Online Appendix for "Governance Through Trading and Intervention: A Theory of Multiple Blockholders"

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A Proofs

Proof of Proposition 1. If the market maker uses a linear pricing rule of the form $p(y) = \mu + \lambda y$, blockholder i maximizes:

$$E[(\tilde{v} - \mu - \lambda y)x_i \,|\, \tilde{v} = v] = (v - \mu - \lambda \sum_{j \neq i} x_j)x_i - \lambda x_i^2.$$

This maximization problem yields

$$x_i(v) = \frac{1}{\lambda}[v - \mu - \lambda \sum_j x_j(v)] \quad \forall i. \tag{40}$$

Summing both sides across i yields

$$\sum_j x_j\,(v) = \frac{I}{\lambda}[v - \mu - \lambda \sum_j x_j\,(v)]$$

$$\sum_j x_j\,(v) = \frac{I}{(I+1)\lambda}\,[v - \mu]$$

Substituting into (40) yields

$$x_i(v) = \frac{1}{(I+1)\lambda}[v - \mu] \quad \forall i,$$

which means that, in a linear equilibrium, blockholders' strategies are symmetric. Total order flow is thus given by

$$y = \frac{I}{(I+1)\,\lambda}\,(v - \mu) + \varepsilon. \tag{41}$$

The market maker takes the blockholders' strategies as given and sets

$$p(y) = E[\tilde{v}|y]. \tag{42}$$

Using the normality of $\tilde{v}$ and $\tilde{y}$ yields

$$\lambda = \frac{\sqrt{I}}{I+1}\frac{\sigma_\eta}{\sigma_\epsilon},$$

$$\mu = \phi_a \log(1+a) + \phi_b \log\left(1 + \textstyle\sum_i b_i\right).$$

From this we obtain:

$$x_i(v) = \frac{1}{\sqrt{I}}\frac{\sigma_\epsilon}{\sigma_\eta}\left(v - \phi_a \log(1+a) - \phi_b \log\left(1 + \textstyle\sum_i b_i\right)\right) \quad \forall i,$$

$$p(y) = \phi_a \log(1+a) + \phi_b \log(1+\textstyle\sum_i b_i) + \frac{\sqrt{I}}{I+1}\frac{\sigma_\eta}{\sigma_\epsilon} y,$$

as required. Blockholder i's trading profits equal $x_i(v-p)$ and can be computed immediately using the above expressions.

Proof of Proposition 2. The result follows from $p(y) = \mu + \lambda y$ and equation (41).

Proof of Proposition 5. Putting equation (18) under a common denominator yields

$$\frac{\phi_a I(I+1) - \phi_b I(I+1)^2 - \phi_a \alpha I^2 + \phi_b \beta (I+1)^2}{I^2 (I+1)^2} = 0. \tag{43}$$

Equation (18) is a cubic, and has at most three roots. The function is discontinuous at $I = -1$ and approaches $-\infty$ either side of $I = -1$ (since the $-\frac{\phi_a \alpha}{(I+1)^2}$ term dominates). It is also discontinuous at $I = 0$ and approaches $+\infty$ either side of $I = 0$ (since the $\frac{\phi_b \beta}{I^2}$ term dominates). It is continuous everywhere else.

As $I \to -\infty$, the $-\frac{\phi_b}{I}$ term in (18) dominates, and so the function asymptotes the x-axis from above. Since it approaches $-\infty$ as I rises to -1, and is continuous between $I = -\infty$ and $I = -1$, there must be one root between these two points. Similarly, since the function tends to $+\infty$ as I rises from just above -1 to just below 0, and is continuous between these two points, there must be a second root within this interval. As $I \to +\infty$, the $-\frac{\phi_b}{I}$ term in (18) again dominates, and so the function asymptotes the x-axis from below. Since the function tends to $+\infty$ as I approaches 0 from above, and is continuous between $I = 0$ and $I = +\infty$, there must be a third root $(\widetilde{I})$ between these two points. There can be no other positive roots, since there are two negative roots and three roots in total. The positive root is a local maximum, since the gradient is positive for $I < \widetilde{I}$ and negative for $I > \widetilde{I}$.

Let $F(I, \theta)$ denote the left-hand side of (43), where θ is a vector of parameters ϕ_a, ϕ_b, α, β. I^*_{soc} is defined by $F = 0$. Differentiating with respect to θ gives

$$\frac{\partial F}{\partial \theta} + \frac{\partial F}{\partial I}\frac{\partial I}{\partial \theta} = 0.$$

Since the gradient F is positive just below I^*_{soc} and negative just above I^*_{soc}, $\frac{\partial F}{\partial I}|_{I=I^*_{soc}} < 0$. Therefore, the sign of $\frac{\partial I}{\partial \theta}$ equals the sign of $\frac{\partial F}{\partial \theta}$, which in turn is the cross-partial derivative of total surplus (19) with respect to I and θ. This generates the comparative statics with respect to α, β, ϕ_a and ϕ_b.

Proof of Proposition 6. Equation (20) can be rewritten

$$2\beta \left(-\frac{\phi_b (I+1)}{\sqrt{I}} + \frac{\phi_a}{\sqrt{I}} + \frac{\phi_b (I+1)}{I^{3/2}} \right) - \frac{I-1}{I+1} \sigma_\eta \sigma_\varepsilon = 0.$$

Let

$$F(I) = 2\beta \left(-\frac{\phi_b (I+1)}{\sqrt{I}} + \frac{\phi_a}{\sqrt{I}} + \frac{\phi_b (I+1)}{I^{3/2}} \right) - \frac{I-1}{I+1} \sigma_\eta \sigma_\varepsilon.$$

We need only consider $I \geq 1$. Since $2\beta \left(-\frac{\phi_b(I+1)}{\sqrt{I}} + \frac{\phi_a}{\sqrt{I}} + \frac{\phi_b(I+1)}{I^{3/2}} \right)$ is decreasing in $I \in [1, \infty)$ and $\frac{I-1}{I+1}\sigma_\eta \sigma_\varepsilon$ is increasing in $I \in [1, \infty)$, $F(I)$ is decreasing in $[1, \infty)$. Then since $F(\infty) < 0$ and $F(1) > 0$, there exists a unique root of $F(I) = 0$ in $[1, \infty)$.

The comparative statics results follow from taking the cross-partial derivatives of the objective function. The cross-partial with respect to I and β is $\frac{\phi_a}{I(I+1)} - \frac{\phi_b}{I} + \frac{\phi_b}{I^2}$, which is positive from equation (20). The other cross-partial derivatives can be immediately signed.

Proof of Proposition 7. The only difference from the previous analysis is that in the action stage of the game, blockholder i now simultaneously chooses her action b_i and whether to become informed.

We proceed by backwards induction. Let J be the number of blockholders that acquire information. In the trading stage, uninformed blockholders cannot expect to make profits and thus do not trade in aggregate. Therefore, only the J informed blockholders trade and the equilibrium is similar to the one derived in Proposition 1.

Now in the action stage, the manager must choose an action a. Using the same arguments as in Proposition 3, the manager's optimal action is

$$a = \phi_a \alpha \left(\frac{J}{J+1} \right) - 1. \tag{44}$$

Blockholders must choose actions b_i and whether to become informed. These decisions can be taken independently since informed trading profits are independent of b_i (which is public), and the choice of b_i depends only on blockholder i's stake β/I. The optimal action of each blockholder is thus

$$b_i = \phi_b \beta \left(\frac{1}{I} \right)^2 - \frac{1}{I}. \tag{45}$$

From equation (6), if there are I informed blockholders, then each blockholder's trading profits are given by:

$$\frac{1}{\sqrt{I}(I+1)} \sigma_\eta \sigma_\epsilon.$$

A blockholder will acquire information if and only if her trading profits are higher than c. This gives the number J of blockholders that decide to become informed in equilibrium.

Proof of Proposition 8. Let n and $J(I)$ be as given in Proposition 7. Using the results of Proposition 3, expected firm value is

$$E[\widetilde{v}] = \phi_a \log \left[\phi_a \alpha \left(\frac{J(I)}{J(I)+1} \right) \right] + \phi_b \log \left[\phi_b \beta \left(\frac{1}{I} \right) \right]. \tag{46}$$

We wish to maximize the above expression with respect to I. Since $J(I) = n$ for $I \geq n$, it is never optimal to increase I beyond n since it reduces the second term in the firm value while keeping the first term constant. Therefore, $I^*_{costly} \leq n$. When $I \leq n$, $J(I) = I$ and the problem is the same as in Proposition 4. From (15) we obtain the desired result.

Proof of Proposition 9. The manager will not exert effort above the level for which

$$\phi_a \log (1+a) = \phi_b \log \left(1 + \textstyle\sum_i \widehat{b}_i \right),$$

i.e.

$$a = \exp\left(\frac{\phi_b}{\phi_a}\log\left(1+\sum_i \widehat{b}_i\right)\right) - 1.$$

This derives the optimal a as given in equation (28). Similarly, blockholder i will not exert effort above the level for which

$$\phi_b \log\left(1 + b_i + \sum_{j\neq i}\widehat{b}_j\right) = \phi_a \log\left(1+\widehat{a}\right),$$

i.e.

$$b_i = \exp\left(\frac{\phi_a}{\phi_b}\log\left(1+\widehat{a}\right)\right) - \sum_{j\neq i}\widehat{b}_j - 1.$$

A Nash equilibrium requires the following three conditions to hold:

$$\begin{aligned}\phi_b \log\left(1+Ib_i\right) = \phi_a \log\left(1+a\right).\\ a \leq \phi_a\alpha\left(\frac{I}{I+1}\right) - 1\\ b_i \leq \phi_b\beta\left(\frac{1}{I}\right)^2 - \frac{1}{I}.\end{aligned}$$

If the first condition was violated, then the party producing the higher output would gain by reducing effort. The two inequality conditions represent the maximum levels of effort that the manager and blockholders will exert, given the marginal cost of effort.

Out of the continuum of potential Nash equilibria, we seek the one that maximizes firm value. Since firm value is increasing in both a and b_i, it is clear that at least one incentive compatibility constraint will bind. If neither constraint binds, then all parties are exerting suboptimal effort. We could raise the effort levels of all parties while maintaining the equality condition and violating neither constraint.

We now show that, in fact, *both* constraints will bind. Consider the case where $b_i = \phi_b\beta\left(\frac{1}{I}\right)^2 - \frac{1}{I}$. (Starting with $a = \phi_a\alpha\left(\frac{I}{I+1}\right) - 1$ leads to the same result). Then we have

$$\begin{aligned}\phi_b \log\left[\phi_b\beta\left(\frac{1}{I}\right)\right] = \phi_a \log\left(1+a\right)\\ a = \exp\left(\frac{\phi_b}{\phi_a}\log\left[\phi_b\beta\left(\frac{1}{I}\right)\right]\right) - 1.\end{aligned}$$

Recall that we also require $a \leq \phi_a\alpha\left(\frac{I}{I+1}\right) - 1$. Hence firm value is optimized by solving:

$$\max_I \exp\left(\frac{\phi_b}{\phi_a}\log\left[\phi_b\beta\left(\frac{1}{I}\right)\right]\right) \text{ s.t. } \exp\left(\frac{\phi_b}{\phi_a}\log\left[\phi_b\beta\left(\frac{1}{I}\right)\right]\right) \leq \phi_a\alpha\left(\frac{I}{I+1}\right).$$

The constraint will bind, and so we obtain

$$\phi_a \log\left[\phi_a\alpha\left(\frac{I}{I+1}\right)\right] = \phi_b \log\left[\phi_b\beta\left(\frac{1}{I}\right)\right]. \tag{47}$$

The firm value optimum setting I to ensure all parties exert their "full" effort levels. The intuition is as follows. Consider a Nash equilibrium where the blockholders are exerting their full effort (i.e. $b_i = \phi_b \beta \left(\frac{1}{I}\right)^2 - \frac{1}{I}$), and the manager is not (i.e. $a < \phi_a \alpha \left(\frac{I}{I+1}\right) - 1$). b_i is thus constrained by I via the equation $b_i = \phi_b \beta \left(\frac{1}{I}\right)^2 - \frac{1}{I}$, and so firm value rises if I is reduced to relax this constraint and allow b_i to rise. Unlike in the core model, we do not have the side-effect that reducing I decreases a. I only determines the upper bound to a, not its level. Since $a < \phi_a \alpha \left(\frac{I}{I+1}\right) - 1$, the upper bound is not a constraint anyway. Rather than declining, a will rise to accompany the increase in b_i and ensure that $\phi_b \log(1 + Ib_i) = \phi_a \log(1 + a)$ still holds.

From equation (47), the optimal number of blockholders is determined implicitly by:

$$\frac{I^2}{I+1} = \frac{\phi_b \beta}{\phi_a \alpha} \exp(\phi_b - \phi_a) = Z.$$

Using the quadratic formula, the unique positive solution is

$$\underline{I} = \frac{Z + \sqrt{Z^2 + 4Z}}{2},$$

which is increasing in ϕ_b and β, and decreasing in ϕ_a and α.

Proof of Proposition 10. We now allow the non-negativity constraints to bind. Deriving $\widetilde{p}$ as in the main model and solving the manager's objective function, he will choose either $a = \phi_a \alpha \frac{I}{I+1} - 1$ or $a = 0$. If $\phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right] < \phi_b \log\left(1 + \sum_i \widehat{b}_i\right)$, exerting $a = \phi_a \alpha \frac{I}{I+1} - 1$ will have no effect on $\widetilde{p}$ and so the manager will choose $a = 0$. Even if $\phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right] \geq \phi_b \log\left(1 + \sum_i \widehat{b}_i\right)$, it is not automatic that the manager will exert effort. Exerting effort increases $\widetilde{p}$ not by $\frac{I}{I+1} \phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right]$, as in the core model, but by only

$$\frac{I}{I+1}\left(\phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right] - \phi_b \log\left(1 + \sum_i \widehat{b}_i\right)\right)$$

because blockholder effort "supports" firm value even if $a = 0$. Hence the manager chooses $a = \phi_a \alpha \frac{I}{I+1} - 1$ if and only if

$$\alpha \frac{I}{I+1}\left(\phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right] - \phi_b \log\left(1 + \sum_i \widehat{b}_i\right)\right) \geq a.$$

and so the optimal a is as given by (32). Blockholder i's effort level is derived similarly.

There are two candidates for a Nash equilibrium:

$$\begin{cases} a = 0, b_i = \phi_b \beta \left(\frac{1}{I}\right)^2 - \frac{1}{I} \\ a = \phi_a \alpha \frac{I}{I+1} - 1, b_i = 0 \end{cases}.$$

Firm value is thus either $\phi_a \log\left[\phi_a \alpha \frac{I}{I+1}\right]$ or $\phi_b \log\left[\phi_b \beta \frac{1}{I}\right]$. The former is monotonically increasing in I, and maximized at $\phi_a \log(\phi_a \alpha)$ for $I = \infty$. The latter is monotonically decreasing in I, and maximized at $\phi_b \log(\phi_b \beta)$ for $I = 1$. Thus I^* is as given in (34).

Proof of Proposition 11. Proceeding as in the main model, the actions are given by

$$a = \phi_a \alpha \left[1 - \frac{\omega}{I+1}\right] - 1 \tag{48}$$

and

$$b_i = \phi_b \beta \left[\frac{\zeta}{I(I+1)} + \frac{1-\zeta}{I^2}\right] - \frac{1}{I}. \tag{49}$$

Firm value is given by:

$$E[\widetilde{v}] = \phi_a \ln \left[\phi_a \alpha \left[1 - \frac{\omega}{I+1}\right]\right] + \phi_b \ln \left[\phi_b \beta \left[\frac{\zeta}{I+1} + \frac{1-\zeta}{I}\right]\right]. \tag{50}$$

The first-order condition is given by (36). Putting this under a common denominator yields

$$F(I, \omega, \zeta) = \frac{I(I+1-\zeta)\phi_a \omega - \phi_b \left[(I+1)^2 - \zeta(2I+1)\right](I+1-\omega)}{I(I+1-\omega)(I+1-\zeta)}.$$

It is a cubic, and has at most three roots. If $I \to \pm\infty$, the numerator becomes dominated by the term containing $(I+1)^2$ and so F tends to $\frac{-\phi_b (I+1)^2}{I(I+1-\zeta)}$. It thus asymptotes the x-axis from below. If $I \to 0$ or $I \to -(1-\zeta)$, then F tends to $\frac{-\phi_b\left[(I+1)^2 - \zeta(2I+1)\right]}{I(I+1-\zeta)}$. For I close to 0, we have $\frac{(I+1)^2-\zeta(2I+1)}{I(I+1-\zeta)} > 0$ and so the sign depends on $-\frac{\phi_b}{I}$. It is positive (negative) as I approaches 0 from below (above). For I close to $-(1-\zeta)$, we have $\frac{(I+1)^2-\zeta(2I+1)}{I} < 0$ and so the sign depends on $\frac{\phi_b}{I+1-\zeta}$. It is negative (positive) as I approaches $-(1-\zeta)$ from below (above). If $I \to -(1-\omega)$, then F tends to $\frac{\phi_a \omega}{I+1-\omega}$ and is negative (positive) as I approaches $-(1-\omega)$ from below (above).

To identify the roots, consider $-(1-\zeta) < -(1-\omega)$. (The same arguments apply for $-(1-\omega) < -(1-\zeta)$.) At $I = -\infty$, F asymptotes the x-axis from below, and declines until it reaches $-\infty$ when I is just below $-(1-\zeta)$, so there are no roots for $I < -(1-\zeta)$. When I is just above $-(1-\zeta)$, $F \to \infty$. It then decreases, crosses through zero and becomes $-\infty$ just below $-(1-\omega)$. There is one root for $-(1-\zeta) < I < -(1-\omega)$. $F \to \infty$ just above $I = -(1-\omega)$ and just below $I = 0$, so there are either 0 or 2 roots for $-(1-\omega) < I < 0$. Thus, there can be at most 2 roots for $I > 0$. $F \to -\infty$ when I is just above 0, and asymptotes the x-axis from below as $I \to \infty$. Therefore, F crosses the x-axis either 0 or 2 times for $I > 0$. If F has no roots, it is negative for all $I > 0$ and so the optimal number of blockholders is its minimum value of 1. If it has two roots greater than 1, the upper root I_u is the maximum since the derivative is positive below I_u and negative above I_u. As in the proof of Proposition 5, the cross-partials are sufficient to determine the sign of the comparative statics. The cross-partials with respect to ϕ_a and ϕ_b are immediate. For ω and ζ, we have:

$$\frac{\partial^2 E[v]}{\partial I \partial \omega} = \frac{\phi_a}{(I+1-\omega)^2} > 0$$

$$\frac{\partial^2 E[v]}{\partial I \partial \zeta} = \frac{\phi_b}{(I+1-\zeta)^2} > 0.$$

Proof of Proposition 14. Suppose the market maker uses a linear pricing rule of the form $p(y) = \mu + \lambda y$ and blockholders use a linear demand of the form $x_i(\nu) = \gamma(\tilde{\nu} - \mu)$. Then blockholder i maximizes:

$$E[(\tilde{v} - \mu - \lambda y)x_i \,|\, \tilde{\nu}_i = \nu] = \left(\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta^2}(\nu - \mu) - \lambda(I-1)\gamma\left(\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta^2}(\nu - \mu) \right) \right) x_i - \lambda x_i^2.$$

This maximization problem yields

$$x_i(\nu) = \frac{1}{2\lambda}\left[\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta^2}(\nu - \mu) - \lambda(I-1)\gamma\left(\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta^2}(\nu - \mu) \right) \right] \quad \forall i.$$

The strategies of the blockholders are symmetric and we thus have

$$x_i(\nu) = \frac{1}{2}\left(\frac{1}{\lambda} - (I-1)\gamma \right) \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta^2}(\nu - \mu) \quad \forall i.$$

which implies that

$$\gamma = \frac{\sigma_\eta^2}{((I+1)\sigma_\eta^2 + 2\sigma_\delta^2)\lambda}$$

The market maker takes the blockholders' strategies as given and sets

$$p(y) = E[\tilde{v}|y]. \tag{51}$$

Using the normality of $\tilde{v}$ and $\tilde{y}$ yields

$$\lambda = \frac{\sqrt{I(\sigma_\delta^2 + \sigma_\eta^2)}\sigma_\eta^2}{\sigma_\epsilon((I+1)\sigma_\eta^2 + 2\sigma_\delta^2)}$$

$$\mu = \phi_a \log(1+a) + \phi_b \log\left(1 + \textstyle\sum_i b_i\right).$$

From this we obtain:

$$x_i(\nu_i) = \frac{\sigma_\epsilon}{\sqrt{I(\sigma_\delta^2 + \sigma_\eta^2)}}\left(\nu_i - \phi_a \log(1+a) - \phi_b \log\left(1 + \textstyle\sum_i b_i\right)\right) \quad \forall i,$$

$$p(y) = \phi_a \log(1+a) + \phi_b \log\left(1 + \textstyle\sum_i b_i\right) + \frac{\sqrt{I(\sigma_\delta^2 + \sigma_\eta^2)}\sigma_\eta^2}{\sigma_\epsilon((I+1)\sigma_\eta^2 + 2\sigma_\delta^2)} y,$$

as required.

Proof of Proposition 12. Dropping terms that do not contain b_i, blockholder i's objective function (37) becomes

$$\max_{b_i} \left(\frac{\beta}{I} \right) \phi_b \log\left(1 + \textstyle\sum_i b_i\right) - b_i + \frac{1}{\sqrt{I}\,(I+1)} \frac{\sigma_\varepsilon}{\sigma_\eta} \left(\phi_a \log(1+a) + \phi_b \log\left(1 + \textstyle\sum_i b_i\right) - \mu\right)^2$$

Given the conjecture $b_i = \frac{\phi_b \beta}{I^2} - \frac{1}{I}$, we have

$$\mu = \phi_a \ln(1+a) + \phi_b \ln\left(\frac{\phi_b \beta}{I} \right)$$

and so the objective becomes

$$\max_{b_i} \left(\frac{\beta}{I}\right) \phi_b \log\left(1+\sum_i b_i\right) - b_i + \frac{1}{\sqrt{I}\,(I+1)} \frac{\sigma_\varepsilon}{\sigma_\eta} E\left[\left(\phi_b \log\left(1+\sum_i b_i\right) - \phi_b \log\left(\frac{\phi_b \beta}{I}\right) + \eta\right)^2\right]$$

with first-order condition

$$\frac{\phi_b \beta}{I\left(1+\sum_i b_i\right)} - 1 + \frac{2}{\sqrt{I}\,(I+1)} \frac{\sigma_\varepsilon}{\sigma_\eta} \left(\phi_b \log\left(1+\sum_i b_i\right) - \phi_b \log\left(\frac{\phi_b \beta}{I}\right)\right) \frac{\phi_b}{\left(1+\sum_i b_i\right)} = 0. \quad (52)$$

where

$$1+\sum_i b_i = \frac{1}{I} + \frac{I-1}{I^2} \phi_b \beta + b_i.$$

One solution is $b_i = \frac{\phi_b \beta}{I^2} - \frac{1}{I}$. The second-order condition is:

$$-\frac{\phi_b \beta}{I\left(1+\sum_i b_i\right)^2} + \frac{2}{\sqrt{I}\,(I+1)} \frac{\sigma_\varepsilon}{\sigma_\eta} \left(\begin{array}{c} -\frac{\phi_b}{\left(1+\sum_i b_i\right)^2}\left(\phi_b \ln\left(1+\sum_i b_i\right) - \phi_b \ln\left(\frac{\phi_b \beta}{I}\right)\right) \\ +\frac{\phi_b}{\left(1+\sum_i b_i\right)} \frac{\phi_b}{\left(1+\sum_i b_i\right)} \end{array} \right)$$

which has the same sign as

$$-\frac{\beta}{I} + \frac{2}{\sqrt{I}\,(I+1)} \frac{\sigma_\varepsilon}{\sigma_\eta} \phi_b \left(1 - \left(\ln\left(\frac{1}{I} + \frac{I-1}{I^2} \phi_b \beta + b_i\right) - \ln\left(\frac{\phi_b \beta}{I}\right)\right)\right).$$

To show that $b_i = \frac{\phi_b \beta}{I^2} - \frac{1}{I}$ is a global maximum, it is sufficient to show that the function is globally concave, i.e. is negative for all b_i. Since the second-order condition is decreasing in b_i, it is sufficient to show that it is negative when b_i is at its lowest possible value of 0. Then, it becomes

$$-\frac{\beta}{I} + \frac{2}{\sqrt{I}\,(I+1)} \frac{\sigma_\varepsilon}{\sigma_\eta} \phi_b \left(1 - \left(\ln\left(\frac{I-1}{I^2} \phi_b \beta + \frac{1}{I}\right) - \ln\left(\frac{1}{I} \phi_b \beta\right)\right)\right) < 0 \quad (53)$$

which is satisfied if

$$-\frac{\beta}{I} + \frac{2}{\sqrt{I}\,(I+1)} \frac{\sigma_\varepsilon}{\sigma_\eta} \phi_b \left(1 + \ln\left(\frac{1}{I} \phi_b \beta\right)\right) < 0.$$

Since $I \geq 1$, this is in turn satisfied if

$$-\beta + \frac{2\sqrt{I}}{(I+1)} \frac{\sigma_\varepsilon}{\sigma_\eta} \phi_b \left(1 + \ln\left(\phi_b \beta\right)\right) < 0.$$

Since $\frac{\sqrt{I}}{(I+1)}$ is decreasing in I, a sufficient condition is

$$\frac{\beta}{\phi_b \left(1 + \ln\left(\phi_b \beta\right)\right)} > \frac{\sigma_\varepsilon}{\sigma_\eta}, \quad (54)$$

i.e. (38).

The alternative sufficient condition is obtained without studying second-order conditions. First, observe that plugging $b_i = \infty$ into the objective function yields a value of $-\infty$, so the global maximum is either $b_i = 0$ or involves b_i satisfying the first-order condition (52). Defining

$$\begin{aligned}
A &= \frac{1}{I} + \frac{I-1}{I^2}\phi_b\beta, \\
B &= \frac{\phi_b\beta}{I}, \\
C &= \frac{\phi_b\beta}{I^2} - \frac{1}{I} = B - A, \\
K &= \frac{2}{\sqrt{I}\,(I+1)}\frac{\sigma_\varepsilon}{\sigma_\eta}\phi_b^2,
\end{aligned}$$

the first-order condition (52) can be rewritten:

$$\begin{aligned}
\frac{B}{A+b_i} - 1 + \frac{K}{A+b_i}\ln\left(\frac{A+b_i}{B}\right) &= 0 \\
C - b_i + K\ln\left(1 + \frac{b_i - C}{B}\right) &= 0. \qquad (55)
\end{aligned}$$

As considered above, $b_i = C$ is a solution to the first-order condition. If $b_i \neq C$, then the first-order condition can be rewritten as

$$-1 + \frac{K}{b_i - C}\ln\left(1 + \frac{b_i - C}{B}\right) = 0. \qquad (56)$$

Note that the function $\ln(1+x)/x$ is decreasing in x, and so $-1+\frac{K}{b_i-C}\ln\left(1+\frac{b_i-C}{B}\right)$ is decreasing in b_i. If $-1-\frac{K}{C}\ln\left(1-\frac{C}{B}\right) < 0$, then (56) has no solution. Then $b_i = C$ is the unique solution for (55). Also note that $-1-\frac{K}{C}\ln\left(1-\frac{C}{B}\right) < 0$ implies that $\frac{\partial f(b_i)}{\partial b_i}|_{b_i=0} > 0$, and so $b_i = 0$ cannot be the global maximum. Hence, if $-1-\frac{K}{C}\ln\left(1-\frac{C}{B}\right) < 0$, the global maximum must be $b_i = C$. This sufficient condition implies $-C < K\ln\left(1-\frac{C}{B}\right)$, which eventually yields:

$$1 > \frac{2\sqrt{I}}{(I+1)}\frac{\sigma_\varepsilon}{\sigma_\eta}\frac{\phi_b}{\beta}\ln\left(1 - \frac{1}{I} + \frac{1}{\phi_b\beta}\right)\frac{1}{\frac{1}{\phi_b\beta} - \frac{1}{I}} \qquad (57)$$

Since $\frac{\ln(1+x)}{x}$ is decreasing in x, the function

$$\ln\left(1 - \frac{1}{I} + \frac{1}{\phi_b\beta}\right)\frac{1}{\frac{1}{\phi_b\beta} - \frac{1}{I}}$$

is decreasing in I. Also note that the function

$$\frac{2\sqrt{I}}{(I+1)}\frac{\sigma_\varepsilon}{\sigma_\eta}\frac{\phi_b}{\beta}$$

is decreasing in I for $I \geq 1$. Thus

$$\frac{2\sqrt{I}}{(I+1)}\frac{\sigma_\varepsilon}{\sigma_\eta}\frac{\phi_b}{\beta}\ln\left(1 - \frac{1}{I} + \frac{1}{\phi_b\beta}\right)\frac{1}{\frac{1}{\phi_b\beta} - \frac{1}{I}}$$

is decreasing in I for $I \geq 1$. Hence a sufficient condition for (57) to hold is that

$$1 > \frac{\sigma_\varepsilon}{\sigma_\eta}\frac{\phi_b}{\beta}\ln\left(\frac{1}{\phi_b\beta}\right)\frac{1}{\frac{1}{\phi_b\beta}-1}$$

$$\frac{\sigma_\varepsilon}{\sigma_\eta} < \frac{\phi_b\beta - 1}{\phi_b^2\ln\left(\phi_b\beta\right)}. \tag{58}$$

Note that sufficient condition (54) or (58) may be weaker, depending on parameter values, so we provide them both in the Proposition.

Proof of Proposition 13. Suppose the conjectured equilibrium actions are $\hat{b}_i$ such that $\sum_i \hat{b}_i \neq \phi_b\beta/I - 1$. Can $b_i = \hat{b}_i$ be an optimal response of blockholder i?

We first analyze the case $\sum_i \hat{b}_i > 0$. In this case, there exists i such that $\hat{b}_i > 0$. Blockholder i's objective function (37) becomes

$$\max_{b_i}\left(\frac{\beta}{I}\right)\phi_b\log\left(1+b_i+\sum_{j\neq i}\hat{b}_j\right)-b_i + \frac{1}{\sqrt{I}\left(I+1\right)}\frac{\sigma_\varepsilon}{\sigma_\eta}E\left[\left(\phi_a\log\left(1+a\right)+\phi_b\log\left(1+b_i+\sum_{j\neq i}\hat{b}_j\right)+\eta-\mu\right)^2\right] \tag{59}$$

with

$$\mu = \phi_a\ln\left(1+a\right)+\phi_b\ln\left(1+\sum_i\hat{b}_i\right).$$

The first-order condition is

$$0 = \left(\frac{\phi_b\beta}{I\left(1+b_i+\sum_{j\neq i}\hat{b}_j\right)}-1\right) + \frac{2}{\sqrt{I}\left(I+1\right)}\frac{\sigma_\varepsilon}{\sigma_\eta}\left(\phi_a\log\left(1+a\right)+\phi_b\log\left(1+b_i+\sum_{j\neq i}\hat{b}_j\right)-\mu\right)\frac{\phi_b}{1+b_i+\sum_{j\neq i}\hat{b}_j} \tag{60}$$

When $b_i = \hat{b}_i$, the second term on the right-hand side of (60) is equal to zero. However, the first term on the right-hand side of (60) is different from zero since $\sum_i \hat{b}_i \neq \phi_b\beta/I - 1$. The first-order condition cannot be satisfied and thus we cannot have $\sum_i \hat{b}_i \neq \phi_b\beta/I - 1$ and $\sum_i \hat{b}_i > 0$ at the same time.

The only other possible symmetric equilibrium in pure strategies involves $\sum_i \hat{b}_i = 0$, which implies $\hat{b}_i = 0$ for all i. For this to be an equilibrium, we would need the right-hand side of (60) to be negative at $b_i = \hat{b}_i = 0$. Since we have $Ib_i = \frac{\phi_b\beta}{I} - 1 > 0$, we have $\phi_b\beta > I$ and so this cannot be the case.

Sufficient Conditions for $a > 0$ and $b_i > 0$. From (7), we have

$$a = \phi_a\alpha\left(\frac{I}{I+1}\right)-1.$$

Since $I \geq 1$, $\frac{I}{I+1} \geq \frac{1}{2}$ and so a sufficient condition for $a \geq 0$ is

$$\phi_a \alpha \geq 2.$$

The sufficient conditions for $b_i \geq 0$ depend on the variant of the model we are considering. We start with the analysis of the firm value optimum in the core model, Proposition 4, which yielded $I^* = \frac{\phi_a - \phi_b}{\phi_b}$. From (9), we have

$$b_i = \phi_b \beta \left(\frac{1}{I}\right)^2 - \frac{1}{I},$$

and so $b_i = 0$ at $I = \phi_b \beta$. Thus, $\frac{\phi_a - \phi_b}{\phi_b} < \phi_b \beta$ is sufficient to guarantee that $b_i > 0$ at $I = \frac{\phi_a - \phi_b}{\phi_b}$. However, in the presence of non-negativity constraints, firm value (16) is no longer a concave function of I and so an additional condition is necessary to guarantee that $I = \frac{\phi_a - \phi_b}{\phi_b}$ is a global, rather than only local, optimum. While increasing I above $\frac{\phi_a - \phi_b}{\phi_b}$ initially reduces firm value (because the detrimental effect on intervention outweighs the beneficial effect on trading), once I hits $\phi_b \beta$, intervention is already at its minimum level of zero. Thus, further increases in I have no negative effect on intervention, but continue to improve trading, and thus unambiguously boost firm value. The global optimum may be either $I = \frac{\phi_a - \phi_b}{\phi_b}$ or $I = \infty$. For $I = \frac{\phi_a - \phi_b}{\phi_b}$, we have

$$\begin{aligned} E[v] &= \phi_a \log\left(\phi_a \alpha \frac{I}{I+1}\right) + \phi_b \log\left(\phi_b \beta \frac{1}{I}\right) \\ &= (\phi_a - \phi_b)\log(\phi_a - \phi_b) + \phi_a \log \alpha + \phi_b \log\left(\phi_b^2 \beta\right), \end{aligned}$$

and for $I = \infty$, we have

$$E[v] = \phi_a \log(\phi_a \alpha).$$

Thus,

$$(\phi_a - \phi_b)\log(\phi_a - \phi_b) + \phi_b \log\left(\phi_b^2 \beta\right) > \phi_a \log \phi_a$$

is sufficient to guarantee that $I^* = \frac{\phi_a - \phi_b}{\phi_b}$ in the presence of non-negativity constraints.

Similar analysis yields sufficient conditions for the analysis of the social optimum, Proposition 5, as

$$I^*_{soc} < \phi_b \beta,$$

$$\phi_a \log\left[\phi_a \alpha \left(\frac{I^*_{soc}}{I^*_{soc}+1}\right)\right] + \phi_b \log\left[\phi_b \beta \left(\frac{1}{I^*_{soc}}\right)\right] - \phi_a \alpha \left(\frac{I^*_{soc}}{I^*_{soc}+1}\right) - \phi_b \beta \frac{1}{I^*_{soc}} > \phi_a \log(\phi_a \alpha) - \phi_a \alpha,$$

where I^*_{soc} is defined by (18). The sufficient conditions for the analysis of the private optimum, Proposition 6, are

$$I^*_{priv} < \phi_b \beta,$$

$$\beta \left\{\phi_a \log\left[\phi_a \alpha \left(\frac{I^*_{priv}}{I^*_{priv}+1}\right)\right] + \phi_b \log\left[\phi_b \beta \frac{1}{I^*_{priv}}\right]\right\} - \phi_b \beta \frac{1}{I^*_{priv}} + \frac{\sqrt{I^*_{priv}}}{I^*_{priv}+1} \sigma_\eta \sigma_\varepsilon > \beta \phi_a \log(\phi_a \alpha),$$

where I^*_{priv} is defined by (20). The sufficient conditions for the model with the general objective function, Proposition 11, are

$$I^*_{gen} < \phi_b \beta$$

$$\phi_a \ln \left[\phi_a \alpha \left[1 - \frac{\omega}{I^*_{gen}+1}\right]\right] + \phi_b \ln \left[\phi_b \beta \left[\frac{\zeta}{I^*_{gen}+1} + \frac{1-\zeta}{I^*_{gen}}\right]\right] > \phi_a \ln (\phi_a \alpha),$$

where I^*_{gen} is defined by (36). The sufficient conditions for the model with imperfect signals, Proposition 17, are

$$\frac{(\phi_a - \phi_b)(2\sigma_\delta^2 + \sigma_\eta^2)}{\phi_b \sigma_\eta^2} > \phi_b \beta$$

$$(\phi_a - \phi_b) \log (\phi_a - \phi_b) + \phi_b \log \left[\frac{{\phi_b}^2 \beta \sigma_\eta^2}{(2\sigma_\delta^2 + \sigma_\eta^2)}\right] > \phi_a \log \phi_a.$$

For the analysis of perfect positive complementarities (Proposition 9), it is automatic that the optimum cannot involve a non-negativity constraint binding, since firm value is zero if a or $\sum_i b_i$ is zero. For perfect negative complementarities (Proposition 10), we do allow for a or $\sum_i b_i$ to be zero, and indeed the optimum involves one of these terms being zero.

B Imperfect Signals

The key mechanism through which we achieve the optimality of a multiple blockholder structure is the positive effect of blockholder numbers on price informativeness. It is therefore important to verify the robustness of this result to other specifications of the information structure. In the core model, blockholders have perfect information about firm value $\tilde{v}$; Appendix C shows that the results hold with imperfect signals when blockholders receive the same signal. Here, we consider the case in which blockholders observe imperfect and uncorrelated signals.

Each blockholder observes a signal $\tilde{\nu}_i = \tilde{v} + \tilde{\delta}_i$ where $\tilde{\delta}_i$, $i \in I$ are independent and $\tilde{\delta}_i \sim N(0, \sigma_\delta^2)$. Propositions 14-17 are the analogs of Propositions 1-4 in the core model.

Proposition 14. ***(Trading Equilibrium)*** *The unique linear equilibrium of the trading stage is symmetric and has the form:*

$$x_i(\tilde{\nu}_i) = \gamma \left(\tilde{\nu}_i - \phi_a \log (1+a) - \phi_b \log (1 + \textstyle\sum_i b_i)\right) \quad \forall i \tag{61}$$

$$p(\tilde{y}) = \phi_a \log (1+a) + \phi_b \log (1 + \textstyle\sum_i b_i) + \lambda \tilde{y}, \tag{62}$$

where

$$\lambda = \frac{\sqrt{I(\sigma_\delta^2 + \sigma_\eta^2)}\sigma_\eta^2}{\sigma_\epsilon((I+1)\sigma_\eta^2 + 2\sigma_\delta^2)} \tag{63}$$

$$\gamma = \frac{\sigma_\epsilon}{\sqrt{I(\sigma_\delta^2 + \sigma_\eta^2)}}, \tag{64}$$

and a and b_i are the market maker's and blockholders' conjectures regarding the actions.

Proposition 15. ***(Price Informativeness)*** *Price informativeness is equal to*

$$\frac{I\sigma_\eta^2}{(I+1)\sigma_\eta^2 + 2\sigma_\delta^2}.$$

Proposition 16. ***(Optimal Actions)*** *The manager's optimal action is*

$$a = \phi_a \alpha \left(\frac{I\sigma_\eta^2}{(I+1)\sigma_\eta^2 + 2\sigma_\delta^2} \right) - 1 \tag{65}$$

and the optimal action of each blockholder is

$$b_i = \phi_b \beta \left(\frac{1}{I} \right)^2 - \frac{1}{I}. \tag{66}$$

Proposition 17. ***(Firm Value Optimum)*** *The optimal number I^* of blockholders maximizes:*

$$E[\widetilde{v}] = \phi_a \log \left[\phi_a \alpha \left(\frac{I\sigma_\eta^2}{(I+1)\sigma_\eta^2 + 2\sigma_\delta^2} \right) \right] + \phi_b \log \left[\phi_b \beta \left(\frac{1}{I} \right) \right]. \tag{67}$$

Solving the maximization problem, we obtain:

$$I^* = \frac{(\phi_a - \phi_b)(2\sigma_\delta^2 + \sigma_\eta^2)}{\phi_b \sigma_\eta^2}. \tag{68}$$

The number of blockholders has exactly the same effects as in the core model. An increase in I raises price informativeness (Proposition 15) and thus managerial effort (Proposition 16), but reduces blockholder effort. Therefore, I^* remains increasing in ϕ_a and decreasing in ϕ_b (Proposition 17). An additional result in the case of imperfect signals is that I^* is also increasing in the noise in the blockholders' signals σ_δ^2 and decreasing in the variance of firm value σ_η^2. Proposition 15 shows that, if σ_η^2 is high, price informativeness is already high under a single blockholder, and so there is less scope to increase it further through augmenting I. The opposite intuition applies to the effect of σ_δ.

The model can also be extended to multiple trading rounds and long-lived private information. Since these extensions have been undertaken in the microstructure literature (albeit without linking price informativeness to manager actions), we can use these prior studies to establish the robustness of our results. Holden and Subrahmanyam (1992) and Foster and Viswanathan (1993) consider the effect of competition among identically informed investors with long-lived private information. As in our model, they find that price discovery is accelerated when compared to Kyle's monopolistic case. Foster and Viswanathan (1996) extend the analysis to the case of heterogeneously informed investors and show that the degree of competition depends on the correlation structure of investors' signals. In particular, competition is more intense when the correlation between initial signals is high.

Back, Cao, and Willard (2000) extend the Kyle model to continuous time and a general correlation structure of investors' signals. They show that price informativeness is again higher under multiple informed traders for some fixed initial period, after which the relationship reverses. This fixed initial period is typically a very long time, and only ends close to the public announcement date. Thus, price informativeness is higher under multiple informed traders for all but the very end of the trading period. It is the initial period that is relevant for our setting: the microfoundations for the manager's stock price concerns discussed in Section 4.3 show that the stock price the manager cares about is a long time before the date when fundamental value is publicly released. For example, the manager can be fired (for a low stock price), headhunted (for a high stock price), sell his own shares or raise equity within a few months. By contrast, the recent corporate scandals and financial crisis highlight that it may take several years for fundamental value to become known.

As discussed in more detail in Section 5, empirical evidence also supports the robustness of our model. In the real world, blockholders have heterogenous signals and there are multiple trading periods. Boehmer and Kelley (2009) and Gallagher, Gardner and Swan (2010) find that competition among blockholders increases price efficiency.

C Precision of Information Varies with I

In the core model, all blockholders observe the value of the firm perfectly. We now allow for blockholders to receive the same noisy signal, the precision of which is increasing in each blockholder's stake (β/I) and thus decreasing in the number of blockholders I. Blockholders now observe a signal $\tilde{\nu} = \tilde{v} + \tilde{\delta}$ where $\tilde{\delta} \sim N\left(0, \sigma_\delta^2\left(I\right)\right)$. We show that the results of the core model are unchanged as long as signal precision does not decline too rapidly with I.[24]

Proposition 18. ***(Trading Equilibrium)*** *The unique linear equilibrium of the trading stage is symmetric and has the form:*

$$x_i(\tilde{\nu}) = \gamma\left(\tilde{\nu} - \phi_a \log\left(1+a\right) - \phi_b \log\left(1 + \textstyle\sum_i b_i\right)\right) \quad \forall i \tag{69}$$

$$p(\tilde{y}) = \phi_a \log\left(1+a\right) + \phi_b \log\left(1 + \textstyle\sum_i b_i\right) + \lambda\tilde{y}, \tag{70}$$

where

$$\lambda = \frac{\sqrt{I}}{I+1}\frac{\sigma_\eta^2}{\sigma_\epsilon(\sqrt{\sigma_\eta^2 + \sigma_\delta(I)^2})} \tag{71}$$

$$\gamma = \frac{1}{\sqrt{I}}\frac{\sigma_\varepsilon}{\sqrt{\sigma_\eta^2 + \sigma_\delta(I)^2}}, \tag{72}$$

and a and b_i are the market maker's and blockholders' conjectures regarding the actions.

Proof If the market maker uses a linear pricing rule of the form $p(y) = \mu + \lambda y$, blockholder i maximizes:

$$E[(\tilde{v} - \mu - \lambda y)x_i \,|\, \tilde{\nu} = \nu] = \left(\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta(I)^2}\nu - \mu - \lambda\sum_{j\neq i} x_j\right) x_i - \lambda x_i^2.$$

[24] Appendix B considers noisy and uncorrelated signals. Here, blockholders receive the same signal. This represents the toughest case for our model, since it means that the amount of information in the economy declines as I rises – there is a single signal which becomes less precise.

This maximization problem yields

$$x_i(\nu) = \frac{1}{\lambda}\left[\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta(I)^2}\nu - \mu - \lambda \sum_j x_j(\nu)\right] \quad \forall i.$$

The strategies of the blockholders are symmetric and we thus have

$$x_i(\nu) = \frac{\sigma_\eta^2}{(I+1)\lambda(\sigma_\eta^2 + \sigma_\delta(I)^2)}(\nu - \mu) \quad \forall i.$$

Total order flow is thus given by

$$y = \frac{I}{(I+1)\,\lambda}\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta(I)^2}\,(v - \mu) + \varepsilon. \tag{73}$$

The market maker takes the blockholders' strategies as given and sets

$$p(y) = E[\tilde{v}|y]. \tag{74}$$

Using the normality of $\tilde{v}$ and $\tilde{y}$ yields

$$\lambda = \frac{\sqrt{I}}{I+1}\frac{\sigma_\eta^2}{\sigma_\epsilon(\sqrt{\sigma_\eta^2 + \sigma_\delta(I)^2})}$$

$$\mu = \phi_a \log(1+a) + \phi_b \log\left(1 + \textstyle\sum_i b_i\right).$$

From this we obtain:

$$x_i(\nu) = \frac{\sigma_\epsilon}{\sqrt{I}}\frac{1}{\sqrt{\sigma_\eta^2 + \sigma_\delta(I)^2}}(\nu - \phi_a \log(1+a) - \phi_b\left(1 + \textstyle\sum_i b_i\right)) \quad \forall i,$$

$$p(y) = \phi_a \log(1+a) + \phi_b\left(1 + \textstyle\sum_i b_i\right) + \frac{\sqrt{I}}{I+1}\frac{\sigma_\eta^2}{\sigma_\epsilon(\sqrt{\sigma_\eta^2 + \sigma_\delta(I)^2})}y,$$

as required. ■

The next proposition calculates price informativeness.

Proposition 19. ***(Price Informativeness)*** *Price informativeness is equal to*

$$\frac{I}{I+1}\frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta(I)^2}$$

Proof The result follows from $p\,(y) = \mu + \lambda y$ and equation (73). ■

It is easy to see that if $\sigma_\delta(I)$ does not increase too quickly, then price informativeness is increasing in I. As in the core model, when I increases, blockholders trade more competitively and impound more information into prices. This outweighs the fact that there is less information in the economy and each blockholder has less precise information. Also as in the core model, liquidity σ_ε has no effect on price informativeness.

We now solve for the actions of the manager and the blockholders in the first stage.

Proposition 20. ***(Optimal Actions)*** *The manager's optimal action is*

$$a = \phi_a \alpha \left(\frac{I}{I+1} \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta(I)^2} \right) - 1 \tag{75}$$

and the optimal action of each blockholder is

$$b_i = \phi_b \beta \left(\frac{1}{I} \right)^2 - \frac{1}{I}. \tag{76}$$

Proof The manager maximizes the market value of his shares, less the cost of effort:

$$E\left[\alpha \widetilde{p} - a\right]. \tag{77}$$

When setting the price $\widetilde{p}$, the market maker uses his conjecture for the manager's action a. Therefore, the manager's actual action affects the price only through its influence on $\tilde{v}$, and thus blockholders' order flow. The manager's first-order condition is given by:

$$\alpha \left(\mathrm{E}\left[\frac{d\tilde{p}}{d\tilde{v}} \right] \right) \left(\frac{\phi_a}{1+a} \right) - 1 = 0. \tag{78}$$

From Proposition 19, we obtain (75). The action of each blockholder is the same as in the paper. ■

If $\sigma_\delta(I)$ does not increase too quickly, the number of blockholders has a positive impact on managerial effort a. The mechanism is the same as in the core model. An increase in the number of blockholders makes prices more informative, increasing the reward to the manager for exerting effort. As in the core model, increasing the number of blockholders always has a negative impact on blockholders effort b_i.

The optimal number I of blockholders maximizes:

$$E[\widetilde{v}] = \phi_a \log \left[\phi_a \alpha \left(\frac{I}{I+1} \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_\delta(I)^2} \right) \right] + \phi_b \log \left[\phi_b \beta \left(\frac{1}{I} \right) \right]. \tag{79}$$

It is easy to see that the optimal number of blockholders is strictly higher than 1 if $\sigma_\delta(I)$ does not increase too quickly. The intuition is similar to the core model. On one hand, an increase in I exacerbates the free-rider problem and hinders intervention. On the other hand, an increase in I can raise price informativeness and thus managerial effort. In this extension, there is an additional negative effect of raising I, which is that each blockholder becomes less informed. The optimal number of blockholders is thus lower than in the core model.

D Measures of Price Informativeness

This section proves that our measure or price informativeness, $\mathrm{E}\left[\frac{d\tilde{p}}{d\tilde{v}}\right]$, is equivalent to the measure commonly used in the microstructure literature, $(\mathrm{Var}(\tilde{v}) - \mathrm{Var}(\tilde{v}|\tilde{p}))\,/\,\mathrm{Var}(\tilde{v})$.

Using the formula for the conditional variance of a bivariate normal distribution

$$\mathrm{Var}(\tilde{v}|\tilde{p}) = (1 - \mathrm{Corr}(\tilde{v}, \tilde{p})^2)\,\mathrm{Var}(\tilde{v}),$$

we have

$$(\mathrm{Var}(\tilde{v}) - \mathrm{Var}(\tilde{v}|\tilde{p}))\,/\,\mathrm{Var}(\tilde{v}) = \mathrm{Corr}(\tilde{v},\tilde{p})^2. \tag{80}$$

Since, in equilibrium, the price is a linear function of $\tilde{v}$ and $\tilde{\epsilon}$,

$$\mathrm{E}\left[\frac{d\tilde{p}}{d\tilde{v}}\right] = \frac{\mathrm{Cov}(\tilde{v},\tilde{p})}{\mathrm{Var}(\tilde{v})}.$$

From the law of iterated expectations and (42),

$$\mathrm{Var}(\tilde{p}) = \mathrm{Cov}(\tilde{v},\tilde{p}).$$

Therefore,

$$\mathrm{Corr}(\tilde{v},\tilde{p})^2 = \mathrm{E}\left[\frac{d\tilde{p}}{d\tilde{v}}\right]. \tag{81}$$

Combining (80) and (81) shows that

$$\mathrm{E}\left[\frac{d\tilde{p}}{d\tilde{v}}\right] = (\mathrm{Var}(\tilde{v}) - \mathrm{Var}(\tilde{v}|\tilde{p}))\,/\,\mathrm{Var}(\tilde{v}).$$

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