

# Inside Debt\*

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**Abstract.** Existing theories advocate the exclusive use of equity-like instruments in executive compensation. However, recent empirical studies document the prevalence of debt-like instruments such as pensions. This paper justifies the use of debt as efficient compensation. Inside debt is a superior solution to the agency costs of debt than the solvency-contingent bonuses and salaries proposed by prior literature, since its payoff depends not only on the incidence of bankruptcy but also firm value in bankruptcy. Contrary to intuition, granting the manager equal proportions of debt and equity is typically inefficient. In most cases, an equity bias is desired to induce effort. However, if effort is productive in increasing liquidation value, or if bankruptcy is likely, a debt bias can improve effort as well as alleviate the agency costs of debt. The model generates a number of empirical predictions consistent with recent evidence.

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## 1. Introduction

Shareholders ultimately bear the agency costs suffered by other stakeholders (Jensen and Meckling, 1976). Therefore, it appears intuitive that they should pay the manager according to firm value, rather than equity value alone. In particular, Jensen and Meckling speculated that granting the manager equal proportions of debt and equity might attenuate the stockholder-bondholder conflicts that arise when the manager is purely equity-aligned. However, this idea of compensating the manager with “inside debt”<sup>1</sup> has not since been pursued further. Instead, the intervening three decades of compensation theories have focused on justifying equity-like

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<sup>1</sup> This paper defines “inside” debt as debt (or any security with payoffs very similar to debt) held by the manager. It contrasts with outside debt, which is held by external investors.

instruments, such as stock and options. In particular, a number of models suggest that bonuses for avoiding bankruptcy, salaries or managerial reputation are adequate remedies to the agency costs of debt, leaving no role for inside debt in efficient compensation (see, e.g., Hirshleifer and Thakor, 1992; Brander and Poitevin, 1992; John and John, 1993). However, the substantial bondholder losses in the recent financial crisis suggest that the agency costs of debt are not fully solved.

Theorists' focus on rationalizing equity pay has likely been driven by the long-standing belief that, empirically, executives do not hold debt (see, e.g., the survey of Murphy, 1999). Accordingly, Dewatripont and Tirole (1994) seek to answer the question "why are managers' monetary incentives . . . traditionally correlated with the value of equity rather than the value of debt?" However, recent empirical studies (Bebchuk and Jackson, 2005; Sundaram and Yermack, 2007; Gerakos, 2007; Wei and Yermack, 2009) find that U.S. CEOs hold substantial defined benefit pensions. These are unsecured, unfunded obligations which, in nearly all cases, have equal priority with other creditors in bankruptcy and thus constitute inside debt.<sup>2</sup> Researchers have also noted the common use of deferred compensation, another form of inside debt, although systematic studies have so far been limited by data availability.<sup>3</sup>

Inside debt is therefore widespread. Such compensation contrasts with existing theories, which do not advocate debt but instead the exclusive use of equity-like compensation. Indeed, Sundaram and Yermack note the lack of a theoretical framework for their results: "the possibility of using debt instruments for management compensation has received little attention. . . . A top priority would appear to be the development of theory that illustrates conditions under which debt-based compensation . . . represent[s] the solution to an optimal contracting problem." Does the absence of a theoretical justification mean that inside debt constitutes rent extraction, as argued by Bebchuk and Jackson?<sup>4</sup> Or can it be part of efficient compensation, and if so, under what conditions? Should the manager's debt-equity ratio equal the firm's, so that he is aligned with firm value as Jensen and Meckling hypothesized? What factors affect the optimal level of inside debt?

These questions are the focus of this paper. We start with a model in which the manager makes a project selection decision; the optimal project depends on a

<sup>2</sup> See Section 4 for further discussion of the priority of pensions in bankruptcy.

<sup>3</sup> Despite limited data, anecdotal evidence suggests that such compensation may be substantial. For example, Roberto Goizueta, the former CEO of Coca-Cola, had over \$1 billion in deferred compensation when he died. Wei and Yermack (2009) consider total inside debt holdings (pensions plus deferred compensation).

<sup>4</sup> An alternative view is that pensions are tax motivated. Bebchuk and Jackson (2005) and Gerakos (2007) provide a number of arguments against this explanation, most notably that executive pensions enjoy a different tax status from employee pensions. This paper takes a neutral stance: if pensions are indeed tax-motivated, the model investigates whether the tax system encourages firms to adopt otherwise inefficient compensation schemes.

signal privately observed by the manager after contracting. We consider a set of standard securities: debt, equity and a fixed bonus that pays off only in solvency, and initially assume that the manager holds an exogenous equity stake to create risk-shifting incentives. We demonstrate that inside debt is a superior remedy to the agency costs of debt than the bonuses advocated by prior research. Bonuses are effective in encouraging the manager to avoid bankruptcy, since they are only received in solvency. However, creditors are concerned with not only the probability of default, but also recovery values in default. Optimal contracts should therefore depend on the value of assets in bankruptcy, as well as the occurrence of bankruptcy. This is the critical difference between inside debt and bonuses: inside debt yields a positive payoff in bankruptcy, proportional to the liquidation value. Thus it renders the manager sensitive to the firm's value in bankruptcy, and not just the incidence of bankruptcy—exactly as desired by creditors. By contrast, bonuses have zero bankruptcy payoffs, regardless of the liquidation value, and so represent binary options rather than debt.

This difference in payoffs is important. Even in situations where bonuses can attenuate risk-shifting, inside debt can be a cheaper solution since its sensitivity to liquidation values renders it a more powerful instrument. Moreover, in some settings, bonuses *aggravate* risk-shifting owing to their binary nature. Since they only pay off in solvency, the manager may inefficiently sacrifice liquidation value to gamble for solvency. The same issues apply to other instruments which have zero payoff in bankruptcy regardless of the liquidation value, e.g. salary (if it is junior to creditors) or reputation (under Hirshleifer and Thakor's (1992) assumption that the labor market can only assess the incidence rather than severity of bankruptcy.) For brevity, we refer to all of these instruments as "bonuses." These alternative measures were shown to be adequate under specific frameworks in which only sensitivity to the incidence of bankruptcy matters, such as where solvency can be guaranteed (John and John, 1993), or liquidation value is always zero (Hirshleifer and Thakor). In the more general setup of this paper, the manager can affect the liquidation value and so his compensation should be sensitive to it.

We then extend the model to incorporate an effort decision, which allows us to endogenize the manager's equity stake. This analysis extends previous models which focus on the agency costs of debt (project selection) and do not incorporate the agency costs of equity (effort). This is a necessary extension, since in the absence of a shirking problem, risk-shifting can be trivially solved by removing the manager's equity and giving him a flat salary; here, equity compensation is optimal to induce effort. The compensation scheme typically does not involve pure equity compensation (as advocated by some existing research) nor giving the manager debt and equity in equal proportions (as intuition might suggest, and as Jensen and Meckling hypothesized). In the most common case, equity is more effective than debt in inducing effort and so an equity bias is desired, where the manager's

percentage equity stake exceeds his percentage debt holding. Even though an equity bias leads to occasional risk-shifting, this is compensated for by greater effort. The optimal debt-equity mix depends on the relative importance of these two agency problems—the ratio of debt to equity is increasing in leverage, the probability of bankruptcy and the manager’s impact on liquidation value, but decreasing in growth opportunities (i.e. the effect of effort on solvency value). However, a debt bias, where the manager’s percentage debt stake exceeds his percentage equity holding, may be optimal if effort has a high expected payoff in bankruptcy, either because bankruptcy is likely, or because effort is particularly productive in enhancing liquidation value. In contrast to the “agency costs of equity” nomenclature, suboptimal effort may result from insufficient inside debt, rather than equity. Indeed, a debt bias is found by Sundaram and Yermack (2007) in 13% of cases.

Finally, we relate the model’s empirical implications to recent findings. Most notably, inside debt compensation is widespread, whereas solvency-contingent bonuses have not yet been documented. Also as predicted, pensions are increasing in firm leverage (Sundaram and Yermack), decreasing in growth opportunities (Gerakos, 2007) and associated with lower risk-taking, as measured by the firm’s “distance to default” (Sundaram and Yermack) or credit rating (Gerakos). Wei and Yermack (2009) find that disclosures of large inside debt holdings lead to an increase in bond prices and a fall in equity prices. In addition, the model provides a theoretical framework underpinning recent normative proposals to reform executive pay by compensating the manager with debt as well as equity, to help prevent the significant bondholder losses that manifested in the recent financial crisis (see, e.g., Bebchuk and Spamann, 2009.)

Jensen and Meckling (1976) were the first to theorize the agency costs of debt. They include a brief verbal section wondering why inside debt (awarded in the same proportion as inside equity) is not used as a solution, but are “unable to incorporate this dimension formally into our analysis in a satisfactory way.” They speculate that the manager’s salary is a sufficient mechanism and thus have no role for inside debt. This paper shows that salaries are problematic given their insensitivity to liquidation value, and that equal proportions of debt and equity are generally suboptimal.

John and John (1993), Brander and Poitevin (1992) and Hirshleifer and Thakor (1992) also demonstrate that the agency costs of debt can be alleviated through certain compensation instruments. Since their goal is to show the effectiveness, rather than optimality, of their proposed solutions, they do not consider whether alternative mechanisms, such as inside debt, would be superior. A second distinction is that this paper incorporates the agency costs of equity as well as of debt. This provides an endogenous justification for the equity compensation that is the cause of asset substitution and allows analysis of the trade-off between effort and project selection, thus leading to empirical predictions on the optimal ratio of debt to

equity.<sup>5</sup> Hirshleifer and Thakor (1992) show that managerial reputation (assumed to be zero in all bankruptcy states) can deter risk-shifting. In their model, liquidation value is always zero and so only sensitivity to the probability of bankruptcy matters. In the more general setup of this paper, the manager can affect the liquidation value and so his compensation should be sensitive to it, which is not achieved by a binary instrument.<sup>6</sup> John and John (1993) advocate two solutions to risk-shifting. The first is a solvency-contingent bonus, which has similar issues to reputation owing to its binary payoff. The second is to reduce the manager's equity. This is possible as their model has no effort decision; indeed, there is no reason to give the manager any incentive pay. Brander and Poitevin (1992) propose a more general fixed bonus, which may be triggered at levels other than solvency. They note that if the firm is sufficiently levered, no bonus can eliminate the agency costs of debt. Here, inside debt is effective even where bonuses are impotent.

Dybvig and Zender (1991) ("DZ") show that an optimal contract can alleviate the Myers and Majluf (1984) "lemons" issue, thus resurrecting the Modigliani-Miller irrelevance theorems. Although they focus on adverse selection rather than risk-shifting, the insight that incentives can achieve first-best is potentially applicable to other agency problems. However, their interest is not on what the contract *is*, but that an optimal contract (whatever form it may take) can render financing irrelevant. By contrast, this paper is focused on the *form* of pay. First, it shows that inside debt can be superior to the instruments advocated by a number of earlier papers, whereas DZ do not compare different contracts. Second, we analyze the optimal relative proportions of debt and equity, generating empirical predictions on the cross-sectional determinants of the inside debt level. While the optimal contract in the core DZ model aligns the manager with firm value, here the manager should not hold debt and equity in equal proportions if there is an effort decision.<sup>7</sup>

This paper is organized as follows. Section 2 considers a project selection model and shows that inside debt can be a superior remedy than bonuses to the

<sup>5</sup> Jensen and Meckling consider the agency costs of debt and equity separately, not simultaneously. Biais and Casamatta (1999) and Hellwig (2009) do consider both agency costs together. They do not analyze executive compensation (which remains pure equity), but an entrepreneur's choice of outside financing. Stoughton and Talmor (1999) also consider contracting under both an effort and investment decision. Investment is undertaken by shareholders (rather than the manager) and does not involve risk-shifting as the firm is unlevered.

<sup>6</sup> While Hirshleifer and Thakor consider reputation in the managerial labor markets, Diamond (1989) considers reputation in debt markets and shows that it can deter risk-shifting; however, it requires the manager to expect to continue to raise debt in the future. The solutions considered here and in earlier papers work in a one-shot game.

<sup>7</sup> When DZ extend their model by introducing an effort decision, they are unable to solve for the optimal contract and note that the solution may not exist. This does not matter for them, since the form of the contract and its comparative static determinants are not the focus of their paper. Their goal is to show that "if there is a solution, it is independent of capital structure."

risk-shifting concerns that arise if the manager is exogenously equity-aligned. Section 3 endogenizes equity compensation by adding an effort decision, and relates the optimal levels of equity and debt to firm characteristics. Section 4 discusses empirical implications and Section 5 concludes. The Appendix contains proofs.

## 2. Debt and Project Selection

### 2.1 THE MODEL

The model consists of four periods. At  $t = -1$ , shareholders offer a contract to the manager. At  $t = 0$ , risky debt is raised with face value of  $F$  and market value  $D_0 < F$ .  $t = 0$  total firm value (gross of expected pay) is  $V_0 = E_0 + D_0$ . All agents are risk-neutral and the risk-free rate and reservation wage are normalized to 0. Bondholders observe the manager's contract when calculating  $D_0$ , and so at  $t = -1$ , shareholders select the contract that maximizes  $V_0$  minus expected pay. The assumption that  $F$  is exogenous is discussed at the end of Section 2.2.

At  $t = 1$ , the manager chooses one of two mutually exclusive projects:  $R$  (risky) or  $S$  (safe). ( $S$  can be thought of as the firm's "status quo" state, and the manager is deciding whether to switch to the riskier project  $R$ .)  $R$  has probability  $p_R$  of "success," in which case the firm is worth  $V_{GR}$  at  $t = 2$ . In "failure" (which occurs with probability  $(1 - p_R)$ ), firm value is  $V_{BR}$ .  $S$  pays  $V_{GS}$  with probability  $p_S$ , and  $V_{BS}$  otherwise. We assume that  $p_R \leq p_S$ ,  $V_{GR} \geq V_{GS} \geq V_{BS} \geq V_{BR}$ ,  $V_{GR} > F$  and  $V_{BS} < F$ : failure of either project leads to bankruptcy, and success of  $R$  leads to solvency. We subdivide the model into two cases, depending on whether the success of  $S$  leads to solvency:  $V_{GS} \geq F$  (Case 1) and  $V_{GS} < F$  (Case 2). At  $t = 2$ , all payoffs are realized and the debt matures.

If all parameters were known at  $t = -1$ , shareholders would know the optimal project and can implement it with certainty. In reality, unforeseen projects often appear after pay is set, and so the contract should induce the manager to accept (reject) any new project  $R$  that appears and offers a higher (lower) NPV than the status quo  $S$ . We therefore assume  $V_{GR} \sim U[V_{GS}, V_{GRH}]$ . The optimal project is not known in advance; either  $R$  or  $S$  may be first-best depending on the realization of  $V_{GR}$ , which is observed privately by the manager at  $t = 1$ . All other parameters are public at  $t = -1$ .

While the core model focuses on project selection, it can easily be extended to involve other agency costs of debt. Hence the terms "asset substitution" and "risk-shifting" should be interpreted as any action that benefits shareholders but reduces total firm value. Other examples include debt overhang, concealing information, failing to disinvest, or paying excessive dividends. Many of these actions were believed to be important in the recent financial crisis.

Prior research on the solutions to (rather than causes of) risk-shifting typically take risk-shifting incentives as given by assuming that the manager exogenously owns a proportion  $\alpha$  of the firm's equity, and shows that certain compensation instruments can remove these incentives (e.g. John and John, 1993). This section follows this approach by also taking  $\alpha$  as exogenous and deriving the optimal accompanying compensation scheme; Section 3 endogenizes  $\alpha$  through the introduction of an effort decision. Our goal is to show whether and under what conditions debt is superior to the bonuses previously advocated, and so we allow the accompanying scheme to consist of a fraction  $\beta$  of the firm's debt and/or a bonus of  $J$ , paid if and only if the firm is solvent. The debt is "locked up" (similar to restricted stock) to prevent its subsequent sale or renegotiation; indeed, pension claims and deferred compensation cannot be sold in practice.  $\beta$  and  $J$  are choice variables, whereas  $\alpha$  is currently fixed. Note that  $\alpha > 0$  can arise exogenously even without the firm granting equity, for instance if the manager's labor market reputation is linked to the equity price (e.g. Hirshleifer and Thakor, 1992; Gibbons and Murphy, 1992).

**Case 1:**  $V_{GS} \geq F$

Incentive compatibility requires the manager to choose  $S$ , i.e., inequality (2) below is satisfied, if and only if it has a higher NPV, i.e., inequality (1) is satisfied:

$$p_R V_{GR} + (1 - p_R)V_{BR} \leq p_S V_{GS} + (1 - p_S)V_{BS}, \tag{1}$$

$$\text{iff } p_R[\alpha(V_{GR} - F) + \beta F + J] + (1 - p_R)\beta V_{BR} \leq p_S[\alpha(V_{GS} - F) + \beta F + J] + (1 - p_S)\beta V_{BS}. \tag{2}$$

The "and only if" requirement ensures that the contract does not lead to excessive conservatism. If and only if  $R$  has a higher NPV, the contract must ensure that the manager chooses  $R$  over  $S$ . The Appendix shows that the above is satisfied if and only if:

$$\beta = \alpha - \frac{J(p_S - p_R)}{F(p_S - p_R) + (1 - p_S)V_{BS} - (1 - p_R)V_{BR}}. \tag{3}$$

Under first-best project selection,  $R$  is chosen if and only if  $V_{GR}$  exceeds a cutoff  $V_{GR}^{**}$ , where

$$V_{GR}^{**} = \frac{p_S V_{GS} + (1 - p_S)V_{BS} - (1 - p_R)V_{BR}}{p_R}.$$

Let  $q = \Pr(V_{GR} > V_{GR}^{**})$ , i.e. the probability that  $R$  is first-best. The optimal compensation scheme is the cheapest incentive compatible contract, i.e. solves

$$\min_{\beta, J} q [p_R (\beta F + J) + (1 - p_R)\beta V_{BR}] + (1 - q) [p_S (\beta F + J) + (1 - p_S)\beta V_{BR}] \text{ s.t. (3)}. \tag{4}$$

**Proposition 1.** *If  $V_{GS} \geq F$ , the optimal compensation scheme  $(\beta^*, J^*)$  is given by*

$$\left\{ \begin{array}{l} (\alpha, 0) \text{ if } p_R(1 - p_S)V_{BS} > p_S(1 - p_R)V_{BR}, \\ \left( \alpha - \frac{J(p_S - p_R)}{F(p_S - p_R) + (1 - p_S)V_{BS} - (1 - p_R)V_{BR}}, J \right) \text{ if } p_R(1 - p_S)V_{BS} = p_S(1 - p_R)V_{BR}, \\ \text{where } J \in \left[ 0, \alpha \left[ F + \frac{(1 - p_S)V_{BS} - (1 - p_R)V_{BR}}{p_S - p_R} \right] \right], \\ \left( 0, \alpha \left[ F + \frac{(1 - p_S)V_{BS} - (1 - p_R)V_{BR}}{p_S - p_R} \right] \right) \text{ if } p_R(1 - p_S)V_{BS} < p_S(1 - p_R)V_{BR}. \end{array} \right. \quad (5)$$

If  $p_S \gg p_R$  and  $V_{BS}$  is close to  $V_{BR}$ , the main advantage of  $S$  over  $R$  is its greater probability of solvency. Hence the bonus should be used exclusively: its zero bankruptcy payoff makes it particularly sensitive to solvency. Indeed, John and John (1993) assume  $p_S = 1$  and find that a bonus can be effective. However, if  $p_S$  is close to  $p_R$  and  $V_{BS} \gg V_{BR}$ , the main advantage of  $S$  is its greater liquidation value. Inside debt should be used exclusively as, unlike the bonus, its payoff is sensitive to liquidation value. Indeed, if  $p_S = p_R$ , the bonus is completely ineffective.

**Case 2:**  $V_{GS} < F$

If  $V_{GS} < F$ , the firm is definitely liquidated if  $S$  is undertaken. Even though it leads to certain liquidation,  $S$  can still be preferred, if (1) is satisfied. The incentive constraint (3) becomes:

$$\beta = \alpha + \frac{p_R J}{p_S V_{GS} + (1 - p_S)V_{BS} - (1 - p_R)V_{BR} - p_R F}. \quad (6)$$

**Proposition 2.** *If  $V_{GS} < F$ , the optimal compensation scheme  $(\beta^*, J^*)$  is given by  $\beta^* = \alpha$ ,  $J^* = 0$ .*

In Case 1, (3) shows that bonuses (partially) alleviate asset substitution and reduce the amount of inside debt required. In Case 2, bonuses *exacerbate* asset substitution owing to their binary nature. Since  $V_{GS} < F$ , the bonus is only received if the risky project is chosen and is successful. It thus induces the manager to choose  $R$ , even if (1) is satisfied and so voluntary liquidation through the choice of  $S$  is efficient. Introducing a bonus increases the level of debt required to achieve optimal project selection (see (6)) and so is counter-productive. The optimal compensation scheme therefore involves zero bonus and only inside debt. This may explain why the solvency-contingent bonuses advocated by prior literature are rarely used. Since inside debt is even more favored in Case 2, we consider Case 1 for the remainder of the paper.



## 2.2 DISCUSSION

We now discuss whether alternative mechanisms can attenuate asset substitution and thus render inside debt unnecessary. Jensen and Meckling (1976) speculated that salaries might constitute inside debt. Most theory papers (e.g. Innes, 1990) assume that salary is junior to creditors and thus not received in bankruptcy; this is indeed the case in countries with pure liquidation bankruptcy codes and little room for renegotiation in debt workouts (Calcagno and Renneboog, 2007). In such a case, salary functions like a bonus and is thus different from inside debt. By contrast, Calcagno and Renneboog cite bankruptcy regulations in certain countries (e.g. US, UK and Germany) that management can use to ensure that salaries are senior to creditors in a bankruptcy, and give a number of examples where this occurred. If salaries are received in all states of nature, they have no effect on incentive constraints and thus the manager's decisions. (They do not mention any cases in which salaries have equal priority to other creditors, nor are we aware of any.)

Private benefits, such as firm-specific human capital, prestige, perks, and the present value of future wages are principally determined by whether the firm is solvent: if the manager is fired upon bankruptcy, he can no longer derive benefits from incumbency, regardless of liquidation value. They therefore have a very similar effect to bonuses  $J$ . Moreover, private benefits plausibly increase with shareholder value and thus may be incorporated into  $\alpha$ , increasing the need for inside debt.

In the model, the face value of debt is set at the first-best level  $F$ , which is optimal in the absence of agency costs of debt. These costs are foreseen by rational creditors and thus shareholders suffer a discount when raising debt. Alternatively, the trade-off theory would advocate lowering debt to a second-best level  $F^{SB} < V_{BR}$ , so that the firm is never bankrupt and no discount is suffered. This loses some of the benefits of debt, such as tax shields. Either way, shareholders have an incentive to reduce the agency costs of debt, to augment tax shields or to reduce the discount. Results would be unchanged (at the cost of complicating the model) if  $F$  was endogenized through the introduction of taxes, as in Hirshleifer and Thakor (1992): inside debt allows the firm to increase  $F$  and thus create additional tax shields.

Covenants are an imperfect solution due to the incompleteness of contracts: see, for example, the discussion in Myers (1977). Covenants may increase asset substitution, as the manager risk-shifts even when the firm is some distance from bankruptcy to avoid breaching the covenant. Also, covenants may not be breached until after the key decision has been made (e.g.,  $R$  was irreversibly chosen and failed, leading to the covenant violation).<sup>8</sup>

<sup>8</sup> Several studies demonstrate that executive compensation has a significant effect on risk, despite covenants. Examples include DeFusco et al. (1990), Guay (1999), and Coles et al. (2006).

### 3. Debt, Equity, Project Selection and Effort

Section 2 followed prior literature by taking  $\alpha$  as exogenous and deriving the optimal accompanying compensation scheme to achieve efficient project selection. However, a complete analysis must provide an endogenous justification for equity compensation, else the optimal contract would be  $\alpha = \beta = 0$ . This section endogenizes  $\alpha > 0$  as the solution to an effort problem. In addition to project selection, the manager now also makes an effort decision. He chooses a pair  $(e_g, e_b)$  where  $e_g, e_b \in [0, e_H]$  and  $e_H < 1$ .  $e_g$  increases the firm's solvency value by  $g$  with probability  $e_g$  and costs the manager  $\frac{1}{2}e_g^2$ ;  $e_b$  increases the firm's bankruptcy value by  $b$  with probability  $e_b$ , where  $V_{BS} + b < F$ , and costs the manager  $\frac{1}{2}e_b^2$ . If the firm has abundant intangible growth opportunities, such as employee training and building customer relationships,  $g$  will be high; if there is scope to scrap investment projects or liquidate assets,  $b$  will be high.

To keep the model tractable in the presence of an effort decision, it is necessary to specialize it to  $p_R = p_S = p$ . This rules out complex feedback effects between effort and project selection.<sup>9</sup> The two decisions can thus be analyzed separately, allowing the effects of compensation on each to be seen cleanly. For example, an increase in  $\beta$  directly raises effort, since the manager benefits more from enhancing liquidation value. Raising  $\beta$  also directly leads to  $S$  being chosen with greater frequency. If  $p_R = p_S = p$ , this change in project selection does not affect the probability of solvency (which always equals  $p$ ), and thus does not feed back into the effort decision (which depends on the probability of solvency). If instead  $p_S > p_R$ , the greater frequency with which  $S$  is chosen leads to a higher probability of solvency. If effort is particularly productive in solvency, this creates a positive feedback effect on effort: in (8) below,  $e_g$  is increasing in the probability of solvency. While such feedback makes it easier to justify inside debt, the model can no longer be solved analytically since project selection and effort cannot be considered separately. A consequence of  $p_R = p_S$  is that the bonus is ineffective (see Proposition 1) and so efficient compensation involves debt and equity alone. Note that even with  $p_S > p_R$ , the bonus provides no incentives to exert effort: a rise in bankruptcy value has no effect on the bonus but increases the value of inside

<sup>9</sup> Tractability concerns also motivate the assumption that effort only affects firm values, not the probability of success. If effort affects both parameters, it is impossible to solve analytically for the manager's effort choice, and to compute the effect of compensation on effort. In reality, it is plausible that effort increases the probability of success. This has a very similar effect to raising  $g$ , the value created by effort in success, in the analysis that follows (i.e. it increases the optimal ratio of  $\alpha$  to  $\beta$ ), and thus has the same comparative static effect.

debt. Thus, if inside debt is preferred to bonuses for solving asset substitution, it remains superior when an effort decision is introduced.

Shareholders maximize firm value net of pay to the manager, i.e. solve

$$\max_{\alpha, \beta} V_0 - \alpha E_0 - \beta D_0 = \max_{\alpha, \beta} (1 - \alpha) E_0 + (1 - \beta) D_0, \tag{7}$$

subject to  $\alpha \geq 0$  and  $\beta \geq 0$ .<sup>10</sup> We first consider the manager’s effort decisions.

**Lemma 1.** *The manager chooses effort levels*

$$\begin{aligned} e_g^* &= p\alpha g, \\ e_b^* &= (1 - p)\beta b. \end{aligned} \tag{8}$$

We now consider project selection. Firm value is maximized if  $R$  is selected if and only if  $V_{GR} > V_{GR}^{**}$ , where  $V_{GR}^{**}$  is defined by

$$V_{GR}^{**} = V_{GS} + \frac{(1 - p)}{p}(V_{BS} - V_{BR}). \tag{9}$$

However, the manager will choose  $R$  if and only if  $V_{GR} > V_{GR}^*$ , where

$$V_{GR}^* = V_{GS} + \frac{\beta(1 - p)}{\alpha p}(V_{BS} - V_{BR}). \tag{10}$$

The cutoff  $V_{GR}^*$  is undefined for  $\alpha = \beta = 0$ , since the manager is indifferent between all projects. We assume that he chooses the efficient project as this is Pareto optimal.<sup>11</sup> The optimal levels of debt and equity are determined by a trade-off between their differential effects on effort and project selection. We are interested not only in the absolute levels of  $\alpha$  and  $\beta$  but also their relative magnitudes—in particular, Jensen and Meckling (1976) speculated the optimal compensation scheme would involve  $\beta = \alpha$ . We thus also study the ratio  $\beta/\alpha$  which we define as  $k$ . In particular, we are interested in whether  $k = 1$  (i.e.  $\beta = \alpha$ ) or whether the contract involves an equity bias ( $k > 1$ ) or a debt bias ( $k < 1$ ). Our main results are summarized in the following Proposition.

<sup>10</sup> This is because of limited liability. In addition, we rule out  $\alpha < 0$  since it is illegal for CEOs to short their own firm. Allowing  $\beta < 0$  would, under some parameters, lead to the uninteresting result that the CEO should borrow to buy the entire firm. It is standard that a moral hazard problem under risk neutrality requires limited liability, otherwise the first-best can always be achieved. In addition,  $\beta < 0$  would correspond to the CEO being given a loan by his firm. In the U.S., executive loans were prohibited by the 2002 Sarbanes-Oxley Act.

<sup>11</sup> If we instead assume that he always takes project  $R$  if  $\alpha = \beta = 0$ , this allows us to drop the first condition in parts (ii) and (iii) of Proposition 3.

**Proposition 3.** *The optimal compensation scheme  $(\beta^*, \alpha^*)$  satisfies the following:*

- (i)  $\beta^* < \frac{1}{2}$  and  $\alpha^* < \frac{1}{2}$ ;
- (ii) If  $p^2 g^2 + (1-p)^2 b^2 > \frac{V_{GRH}+V_{GS}}{2} + \frac{1}{2} \frac{(1-p)^2}{p} \frac{(V_{BS}-V_{BR})^2}{V_{GRH}-V_{GS}} + (1-p)V_{BR}$  and  $(1-p)^2 b^2 - [pF + (1-p)(V_{BS} - V_{BR})] \leq 0$ , then  $\alpha^* > 0$ ;
- (iii) If  $p^2 g^2 + (1-p)^2 b^2 > \frac{V_{GRH}+V_{GS}}{2} + \frac{1}{2} \frac{(1-p)^2}{p} \frac{(V_{BS}-V_{BR})^2}{V_{GRH}-V_{GS}} + (1-p)V_{BR}$  and  $\frac{2(1-p)^2}{p} \frac{(V_{BS}-V_{BR})^2}{V_{GRH}-V_{GS}} + (1-p)^2 b^2 > pF + (1-p)V_{BR}$ , then  $\beta^* > 0$ .

If the conditions in (ii) and (iii) are satisfied, we also have the following comparative statics:

- (iv)  $\beta^*$  is increasing in  $b$ ;  $\alpha^*$  is increasing in  $g$ ;
- (v) If  $pF + (1-p)V_{BR} > 2(1-p)b^2$ , then  $k^* < 1$ .  $k^*$  is increasing in  $V_{BS} - V_{BR}$  and decreasing in  $g$  and  $p$ ;
- (vi) If  $p[\frac{V_{GRH}+V_{GS}}{2} - F] > p^2 g^2 + \frac{1}{2} \frac{(1-p)^2}{p} \frac{(V_{BS}-V_{BR})^2}{V_{GRH}-V_{GS}}$ , then  $k^* > 1$ .  $k^*$  is increasing in  $b$ , and decreasing in  $V_{BS} - V_{BR}$  and  $F$ .
- (vii)  $D_0$  is increasing in  $\beta$ ,  $E_0$  is decreasing in  $\beta$ .

We now discuss the intuition behind each component of Proposition 3. Starting with part (i), increasing  $\alpha$  augments effort, but is costly to the firm. Given the convex cost function, equity has a diminishing marginal effect on effort; when  $\alpha \geq \frac{1}{2}$ , this benefit is insufficient to outweigh the costs and so the optimal  $\alpha$  is less than  $\frac{1}{2}$ . A similar argument applies for  $\beta$ .

The common condition in parts (ii) and (iii),  $p^2 g^2 + (1-p)^2 b^2 > \frac{V_{GRH}+V_{GS}}{2} + \frac{1}{2} \frac{(1-p)^2}{p} \frac{(V_{BS}-V_{BR})^2}{V_{GRH}-V_{GS}} + (1-p)V_{BR}$ , is sufficient to rule out the optimum involving  $\alpha^* = \beta^* = 0$ . Intuitively, if effort is very unproductive ( $g$  and  $b$  are small), then it is not worth giving the manager incentive compensation to induce effort; instead, the firm should set  $\alpha = \beta = 0$  to guarantee optimal project selection. The condition guarantees that effort is sufficiently important for at least one of  $\alpha$  or  $\beta$  to be strictly positive.

Combining this with the second condition in part (ii) is sufficient to guarantee  $\alpha^* > 0$ . The latter is a technical condition to rule out a boundary case. If  $b$  is high, then effort is effective in bankruptcy; if  $p$  is low, bankruptcy is likely. Both factors mean that effort has a high expected productivity in bankruptcy; since debt is sensitive to the bankruptcy payoff, the optimal  $\beta$  is high. The disadvantage of high  $\beta$  is that it leads to excessive conservatism in project selection, and so  $\alpha > 0$  is typically optimal to counterbalance this. However, if  $\beta$  is sufficiently high that  $\frac{\beta(1-p)}{\alpha p} \frac{V_{BS}-V_{BR}}{V_{GRH}-V_{GS}} > 1$ , then we are at the boundary case where  $S$  is always selected. From (10) the cutoff  $V_{GR}^*$  exceeds  $V_{GRH}$ , and so  $R$  is never chosen. Even if  $\alpha$

increases by a small amount, the cutoff remains above  $V_{GRH}$  and so project selection is unchanged. The second condition in part (ii) places an upper bound on  $b$  and  $(1 - p)$  to rule this out.

Part (iii) gives a sufficient condition for  $\beta^* > 0$ , and thus pure equity compensation to be suboptimal. The condition is more likely to be satisfied if  $V_{BS} - V_{BR}$  and  $b$  are high, and  $p$  is low.  $V_{BS} - V_{BR}$  measures the manager's ability to destroy liquidation value by inefficiently choosing  $R$ , and thus the magnitude of the asset substitution effect. If the firm has few tangible assets, creditors recover very little regardless of the severity of liquidation: both  $V_{BS}$  and  $V_{BR}$  are close to zero. If the firm has illiquid, tangible assets (such as buildings) that cannot be eroded by risk-taking, both  $V_{BS}$  and  $V_{BR}$  are high. In both cases,  $V_{BS} - V_{BR}$  is low: there are few gains from making the manager sensitive to the liquidation value, since he has little effect on it. On the other hand, if  $R$  reduces liquidation value (such as an advertising campaign, which transforms tangible cash into an intangible asset),  $V_{BS} - V_{BR}$  is high and so the optimal level of inside debt is strictly positive. Similarly, if effort is effective at improving liquidation value ( $b$  is high), then debt is effective at increasing effort and so again inside debt is justified. If  $p$  falls, bankruptcy becomes likelier. This increases the severity of the asset substitution issue; it also augments the effectiveness of debt in inducing effort, because debt is sensitive to the firm's value in bankruptcy which is enhanced by effort. Both factors lead to  $\beta^* > 0$ .

In sum, the previous literature's justification of exclusively equity-linked incentives is warranted for firms where the agency costs of debt are low and effort considerations are first-order, such as start-ups with high growth opportunities. However, inside debt is desirable in companies with a significant risk of bankruptcy (low  $p$ ), where the investment decision affects liquidation value (high  $V_{BS} - V_{BR}$ ), and where effort can improve liquidation value (high  $b$ ). One example is LBOs, which are frequently undertaken in mature firms where the main agency problem is excessive investment. Indeed, the private equity firm can be considered the "manager" in LBOs, given its close involvement in operations, and typically holds strips of debt and equity to minimize conflicts.

If the conditions in parts (ii) and (iii) are satisfied, then we have interior solutions for  $\alpha^*$  and  $\beta^*$ . This permits comparative statics, which are given in parts (iv)–(vii). The intuition for part (iv) is standard. Parts (v) and (vi) consider the optimal ratio of debt to equity,  $k = \beta/\alpha$ . The optimal  $k$  is a trade-off between its differential effects on project selection and effort. For project selection,  $k = 1$  (i.e.  $\beta = \alpha$ ) is optimal as then  $V_{GR}^* = V_{GR}^{**}$ . However, effort considerations may cause the optimal  $k$  to deviate from 1. To illustrate this, define "output" as

$$\eta = pge_g + (1 - p)be_b - \alpha E_0 - \beta D_0,$$

i.e. the contribution to firm value provided by effort, minus the manager's pay. We prove in the Appendix that

$$\frac{\partial \eta}{\partial k} = 2p^2 g^2 \alpha \frac{\partial \alpha}{\partial k} + 2(1-p)^2 b^2 k \alpha \frac{\partial(k\alpha)}{\partial k}, \quad \frac{\partial \alpha}{\partial k} < \frac{\partial(k\alpha)}{\partial k}. \quad (11)$$

If effort has a high expected productivity in solvency (either because solvency is likely ( $p$  is high) or effort is particularly effective in solvency states ( $g$  is high relative to  $b$ )), then equity is more effective than debt in inducing effort, since it is sensitive to the payoff in solvency. Thus, reducing debt and increasing equity augments effort; indeed, inspecting (11) shows that  $\frac{\partial \eta}{\partial k} < 0$  if  $p$  is sufficiently high and  $g$  is sufficiently larger than  $b$ .<sup>12</sup> The Appendix proves that, if and only if  $\frac{\partial \eta}{\partial k}|_{k=k^*} < 0$ ,  $k^* < 1$  and so the contract involves an equity bias ( $\beta^* < \alpha^*$ ). The condition in part (v),  $pF > 2(1-p)b^2$ , is a sufficient condition for  $\frac{\partial \eta}{\partial k}|_{k=k^*} < 0$  and is indeed satisfied if  $p$  is high and  $b$  is low. Even though increasing  $k$  towards 1 would improve project selection, it would also reduce output, and so the optimal  $k$  is less than 1. The actual value of  $k^*$  is a trade-off between the positive effect on project selection and the negative effect on effort, and thus depends on the magnitude of the two agency problems. If  $V_{BS} - V_{BR}$  is high, asset substitution is relatively important and so  $k^*$  is closer to the level of 1 that optimizes project selection. If  $g$  rises, the benefits from effort are more concentrated in solvency and so equity is more effective at inducing effort, reducing  $k^*$ . An increase in  $p$  augments the potency of equity in inducing effort and reduces the severity of asset substitution; both factors reduce  $k^*$ .

Conversely, if  $p$  is low or  $b$  is high relative to  $g$ , then effort has a high expected productivity in liquidation and so debt is more effective than equity in inducing effort, since it is sensitive to the liquidation payoff. If and only if  $\frac{\partial \eta}{\partial k}|_{k=k^*} > 0$ , we have  $k^* > 1$  and the contract involves a debt bias ( $\beta^* > \alpha^*$ ). The condition in part (vi) is a sufficient condition for  $\frac{\partial \eta}{\partial k}|_{k=k^*} > 0$ , and is indeed satisfied if  $g$  and  $p$  are low. Even though increasing  $k$  above 1 leads to excessive conservatism in project selection, it also improves output and so the optimal  $k$  exceeds 1. This debt bias contrasts the traditional view that insufficient effort results from the "agency costs of equity", i.e. raising too much outside equity leaves the manager with too few shares. For a firm close to bankruptcy, effort may be more efficiently induced by giving the manager more debt. For the comparative statics, the effect of  $V_{BS} - V_{BR}$  is reversed: if it increases, project selection becomes more important and so  $k$  should be closer to 1; since  $k^* > 1$ , this involves a *reduction* in  $k^*$ . A rise in  $F$  increases the importance of asset substitution and has the same effect. By contrast, an increase in  $b$  augments the importance of effort and so raises  $k^*$ .

<sup>12</sup> Habib and Johnsen (2000) also feature the idea that equity induces an agent to improve firm value in solvency, and debt induces an agent to improve firm value in bankruptcy. They consider a different setting where equity (debt) is given to outside investors rather than the manager, to induce them to credibly assess the firm's value in its primary (secondary) use.

While parts (i) to (vi) of Proposition 3 consider the optimal compensation scheme, part (vii) addresses the effect of the compensation scheme on security prices. An increase in  $\beta$  leads to greater conservatism in project selection and thus increases (decreases) the value of debt (equity).

#### 4. Empirical Implications

The model generates a number of empirical predictions involving inside debt as both an independent and dependent variable, i.e. implications for the effects of inside debt holdings and the firm characteristics that affect the optimal debt level. We outline the predictions below. Existing evidence on pensions appears to be consistent with the first five; the other predictions are unexplored and thus may be fruitful topics for future research.

The “big picture” prediction is that inside debt should be used in executive compensation, whereas prior theories do not advocate debt. Indeed, Bebchuk and Jackson (2005), Sundaram and Yermack (2007), and Gerakos (2007) document the extensive use of pensions. While pensions for rank-and-file employees are typically insured by the Pension Benefit Guaranty Corporation and thus insensitive to bankruptcy, executive pensions typically substantially exceed the maximum insured amount. In a bankruptcy, they represent unsecured, unfunded debt claims with equal priority to other unsecured creditors.<sup>13</sup> Thus, pensions constitute inside debt. Moreover, Sundaram and Yermack find the percentage debt stake exceeds the percentage equity stake for 13% of CEOs. This finding represents an even sharper disparity with theories that advocate only equity, whereas this paper shows that a debt bias is sometimes optimal. Moreover, current evidence understates the extent of inside debt as it typically focuses on executive pensions and ignores deferred compensation, owing to data limitations thus far. Section 409a of the Internal Revenue Code has recently increased the reporting requirements for deferred compensation, and it would be useful to test the predictions below using the CEO’s total inside debt holdings; Wei and Yermack (2009) is one such paper that does this. By contrast, there appears to be very little evidence for the fixed bonuses advocated by previous theories of stockholder-bondholder conflicts. Murphy’s (1999) survey documents that bonuses in practice are instead typically increasing in equity value (up to an upper limit), and thus augment risk-shifting tendencies.

In addition to this “high-level” prediction on the existence of debt, there are a number of “detail-level” comparative statics predictions. In reality,  $g$  is likely to be significantly higher than  $b$  and so  $k^* < 1$ ; indeed, empirically,  $\alpha > \beta$  for 87% of CEOs, so we use part (v) of Proposition 3 to form the empirical predictions.

<sup>13</sup> See Sundaram and Yermack (2007) and Gerakos (2007) for further detail on the priority of pensions in bankruptcy.

From (10), an increase in  $k$  augments the “cutoff”  $V_{GR}^*$  for the manager to choose the risky project. Therefore, firm risk should decrease with the manager’s personal leverage (i.e. his ratio of inside debt to equity). This is indeed found by Sundaram and Yermack, using distance to default as a measure of firm risk. Similarly, Gerakos finds that CEO pension holdings are associated with higher credit ratings.

Empiricists sometimes measure the manager’s personal leverage not using percentages of debt and equity ( $\frac{\beta}{\alpha}$ ) but using dollar values ( $\frac{\beta}{\alpha} \frac{D_0}{E_0}$ ). Using this measure, the model predicts a positive relationship between dollar personal leverage and firm leverage ( $\frac{D_0}{D_0+E_0}$ ) for two reasons. First, an increase in firm leverage ( $\frac{D_0}{D_0+E_0}$ ) mechanically increases personal leverage through augmenting the second term. Second, changes in the underlying parameters can jointly increase both firm and personal leverage. From Proposition 3, an increase in  $b$  and a decrease in  $g$  both augment  $k$ . From Equations (A1) and (A2) in the Appendix, these changes also increase  $D_0$  and reduce  $E_0$ . Indeed, Sundaram and Yermack find a strong correlation between firm leverage and personal leverage.

In firms with growth options, the effort decision is first-order ( $g$  is high). Consequently, pension entitlements should fall, as found by Gerakos. As with the leverage association, this relationship is difficult to reconcile with tax or stealth compensation justifications for pensions, but consistent with efficient contracting. Indeed, Gerakos concludes that contracting variables explain a greater proportion of pension levels than do measures of CEO power.

Turning from the determinants of the compensation scheme to its effects, part (vii) of Proposition 3 predicts that the value of debt (equity) should rise (fall) with inside debt holdings. SEC regulations mandated disclosure of CEO’s inside debt holdings (both pensions and deferred compensation) in Spring 2007. Wei and Yermack (2009) find that firms that disclosed large inside debt holdings indeed experienced increases in bond prices and decreases in equity prices, and that post-disclosure bond yields are significantly positively related to  $k$ .

We now move to untested predictions. In Section 2. Case 1 (2) predicts that inside debt is decreasing (increasing) in private benefits. Case 2 depicts a highly levered firm for which liquidation is very likely, and so Case 1 likely applies to the majority of firms. Sundaram and Yermack find that personal leverage is significantly increasing in firm age, which is consistent since the present value of future salary is lower for older managers closer to retirement. Note that this link is not automatically mechanical—while pension benefits naturally increase over time with the CEO’s tenure, the same is true for equity compensation (e.g. Gibbons and Murphy, 1992.) However, sharper tests of this prediction may be possible with measures of private benefits that do not depend on CEO age.

The last implication comes from Proposition 3, which predicts that inside debt is most valuable where the manager has greatest effect on liquidation values. Note that this is different from raw asset tangibility: if assets are highly intangible (tangible),



liquidation values are low (high) regardless of the manager's actions. A possible proxy could be the intensity of covenants.

It is important to note some caveats with interpreting recent pensions findings as being fully consistent with the model and thus evidence that real-life practices are optimal. In many firms, pensions are sufficiently large, and have sufficiently similar payoffs to debt, that they fulfill the role of inside debt advocated by the model and explain why executives do not need to hold actual debt securities in addition. However, these conditions may not be fulfilled in certain circumstances, in which case there may be an argument for supplementing pensions with actual debt, as advocated by Bebchuk and Spamann (2009) in a normative proposal for compensation reform.

First, existing studies are focused on large firms in the U.S. It is not clear whether these findings are representative of all firms, or of firms overseas. While Sundaram and Yermack find that the CEO's debt-equity mix is independent of firm size, potentially implying their results may also apply to smaller firms outside their sample, this has yet to be directly shown. Further research is necessary to investigate the generality of recent results.

Second, the model illustrates that the payoff of a pension has to be very similar to debt for it to be effective: small departures may lead to pensions either not affecting or exacerbating the issue. If debt is secured, pensions are junior and thus similar to the bonus  $J$ ; they may therefore encourage risk-shifting (Proposition 3). In other cases, the payoff may be close to risk-free and thus pensions do not affect managerial incentives. Executives can put pension fund assets into a "secular" trust fund, ring-fenced from the reach of creditors, or a "springing" trust which converts into a secular trust upon trigger events, such as a credit downgrade. Note, however, that these trusts are very rare. The CEOs of Delta Airlines and AMR (the parent of American Airlines) lost their jobs after such trusts were disclosed (Sundaram and Yermack, 2007.)

Finally, debt securities may have a role even in companies where pensions are currently sufficient to mitigate risk-shifting. Such firms may have low agency costs of debt because they are addressing them by reducing leverage or inside equity.<sup>14</sup> Since these measures are costly, the importance of the agency costs of debt in practice cannot be ascertained solely by looking at actual cases of risk-shifting. Debt grants may allow leverage or equity to increase, and thus be a less costly solution.

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<sup>14</sup> There is a widely documented negative relationship between  $\alpha$  and leverage (Friend and Lang, 1988; Agrawal and Nagarajan, 1990; Ortiz-Molina, 2007). One interpretation is that firms for which high debt is optimal are reducing the manager's equity stake to attenuate risk-shifting (as predicted by John and John, 1993), or firms that require high  $\alpha$  to induce effort are under-leveraging for the same reason.

## 5. Conclusion

The simplest theory of executive compensation would advocate aligning the manager with firm value. Since empiricists have long believed that managers are compensated exclusively with cash and equity in practice, a number of theory papers rationalize such a scheme. However, recent research has shown that debt-like instruments such as pensions are in fact substantial components of executive compensation. Debt is critically different from other instruments as it is sensitive to the value of assets in bankruptcy. These findings suggest the need for new theories to explain why and when inside debt has a role in efficient compensation, and how much debt should be used.

This paper is a first step in this direction. Inside debt can be a more effective solution to creditor expropriation than salaries, bonuses, reputation and private benefits, owing to its sensitivity to liquidation value. When equity compensation is endogenized via an effort decision, the optimal level of inside debt is typically strictly positive and so pure equity compensation is inefficient. However, contrary to intuition, it typically does not equal the fraction of inside equity due to a trade-off between effort and project selection. An equity bias is usually optimal; the manager's debt-to-equity ratio is increasing in his effect on the liquidation value and the probability of bankruptcy, and decreasing in growth opportunities.

The model generates a number of empirical predictions, many of which appear to be supported with existing findings. However, since data on debt compensation has only recently become available, there are a number of untested predictions that may be interesting topics for future empirical research. In terms of future theoretical directions, this paper has derived conditions under which inside debt is superior to bonuses, and analyzed the optimal relative proportions of debt and equity when compensation comprises of these instruments. It would be fruitful to study the conditions under which debt is optimal in a general contract design setting.

## Appendix

*Proof of Equation (3).* Rearranging Equations (1) and (2) yields

$$\begin{aligned}
 p_R V_{GR} - p_S V_{GS} &\leq (1 - p_S) V_{BS} - (1 - p_R) V_{BR} \\
 p_R V_{GR} - p_S V_{GS} &\leq F(p_R - p_S) + \frac{1}{\alpha} [(p_S - p_R)(\beta F + J) \\
 &\quad + (1 - p_S)\beta V_{BS} - (1 - p_R)\beta V_{BR}].
 \end{aligned}$$

Equating the right-hand sides of each inequality leads to (3). To prove that the denominator of (3) is positive, we have:

$$\begin{aligned}
 &F(p_S - p_R) + (1 - p_S)V_{BS} - (1 - p_R)V_{BR} \\
 &= (V_{BS} - V_{BR})(1 - p_R) + (p_S - p_R)(F - V_{BS}) > 0.
 \end{aligned}$$

*Proof of Proposition 1.* To find the cheapest contract that satisfies (3), we first calculate the cost of debt and the bonus. The firm is solvent with probability  $p = p_R q + p_S(1 - q)$ . A bonus of  $J$  costs  $pJ$ ; debt of  $\beta$  costs

$$\beta[pF + q(1 - p_R)V_{BR} + (1 - q)(1 - p_S)V_{BS}].$$

Hence an incentive compatible contract will cost

$$W = pJ + \left[ \alpha - \frac{J(p_S - p_R)}{F(p_S - p_R) + (1 - p_S)V_{BS} - (1 - p_R)V_{BR}} \right] \times [pF + q(1 - p_R)V_{BR} + (1 - q)(1 - p_S)V_{BS}],$$

where

$$\frac{\partial W}{\partial J} = p - \frac{(p_S - p_R)[pF + q(1 - p_R)V_{BR} + (1 - q)(1 - p_S)V_{BS}]}{F(p_S - p_R) + (1 - p_S)V_{BS} - (1 - p_R)V_{BR}}.$$

Since the derivative is constant, we have a corner solution. The manager is paid entirely with debt if  $\frac{\partial W}{\partial J} > 0$ , i.e.,

$$p_R(1 - p_S)V_{BS} > p_S(1 - p_R)V_{BR},$$

and entirely with the bonus if  $\frac{\partial W}{\partial J} < 0$ .

*Proof of Equation (6).* Rearranging (1) and (2) yields

$$p_R V_{GR} \leq p_S V_{GS} + (1 - p_S)V_{BS} - (1 - p_R)V_{BR}$$

$$p_R V_{GR} \leq p_R F - \frac{1}{\alpha} p_R J + \frac{\beta}{\alpha} [p_S V_{GS} + (1 - p_S)V_{BS} - (1 - p_R)V_{BR} - p_R F].$$

Equating the left-hand sides of each inequality leads to (6).

*Proof of Lemma 1.* The manager's objective function for effort is given by:

$$\alpha p g e_g + \beta(1 - p) b e_b - \frac{1}{2} e_g^2 - \frac{1}{2} e_b^2$$

and differentiating with respect to  $e_g$  and  $e_b$  gives Equation (8).

*Proof of Proposition 3.* For conciseness, it is helpful to define the following:

$$X = \frac{(1 - p)^2 (V_{BS} - V_{BR})^2}{p (V_{GRH} - V_{GS})}$$

$$Y = p \left[ \frac{V_{GRH} + V_{GS}}{2} - F \right].$$

From (10), the manager will choose  $S$  if and only if

$$V_{GR} < V_{GS} + \frac{\beta(1 - p)}{\alpha p} (V_{BS} - V_{BR}).$$

Since  $V_{GR} \sim U[V_{GS}, V_{GRH}]$ , this occurs with probability  $\min(1, \frac{\beta(1-p)}{\alpha p} \frac{V_{BS}-V_{BR}}{V_{GRH}-V_{GS}})$ . There are three cases to consider.

*Case 1:*  $\frac{\beta(1-p)}{\alpha p} \frac{V_{BS}-V_{BR}}{V_{GRH}-V_{GS}} \leq 1$  and at least one of  $\alpha$  and  $\beta$  is strictly positive. Here, both  $R$  and  $S$  are selected with strictly positive probability, and we need not worry about boundary cases. Then the values of equity and debt are given by:

$$\begin{aligned}
 E_0 &= p \frac{V_{GRH} + V_{GS} + \frac{\beta(1-p)}{\alpha p}(V_{BS} - V_{BR})}{2} \times \frac{V_{GRH} - V_{GS} - \frac{\beta(1-p)}{\alpha p}(V_{BS} - V_{BR})}{V_{GRH} - V_{GS}} \\
 &\quad + pV_{GS} \times \frac{\beta(1-p)}{\alpha p} \frac{V_{BS} - V_{BR}}{V_{GRH} - V_{GS}} - pF + p^2g^2\alpha \\
 &= Y - \frac{\beta^2}{2\alpha^2}X + p^2g^2\alpha. \tag{A1}
 \end{aligned}$$

$$\begin{aligned}
 D_0 &= [pF + (1-p)V_{BR}] \times \frac{V_{GRH} - V_{GS} - \frac{\beta(1-p)}{\alpha p}(V_{BS} - V_{BR})}{V_{GRH} - V_{GS}} \\
 &\quad + [pF + (1-p)V_{BS}] \times \frac{\beta(1-p)}{\alpha p} \frac{V_{BS} - V_{BR}}{V_{GRH} - V_{GS}} + (1-p)^2b^2\beta \\
 &= pF + (1-p)V_{BR} + \frac{\beta}{\alpha}X + (1-p)^2b^2\beta. \tag{A2}
 \end{aligned}$$

Differentiating the objective function (7) with respect to  $\beta$  and  $\alpha$  yields:

$$(\beta) : -(1-\alpha)\frac{\beta}{\alpha^2}X - D_0 + (1-\beta)\left[\frac{X}{\alpha} + (1-p)^2b^2\right] = 0 \tag{A3}$$

$$(\alpha) : -E_0 + (1-\alpha)\left[\frac{\beta^2}{\alpha^3}X + p^2g^2\right] + (1-\beta)\left[-\frac{\beta}{\alpha^2}X\right] = 0. \tag{A4}$$

Using the expressions for  $E_0$  and  $D_0$  in (A1) and (A2), the above first-order conditions become:

$$(\beta) : -[pF + (1-p)V_{BR}] + \left[\frac{1-\beta}{\alpha} - \frac{\beta}{\alpha^2}\right]X + (1-2\beta)(1-p)^2b^2 = 0 \tag{A5}$$

$$(\alpha) : -Y + \left[\frac{\beta^2}{2\alpha^2} + \frac{\beta^2}{\alpha^3} - \frac{\beta}{\alpha^2}\right]X + (1-2\alpha)p^2g^2 = 0. \tag{A6}$$

*Case 2:*  $\frac{\beta(1-p)}{\alpha p} \frac{V_{BS}-V_{BR}}{V_{GRH}-V_{GS}} > 1$  and at least one of  $\alpha$  and  $\beta$  is strictly positive. In this case,  $S$  is always selected. Shareholders' payoff (7) is

$$(1-\alpha)(pV_{GS} - pF + p^2g^2\alpha) + (1-\beta)(pF + (1-p)V_{BS} + (1-p)^2b^2\beta).$$

Differentiating this yields

$$\alpha^* = \max \left( 0, \frac{pg^2 - (V_{GS} - F)}{2pg^2} \right) \tag{A7}$$

$$\beta^* = \frac{(1-p)^2b^2 - (pF + (1-p)V_{BS})}{2(1-p)^2b^2}. \tag{A8}$$

There is no  $\max(0, \cdot)$  function for  $\beta^*$  since  $\frac{\beta(1-p)}{\alpha p} \frac{V_{BS}-V_{BR}}{V_{GRH}-V_{GS}} > 1$  rules out  $\beta^* = 0$ .

*Case 3:*  $\alpha = \beta = 0$ . This must be considered separately from Cases 1 and 2 since the expression  $\frac{\beta(1-p)}{\alpha p} \frac{V_{BS}-V_{BR}}{V_{GRH}-V_{GS}}$  is undefined. We assume that the manager takes the efficient project in this case.

*Proof of part (i).* For Cases 2 and 3, it is immediate that  $\alpha^* < \frac{1}{2}$  and  $\beta^* < \frac{1}{2}$ , so we only need to tackle Case 1.

First, we consider the case in which the optimum is on the boundaries of Case 1, and so the first-order conditions do not apply. The boundaries of Case 1 are  $\{(\alpha, \beta) : \beta = 0, \alpha > 0\}$  and  $\{(\alpha, \beta) : \frac{\beta(1-p)}{\alpha p} \frac{V_{BS}-V_{BR}}{V_{GRH}-V_{GS}} = 1\}$ . On the boundary  $\{(\alpha, \beta) : \beta = 0, \alpha > 0\}$ , shareholders' payoff is

$$(1 - \alpha)(Y + p^2g^2\alpha) + pF + (1 - p)V_{BR}.$$

Differentiating this yields

$$\alpha^* = \frac{p^2g^2 - Y}{2p^2g^2}$$

$$\beta^* = 0,$$

and so  $\alpha^* < \frac{1}{2}$  and  $\beta^* < \frac{1}{2}$ .

The second boundary can be rewritten  $\{(\alpha, k) : \frac{k(1-p)}{p} \frac{V_{BS}-V_{BR}}{V_{GRH}-V_{GS}} = 1\}$ . Shareholders solve

$$\max_{\alpha, \beta} (1 - \alpha)(pV_{GS} - pF + p^2g^2\alpha) + (1 - \beta)(pF + (1 - p)V_{BS} + (1 - p)^2b^2\beta)$$

subject to the constraint

$$\frac{\beta(1-p)}{\alpha p} \frac{V_{BS} - V_{BR}}{V_{GRH} - V_{GS}} = 1.$$

Defining  $A = \frac{p(V_{GRH}-V_{GS})}{(1-p)(V_{BS}-V_{BR})}$ , we have

$$\alpha^* = \frac{p^2g^2 - p(V_{GS} - F) + A[(1-p)^2b^2 - pF - (1-p)V_{BS}]}{2[p^2g^2 + A^2(1-p)^2b^2]}$$

$$\beta^* = A\alpha^*.$$

It is automatic that  $\alpha^* < \frac{1}{2}$ . A necessary condition for the optimum to be on this boundary is that

$$\frac{\partial}{\partial k} \left[ (1 - \alpha) \left( Y - \frac{1}{2}k^2X + p^2g^2\alpha \right) + (1 - k\alpha)(pF + kX + (1 - p)^2b^2k\alpha) \right] \geq 0$$

at  $k = A$ . This derivative at  $k = A$  equals:

$$\begin{aligned} & -(1 - \alpha)kX - \alpha[pF + kX + (1 - p)^2b^2k\alpha] + (1 - k\alpha)(X + (1 - p)^2b^2\alpha) \\ & \leq -\alpha[pF + (1 - p)(V_{BS} - V_{BR}) - (1 - 2k\alpha)(1 - p)^2b^2]. \end{aligned} \tag{A9}$$

For the derivative to be non-negative,  $1 - 2k\alpha$  must be positive and so  $\beta^* = k^*\alpha^* < \frac{1}{2}$ .

We now move to the interior of Case 1, which allows us to use first-order conditions. If  $\beta \geq \frac{1}{2}$ , then from (A5) we must have  $\frac{1-\beta}{\alpha} - \frac{\beta}{\alpha^2} > 0$ . This yields

$$\alpha > \frac{\beta}{1 - \beta} \geq \frac{1/2}{1 - 1/2} = 1.$$

This is a contradiction, so  $\beta < \frac{1}{2}$ .

Similarly, from (17), we have

$$\left[ -Y + \frac{\beta^2}{2\alpha^2}X \right] + \left[ \frac{\beta^2}{\alpha^3} - \frac{\beta}{\alpha^2} \right] X + (1 - 2\alpha)p^2g^2 = 0.$$

From (A1), the first term is the negative of the value of equity if  $g = 0$ . Since equity value must be positive, this first term must be negative. Thus if  $\alpha \geq \frac{1}{2}$ , we must have  $\frac{\beta^2}{\alpha^3} - \frac{\beta}{\alpha^2} > 0$ , i.e.  $\alpha < \beta$ . However, since  $\beta < \frac{1}{2}$ , this is inconsistent with  $\alpha \geq \frac{1}{2}$ . Hence  $\alpha < \frac{1}{2}$ .

*Proof of part (ii).* First, we derive a sufficient condition to rule out Case 3 being optimal. Under Case 3, shareholders' payoff is

$$\frac{V_{GRH} + V_{GS}}{2} + \frac{1}{2}X + (1 - p)V_{BR}.$$

To show that Case 3 is suboptimal, it is sufficient to prove that shareholders' payoff is lower than under Case 1 with an arbitrary contract – then it will definitely be lower than under Case 1 with the optimal contract. Consider the contract  $\alpha = \beta = \epsilon$ . Then, in Case 1, shareholders' payoff is

$$(1 - \epsilon) \left[ p^2g^2\epsilon + (1 - p)^2b^2\epsilon + \frac{V_{GRH} + V_{GS}}{2} + \frac{1}{2}X + (1 - p)V_{BR} \right].$$

The derivative of this function with respect to  $\epsilon$  at  $\epsilon = 0$  is

$$p^2g^2 + (1 - p)^2b^2 - \frac{V_{GRH} + V_{GS}}{2} - \frac{1}{2}X - (1 - p)V_{BR}.$$

Thus, to rule out  $\alpha = \beta = 0$ , it is sufficient to show that

$$p^2g^2 + (1 - p)^2b^2 > \frac{V_{GRH} + V_{GS}}{2} + \frac{1}{2}X + (1 - p)V_{BR}. \tag{A10}$$

In addition, if  $(1 - p)^2b^2 - [pF + (1 - p)(V_{BS} - V_{BR})] < 0$ , then  $\beta^*$  in Equation (A8) is negative and so Case 2 is not feasible. Hence, the optimum must be Case 1. Since Case 1 involves  $\frac{\beta(1-p)}{\alpha p} \frac{V_{BS}-V_{BR}}{V_{GRH}-V_{GS}} \leq 1$ , we have  $\alpha^* > 0$ .

*Proof of part (iii).* Condition (A10) is sufficient to rule out Case 3. Case 2 requires  $\frac{\beta(1-p)}{\alpha p} \frac{V_{BS}-V_{BR}}{V_{GRH}-V_{GS}} > 1$  so  $\beta^* > 0$  automatically holds. We thus only need to consider Case 1. Differentiating the objective (7) with respect to  $\beta$  yields:

$$-[pF + (1 - p)V_{BR}] + \left[ \frac{1 - \beta}{\alpha} - \frac{\beta}{\alpha^2} \right] X + (1 - 2\beta)(1 - p)^2b^2.$$

At  $\beta = 0$ , this becomes

$$-[pF + (1 - p)V_{BR}] + \frac{1}{\alpha}X + (1 - p)^2b^2.$$

Since  $\alpha^* < \frac{1}{2}$ ,  $2X > pF + (1 - p)V_{BR} - (1 - p)^2b^2$  is sufficient to guarantee that this derivative is positive and so  $\beta^* > 0$ .

*Proof of part (iv).* First, note that the condition  $(1 - p)^2b^2 - [pF + (1 - p)(V_{BS} - V_{BR})] \leq 0$  in part (ii) of the Proposition guarantees that (A9), the derivative at the boundary where  $\frac{\beta(1-p)}{\alpha p} \frac{V_{BS}-V_{BR}}{V_{GRH}-V_{GS}} = 1$ , is negative. Thus, the optimum is interior and so we can use first-order conditions.

Differentiating the objective (7) with respect to  $\beta$ , and treating  $\alpha$  as a function of  $\beta$ , yields

$$-Y \frac{\partial \alpha}{\partial \beta} - pF - (1 - p)V_{BR} + X \left[ \frac{\alpha - \frac{\partial \alpha}{\partial \beta} \beta}{\alpha^2} \left( 1 - \frac{\beta}{\alpha} \right) + \frac{\beta^2}{2\alpha^2} \frac{\partial \alpha}{\partial \beta} - \frac{\beta}{\alpha} \right] + (1 - 2\alpha)p^2g^2 \frac{\partial \alpha}{\partial \beta} + (1 - 2\beta)(1 - p)^2b^2 = 0. \tag{A11}$$

If we solve for  $\alpha$  from (A6), then the solution for  $\alpha$  is independent of  $b$ . Let  $h$  denote the left hand side of (A11). Thus, the partial derivative of  $h$  with respect to  $b$  is  $2(1 - 2\beta)(1 - p)^2b$ . From  $h(\beta, b) = 0$ , we have  $\frac{\partial h}{\partial \beta} \frac{\partial \beta}{\partial b} + \frac{\partial h}{\partial b} = 0$ ; since  $\frac{\partial h}{\partial \beta} < 0$  at the optimum, the sign of  $\frac{\partial \beta}{\partial b}$  is the same as the sign of  $\frac{\partial h}{\partial b}$ . The latter is positive since  $\beta < \frac{1}{2}$ . Thus,  $\beta^*$  is increasing in  $b$ . A similar analysis proves that  $\alpha^*$  is increasing in  $g$ .

*Proof of parts (v) and (vi).* Defining  $k = \frac{\beta}{\alpha}$  and plugging into (A5) and (A6) yields the first-order conditions:

$$(\beta) : -[pF + (1 - p)V_{BR}] + \left[ \frac{1 - k\alpha}{\alpha} - \frac{k\alpha}{\alpha^2} \right] X + (1 - 2\alpha k)(1 - p)^2 b^2 = 0 \tag{A12}$$

$$(\alpha) : -Y + \left[ \frac{k^2}{2} + \frac{k^2}{\alpha} - \frac{k}{\alpha} \right] X + (1 - 2\alpha)p^2 g^2 = 0. \tag{A13}$$

Multiplying (A3) by  $k$  and adding it to (A4) yields:

$$\alpha E_0 + \beta D_0 = (1 - \beta)\beta(1 - p)^2 b^2 + (1 - \alpha)\alpha p^2 g^2. \tag{A14}$$

Using the expressions for  $E_0$  and  $D_0$  in (A1) and (A2) yields

$$Y\alpha + \frac{1}{2}k^2\alpha X + [pF + (1 - p)V_{BR}]k\alpha = p^2 g^2\alpha(1 - 2\alpha) + (1 - p)^2 b^2(k\alpha)(1 - 2k\alpha), \tag{A15}$$

and so

$$\alpha = \frac{p^2 g^2 + (1 - p)^2 b^2 k - Y - \frac{1}{2}k^2 X - k[pF + (1 - p)V_{BR}]}{2[p^2 g^2 + (1 - p)^2 b^2 k^2]}. \tag{A16}$$

Inserting (A14) into (7) gives the shareholders' objective function as:

$$V_0 - [(1 - \beta)\beta(1 - p)^2 b^2 + (1 - \alpha)\alpha p^2 g^2].$$

Differentiating this with respect to  $k$ , and treating  $\alpha$  as a function of  $k$ , yields:

$$f(k, \theta) = X(1 - k) + 2p^2 g^2 \alpha \frac{\partial \alpha}{\partial k} + 2(1 - p)^2 b^2 k \alpha \frac{\partial(k\alpha)}{\partial k} = 0. \tag{A17}$$

(All of these derivatives are being evaluated at  $k = k^*$ ; we suppress “ $|_{k=k^*}$ ” notation for brevity.) Divide the left side of the equation into two parts:

$$f_1 = X(1 - k)$$

$$f_2 = 2p^2 g^2 \alpha \frac{\partial \alpha}{\partial k} + 2(1 - p)^2 b^2 k \alpha \frac{\partial(k\alpha)}{\partial k}.$$

The first part represents the effect of  $k$  on project selection; the second represents the effect on effort, i.e.  $f_2 = \frac{\partial \eta}{\partial k}$ . From (A17), it is easy to see that  $k^* < 1$  if and only if  $\frac{\partial \eta}{\partial k} < 0$ .

We are interested in the relationship between  $k^*$  and a parameter  $\theta$ . From  $f(k, \theta) = 0$ , we have  $\frac{\partial f}{\partial k} \frac{\partial k}{\partial \theta} + \frac{\partial f}{\partial \theta} = 0$ . Since  $\frac{\partial f}{\partial k} < 0$  at a maximum, the sign of



$\frac{\partial k}{\partial \theta}$  is the same as the sign of  $\frac{\partial f}{\partial \theta} \cdot \frac{\partial f_1}{\partial \theta}$  is simple to calculate; for  $\frac{\partial f_2}{\partial \theta}$ , we have

$$\begin{aligned} \frac{\partial f_2}{\partial \theta} &= \frac{\partial}{\partial \theta} \left[ \frac{\partial}{\partial k} (p^2 g^2 \alpha^2 + (1-p)^2 b^2 (k\alpha)^2) \right] \\ &= \frac{\partial}{\partial k} \left[ \frac{\partial}{\partial \theta} (p^2 g^2 \alpha^2 + (1-p)^2 b^2 (k\alpha)^2) \right]. \end{aligned}$$

From (A16), we have

$$\begin{aligned} &p^2 g^2 \alpha^2 + (1-p)^2 b^2 (k\alpha)^2 \\ &= \frac{[p^2 g^2 + (1-p)^2 b^2 k - Y - \frac{1}{2} k^2 X - k[pF + (1-p)V_{BR}]]^2}{4[p^2 g^2 + (1-p)^2 b^2 k^2]}, \end{aligned}$$

and so

$$\frac{\partial f_2}{\partial \theta} = \frac{\partial}{\partial k} \frac{\partial}{\partial \theta} \left( \frac{[p^2 g^2 + (1-p)^2 b^2 k - Y - \frac{1}{2} k^2 X - k[pF + (1-p)V_{BR}]]^2}{4[p^2 g^2 + (1-p)^2 b^2 k^2]} \right).$$

Differentiating (A16) with respect to  $k$  gives

$$\begin{aligned} \frac{\partial \alpha}{\partial k} &= \frac{(1-p)^2 b^2 - kX - [pF + (1-p)V_{BR}]}{2[p^2 g^2 + (1-p)^2 b^2 k^2]} \\ &\quad - \frac{p^2 g^2 + (1-p)^2 b^2 k - Y - \frac{1}{2} k^2 X - k[pF + (1-p)V_{BR}]}{2[p^2 g^2 + (1-p)^2 b^2 k^2]} \times 2(1-p)^2 b^2 k \\ &= \frac{(1-p)^2 b^2 - kX - [pF + (1-p)V_{BR}]}{2[p^2 g^2 + (1-p)^2 b^2 k^2]} - \frac{2(1-p)^2 b^2 k\alpha}{p^2 g^2 + (1-p)^2 b^2 k^2}. \end{aligned} \tag{A18}$$

From (A12) we have:

$$(1-p)^2 b^2 - kX - [pF + (1-p)V_{BR}] = \frac{k-1}{\alpha} X + 2k\alpha(1-p)^2 b^2.$$

Inserting this into (A18) yields:

$$\frac{\partial \alpha}{\partial k} = \frac{(k-1)X}{2\alpha[p^2 g^2 + (1-p)^2 b^2 k^2]} - \frac{(1-p)^2 b^2 k\alpha}{p^2 g^2 + (1-p)^2 b^2 k^2}. \tag{A19}$$

We commence with the case of  $pF + (1-p)V_{BR} > 2(1-p)b^2$ , considered in part (v) of the Proposition. (A18) yields  $\frac{1-k\alpha}{\alpha} - \frac{k\alpha}{\alpha^2} > 0$ . This implies  $1-k-k\alpha > 0$ , and so  $k^* < \frac{1}{1+\alpha} < 1$ . From (A19), we have  $\frac{\partial \alpha}{\partial k} < 0$ . We now calculate  $\frac{\partial f}{\partial \theta}$  for various parameters  $\theta$ . For  $\theta = V_{BS} - V_{BR}$ , we have

$$\begin{aligned} \frac{\partial f_1}{\partial \theta} &= \frac{2X}{V_{BS} - V_{BR}} (1-k) \\ \frac{\partial f_2}{\partial \theta} &= -\frac{X}{V_{BS} - V_{BR}} \frac{\partial(\alpha k^2)}{\partial k}. \end{aligned}$$

Hence,

$$\frac{\partial f}{\partial \theta} = \frac{X}{V_{BS} - V_{BR}} \left[ 2(1 - k) - \frac{\partial(\alpha k^2)}{\partial k} \right] = \frac{X}{V_{BS} - V_{BR}} \left[ 2(1 - k - \alpha k) - k^2 \frac{\partial \alpha}{\partial k} \right], \tag{A20}$$

Since  $(1 - k - \alpha k) > 0$  and  $\frac{\partial \alpha}{\partial k} < 0$ , this is positive.

For  $\theta = g$ ,  $\frac{\partial f}{\partial \theta}$  depends on the sign of

$$(1 - 2\alpha) \frac{\partial \alpha}{\partial k}.$$

Since  $\alpha < \frac{1}{2}$  and  $\frac{\partial \alpha}{\partial k} < 0$ , this is negative.

For  $\theta = p$ , it depends on the sign of

$$\begin{aligned} & \frac{1}{2} \frac{\partial X}{\partial p} \left[ 2(1 - k) - \frac{\partial(\alpha k^2)}{\partial k} \right] - 2\alpha(1 - p)b^2(1 - 2\alpha k) - (F - V_{BR})\alpha \\ & + 2 \frac{\partial \alpha}{\partial k} \left[ (1 - 2\alpha)pg^2 - (1 - p)b^2k(1 - 2\alpha k) \right. \\ & \left. - \frac{1}{2} \left[ k(F - V_{BR}) + \left( \frac{V_{GRH} + V_{GS}}{2} - F \right) \right] \right]. \end{aligned}$$

From (A20), we know  $2(1 - k) - \frac{\partial(\alpha k^2)}{\partial k} > 0$ ; since also  $\beta = \alpha k < \frac{1}{2}$ , the first three terms of the above expression are negative. For the whole expression to be negative, it is sufficient to show that  $(1 - 2\alpha)pg^2 - (1 - p)b^2k(1 - 2\alpha k) - \frac{1}{2}[k(F - V_{BR}) + (\frac{V_{GRH} + V_{GS}}{2} - F)] > 0$  (since  $\frac{\partial \alpha}{\partial k} < 0$ ). From (A15), we know that

$$(1 - 2\alpha)pg^2 = \frac{1}{p} \left( Y + \frac{1}{2}k^2X \right) + \frac{1}{p} [pF + (1 - p)V_{BR}]k - \frac{(1 - p)^2}{p} b^2k(1 - 2k\alpha),$$

and so

$$\begin{aligned} & (1 - 2\alpha)pg^2 - (1 - p)b^2k(1 - 2\alpha k) - \frac{1}{2} \left[ k(F - V_{BR}) + \left( \frac{V_{GRH} + V_{GS}}{2} - F \right) \right] \\ & = \frac{1}{p} \left( \frac{1}{2}Y + \frac{1}{2}k^2X \right) + k \left[ \frac{1}{2}F + \frac{1 - p}{p}V_{BR} + \frac{1}{2}V_{BR} \right] - \frac{1 - p}{p} b^2k(1 - 2\alpha k). \end{aligned}$$

This is positive, since the first term is positive, and the combination of the second and third terms is positive because  $pF + (1 - p)V_{BR} > 2(1 - p)b^2$ . Thus  $k^*$  is decreasing in  $p$ .

We now turn to the case of  $Y > \frac{1}{2}X + p^2g^2$ , considered in part (vi) of the Proposition. From (A13), we have

$$\left( \frac{k^2}{\alpha} - \frac{k}{\alpha} \right) X = Y - \frac{k^2}{2}X - (1 - 2\alpha)p^2g^2.$$

If  $k \leq 1$ , then the right hand side of the above equation is positive, which implies that  $\frac{k^2}{\alpha} - \frac{k}{\alpha} > 0$ , i.e.  $k > 1$ . This is a contradiction, so we must have  $k > 1$ . We also have

$$\begin{aligned} \frac{\partial(\alpha k)}{\partial k} &= \alpha + k \frac{\partial \alpha}{\partial k} \\ &= \alpha + \frac{k(k-1)X}{2\alpha[p^2g^2 + (1-p)^2b^2k^2]} - \frac{(1-p)^2b^2k^2\alpha}{p^2g^2 + (1-p)^2b^2k^2} \\ &= \frac{k(k-1)X}{2\alpha[p^2g^2 + (1-p)^2b^2k^2]} + \frac{p^2g^2\alpha}{p^2g^2 + (1-p)^2b^2k^2} > 0 \text{ when } k > 1. \end{aligned} \tag{A21}$$

Therefore,  $\frac{\partial(\alpha k^2)}{\partial k}$  must be also positive. We also have  $\frac{\partial(\alpha k)}{\partial k} > \frac{\partial \alpha}{\partial k}$ . For  $k > 1$ , this is immediate by comparing (A19) with (A21). For  $k < 1$ , note that  $\frac{\partial(k\alpha)}{\partial k} > \frac{\partial \alpha}{\partial k}$  if and only if  $\alpha > (1 - \frac{b}{\alpha})\frac{\partial \alpha}{\partial k}$ . If  $k < 1$ ,  $\frac{\partial \alpha}{\partial k} < 0$  and so the right-hand side of this inequality is negative and less than the left-hand side.

From (A20),  $k^*$  is decreasing in  $V_{BS} - V_{BR}$ . For  $\theta = b$ ,  $\frac{\partial f}{\partial \theta}$  depends on the sign of

$$(1 - 2\alpha k) \frac{\partial(\alpha k)}{\partial k},$$

which is positive, and so  $k^*$  is increasing in  $b$ . For  $\theta = F$ ,  $\frac{\partial f}{\partial \theta}$  depends on the sign of

$$\frac{\partial}{\partial k} (\alpha(1 - k)) = \frac{\partial \alpha}{\partial k} - \frac{\partial(\alpha k)}{\partial k},$$

which is negative, and so  $k^*$  is decreasing in  $F$ .

*Proof of part (vii).* Immediate from Equations (A1) and (A2).

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