A Proofs

Proof of Lemma 1
The IC condition (9) is stronger than the ND condition (7) if and only if
\[
\frac{A_L(C_H + A_H)(1 + \bar{k}) - \mathbb{E}[A](C_L + A_L)}{\mathbb{E}[A] - A_L(1 + \bar{k})} < \frac{(C_H + A_H)\mathbb{E}[A] - (C_L + A_L)A_H(1 + \bar{k})}{A_H(1 + \bar{k}) - \mathbb{E}[A]}
\]
This yields \(1 + \bar{k} < \frac{\mathbb{E}[A]}{\sqrt{\pi}}\). Note that the RHS is always greater than one since \(\pi > \frac{1}{2}\).

Proof of Lemma 2
\(F^{EPE,IC}\) is greater than \(F^{EPE,ND,H}\) if and only if
\[
\frac{A_L\mathbb{E}[C + A](1 + \bar{k}) - A_H(C_L + A_L)}{A_H - A_L(1 + \bar{k})} > \frac{A_L(C_H + A_H) - A_H\mathbb{E}[C + A](1 + \bar{k})}{A_H(1 + \bar{k}) - A_L}
\]
which becomes
\[
1 + \bar{k} > \frac{A_H A_L}{\pi A_H^2 + (1 - \pi)A_L^2} = \frac{A_H A_L}{\mathbb{E}[A^2]}
\]
Note that the RHS is always less than one since \(\pi > \frac{1}{2}\).

Proof of Lemma 3
A type \((q,k)\) will prefer equity if and only if its unit cost of financing is no greater:
\[
\frac{C_q + A_q + F}{\mathbb{E}[E|X = E]} \leq \frac{A_q(1 + k)}{\mathbb{E}[A|X = A]}. \tag{22}
\]
The cutoff \(k_q^*\) is that which allows (22) to hold with equality. Thus, it is defined by:
\[
1 + k_q^* = \frac{C_q + A_q + F}{A_q} \frac{\mathbb{E}[A|X = A]}{\mathbb{E}[E|X = E]} \tag{23}
\]
Although \(k_q^*\) is not attainable in closed form, we can study whether \(k_H^* \leq k_L^*\). Since only the \(\frac{C_q + A_q + F}{A_q}\) term on the RHS depends on \(q\), the higher cutoff \(k_q^*\) belongs to the quality \(q\) for which this term is higher. Thus, \(k_H^* > k_L^*\) if and only if
\[
\frac{C_H + A_H + F}{C_L + A_L + F} > \frac{A_H}{A_L} \tag{24}
\]
\[ F < F^* = \frac{C_H A_L - C_L A_H}{A_H - A_L}. \]  

From the cutoff equation (23), we also have

\[ \frac{A_L(1 + k_L^*)}{E_L} = \frac{A_H(1 + k_H^*)}{E_H} = \frac{\mathbb{E}[A|X = A]}{\mathbb{E}[E|X = E]}. \]

These equations mean that, in any SE, \( k_L^* \) and \( k_H^* \) obey the following relationship:

\[ 1 + k_H^* = \lambda(F)(1 + k_L^*), \tag{26} \]

where \( \lambda(F) \equiv \frac{A_L E_H}{A_H E_L} \), which is decreasing in \( F \). If \( F < (>) F^* \), then \( \lambda > (<) 1 \) so \( k_H^* > (<) k_L^* \) from (26).

These general results hold regardless of whether the cutoffs \( k_q^* \) are interior or are at the boundaries \( \bar{k} \) or \( \underline{k} \). However, to formally prove existence of any of these equilibria, we must deal separately with the cases where cutoffs are interior or are at the boundaries. Results are summarized in the following Lemma.

**Lemma 9.** A full semi-separating equilibrium where both qualities \( q \) strictly separate \((\underline{k} < k_q^* < \bar{k})\) is sustainable under the following conditions:

(ia) If \( F < F^* \), a necessary condition is \( 1 + \bar{k} > \frac{E_H \mathbb{E}[A]}{A_H \mathbb{E}[E]} \) and a sufficient condition is \( 1 + \bar{k} \geq \frac{E_H \mathbb{E}[A]}{A_H \mathbb{E}[E]} \).

(ib) If \( F > F^* \), a necessary condition is \( 1 + \underline{k} < \frac{E_H \mathbb{E}[A]}{A_H \mathbb{E}[E]} \) and a sufficient condition is \( 1 + \underline{k} \leq \frac{A_L}{A_H} \).

(ic) If \( F = F^* \), this is sufficient for existence.

A partial semi-separating equilibrium where \( H \)'s cutoff is at a boundary is sustainable in the following cases:

(iia) If \( F < F^* \), we can sustain a SSE where all \( H \)-firms sell assets \((k_H^* = \bar{k})\) and \( L \)-firms strictly separate \((\underline{k} < k_L^* < \bar{k})\), where \( k_L^* > 0 \). A necessary condition is \( \frac{E[A]}{A_L} < 1 + \bar{k} < \frac{E_H \mathbb{E}[A]}{A_H \mathbb{E}[E]} \) and a sufficient condition is \( \frac{A_L}{A_H} \leq 1 + \bar{k} \leq \frac{E[A]}{E[H]} \).

(iib) If \( F > F^* \), we can sustain a SSE where all \( H \)-firms sell equity \((k_H^* = \underline{k})\) and \( L \)-firms strictly separate \((\underline{k} < k_L^* < \bar{k})\), where \( k_L^* < 0 \). A necessary condition is \( \frac{A_L}{A_H} < 1 + \bar{k} < \frac{E_H \mathbb{E}[E]}{A_H \mathbb{E}[H]} \) and a sufficient condition is \( \frac{A_L}{A_H} \leq 1 + \bar{k} \leq \frac{E_L \mathbb{E}[E]}{E_H \mathbb{E}[H]} \).

A partial semi-separating equilibrium where \( L \)'s cutoff is at a boundary is sustainable in the following cases:

(iia) If \( F < F^* \), we can sustain a SSE where all \( L \)-firms sell equity \((k_L^* = \bar{k})\) and \( H \)-firms strictly separate \((\underline{k} < k_H^* < \bar{k})\). A set of sufficient conditions is \( \underline{k} = 0 \),
1 + \overline{k} > \frac{E_H}{E_L}, \text{ and } \pi \text{ sufficiently close to } 1.

(iiib) If } F < F^*, \text{ we can sustain a SSE where all } L\text{-firms sell assets } (k^*_L = \overline{k}) \text{ and } H\text{-firms strictly separate } (\underline{k} < k^*_H < \overline{k}). \text{ A set of sufficient conditions is } \overline{k} = 0, 
1 + k < \frac{A_H}{\lambda_H}, \text{ and } \pi \text{ sufficiently close to } 1.

If } F \text{ is close to } F^* \text{ and } k \text{ or } k^* \text{ is extreme, synergy motives are strong, and so firms of the same quality issue different claims depending on } k. \text{ We thus have a full SSE, where firms of both quality separate. If synergies are moderate relative to information asymmetry, we have a partial SSE where all firms of one quality issue the same claim, regardless of } k, \text{ and firms of the other quality strictly separate.}

We first derive sufficient conditions under which a full SSE exists (cases (ia), (ib), and (ic) of Lemma 9). Our general proof strategy is to show existence of a pair of interior cutoffs \((k^*_H, k^*_L)\) such that neither } H \text{ nor } L \text{ has an incentive to deviate.}

We start by observing that, given any \(k^*_L\), the corresponding \(k^*_H\) in any equilibrium must be given by (26) above. The problem therefore reduces to proving the existence of a \(k^*_L \in (\underline{k}, \overline{k})\) such that \(L\) has no incentive to deviate, and such that the corresponding \(k^*_H\) is also in \((\underline{k}, \overline{k})\).

In each case, our proof technique will apply the Intermediate Value Theorem ("IVT"). This will prove the existence of a \(k^*_L\) such that \(L\) has no incentive to deviate, but will not deliver the actual value of that \(k^*_L\), so the condition \(k^*_H(k^*_L) \in (\underline{k}, \overline{k})\) cannot be checked explicitly. Instead, we will provide necessary and sufficient conditions for \(k^*_H\) to be interior regardless of the value of \(k^*_L\). These are the conditions we state in the Lemma. For each case, we provide the necessary and sufficient conditions first, then prove the existence of \(k^*_L\).

We start with part (ia), where } F < F^*. \text{ The ND condition for } (H, k^*_H) \text{ is } 1 + k^*_H = \frac{E_H}{A_H} \frac{E[A|X = A]}{E[E|X = E]}, \text{ and associated valuations } E[A|X = A] \text{ and } E[E|X = E], \text{ for some } H\text{-firms to be willing to issue equity (so that } k^*_H \text{ is interior), we must have}

\[1 + \overline{k} > \frac{E_H}{A_H} \frac{E[A|X = A]}{E[E|X = E]]. \quad (27)\]

The RHS is bounded below by \(\frac{E_H}{A_H} \frac{E[A]}{E[E]}\) and above by \(\frac{E_H}{E_L}\). Thus, a sufficient condition for some } H\text{-firms to issue equity is } 1 + \overline{k} > \frac{E_H}{E_L} \text{ and a necessary condition is } 1 + \overline{k} > \frac{E_H}{A_H} \frac{E[A]}{E[E]}]. \text{ These quantities no longer depend on } k^*_L \text{ since they contain no conditional expectations. Now, given that } k^*_H(k^*_L) \in (\underline{k}, \overline{k}) \text{ for any } k^*_L, \text{ we demonstrate the existence of an equilibrium cutoff } k^*_L. \text{ For a candidate cutoff } k'_L, \text{ the indifference condition for}
\((L, k'_L)\) is

\[
1 + k'_L = \frac{E_L \mathbb{E}[A|X = A]}{A_L \mathbb{E}[E|X = E]}
\] (28)

Using this, we can rewrite the incentive of \((L, k'_L)\) to sell assets rather than issue equity as a function continuous in \(k'_L\):

\[
f(k'_L) \equiv \frac{E_L}{\mathbb{E}[E|X = E]} - \frac{A_L(1 + k'_L)}{\mathbb{E}[A|X = A]}.
\]

If \(f(k'_L) > (\leq) 0\), \((L, k'_L)\) will sell assets (equity). Thus, \(k'_L\) is an equilibrium cutoff if and only if \(f(k'_L) = 0\). We show that \((L, k'_L)\) sells assets if \(1 + k'_L = \frac{E_L \mathbb{E}[A]}{A_L \mathbb{E}[E]}\), and equity if \(1 + k'_L = \frac{1 + \lambda}{A_L \mathbb{E}[E]}\). (The latter yields the highest possible \(k'_L\), since \(k'_L\) and \(k'_H\) are related by (26), and \(k'_H\) is capped at \(\bar{k}\).) Then, by the IVT, there exists a \(k'_L\) between these two values of \(k'_L\) for which \(f(k'_L) = 0\) and so the firm is indifferent. (Note that \(f\) is everywhere defined and continuous on the interval \([k, \bar{k}]\).)

To show that \((L, k'_L)\) sells assets if \(1 + k'_L = \frac{E_L \mathbb{E}[A]}{A_L \mathbb{E}[E]}\), we use the fact that \(F < F^*\) implies \(\lambda(F) > 1\) and so \(k'_H > k'_L\). We thus have \(\mathbb{E}[A|X = A] > \mathbb{E}[A]\) and \(\mathbb{E}[E|X = E] < \mathbb{E}[E]\), which yields \(f(k'_L) > 0\). Similarly, \(1 + k'_L = \frac{1 + \lambda}{A_L \mathbb{E}[E]}\) yields

\[
f(k'_L) = \frac{E_L}{\mathbb{E}[E|X = E]} - \frac{A_H(1 + \bar{k})}{\mathbb{E}[A|X = A]} \mathbb{E}[E],
\]

and so \(f(k'_L) < 0\) holds if and only if \(1 + \bar{k} > \frac{E_L \mathbb{E}[A|X = A]}{A_H \mathbb{E}[E|X = E]}\), which is the same condition as (27). Thus, the sufficient condition for \(H\) to follow an interior cutoff, \(1 + \bar{k} > \frac{E_H \mathbb{E}[A]}{A_H \mathbb{E}[E]}\), is also sufficient for the IVT to imply an equilibrium \(k'^*_L\), and so is sufficient for the SSE to exist.

The analysis for part (ib) \((F > F^*)\) is analogous. The ND condition is now

\[
1 + k < \frac{E_H \mathbb{E}[A|X = A]}{A_H \mathbb{E}[E|X = E]}.
\] (29)

With \(F > F^*\) we now have \(\mathbb{E}[A] > \mathbb{E}[A|X = A]\) and \(\mathbb{E}[E] < \mathbb{E}[E|X = E]\), so the RHS of (29) is bounded above by \(\frac{E_H \mathbb{E}[A]}{A_H \mathbb{E}[E]}\). Thus, a sufficient condition for some \(H\)-firms to sell assets is \(1 + k \leq \frac{A_H}{A_H} \mathbb{E}[E]\), and a necessary condition is \(1 + k < \frac{E_H \mathbb{E}[A]}{A_H \mathbb{E}[E]}\). We now turn to the ND condition for \((L, k'_L)\), which remains (28), and again use the IVT. We can easily show that \((L, k'_L)\) will deviate to equity at \(1 + k'_L = \frac{E_L \mathbb{E}[A|X = A]}{A_L \mathbb{E}[E|X = E]}\). A sufficient condition for \((L, k'_L)\) to deviate to asset sales at \(1 + k'_L = \frac{1 + \lambda}{A_L \mathbb{E}[E]}\) is \(1 + k \leq \frac{A_H}{A_H} \mathbb{E}[E]\), which is the same as the sufficient condition for some \(H\)-firms to sell assets, and so is sufficient for the SSE to exist.
We next to the partial SSEs in parts (ii) and (iii), where one cutoff is at a boundary. In case (iia), all $H$-firms sell assets and $L$-firms choose an interior cutoff. Assets are priced at $E[A|X = A] > E[A]$ and equity is priced at $E_L$. The ND condition for $H$-firms is:

$$1 + \bar{k} \leq \frac{E[A|X = A]}{A_H} \frac{E_H}{E_L} = \lambda(F) \frac{E[A|X = A]}{A_L}. \quad (30)$$

A sufficient condition for (30) is $1 + \bar{k} \leq \frac{E_H}{E_L} \frac{E[A]}{A_H}$ and a necessary condition is $1 + \bar{k} < \frac{E_H}{E_L}$. The indifference condition for $(L, k^*_L)$ yields

$$1 + k^*_L = \frac{E[A|X = A]}{A_L}, \quad (31)$$

and so $k^*_L > 0$: since assets are priced above their unconditional mean, $L$ is willing to sell them even if they are synergistic. For (31) to hold, we must have $1 + \bar{k} > \frac{E[A|X = A]}{A_L}$, for which $1 + \bar{k} > \frac{A_H}{A_L}$ is a sufficient condition and $1 + \bar{k} > \frac{E[A]}{A_H}$ is a necessary condition. Combining (30) with (31), we have $\frac{E[A|X = A]}{A_L} < 1 + \bar{k} \leq \lambda(F) \frac{E[A|X = A]}{A_L}$. Since $\lambda(F^*) = 1$ and $\lambda'(F) < 0$, both conditions can be simultaneously satisfied only if $F < F^*$.

Finally, we need to show that a cutoff $k^*_L$ actually exists at which the cutoff type $(L, k^*_L)$ is indifferent between asset sales and equity (at which the equilibrium condition (31) holds). We again employ the IVT. If we specify a cutoff $1 + k'_L$ equal to the necessary lower bound $\frac{E[A]}{A_L}$ on $1 + \bar{k}$, $(L, k'_L)$ deviates to asset sales. Meanwhile, if we specify $1 + k'_L = \frac{A_H}{A_L}$, $(L, k'_L)$ deviates to equity. Thus, a pair of sufficient conditions for existence of the equilibrium is $1 + \bar{k} \geq \frac{A_H}{A_L}$ and $1 + \bar{k} \leq \frac{E_H}{E_L} \frac{E[A]}{A_L}$.

In case (iib), all $H$-firms issue equity and $L$-firms choose an interior cutoff. Assets are priced at $A_L$ and equity is priced at $E[E|X = E] > E[E]$. The ND condition for $H$-firms is

$$1 + k \geq \frac{A_L}{A_H} \frac{E_H}{E[E|X = E]} = \lambda(F) \frac{E_L}{E[E|X = E]} \quad (32)$$

A sufficient condition for (32) is $1 + k \geq \frac{A_H}{A_L} \frac{E_H}{E[E]}$ and a necessary condition is $1 + k > \frac{A_L}{A_H}$. The indifference condition for $(L, k^*_L)$ yields

$$1 + k^*_L = \frac{E_L}{E[E|X = E]}, \quad (33)$$

and so $k^*_L < 0$. For (33) to hold, we must have $1 + \bar{k} < \frac{E_L}{E[E|X = E]}$, for which $1 + \bar{k} \leq \frac{E_L}{E[H]}$ is a sufficient condition and $1 + \bar{k} < \frac{E_L}{E[E]}$ is a necessary condition. Combining (32) with (33), we have $\lambda(F) \frac{E_L}{E[E|X = E]} \leq 1 + \bar{k} < \frac{E_L}{E[E|X = E]}$. Since $\lambda(F^*) = 1$ and $\lambda'(F) < 0$, both conditions can be simultaneously satisfied only if $F > F^*$.  

44
Finally, we need to show that a cutoff \( k_L^* \) actually exists at which the cutoff type \((L, k_L^*)\) is indifferent given the resulting equilibrium valuations. We again employ the IVT. If we specify a cutoff \( 1 + k_L \) equal to the necessary upper bound \( \frac{E_L}{E[H]} \) on \( 1 + k \), \((L, k_L^*)\) deviates to equity. Meanwhile, if we specify \( 1 + k_L = \frac{E_L}{E[H]} \), \((L, k_L^*)\) deviates to asset sales. Thus, a pair of sufficient conditions for existence of the equilibrium is \( 1 + k < \frac{E_L}{E[H]} \) and \( 1 + k > \frac{A_L}{A_H} \frac{E_H}{E[L]} \).

In case (iiiA), assets are priced at \( A_H \) and equity is priced at \( E[E|X = E|] < E[E] \). The ND condition for \( L \) is \( \frac{A_L(1 + k)}{A_H} \geq \frac{E_L}{E[L]} \), or equivalently

\[
\left( 1 + Pr(q = H|X = E) \frac{E_H - E_L}{E_L} \right) (1 + k) \geq 1 + \frac{A_H - A_L}{A_L}.
\]

Note that \( \frac{E_H - E_L}{E[L]} > \frac{A_H - A_L}{A_L} \) if and only if \( F < F^* \). Then the inequality is satisfied if \( F \leq F^* \), \( k = 0 \), and \( Pr(q = H|X = E) \) is sufficiently high. \( Pr(q = H|X = E) \) approaches 1 from below as \( \pi \to 1 \) (in the limit, there are only \( H \) firms remaining), so for \( \pi \) sufficiently close to 1 we can satisfy the inequality and all \( L \) firms will cooperate with the equilibrium. It remains to show that there is an equilibrium \( k_H^* \) at which type \((H, k_H^*)\) is indifferent between selling assets and issuing equity. We again apply the IVT. First, we show that there is a candidate value \( k_H^* \) at which \((H, k_H^*)\) deviates to selling assets. This occurs if \( 1 + k_H^* < \frac{E_H}{E[L]} \), which will be satisfied if, for example, \( k_H^* = 0 \). Next, we find a candidate \( k_H^* \) at which \((H, k_H^*)\) deviates to issuing equity: this happens if \( 1 + k_H^* > \frac{E_H}{E[L]} \). A sufficient condition for such a \( k_H^* \) to exist is that potential synergies be very high: if \( 1 + \bar{k} > \frac{E_H}{E[L]} \), then we can specify a \( k_H^* \) that will deviate to equity issuance regardless of the price reaction. Then an equilibrium \( k_H^* \) exists between 0 and \( \frac{E_H}{E[L]} \), allowing the equilibrium to exist.

The proof for case (iiib) is analogous. Assets are priced at \( E[A|X = A] < E[A] \) and equity is priced at \( E_H \). The ND condition for \( L \) is \( \frac{A_L(1 + \pi)}{A_H} \geq \frac{E_L}{E[H]} \), or equivalently

\[
\left( 1 + \frac{E_H - E_L}{E_L} \right) (1 + \bar{k}) \leq 1 + Pr(q = H|X = A) \left( \frac{A_H - A_L}{A_L} \right).
\]

This will be satisfied if \( F \geq F^* \), \( \bar{k} = 0 \), and \( \pi \) is sufficiently close to 1 so that \( Pr(q = H|X = A) \) is also close to 1. It remains to show that an equilibrium \( k_H^* \) exists. Type \((H, k_H^*)\) will deviate to asset sales if \( 1 + k_H^* < \frac{E[A|X = A]}{A_H} \), A sufficient condition for such a \( k_H^* \) to exist is \( 1 + k_H^* < \frac{A_L}{A_H} \). Type \((H, k_H^*)\) will deviate to equity issuance if \( 1 + k_H^* > \frac{E[A|X = A]}{A_H} \), which is satisfied for \( k_H^* = 0 \). Thus all the conditions stated in the Lemma are sufficient for the equilibrium to exist.
Proof of Proposition 1

Parts (ia), (ib), and (ii) follow from the discussion of the various equilibria in Lemmas 1-3.

For (ic), we first prove $F^{EPE,IC} < F^* < F^{APE,IC}$. Suppose $F \leq F^{EPE,IC}$. This means that the IC is violated for $EPE$, so that $\frac{A_L(1+k)}{A_H} \geq \frac{E_L}{E_H}$. This implies $\frac{A_L}{A_H} > \frac{E_L}{E_H}$ and so $F < F^*$. Thus $F^{EPE,IC} < F^*$. Similarly, suppose $F \geq F^{APE,IC}$. This means that the IC is violated for $APE$, so that $\frac{E_L}{E_H} \geq \frac{A_L(1+k)}{E[A]}$. This implies $\frac{E_L}{E_H} > \frac{A_L}{A_H}$, and so $F > F^*$. Thus $F^{APE,IC} > F^*$.

Next, we prove that $F^* \leq F^{APE,ND,H}$. $F^* \leq F^{APE,ND,H}$ if $F \geq F^{APE,ND,H}$ implies $F \geq F^*$; and the inequality is strict if $F \geq F^{APE,ND,H}$ implies $F > F^*$. Suppose that $F \geq F^{APE,ND,H}$, so that some $H$-firm would deviate under $APE$, i.e. $\frac{A_H(1+E)}{E[A]} \geq \frac{E_H}{E_L}$. If $1 + k < \frac{E[A]}{A_L}$, then $\frac{A_H(1+E)}{E[A]} < \frac{A_H}{A_L}$ and thus $F > F^*$. If we only have $1 + k \leq \frac{E[A]}{A_L}$, then $\frac{A_H(1+E)}{E[A]} \leq \frac{A_H}{A_L}$ and thus $F \geq F^*$. Recall that $1 + k \leq \frac{E[A]}{A_L}$ was a necessary condition for $APE$ to be sustainable, from part (i). Thus, $F^* \leq F^{APE,ND,H}$ whenever $APE$ is sustainable, and the inequality is strict except when $1 + k$ exactly equals $\frac{E[A]}{A_L}$.

Finally, we prove $F^{APE,ND,H} \leq F^*$. $F^{APE,ND,H} \leq F^*$ if $F \leq F^{APE,ND,H}$ implies $F < F^*$ Suppose $F \leq F^{APE,ND,H}$, so that some $H$-firm weakly prefers to deviate under $EPE$, i.e. $\frac{E_L}{E[H]} \geq \frac{A_H(1+E)}{E[A]}$. If $1 + k > \frac{E[L]}{E[H]}$, then $\frac{E_L}{E[H]} > \frac{A_H}{A_L}$ and thus $F < F^*$. If we only have $1 + k \geq \frac{E[L]}{E[H]}$, then $\frac{E_L}{E[H]} \geq \frac{A_H}{A_L}$ and thus $F \leq F^*$. Recall that $1 + k \geq \frac{E[L]}{E[H]}$ was a necessary condition for $EPE$ to be sustainable, from part (i). Thus, whenever $EPE$ is sustainable, we have $F^{APE,ND,H} \leq F^*$, and the inequality is strict except when $1 + k$ exactly equals $\frac{E[L]}{E[H]}$.

Taking these three points together, whenever both $PE$s are sustainable, $F^{EPE} \leq F^{APE}$. The inequality is strict unless $1 + k = \frac{E[L]}{A_L}$ and $1 + k = \frac{E[L]}{E[H]}$.

Proof of Lemma 4

This Lemma differs from Lemma 1 in two ways: it contains $r_q$ terms and inaction has been added to the action space for each firm. The $r_q$ terms modify the bounds stated in (iii) and (iv), but their derivation is otherwise exactly analogous to that in Lemma 1. It only remains to examine any new conditions that arise due to the firm’s option to do nothing.

The equilibrium requires three new conditions due to the expanded action space: one each to guarantee that no $H$- or $L$-firms deviate to inaction, and one to guarantee that the IC continues to hold (that, if equity issuance is inferred as stemming from $H$, there is some type $(L,k)$ that will deviate to equity issuance rather than continuing
to sell assets or deviating to inaction). Of these, the only nontrivial condition is the
first, that no $H$-firm deviates to inaction, which gives rise to the new condition (ii).
Intuitively, $H$ suffers a fundamental loss from asset sales, so if the investment opportu-
nity is sufficiently small, he will deviate to inaction. The condition to prevent $L$ from
deviating to inaction is trivial, since he benefits from both the growth opportunity and
the fundamental gain. The same logic implies that any $L$ with non-positive synergies
will prefer equity issuance to inaction if inferred as $H$, satisfying the IC.

**Proof of Lemma 5**

The derivation of the conditions is exactly analogous to that in Lemma 2, reworked
to include the $r_q$ terms. The only new condition is part (ii), which prevents $H$ from
deviating to inaction.

**Proof of Proposition 2**

Parts (ia) and (ib) repeat the results of the previous two Lemmas. In part (ic),
we start with the $SSE$, which is similar to Lemma 3. $L$-firms will not deviate to
inaction, as they are enjoying a fundamental gain and exploiting a desirable investment
opportunity. A high-quality equity issuer will not deviate to inaction if

$$1 + r_H \geq \frac{E_H}{\mathbb{E}[E|X = E]},$$

i.e., the capital loss from selling undervalued equity is less than the value of the growth
opportunity. Similarly, a high-quality asset seller will not deviate if

$$1 + r_H \geq \frac{A_H (1 + k_H)}{\mathbb{E}[A|X = A]}.$$\

Since $k^*_H$ is defined by $\frac{E_H}{\mathbb{E}[E|X = E]} = \frac{A_H (1 + k^*_H)}{\mathbb{E}[A|X = A]}$, we have $\frac{A_H (1 + k_H)}{\mathbb{E}[A|X = A]} \leq \frac{E_H}{\mathbb{E}[E|X = E]}$ for all asset
sellers (because $k_H \leq k^*_H$). Thus, (34) is the applicable lower bound on $r_H$ for no firm
to deviate. Since $\mathbb{E}[E|X = E]$ is an equilibrium value, the sufficient condition in terms
of model primitives is $1 + r_H \geq \frac{E_H}{E_L}$.

Turning to part (ii), we start by considering the case of interior cutoffs. The definitions
of $k^*_H$ and $k^*_L$ in the Proposition are given by the indifference conditions. $L$-firms
will not deviate to inaction, as they are enjoying a (weakly positive) fundamental gain
and exploiting a desirable investment opportunity. An inactive $H$-firm will not deviate
to equity issuance if

$$1 + r_H \leq \frac{E_H}{E_L}.$$
i.e., the capital loss from selling undervalued equity exceeds the value of the growth opportunity. If the above is satisfied, it is easy to show that a high-quality asset seller will not deviate either to inaction or equity issuance.

Combining \(1 + r_H = \frac{A_H(1+k_H^*)}{E[A|X=A]}\) and \(1 = \frac{A_L(1+k_L^*)}{E[A|X=A]}\) (the definition of the cutoffs if they are interior) yields

\[
(1 + r_H) \frac{A_L}{A_H} = \frac{1 + k_H^*}{1 + k_L^*}.
\]

When \(r_H\) is high (specifically, \(1 + r_H > \frac{A_H}{A_L}\)), we have \(k_H^* > k_L^*\): \(H\) is more willing to sell assets than \(L\) because, if it switches to doing nothing, it loses the growth opportunity (whereas \(L\) continues to exploit the growth opportunity if it does not sell assets, since it issues equity instead). When \(1 + r_H \leq \frac{A_H}{A_L}\), we have \(k_H^* \leq k_L^*\): \(H\) is less willing to sell assets than \(L\), because they are undervalued. Note that \(r_H\) is bounded above, since \(1 + r_H < \frac{E_H}{E_L}\) for this equilibrium to hold. Thus, we have

\[
1 + r_H = \frac{1 + k_H^* A_H}{1 + k_L^* A_L}
\]

Finally, if \(1 + r_H < \frac{A_H}{A_L}(1 + k)\), then all \(H\)-firms do nothing: we have a boundary cutoff. The investment opportunity is sufficiently unattractive, and dissynergies are sufficiently weak, that no \(H\)-firm wishes to sell its high-quality assets for a low price.

**Proof of Lemma 6**

As discussed in the text, it is trivial that \(H\) will cooperate. The condition stated in the Lemma is \(L\)’s ND condition. If \(L\) issues equity, it is correctly valued. Both fundamental value and the stock price, and thus the manager’s objective function, equal \(C_L + A_L\). If \(L\) deviates to selling assets, its stock price will be \(C_H + A_H\), and it will receive a price of \(\frac{A_L}{A_H}\) for each dollar of assets sold. \(L\) will thus cooperate with equity issuance if

\[
C_L + A_L \geq \omega(C_H + A_H) + (1 - \omega) \left( C_L + A_L + F - F \frac{A_L}{A_H} \right),
\]

which simplifies to the condition stated in the text.
Proof of Lemma 7

As discussed in the text, it is trivial that $H$ will cooperate. The condition stated in the Lemma is $L$’s ND condition. If $L$ sells assets, these are valued at the pooled price of $\pi A_H + (1 - \pi) A_L$. Similarly, its stock price is $\pi(C_H + A_H) + (1 - \pi)(C_L + A_L)$. If $L$ deviates to issuing equity, this will be valued correctly at $C_L + A_L + F$ and its stock price will be correct at $C_L + A_L$, so its objective function is simply $C_L + A_L$. $L$ will thus cooperate with asset sales if

$$\omega(\pi(C_H + A_H) + (1 - \pi)(C_L + A_L)) + (1 - \omega)\left( C_L + A_L + F - F\left( \frac{A_L}{\pi A_H + (1 - \pi)A_L} \right) \right) \geq C_L + A_L,$$

which simplifies to the condition stated in the text.

Proof of Lemma 8

As discussed in the text, it is trivial that $L$ will cooperate. The condition stated in the Lemma is $H$’s ND condition. If $H$ issues equity, it is valued at $\pi(C_H + A_H + F) + (1 - \pi)(C_L + A_L + F)$ and its stock price is $\pi(C_H + A_H) + (1 - \pi)(C_L + A_L)$, since $H$ pools with all other firms. If $H$ deviates to selling assets, the assets sold will be valued at $A_L$, and its stock price will be $C_L + A_L$. $H$ will cooperate with equity issuance if

$$\omega(\mathbb{E}[C + A]) + (1 - \omega)\left( C_H + A_H + F - F\left( \frac{A_L}{\mathbb{E}[C + A] + F} \right) \right) \geq \omega(C_L + A_L) + (1 - \omega)\left( C_H + A_H + F - F\left( \frac{A_L}{A_H} \right) \right).$$

To derive the IC condition, $L$ will sell assets if inferred as $H$ if

$$\omega(C_H + A_H) + (1 - \omega)\left( C_L + A_L + F - F\left( \frac{A_L}{A_H} \right) \right) > \omega\mathbb{E}[C + A] + (1 - \omega)\left( C_L + A_L + F - F\left( \frac{C_L + A_L + F}{\mathbb{E}[C + A] + F} \right) \right).$$

Both inequalities simplify to the conditions stated in the text.

To show that the IC condition is stronger, we compare the conditions as stated in the text. Since $\pi > 1/2$ and so $1 - \pi < \pi$, it is sufficient to show that

$$\frac{A_L}{A_H} - \frac{C_L + A_L + F}{\mathbb{E}[C + A] + F} > \frac{C_H + A_H + F}{\mathbb{E}[C + A] + F} - \frac{A_H}{A_L}.$$
This inequality can be rearranged to
\[
\frac{A_L + A_H}{A_H} > \frac{(C_H + A_H + F) + (C_L + A_L + F)}{E[C + A] + F}.
\]
The LHS takes the form \(a + \frac{1}{a}\) with \(a > 0\), and therefore exceeds 2. The RHS is less than 2, because \(\pi > 1/2\) means that the uninformed valuation of equity (the denominator) is greater than the equal-weighted average of the high and low values (half of the numerator). Thus, the inequality holds, and so the IC condition is stronger.

**Proof of Proposition 3**

We first show that \(\omega^{APE} < \omega^{SE}\). To facilitate the comparison, we multiply by \(\pi\) the top and bottom of the bound \(\omega^{SE}\) stated in Lemma 6:
\[
\omega^{SE} = \frac{F^{\frac{\pi(A_L-A_H)}{A_H}}}{F^{\frac{\pi(A_L-A_H)}{A_H}} + \pi(C_H - C_L) - (A_L - A_H)}.
\]
Now both \(\omega^{SE}\) and \(\omega^{APE}\) are of the form \(\frac{a}{a + \frac{1}{a}}\), with a common value of \(x \equiv (C_H - C_L) - (A_L - A_H)\). They are increasing in \(\alpha\) since \(x > 0\) (by assumption (1), \(C_H - C_L > A_L - A_H\)), so it only remains to show that the numerator of \(\omega^{SE}\) is greater than that of \(\omega^{APE}\). Note that in the numerator of \(\omega^{SE}\), \(\pi(A_L - A_H) = A_L - \mathbb{E}[A]\), so we wish to show \(\frac{A_L - \mathbb{E}[A]}{\mathbb{E}[A]} < \frac{A_L - \mathbb{E}[A]}{A_H}\), which follows from \(A_H < \mathbb{E}[A]\).

We next show that \(\omega^{SE} < \omega^{EPE}\). First, since \(\pi > \frac{1}{2} > 1 - \pi\), we have
\[
\omega^{EPE} > \frac{F\left(\frac{A_L}{A_H} - \frac{E_L}{\mathbb{E}[E]}\right)}{\pi((C_H - C_L) - (A_L - A_H)) + F\left(\frac{A_L}{A_H} - \frac{E_L}{\mathbb{E}[E]}\right)}.
\]
As before, the form of the expressions involved mean that it is sufficient to show
\[
\frac{A_L}{A_H} - \frac{E_L}{\mathbb{E}[E]} > \frac{\pi(A_L - A_H)}{A_H} = \frac{A_L}{A_H} - \frac{\mathbb{E}[A]}{A_H},
\]
i.e., \(\frac{\mathbb{E}[A]}{A_H} > \frac{E_L}{\mathbb{E}[E]}\). This holds because \(A_H < \mathbb{E}[A]\) and \(\mathbb{E} > E_L\).
B Selling the Core Asset

B.1 Positive Correlation

This subsection extends the core positive correlation model of Section 2.1 to allow the firm to sell the core asset (in addition to the non-core asset and equity). Proposition 4 below characterizes which equilibria are sustainable and when. For simplicity of exposition, we shut down synergies ($\bar{k} = \bar{k} = 0$), but the results extend to the case of general $\bar{k}$ and $\bar{k}$.

**Proposition 4.** (Positive correlation, selling the core asset.) Consider a pooling equilibrium where all firms sell non-core assets ($X = A$) and a firm that sells equity or the core asset is inferred as $L$. The prices of core assets, non-core assets, and equity are $C_L$, $\pi A_H + (1 - \pi) A_L$, and $C_L + A_L + F$, respectively. This equilibrium is sustainable if the following conditions hold:

\[
F \leq F^{APE, IC} = \frac{A_L(C_H + A_H) - \mathbb{E}[A](C_L + A_L)}{\mathbb{E}[A] - A_L}
\]

(35)

\[
\frac{\mathbb{E}[A]}{A_L} \leq \frac{C_H}{C_L}.
\]

(36)

Consider a pooling equilibrium where all firms sell core assets ($X = C$) and a firm that sells equity or the non-core asset is inferred as $L$. The prices of core assets, non-core assets, and equity are $\pi C_H + (1 - \pi) C_L$, $A_L$, and $C_L + A_L + F$, respectively. This equilibrium is sustainable if the following conditions hold:

\[
F \leq \frac{(C_H + A_H) C_L - (C_L + A_L) \mathbb{E}[C]}{\mathbb{E}[C] - C_L}
\]

(37)

\[
\frac{A_L}{A_H} \leq \frac{C_L}{\mathbb{E}[C]}.
\]

(38)

Consider a pooling equilibrium where all firms sell equity ($X = E$) and a firm that sells either asset is inferred as $L$. The prices of core assets, non-core assets, and equity are $C_L$, $A_L$, and $\pi (C_H + A_H) + (1 - \pi) (C_L + A_L) + F$, respectively. This equilibrium is sustainable if the following conditions hold:

\[
F \geq F^{EPE, IC} = \frac{A_L \mathbb{E}[C + A] - A_H (C_L + A_L)}{A_H - A_L}
\]

(39)

\[
F \geq \frac{C_L \mathbb{E}[C + A] - C_H (C_L + A_L)}{C_H - C_L}.
\]

(40)
For the APE, equation (35) is the same as (9) in the core model: it means that the OEPB that an equity issuer is of quality L satisfies the IC. Equation (36) is new and guarantees that the belief that a core asset seller is of quality L satisfies the IC. This condition is stronger than the condition that prevents H deviating to sell the core asset. For the core-asset-pooling equilibrium (CPE), equations (37) and (38) guarantee that the OEPB, that a seller of the non-core asset or equity is of quality L, satisfies the IC. Again, these conditions are stronger than the conditions preventing H from deviating to either of these actions.

The main result of Proposition 4 is to show that an EPE is still sustainable. Equations (39) is the same as (12) in the core model: it means that the OEPB that a seller of the non-core asset is of quality L satisfies the IC. Equation (40) is new and guarantees that the belief that a core-asset seller is of quality L is also consistent with the IC. Again, it is stronger than the condition preventing H from deviating to this action. It is possible for both inequalities to be satisfied: thus, equity issuance may be sustainable even though it does not exhibit the least information asymmetry (absent the certainty effect). One of the assets (core or non-core) will exhibit more information asymmetry than the other; since equity is a mix of both assets, its information asymmetry will lie in between. Even though equity is never the safest claim, it may still be issued, if F is sufficiently large, due to the certainty effect.

Comparing the conditions from CPE and APE that prevent H from deviating to non-core and core assets, respectively, the former is harder to satisfy if

\[
\frac{A_H}{A_L} < \frac{C_H}{C_L}.
\]

Thus, as is intuitive, if the core asset exhibits greater information asymmetry, it is more difficult to sustain its sale. This result is a natural extension of MM.

B.2 Negative Correlation

We now move to the negative correlation case. Proposition 5 characterizes the equilibria.

Proposition 5. (Negative correlation, selling the core asset.) Consider a pooling equilibrium where all firms sell non-core assets \((X = A)\) and a firm that sells equity or the core asset is inferred as L. The prices of core assets, non-core assets, and equity are \(C_L, \pi A_H + (1 - \pi)A_L,\) and \(C_L + A_L + F,\) respectively. This equilibrium is sustainable
if the following conditions hold:

\[ \omega \geq \frac{F \left( \frac{A_L}{E[A]} - 1 \right)}{\pi((C_H - C_L) - (A_L - A_H)) + F \left( \frac{A_L}{E[A]} - 1 \right)}. \]  

(41)

Consider a pooling equilibrium where all firms sell core assets \((X = C)\) and a firm that sells equity or the non-core asset is inferred as \(L\). The prices of core assets, non-core assets, and equity are \(\pi C_H + (1 - \pi) C_L, A_L, \) and \(C_L + A_L + F\), respectively. This equilibrium is sustainable if the following conditions hold:

\[ \omega \geq \frac{F \left( \frac{C_H}{E[C]} - \frac{A_H}{A_L} \right)}{\pi((C_H - C_L) - (A_L - A_H)) + F \left( \frac{C_H}{E[C]} - \frac{A_H}{A_L} \right)}. \]  

(42)

\[ \omega \geq \frac{F \left( \frac{A_L}{A_H} - \frac{C_L}{E[C]} \right)}{(1 - \pi)((C_H - C_L) - (A_L - A_H)) + F \left( \frac{A_L}{A_H} - \frac{C_L}{E[C]} \right)}. \]  

(43)

Consider a pooling equilibrium where all firms sell equity \((X = E)\) and a firm that sells either asset is inferred as \(L\). The prices of core assets, non-core assets, and equity are \(C_L, A_L, \) and \(\pi (C_H + A_H) + (1 - \pi) (C_L + A_L) + F\), respectively. This equilibrium is sustainable if the following conditions hold:

\[ \omega \geq \frac{F \left( \frac{C_H + A_H + F}{E[C+A]} - \frac{A_H}{A_L} \right)}{\pi((C_H - C_L) - (A_L - A_H)) + F \left( \frac{C_H + A_H + F}{E[C+A]} - \frac{A_H}{A_L} \right)}. \]  

(44)

\[ \omega \geq \frac{F \left( \frac{A_L}{A_H} - \frac{E_I}{E[I]} \right)}{(1 - \pi)((C_H - C_L) - (A_L - A_H)) + F \left( \frac{A_L}{A_H} - \frac{E_I}{E[I]} \right)}. \]  

(45)

Starting with the APE, equation (41) is the ND condition for \(L\) not to deviate to equity, and is the same as equation (19) in the core model. If \(L\) deviates to selling the core asset, his objective function is also \(C_L + A_L\) and so we have the same condition. This is intuitive: regardless of whether he deviates to the core asset or equity, the claim he issues is fairly priced as he is revealed as \(L\). The IC condition that a seller of the core asset or equity is of quality \(L\) is trivially satisfied.

Moving to a CPE, \(L\) will automatically not deviate. If \(H\) deviates to sell the non-core asset, his unit cost of financing is \(\frac{A_H}{A_L} < 1\), but if he deviates to sell equity, his unit cost of financing is \(\frac{C_H + A_H + F}{C_L + A_L + F} > 1\). Thus, he will prefer to deviate to non-core assets,
and we have the ND condition stated. The IC condition that a seller of the core asset is of quality \( L \) is again trivially satisfied; the IC condition that a seller of the non-core asset is of quality \( L \) is satisfied if (43) holds.

Finally, in the \( EPE \), \( L \) will automatically not deviate. \( H \) will deviate to non-core assets rather than the core asset, since \( \frac{A_H}{A_L} < 1 < \frac{C_H}{C_L} \), and so we have the same ND condition as before. The IC condition that a seller of the core asset is of quality \( L \) is again trivially satisfied; the IC condition that a seller of the non-core asset is of quality \( L \) is satisfied if (45) holds. Equation (45) is stronger than (41), the \( APE \) lower bound, if and only if:

\[
\pi \left( \frac{A_L}{A_H} - \frac{C_L}{E[C]} \right) > (1 - \pi) \left( \frac{A_L - E[A]}{E[A]} \right) \\
\pi \left( \frac{C_L}{E[C]} \right) < \frac{\pi A_L E[A] - (1 - \pi) A_H (A_L - E[A])}{A_H E[A]}. 
\]

Since \( C_L < E[C] \), it is sufficient that

\[
\pi A_H E[A] < \pi A_L E[A] - (1 - \pi) A_H (A_L - E[A]) \\
0 < (A_L - A_H) \pi E[A] - (1 - \pi) A_H (A_L - E[A]),
\]

which is true since \( \pi > 1 - \pi, (A_L - A_H) > (A_L - E[A]), \) and \( E[A] > A_H \).

Thus, the \( APE \) is easier to sustain than the \( CPE \). This is a simple extension of the camouflage effect of the core model. A deviation from the \( APE \) to either the core asset or equity is relatively unattractive, since the firm suffers a “lemons” discount on both the security being issued and the rest of the firm as a whole. This is because both the core asset and equity are positively correlated with the value of the firm. In contrast, a deviation from either the \( CPE \) or the \( EPE \) to selling the non-core asset is harder to rule out: even if a high price is received for the non-core asset, this does not imply a high valuation for the firm as a whole, and so it is difficult to satisfy the IC.

The \( SEs \) are very similar to the core model. As in the core model, there is a \( SE \) where \( H \) sells non-core assets and \( L \) issues equity. There is also a \( SE \) where \( H \) sells non-core assets and \( L \) sells core assets. The conditions for this equilibrium to hold are exactly the same as in the core model. In both equilibria, by deviating, \( L \)’s stock price increases but his fundamental value falls by \( \frac{E(A_L - A_H)}{A_H} \). Regardless of whether \( L \) sells equity or core assets in the \( SE \), deviation involves him selling his highly-valued non-core assets and thus suffering a loss. There is no \( SE \) where \( H \) sells core assets and \( L \) sells equity, or when \( H \) sells equity or \( L \) sells the core asset, since \( L \) will mimic \( H \) in
both cases. The only possible SE is where \( H \) sells non-core assets, as \( L \) will not wish to mimic him as this will involve selling assets at a fundamental loss.

**B.3 A Three-Asset Model**

The previous sub-section showed that, in the case of negative correlation, it is easier to sustain an equilibrium in which all firms sell the non-core asset than one in which all firms sell the core asset. While this result is suggestive of the correlation effect, it may also arise from the fact that the non-core asset exhibits less information asymmetry, because \( A_L - A_H < C_H - C_L \). If we reversed this assumption, then firm value would be higher for \( L \) than \( H \), and so we would have the same model but with reversed notation. Since firm value is higher for \( L \), then \( L \) is effectively \( H \). Since \( A \) is positively correlated with firm value, \( A \) is effectively \( C \) and \( C \) is effectively \( A \). We will obtain the result that it is easier to sustain a CPE than an APE, but this would be because \( C \) exhibits less information asymmetry rather than \( C \) being negatively correlated.

Thus, to allow for both positively and negatively correlated assets, and also for either asset to exhibit higher information asymmetry, we need to move to a 3-asset model. Let the three assets be \( C \), \( P \), and \( N \). Asset \( C \) cannot be sold as it is the core asset, but assets \( P \) and \( N \) can be. Asset \( P \) is the positively correlated asset (\( P_H \geq P_L \)) and asset \( N \) is the negatively correlated asset (\( N_H \leq N_L \)). We allow for both \( P_H - P_L > N_L - N_H \) and \( P_H - P_L < N_L - N_H \): either asset may exhibit more information asymmetry. We only assume \( C_H + P_H + N_H > C_L + P_L + N_L \): the existence of the third asset \( C \) means that \( H \) has a higher firm value than \( L \), even if \( N \) exhibits more information asymmetry than \( P \). Let \( A = P + N \) be the total value of the two non-core assets.

Proposition 6 characterizes the equilibria.

**Proposition 6.** (Three-asset model.) Consider a pooling equilibrium where all firms sell the negatively-correlated asset \( (X = N) \) and a firm that sells equity or the positively-correlated asset is inferred as \( L \). The prices of the positively-correlated asset, negatively-correlated asset, and equity are \( P_L \), \( \pi N_H + (1-\pi)N_L \), and \( C_L + P_L + N_L + F \), respectively. This equilibrium is sustainable if the following conditions hold:

\[
\omega \geq \frac{F \left( \frac{N_L}{E[N]} - 1 \right)}{\pi (E_H - E_L) + F \left( \frac{N_L}{E[N]} - 1 \right)}.
\]

Consider a pooling equilibrium where all firms sell the positively-correlated asset \( (X = \)
and a firm that sells equity or the negatively-correlated asset is inferred as \(L\). The prices of the positively-correlated asset, negatively-correlated asset, and equity are \(\pi P_H + (1-\pi)P_L, N_L,\) and \(C_L + P_L + N_L + F\), respectively. This equilibrium is sustainable if the following conditions hold:

\[
\omega \geq \frac{F \left( \frac{P_H}{E[H]} - \frac{N_H}{N_L} \right)}{\pi (E_H - E_L) + F \left( \frac{P_H}{E[H]} - \frac{N_H}{N_L} \right)}
\tag{47}
\]

\[
\omega \geq \frac{F \left( \frac{E_L}{E[H]} - \frac{P_L}{E[P]} \right)}{(1-\pi)(C_H - C_L + A_H - A_L) + F \left( \frac{E_L}{E[H]} - \frac{P_L}{E[P]} \right)}
\tag{48}
\]

Consider a pooling equilibrium where all firms sell equity \((X = E)\) and a firm that sells either asset is inferred as \(L\). The prices of the positively-correlated asset, negatively-correlated asset, and equity are \(P_L, N_L,\) and \(\pi (C_H + P_H + N_H) + (1-\pi)(C_L + P_L + N_L) + F\), respectively. This equilibrium is sustainable if the following conditions hold:

\[
\omega \geq \frac{F \left( \frac{N_L}{N_H} - \frac{E_L}{E[H]} \right)}{(1-\pi)(C_H - C_L + A_H - A_L) + F \left( \frac{N_L}{N_H} - \frac{E_L}{E[H]} \right)}
\tag{49}
\]

\[
\omega \geq \frac{F \left( \frac{E_H}{E[H]} - \frac{N_H}{N_L} \right)}{\pi (C_H - C_L + A_H - A_L) + F \left( \frac{E_H}{E[H]} - \frac{N_H}{N_L} \right)}
\tag{50}
\]

Starting with the \(N\)-pooling equilibrium, \(H\) will not deviate; equation (46) gives the condition for \(L\) not to deviate to either equity or \(P\). The IC conditions that \(L\) will deviate to \(P\) or equity if it were revealed good are trivially satisfied. \(L\) would make a capital gain on selling low-quality \(P\) or low-quality equity, compared to its capital loss on selling high-quality \(N\), and enjoy a higher stock price.

Moving to the \(P\)-pooling equilibrium, \(L\) will not deviate. \(H\) will always deviate to sell \(N\) rather than equity, and equation (47) is the ND condition for him not to do so. Equation (48) is the IC condition for \(L\) to be willing to deviate to equity if he were revealed good, which is stronger than the IC condition for deviation to \(N\).

The IC condition for the \(P\)-pooling equilibrium (equation (48)) is stronger than the ND condition for the \(N\)-pooling equilibrium (46) if and only if

\[
\pi \left( \frac{N_L}{N_H} - \frac{P_L}{E[P]} \right) > (1-\pi) \left( \frac{N_L}{E[N]} - 1 \right).
\]
This always holds, since \( \pi > 1 - \pi \), and \( \frac{N_L}{N_H} > \frac{N_L}{E[N]} \), and \( \frac{P_L}{E[P]} < 1 \). Thus, it is easier to sustain an equilibrium in which all firms sell negatively-correlated assets than one in which all firms sell positively-correlated assets, due to the correlation effect.

Finally, for the EPE, \( L \) will not deviate. There are two IC conditions, one to ensure deviation to \( P \) and one to \( N \), but the latter condition is stronger and is the first of the two conditions listed. There are similarly two ND conditions for \( H \), one to prevent deviation to \( P \) and the other to \( N \), but again the latter is stronger and is the second of the two conditions listed. The ND condition for \( L \) is trivial. Since every type has the potential to deviate to an asset that will be valued highly, we require high stock price concerns to deter such a deviation.

### C Financing from Multiple Sources

The core model assumes that firms can only raise financing from a single source. One potential justification is that the transactions costs from using multiple sources of financing are prohibitive. Alternatively, the assumption can be endogenized with the OEPB that any firm that issues multiple financing sources is of quality \( L \). This section studies conditions under which this belief satisfies the IC.

For three of the four pooling equilibria studied in the paper, our existing IC condition (that \( L \) would deviate to a claim consisting entirely of the off-equilibrium security choice, if valued as \( H \) by doing so) is already sufficient to achieve the new desired result (that \( L \) would deviate to any mixture of asset sales and equity issuance, if valued as \( H \) by doing so). For the equilibria in the positive correlation model, this happens because \( L \) receives a high price for both components of the mixture, instead of receiving a pooled price from cooperating with the pooling equilibrium. For the EPE in the negative correlation model, \( L \) receives a low price if he deviates to sell assets and is inferred as \( H \), but this loss will be lower if he mixes the asset sale with an equity issue, since the latter will fetch a high price. Thus, his fundamental value is higher when selling the mixture than when selling assets only. As a result, the existing IC condition, which guarantees that he will deviate to assets if inferred as \( H \), ensures that he will deviate to any mixture if inferred as \( H \).

For the APE in the negative correlation model, we require an additional condition (a lower bound on \( \omega \), given in (51) below) to ensure that the IC condition is satisfied. This additional condition is only necessary because, unlike the other three pooling equilibria, this pooling equilibrium required no IC condition in the core model. It was automatic that \( L \) would deviate to pure equity if he was inferred as \( H \), since he avoids
the capital loss from selling high-quality assets and also enjoys a high stock price. When financing mixtures are possible, there may be mixtures that comprise such a high proportion of asset sales that $L$ would suffer a large fundamental loss by selling this mixture and so will not deviate despite enjoying a high stock price. This is the case we rule out with (51) below; the results and economic intuition of the model do not change in response to this new condition.

We now proceed with formal proofs of the above statements. Consider a deviation by $L$ to raise $\alpha F$ from asset sales and $(1 - \alpha) F$ from equity issuance. We wish to study whether, if he is inferred as $H$ from such a deviation, his payoff is higher than in the pooling equilibrium, for any $\alpha$. We consider the four pooling equilibria in turn.

**Positive correlation, APE.** The existing IC condition (9) implies that the capital gain to $(L, \bar{k})$ from selling equity at a high price (if he deviates and is inferred as $H$) exceeds his gain from selling assets at a pooled price in the pooling equilibrium:

$$\frac{E_L}{E_H} \leq \frac{A_L(1 + \bar{k})}{\pi A_H + (1 - \pi)A_L}$$

We wish to show that his gain from selling any mix of assets and equity at a high price exceeds his gain from selling assets at a pooled price:

$$\alpha \frac{A_L(1 + \bar{k})}{A_H} + (1 - \alpha) \frac{E_L}{E_H} \leq \frac{A_L(1 + \bar{k})}{\pi A_H + (1 - \pi)A_L}$$

The existing IC condition (9) establishes the inequality for $\alpha = 0$. The LHS is linear in $\alpha$, and for $\alpha = 1$ it simplifies to $\frac{A_L(1 + \bar{k})}{A_H} \leq \frac{A_L(1 + \bar{k})}{E[A] | H}$, which holds because positive correlation implies $A_H > E[A]$. Thus, the inequality is satisfied for all $\alpha \in [0, 1]$.

**Positive correlation, EPE.** We wish to show that, for all $\alpha$:

$$\alpha \frac{A_L(1 + \bar{k})}{A_H} + (1 - \alpha) \frac{E_L}{E_H} \leq \frac{E_L}{E[E]}$$

The LHS is again linear in $\alpha$. The IC condition (12) in the core model establishes the inequality for $\alpha = 1$, and for $\alpha = 0$ the LHS simplifies to

$$\frac{E_L}{E_H} \leq \frac{E_L}{E[E]}$$

which always holds. Thus, the inequality is satisfied for all $\alpha \in [0, 1]$. 

58
Negative correlation, APE. We wish to show that, for all $\alpha$:

\[
\omega \mathbb{E} [C + A] + (1 - \omega) \left( C_L + A_L + F - F \left( \frac{A_L(1 + \bar{k})}{\mathbb{E}[A]} \right) \right) \\
\leq \omega (C_H + A_H) + (1 - \omega) \left( C_L + A_L + F - F \left( \alpha \frac{A_L(1 + \bar{k})}{A_H} + (1 - \alpha) \frac{E_L}{E_H} \right) \right)
\]

We can rearrange this condition to $\omega \geq \frac{MF}{\kappa + MF}$, where $\kappa \equiv (C_H - C_L) - (A_L - A_H)$ and $M$ is defined appropriately. For $\alpha = 0$, $M < 0$ and the lower bound on $\omega$ is negative, which is why the IC condition was trivial in the core model. However, the derivative of the bound with respect to $\alpha$ is positive: it equals the sign of $\frac{\partial M}{\partial \alpha}$, which is positive since $A_L > A_H$ under negative correlation. Thus, as $\alpha$ increases, the lower bound on $\omega$ rises and can eventually become positive and constitute a nontrivial IC condition.

Intuitively, if $L$ is inferred as $H$, his capital loss to selling assets is higher than under the pooling equilibrium, since he receives a low price ($A_H$) rather than a pooled price. Thus, financing mixes that are primarily comprised of assets (high $\alpha$) are particularly costly to him and he may be unwilling to deviate even if he is inferred as $H$. To ensure that $L$ is willing to deviate for all $\alpha$, we require the condition to be satisfied for $\alpha = 1$. This in turn requires:

\[
\omega \geq \frac{F \left( \frac{A_L(1 + \bar{k})}{A_H} - \frac{A_L(1 + \bar{k})}{\mathbb{E}[A]} \right)}{(C_H - C_L) - (A_L - A_H) + F \left( \frac{A_L(1 + \bar{k})}{A_H} - \frac{A_L(1 + \bar{k})}{\mathbb{E}[A]} \right)}. \tag{51}
\]

To encourage deviation, $L$ must have a high weight on the stock price to offset his capital loss from asset sales. Condition (51) is stronger than (19) (the ND condition for APE) if and only if $(1 - \pi)A_L > A_H$, which is not imposed or ruled out by any of our assumptions thus far. Thus, if $(1 - \pi)A_L > A_H$, the additional condition (51) is needed for the OEPB, that an issuer of multiple financing sources is of quality $L$, to satisfy the IC.

Negative correlation, EPE. We wish to show that, for all $\alpha$:

\[
\omega \mathbb{E} [C + A] + (1 - \omega) \left( C_L + A_L + F - F \left( \frac{E_L}{\mathbb{E}[E]} \right) \right) \\
\leq \omega (C_H + A_H) + (1 - \omega) \left( C_L + A_L + F - F \left( \alpha \frac{A_L(1 + \bar{k})}{A_H} + (1 - \alpha) \frac{E_L}{E_H} \right) \right).
\]

The existing IC condition establishes the inequality for $\alpha = 1$. If $\alpha = 0$, the inequality also holds since $E_H > E_L$. Since the RHS is linear in $\alpha$, the inequality also holds at
any value of $\alpha$ between these extremes.

D Voluntary Capital Raising: Additional Material

D.1 Positive Correlation, Positive-NPV Investment, Relaxing Assumptions (15) and (16)

Assumption (15) was sufficient for the RHS of the conditions in parts (iii) and (iv) of Lemma 4 to be positive. If (15) does not hold, i.e., assets are so volatile that $APE$ is never sustainable in the core model, then the RHS of one of these conditions is negative. For example, if $C_H + A_H > C_L + A_L$ (the information asymmetry of investment is lower than that of assets), the LHS is positive and so $APE$ is never sustainable for any $F$, just as in the core model. Intuitively, if assets exhibit higher information asymmetry than both equity and the new investment, then no portfolio of equity and the new investment will have greater information asymmetry than assets, and so $APE$ cannot be sustained. If, on the other hand, the extreme case in which (15) is violated is combined with $A_H(1 + \kappa) \leq \frac{1 + \mu}{1 + r_L}$, the LHS is also negative and so we now have a lower bound: the ND condition becomes $F > E[A(C_H + A_H) - A_H(C_L + A_L)(1 + \kappa)]$. Intuitively, if the new investment has high information asymmetry, then the portfolio of equity plus the new investment will also have high information asymmetry (allowing the $APE$ to hold) if the weight placed on the new investment is sufficiently high, reversing the usual upper bound for $APE$. Similar intuition applies to the IC condition.

Assumption (16) was sufficient for the RHS of the conditions in parts (iii) and (iv) of Lemma 5 to be positive. If (16) does not hold, i.e., assets are so volatile that $EPE$ is always sustainable in the core model, then the RHS of one of these conditions is negative. For example, if $C_H + A_H > A_H(1 + \kappa) E[A](1 + \kappa)$, the RHS of the ND condition is negative. Then, if $\frac{A_H(1 + \kappa)}{E[A]} > \frac{1 + \mu}{1 + r_L}$, the LHS is positive and so $EPE$ is sustainable for all $F$, just as in the core model. Intuitively, if assets exhibit higher information asymmetry than both equity and the new investment, then no portfolio of equity and the new investment will have greater information asymmetry than assets, and so $EPE$ can always be sustained. If, on the other hand, the extreme case in which (16) is violated is combined with $\frac{A_H(1 + \kappa)}{A_L} \leq \frac{1 + \mu}{E[1 + \kappa]}$, the LHS is also negative and so we now have an upper bound: the ND condition becomes $F < \frac{A_L(C_H + A_H) - A_H E[C + A](1 + \kappa)}{A_H(1 + \kappa) E[1 + \kappa] - A_L(1 + r_L)}$. Intuitively, if the new investment has high information asymmetry, the portfolio of equity plus the
new investment will also have high information asymmetry (allowing the EPE to hold) unless the weight on the new investment is sufficiently low. Similar intuition applies to the IC condition.

### E Negative Correlation With Synergies

This section repeats the analysis of Section 3 allowing for synergies. The primary difference is that we can now have semi-separating equilibria in which $H$ and $L$ separate between equity issuance and asset sales based on their level of synergies. The conditions for the other equilibria are more complex but intuitively analogous to those in the main text of the paper. Proofs are presented in Section E.3.

#### E.1 Separating and Semi-Separating Equilibria

As in Section 2.1.3, we have a SSE characterized by a cutoff $k_q^*$. The prices paid for assets and equity are again given by (13) and (14). Since the manager now places weight on the firm’s stock price, we need to calculate the stock prices of asset sellers and equity issuers. These are, respectively:

\[
\mathbb{E}[V|X=A] = \pi \left( \frac{k_H - k}{\mathbb{E}[k_q^*] - k} \right) (C_H + A_H) + (1 - \pi) \left( \frac{k_L^* - k}{\mathbb{E}[k_q^*] - k} \right) (C_L + A_L) \\
- \frac{1}{2} F \left( \frac{\mathbb{E}[(k_q^*)^2] - k^2}{\mathbb{E}[k_q^*] - k} \right),
\]

(52)

\[
\mathbb{E}[V|X=E] = \pi \left( \frac{k - k_H^*}{k - \mathbb{E}[k_q^*]} \right) (C_H + A_H) + (1 - \pi) \left( \frac{k - k_L^*}{k - \mathbb{E}[k_q^*]} \right) (C_L + A_L).
\]

(53)

The stock price of an asset seller includes an additional term, $-F \mathbb{E}[k|X=A] = -\frac{1}{2} F \left( \frac{\mathbb{E}[(k_q^*)^2] - k^2}{\mathbb{E}[k_q^*] - k} \right)$, which reflects the expected synergy loss (which may be negative). Note that $\mathbb{E}[k|X=A] < \mathbb{E}[k]$, since the decision to sell assets suggests that synergies are low. The stock price is higher for an asset seller than an equity issuer ($\mathbb{E}[V|X=A] > \mathbb{E}[V|X=E]$) if and only if

\[
[\Pr (q = H|X = A) - \Pr (q = H|X = E)] \times [(C_H - C_L) - (A_L - A_H)] > F \mathbb{E}[k|X = A].
\]

(54)
The cutoff $k_q^*$ for a particular quality $q$ is defined by:

$$
\omega \left( \mathbb{E}[V|X = A] - \mathbb{E}[V|X = E] \right) = (1 - \omega)F \left( \frac{A_q(1 + k_q^*)}{\mathbb{E}[A|X = A]} - \frac{C_q + A_q + F}{\mathbb{E}[E|X = E]} \right) .
$$

(55)

Only the parenthetical term on the RHS differs by quality $q$. Ignoring $k$, this term will be higher for $L$, and so $k_H^* > k_L^*$. This is intuitive: since $H$ has low-quality assets but high-quality equity, he is more willing to sell assets. Under positive correlation, $k_H^* > k_L^*$ only if assets exhibit less (certainty effect-adjusted) information asymmetry than equity, as then the capital loss from asset sales is lower. With negative correlation, the capital loss from asset sales is always lower since it is negative (i.e., a capital gain), and so we always have $k_H^* > k_L^*$. From (52) and (53), $k_H^* > k_L^*$ implies that asset (equity) sales lead to a positive (negative) inference about firm quality, i.e. $\Pr(q = H|X = A) > \Pr(q = H|X = E)$, and so the LHS of (54) positive. Thus, in the absence of the additional term $F \mathbb{E}[k|X = A]$ on the RHS, (54) will hold: the stock price is higher for an asset seller, since $H$ is more likely to sell assets than $L$. However, if synergies become extremely strong so that $F \mathbb{E}[k|X = A]$ is very large, this could theoretically lead to a violation of (54): an asset seller is expected to lose very large synergies, swamping the positive quality inference. Since this paper considers the trade-off between information asymmetry and synergies, to ensure that synergies are not so strong that they dominate the trade-off, we assume that (54) holds. One sufficient condition for this is symmetric synergies ($\bar{k} = \bar{k}$). (In this case, we have $\mathbb{E}[k] = 0$ and so $\mathbb{E}[k|X = A] < \mathbb{E}[k] = 0$; thus, the RHS of (54) is negative and (54) holds.) In turn, (54) implies that the LHS of (55) is positive. Setting $q = H$ on the RHS yields $k_H^* > 0$ for the equality to hold. Intuitively, $H$ will sell assets even if they are moderately synergistic, as he benefits from both the capital gain and the higher stock price.

The amount of financing $F$ has three effects on the cutoffs in (55). To illustrate, consider $L$’s decision. First, an increase in $F$ augments the certainty effect and makes equity less attractive, because $L$ enjoys a smaller capital gain. This tends to increase $k_L^*$. Second, an increase in $F$ augments fundamental value considerations due to the base effect, which tends to decrease $k_L^*$. Third, $F$ multiplies the expected synergy loss of an asset seller. If the expected synergy loss $\mathbb{E}[k|X = A]$ is negative (on average, sold assets are disynergistic), a higher $F$ magnifies this expected gain, increasing the stock price reaction to selling assets and raising $k_L^*$.

It is possible to have a separating equilibrium by quality only (a “SEq”), i.e. where all high (low)-quality firms sell assets (equity) as in Lemma 6. This equilibrium corre-
sponds to $k_H^* = \bar{k}$ and $k_L^* = \bar{k}$. The required conditions are as follows:

$$\omega \geq \omega^{SE^{q},H} = \frac{F \left( (1 + \bar{k}) - \frac{E_H}{E_L} \right)}{\left( (C_H - C_L) - (A_L - A_H) \right) - \frac{1}{2} F (\bar{k} + k) + F \left( (1 + \bar{k}) - \frac{E_H}{E_L} \right)} \quad (56)$$

$$\omega \leq \omega^{SE^{q},L} = \frac{F \left( \frac{A_L(1 + k)}{A_H} - 1 \right)}{\left( (C_H - C_L) - (A_L - A_H) \right) - \frac{1}{2} F (\bar{k} + k) + F \left( \frac{A_L(1 + k)}{A_H} - 1 \right)} \quad (57)$$

The lower bound on $\omega$ ensures that $H$ will not deviate. Type $(H, \bar{k})$ may wish to deviate to retain its synergistic assets; a high $\omega$ is needed for stock price concerns to deter deviation. There are three effects of changing $F$ on the lower bound, analogous to the three effects on the cutoffs in (55). First, a rise in $F$ increases $\omega^{SE^{q},H}$ due to the certainty effect (reducing $\frac{E_H}{E_L}$). Second, it reduces it due to the base effect. Third, $F$ multiplies the expected synergy loss of an asset seller. If the expected synergy loss $\frac{F (1 + k)}{2}$ is negative, a higher $F$ magnifies this expected gain, reducing $\omega^{SE^{q},H}$. The upper bound ensures that $L$ will not deviate. If $1 + k \leq \frac{A_H}{A_L}$, i.e. the benefits of getting rid of a disynergistic asset exceed the capital loss from selling high-quality assets, deviation to asset sales yields $(L, \bar{k})$ a fundamental gain and so $SE^{q}$ can never hold. However, if $1 + k > \frac{A_H}{A_L}$, deviation yields $(L, \bar{k})$ a fundamental loss. Since it also leads to a stock price increase, $\omega$ must be low to deter deviation. Unlike the lower bound, there are only two effects of changing $F$ on the upper bound as there is no certainty effect. The range of $\omega$’s that satisfy (56) and (57) is increasing in $k$ and decreasing in $\bar{k}$: the weaker the synergy motive, the easier it is to sustain $SE^{q}$.

Finally, we may have partial SSEs where one quality pools and the other separates. As in the positive correlation case, if $\bar{k}$ is sufficiently low, we have a partial SSE where all $H$-firms sell assets and $L$-firms strictly separate. Unlike the positive correlation case, we cannot have a partial SSE where $H$-firms issue equity and $L$-firms strictly separate. Such an equilibrium would require some $L$-firms to be willing to sell assets but all $H$-firms not to be. However, since $H$’s assets are lower-quality under negative correlation, $H$ is more willing to sell assets than $L$. Similarly, if $\bar{k}$ and $\bar{k}$ are high and $\omega$ is low, we have a partial SSE where all $L$-firms issue equity and $H$-firms strictly separate. We cannot have a partial SSE where all $L$-firms sell assets and $H$-firms strictly separate: since some $H$-firms are issuing equity, $L$-firms will enjoy both a capital gain and a stock

\[\text{19If } 1 + \bar{k} < \frac{E_H}{E_L}, \text{ then the loss of synergies is less than the capital loss that } (H, \bar{k}) \text{ would suffer by issuing equity. Thus, } H\text{'s fundamental value and stock price are both higher under asset sales, and the lower bound on } \omega \text{ is trivially satisfied.}\]
price increase by deviating to equity. Thus, the only feasible partial SSEs involve all \(H\)-firms selling assets, or all \(L\)-firms issuing equity, which is intuitive since \(H\)'s assets and \(L\)'s equity are both low-quality.

The results of this section are summarized in Lemma 10 below.

Lemma 10. (Negative correlation, semi-separating equilibrium.) Assume that (54) holds.

(i) A full semi-separating equilibrium is sustainable where quality \(q\) sells assets if \(k \leq k_q^*\) and equity if \(k > k_q^*\), where \(k_q^*\) is defined by (55), if \(k\) is sufficiently low and \(\bar{k}\) is sufficiently high. We have \(k_H^* > k_L^* > 0\); the sign of \(k_L^*\) depends on parameter values. The stock prices of asset sellers and equity issuers are given by (52) and (53) respectively.

(ii) A partial semi-separating equilibrium in which all firms of quality \(H\) (\(L\)) sell assets (equity) is sustainable if the following two conditions hold:

\[
\omega \geq \omega_{SEq_H} = \frac{F \left( (1 + \bar{k}) - \frac{E_H}{E_L} \right)}{((C_H - C_L) - (A_L - A_H)) - \frac{1}{2} F(\bar{k} + k) + F \left( (1 + \bar{k}) - \frac{E_H}{E_L} \right)} - \frac{1}{2} F(\bar{k} + k) + F \left( \frac{A_L (1 + k)}{A_H} - 1 \right)
\]

\[
\omega \leq \omega_{SEq_L} = \frac{F \left( (1 + \bar{k}) - \frac{E_H}{E_L} \right)}{((C_H - C_L) - (A_L - A_H)) - \frac{1}{2} F(\bar{k} + k) + F \left( \frac{A_L (1 + k)}{A_H} - 1 \right)} - \frac{1}{2} F(\bar{k} + k) + F \left( \frac{A_L (1 + k)}{A_H} - 1 \right)
\]

(iii) A partial semi-separating equilibrium where all \(H\)-firms sell assets (\(k_H^* = \bar{k}\)) and \(L\)-firms strictly separate (\(k < k_L^* < \bar{k}\)) is sustainable if \(k\) is sufficiently low, \(\bar{k}\) is sufficiently high, and \(\omega > \omega_{SEq_H}^\prime\).

(iv) A partial semi-separating equilibrium where all \(L\)-firms issue equity (\(k_L^* = k\)) and \(H\)-firms strictly separate (\(k < k_H^* < \bar{k}\)) is sustainable if \(k\) is sufficiently high, \(\bar{k}\) is sufficiently high, and \(\omega < \omega_{SEq_L}^\prime\).

E.2 Pooling Equilibria

The \(APE\) and \(EPE\) are analogous to those in the main text and are summarized by the following two Lemmas:

Lemma 11. (Negative correlation, pooling equilibrium, all firms sell assets.) Consider a pooling equilibrium where all firms sell assets (\(X_H = X_L = A\)) and a firm that sells equity is inferred as type \((L, \bar{k})\). The prices of assets and equity are \(\pi A_H + (1 - \pi) A_L\) and \(E_L\) respectively. The stock prices of asset sellers and equity issuers are \(E[C + A] -
\( F \) \( E[k] \) and \( C_L + A_L \), respectively. This equilibrium is sustainable if

\[
\omega \geq \omega^{APE,ND,L} = \frac{F \left( \frac{A_L}{E[A]} (1 + k) - 1 \right)}{\pi((C_H - C_L) - (A_L - A_H)) - \frac{1}{2} F(k + k) + F \left( \frac{A_L}{E[A]} (1 + k) - 1 \right)}.
\]

For the EPE, the IC condition is stronger than the ND condition if and only if:

\[
F(-k)(N_1 + N_2) < [(C_H - C_L) - (A_L - A_H)]\pi N_2 - (1 - \pi)N_1, \quad (58)
\]

where \( N_1 \) and \( N_2 \) are the parenthetical terms in the numerators of the IC and ND bounds, respectively (these bounds are presented in the Lemma below):

\[
N_1 \equiv \frac{A_L (1 + k)}{A_H} - \frac{E_L}{E[E]} > 0
\]

\[
N_2 \equiv \frac{E_H}{E[E]} - \frac{A_H (1 + k)}{A_L} > 0.
\]

The EPE is summarized in the following Lemma:

**Lemma 12.** (Negative correlation, pooling equilibrium, all firms sell equity.) Consider a pooling equilibrium where all firms sell assets \( (X_H = X_L = A) \) and a firm that sells assets is inferred as type \( (L,k) \). The prices of assets and equity are given by \( A_L \) and \( \pi(C_H + A_H) + (1 - \pi)(C_L + A_L) + F \) respectively. The stock prices of asset sellers and equity issuers are \( C_L + A_L - Fk \) and \( E[E + A] \), respectively. This equilibrium is sustainable if \( \omega \geq \omega^{EPE} \), where

\[
\omega^{EPE} = \begin{cases} 
\omega^{EPE,IC} = \frac{F \left( \frac{A_L (1 + k)}{A_H} - \frac{E_L}{E[E]} \right)}{(1 - \pi)(C_H - C_L - (A_L - A_H)) + F \left( k + \frac{A_L (1 + k)}{A_H} - \frac{E_L}{E[E]} \right)} & \text{if (58) holds;} \\
\omega^{EPE,ND,H} = \frac{F \left( \frac{E_H}{E[E]} - \frac{A_H (1 + k)}{A_L} \right)}{\pi(C_H - C_L - (A_L - A_H)) + F \left( k + \frac{E_H}{E[E]} - \frac{A_H (1 + k)}{A_L} \right)} & \text{if (58) does not hold.}
\end{cases}
\]

\[(59)\]

**E.3 Proofs for Appendix E**

**Proof of Lemma 10**

For part (i), the logic is as follows. We seek a pair of cutoffs \( (k_H^*, k_L^*) \) for which both types \( (q, k_q^*) \) are indifferent between the two financing sources. As before, we use \( k_q^* \) to denote candidate cutoffs that may not be equilibria, in response to which we will derive the optimal action of the types.
First we show (under certain assumptions) that, given any candidate cutoff \( k_H' \), there will be a \( k_L' \) at which type \( (L, k_L') \) is indifferent, with this value of \( k_L' \) implicitly determined as a continuous function of \( k_H' \). Then we consider candidate equilibria such that \( k_L' \) is chosen conditional on \( k_H' \) in this manner, and we show that there exists a \( k_H' \) where \( (H, k_H') \) is indifferent as well. This method will show that an equilibrium exists.

To prove the first statement, we take as given a cutoff \( k_H' > 0 \), and we employ the IVT as before, showing that for a sufficiently low (high) \( k_L' \), type \( (L, k_L') \) will deviate to assets (equity). Quality \( L \) deviates to asset sales if the price difference between an asset seller and an equity issuer exceeds:

\[
(1 - \omega) F \left( \frac{A_L(1 + k_L')}{E[A|X = A]} - \frac{E_L}{E[E|X = E]} \right). \tag{60}
\]

Recall that the difference in stock price is positive by assumption (54). If \( 1 + k_L' < \frac{E_L}{A_H - A_L} \), then expression (60) is negative, and \( (L, k_L') \) will then deviate to asset sales. On the other hand, as we increase \( k_L' \), the stock price reaction to an asset seller relative to an equity issuer falls to a negative value (the difference in posterior probabilities \( Pr(q = H|X = A) - Pr(q = H|X = E) \) falls to zero, and the expected synergy loss grows), while expression (60) is positive and increasing.

Thus, there will be values of \( k_L' \) high enough that type \( (L, k_L') \) issues equity rather than sell assets. Note that both of these conclusions hold regardless of the value of \( k_H' \). Thus, applying the IVT, and allowing sufficiently strong dissynergies that \( 1 + k_L' < \frac{E_L}{A_H - A_L} \) is feasible, we conclude that for any candidate value of \( k_H' \), there is a value of \( k_L' \) at which type \( (L, k_L') \) is indifferent between asset sales and equity. Moreover, since there are no discontinuities in the model, the function implicitly determining this value is continuous.

Turning to the second statement, let us consider different candidate values \( k_H' \), and choose \( k_L' \) such that \( (L, k_L') \) is indifferent as described above. Type \( (H, k_H') \) will deviate to asset sales if the (positive) stock price reaction to asset sales relative to equity is greater than

\[
(1 - \omega) F \left( \frac{A_H(1 + k_H')}{{E[H|X = H]} - {E[E|X = E]} \right). \tag{60}
\]

This expression is negative if \( 1 + k_H' < \frac{E_L}{A_H - A_L} \). Since the RHS of this inequality is greater than 1, there will be values \( k_H' > 0 \) such that type \( (H, k_H') \) deviates to asset sales. On the other hand, the above expression grows without bound in \( k_H' \), while the difference in the stock price reactions to asset sales and equity is bounded above by \( (C_H - C_L) - (A_L - A_H) - F \). Thus, after \( \tilde{k} \) crosses some threshold \( \tilde{k}_H \), there
will be values of $k'_H$ high enough that type $(H, k'_H)$ issues equity rather than sell its highly-synergistic assets. (As described above, $k'_L$ adjusts in both cases such that type $(L, k'_L)$ remains indifferent.) We conclude that with synergies strong enough such that $k > \bar{k}_H$ and $1 + k < \frac{E_H}{E_H - A_L}$ are both feasible, then there will be at least one pair of cutoff values $k^*_q$ at which types $(H, k^*_H)$ and $(L, k^*_L)$ are both indifferent between equity and asset sales, giving rise to the existence of a full SSE.

To prove (ii), it suffices to write out the ND conditions for both qualities, solve for $\omega$, and state the bounds in terms of the type with the synergy value that is most likely to issue a different claim.

To prove (iii), first we examine $H$’s ND condition, which is:

$$\omega \left( Pr(q = H | X = A) \left( (C_H - C_L) - (A_L - A_H) \right) - F \times \mathbb{E}[k | k < k^*_q] \right) > (1 - \omega) F \left( \frac{A_H (1 + \bar{k})}{A_L [A | X = A]} - \frac{E_H}{E_L} \right).$$

In general, the condition is that $\omega$ be sufficiently high that even managers with the highest level of synergies cooperate with asset sales. To obtain a condition that is sufficient regardless of the equilibrium value of $k^*_L$, we consider the limiting case $k^*_L \to \bar{k}$ (the strictest possible condition on $\omega$, where all $L$-firms are issuing equity). Then the bound on $\omega$ is

$$\omega \geq \frac{F \left( 1 + \bar{k} - \frac{E_H}{E_L} \right)}{\left( (C_H - C_L) - (A_L - A_H) \right) - \frac{1}{2} F(\bar{k} + k) + F \left( 1 + \bar{k} - \frac{E_H}{E_L} \right)}.$$

Note that this bound is identical to $\omega^{SSEq,H}$. In this limiting case, we require the same behavior of $H$ as in the $SSEq$: all $H$-firms must cooperate with asset sales, which perfectly reveal their quality, while equity would perfectly “reveal” them to be $L$.

Next, we again apply the IVT to prove existence of an equilibrium. We first seek a candidate cutoff value $k'_L$ at which $(L, k'_L)$ will deviate to asset sales, given the price reactions that result from this cutoff. This happens if the (positive) difference in stock price reactions between asset sales and equity is greater than

$$(1 - \omega) F \left( \frac{A_L (1 + k'_L)}{A_L [A | X = A]} - 1 \right).$$

When $1 + k'_L = \frac{A_L}{A_L}$, the above expression is negative. Thus if $1 + k \geq \frac{A_L}{A_L}$, there will be an $L$-firm that deviates to asset sales.
Finally, we must find a candidate cutoff value \( k'_L \) at which \((L, k'_L)\) will deviate to equity. Clearly, \( L \) will do this if \( k'_L \) is sufficiently high, and as we have imposed no upper bound on \( \overline{k} \), we conclude that for sufficiently high \( \overline{k} \) (along with the previously-imposed bounds on \( \omega \) and \( k \)), there will be values of \( k'_L \) such that \( L \) deviates to equity, allowing the equilibrium to exist. (Note that the lower bound on \( \omega \) increases as we raise \( \overline{k} \). This does not invalidate the equilibrium, as that lower bound is still strictly less than 1.)

To prove part (iv), we first examine the ND condition for \( L \):

\[
\omega \left( 1 - Pr(q = H|X = E) \right) \left( (C_H - C_L) - (A_L - A_H) \right) - \frac{1}{2} F(k + k'_H) \leq (1 - \omega) F \left( \frac{A_L(1 + k)}{A_H} - \frac{E_L}{E[X = E]} \right)
\]

To satisfy this, we require \( \omega \) to be sufficiently low. Consider the limiting case \( k'_H \to \overline{k} \). If

\[
\omega \leq \frac{F \left( \frac{A_L(1 + k)}{A_H} - 1 \right)}{((C_H - C_L) - (A_L - A_H)) - \frac{1}{2} F(\overline{k} + k) + F \left( \frac{A_L(1 + k)}{A_H} - 1 \right)}
\]

then all \( L \)-firms will cooperate with equity issuance. The bound on \( \omega \) is identical to \( \omega^{SEq, L} \): In this limiting case, we require the same behavior of \( L \) as in \( SEq \): all \( L \)-firms must cooperate with equity issuance even though it perfectly reveals their quality, while asset sales would perfectly “reveal” them to be \( H \).

Note also that we must also have \( 1 + k > \frac{A_H}{A_L} \) for this to be possible, the reverse of the condition that was imposed in (iii) to ensure that some \( L \)-firms sell assets.

Given these conditions, we proceed as before. We find candidate cutoffs \( k'_H \) at which \((H, k'_H)\) deviates to asset sales and to equity, and then apply the IVT to conclude that an equilibrium cutoff \( k'_H^* \) exists between them. \( H \) will deviate to asset sales if the positive stock price incentive to sell assets is greater than

\[
(1 - \omega) F \left( (1 + k'_H) - \frac{E_H}{E[E|X = E]} \right).
\]

Since \( E_H > E[E|X = E] \), the above expression is negative, and the inequality holds for any \( k'_H \leq 0 \).
Finally, $H$ will deviate to equity if the opposite is true:

$$\omega \left( 1 - Pr(q = H|X = E))((C_H - C_L) - (A_L - A_H)) - \frac{1}{2} F(k + k^*_H) \right)$$

$$\leq (1 - \omega) F \left( (1 + k'_H) - \frac{E_L}{E[E|X = E]} \right)$$

With no upper bound imposed on synergies, we can choose $k$ sufficiently high that there will be values of $k'_H$ satisfying this inequality.

**Proofs of Lemmas 11 and 12**

These Lemmas can be derived analogously to Lemmas 7 and 8 in the main text, simply adding the synergy term $k$ to all asset sales, and expressing the final condition in terms of the type most likely to deviate (either $\kbar$ or $\ktilde$).
Table 1: Key variables and abbreviations in the model.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>Firm type</td>
</tr>
<tr>
<td>$q$</td>
<td>Firm quality</td>
</tr>
<tr>
<td>$k$</td>
<td>Firm synergy</td>
</tr>
<tr>
<td>$F$</td>
<td>Level of financing required</td>
</tr>
<tr>
<td>$C_q$</td>
<td>Value of core asset in firm of quality $q$</td>
</tr>
<tr>
<td>$X$</td>
<td>Claim issued by firm</td>
</tr>
<tr>
<td>$A_q$</td>
<td>Value of non-core asset in firm of quality $q$</td>
</tr>
<tr>
<td>$APE$</td>
<td>Asset-pooling equilibrium</td>
</tr>
<tr>
<td>$EPE$</td>
<td>Equity-pooling equilibrium</td>
</tr>
<tr>
<td>$SE$</td>
<td>Semi-separating equilibrium</td>
</tr>
<tr>
<td>$k^*$</td>
<td>“Threshold” synergy level. Quality $q$ with $k \leq (&gt; ) k^*_q$ sell assets (equity)</td>
</tr>
</tbody>
</table>

**Positive Correlation Model**

| $F^{APE,IC}$ | Upper bound on $F$ in an $APE$ to satisfy the intuitive criterion condition |
| $F^{APE,ND,q}$ | Upper bound on $F$ in an $APE$ to satisfy the no-deviation condition for all firms of quality $q$ |
| $F^{APE}$ | The applicable upper bound on $F$, $\min \{ F^{APE,IC}, F^{APE,ND,H} \}$ |
| $F^{EPE,IC}$ | Lower bound on $F$ in an $EPE$ to satisfy the intuitive criterion condition |
| $F^{EPE,ND,q}$ | Lower bound on $F$ in an $EPE$ to satisfy the no-deviation condition for all firms of quality $q$ |
| $F^{EPE}$ | The applicable lower bound on $F$, $\max \{ F^{EPE,IC}, F^{EPE,ND,H} \}$ |
| $F^{APE,IC,I}$ | Upper bound on $F$ in an $APE$ to satisfy the intuitive criterion condition in investment model. Other variables with superscript $I$ defined analogously |

**Negative Correlation Model**

| $\omega$ | Manager’s weight on the stock price |
| $\omega^{APE,IC}$ | Upper bound on $\omega$ in an $APE$ to satisfy the intuitive criterion condition. Other $\omega$ variables defined analogously |