Online Appendix for "A Multiplicative Model of Optimal CEO Incentives in Market Equilibrium"

Appendix C: Multiperiod Model

This section underpins Section 1.4, which extends the pay-performance sensitivity results of Sections 1.1-1.3 to wealth-performance sensitivity in an intertemporal framework. It shows that the key results of the one-period model still hold: since effort continues to have multiplicative costs and benefits, %-% incentives remain relevant.

As in the core model, we retain risk neutrality, except that we continue to choose the incentive-compatible contract that involves the fewest number of shares. However, we also require an incentive to smooth consumption over time to create a meaningful intertemporal model. Therefore, we use the framework of Weil (1989) and Epstein and Zin (1990), which allows us to disentangle risk aversion and intertemporal substitution.

Let W_t denote the CEO's wealth, and the value function V_t denote the discounted utility of future consumption:

$$\ln V_t = (1-\delta)\ln(c_t) + \delta \ln E_t[V_{t+1}] - \Lambda e_t \Delta t.$$

For instance, if consumption and effort are deterministic, $\ln V_t = \sum_{s=0}^{\infty} \delta^s ((1-\delta)\ln c_{t+s} - \Lambda e_{t+s})$. Note that this is still a multiplicative model, as in (1) and (23). The model is most suited for continuous time analysis, but for expositional clarity, we proceed in discrete time and take the continuous time limit where applicable.

With a logarithmic utility function, the indirect utility of wealth is $\ln V_t = \ln W_t + k$, where k is a constant independent of wealth. Therefore, the optimal consumption policy is $c_t = (1-\delta) W_t$.

The firm's return is $r_{t+1} = r_f + e_t - \overline{e} + \eta_{t+1}$, where r_f is the risk-free rate, $e_t \in \{0, \overline{e}\}$, and η_{t+1} is a mean random shock. The CEO's wealth evolves according to:

$$W_{t+1} = W_t(1 + r_f + \theta_t(e_t - \overline{e}) + \theta_t \eta_{t+1}) - c_{t+1}, \tag{43}$$

where θ_t is a performance sensitivity to be chosen optimally by the firm.

If the CEO shirks at time t, his utility $\ln V_t$ increases by $\Lambda \overline{e} \Delta t$. On the other hand, his wealth at $t+1$ is reduced by $\Delta W_{t+1} = -W_t \theta_t \overline{e} \Delta t$. Therefore, shirking increases utility $\ln V_t$ by:

$$\Delta \ln V_t = \Lambda \overline{e} \Delta t + \ln(W_t + \Delta W_t) - \ln W_t = \Lambda \overline{e} \Delta t + \ln(1 - \theta_t \overline{e} \Delta t) = \overline{e} \Delta t (\Lambda - \theta_t) + o(\Delta t).$$

We take the continuous time limit, $\Delta t \to 0$. The CEO exerts high effort if and only if $\Lambda - \theta_t \leq 0$, i.e., $\theta_t \geq \Lambda$. The pay-performance sensitivity has to be higher than Λ. As in Section 1, we select the contract that minimizes the risk in the CEO's pay. It is given by $\theta_t = \Lambda$. The wealth-performance sensitivities in Proposition 6 can be derived using Definition 2.

Appendix D: Options and Nonlinear Incentive Contracts

We return to the basic model with a binary effort decision, and generalize from stocks to a broader range of compensation instruments. The CEO receives fixed pay f, and ν units of a security; one unit of the security pays $V(P_1)$. For instance, for an option with strike price K, $V(P_1) = \max(0, P_1 - K)$. Total compensation is $c = f + \nu V(P_1)$.

In equilibrium, the CEO should be paid $w \equiv E[cg(\overline{e}) \mid e = \overline{e}]$. If the CEO shirks ($e = \underline{e}$), the CEO's utility is:

$$\begin{aligned} E[cg(0) \mid e = 0] &= E[f + \nu V(P_1(1-\overline{e}))]\, g(0) = E[f + \nu V(P_1) - \nu\Delta\overline{e}]\, g(0) \\ &= (w - \nu\Delta\overline{e}) / (1 - \Lambda\overline{e}), \end{aligned}$$

with

$$\Delta \equiv (E[V(P_1)] - E[V(P_1(1-\overline{e}))]) / \overline{e}. \tag{44}$$

Hence, the CEO works if $E[cg(\overline{e}) \mid e = \overline{e}] \geq E[cg(0) \mid e = 0]$, i.e.,

$$w \geq (w - \nu\Delta\overline{e}) / (1 - \Lambda\overline{e}) \Leftrightarrow \nu \geq \nu^* = w\frac{\Lambda}{\Delta}.$$

This leads to the following generalization of Proposition 1.

Proposition 10. ***(General incentive contracts.)*** *Using general incentive contracts, the conclusions of Proposition 1 remain the same, with a change of notation. The CEO's expected pay is w, which comprises fixed base salary f^*, and $\nu^* E[V(P_1)]$ worth of securities, with:*

$$\begin{aligned} \textit{Incentivized pay} &= \nu^* E[V(P_1)] = w\Lambda', \\ \textit{Fixed pay} &= f^* = w(1 - \Lambda'), \end{aligned} \tag{45}$$

where $\Lambda' = \Lambda E[V(P_1)] / \Delta$, $\nu^* = w\frac{\Lambda}{\Delta}$, and Δ is given by (44). Realized pay is:

$$c = w + \nu^* (V(P_1) - E[V(P_1)]).$$

Regressing the ex post compensation c on the firm return $r = P_1/P_0 - 1$ yields,

$$b^{III} = E\left[\frac{\partial c}{\partial r}\right] = \nu^* E\left[\frac{\partial V(P_0(1+r))}{\partial r}\right] = \nu^* P_0 E[V'(P_1)] = w\frac{\Lambda}{\Delta} P_0 E[V'(P_1)] = w\Lambda\xi,$$

with:

$$\xi = \frac{P_0 E[V'(P_1)]}{\Delta}. \tag{46}$$

For instance, if the security is a stock, $V(P) = P$, $\Delta = 1$, $E[V'(P_1)] = 1$, and $\xi = 1$. For general securities and small $P_1/P_0 - 1$ and $\overline{e}$, Taylor expansion yields $\Delta \to E[V'(P_1)]\, P_0$ and $\xi \to 1$. We can therefore think of ξ as approaching 1, and so the broader economics are unchanged.

Proposition 11. ***(Pay-performance sensitivities, general incentive contracts.)*** *Using general incentive contracts, the conclusions of Proposition 3 remain the same, modified only by the introduction of a parameter ξ. The pay-performance sensitivities are:*

$$b_n^I = \xi \frac{\Lambda}{L}$$
$$b_n^{II} = \xi \frac{\Lambda}{L} \frac{w}{S}$$
$$b_n^{III} = \xi \frac{\Lambda}{L} w,$$

with ξ given in (46). In many cases, $\xi \simeq 1$. Propositions 4 and 5 remain the same.

References

Epstein, L. and S. Zin. 1989. Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. *Econometrica* 57:937-968.

Weil, P. 1990. Nonexpected Utility in Macroeconomics. *Quarterly Journal of Economics* 105:29-42.