\left(c^*_{con,end}\right)\right)$ and $\phi_{end}(\tau)<\zeta_{end}(\tau)$ for all τ, and since $\phi_{end}(\tau)$ is decreasing in τ, all equilibria under diversification are weakly more efficient than under concentration, that is, $\beta^*(L)=0$.

Suppose $L/n<\underline{v}\left(1-F\left(\Delta\right)\right)$. Let $\underline{c}^*\left(\beta,L\right)\equiv\underline{c}^*_{div,end}$ as a function of β and L, where $\underline{c}^*_{div,end}$ is defined in Lemma 8. Note that $\underline{c}^*\left(\beta,L\right)$ is strictly increasing in β and $\underline{c}^*\left(0,L\right)=\Delta<\underline{c}^*\left(1,L\right)$. Also note that $c^*_{con,end}(\beta)$, the unique concentration equilibrium cutoff as a function of β, is strictly decreasing in β, where $c^*_{con,end}(0)>\Delta=c^*_{con,end}(1)$. Thus, there exists a unique $\lambda(L)\in(0,1)$ such that

$$\underline{c}^*\left(\beta,L\right)\geq c^*_{con,end}\left(\beta\right)\Leftrightarrow\beta\geq\lambda\left(L\right). \tag{35}$$

Suppose $\beta\geq\lambda(L)$. Consider the following two cases.

1. If $L/n\leq\underline{v}\left(1-F\left(\underline{c}^*\left(\beta,L\right)\right)\right)$ then, based on Lemma 8, an equilibrium with cutoff rule

$\underline{c}^*(\beta, L)$ exists. Moreover, based on Lemma 8, $\underline{c}^*(\beta, L)$ is the lowest cutoff among all equilibria that satisfy $L/n \leq \underline{v}(1 - F(c^*))$. Any other equilibrium that satisfies $L/n > \underline{v}(1 - F(c^*))$, implies $F(c^*) > 1 - \frac{L/n}{\underline{v}} \geq F(\underline{c}^*(\beta, L))$, and hence, is strictly more efficient. So, $\underline{c}^*(\beta, L)$ is the cutoff of the least efficient equilibrium under diversification. Since $\beta \geq \lambda(L) \Rightarrow \underline{c}^*(\beta, L) \geq c^*_{con,end}(\beta)$, all equilibria under diversification are more efficient than the concentration benchmark.

2. If $L/n > \underline{v}(1 - F(\underline{c}^*(\beta, L)))$ then, based on Lemma 8, any equilibrium under diversification satisfies $L/n > \underline{v}(1 - F(c^*))$. In this case, the equilibrium cutoff rules are the set $\{c^{**}_{ii,end}, c^*_{con,end}(\beta)\}$. According to Proposition 7, $c^{**}_{ii,end} > c^*_{con,end}(\beta)$, and hence, the equilibrium cutoff is weakly larger than $c^*_{con,end}(\beta)$. Therefore, diversification is weakly more efficient than concentration for $\beta \geq \lambda(L)$.

Next, suppose instead that $\beta < \lambda(L)$. We proceed in several steps.

1. We first argue that there is a unique $L^{**} \in (0, n\underline{v}(1 - F(\Delta)))$ s.t.

$$L \leq n\underline{v}\left(1 - F(c^*_{con,end}(\lambda(L)))\right) \Leftrightarrow L \leq L^{**}. \tag{36}$$

To see why, note that $\underline{c}^*(\beta, L)$ weakly increases in L, and so $\lambda(L)$ weakly decreases in L. Since $c^*_{con,end}(\beta)$ decreases in β, $c^*_{con,end}(\lambda(L))$ weakly increases in L. Moreover, since $\lim_{L\to 0} \underline{c}^*(\beta, L) = \Delta$ and $\beta < 1 \Rightarrow c^*_{con,end}(\beta) > \Delta$, $\lim_{L\to 0} \lambda(L) = 1$. Therefore,

$$\lim_{L\to 0} \frac{1 - F(c^*_{con,end}(\lambda(L)))}{L} = \frac{1 - F(c^*_{con,end}(1))}{\lim_{L\to 0} L} = \frac{1 - F(\Delta)}{\lim_{L\to 0} L} > \frac{1}{n\underline{v}}.$$

Note also that $L > 0 \Rightarrow \lambda(L) < 1$, and so $c^*_{con,end}(\lambda(L)) > \Delta$. Therefore,

$$\begin{aligned} \lim_{L\to n\underline{v}(1-F(\Delta))} \frac{1 - F(c^*_{con,end}(\lambda(L)))}{L} &= \frac{1 - F\left(c^*_{con,end}(\lambda(n\underline{v}(1 - F(\Delta))))\right)}{n\underline{v}(1 - F(\Delta))} \\ &< \frac{1 - F(\Delta)}{n\underline{v}(1 - F(\Delta))} = \frac{1}{n\underline{v}}. \end{aligned}$$

By the intermediate value theorem, there is a unique $L^{**} \in (0, n\underline{v}(1 - F(\Delta)))$ s.t. $L \leq n\underline{v}\left(1 - F\left(c^*_{con,end}(\lambda(L))\right)\right) \Leftrightarrow L \leq L^{**}$, as required.

2. We next argue $L \leq n\underline{v}\left(1-\underline{\tau}^*(\beta, L)\right)$, with $\underline{\tau}^* \equiv F\left(\underline{c}^*(\beta, L)\right)$, if and only if:

   - $L \in (0, L^{**}]$ and $\beta < \lambda(L)$, or
   - $L \in \left(L^{**}, (1-F(\Delta))\, n\underline{v}\right)$ and $\beta < \varphi(L)$, where $\varphi(L) \in (0, \lambda(L))$ solves $L = n\underline{v}\left(1-\underline{\tau}^*(\varphi(L), L)\right)$.

   To see why, recall $\underline{c}^*(\beta, L)$ is increasing in β where $\underline{c}^*(0, L) = \Delta$. Therefore, $1 - \underline{\tau}^*(\beta, L)$ is decreasing in β. Since $\underline{c}^*(0, L) = \Delta$ and $L < n\underline{v}\left(1-F(\Delta)\right)$, if $\beta = 0$ then $L \leq n\underline{v}\left(1-\underline{\tau}^*(\beta, L)\right)$ holds. Recall also that $\underline{c}^*(\lambda(L), L) = c^*_{con,end}(\lambda(L))$ and $L \leq n\underline{v}\left(1-\tau^*_{con,end}(\lambda(L))\right) \Leftrightarrow L \leq L^{**}$. Therefore, if $L \in (0, L^{**}]$ then $L \leq n\underline{v}\left(1-\underline{\tau}^*(\beta, L)\right)$ $\forall\ \beta < \lambda(L)$. Suppose $L \in \left(L^{**}, n\underline{v}\left(1-F(\Delta)\right)\right)$. Then $L > n\underline{v}\left(1-\tau^*_{con,end}(\lambda(L))\right)$ and so $L \leq n\underline{v}\left(1-\underline{\tau}^*(\beta, L)\right) \Leftrightarrow \beta < \varphi(L)$, where $\varphi(L) \in (0, \lambda(L))$ solves $L = n\underline{v}\left(1-\underline{\tau}^*(\varphi(L), L)\right)$, as required.

3. Let

$$\beta^*(L) = \begin{cases} \lambda(L) & \textit{if } L \leq L^{**} \\ \varphi(L) & \textit{if } L \in \left(L^{**}, n\underline{v}\left(1-F(\Delta)\right)\right) \\ 0 & \textit{if } L \geq n\underline{v}\left(1-F(\Delta)\right). \end{cases} \tag{37}$$

   Based on the analysis of the case where $L \geq n\underline{v}\left(1-F(\Delta)\right)$, and the steps for the case where $L < n\underline{v}\left(1-F(\Delta)\right)$, $\beta^*(L)$ is as stated in the proposition. Note that, if $\beta < \beta^*(L)$, we proved that an equilibrium with cutoff $\underline{c}^*(\beta, L)$ exists, and its properties are as stated in the proposition.

4. Consider the properties of $\beta^*(L)$. Recall that we proved $\lim_{L\to 0} \lambda(L) = 1$, and so $\beta^*(0) = 1$. Note also that $L \geq n\underline{v}\left(1-F(\Delta)\right) \Rightarrow \beta^*(L) = 0$. We argue that $\beta^*(L)$ decreases with L. To see this, first note that $\lambda(L) = \varphi(L) \Leftrightarrow L = L^{**}$, and $\varphi(L) = 0 \Leftrightarrow L = n\underline{v}\left(1-F(\Delta)\right)$. Recall we proved earlier that $\lambda(L)$ weakly decreases in L. Note that $\varphi(L)$ is defined by $L = 1 - F\left(\underline{c}^*(\varphi(L), L)\right)$. Since $\underline{c}^*(\beta, L)$ increases in β and in L, $1 - F\left(\underline{c}^*(\beta, L)\right)$ decreases in β and L. Therefore, $\frac{1-F(\underline{c}^*(\beta,L))}{L}$ decreases in L, and so $\varphi(L)$ must decrease in L as well.

■

C Extensions

C.1 Two Assets

We define $\mathbf{x} \equiv (x_i, x_j)$, $\mathbf{v} \equiv (v_i, v_j)$, $\mathbf{x}^T \equiv (x_j, x_i)$, $\mathbf{v}^T \equiv (v_j, v_i)$, $\overline{\mathbf{v}} \equiv (\overline{v}, \overline{v})$, and $\underline{\mathbf{v}} \equiv (\underline{v}, \underline{v})$. We denote by $\mathbf{e}(\theta, \mathbf{v}) \equiv (e_i(\theta, \mathbf{v}), e_j(\theta, \mathbf{v}))$ the equilibrium strategy of type $(\theta, \mathbf{v})$. By symmetry, $p_i(\mathbf{x}) = p_j(\mathbf{x}^T)$ for all x_i and x_j, and $e_j(\theta, \mathbf{v}) = e_i(\theta, \mathbf{v}^T)$. We therefore omit the subscript whenever there is no risk of confusion. Let $\Pi^*(\theta, \mathbf{v}) \equiv \Pi(\mathbf{e}(\theta, \mathbf{v}), \mathbf{v})$ denote the equilibrium payoff of type $(\theta, \mathbf{v})$, where

$$\Pi(\mathbf{x}, \mathbf{v}) = x_i p_i(\mathbf{x}) + (n/2 - x_i) v_i + x_j p_i(\mathbf{x}^T) + (n/2 - x_j) v_j$$

We focus on symmetric, pure strategy equilibria. We start by analyzing the case where each asset has a different buyer, and then the case of a single buyer. We focus on the case of small liquidity shocks ($L/n \leq \underline{v}/2$, so that a shock can be met by fully selling one bad asset) since this is where our results are strongest.

C.1.1 Separate buyers

Proposition 6 *(Diversification, two assets, separate buyers): Suppose* $0 < L/n \leq \underline{v}/2$. *An equilibrium under diversification always exists and is unique.*[27]

(i) If $\tau \geq \frac{1}{1+\beta}$ *then*

$$x^*_{div,i}(v_i, v_j, \theta) = \begin{cases} 0 & \textit{if } v_i = \overline{v} \textit{ and } \theta = 0 \\ \overline{x}_{div}(\tau) = \frac{L/2}{\overline{p}_{div}(\tau)} & \textit{else} \end{cases} \tag{38}$$

and prices of asset i are

$$p^*_i(x_i) = \begin{cases} \overline{v} & \textit{if } x_i = 0 \\ \overline{p}_{div}(\tau) = \underline{v} + \frac{\beta\tau}{\beta\tau + 1 - \tau}\Delta & \textit{if } x_i \in (0, \overline{x}_{div}(\tau)], \\ \underline{v} & \textit{if } x_i > \overline{x}_{div}(\tau). \end{cases}$$

[27] If $L/n \leq \underline{v}/2$ and $\tau = \frac{1}{1+\sqrt{\beta}}$ then the equilibria in part (i) and part (ii) coexist.

(ii) If $\tau < \frac{1}{1+\beta}$ *then*

$$x^*_{div,i}(\mathbf{v},\theta) = \begin{cases} 0 & \text{if } \mathbf{v} = (\overline{v},\underline{v}), \text{ or } \mathbf{v} = \overline{\mathbf{v}} \text{ and } \theta = 0 \\ \overline{x}_{div}(\tau) = \frac{L/2}{\overline{p}_{div}(\tau)} & \text{if } \mathbf{v} = (\underline{v},\overline{v}) \text{ and } \theta = 0, \mathbf{v} = \underline{\mathbf{v}}, \text{ or } \mathbf{v} = \overline{\mathbf{v}} \text{ and } \theta = L \\ n/2 & \text{if } \mathbf{v} = (\underline{v},\overline{v}) \text{ and } \theta = L, \end{cases} \tag{39}$$

and prices of asset i *are*

$$p^*_i(x_i) = \begin{cases} \overline{v} & \text{if } x_i = 0 \\ \overline{p}_{div}(\tau) = \underline{v} + \frac{\tau^2\beta}{(1-\beta)(1-\tau)+(\tau^2+(1-\tau)^2)\beta}\Delta & \text{if } x_i \in (0,\overline{x}_{div}(\tau)], \\ \underline{v} & \text{if } x_i > \overline{x}_{div}(\tau). \end{cases} \tag{40}$$

Proof of Proposition 6. We start by proving several claims.

1. *In any equilibrium, there is a unique* $\overline{x}_G \in (0,n/2)$ *such that* $\mathbf{e}(L,\overline{\mathbf{v}}) = \overline{\mathbf{x}}_G$.

   Proof: A symmetric equilibrium requires the seller to sell the same quantities from both assets if they have the same value. A pure strategy equilibrium requires $\overline{x}_G$ to be unique. Since $\beta > 0$, such $\overline{x}_G$ exists. Since $L > 0$, if $\overline{x}_G = 0$ then the seller has a profitable deviation to selling $n/2$ units from each asset and raising more revenue, a contradiction. If $\overline{x}_G = n/2$, since $p(\overline{x}_G) \geq \underline{v}$, $\overline{x}_G p(\overline{x}_G) \geq n/2\underline{v} \geq L > L/2$, which contradicts $\overline{x}_G p(\overline{x}_G) = L/2$ that we prove in claim 2 below.

2. *In any equilibrium, (i)* $p_i(\overline{x}_G) \in (\underline{v},\overline{v})$*; (ii)* $\overline{x}_G p(\overline{x}_G) = L/2$.

   Proof of part (i): Since $\beta > 0$, a buyer with rational expectations must set $p(\overline{x}_G) > \underline{v}$. Suppose on the contrary $p(\overline{x}_G) = \overline{v}$. Since prices are non-increasing, there is $\bar{x} \geq \overline{x}_G$ such that $x \leq \bar{x} \Rightarrow p(x) = \overline{v}$ and either $\bar{x} = n/2$ or $x > \bar{x} \Rightarrow p(x) < \overline{v}$. Let $\overline{x}_B = e_i(L,\underline{\mathbf{v}}) = e_j(L,\underline{\mathbf{v}})$. Then, $p(\overline{x}_B) < \overline{v}$. Moreover, since $x \leq \bar{x} \Rightarrow p(x) = \overline{v}$, it must be $\overline{x}_B > \bar{x}$. If $p(\overline{x}_B) = \underline{v}$ then type $\underline{\mathbf{v}}$ has a profitable deviation to $(\bar{x},\bar{x})$. Therefore, it must be $p(\overline{x}_B) > \underline{v}$. However, $p(\overline{x}_B) > \underline{v}$ is possible only if $\mathbf{e}(L,(\overline{v},\underline{v})) = \overline{\mathbf{x}}_B$ or $\mathbf{e}(0,(\overline{v},\underline{v})) = \overline{\mathbf{x}}_B$

as well. Type $(\overline{v}, \underline{v})$ prefers $\overline{\mathbf{x}}_B$ over $\overline{\mathbf{x}}_G$ only if

$$\begin{aligned} 2\overline{x}_B p(\overline{x}_B) + (n/2 - \overline{x}_B)(\underline{v} + \overline{v}) &\geq 2\overline{x}_G \overline{v} + (n/2 - \overline{x}_G)(\underline{v} + \overline{v}) \Leftrightarrow \\ p(\overline{x}_B) &\geq \frac{\overline{x}_G}{\overline{x}_B} \frac{\overline{v} - \underline{v}}{2} + \frac{\overline{v} + \underline{v}}{2} \end{aligned}$$

which is strictly greater than $\frac{\overline{v}+\underline{v}}{2}$. However, note that

$$p(\overline{x}_B) \leq \underline{v} + \frac{\tau(1-\tau)\Delta}{\tau(1-\tau) + (1-\tau)\tau + \beta(1-\tau^2)}$$

i.e., the best case scenario is that type $(\overline{v}, \underline{v})$ chooses $\overline{\mathbf{x}}_B$ regardless of her liquidity needs and type $\underline{\mathbf{v}}$ chooses $\overline{\mathbf{x}}_B$ only when there is a shock. In addition,

$$\underline{v} + \frac{\tau(1-\tau)\Delta}{\tau(1-\tau) + (1-\tau)\tau + \beta(1-\tau^2)} < \frac{\overline{v} + \underline{v}}{2} \Leftrightarrow 0 < \beta\left(1 - \tau^2\right)$$

which always holds. Therefore, type $(\overline{v}, \underline{v})$ never chooses $\overline{\mathbf{x}}_B$, a contradiction.

Proof of part (ii): Suppose on the contrary $\overline{x}_G p(\overline{x}_G) > L/2$. Since prices are non-increasing, there is $\varepsilon > 0$ such that $(\overline{x}_G - \varepsilon) p(\overline{x}_G - \varepsilon) \geq L/2$. Since $p(\overline{x}_G) < \overline{v}$, if $\theta = L$ then type $\overline{\mathbf{v}}$ has a profitable deviation from $(\overline{x}_G, \overline{x}_G)$ to $(\overline{x}_G - \varepsilon, \overline{x}_G - \varepsilon)$, a contradiction. Suppose on the contrary $\overline{x}_G p(\overline{x}_G) < L/2$. Since $L/n \leq \underline{v}/2$, the seller can sell $n/2$ from both firms, get a price which is no lower than $\underline{v}$, and therefore, raise enough liquidity. This creates a profitable deviation, and therefore, a contradiction.

3. *In any equilibrium,* $e_i(0, \overline{\mathbf{v}}) < \overline{x}_G$ *and* $p(e_i(0, \overline{\mathbf{v}})) = \overline{v}$.

   Proof: Since the seller can fully retain both assets, it must be $p(e_i(0, \overline{\mathbf{v}})) = \overline{v}$. Suppose on the contrary $e_i(0, \overline{\mathbf{v}}) \geq \overline{x}_G$. Since $p(e_i(0, \overline{\mathbf{v}})) = \overline{v}$ and $p(\overline{x}_G) < \overline{v}$ (from claim 2), it cannot be $e_i(0, \overline{\mathbf{v}}) = \overline{x}_G$. Suppose $e_i(0, \overline{\mathbf{v}}) > \overline{x}_G$. Since $p(e_i(0, \overline{\mathbf{v}})) = \overline{v} > p(\overline{x}_G)$, if $\theta = L$ then type $\overline{\mathbf{v}}$ has a profitable deviation from $\overline{\mathbf{x}}_G$ to $\mathbf{e}(0, \overline{\mathbf{v}})$, a contradiction.

4. *In any equilibrium,* $e_i(0, (\overline{v}, \underline{v})) < \overline{x}_G$ *and* $p(e_i(0, (\overline{v}, \underline{v}))) = \overline{v}$.

   Proof: the same as claim 3.

5. *In any equilibrium, if the seller sells* x_B *from a bad asset, either* $x_B = \overline{x}_G$ *or* $p(x_B) = \underline{v}$.

Proof: Suppose on the contrary the seller sells $x_B \neq \overline{x}_G$ from the bad asset w.p. $\gamma > 0$ in equilibrium, and $p(x_B) > \underline{v}$. Therefore, $p(x_B) < \overline{v}$. Based on claims 1-4, $p(x_B) > \underline{v}$ requires the seller to sell x_B from the good asset when $\theta = L$ and the other asset is bad. Let $\hat{x}$ be the quantity sold from the bad asset in this case. That is, $\mathbf{e}(L, (\overline{v}, \underline{v})) = (x_B, \hat{x})$ in this equilibrium. Note that $\hat{x} \geq x_B$. Otherwise, type $(\overline{v}, \underline{v})$ has a strictly optimal deviation from $(x_B, \hat{x})$ to $(\hat{x}, x_B)$, that is, selling more from the bad asset and still meeting her liquidity needs. Moreover, note that $\hat{x} = x_B$. Indeed, if $\hat{x} \neq x_B$ then based on claims 1-4, it must be $x_B < \hat{x} = \overline{x}_G$ or $p(\hat{x}) = \underline{v}$. We deal with each case separately.

(a) Suppose $x_B < \hat{x} = \overline{x}_G$. If type $(\overline{v}, \underline{v})$ prefers $(x_B, \overline{x}_G)$ over $(\overline{x}_G, \overline{x}_G)$ when $\theta = L$, then it must be that type $(\overline{v}, \overline{v})$ also prefers $(x_B, \overline{x}_G)$ over $(\overline{x}_G, \overline{x}_G)$ when $\theta = L$. However, since $\mathbf{e}(L, (\overline{v}, \overline{v})) = (\overline{x}_G, \overline{x}_G)$, type $(\overline{v}, \overline{v})$ must be indifferent between the two. Indifference implies that both strategies generate the same payoff, but also raise a revenue of exactly L: otherwise, the seller can always sell a bit less from the good asset and still meet her liquidity needs. However, this implies $x_B = \overline{x}_G$, a contradiction.

(b) Suppose $p(\hat{x}) = \underline{v}$. However, in this case, type $(\overline{v}, \underline{v})$ has a strictly profitable deviation from $(x_B, \hat{x})$ to $(0, n/2)$: in both cases, she raises enough revenue (recall $\underline{v}n/2 \geq L$). However, in the latter case her payoff is $n/2(\underline{v} + \overline{v})$, and in the former case her payoff is strictly smaller: she is getting a payoff $\underline{v}$ for the bad asset, but since $p(x_B) < \overline{v}$, her payoff for the good asset is strictly smaller than $\overline{v}$, a contradiction.

Therefore, $\mathbf{e}(L, (\overline{v}, \underline{v})) = (x_B, x_B)$. By revealed preference,

$$\Pi((x_B, x_B), (\overline{v}, \underline{v})) \geq n/2(\underline{v} + \overline{v}) \Leftrightarrow p(x_B) \geq \frac{\underline{v} + \overline{v}}{2}$$

Suppose type $(\overline{v}, \underline{v})$ chooses (x_B, x_B) w.p. $\sigma > 0$, then

$$p(x_B) = \underline{v} + \Delta \frac{\sigma\tau(1-\tau)}{\sigma[\tau(1-\tau) + (1-\tau)\tau] + \gamma(1-\tau)} < \frac{\underline{v} + \overline{v}}{2},$$

a contradiction.

6. *In any equilibrium, if* $\theta = 0$ *then the seller sells* $\overline{x}_G$ *from a bad asset.*

Proof: Based on claim 5, if the seller sells x_B from a bad asset, either $x_B = \overline{x}_G$ or $p(x_B) = \underline{v}$. Since $p(\overline{x}_G) > \underline{v}$, if the seller does not need liquidity, she will maximize her payoff by choosing $\overline{x}_G$.

7. *In any equilibrium,* $\mathbf{e}(L, \underline{\mathbf{v}}) = (\overline{x}_G, \overline{x}_G)$.

   Proof: By symmetry, the seller must sell in equilibrium the same quantity from both assets. Based on claim 5, if $\mathbf{e}(L, \underline{\mathbf{v}}) \neq (\overline{x}_G, \overline{x}_G)$ then the price the seller gets for the assets must be $\underline{v}$, and therefore, her payoff is $\underline{v}$. Since $p(\overline{x}_G) > \underline{v}$ and $\overline{x}_G p(\overline{x}_G) = L/2$, the seller maximizes her payoff by choosing $(\overline{x}_G, \overline{x}_G)$.

8. *In any equilibrium, if* $p(\overline{x}_G) > \frac{\underline{v}+\overline{v}}{2}$ *then* $\mathbf{e}(L, (\overline{v}, \underline{v})) = (\overline{x}_G, \overline{x}_G)$, *and if* $p(\overline{x}_G) < \frac{\underline{v}+\overline{v}}{2}$ *then* $e_i(L, (\overline{v}, \underline{v})) < \overline{x}_G < e_j(L, (\overline{v}, \underline{v}))$.

   Proof: Recall $L/n \leq \underline{v}/2$ implies that by choosing $(0, n/2)$ type $(\overline{v}, \underline{v})$ can obtain a payoff of $n/2(\underline{v} + \overline{v})$ and enough revenue to cover her liquidity needs. Therefore, she chooses $(\overline{x}_G, \overline{x}_G)$ if and only if

$$\Pi((\overline{x}_G, \overline{x}_G), (\overline{v}, \underline{v})) \geq n/2(\underline{v} + \overline{v}) \Leftrightarrow p(\overline{x}_G) \geq \frac{\underline{v} + \overline{v}}{2}.$$

   Suppose $p(\overline{x}_G) < \frac{\underline{v}+\overline{v}}{2}$. If the seller chooses balanced exit $(x, x) \notin \{(\overline{x}_G, \overline{x}_G), (0, 0)\}$, then by symmetry it must be $p(x) = \frac{\underline{v}+\overline{v}}{2}$. Moreover, the revenue raised by the seller is exactly L, otherwise, she can sell slightly less from the good asset and make a strictly higher profit. Therefore, it cannot be $x < \overline{x}_G$, since then type $(\overline{v}, \overline{v})$ has a profitable deviation from $(\overline{x}_G, \overline{x}_G)$ to (x, x) when $\theta = L$. Also, it cannot be $x > \overline{x}_G$. Indeed, if $x > \overline{x}_G$ then $\Pi((x, x), (\overline{v}, \underline{v})) \geq \Pi((\overline{x}_G, \overline{x}_G), (\overline{v}, \underline{v}))$ implies $\Pi((\overline{x}_G, \overline{x}_G), (\underline{v}, \underline{v})) > \Pi((\overline{x}_G, \overline{x}_G), (\underline{v}, \underline{v}))$, which means that type $(\underline{v}, \underline{v})$ has a profitable deviation from $(\overline{x}_G, \overline{x}_G)$ to (x, x) when $\theta = L$. Therefore, the seller cannot choose balanced exit when $p(\overline{x}_G) < \frac{\underline{v}+\overline{v}}{2}$.

   Next, suppose type $(\overline{v}, \underline{v})$ chooses an imbalanced exit strategy (x_G, x_B). Note that the seller always sells more from the bad asset, that is, $x_G < x_B$. Also, since the seller can always meet her liquidity needs choosing $(0, n/2)$, she must raise enough liquidity by choosing (x_G, x_B). Suppose on the contrary $x_B \leq \overline{x}_G$. Since prices are non-increasing, $p(x_B) \geq p(x_G) \geq p(\overline{x}_G)$, and type $(\overline{v}, \overline{v})$ has a profitable deviation from $(\overline{x}_G, \overline{x}_G)$ to (x_G, x_B) when $\theta = L$, a contradiction. Therefore, it must be $\overline{x}_G < x_B$. Suppose on the contrary $\overline{x}_G \leq x_G$. If $x_G > \overline{x}_G$ then the based on all the claims above, no other type is choosing x_G, and hence, $p(x_G) = \overline{v}$. However, since $x_G > \overline{x}_G$ type $(\underline{v}, \underline{v})$ has a profitable deviation from $(\overline{x}_G, \overline{x}_G)$ to (x_G, x_G), a contradiction. Suppose $x_G = \overline{x}_G$. Note that according to claim 5, $x_B > \overline{x}_G$ implies $p(x_B) = \underline{v}$. Therefore, the seller's payoff on the bad asset is $\underline{v}$. Since $p(\overline{x}_G) < \overline{v}$, the seller's payoff on the good asset is strictly lower than $\overline{v}$. Therefore, the seller's overall payoff is strictly smaller than $n/2(\underline{v} + \overline{v})$, which means that she has a profitable deviation to $(0, n/2)$, a contradiction. We conclude that $x_G < \overline{x}_G < x_B$, as required.

Given the claims above, there are two types of equilibria, those which involve balanced exit and those which involve imbalanced exit. We consider each of the above equilibria separately. Based on claims 3 and 4, we can assume without any loss of generality that $e_i(0, \overline{\mathbf{v}}) = e_i(0, (\overline{v}, \underline{v})) = 0$. Consider first the equilibrium with balanced exit. In this case, the equilibrium strategies and prices (from Bayes' rule) are described in part (i) of the proposition. Note that, based on claim 8, it must be $p(\overline{x}_G) \geq \frac{\underline{v}+\overline{v}}{2}$, and since $\overline{x}_G = \overline{x}_{div}(\tau)$, given the expression of $\overline{p}_{div}(\tau)$ in part (i) it must be $\tau \geq \frac{1}{1+\beta}$.

To show that this is indeed an equilibrium, we need to show profitable deviations do not exist. There are four cases to consider:

1. If $\theta = 0$, the seller cannot receive more than $\overline{v}$ from a good asset and $\overline{x}_{div}(\tau)\overline{p}_{div}(\tau) + (n/2 - \overline{x}_{div}(\tau))\underline{v}$ from a bad asset, which she does by following the equilibrium strategies.

2. Suppose $\theta = L$ and both assets are bad. Then, by following the equilibrium strategy the seller gets the highest payoff she can get given prices, and no deviation is profitable.

3. Suppose $\theta = L$ and both assets are good. The only possible profitable deviation is selling $\varepsilon \in (0, n/2 - \overline{x}_{div}(\tau)]$ more from one asset and $\delta \in (0, \overline{x}_{div}(\tau)]$ less from the other asset, while still raising enough revenue. This deviation generates at least L in revenue if and only if

$$(\overline{x}_{div}(\tau) + \varepsilon)\underline{v} + (\overline{x}_{div}(\tau) - \delta)\overline{p}_{div}(\tau) \geq L \Leftrightarrow \delta \leq \frac{(\overline{x}_{div}(\tau) + \varepsilon)\underline{v} - L/2}{\overline{p}_{div}(\tau)}. \tag{41}$$

This deviation increases the seller's payoff if and only if

$$
\begin{aligned}
&(\overline{x}_{div}(\tau)+\varepsilon)\underline{v}+(n/2-(\overline{x}_{div}(\tau)+\varepsilon))\overline{v}+(\overline{x}_{div}(\tau)-\delta)\overline{p}_{div}(\tau)+(n/2-(\overline{x}_{div}(\tau)-\delta))\overline{v}\\
&>2\overline{x}_{div}(\tau)\overline{p}_{div}(\tau)+2(n/2-\overline{x}_{div}(\tau))\overline{v}\Leftrightarrow\\
\delta &> \frac{\varepsilon\Delta+\overline{x}_{div}(\tau)(\overline{p}_{div}(\tau)-\underline{v})}{\overline{v}-\overline{p}_{div}(\tau)}.
\end{aligned}
$$

However, note that

$$
\frac{\varepsilon\Delta+\overline{x}_{div}(\tau)(\overline{p}_{div}(\tau)-\underline{v})}{\overline{v}-\overline{p}_{div}(\tau)}>\frac{(\overline{x}_{div}(\tau)+\varepsilon)\underline{v}-L/2}{\overline{p}_{div}(\tau)}\Leftrightarrow\overline{p}_{div}(\tau)>\underline{v},
$$

and therefore, a profitable deviation does not exist.

4. Suppose $\theta = L$, one asset is bad, and one asset is good. The only possible profitable deviation is selling $\varepsilon \in (0, n/2-\overline{x}_{div}(\tau)]$ more from the bad asset and $\delta \in (0, \overline{x}_{div}(\tau)]$ less from the good asset. As in case 3, this deviation generates at least L in revenue if and only if $\delta \leq \frac{(\overline{x}_{div}(\tau)+\varepsilon)\underline{v}-L/2}{\overline{p}_{div}(\tau)}$. However, this deviation increases profit if and only if

$$
\begin{aligned}
&(\overline{x}_{div}(\tau)+\varepsilon)\underline{v}+(n/2-(\overline{x}_{div}(\tau)+\varepsilon))\underline{v}+(\overline{x}_{div}(\tau)-\delta)\overline{p}_{div}(\tau)+(n/2-(\overline{x}_{div}(\tau)-\delta))\overline{v}\\
&>2\overline{x}_{div}(\tau)\overline{p}_{div}(\tau)+(n/2-\overline{x}_{div}(\tau))(\underline{v}+\overline{v})\Leftrightarrow\\
\delta &> \overline{x}_{div}(\tau)\frac{\overline{p}_{div}(\tau)-\underline{v}}{\overline{v}-\overline{p}_{div}(\tau)}.
\end{aligned}
$$

Overall, such deviation is feasible and profitable if and only if

$$
\overline{x}_{div}(\tau)\frac{\overline{p}_{div}(\tau)-\underline{v}}{\overline{v}-\overline{p}_{div}(\tau)}<\delta\leq\min\left\{\overline{x}_{div}(\tau),\frac{(\overline{x}_{div}(\tau)+\varepsilon)\underline{v}-L/2}{\overline{p}_{div}(\tau)}\right\}.
$$

Note that larger δ implies more profitable deviation, and that the profitability of the deviation is invariant to ε as long as $\varepsilon > 0$. Therefore, it is sufficient to focus on

$\varepsilon = n/2 - \overline{x}_{div}(\tau)$. Therefore, a profitable deviation exists if and only if

$$\begin{aligned}
\overline{x}_{div}(\tau)\frac{\overline{p}_{div}(\tau)-\underline{v}}{\overline{v}-\overline{p}_{div}(\tau)} &< \min\left\{\overline{x}_{div}(\tau), \frac{(\overline{x}_{div}(\tau)+n/2-\overline{x}_G)\underline{v}-L/2}{\overline{p}_{div}(\tau)}\right\} \Leftrightarrow \\
\frac{\overline{p}_{div}(\tau)-\underline{v}}{\overline{v}-\overline{p}_{div}(\tau)} &< \min\left\{1, \frac{n\underline{v}-L}{L}\right\} \Leftrightarrow \\
\frac{\overline{p}_{div}(\tau)-\underline{v}}{\overline{v}-\overline{p}_{div}(\tau)} &< 1 \Leftrightarrow \\
\tau &< \frac{1}{1+\beta}
\end{aligned}$$

Next, consider the equilibrium with imbalanced exit. Given claim 8, we assume without loss of generality that $\mathbf{e}(L, (\overline{v}, \underline{v})) = (0, n/2)$. In this case, the equilibrium strategies and prices (from Bayes' rule) are described in part (ii) of the statement of the proposition. Note that, based on claim 8, it must be $p(\overline{x}_G) < \frac{\underline{v}+\overline{v}}{2}$, and since $\overline{x}_G = \overline{x}_{div}(\tau)$, given the expression of $\overline{p}_{div}(\tau)$ in part (ii), it must be $\tau < \frac{1}{1+\beta}$. To show that this is indeed an equilibrium, we need to show that profitable deviations do not exist. As in the equilibrium with balanced exit, it is straightforward to see that there is no profitable deviation when $\theta = 0$, when $\theta = L$ and both assets are bad, or when $\theta = L$ and both assets are good.

Suppose $\theta = L$, one asset is bad, and one asset is good. First note that selling more of the good asset and less from the bad asset (but still more than $\overline{x}_{div}(\tau)$ units) is necessarily suboptimal: the payoff of the seller on the bad asset does not change, but since $x > 0 \Rightarrow p(x) < \overline{v}$, her payoff on the good asset decreases. Therefore, a profitable deviation requires selling $\overline{x}_{div}(\tau)$ from the bad asset. However, since by construction $\overline{x}_{div}(\tau)\overline{p}_{div}(\tau) = L/2$, this deviation generates a revenue of at least L if and only if the seller also sells $\overline{x}_{div}(\tau)$ from the good asset. Such deviation is profitable if and only if

$$\begin{aligned}
&\underline{v}n/2 + \overline{v}n/2 < \overline{x}_{div}(\tau)\overline{p}_{div}(\tau) + (n/2 - \overline{x}_{div}(\tau))\underline{v} + \overline{x}_{div}(\tau)\overline{p}_{div}(\tau) + (n/2 - \overline{x}_{div}(\tau))\overline{v} \Leftrightarrow \\
&\quad \frac{\underline{v}+\overline{v}}{2} < \overline{p}_{div}(\tau) \Leftrightarrow \frac{1}{1+\beta} < \tau,
\end{aligned}$$

as required. ■

The intuition is as follows. If $\tau \geq \frac{1}{1+\beta}$, then the seller retains an asset only if it is good and she suffers no liquidity shock. If the asset is bad, or if she suffers a shock, she sells the asset to the same degree ($\overline{x}_{div}(\tau)$ in this case), just as in the equilibrium under concentration.

In particular, even if asset i is good and asset j is bad, she still sells asset i upon a shock. Even though she could meet her liquidity need by selling only asset j, doing so would lead to the lowest possible price of $\underline{v}$. When $\tau \geq \frac{1}{1+\beta}$, the probability of a good asset (τ) and the probability of a liquidity shock (β) are both high, and so the price of a partially sold asset $\overline{p}_{div}(\tau)$ is also high as there is a sufficiently large probability that this is a good asset sold due to a shock. Since $\overline{p}_{div}(\tau)$ is so much higher than $\underline{v}$, the seller prefers to meet the shock by selling only $\overline{x}_{div}(\tau)$ of the bad asset, even though doing so also requires her to sell $\overline{x}_{div}(\tau)$ of the good asset to meet the shock. Put differently, even though diversification gives the investor the flexibility to meet the liquidity shock by selling only bad assets, she chooses not to take advantage of this flexibility. This issue does not arise with a continuum of firms since it is never the case that all assets are good. Thus, a sold asset cannot be a good asset sold due to a shock, and so it receives the lowest possible price of $\underline{v}$.

If $\tau < \frac{1}{1+\beta}$, then the price of a partially-sold asset is sufficiently low that a seller with exactly one bad asset chooses to meet a liquidity need by fully selling the bad asset and fully retaining the good asset. Thus, she does take advantage of her flexibility to meet a liquidity shock by selling only the bad asset. Unlike in the case of $\tau \geq \frac{1}{1+\beta}$, a good asset is now retained upon a liquidity shock, if the other asset is bad.

C.1.2 Single Buyer

Here, the buyer can condition p_i on x_j. We focus on pure strategy symmetric equilibria with non-increasing prices in the following sense: If $0 \leq x < x+\varepsilon \leq n/2$ then $p_i(x,x) \geq p_i(x+\varepsilon, x+\varepsilon)$ and $p_j(x,x) \geq p_j(x+\varepsilon, x+\varepsilon)$. Intuitively, if the seller sells the same quantity from both assets, the price of each asset cannot be higher if the seller increases the quantity sold from each asset by the same amount.

Proposition 7 *(Diversification, two assets, one buyer): Suppose* $0 < L/n \leq \underline{v}/2$*. An equilibrium under diversification always exists.*[28]

[28] If $L/n \leq \underline{v}/2$ and $\tau = \frac{1}{1+\sqrt{\beta}}$ then the equilibria in part (i) and part (ii) coexist.

(i) If $\tau > \frac{1}{1+\sqrt{\beta}}$ *then the equilibrium is unique where:*

$$x^*_{div,i}\left(v_i, v_j, \theta\right) = \begin{cases} 0 & \text{if } \mathbf{v} = \left(\overline{v}, \overline{v}\right) \text{ and } \theta = 0 \\ \overline{x}_{div}\left(\tau\right) = \frac{L/2}{\overline{p}_{div}(\tau)} & \text{else} \end{cases} \tag{42}$$

and prices of asset i *are*

$$p^*_i\left(x_i, x_j\right) = \begin{cases} \overline{v} & \text{if } x_i = 0 \\ \overline{p}_{div}\left(\tau\right) = \underline{v} + \tau \frac{\beta + (1-\beta)(1-\tau)}{\beta + (1-\beta)(1-\tau^2)} \Delta & \text{if } 0 < x_i \leq \min\left\{\overline{x}_{div}\left(\tau\right), x_j\right\}, \\ \underline{v} & \text{if } \min\left\{\overline{x}_{div}\left(\tau\right), x_j\right\} < x_i \end{cases}$$

(ii) If $\tau < \frac{1}{1+\sqrt{\beta}}$ *then in any equilibrium*

$$x^*_{div,i}\left(\mathbf{v}, \theta\right) = \begin{cases} 0 & \text{if } \mathbf{v} = \overline{\mathbf{v}} \text{ and } \theta = 0, \text{ or } \mathbf{v} = \left(\overline{v}, \underline{v}\right) \text{ and } \theta = L \\ \overline{x}_{div}\left(\tau\right) = \frac{L/2}{\overline{p}_{div}(\tau)} & \text{if } \mathbf{v} = \overline{\mathbf{v}} \text{ and } \theta = L, \text{ or } \mathbf{v} = \underline{\mathbf{v}} \\ n/2 & \text{if } \mathbf{v} = \left(\underline{v}, \overline{v}\right) \text{ and } \theta = L \end{cases} \tag{43}$$

and

$$\left(x^*_{div,i}\left(\left(\underline{v}, \overline{v}\right), 0\right), x^*_{div,j}\left(\left(\underline{v}, \overline{v}\right), 0\right)\right) \neq \left(\overline{x}_{div}\left(\tau\right), \overline{x}_{div}\left(\tau\right)\right).$$

Moreover,

$$\overline{p}_{div}\left(\tau\right) = \underline{v} + \frac{\beta\tau^2}{\beta\left(\left(1-\tau\right)^2 + \tau^2\right) + \left(1-\beta\right)\left(1-\tau\right)^2}\Delta.$$

The equilibrium with the lowest price informativeness satisfies

$$\left(x^*_{div,i}\left(\left(\underline{v}, \overline{v}\right), 0\right), x^*_{div,j}\left(\left(\underline{v}, \overline{v}\right), 0\right)\right) = (0, 0)$$

and

$$p^*_i\left(x_i, x_j\right) = \begin{cases} p\left(0\right) = \underline{v} + \Delta \frac{\tau(1-\tau) + \tau^2}{\tau(1-\tau) + (1-\tau)\tau + \tau^2} & \text{if } x_i = 0 \\ \overline{p}_{div}\left(\tau\right) & \text{if } 0 < x_i \leq \min\left\{\overline{x}_{div}\left(\tau\right), x_j\right\}. \\ \underline{v} & \text{if } \min\left\{\overline{x}_{div}\left(\tau\right), x_j\right\} < x_i \end{cases}$$

Proof of Proposition 7. We start by proving several claims.

1. *In any equilibrium, if* $\mathbf{x}$ *such that* $x_i < x_j$ *is on the path then* $p_i(\mathbf{x}) = \overline{v}$ *and* $p_j(\mathbf{x}) = \underline{v}$*.*

   Proof: by symmetry it must be $\mathbf{v} \in \{(\underline{v}, \overline{v}), (\overline{v}, \underline{v})\}$. The profit of the seller is $\Pi(\mathbf{x}, \mathbf{v})$. Note that for $\mathbf{x}$ satisfying $x_i < x_j$

   $$\Pi(\mathbf{x}, \mathbf{v}) > \Pi\left(\mathbf{x}^T, \mathbf{v}\right) \Leftrightarrow v_i > v_j.$$

   Since $\mathbf{x}$ is on the equilibrium path (by symmetry, so is x^T), a buyer with rational expectations will set $p_i(\mathbf{x}) \geq p_j(\mathbf{x})$. Since $x_i < x_j$ implies $v \in \{(\underline{v}, \overline{v}), (\overline{v}, \underline{v})\}$, it must be $\mathbf{v} = (\overline{v}, \underline{v})$, and therefore, $p_i(\mathbf{x}) = \overline{v}$ and $p_j(\mathbf{x}) = \underline{v}$, as required.

2. *In any equilibrium, if* $v_i > v_j$ *then* $x_i \leq x_j$*.*

   Proof: suppose on the contrary $v_i > v_j$ and $x_i < x_j$. Since $\Pi(\mathbf{x}, \mathbf{v}) > \Pi\left(\mathbf{x}^T, \mathbf{v}\right) \Leftrightarrow v_i > v_j$, the seller has a profitable deviation to sell x_i units from asset v_j and x_j units from v_i, a contradiction.

3. *In any equilibrium, there is a unique* $\overline{x}_G \in (0, n/2)$ *such that* $\mathbf{e}(L, \overline{\mathbf{v}}) = \overline{\mathbf{x}}_G$*.*

   Proof: the same as claim #1 in the proof of Proposition 6, where the price is $p(x, x)$ instead of $p(x)$.

4. *In any equilibrium, (i)* $p_i(\overline{\mathbf{x}}_G) \in (\underline{v}, \overline{v})$*; (ii)* $\overline{x}_G p(\overline{\mathbf{x}}_G) = L/2$*.*

   Proof: the same as claim #2 in the proof of Proposition 6, where the price is $p(x, x)$ instead of $p(x)$.

5. *In any equilibrium,* $e_i(0, \overline{\mathbf{v}}) < \overline{x}_G$ *and* $p(\mathbf{e}(0, \overline{\mathbf{v}})) = \overline{v}$*.*

   Proof: the same as claim #3 in the proof of Proposition 6, where the price is $p(x, x)$ instead of $p(x)$.

6. *In any equilibrium,* $e_i(0, \underline{\mathbf{v}}) = e_i(L, \underline{\mathbf{v}}) = \overline{x}_G$*.*

   Proof: By symmetry, if $v_i = v_j = \underline{v}$ then $e_i(\theta, \underline{\mathbf{v}}) = e_j(\theta, \underline{\mathbf{v}})$. Let $\mathbf{x}_B(\theta) \equiv \mathbf{e}(\theta, \underline{\mathbf{v}})$. Note that if $x < \overline{x}_G$ then $xp(x, x) < L/2$. Otherwise, since $p(\overline{\mathbf{x}}_G) < \overline{v}$, requirement #1 implies that type $\overline{\mathbf{v}}$ has strict incentives to deviate and sell less than $\overline{x}_G$ from each asset when

$\theta = L$. Therefore, $x_B(L) \geq \overline{x}_G$. Suppose on the contrary $x_B(L) > \overline{x}_G$. Since $p(\overline{\mathbf{x}}_G) > \underline{v}$, type $\underline{\mathbf{v}}$ chooses $\mathbf{x}_B(L)$ only if $p(\mathbf{x}_B(L)) > \underline{v}$. This is possible only if $\mathbf{e}(L, (\overline{v}, \underline{v})) = \mathbf{x}_B(L)$ or $\mathbf{e}(0, (\overline{v}, \underline{v})) = \mathbf{x}_B(L)$. Note that type $(\overline{v}, \underline{v})$ can always obtain a payoff of $n/2(\underline{v} + \overline{v})$ and a revenue of $n/2\underline{v} \geq L$ by choosing $(0, n/2)$. Therefore, she would choose $\mathbf{x}_B(L)$ only if

$$2x_B(L)\,p(\mathbf{x}_B(L)) + (n/2 - x_B(L))(\underline{v} + \overline{v}) \geq n/2(\underline{v} + \overline{v}) \Leftrightarrow p(\mathbf{x}_B(L)) \geq \frac{\underline{v} + \overline{v}}{2}$$

Let $\gamma > 0$ be the probability type $\underline{\mathbf{v}}$ chooses $\mathbf{x}_B(L)$, then at the best case scenario (the highest value that can be obtained by $p(\mathbf{x}_B(L))$), type $(\overline{v}, \underline{v})$ chooses $\mathbf{x}_B(L)$ w.p. one, and in this case,

$$p(\mathbf{x}_B(L)) = \underline{v} + \Delta \frac{\tau(1-\tau)}{[\tau(1-\tau) + (1-\tau)\tau] + \gamma(1-\tau)^2} < \frac{\underline{v} + \overline{v}}{2}$$

a contradiction.

7. *In any equilibrium, if* $p(\overline{\mathbf{x}}_G) > \frac{\underline{v}+\overline{v}}{2}$ *then* $\mathbf{e}(0, (\overline{v}, \underline{v})) = \mathbf{e}(L, (\overline{v}, \underline{v})) = \overline{\mathbf{x}}_G$, *and if* $p(\overline{\mathbf{x}}_G) < \frac{\underline{v}+\overline{v}}{2}$ *then* $\mathbf{e}(0, (\overline{v}, \underline{v})) \neq \overline{\mathbf{x}}_G$ *and* $\mathbf{e}(L, (\overline{v}, \underline{v})) \neq \overline{\mathbf{x}}_G$.

   Proof: Note that if type $(\overline{v}, \underline{v})$ chooses $(0, n/2)$ then she obtains a payoff of $n/2(\underline{v} + \overline{v})$, and since $\underline{v}n/2 \geq L$, this strategy raises enough revenue to meet the liquidity needs. Therefore, regardless of her liquidity needs, type $(\overline{v}, \underline{v})$ chooses $\overline{\mathbf{x}}_G$ if and only if

$$2\overline{x}_G p(\overline{\mathbf{x}}_G) + (n/2 - \overline{x}_G)(\underline{v} + \overline{v}) > n/2(\underline{v} + \overline{v}) \Leftrightarrow p(\overline{\mathbf{x}}_G) > \frac{\underline{v} + \overline{v}}{2}, \tag{44}$$

   as required.

Based on the claims above, there are two types of equilibria, those in which $\mathbf{e}(\theta, (\overline{v}, \underline{v})) = \overline{\mathbf{x}}_G$ (balanced exit), and those in which $\mathbf{e}(\theta, (\overline{v}, \underline{v})) \neq \overline{\mathbf{x}}_G$ (which can potentially be imbalanced exit). We consider each of the above equilibria separately.

Consider first the equilibrium with balanced exit. In this case, the equilibrium strategies and prices (from Bayes' rule) are described in part (i) of the proposition. In particular $\overline{x}_G = \overline{x}_{div}(\tau)$. Note that condition (44) and $p(\overline{\mathbf{x}}_G) = \overline{p}_{div}(\tau)$ imply that $\tau \geq \frac{1}{1+\sqrt{\beta}}$ is necessary. To show that this is indeed an equilibrium, we need to show profitable deviations do not exist. There are four cases to consider:

1. If $\theta = 0$ and $\mathbf{v} = \overline{\mathbf{v}}$, the seller cannot receive more than $\overline{v}$ on each of her good assets, and so fully retains both.

2. Suppose $\mathbf{v} = \underline{\mathbf{v}}$. The equilibrium payoff is

$$\Pi^* (\underline{\mathbf{v}}) = 2\overline{x}_{div} (\tau) \overline{p}_{div} (\tau) + 2 (n/2 - \overline{x}_{div} (\tau)) \underline{v} > \underline{v}$$

Based on the pricing function, any deviation to sell $x > \overline{x}_{div} (\tau)$ from one of the assets generates a strictly lower payoff: the price of the sold asset drops to $\underline{v}$, and the price of the other assets does not increase unless it is fully retained, in which case it generates a payoff of $\underline{v}$. Any deviation to sell $x < \overline{x}_{div} (\tau)$ from one of the assets also generates a strictly lower payoff: the price of either asset does not increase as long as the asset is not fully retained. Therefore, there is no profitable deviation.

3. Suppose $\mathbf{v} = (\overline{v}, \underline{v})$. Note that by construction $\overline{\mathbf{x}}_G p (\overline{\mathbf{x}}_G) = L/2$, and so the seller can meet her liquidity needs by choosing $\overline{\mathbf{x}}_G$. Since (44) holds, by choosing $\overline{\mathbf{x}}_G$ the seller obtains a payoff higher than $n/2 (\underline{v} + \overline{v})$. Based on the pricing function in part (i), any deviation that involves selling strictly more than $\overline{x}_{div}$ from at least one asset is dominated by fully selling the bad asset and fully retaining the good asset, a strategy that generates a payoff of $n/2 (\underline{v} + \overline{v})$. Consider a deviation that involves selling strictly $x < \overline{x}_{div}$ from at least one of the assets. If it is a balanced exit strategy, it generates a payoff of $2x\overline{p}_{div} (\tau) + 2 (n/2 - x) \frac{\overline{v}+\underline{v}}{2}$. However,

$$\begin{aligned} x\overline{p}_{div} (\tau) + (n/2 - x) \frac{\overline{v} + \underline{v}}{2} &< \overline{x}_{div} (\tau) \overline{p}_{div} (\tau) + (n/2 - \overline{x}_{div} (\tau)) \frac{\overline{v} + \underline{v}}{2} \Leftrightarrow \\ x[\overline{p}_{div} (\tau) - \frac{\overline{v} + \underline{v}}{2}] &< \overline{x}_{div} (\tau) [\overline{p}_{div} (\tau) - \frac{\overline{v} + \underline{v}}{2}] \end{aligned}$$

Since $x < \overline{x}_{div}$ and $\overline{p}_{div} (\tau) - \frac{\overline{v}+\underline{v}}{2} \geq 0$, choosing $x < \overline{x}_{div}$ is suboptimal. If it is an imbalanced exit strategy, the seller can profit from a deviation only if she sells less from the good asset. However, any such deviation lowers the price of the bad asset to $\underline{v}$. Therefore, if there is an optimal deviation, it is one in which the seller fully retains the good the asset. However, this deviation creates a payoff of $n/2 (\underline{v} + \overline{v})$, which is lower than the equilibrium payoff.

4. Suppose $\theta = L$ and $\mathbf{v} = \overline{\mathbf{v}}$. The equilibrium payoff is

$$\Pi^* (\overline{\mathbf{v}}) = 2\overline{x}_{div} (\tau) \overline{p}_{div} (\tau) + 2 (n/2 - \overline{x}_{div} (\tau)) \overline{v},$$

Based on the pricing function, any deviation to selling more than $\overline{x}_{div} (\tau)$ from both assets requires the seller to sell more of the good asset but get a price which is strictly lower than $\overline{p}_{div} (\tau)$ and $\overline{v}$, and hence generates a strictly lower payoff. Moreover, since $\overline{x}_{div} (\tau) \overline{p}_{div} (\tau) = L/2$, any deviation to selling less than $\overline{x}_{div} (\tau)$ from both assets does not generate enough revenue to meet the liquidity needs, and hence, suboptimal. Consider a deviation to imbalanced exit (x_i, x_j) such that $x_i < \overline{x}_{div} (\tau) < x_j$. The payoff and revenue are, respectively,

$$\hat{\Pi} = x_i \overline{p}_{div} (\tau) + x_j \underline{v} + (n - (x_i + x_j)) \overline{v},$$

and

$$\hat{R} = x_i \overline{p}_{div} (\tau) + x_j \underline{v}.$$

This deviation is optimal only if $\hat{\Pi} \geq \Pi^* (\overline{\mathbf{v}})$ and $\hat{R} \geq L$. Clearly, if there is such a deviation, then there is another optimal deviation in which $\hat{R} = L$ (selling less from either asset does not decrease the price). Since $2\overline{x}_{div} (\tau) \overline{p}_{div} (\tau) = L$, $\hat{\Pi} \geq \Pi^* (\overline{\mathbf{v}})$ requires

$$(n - (x_i + x_j)) \overline{v} \geq 2 (n/2 - \overline{x}_{div} (\tau)) \overline{v} \Leftrightarrow (x_i + x_j) \leq 2\overline{x}_{div} (\tau)$$

Using $x_i \overline{p}_{div} (\tau) + x_j \underline{v} = L$ and $\overline{x}_{div} (\tau) = \frac{L/2}{\overline{p}_{div}(\tau)}$, the condition becomes $x_i \geq 2\overline{x}_{div} (\tau)$, which contradicts $x_i < \overline{x}_{div} (\tau)$. Therefore, there is no profitable deviation.

Next, consider the equilibrium with $\mathbf{e} (\theta, (\overline{v}, \underline{v})) \neq \overline{\mathbf{x}}_G$. This implies that $\overline{x}_G = \overline{x}_{div} (\tau)$, and from Bayes' rule, $p (\overline{\mathbf{x}}_G) = \overline{p}_{div} (\tau)$ is given as in part (ii) of the Proposition. Since we require $p (\overline{\mathbf{x}}_G) < \frac{\underline{v}+\overline{v}}{2}$, the expression for $\overline{p}_{div} (\tau)$ in part (ii) implies that $\tau < \frac{1}{1+\sqrt{\beta}}$.

Next, we argue that if $\theta = L$ then type $(\overline{v}, \underline{v})$ must follow imbalanced exit, that is $e_i (L, (\overline{v}, \underline{v})) < e_j (L, (\overline{v}, \underline{v}))$. Suppose on the contrary $\mathbf{e} (L, (\overline{v}, \underline{v})) = \mathbf{x} = (x, x)$ where $x \neq \overline{x}_G$. Based on claims 3, 5, and 6, no other type is choosing $\mathbf{x}$. Therefore, $p (\mathbf{x}) = \frac{\underline{v}+\overline{v}}{2}$. Since $p (\overline{\mathbf{x}}_G) < \frac{\underline{v}+\overline{v}}{2}$, if $x > \overline{x}_G$ then type $\underline{\mathbf{v}}$ has a strictly profitable deviation from $\overline{\mathbf{x}}_G$ to $\mathbf{x}$: she can sell a strictly larger number of units from each assets for a price which is higher than $p (\overline{\mathbf{x}}_G)$, and thereby get a strictly larger payoff, yielding a contradiction. Suppose on the contrary $x < \overline{x}_G$. Note that it must be $xp(\mathbf{x}) < L/2$, otherwise, type $\overline{\mathbf{v}}$ would have a profitable deviation from $\overline{\mathbf{x}}_G$ to $\mathbf{x}$ when $\theta = L$. Therefore, it cannot be $x < \overline{x}_G$. We conclude, it must be $e_i(L, (\overline{v}, \underline{v})) \neq e_j(L, (\overline{v}, \underline{v}))$. Given claims 1 and 2, we can assume without loss of generality (and to ease the exposition) that the imbalanced strategy involves $\mathbf{e}(L, (\overline{v}, \underline{v})) = (0, n/2)$. Since $\underline{v}n/2 \geq L$, this strategy raises enough revenue to meet the liquidity needs.

The equilibrium therefore hinges on the strategy of type $(\overline{v}, \underline{v})$ when $\theta = 0$. The lowest price informativeness arises when type $(\overline{v}, \underline{v})$ follows balanced exit. There are two cases to consider:

1. If $\mathbf{e}(0, (\overline{v}, \underline{v})) \neq \mathbf{e}(0, (\overline{v}, \overline{v}))$ then

$$\begin{aligned}
P'_{div,single}(\underline{v}) &= \tau\left(\beta\underline{v} + (1-\beta)\frac{\overline{v}+\underline{v}}{2}\right) + (1-\tau)\overline{p}_{div}(\tau) \\
P'_{div,single}(\overline{v}) &= \beta\tau\overline{p}_{div}(\tau) + (1-\beta)\tau\overline{v} + \beta(1-\tau)\overline{v} + (1-\beta)(1-\tau)\frac{\overline{v}+\underline{v}}{2}
\end{aligned}$$

2. If $\mathbf{e}(0, (\overline{v}, \underline{v})) = \mathbf{e}(0, (\overline{v}, \overline{v}))$ then

$$\begin{aligned}
P''_{div,single}(\underline{v}) &= \tau(\beta\underline{v} + (1-\beta)p(0)) + (1-\tau)\overline{p}_{div}(\tau) \\
P''_{div,single}(\overline{v}) &= \beta\tau\overline{p}_{div}(\tau) + (1-\beta)\tau p(0) + \beta(1-\tau)\overline{v} + (1-\beta)(1-\tau)p(0)
\end{aligned}$$

Note that

$$p(0) \in \left(\frac{\overline{v}+\underline{v}}{2}, \tau\overline{v} + (1-\tau)\frac{\overline{v}+\underline{v}}{2}\right)$$

and

$$\begin{aligned}
P''_{div,single}(\underline{v}) &> P'_{div,single}(\underline{v}) \Leftrightarrow p(0) > \frac{\overline{v}+\underline{v}}{2}, \text{ and} \\
P''_{div,single}(\overline{v}) &< P'_{div,single}(\overline{v}) \Leftrightarrow p(0) < \tau\overline{v} + (1-\tau)\frac{\overline{v}+\underline{v}}{2}
\end{aligned}$$

Therefore, equilibrium with the lowest price informativeness, if exists, is obtained when type $(\overline{v}, \underline{v})$ chooses $(0, 0)$ when $\theta = 0$.

We now show that an equilibrium with $\mathbf{e}(0, (\overline{v}, \underline{v})) = \mathbf{e}(0, (\overline{v}, \overline{v}))$ indeed exists. In this case, the equilibrium strategies and prices (from Bayes' rule) are described in part (ii) of the proposition. We need to show that profitable deviations do not exist. As in the equilibrium with balanced exit, it is straightforward to see that there is no profitable deviation when $\mathbf{v} = \overline{\mathbf{v}}$

or $\mathbf{v} = \underline{\mathbf{v}}$ (cases 1,2, and 4 above). Indeed, the equilibrium strategies of these type do not change. Moreover, the proof only depend on the property $\overline{x}_{div}(\tau) = \frac{L/2}{\overline{p}_{div}(\tau)}$, which holds. One difference is that relative to the balanced exit equilibrium, the price $p(0,0) < \overline{v}$. However, since the seller's payoff does not depend on the price when she fully retains the assets, $p(0,0)$ has no effect on her incentives.

There is only one case to consider. Suppose $\mathbf{v} = (\overline{v}, \underline{v})$. The equilibrium payoff is $n/2(\underline{v} + \overline{v})$ and the revenue is when $\theta = L$ is $\underline{v}n/2 \geq L$ (the seller can meet her liquidity needs by choosing $(0, n/2)$). Note that any deviation to imbalanced exit will continue to generate a payoff of $\underline{v}$ on the bad asset and a payoff of at most $\overline{x}_{div}(\tau)\overline{p}_{div}(\tau) + (n/2 - \overline{x}_{div}(\tau))\overline{v} < \overline{v}$, on the good asset. Therefore, such a deviation is suboptimal. Any deviation to balanced exit (x, x) where $x > \overline{x}_{div}(\tau)$ is going to generate a payoff of $\underline{v}$ on the bad asset and a payoff of at most $\overline{x}_{div}(\tau)\underline{v} + (n/2 - \overline{x}_{div}(\tau))\overline{v} < \overline{v}$, on the good asset, which is suboptimal. Consider a deviation to a balanced exit (x, x) where $x \leq \overline{x}_{div}(\tau)$. The payoff to the investor is

$$2x\overline{p}_{div}(\tau) + (n/2 - x)(\overline{v} + \underline{v}),$$

which is smaller than $n/2(\underline{v} + \overline{v})$ if and only if $\overline{p}_{div}(\tau) \leq \frac{\overline{v}+\underline{v}}{2}$, which always if $\tau < \frac{1}{1+\sqrt{\beta}}$. Therefore, the seller has no profitable deviation, as required. Note that that monotonicity condition is preserved:

$$p(0) > \overline{p}_{div}(\tau) \Leftrightarrow \frac{(1-\tau)}{\tau^2} > \beta,$$

which always holds if $\tau < \frac{1}{1+\sqrt{\beta}}$. ■

The intuition is as follows. The equilibria are similar to the separate buyer case, except for when the seller has exactly one bad asset and does not suffer a liquidity shock. With separate buyers, the seller would voluntarily sell $\overline{x}_{div}(\tau)$ of the bad asset, to disguise the sale as being of a good asset and motivated by a liquidity shock, and fully retain the good asset. With a single buyer, such disguise is no longer possible, since the buyer would see that the good asset is fully retained, and thus infer that the partial sale of the bad asset is due to its low quality, rather than a shock. Thus, the seller would receive the lowest possible price of $\underline{v}$ by partially selling only the bad asset. She may thus choose to engage in balanced exit, i.e. sell both the bad and good asset to the same degree, to disguise their sale as being motivated by a shock. While this leads to losses on the good asset, which is partially sold, she gains on the bad asset, which now receives a price strictly greater than $\underline{v}$. The lowest level of price informativeness arises when the seller retains both assets upon no shock, because now being fully retained no longer fully reveals that an asset is high-quality.

C.1.3 Price Informativeness

Proposition 8 *(Price informativeness, two assets): Suppose* $0 < L/n \leq \underline{v}/2$.

(i) *Separate buyers: If* $\tau < \frac{1}{1+\beta}$ *then in any equilibrium under concentration and diversification*

$$P_{div,Separate}\left(\overline{v},\tau\right) > P_{con}\left(\overline{v},\tau\right) \text{ and } P_{div,Separate}\left(\underline{v},\tau\right) < P_{con}\left(\underline{v},\tau\right). \tag{45}$$

(ii) *Single buyer: There is* $\overline{\beta} \in (0,1)$ *such that if* $\tau < 1/2$ *and* $\beta \in (\overline{\beta}, 1]$ *then in any equilibrium under concentration and diversification*

$$P_{div,Single}\left(\overline{v},\tau\right) > P_{con}\left(\overline{v},\tau\right) \text{ and } P_{div,Single}\left(\underline{v},\tau\right) < P_{con}\left(\underline{v},\tau\right). \tag{46}$$

Proof. Recall that according to Proposition 1, $P_{con}\left(\underline{v}\right) = \overline{p}_{con}\left(\tau\right)$ and $P_{con}\left(\overline{v}\right) = \beta\overline{p}_{con}\left(\tau\right) + (1-\beta)\,\overline{v}$ where

$$\overline{p}_{con}\left(\tau\right) = \underline{v} + \Delta\frac{\beta\tau}{\beta\tau + 1 - \tau}.$$

Consider part (i). Based on part (i) if Proposition 6, if $\tau \geq \frac{1}{1+\beta}$ then the equilibrium is the same as under the benchmark, and therefore, price informativeness is the same. However, if $\tau < \frac{1}{1+\beta}$ then

$$\begin{aligned}
P_{div,Separate}\left(\underline{v},\tau\right) &= \beta\tau\underline{v} + (1-\beta\tau)\,\overline{p}_{div}\left(\tau\right) \\
P_{div,Separate}\left(\overline{v},\tau\right) &= \beta\tau\overline{p}_{div}\left(\tau\right) + (1-\beta\tau)\,\overline{v}
\end{aligned}$$

Using

$$\overline{p}_{div}\left(\tau\right) = \underline{v} + \frac{\tau^2\beta}{(1-\beta)\,(1-\tau) + (\tau^2 + (1-\tau)^2)\beta}\Delta$$

Using plain algebra, it can be verified that

$$P_{div,Separate}\left(\overline{v},\tau\right) > P_{con}\left(\overline{v},\tau\right) \text{ and } P_{div,Separate}\left(\underline{v},\tau\right) < P_{con}\left(\underline{v},\tau\right). \tag{47}$$

Consider part (ii). Based on Proposition 7, price informativeness is higher with two firms only if $\tau < \frac{1}{1+\sqrt{\beta}}$. Part (ii) of Proposition 7 also gives the equilibrium with the worst price informativeness with two assets. In this equilibrium

$$\begin{aligned} P_{div,Single}\left(\underline{v},\tau\right) &= \tau\left(\beta\underline{v} + (1-\beta)\,p\left(0\right)\right) + (1-\tau)\,\overline{p}_{div}\left(\tau\right) \\ P_{div,Single}\left(\overline{v},\tau\right) &= \beta\tau\overline{p}_{div}\left(\tau\right) + \beta\left(1-\tau\right)\overline{v} + \left((1-\beta)\,\tau + (1-\beta)\left(1-\tau\right)\right)p\left(0\right) \end{aligned}$$

where

$$\begin{aligned} p\left(0\right) &= \underline{v} + \Delta\frac{\tau\left(1-\tau\right)+\tau^2}{\tau\left(1-\tau\right)+\left(1-\tau\right)\tau+\tau^2} = \underline{v} + \Delta\frac{1}{2-\tau} \\ \overline{p}_{div}\left(\tau\right) &= \underline{v} + \frac{\beta\tau^2}{\beta\left(\left(1-\tau\right)^2+\tau^2\right)+\left(1-\beta\right)\left(1-\tau\right)^2}\Delta = \underline{v} + \frac{\beta\tau^2}{\beta\tau^2+\left(1-\tau\right)^2}\Delta \end{aligned}$$

Using plain algebra, it can be verified that

$$\begin{aligned} P_{div,Single}\left(\underline{v},\tau\right) &< P_{con}\left(\underline{v},\tau\right) \Leftrightarrow \\ \frac{1-\beta}{2-\tau} + \frac{\left(1-\tau\right)\beta\tau}{\beta\tau^2+\left(1-\tau\right)^2} &< \frac{\beta}{\beta\tau+1-\tau} \end{aligned}$$

and

$$\begin{aligned} P_{div,Single}\left(\overline{v},\tau\right) &> P_{con}\left(\overline{v},\tau\right) \Leftrightarrow \\ \frac{\beta^2\tau^3}{\beta\tau^2+\left(1-\tau\right)^2} + \left(1-\tau\right)\frac{3\beta-1-\tau\beta}{2-\tau} &> \frac{\beta^2\tau}{\beta\tau+1-\tau} \end{aligned}$$

Note that as $\beta \rightarrow 1$, the two conditions hold for any τ. Also note that if $\tau < \frac{1}{2}$ then $\tau < \frac{1}{1+\sqrt{\beta}}$ for all $\beta \in [0,1]$. Combined, it concludes the proof. ■

The intuition is as follows. Price informativeness can only be higher under diversification only if $\tau < \frac{1}{1+\sqrt{\beta}}$, because only in this case does the seller take advantage of her flexibility to meet a liquidity shock by selling only the bad asset asset (if she has exactly one bad asset). For price informativeness to be higher under diversification in *any* equilibrium, we need the price of a bad firm to be lower than under concentration. Under concentration, a bad firm's price is increasing in β, since the higher the probability of a liquidity shock, the higher the probability that a partial sale is of a good firm due to a shock. Thus, the combination of a high β (to increase the price of a bad firm under concentration) and a low τ (so that $\tau < \frac{1}{1+\sqrt{\beta}}$) ensures that price informativeness is higher under diversification in any equilibrium.

C.2 Single Buyer

This section considers the case of a single buyer, who observes the order flows of all assets when setting prices. Since liquidity shocks are i.i.d. across sellers under concentration, x_j contains no information relevant for the pricing of asset $i \neq j$. Therefore, the analysis of concentration does not change. Below we derive the equilibria under diversification.

We consider only those equilibria with monotonic prices. Here, we define monotonicity similar to how it is defined in the two asset, single buyer case. That is, prices are monotonic if, for all $\mathbf{x}$ such that $x_i = x$ for all i, $p_i(\mathbf{x}) \geq p_i(\mathbf{x} + \varepsilon)$ for all $\varepsilon > 0$. In other words, among balanced exit strategies, if an investor sells less of all firms, the prices of all firms must (weakly) increase.

Proposition 9 *(Single buyer) For all L, there exists an equilibrium under diversification where price informativeness is strictly higher than under concentration. There also exists an equilibrium under diversification where price informativeness is strictly lower than under concentration.*

Proof of Proposition 9. First, suppose $L/n \leq \underline{v}(1-\tau)$. Since the single buyer observes all trades, the pricing function for asset i will be a function of $x_j \; \forall \; j$. This also implies that in equilibrium the seller will, ex-post, receive the expected value of her portfolio ($\underline{v} + \tau\Delta$) in all states. This is because the buyer (ex-post) knows the exact measure of bad firms sold across all shares he/she buys, and therefore the expected price over all shares must be their expected value. This contrasts with the case of separate buyers, in which a buyer does not observe the entire vector of trades, and thus may not know the exact measure of bad firms among the shares purchased. We show that there is an equilibrium in which, under diversification, $x^*(\overline{v}, \theta) = 0$ and $x^*(\underline{v}, \theta) = \overline{x} \; \forall \; \theta \in \{0, L\}$, with $\overline{x} = \frac{L/n}{\underline{v}(1-\tau)} \leq 1$. For this proposed equilibrium, prices can be:

$$p_i(\mathbf{x}) = \overline{v} \text{ if } x_i = 0$$

$$p_i(\mathbf{x}) = \underline{v} \text{ otherwise.}$$

Clearly, there are no profitable deviations, and when $\theta = L$, the seller generates revenue of $\overline{x}(1-\tau)\underline{v}n = L$, which is sufficient to cover her liquidity shock. Finally, prices are clearly monotonic. Thus, this is an equilibrium.

Next, suppose $L/n \in (\underline{v}(1-\tau), \underline{v} + \Delta\tau)$. Then, the seller must sell a positive amount of good assets upon a liquidity shock. Consider the potential equilibrium in which $x^*(\overline{v}, L) = \overline{x}$ and $x^*(\underline{v}, L) = 1$, with $\overline{x} = \frac{L/n - \underline{v}(1-\tau)}{\overline{v}\tau} < 1$, and $x^*(\overline{v}, 0) = 0$ and $x^*(\underline{v}, 0) = 1$. Prices in this equilibrium can be:

$$p_i(\mathbf{x}) = \overline{v} \text{ if } x_i \leq \overline{x} \text{ and } \int_{j:x_j > \overline{x}} dj = (1-\tau)\,n$$
$$p_i(\mathbf{x}) = \underline{v} \text{ otherwise.}$$

Clearly, there are no profitable deviations, and when $\theta = L$, the seller by construction generates revenue of L, which is sufficient to cover her liquidity shock. Finally, prices in this equilibrium are monotonic. Thus, this is an equilibrium.

Given these first two cases, if $L/n < \underline{v} + \Delta\tau$, then there exists an equilibrium under diversification in which $P_{div}(\overline{v}, \tau) = \overline{v}$ and $P_{div}(\underline{v}, \tau) = \underline{v}$, i.e. prices are fully informative. Furthermore, from the analysis of trade under concentration (Section 1.2), we have $P_{con}(\overline{v}, \tau) < \overline{v}$ and $P_{con}(\underline{v}, \tau) > \underline{v}$. Thus, for $L/n < \underline{v} + \Delta\tau$, there is an equilibrium under diversification that features higher price informativeness than under concentration.

Moreover, there is also an equilibrium under diversification with *no* price informativeness when $L/n < \underline{v} + \Delta\tau$. Consider a candidate equilibrium in which $x^*(v, \theta) = \overline{x} \equiv \frac{L/n}{\underline{v} + \Delta\tau} < 1$ for all v and θ. Let equilibrium prices be

$$p_i(\mathbf{x}) = \underline{v} + \Delta\tau \text{ if } x_i = x_j \;\forall i, j$$
$$p_i(\mathbf{x}) = \underline{v} \text{ otherwise.}$$

Under these strategies and prices, note that the seller raises $L = n\overline{xp}$ and receives a payoff of $\underline{v} + \Delta\tau$. Furthermore, there are no profitable deviations: switching to another balanced exit strategy will still give a payoff of $\underline{v} + \Delta\tau$, while switching to imbalanced exit will yield a payoff of, at most, $\underline{v} + \Delta\tau$. Finally, since the prices for balanced exit are always $\underline{v} + \Delta\tau$, they satisfy monotonicity. Thus, there is an equilibrium in which ex-post prices are always $\underline{v} + \Delta\tau$ for all shares, and therefore there is no price informativeness. This equilibrium under diversification necessarily has lower price informativeness than in any equilibrium under concentration.

Finally, consider the case when $L/n \geq \underline{v} + \Delta\tau$. Then, in any equilibrium under diversification, the seller must fully sell all assets when $\theta = L$. This implies that $p_i(\mathbf{x}) = \underline{v} + \Delta\tau$ when $x_i = 1$ for all i. The seller has two potential strategies when $\theta = 0$: balanced exit where she sells a constant amount of all assets regardless of v_i, or imbalanced exit where she sells $\underline{x}$ when $v_i = \overline{v}$, and $\overline{x} > \underline{x}$ when $v_i = \underline{v}$. With balanced exit, the the seller sells the same amount of all assets regardless of whether there is a shock. Therefore, prices are fully uninformative and, thus, are strictly more informative under concentration.

With imbalanced exit, expected prices under diversification are:

$$P_{div}(\overline{v}, \tau) = (1-\beta)\overline{v} + \beta(\underline{v} + \Delta\tau)$$
$$P_{div}(\underline{v}, \tau) = (1-\beta)\underline{v} + \beta(\underline{v} + \Delta\tau) = \underline{v} + \Delta\tau\beta.$$

Under concentration, expected prices are:

$$P_{con}(\overline{v}, \tau) = (1-\beta)\overline{v} + \beta\left(\underline{v} + \Delta\frac{\beta\tau}{\beta\tau + (1-\beta)}\right)$$
$$P_{con}(\underline{v}, \tau) = \underline{v} + \Delta\frac{\beta\tau}{\beta\tau + (1-\beta)}.$$

Since $P_{con}(\underline{v}, \tau) > P_{div}(\underline{v}, \tau)$, prices in this equilibrium under diversification are strictly more informative than under concentration. ■

C.3 Heterogeneous Assets

In the core model, all assets have the same valuation distribution Δ. We now analyze the case in which assets have different valuation distributions, and thus differ in their information asymmetry and the price impact of selling. For brevity, we consider the small shock case, since this is where our results are strongest.

Let there be $J \geq 1$ classes of assets. The valuation of asset i in class $j \in \{1, ..., J\}$ is $v_j \in \{\underline{v}_j, \overline{v}_j\}$ where $\Delta_j \equiv \overline{v}_j - \underline{v}_j > 0$. We assume $\underline{v}_{j'} < \overline{v}_{j''}$ for all $j' \in J$ and $j'' \in J$, i.e. the worst good asset is more valuable than the best bad asset. We also index by j the exogenous parameters τ, $\overline{v}$, and $\underline{v}$. As in the core model, each asset has its own buyer, and the class to which asset i belongs is common knowledge. All random variables are independent across assets and classes.

The analysis of concentration remain unchanged, with the addition of a subscript j to denote that quantities apply to an asset of class j. Under diversification, we assume the seller owns a mass of $n_j \geq 0$ assets from class j.

Proposition 10 *(Heterogeneous assets): Suppose* $L \leq \sum_{j=1}^{J} n_j \underline{v}_j (1 - \tau_j)$. *For asset class* j*, if* $\gamma_j \leq \frac{\beta\tau_j}{\beta\tau_j + (1-\beta)(1-\tau_j)}$*, price informativeness is weakly higher under diversification than concentration. Conversely, if* $\gamma_j > \frac{\beta\tau_j}{\beta\tau_j + (1-\beta)(1-\tau_j)}$*, price informativeness is strictly higher under concentration..*

Proof of Proposition 10. Since $L \leq \sum_{j=1}^{J} n_j \underline{v}_j (1 - \tau_j)$, the seller can meet her liquidity need by selling bad assets alone. Then, in any equilibrium, as in the core model it must be that for asset i in class j, $x_i > 0 \rightarrow p_{j,i}(x_i) = \underline{v}_j$. Then, if $v_i = \overline{v}$, $x^*_{j,div}(v_i, \theta) = 0$; if $v_i = \underline{v}$, similar to the core model, $x^*_{j,div}(v_i, \theta) = \tilde{\underline{x}}_j$, with $\tilde{\underline{x}}_j$ such that $\sum_{j=1}^{J} E[\tilde{\underline{x}}_j] n_j \underline{v}_j (1 - \tau_j) \in \left[\theta, \sum_{j=1}^{J} n_j \underline{v}_j (1 - \tau_j)\right]$. Letting $\gamma_j = Pr(\tilde{\underline{x}}_j = 0)$, prices on the equilibrium path are then:

$$
\begin{aligned}
p_{j,i}(x_i) &= \underline{v}_j \text{ if } x_i > 0 \\
p_{j,i}(x_i) &= \underline{v}_j + \Delta_j \left(\frac{\tau_j}{\tau_j + (1 - \tau_j)\gamma_j} \right) \text{ if } x_i = 0.
\end{aligned}
$$

Then, expected prices under diversification are:

$$
\begin{aligned}
P_{j,div}(\overline{v}_j, \tau_j) &= \underline{v}_j + \Delta_j \left(\frac{\tau_j}{\tau_j + (1 - \tau_j)\gamma_j} \right) \\
P_{j,div}(\underline{v}_j, \tau_j) &= \underline{v}_j + \Delta_j \gamma_j \left(\frac{\tau_j}{\tau_j + (1 - \tau_j)\gamma_j} \right).
\end{aligned}
$$

Under concentration, expected prices are:

$$
\begin{aligned}
P_{j,con}(\overline{v}_j, \tau_j) &= \underline{v}_j + \Delta_j \left(\frac{\beta\tau_j + (1 - \beta)(1 - \tau_j)}{\beta\tau_j + (1 - \tau_j)} \right) \\
P_{j,con}(\underline{v}_j, \tau_j) &= \underline{v}_j + \Delta_j \left(\frac{\beta\tau_j}{\beta\tau_j + (1 - \tau_j)} \right).
\end{aligned}
$$

Then, we have

$$
\begin{aligned}
P_{j,div}(\underline{v}_j, \tau_j) \leq P_{j,con}(\underline{v}_j, \tau_j) &\iff \\
\frac{\gamma_j \tau_j}{\tau_j + (1-\tau_j)\gamma_j} \leq \frac{\beta \tau_j}{\beta \tau_j + (1-\tau_j)} &\iff \\
\beta \tau_j \gamma_j + (1-\tau_j)\gamma_j \leq \beta \tau_j + \beta(1-\tau_j)\gamma_j &\iff \\
\frac{\beta \tau_j}{\beta \tau_j + (1-\beta)(1-\tau_j)} \geq \gamma_j.
\end{aligned}
$$

Price informativeness is higher under diversification if this inequality holds, which is the same as in the core model (with subscript j added). ■

The intuition is as follows. Regardless of whether the assets have the same or different payoff distributions, it remains the case that, for a sufficiently small shock, diversification allows the seller to fully retain good assets upon a shock, thus maximizing the payoff to monitoring. As a result, the adverse selection problem upon selling is severe, thus minimizing the payoff to not monitoring.

C.4 Discontinuing Relationships

In this section, we apply our model to situations in which the seller decides whether to (partly) discontinue a relationship with the asset. Examples include a bank terminating a lending relationship with a borrower, or a venture capital seller choosing not to invest in a future financing round. We thus distinguish between two concepts. The price $p_i(x_i)$ reflects the impact of (dis)continuation on asset i's reputation. In the core model, this equalled the seller's payoff upon selling. Here, since we consider a discontinuation rather than a sale decision, the seller receives her outside option, $r < \overline{v}$, upon sale.[29] Importantly, unlike in the core model, this reservation payoff is fixed and independent of the impact of sale on the asset's reputation. However, we nevertheless show that diversification can still improve price informativeness. Note that this extension can be applied to stakeholders other than sellers, e.g. a supplier or customer's decision to terminate its relationship with a firm, in which case it also receives a fixed reservation payoff.

[29] If $r \geq \overline{v}$, the analysis is trivial since the investor is weakly better off exiting all firms, regardless of their value and her liquidity needs. This behavior will result in identical price informativeness under separate and common ownership, and it particular, expected prices will be the same for both good and bad firms.

We consider two cases based on the magnitude of r. In the first case, $r < \underline{v}$, and so the seller exits only if she needs liquidity. In the second case, $r \in (\underline{v}, \overline{v})$, and so the seller wishes to terminate her relationship with bad assets, but retain it in good assets.

Proposition 11 *(Discontinuing relationships, low reservation payoff): Suppose $r < \underline{v}$. Price informativeness under diversification is always weakly higher than under concentration, and strictly higher when $L/n < r$.*

Proof of Proposition 11. First, consider the case under concentration. The optimal strategies are $x_i^*(v_i, L) = \overline{x} \equiv \min\left\{\frac{L/n}{r}, 1\right\}$ for all v_i, and $x_I^*(v_i, 0) = 0$ for all v_i. Then, prices will be completely uninformative.

Next, consider the case under diversification. Note first that, for all L/n, it is optimal to sell nothing when $\theta = 0$. Now, suppose $L/n \leq r(1-\tau)$. Then, when $\theta = L$, the optimal strategies are $x_i^*(\overline{v}, L) = 0$ and $x_i^*(\underline{v}, L) = \overline{x} \equiv \frac{L/n}{(1-\tau)r}$. In that case $p(x_i) = \underline{v}$ for $x_i = \overline{x}$, and $p(x_i) = \underline{v} + \Delta\frac{\tau}{\tau+(1-\beta)(1-\tau)}$ for $x_i = 0$. If, instead, $L/n \in (r(1-\tau), r)$, optimal strategies when $\theta = L$ are $x_i^*(\underline{v}, L) = 1$ and $x_i^*(\overline{v}, L) = \frac{L/n - r(1-\tau)}{r\tau}$. Thus, again prices are partially informative. Finally, if $L/n \geq r$, we have $x_i^*(v_i, L) = 1$ for all v_i. Therefore, in this case, prices are fully uninformative, as in the concentrated case.

Therefore, prices are strictly more informative under diversification when $L/n < r$, and equally as informative when $L/n \geq r$. ■

The intuition is as follows. Since the reservation payoff is so low, the seller never sells an asset, even if it is bad, unless she is forced to do so by a shock. Upon a shock, a concentrated investor sells the minimum amount possible, regardless of asset quality. A diversified investor sells bad assets more and good assets less, and so price informativeness is higher than under concentration. As in the core model, diversification gives sellers a choice over what asset to sell when they suffer a shock, and so their sale decisions convey information.

Proposition 12 *(Discontinuing relationships, high reservation payoff): Suppose $r \in (\underline{v}, \overline{v})$. Price informativeness under diversification is always identical to that under concentration.*

Proof of Proposition 12. Note that when $r \in (\underline{v}, \overline{v})$, the seller always wants to sell bad assets fully. First, consider concentration. Then, $x_i^*(\underline{v}, \theta) = 1$ for all θ,$x_i^*(\overline{v}, 0) = 0$, and $x_i^*(\overline{v}, L) = \overline{x} \equiv \min\left\{\frac{L/n}{r}\right\}$. Then, if $L/n < r$, post-sale prices are $p_i(x_i) = \underline{v}$ if $x_i = 1$, and

$p_i(x_i) = \overline{v}$ is $x_i \in \{0, \overline{x}\}$.[30] Otherwise, they are $p_i(x_i) = \underline{v} + \Delta\frac{\beta\tau}{\beta\tau+1-\tau}$ if $x_i = 1$ and $p_i(x_i) = \overline{v}$ if $x_i = 0$.

Under diversification, $x_i^*(\underline{v}, \theta) = 1$ for all θ, and $x_i^*(\overline{v}, 0) = 0$. If $\frac{L}{n} < r$, then $x_i^*(\overline{v}, L) = \overline{x} \equiv \max\left\{0, \frac{L/n - r(1-\tau)}{\tau r}\right\}$. In this case, prices are $p_i(x_i) = \overline{v}$ if $x_i \in \{0, \overline{x}\}$, and $p_i(x_i) = \underline{v}$ if $x_i = 1$.

Alternatively, if $\frac{L}{n} \geq r$, then $x_i^*(\overline{v}, L) = 1$. Prices are $p_i(x_i) = \overline{v}$ if $x_i = 0$, and $p_i(x_i) = \underline{v} + \Delta\tau$ if $x_i = 1$.

Thus, if $\frac{L}{n} < r$,

$$\begin{aligned} P_{con}(\overline{v}, \tau) &= \overline{v} \\ P_{con}(\underline{v}, \tau) &= \underline{v}, \end{aligned}$$

and

$$\begin{aligned} P_{div}(\overline{v}, \tau) &= \overline{v} \\ P_{div}(\underline{v}, \tau) &= \underline{v}. \end{aligned}$$

If instead $\frac{L}{n} \geq r$,

$$\begin{aligned} P_{con}(\overline{v}, \tau) &= \underline{v} + \Delta\frac{\beta\tau + (1-\beta)(1-\tau)}{\beta\tau + 1 - \tau} = P_{div}(\overline{v}, \tau) \\ P_{con}(\underline{v}, \tau) &= \underline{v} + \Delta\frac{\beta\tau}{\beta\tau + 1 - \tau} = P_{div}(\underline{v}, \tau) \end{aligned}$$

Thus, price informativeness under concentration is identical to that under diversification. ■

The intuition is as follows. Since the seller's payoff upon sale is independent of how much she sells, she is unconcerned with price impact. As a result, she sells a bad asset completely, and thus receives the lowest possible price of $\underline{v}$ under any ownership structure. This contrasts the core model where, in some equilibria, the seller only partially sells a bad asset, to disguise the sale as being motivated by a shock and receive a price exceeding $\underline{v}$. Thus, if the liquidity shock is not large enough to force the sale of her entire portfolio ($\frac{L}{n} < r$), prices are already fully informative under concentration and so no higher under diversification.

[30] Since off-equilibrium prices are irrelevant for the seller's exit decision, we do not specify them to ease the exposition.